14 January 2021 12:04

Errors

→ Problem (system, under certain condition)

↓

[equation (& condition)

Nonlinear | differential equation | Integration

↓

We want the solution

↓

→ Behaviour of system

equation

- simpler form
  - ↓ Method
  - ✓ Analytical Method
- complex form (complicated)
  - ↓ Not easier to solve by using analytic method

→ $ax^2+bx+c=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

→ $(x-\alpha)(x-\beta)=0$

$x=\alpha,\ x=\beta$]

Analytical Method $\xrightarrow{\text{Solution}}$ exactly correct

↓

Numerical Method

- Use certain approximation to simply the problem and to get the solution
- Result → approximate result

Accuracy of result is going to depend on the approx.:

error ↓ Result ↑ (accuracy)

- various sources of error
- How to minimise it
- How to calculate the maximum possible error

Accuracy of Numbers

Exact Number :- $2, 4, \frac{7}{2}, 10$ → exact number

approximate number - $\sqrt{2}$, π, $4/3$

$\sqrt{2} = 1.414213 \ldots$

$\pi = 3.141592 \ldots$

$\frac{4}{3} = 1.333333 \ldots$

leads to errord

Significant figures :- digit used to express a nuber are called significant figures.

7835 — 4	3.575 — 4
2345 — 4	0.4423 — 4

0.0026 — 2	4500 — 2	4503 — 4
0.000035 — 2	6320 — 3	

Approximate no. ⟶ Round off the no. upto certain decimal point.

Rule :- to round off no. upto "n" significant no.

(i) Discard all digits to the right of n^{th} digit

(ii) if discarded no. is

(a) less than half a unit in the n^{th} place, leave the n^{th} digit unchanged

(b) greater than half a unit in the n^{th} place, increase the n^{th} digit by unity.

→ (c) exactly half a unit in the n^{th} place, increase the n^{th} digit by unity if it is odd otherwise leave it unchange.

Round off upto 3 significant figures

3 < 5 $\quad$ 7.893 ⟶ 7.89

6 > 5 $\quad$ 12.865 ⟶ 12.8 ⟶ 12.9

$$\begin{bmatrix} (n+1)^{th} \to & < 5 \\ & > 5 \\ & = 5 \end{bmatrix}$$

7 > 5 $\quad$ 3.567 ⟶ 3.56 ⟶ 3.57 (+1)

→ 84767 ⟶ 84700

5 = 5 $\quad$ 5.8254 ⟶ 5.82

5 = 5 $\quad$ 5.8354 ⟶ 5.83 (+1) ⟶ 5.84

Error

Error

1. **Inherent Error** :- the error which is present in the statement of the problem is called inherent error

B. S. Grewal

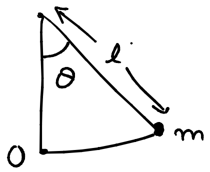
2. **Rounding error** :- rounding off no. leads to error

3. **Truncation error** :- infinite series $\longrightarrow$ finite term

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \underbrace{- - -}_{R_n}$$

R_n lead to error

(1) $$l\frac{d^2\theta}{dt^2} + g\theta = 0$$ θ small, $\sin\theta \simeq \theta$



(2) $$l\frac{d^2\theta}{dt^2} + g\sin\theta = 0$$

(3) $$l\frac{d^2\theta}{dt^2} + m\frac{d\theta}{dt} + g\sin\theta = 0$$

$$l\ddot{\theta} + m\dot{\theta} + g\sin\theta = 0$$

method — Numerical method $\longrightarrow$ approximation $\longrightarrow$ error

$\theta(t=0) = ?? \rightarrow (a)$, $\dot{\theta}(t=0) = ?? \rightarrow (b)$

(1) how you have measured these value.

(2) a & b are not measured; its given to you.

a & b $\rightarrow$ exact no.
$\searrow$ approximate no.

