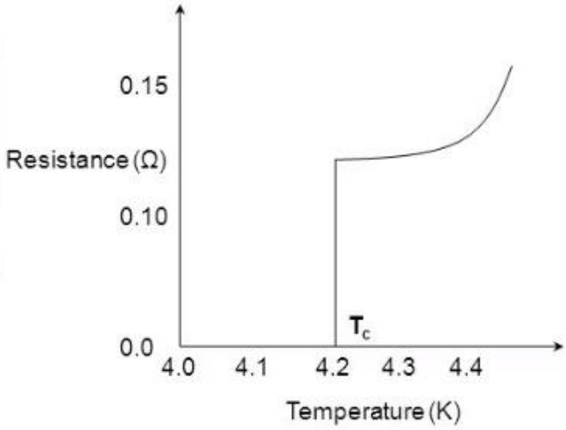
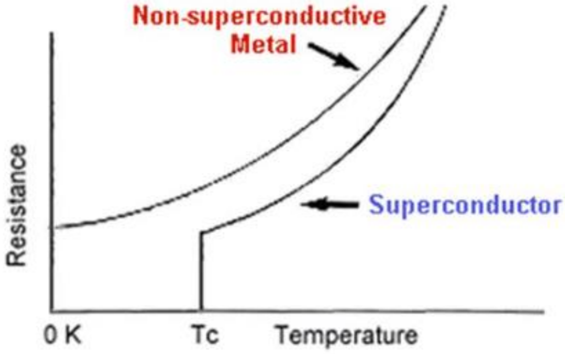
Superconductivity

➢ H. Kammerlingh Onnes – 1911 – Pure Mercury





◆Zero resistivity
◆Perfect diamagnetism

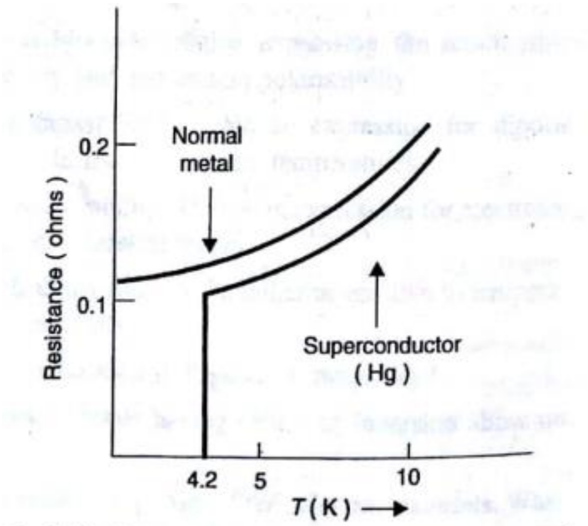


Fig. 10.1. Temperature dependence of the resistance of a normal metal and a superconductor like Hg.

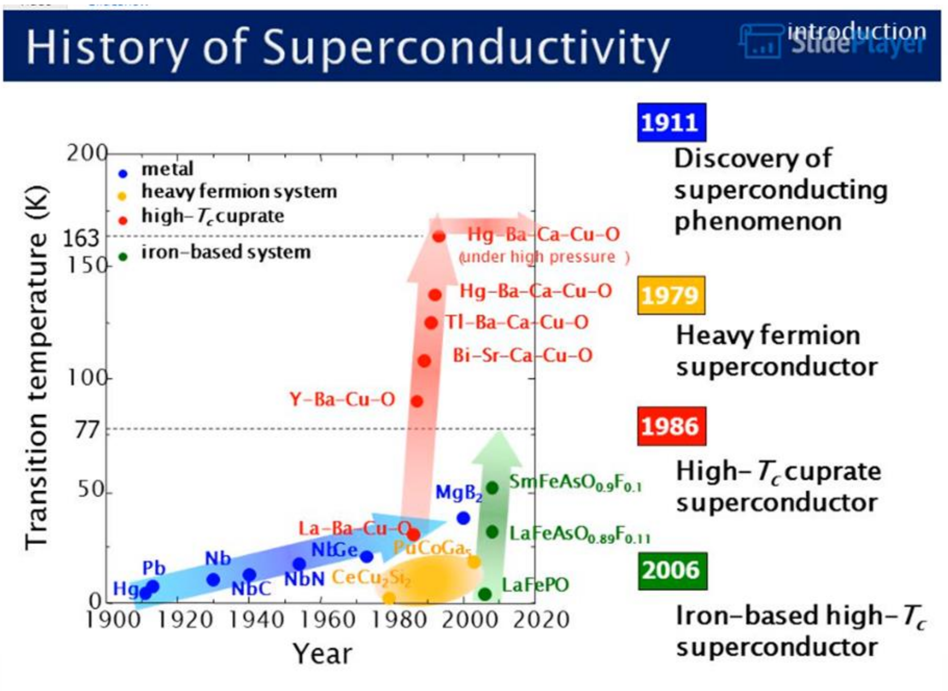


TABLE 10.1. Properties of some selected superconductors in chronological order

Year	T_c (K)	Material	Class	Crystal Structure	Type
1911	4.2	Hg	Metal	Tetragonal	I
1913	6.2	Pb	Metal	*fcc*	I
1930	9.25	Nb	Metal	*bcc*	II
1940	15	NbN	Interstitial compound	NaCl	II
1950	17	V_3Si	Intermetallic compound	β-tungsten (W_3O)	II
1954	18	Nb_3Sn	Intermetallic compound	W_3O	II
1960	10	Nb-Ti	Alloy	*bcc*	II
1964	0.7	$SrTiO_3$	Ceramic	Perovskite	II
1970	20.7	Nb_3 (Al, Ge)	Intermetallic compound	W_3O	II
1977	23	Nb_3Ge	Intermetallic compound	W_3O	II
1986	34	$La_{1.85}Ba_{0.15}CuO_4$	Ceramic	Tetragonal	II
1987	90	$YBa_2Cu_3O_7$	Ceramic	Orthorhombic	II
1988	108	Bi cuprates	Ceramic	Orthorhombic	II
1988	125	Tl cuprates	Ceramic	Orthorhombic	II

10.3 PERFECT DIAMAGNETISM OR MEISSNER EFFECT

Meissner and Ochsenfeld discovered in 1933 that a superconductor expelled the magnetic flux as the former was cooled below T_c in an external magnetic field, i.e., it behaved as a perfect diamagnet. This phenomenon is known as the *Meissner effect.* Such a flux exclusion is also observed if the superconductor is first cooled below T_c and then placed in the magnetic field. It thus follows that the diamagnetic behaviour of a superconductor is independent of its history as illustrated by Fig. 10.2. It also follows from this figure that the Meissner effect is a reversible phenomenon. Since $\mathbf{B} = 0$ inside the superconductor, we can write

$$\mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M}) = 0$$

i.e.,

$$\mathbf{H} = -\mathbf{M}$$

Therefore, the susceptibility is given by

$$\chi = \mathbf{M}/\mathbf{H} = -1 \tag{10.1}$$

which is true for a perfect diamagnet.

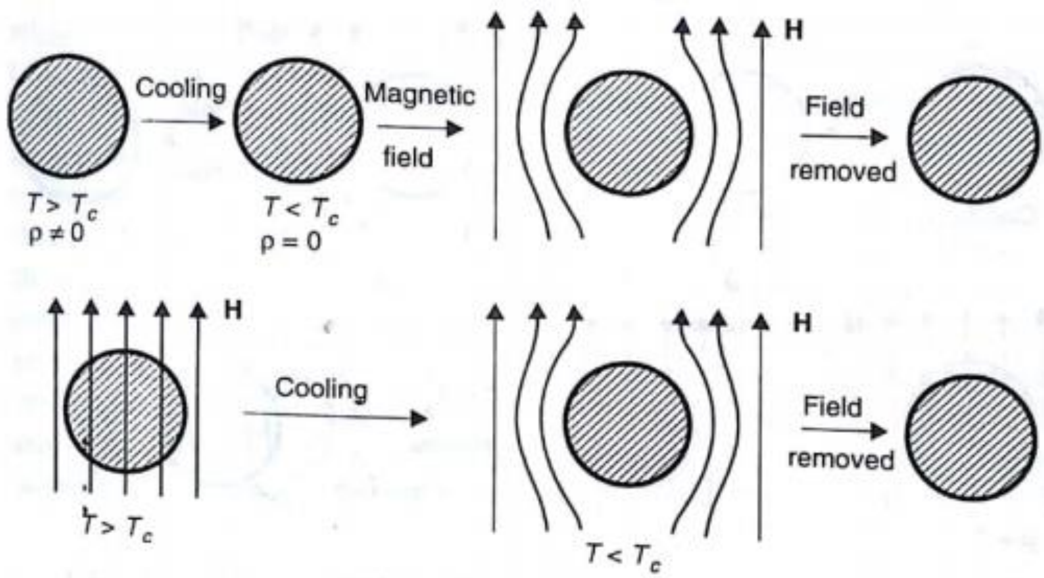


Fig. 10.2. *A superconductor showing a perfect diamagnetism independent of its history.*

It is interesting to note that the perfect diamagnetic behaviour of a superconductor cannot be explained simply by considering its zero resistivity. Such a perfect conductor would behave differently under different conditions as illustrated by Fig. 10.3. Since the resistivity, ρ, is zero for a perfect conductor, the application of Ohm's law ($E = \rho J$) indicates that no electric field can exist inside the perfect conductor. Using one of the Maxwell's equations, i.e.,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

we obtain

$$\mathbf{B} = \text{constant}$$

Thus the magnetic flux density passing through a perfect conductor becomes constant. This means that when a perfect conductor is cooled in the magnetic field until its resistance becomes zero, the magnetic field in the material gets frozen in and cannot change subsequently irrespective of the applied field. This is in contradiction to the Meissner effect.