$\left[g - \begin{smallmatrix} 9.8 \\ 10 \end{smallmatrix} \right]$; l, m] measurement $\downarrow$ insturement $\downarrow$ least count $\downarrow$ lead the error

l: scale LC = 0.001 cm $\rightarrow$ 2.131 cm

scale LC = 0.01 cm $\rightarrow$ 2.13 cm, × (2.135 cm)

scale LC = 0.1 cm $\rightarrow$ 2.1 cm ; 2.12 cm ×

(2.1 ± 0.1)

(2.13 ± 0.01)

2 — 2.2

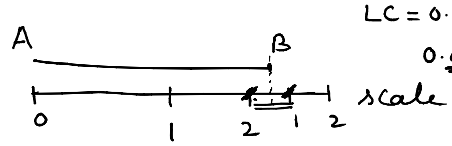
LC = 0.1

1.9 - 2.2

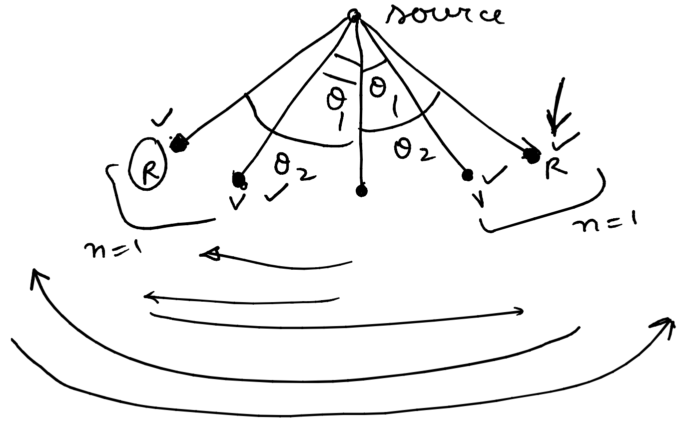
(2.1 ± 0.1)
(2.13 ± 0.01)
(2.131 ± 0.001)

read the error

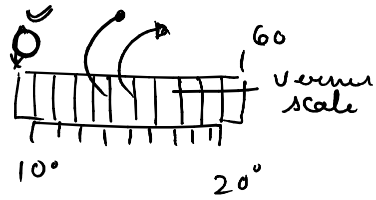
LC = 0.1
0.01



instrument ⟶ least count
↘ Back less error



$(e+d)\sin\theta = n\lambda$



- MS :
VS : 10 × LC

Absolute error :- X — true value
X' — approximate value

$$E_a = |X - X'|$$

0.05

Relative error :- $E_r = \left|\dfrac{X - X'}{X}\right|$

Percentage Error :- $E_p = 100\left|\dfrac{X - X'}{X}\right|$

1% 10%

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Q. Roundoff the number 865250 & 37.46235 to four significant figures and calculate E_a, E_r, E_p.

Solution.

$X = 865250$

$X' = 865200$ (four significant figure)

$E_a = |X - X'| = |865250 - 865200| = 50$

$E_r = \left|\frac{X - X'}{X}\right| = \left|\frac{50}{865250}\right| = 5.77\times10^{-5}$

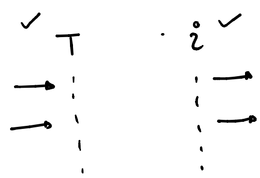
$E_p = E_r \times 100 = 5.77\times10^{-3}$

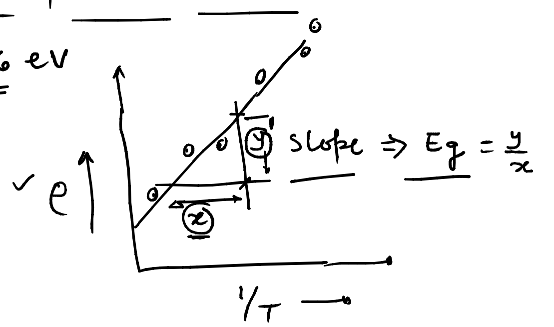
Maximum Possible Error

New material (semiconductor)

E_g ?? ; four probe method

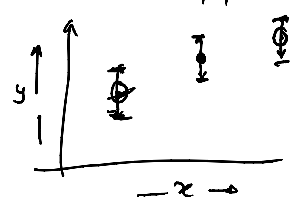
E_g = Band gap = 1.6 eV





Band gap $= (E_g \pm dE_g)$

$= [E_g - dE_g$ to $E_g + dE_g]$



λ ; $d\lambda$

result : wavelength — $(\lambda \pm d\lambda)$ $[\lambda - d\lambda$ — $\lambda + d\lambda]$

$\frac{(e+d)\sin\theta}{n} = \lambda$

$u = u(x_1, x_2, x_3, x_4, \ldots x_n)$ — (1)

x_1 — $x_1 \pm dx_1$

x_2 — $x_2 \pm dx_2$

⋮

$x_n \rightarrow x_n \pm dx_n$

$u \rightarrow u \pm du$

$u + du = u(x_1 + dx_1,\ x_2 + dx_2,\ x_3 + dx_3 \ldots,\ x_n + dx_n)$ — (2)

$$= u(x_1, x_2, x_3, \dots x_n) + \left[\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 + \dots\dots\right]$$

$$+ \frac{1}{2}\left[\underbrace{\frac{\partial^2 u}{\partial x_1^2}(dx_1)^2}_{0} + \underbrace{\frac{\partial^2 u}{\partial x_2^2}(dx_2)^2}_{0} + \dots\right]$$

$$\cancel{u}+du = \cancel{u(x_1, x_2, \dots x_n)} + \left[\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 + \frac{\partial u}{\partial x_3} dx_3 + \dots\right]$$

$$\boxed{du = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 + \frac{\partial u}{\partial x_3} dx_3 + \dots}$$ Total derivative

$\rightarrow (u \pm du) \rightarrow (u - du \leftrightarrow u + du)$

Q. $u = \frac{5xy}{z^2}$; $x = 1, y = 1, z = 2$.

$[dx = 0.1 ; dy = 0.1, dz = 0.1]$ - case ①

$dx = dy = 0.1 ; dz = 0.01$ — case ②

$dx = dy = dz = 0.01$ — case ③

Sol. $u = \frac{5xy}{z^2}$, $x = 1, y = 1, z = 2$; $u = 1.25$; $dx = dy = dz = 0.1$

$\log u = \log 5 + \log x + \log y - 2\log z$ → take log of both sides

$\log u = \log 5 + \log x + \log y + 2\log z$ → Convert all negative term to positive

$\frac{du}{u} = 0 + \frac{dx}{x} + \frac{dy}{y} + \frac{2dz}{z}$ → take derivative

$\frac{du}{u} = \frac{dx}{x} + \frac{dy}{y} + 2\frac{dz}{z}$

$$\boxed{du = u\left\{\frac{dx}{x} + \frac{dy}{y} + 2\frac{dz}{z}\right\}}$$

$du = 1.25\left\{\frac{0.1}{1} + \frac{0.1}{1} + \frac{\cancel{2} \times 0.1}{\cancel{2}}\right\} = 1.25\,(0.3) = 0.375$

case (i) — $(1.25 \pm 0.375) \rightarrow [1.25 - 0.375 \leftrightarrow 1.25 + 0.375]$

Case 2. $dx = dy = 0.1$, $dz = 0.01$

$du = 1.25\left\{\frac{0.1}{1} + \frac{0.1}{1} + \frac{\cancel{2} \times 0.01}{\cancel{2}}\right\} = 1.25\{0.21\} = 0.2625$

Case 3. $dx = dy = dz = 0.01$

$du = 1.25\left\{\frac{0.01}{1} + \frac{0.01}{1} + \frac{\cancel{2}\ 0.01}{\cancel{2}}\right\} = 1.25 \times (0.03)$

$= 0.0375$

→ Result = (1.25 ± 0.0375)

→ Result = (1.25 ± 0.0375)