Thus we conclude that the behaviour of a superconductor is different from that of a perfect conductor and the superconducting state may be considered as a characteristic thermodynamic phase of a substance in which the substance cannot sustain steady electric and magnetic fields. Hence the two mutually independent properties defining the superconducting state are the zero resistivity and perfect diamagnetism, i.e.,

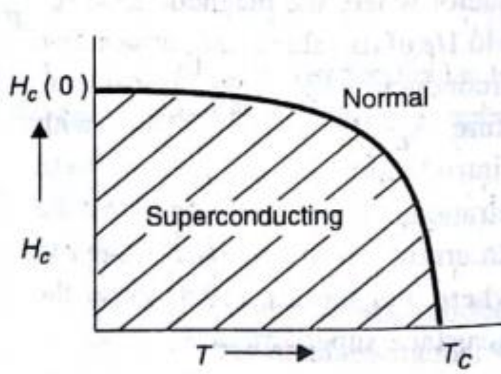
$$\mathbf{E} = 0 \quad \text{and} \quad \mathbf{B} = 0 \qquad (10.2)$$

Critical filed or Effects of Magnetic field

$$H_c = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right) \qquad (10.6)$$

where $H_c(0)$ is the critical field at 0 K. Thus, at the critical temperature, the critical field becomes zero, i.e.,

$$H_c(T_c) = 0 \qquad (10.7)$$



below these two types of superconductors.

10.6.1. Type I or Soft Superconductors

The superconductors which strictly follow the Meissner effect are called *type I superconductors.* The typical magnetic behaviour of lead, a type I superconductor, is shown in Fig. 10.8. These superconductors exhibit perfect diamagnetism below a critical field H_c which, for most of the cases, is of the order of 0.1 tesla. As the applied magnetic field is increased beyond H_c, the field penetrates the material completely and the latter abruptly reverts to its normal resistive state. These materials give away their superconductivity at lower field strengths and are referred to as the *soft superconductors.* Pure specimens of various metals exhibit this type of behaviour. These materials have very limited technical applications owing to the very low values of H_c.

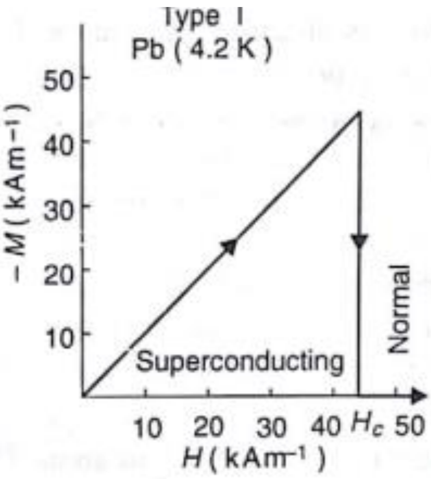


Fig. 10.8. *Magnetization curve of pure lead at 4.2 K.*

10.6.2. Type II or Hard Superconductors

These superconductors do not follow the Meissner effect strictly, i.e., the magnetic field does not penetrate these materials abruptly at the critical field. The typical magnetization curve for Pb-Bi alloy shown in Fig. 10.9 illustrates the magnetic behaviour of such a superconductor. It follows from this curve that for fields less than H_{c1}, the material exhibits perfect diamagnetism and no flux penetration takes place. Thus for $H < H_{c1}$, the material exists in the superconducting state. As the field exceeds H_{c1}, the flux begins to penetrate the specimen and, for $H = H_{c2}$, the complete penetration occurs and the material becomes a normal conductor. The fields H_{c1} and H_{c2} are called the lower and upper critical fields respectively. In the region between the fields H_{c1}

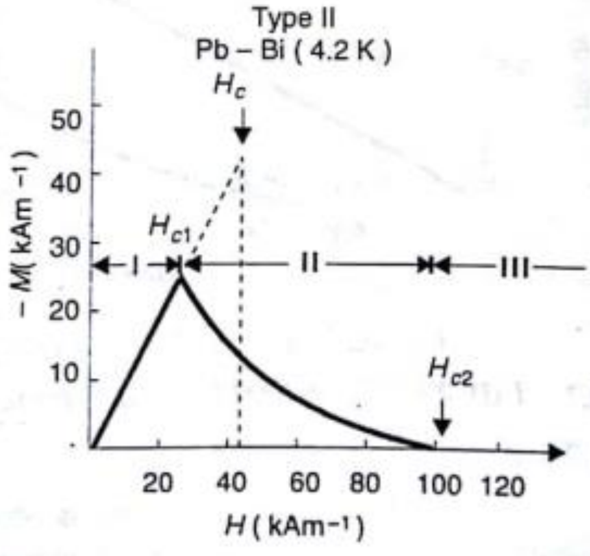


Fig. 10.9. *Magnetization curve of a lead-bismuth alloy at 4.2 K.*

I : Superconducting state

II : Vortex or mixed or intermediate state

III : Normal state

and H_{c2}, the diamagnetic behaviour of the material vanishes gradually and the flux density **B** inside the specimen remains non-zero, i.e., the Meissner effect is not strictly followed. The specimen in this region is said to be existing in the *vortex* or *intermediate state* which has a complicated distribution of superconducting and non-superconducting regions and may be regarded as a mixture of superconducting and normal states. The type II superconductors are also called the *hard superconductors* because relatively large fields are needed to bring them back to the normal state. Also, large magnetic hysteresis can be induced

The critical field is relatively low for type I superconductors. They would generate magnetic fields of about 100 to 2000 G only. Hence, they are not of much use in production of high magnetic fields. Type I superconductors are also called ***soft superconductors***. We summarize here the characteristics of Type I superconductors.

Characteristics of Type-I superconductors:

(*i*) They are perfectly diamagnetic and exhibit complete Meissner effect.

(*ii*) They have only one critical field. At the critical field the magnetization drops to zero.

(*iii*) The maximum critical field for type I superconductor is of the order of 0.1 Wb/m^2.

(*iv*) The transition at H_C is reversible. Below H_C the material behaves as a superconductor and above H_C it behaves as a normal conductor.

Disadvantages: Type I superconductors cannot carry large currents and hence are not of much use in producing high magnetic fields.

Characteristics of Type-II superconductors

(*i*) They have two critical magnetic fields, H_{C1} and H_{C2}.

(*ii*) The material is perfect diamagnetic below the *lower critical field*, H_{C1}. Meissner effect is complete in this region. Above the *upper critical field*, H_{C2}, magnetic flux enters the specimen.

(*iii*) Above H_{C1} they do not show complete Meissner effect and therefore do not behave as perfect diamagnetic materials.

(*iv*) They exist in an *intermediate state* in between the critical fields, H_{C1} and H_{C2}. The intermediate state is a mixture of the normal and superconducting states, magnetically but electrically the material is a superconductor.

(*v*) At H_{C2} the magnetization vanishes and the specimen returns to normal conducting state.

(*vi*) The upper critical field is very high and is of the order of 30 Wb/m^2.

magnetic fields.

Table 42.2: *Comparison between Type I And Type II Superconductors*

Sl.No.	Type-I Superconductors	Type-II Superconductors
1.	They exhibit complete Meissner effect	They do not exhibit complete Meissner effect
2.	They show perfect diamagnetic behaviour	They do not show perfect diamagnetic behaviour
3.	They have only one critical magnetic field, H_C	They have two critical magnetic fields, lower critical magnetic field, H_{C1} and upper critical magnetic field, H_{C2}
4.	There is no mixed state or intermediate state in case of these materials	Mixed state or intermediate state is present in these materials
5.	The material loses magnetization abruptly	The material loses magnetization gradually
6.	Highest value for H_C is about 0.1 Wb/m^2	Upper critical field is of the order of 30 Wb/m^2
7.	They are known as soft superconductors	They are known as hard superconductors
8.	Lead, tin, mercury are examples	Nb-Sn, Nb-Ti, Nb-Zr, Va-Ga are examples

...ing magnets.

Difference between Type-I and Type-II superconductors

Type-I superconductors	Type-II superconductors
Type-I superconductors exhibit complete Meissner effect.	Type-II superconductors show complete Meissner effect below H_{C_1} and allow the flux to penetrate the superconductor between H_{C_1} and H_{C_2}. Between H_{C_1} and H_{C_2}, the material shows a region of mixed state.
Above critical field H_C, the superconductor becomes normal conductor.	Between H_{C_1} and H_{C_2}, the superconductor exists in a mixed state called as vortex state and above H_{C_2}, it comes in normal state.
Type-I superconductors are known as soft superconductors.	Type-II superconductors are known as hard superconductors.
The critical field H_C is relatively low. They can generate fields about 100 to 1000 gauss.	The value of H_{C_2} is very large. They are able to produce very high magnetic field. They can carry larger current when the magnetic field is between H_{C_1} and H_{C_2}.
Type-I superconductors are materials such as Al, Zn, Ga, etc.	Type-II are alloys like lead-indium alloy, etc.