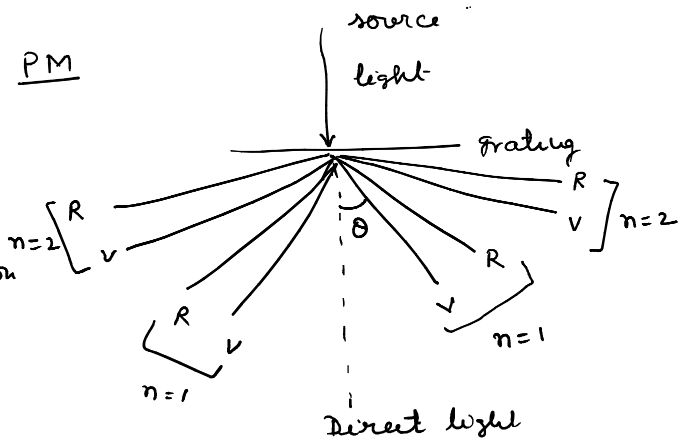
Grating

$$\boxed{(e+d)\sin\theta = n\lambda}$$ PM

$(e+d)$ = grating element

θ = diffraction angle

n = order of diffraction



$n\lambda = (e+d)\sin\theta$

$\lambda = \dfrac{(e+d)\sin\theta}{n}$

$\log\lambda = \log(e+d) + \log\sin\theta - \log n$

$= \log(e+d) + \log\sin\theta + \log n$

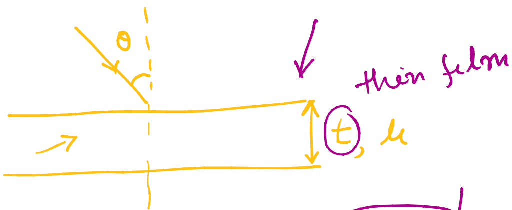
$\dfrac{d\lambda}{\lambda} = 0 + \cot\theta\, d\theta$

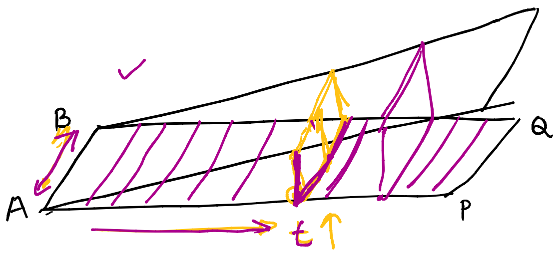
$d\theta = 1'$, $d\theta = 10''$

$$\boxed{d\lambda = \lambda\cot\theta\, d\theta}$$

$d\theta = ??$
least count of spectrometer

1. Circular
2. straight; hyperbolic





Wedge Shaped Air film

"locus of constant thickness"

① "t", "θ"