Thermal Properties:

Entropy:

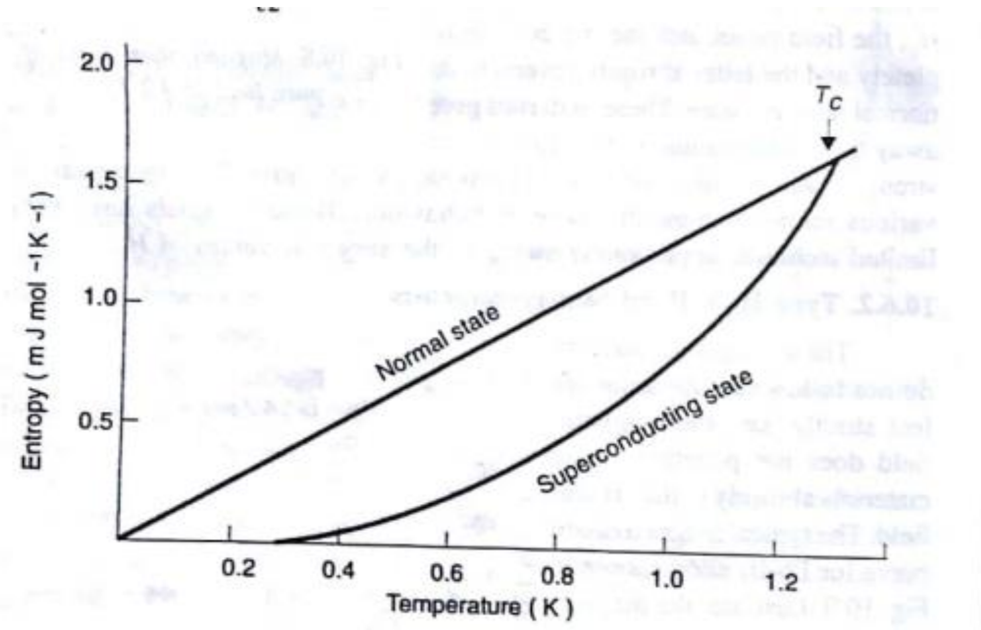


Fig. 10.10. ***Entropy versus temperature for aluminium.***

Specific heat:

$$C_n(T) = \gamma T + \beta T^3 \tag{8.5}$$

The first term is equation (8.5) is the specific heat of electrons in the metal and the second term is the contribution of lattice vibrations at low temperatures. The specific heat of the superconductor shows a jump at T_c. Since the superconductivity affects electrons mainly, it is natural to assume that the lattice vibration part remains unaffected, *i.e.*, it has the same value βT^3 in the normal and superconducting states. On subtracting this, we notice that the electronic specific heat C_{es} is not linear with temperature. It rather fits an exponential form

$$\boxed{C_{es}(T) = A\exp(-\Delta/k_B T)} \tag{8.6}$$

This exponential form is an indication of the existence of a finite gap in the energy spectrum of electrons separating the ground state from the lowest state (Fig. 8.9).

The number of electrons thermally excited across the gap varies exponentially with the reciprocal of temperature. The energy gap is believed to be a characteristic feature of the superconducting state which determines the thermal properties as well as high frequency electromagnetic response of all superconductors.

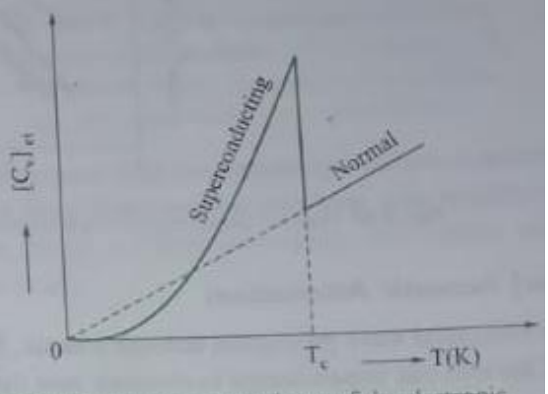


Fig. 8.8 Temperature dependence of the electronic specific heat in the normal and superconducting states

IX. THE ENERGY GAP

The heat capacity in the superconducting state varies with temperature in an exponential manner; that is, it is of the form exp $(-\Delta/k_B T)$ with $\Delta = b\, k_B\, T_c$, where b is a constant. This indicates, in accordance with the fact that the exponential form is compatible with the thermal excitation across a gap in energy that an energy gap may exist in the superconducting electron levels. The jump in the heat capacity at the critical temperature T_c supports this idea of the existence of the energy gap further. See Fig. 8.9. The energy gap in superconductors differs from the gap in semiconductors or insulators in a very fundamental way. The energy gap in superconductors is often an entirely different nature than the energy gap in insulators because in the former, the gap is attached to the Fermi gas whereas in the later the gap is tied to the lattice. In semiconductors, the gap prevents the flow of electrical current. Energy must be added to lift electrons from the valence band into the conduction band before current can flow. In a superconductor, on the other hand, current flows despite the presence of a gap. The energy gap has no effect upon the behaviour of the special electrons that carry current in a superconductor. Superconductors contain normal electrons as well, and it is these electrons that are affected by the gap.

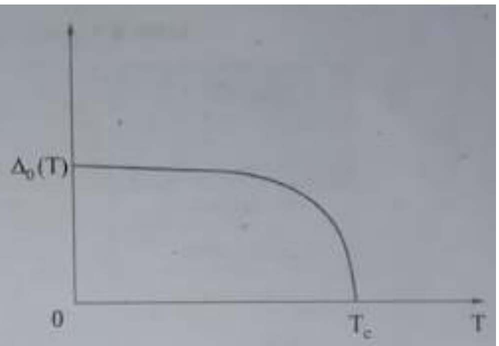


Fig. 8.11 A plot of the temperature dependence of the energy gap parameter $\Delta(T)$. Note that $\Delta(T)$ vanishes with infinite slope as $T \to T_c$ leading to the second-order phase transition

The existence of energy gap in superconductors has been confirmed by a number of experiments: Electron tunnelling observation across the superconducting junctions by Giaever being one of them. Some experiments have been employed for the experimental determination of its value. From theory and from comparison with optical and other methods of determination of gap, it is concluded that

$$\boxed{\begin{aligned} E_g &= 2\Delta = 2b\,(k_B T_c) \\ \text{or} \quad [E_g/k_B T_c] &= 2b \end{aligned}} \tag{8.9}$$

$2b$ is about 3.5 *i.e.*, the gap decreases from a value of about 3.5 $k_B T_c$ at 0 K to zero at the transition temperature. Values of energy gap of some selected superconductors are given in Table 8.3.

X. ISOTOPE EFFECT

It has been observed that the critical temperature of superconductors varies with isotopic mass. The observation was first made by Maxwell and others, who used mercury isotopes. To give an idea of the magnitude of the effect, for mercury T_c varies from 4.185 K to 4.416 K as the isotopic mass M varies from 199.5 to 203.4. The argument was that isotopic mass can enter in the process of the formation of the superconducting phase of the electron states only through the electron-phonon interaction.

In the early years of the development of the BCS theory the simple law

$$T_c \propto M^{-\beta}$$

with $\beta = +0.5$ was thought to be valid for most materials.

10.8 ISOTOPE EFFECT

It was observed in the year 1950 that the transition temperatures of a superconductor varies with its isotopic mass M as

$$T_c \propto M^{-1/2}$$

or $$T_c M^{1/2} = \text{constant} \tag{10.11}$$

Thus larger the isotopic mass, lower is the transition temperature. For example, the transiton temperature of mercury changes from 4.185 K to 4.146 K when its isotopic mass is changed from 199.5 to 203.4 amu.

XIV. LONDON EQUATIONS: ELECTRODYNAMICS

The London theory is based on rather the old ideas of the *two fluid model*, according to which a superconductor can be thought to be composed of both normal and superfluid electrons. The Maxwell's electromagnetic equations are inadequate to explain the *electrodynamics* of superconductors, namely, the zero resistance state coupled with perfect diamagnetism. The London brothers in 1935 derived two field equations though successful in explaining the two important experimental facts about superconductors, do not give any insight into the underlying electronic process in superconductors anything more than what Ohm's law does for normal conductors. In this respects London's theory is purely macroscopic.

According to London's theory, it is assumed that there are two types of conduction electrons in a superconductor, namely, the *superelectrons* and the *normalelectrons*. At 0 K a superconductor contains only superconducting electrons, but as temperature increases the ratio of the normal electrons to superconducting electrons increases, until at the transition temperature all the electrons are normal. At any temperature the sum of the superconducting electrons and the normal electrons is equal to the conduction electron density in the material in the normal state. The superconducting electrons are not subjected to any lattice scattering and therefore are merely accelerated in electric field.

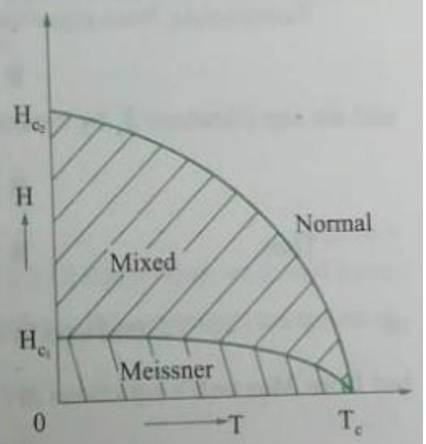
Let n_n $\mathbf{v}_n$ and n_s $\mathbf{v}_s$ be respectively the density, and velocity of the normal and superfluid electrons. If n is the number of electrons m^3 then on the average

$$n = n_n + n_s$$

The current density is given by

$$\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s = en_n \mathbf{v}_n + en_s \mathbf{v}_s \tag{8.12}$$

In a superconducting metal the superconducting electrons encounter no resistance to their motion, so, if a constant electric field $\mathbf{E}$ is maintained in the material, the electrons accelerate steadily under the action of this field.



(i) First London equation : Let n_s be the super electron density at a temperature less than the transition temperature. Then super current density will be

$$\mathbf{J}_S = -e\, n_s\, \mathbf{v}_S,$$

where $\mathbf{v}_s$ is the drift velocity of super electrons. Since super electrons are not subjected to any lattice scattering, they are continuously accelerated by the electric field. With force on super electron being $e\,\mathbf{E}$, we can write the equation of motion as

$$m \frac{d\mathbf{v}_S}{dt} = -e\,\mathbf{E}$$

Differentiating (1),

$$\frac{d\mathbf{J}_S}{dt} = -e\, n_S \frac{d\mathbf{v}_S}{dt}$$

which with equation (2) gives

$$\frac{d\mathbf{J}_S}{dt} = \frac{n_s e^2}{m}\mathbf{E}. \qquad ...(3)$$

This is the first London equation which describes the absence of resistance. The equation shows that it is possible to have steady currents in the absence of electric field (for $\mathbf{E} = 0$, $\mathbf{J}_S$ is finite and steady) which is the phenomenon of superconductivity. The corresponding expression for normal current density is

$$\mathbf{J}_N = \sigma\,\mathbf{E}$$

which shows that no current is possible in the absence of an electric field which is in line with the behaviour of the material in the normal state.

(ii) Second London Equations : From Maxwell's equation, we write

$$\text{curl}\,\mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

or

$$\nabla \times \mathbf{E} = -\mu_0 \frac{d\mathbf{H}}{dt} \qquad ...(4)$$

Taking curl of equation (3), we write

$$\vec{\nabla} \times \frac{d\mathbf{J}_S}{dt} = \frac{n_S e^2}{m}(\vec{\nabla} \times \mathbf{E})$$

$$= -\frac{\mu_0 n_S e^2}{m}\frac{d\mathbf{H}}{dt}, \qquad ...(5)$$

on applying equation (4).

On integrating equation (5), we arrive at

$$\text{curl}\,\mathbf{J}_S = -\frac{\mu_0 n_S e^2}{m}(\mathbf{H} - \mathbf{H}_0), \qquad ...(6)$$

where H_0 is a constant of integration. Since Meissner effect prohibits magnetic fields inside the superconductor, H_0 must be zero. Therefore

$$\text{curl}\,\mathbf{J}_S = -\frac{\mu_0 n_S e^2}{m}\mathbf{H}. \qquad ...(7)$$

which is second London equation and explains Meissner effect.

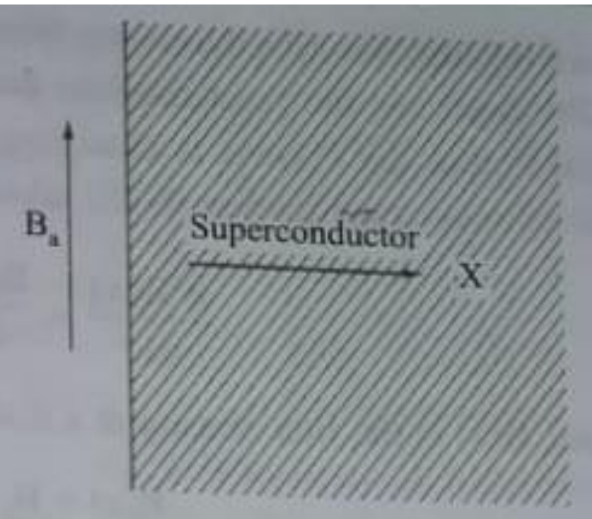


Fig. 8.15 (a) Magnetic field applied parallel to the boundary of the superconductor

$$\nabla \times J_s = -\frac{n_s e^2}{m} B \tag{37}$$

This is called the second London equation and leads to results that are in agreement with the experiment.

17.9 LONDON PENETRATION DEPTH

London brothers successfully explained the Meissner effect and zero resistivity by adding two new equations to the four Maxwell's equations used in electrodynamics. According to the London equations, the flux does not suddenly drops to zero at the surface of Type I superconductors, but decreases exponentially. In order to see the actual variation of flux with distance, let us start with the Maxwell's equation

$$\nabla \times B = \mu_0 J_s \tag{38}$$

Taking curl of this equation, we obtain

$$\nabla \times \nabla \times B = \mu_0 \nabla \times J_s \tag{39}$$

Making use of the identity $\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B = -\nabla^2 B$, where $\nabla \cdot B = 0$ for a superconductor, eq. 39 becomes

$$-\nabla^2 B = \mu_0 \nabla \times J_s \tag{40}$$

Now, from eqs. 37 and 40, we have

$$\nabla^2 B = -\frac{\mu_0 n_s e^2}{m} B = \frac{1}{\lambda^2} B \tag{41}$$

where $\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$ and is called the London penetration depth. One dimensional form of eq. 41 is

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{\lambda^2} B_z \tag{42}$$

Here it is supposed that the specimen is semi-infinite with its surface lying in the yz-plane and the field is applied in the z-direction. The solution of this simple differential equation is assumed to be of the form

$$B_z(x) = B_z(0) \exp\left(-\frac{x}{\lambda}\right) \tag{43}$$

The graphical representation of eq. 43 is shown in Fig. 17.13. This indicates that the flux density decreases exponentially inside the superconductor, falling to $1/e$ of its initial value at a distance λ, the London penetration depth. It further indicates that the flux inside the bulk of superconductor is zero and hence is in agreement with the Meissner effect.

$$\boxed{\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}} \tag{8.30}$$

λ will be of the order of 1 nm.

We can now write the London equations (8.27) and (8.28) as

$$\text{curl } \mathbf{J}_s = -\frac{\mathbf{B}}{\mu_0 \lambda^2} \tag{8.31}$$

$$\dot{\mathbf{J}}_s = \frac{\mathbf{E}}{\mu_0 \lambda^2} \tag{8.32}$$

London equations do not replace Maxwell's equations, but are additional conditions obeyed by the supercurrents. The penetration depth is found to depend strongly on temperature and to become much larger as T approaches T_c. At low temperatures, it is nearly independent of temperature and has a

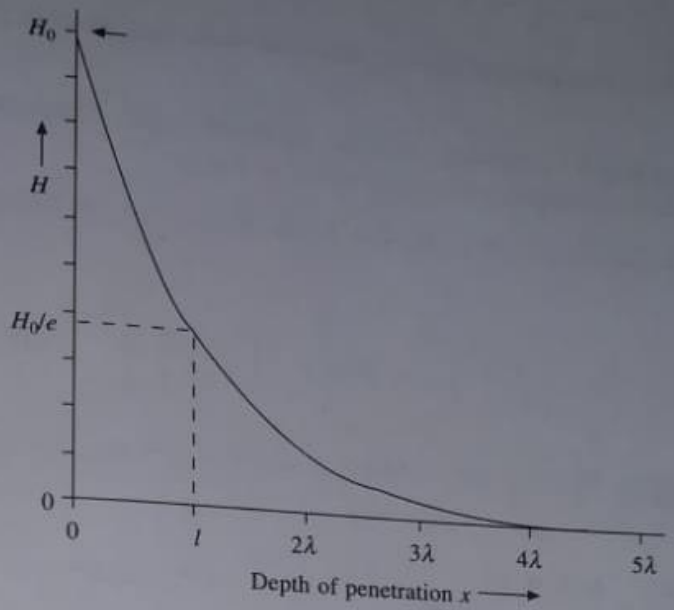


Fig. 17.13 Exponential decrease of the magnetic field inside a superconductor

The penetration depth has been verified and measured in a number of cases and is found to be in agreement with the theoretical value which is about 500 Å. The penetration depth is also found to depend on temperature according to the relation

$$\lambda(T) = \lambda(0)\left(1 - \frac{T^4}{T_c^4}\right)^{-1/2} \tag{44}$$

where $\lambda(0) = \lambda$, the penetration depth at $T = 0$K. According to eq. 44, λ increases with the increase of T and becomes infinite at $T = T_c$ (Fig. 17.14).