$2\mu t\cos\theta = n\lambda$; normal incidence ; $\theta = 0$

② ①

$\theta = 0$

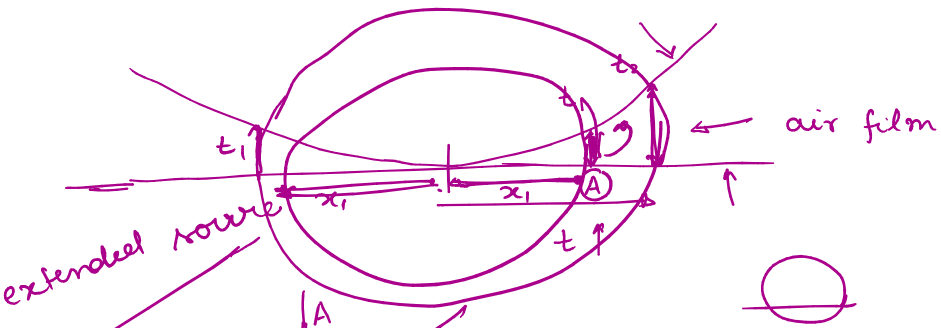
$2\mu t = n\lambda$ → Maxima minima

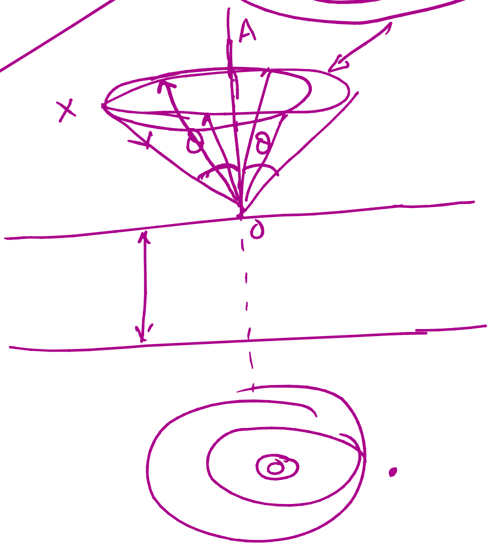
$2\mu t\cos\theta = n\lambda$

all values of "t", $2\mu t$ = λ, 2λ, 3λ, 4λ Maxima

$\theta = 0$

$2\mu t = n\lambda$





$$2\mu t \cos\theta = n\lambda$$

$y = mx + c$

1 circular fringes

2

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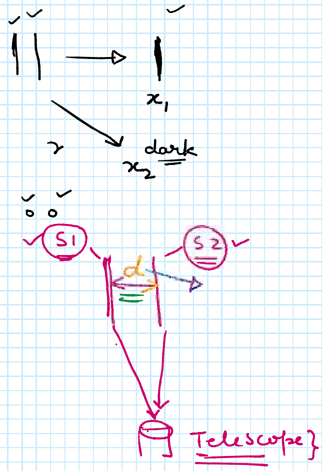
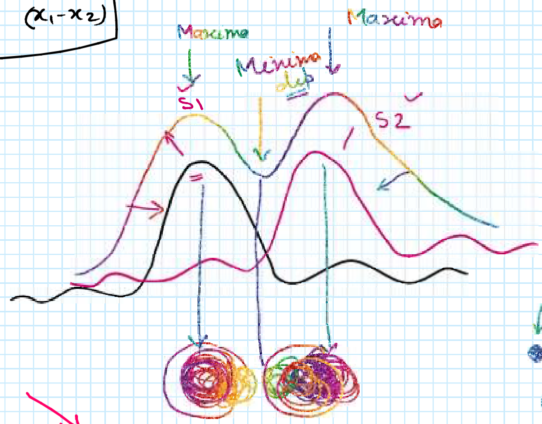
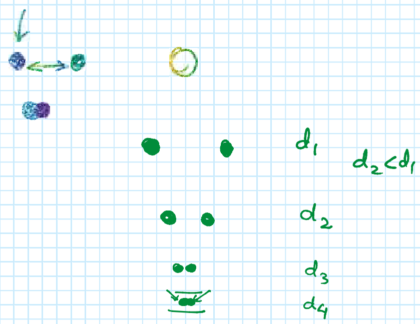
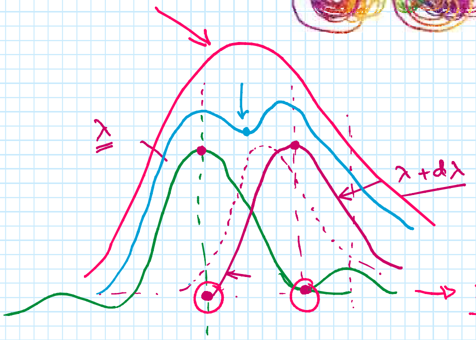
$$P = \boxed{\frac{\lambda}{d\lambda}} \frac{W}{(x)} = K = \frac{K}{(x_1 \sim x_2)}$$

$$\rightarrow \boxed{dP = \frac{2P\,dx}{(x_1 - x_2)}}$$

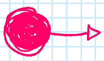








→ Just resolving condition


→ Single λ

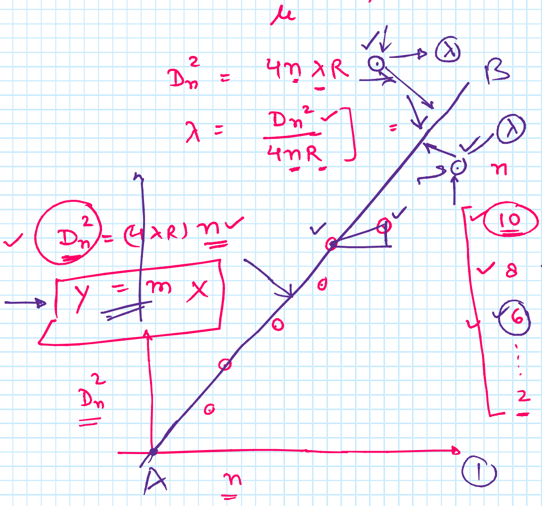
$$D_n^2 = \frac{4n\lambda R}{\mu} \quad ; \quad \mu = 1$$