This is expected because at $T = T_c$, the substance changes from superconducting state to normal state when the field can penetrate to the whole specimen, i.e. the specimen has an infinite depth of penetration. Further, since the London penetration depth and the number of superelectrons n_s is inversely related to each other and is also temperature dependent, therefore a similar equation like eq. 44 can be obtained for superelectrons, i.e.

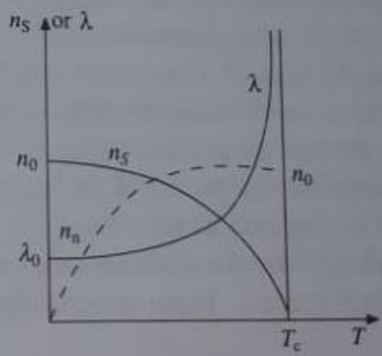


Fig. 17.14 The penetration depth as a function of temperature

$$n_s = n_0\left(1 - \frac{T^4}{T_c^4}\right) \tag{45}$$

or

$$w = \frac{n_s}{n_0} = \left(1 - \frac{T^4}{T_c^4}\right) \tag{46}$$

where w is called the order parameter which characterizes the degree of order in the superconducting state. The variation of superelectrons and normal electrons with temperature is also depicted in Fig. 17.14.

Example: The penetration depth of mercury at 3.5 K is about 750 Å. Estimate the penetration depth at 0 K. Also calculate the superconducting electron density.

Solution: Given: $\lambda(3.5) = 750$ Å, $T_c = 4.12$ K, $T = 3.5$ K, $\lambda(0) = ?$, $n_s = ?$ From eq. 44, we can write

$$\lambda(0) = \lambda(T)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2} = 750\left[1 - \left(\frac{3.5}{4.12}\right)^4\right]^{1/2} = 519 \text{ Å}$$

The normal electron density in mercury can be found in terms of molecular weight and molecular density. Therefore,

$$n_0 = \frac{N\rho}{M} = \frac{6.02 \times 10^{26} \times 13.55 \times 10^3}{200.6} = 4.06 \times 10^{28}\,/\text{m}^3$$

Now, making use of eq. 45 and substituting the desired values, we have

$$n_s = n_0\left[1 - \left(\frac{T}{T_c}\right)^4\right] = 4.06 \times 10^{28}\left[1 - \left(\frac{3.5}{4.12}\right)^4\right]$$

$$= 4.06 \times 10^{28} \times 0.479 = 1.95 \times 10^{28}/\text{m}^3$$

Coherence Length:

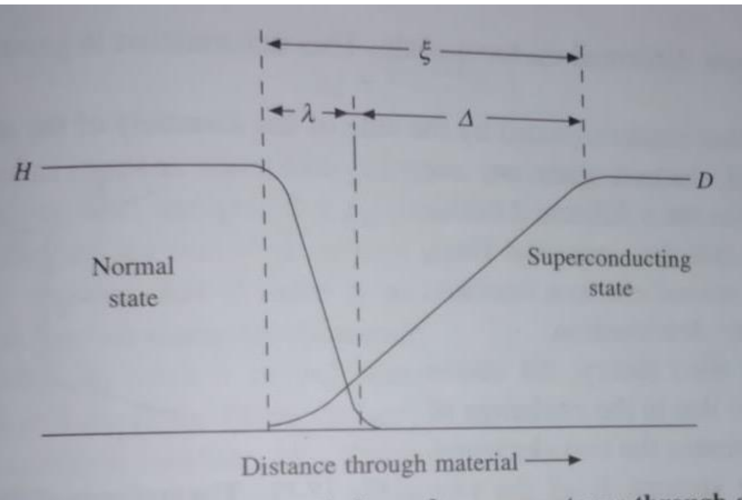


Fig. 17.15 **Variation of magnetic field H and the order parameter w through an interface between the normal and superconducting regions**

$$\xi_0 = \frac{2\hbar v_F}{\pi \Delta} \cong 10^{-6} \text{ m} \tag{48}$$

Like London penetration depth λ, the coherence length ξ is also a function of temperature and also vary in the same manner (Fig. 17.14).

BCS theory, Cooper Pair, BCS Ground state:

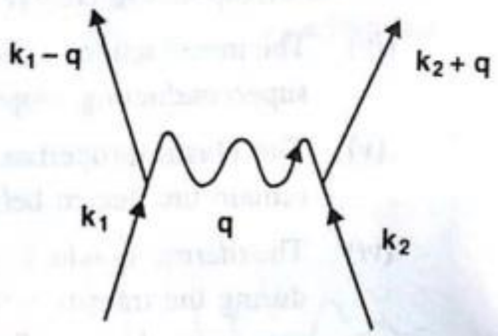
10.12 THEORETICAL ASPECTS

A number of theories have been proposed to explain the phenomenon of superconductivity with varying success. These are, for example, the phenomenological theory by London and London (1935), semiphenomenological theory of Ginzburg and Landau (1950) and the microscopic theory by Bardeen, Cooper and Schrieffer (1957), also called the BCS theory. Of these theories, the BCS theory is the most successful one and explains all the properties of superconductors except those of high-T_c ceramic superconductors. Bardeen, Cooper and Shrieffer were awarded the Nobel prize in 1972 for this work. The BCS theory is briefly presented in the following section.

10.12.1. The BCS Theory

The superconducting state of a metal may be considered to be resulting from a cooperative behaviour of conduction electrons. Such a cooperation or coherence of electrons takes place when a number of electrons occupy the same quantum state. This, however, appears to be impossible for both statistical and dynamic reasons. Statistically, electrons are fermions

and hence occupy the quantum states singly. Secondly, the repulsive force among electrons tends to take them away from one another. In metals, however, the repulsive forces are not very strong owing to screening. According to the BCS theory, both these difficulties can be overcome under certain circumstances. In such a case, the electrons attract each other in a certain energy range and form pairs. A pair of electrons behaves like a boson. Thus a number of pairs can occupy the same quantum state which causes coherence among electrons. The complete BCS theory is too technical to be described here. We present below some physical arguments and ideas underlying this theory.



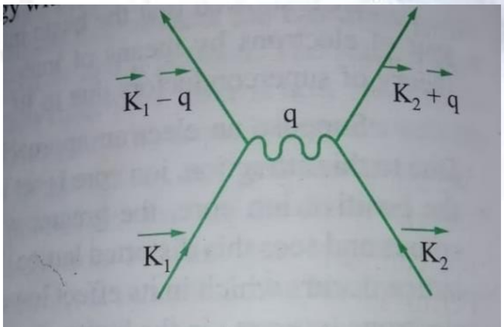


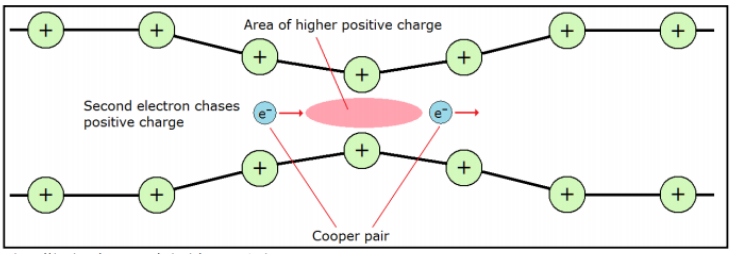
Fig. 8.19 Electron-electron interaction through lattice phonons

(i) Electron-phonon interaction

Frohlich, in 1950, realized that electrons could attract each other via distortion of the lattice. When an electron moves through a crystal, it produces lattice distortion and sets the heavier ions into slow forced oscillations. Since the electron moves very fast it leaves this region much before the oscillations can die off. Meanwhile, if another electron happens to pass through this distorted region, it experiences a force which is one of attraction and is of the type of polarization force. This attractive force lowers the energy of the second electron. The repulsive force between the electrons is small since the Coulomb's repulsion is instantaneous while the attraction mediated by lattice distortion is highly retarded in time. Therefore, the attraction caused by even a weak lattice distortion can overcome a stronger Coulomb's repulsion. Thus the net effect is the attraction of two electrons via a lattice distortion (or phonon) to form a pair of electrons known as the *Cooper pair.*

In quantum-mechanical terms, the first electron of wave vector $\mathbf{k}_1$ creates a virtual phonon $\mathbf{q}$ and loses momentum while the second electron of wave vector $\mathbf{k}_2$ acquires this momentum during its collision with the virtual phonon so that the overall momentum remains conserved. This is depicted in Fig. 10.18. The phonons involved are called *virtual phonons* due to their very short life time which renders it unnecessary to conserve energy during interaction in accordance with the uncertainty principle. Infact, the nature of the resulting electron-electron interaction depends on the relative magnitudes of the electronic energy change and the phonon energy. If the phonon energy exceeds the electronic energy change, the interaction is attractive. Also the