$$D_n^2 = 4n\lambda R$$

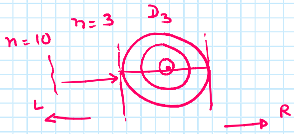
$$\lambda = \frac{D_n^2}{4nR}$$

$$D_n^2 = (4\lambda R)\, n$$

$$\boxed{y = m\,x}$$



$$= \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$



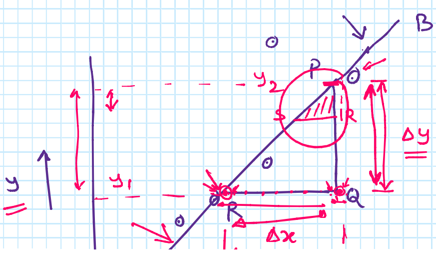
L	R	Dia ($x \sim y$)	λ
x_1	y_1	d_1	λ_1
x_2	y_2	$\frac{y_2 - y_1}{x_2 - x_1}$ d_2	λ_2
x_3	y_3	$\frac{y_3 - y_2}{x_3 - x_2}$ d_3	λ_3
x	y_1		

1

2 slope cal.

1 measurement

2 error due to graphical



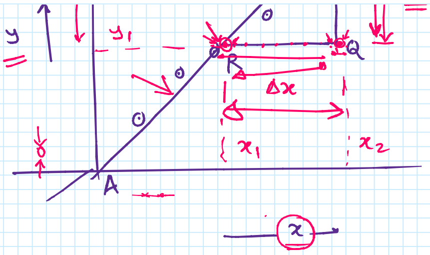
1

$$\boxed{\text{slope} = \frac{PQ}{QR}}$$

$$\boxed{\text{slope} = \frac{PR}{SR}}$$

2

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



② $\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

Big Area

$\left[\begin{array}{l} PQ = 8 \text{ unit} \\ QR = 8 \text{ unit} \end{array}\right] \rightarrow 1 \text{ unit}$

small area

$\left[\begin{array}{l} PR = 3 \text{ unit} \\ QR = 8 \text{ unit} \end{array}\right] \rightarrow 1 \text{ unit}$

$y = m x$

$m = \dfrac{y}{x} \rightarrow \log m = \log y + \log x$

$dx = dy = 1$

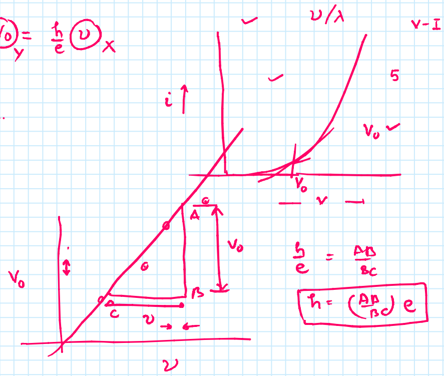
$\dfrac{dm}{m} = \dfrac{dy}{y} + \dfrac{dx}{x}$

$dm = m\left[\dfrac{dy}{y} + \dfrac{dx}{x}\right] = \left[\dfrac{1}{8} + \dfrac{1}{8}\right] \rightarrow \text{Big area}$

$= \dfrac{1}{3} + \dfrac{1}{8}$

$\boxed{h\nu = e V_0}$ $\quad V_0 = \dfrac{h}{e}\nu$

ν	V_0
ν_1	u_1
ν_2	u_2
ν_3	u_3
ν_4	u_4
ν_5	u_5



$\dfrac{h}{e} = \dfrac{AB}{BC}$

$\boxed{h = \left(\dfrac{AB}{BC}\right) e}$

$h = e\dfrac{V_0}{\nu}$

$\log h = \log e + \log V_0 + \log \nu$

$\boxed{dh = h\left[\dfrac{dV_0}{V_0} + \dfrac{d\nu}{\nu}\right]}$