- Two electrons (fermions) can interact due to the presence of a crystal lattice and form a **bosonic pair** – these bosonic pairs condense into one common ground state
- Wikipedia: *"Although Cooper pairing is a quantum effect, the reason for the pairing can be seen from a simplified classical explanation. An electron in a metal normally behaves as a free particle. The electron is repelled from other electrons due to their negative charge, but it also attracts the positive ions that make up the rigid lattice of the metal. This attraction distorts the ion lattice, moving the ions slightly toward the electron, increasing the positive charge density of the lattice in the vicinity. This positive charge can attract other electrons. At long distances, this attraction between electrons due to the displaced ions can overcome the electrons' repulsion due to their negative charge, and cause them to pair up. The rigorous quantum mechanical explanation shows that the effect is due to electron–phonon interactions, with the phonon being the collective motion of the positively-charged lattice"*



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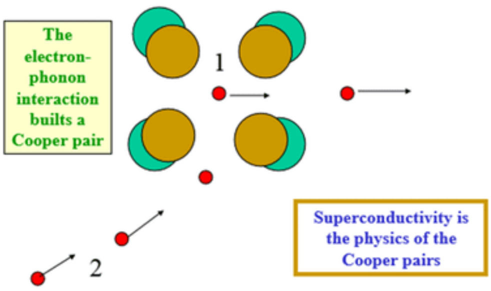
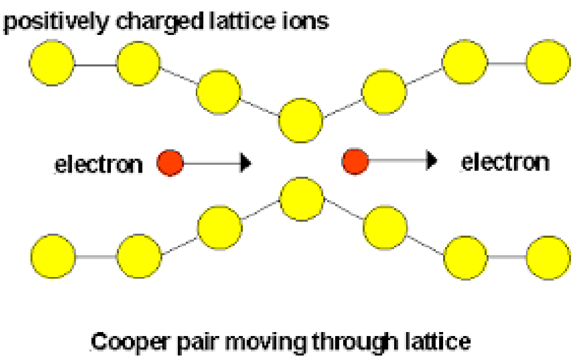




interaction is the strongest when the two electrons have equal and opposite momenta and spins, i.e., $\mathbf{k}_1 = -\mathbf{k}_2$ and $\mathbf{s}_1 = -\mathbf{s}_2$. Such a pair of electrons is called the Cooper pair as L.N. Cooper first discovered that it was energetically favourable for such electrons to enter into an attractive interaction of this type.

It is now obvious that the mass of an ion has an important role in superconductivity. The smaller the mass of ions, the larger is the energy of phonons emitted and hence larger should be the transition temperature. The same conclusion is drawn from the isotope effect discussed earlier. In fact, it was the isotope effect which led Frohlich to consider the electron-phonon interaction as the possible cause of supercoductivity.

(*ii*) *Cooper pair*

As described above, a Cooper pair is formed when the phonon mediated attractive interaction between two electrons dominates the usual repulsive Coulombic interaction. The energy of such a pair of electrons in the bound state is less than the energy of two unbound or free electrons. The difference in energy is the binding energy, E_B, of the electron pair and is basically the same as the energy gap parameter, which was discussed in Sec. 10.7.2. Its typical value is of the order of 10^{-3} eV or about 10 K in temperature units. The binding is the strongest when the total momentum of the pair is zero and the pairs are in a spin singlet state with symmetrical spatial wave function. Cooper calculated the size of the Cooper pair as

$$r_o = \frac{\hbar v_F}{E_B} \tag{10.15}$$

wher v_F is the characteristic velocity of an electron in a metal and is called the *Fermi velocity*. In metals, v_F is related to the conduction electron concentration and is typically of the order of 10^6 ms^{-1}. Using this value of v_F along with E_B equal to 10^{-3} eV in Eq. (10.15), we obtain $r_o = 4 \times 10^{-7}$ m. This is rather a large value compared to the typical distance between two electrons which is of the order of 10^{-10} m. Hence the Cooper pairs overlap with each other considerably and the coherence between them becomes important. It was shown by Barden, Cooper and Schrieffer that the energy of the system is the lowest when total momentum of each pair is the same and is zero. This is the single quantum-mechanical state into which the electron pairs condense. The flow of Cooper pairs constitutes the supercurrent.

(*iii*) *Existence of energy gap*

The Cooper pairs are bound together by a very small energy, Δ , and form a new ground state which is superconducting and is separated by an energy gap 2Δ from the next lowest excited state above it. The Fermi level lies at the middle of the gap. The normal electron states lie above the energy gap and the superconducting electron states lie below the gap at the Fermi surface.

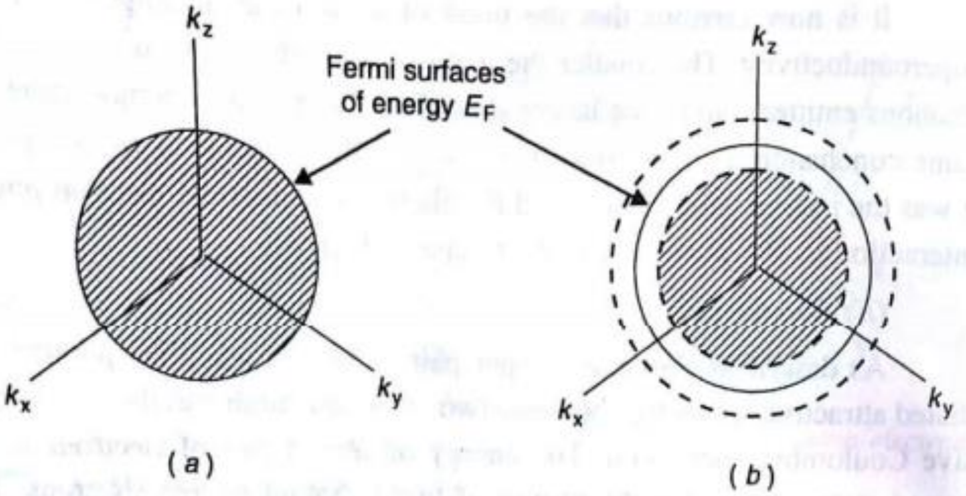
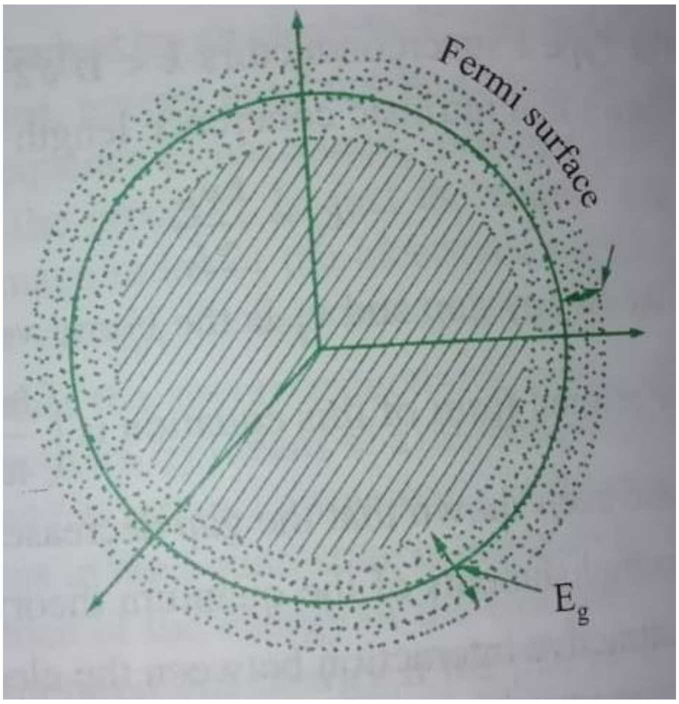


Fig. 10.19. *Ground state of non-interacting Fermi gas (a), and BCS ground state (b) in three dimensions.*

BCS ground state

The BCS ground state differs from the ground state of the non-interacting Fermi gas as shown in Fig. 10.19. In the non-interacting Fermi gas, all the states below the Fermi surface are occupied and all above it are vacant. The lowest excited state is separated from the ground state by an arbitrary low excitation energy, i.e., one can form an excited state by taking an electron from the Fermi surface and raising it just above this surface. As described earlier, the phonon assisted attractive interaction between electrons gives rise to the BCS ground state which is superconducting. This state is separated from the lowest excited state by a finite energy gap E_g. The formation of the BCS ground state can be understood from Fig. 10.20. The BCS state appears to have a higher kinetic energy than the Fermi state, but the attractive potential energy (not shown) of the BCS state acts to decrease the total energy of the BCS state relative to the Fermi state. Thus the BCS state is more stable than the Fermi state and superconductivity persists. The occupancy of one-particle orbitals near E_F in the BCS state resemble somewhat like that obtained from the Fermi-Dirac distribution at a finite temperature. However, in the BCS state, the one particle orbitals are occupied in pairs which are called Cooper pairs. If an orbital with wave vector **k** and spin up is occupied, then the one with wave vector – **k** and spin down is also occupied. Likewise, if **k** (↑) is vacant, then – **k** (↓) is also vacant.



Flux quantization, Josephson effect and tunneling:

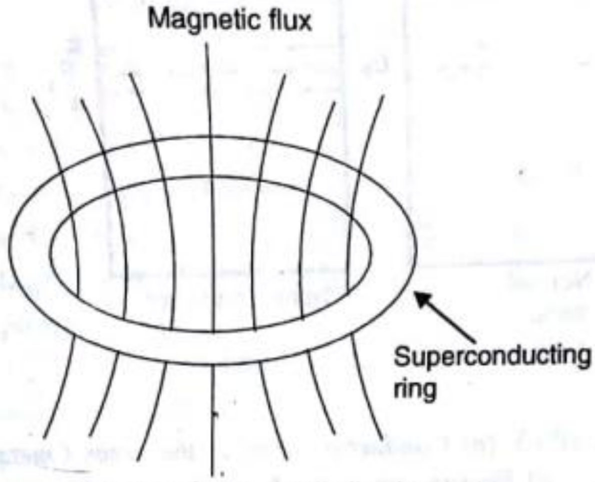


Fig. 10.15. *Magnetic flux through a superconducting ring.*

The Eqs. (10.13) and (10.14) indicate that the lattice vibrations are likely to be involved in causing superconductivity, i.e., the electron-phonon interactions might be playing an important role for the occurrence of superconductivity. This led Frohlich to show that two electrons in a metal can effectively attract each other, the attraction being mediated by lattice vibrations. Later, in 1957, Bardeen, Cooper and Schrieffer developed a complete atomic theory of superconductivity which was based on the formation of such electron pairs known as *Cooper pairs* and the coherent superposition of the pairs into a single quantum state.

10.9 FLUX QUANTIZATION

London, in 1950, speculated that the magnetic flux passing through a superconducting ring (Fig. 10.15) or a hollow superconducting cylinder can have values equal to nh/e (nhc/e in cgs units) where n is an integer. This flux quantization in the non-superconducting region is simply the consequence of the fact that the non-superconducting region is surrounded by the superconducting region. The flux quantization has been confirmed experimentally but the quantum of flux has been found to be $h/2e$ rather than h/e. This unit of flux is called a *fluxoid* and is nearly equal to 2.07×10^{-15} Weber.

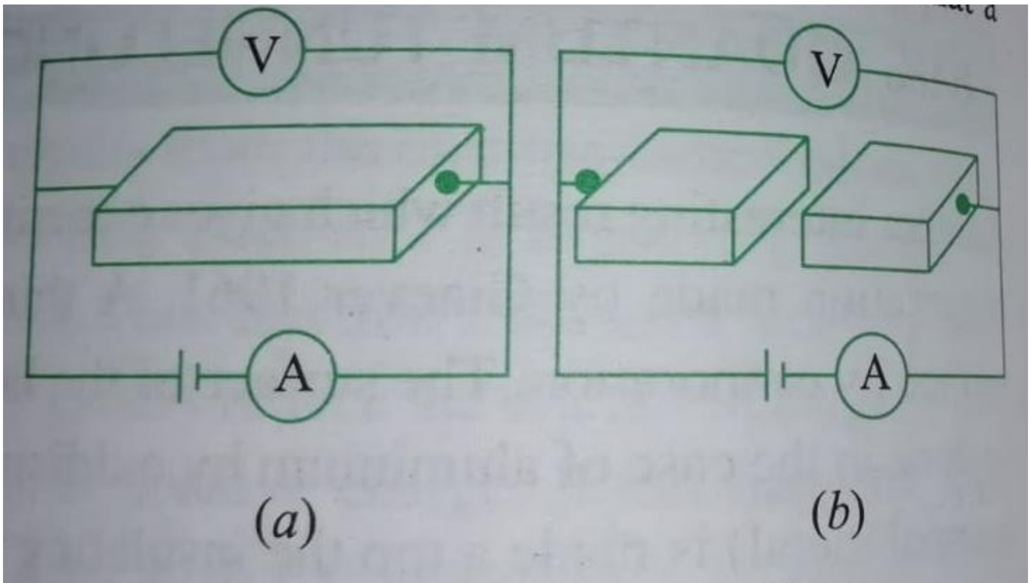


Fig. 8.24 Josephson's effect

10.10 THE JOSEPHSON EFFECTS AND TUNNELLING

Josephson observed some remarkable effects associated with the tunnelling of superconducting electrons through a very thin insulator (1–5 nm) sandwiched between two superconductors. Such an insulating layer forms a weak link between the superconductors which is referred to as the *Josephson junction.* The effects observed by Josephson are given as follows :

Suppose the bar is cut into two pieces and the two pieces are separated by say 1 cm as shown in Fig. 8.24 (*b*). No current will flow and the voltmeter will indicate a voltage equal to the open circuit voltage of the current source. If the distance between the pieces is reduced to 1 nm, the voltmeter suddenly shows zero voltage showing thereby that a current flows across the gap in a superconducting way. This is known as the d.c. *Josephson's effect*. Another effect which is observed if that the voltmeter indicates a voltage, but at the same time a very high frequency electromagnetic radiation emanates from the gap, indicating the presence of a very high frequency alternating current in the gap.

(ii) The ac Josephson effect

If a dc voltage is applied across the junction, *rf* current oscillations of frequency $f = 2eV/h$ are set up across it. For example, a *dc* voltage of 1μV produces a frequency of 483.6 MHz. By measuring the frequency and the voltage, the value of *e*/*h* can be determined. Hence this effect has been utilized to measure *e*/*h* very precisely and may be used as a means of establishing a voltage standard. Furthermore, an application of *rf* voltage along with the *dc* voltage can result in the flow of direct current through the junction.

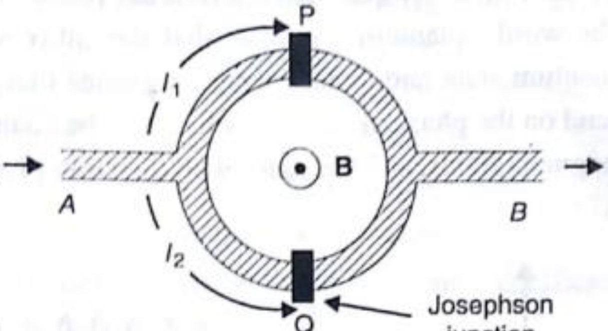


Fig. 10.16. *A SQUID.*

High Tc-Supercoductors:

In 1986, Bednorz and Muller reported their discovery on the La-Ba-Cu-O system of ceramic superconductors which showed T_c equal to 34 K. Thus, contrary to the previous findings, a new class of ceramic superconductors was discovered which showed critical temperature considerably greater than that of the metallic superconductors. They named these materials as *high-T_c ceramic superconductors.* They were awarded the Nobel Prize in 1988 for such an important discovery which created an unprecedented world wide interest in the field of oxide ceramic superconductors. In 1987, a ceramic superconductor of the composition $YBa_2Cu_3O_7$ was discovered which showed T_c equal to 90 K. In 1988, the value of T_c further shot up to about 125 K for thallium cuprates. Table 10.1 gives some data on superconductors in

...ves are at 0 K).

10.13 HIGH TEMPERATURE CERAMIC SUPERCONDUCTORS

As described in Sec 10.1, a new class of oxide ceramic superconductors having the critical temperature greater than 30 K was discovered by Bednorz and Muller in 1986 which ushered in a new era in the field of superconductivity. These are called high-T_c superconductors. The first group of such superconductors discovered was $La_{2-x}M_xCuO_4$ (M = Ba, Sr, Ca) with T_c ranging from 25 to 40 K and is usually referred to as *'214' system.* This system possesses K_2NiF_4 structure with an orthorhombic distortion. This discovery was followed by the discovery of another important system having the general formula $LnBa_2Cu_3O_{7-x}$ (Ln = Y, Nd, Sm, Eu, Gd, Dy, Ho, Er, Tm, Yb) with $x \approx 0.2$. This is called *'123' system* and has orthorhombic structure. In 1988, several other non-rare earth based copper oxide systems involving Bi and Tl were discovered which showed superconductivity between 60 K and 125 K. Some data on superconductors in chronological order is given in Table 10.1.

Many of the properties of these conventional high-T_c superconductors are identical to those of conventional low-T_c metallic superconductors. These include, for example, the existence of energy gap over the entire Fermi surface below T_c and the Josephson tunnelling. These superconductors, however, possess certain properties which do not match with those of conventional ones. These are, for example, small isotope effect, small coherence lengths (~ a few lattice spacings) and unconventional temperature dependencies of normal state response functions. Also, the pressure is found to increase the transition temperature in high-T_c superconductors, whereas usually an opposite effect is observed in conventional superconductors. Thus there appears to be something essentially new in these high-T_c superconductors which has not yet been clearly understood. The identification of the possible conduction mechanism in the high-T_c superconductors is perhaps the most challenging problem in condensed matter physics these days.

and copper)

Continuous search is going on to discover materials that may exhibit superconducting state around room temperature. Interesting practical applications of superconductors are visualized in many fields, but they could not be used on large scale because of the requirement of liquid helium, which is highly expensive. Therefore, they remained as a laboratory curiosity for a long time. The discovery of high temperature materials opened up possibilities of putting superconductors to large-scale use. They require only liquid nitrogen, which is easily available and also cheaper.

properties of high T_c supercon-

The high temperature superconductors discovered so far belong to five chemical systems having the following general formulae.

1. $BaPb_{1-x}Bi_xO_3$
2. $La_{2-x}M_xCuO_{4-x}$ (M = Ba, Sr)
3. $Ba_2MCu_3O_{7-x}$ (M = Y or rare earth metals such as Gd, Eu, etc.)
4. $Ba_{2-x}La_{1+x}Cu_3O_\delta$
5. $Bi_2CaSr_2Cu_2O_\delta$

Table 17.2 High T_c Superconductors

Compounds	$T_c(K)$	Year of discovery
$SrTiO_{3-x}$	~1	1964
$AxWO_3$	6	1965
$LiTi_2O_4$	13	1974
$Ba(Pb, Bi)O_3$	13	1975
$(La, Sr)_2 CuO_4$	37	1986
$YBa_2 Cu_3O_{7-x}$	95	1987
$NdBa_2 Cu_3O_{7-x}$	80	
$SmBa_2 Cu_3O_{7-x}$	90	
$EaBa_2 Cu_3O_{7-x}$	98	
$GdBa_2 Cu_3O_{7-x}$	92	
$DyBa_2 Cu_3O_{7-x}$	95	
$HoBa_2 Cu_3O_{7-x}$	96	
$YbBa_2 Cu_3O_{7-x}$	93	
$LuBa_2 Cu_3O_{7-x}$	45	

25.10 HIGH TEMPERATURE SUPERCONDUCTORS

In the year 1986, Bednorz and Alex Muller in Zurich (Switzerland) discovered the high temperature superconductivity in ceramics. They made a particular type of ceramic material which remained a superconductor at a temperature as high as 30 K. The importance of the discovery was recognised immediately and they were awarded the Nobel prize in Physics in 1987.

The high temperature superconductors are also called as high-T_C materials. By 1988, the long standing 30 K ceiling of T_C in intermetallic compound has been elevated to 125 K in bulk superconducting oxides. All high temperature superconductors are different types of oxides of copper. By 1994 high temperature superconductor showed promise in pre-commercial applications, as in thin film devices, and wires being fabricated. Few examples are:

$Ba\,Pb_{0.75}\,Bi_{0.25}\,O_3$	$T_C = 12\text{ K}$	[BPBO]
$La_{1.85}\,Ba_{0.15}\,Cu\,O_4$	$T_C = 36\text{ K}$	[LBCO]
$Y\,Ba_2\,Cu_3\,O_7$	$T_C = 90\text{ K}$	[YBCO]
$Tl_2\,Ba_2\,Ca_2\,Cu_3\,O_{10}$	$T_C = 120\text{ K}$	[TBCO]

Important Observations

1. All high temperature superconductors bear a particular type of crystal structure called the Perovskite structure.
2. The addition of extra copper oxygen layers into the structure unit of superconducting copper oxide complexes pushes the critical temperature to higher values.
3. The addition of any atom into copper oxide layer either brings down or destroy the effect of superconductivity.
4. The important observation is that the formation of supercurrents in high T_C superconductors is direction dependent. The supercurrents are strong in copper-oxygen planes and weak in direction perpendicular to the planes.
5. In bulk materials, since they are ceramics, the flow of supercurrents have a restriction due to grain boundary effects. As a result, the critical current value is pushed down.

25.11 CHARACTERISTICS OF SUPERCONDUCTORS IN SUPERCONDUCTING STATES

We shall study the characteristics under the following two aspects:

1. Characteristics which do not change in superconducting transition.
2. Characteristics which change in superconducting transition.

1) Characteristics which do not change in superconducting transition

(i) There is no change in crystal structure as revealed by X-ray diffraction studies. This suggests that superconductivity is more connected with the conduction electrons than with atoms themselves.

(ii) The photoelectric properties are unchanged, i.e., no change in absorption of fast or slow electrons.

(iii) The thermal expansion and elastic properties do not change in transition.

(iv) In absence of magnetic field, there is no change of latent heat and no change of volume in transition.

Properties:- Following are the few properties of superconducting materials.

(i) At room temperature, superconducting materials have greater resistivity than other elements.

(ii) The transition temperature T_c is different for different isotopes of an element. It decreases with increasing atomic weight of the isotopes.

(iii) The superconducting property of a superconducting ~~material~~ element is not lost by adding impurities to it but the critical temperature is lowered.

(iv) There is no change in the crystal structure as revealed by X-ray diffraction studies. This shows that superconductivity may be more concerned with the conduction electrons than with the atoms itself.

(v) The superconductor is characterized by zero electrical resistance. It can conduct electric current even in the absence of an applied voltage and the current can persist for years without any detectable decay.

(vi) The thermal properties (entropy, heat capacity, thermal conductivity etc.) of a metal change sharply at transition temperature of supercond as the temperature is lowered.

→ The thermal conductivity in superconducting state varies as exponential with an argument proportional to $1/T$

→ The thermal conductivity of metals decreases markedly when superconductivity sets in.

(vii) The elastic properties dont change in transition.

(viii) All ~~thermo ele~~ thermoelectric effects disappear in superconducting state.

(ix) When a sufficient strong magnetic field is applied to a superconductor below critical temperature its superconductivity is destroyed. At any given temperature below T_c, there is a critical magnetic field H_c such that the superconductivity property is destroyed the application of magnetic field.

The value of H_c decreases as the temperature increases.

Type II sup...

XXIV. POTENTIAL APPLICATIONS OF SUPERCONDUCTIVITY

The potential applications of superconductors in vital areas are given in the following Table 8.5.

Table 8.5 Applications of superconducting materials in some important fields

Magnets	• High-field magnet applications • Nuclear magnetic resonance (medical diagnostics and spectroscopy) • Ore refining (magnetic separators) • Magnetic levitation • Magnetic shielding • Large physics machines (colliders, fusion confinement, r.f. cavities)
Energy related	• Production by magnetic fusion and magnetohydrodynamics • Energy storage (magnetic) • Electric power transmission
Transportation	• High speed trains • Ship-drive system
Electronics and small devices	• SQUIDS (superconducting quantum interference devices) • Josephson devices (square-law detector, parametric amplifier, mixer) • Bolometer • Electromagnetic shielding
Computers and information processing	• Semiconductor-superconductor hybrids (A-D converters) • Active superconducting elements (FETs) • Voltage standard • Optoelectronics • Malched filters

Of the many possible...

42.10 APPLICATIONS SUPERCONDUCTIVITY

Utilization of superconductivity in practical applications is severely limited by the very low temperatures required to maintain the superconducting material in the superconducting state. Till 1986 the highest critical temperature known was about 27 K. Only using liquid helium as the coolant, which is very costly, one can attain such low temperatures. In the last two decades certain high T_C ceramic materials have been discovered. These materials are brittle, difficult to be drawn into wires and cannot carry large currents. Vigorous research is going on around the world to overcome the drawbacks of high T_C materials and to gainfully utilize them in different applications. We discuss here some of the interesting applications.

(*i*) **Power transmission:** The most obvious application of superconductors is in power transmission. In a country, if the electrical grid is made of superconductor wires rather than aluminium, there would be no need to transform the electricity to a higher voltage and then back down again. The resulting savings in cost and materials would be enormous.

(*ii*) **Transformers and electrical machines:** The transformers and electrical machines with superconducting coils generate greatly stronger magnetic fields than magnetic circuits that use ferromagnetic materials produce.

The normal eddy current losses and hysteresis losses will not be present in superconducting devices and hence the size of motors and generators will be drastically reduced. Thus, superconductors are likely to revolutionize the whole range of rotating electrical machines, making them smaller, lighter and highly efficient. For example, a superconducting generator about half the size of a copper wire generator is about 99% efficient; typical generators are around 50% efficient.

(*iii*) **Diagnostic equipments:** High magnetic fields are essential in many areas of research and diagnostic equipments in medicine. They are at present produced using big electromagnets. The electromagnets are unwieldy being very big; require large electrical power and continuous cooling. On the other hand, superconducting solenoids produce very strong magnetic fields.

They are small in size and are less cumbersome. They do not need either large power supplies or the means of removing heat. The only power required is to bring the solenoid into the superconducting state and maintain it in that state. The low power requirement and simple cooling technique leads to a large saving in cost. The superconductor magnets have improved the MRI, as they can be smaller and more efficient than an equivalent conventional magnet.

(*iv*) **Electronic switches:** Type II superconductors can be used as very fast electronic switches. This has allowed building a 4-bit computer microchip operating at about 5[illegible] times the speed of earlier processors.

(v) **Cryotrons:** The application of a magnetic field greater than its critical magnetic field changes the superconducting state of a superconducting material to normal state and removal of the field brings the material back from normal state to the superconducting state. This fact is used in developing *cryotron switches*.

Fig. 42.19 (a) shows a schematic diagram of a simple cryotron. It consists of a superconducting material (control) coil, B, wound around another superconducting (gate) wire, A. The current in the central wire A can be controlled by the current in the coil B. Thus, whenever the current passing through the coil B exceeds the critical current value, the wire A will become a normal conductor exhibiting a finite resistance. This closes the gate for the flow of current through the core wire A. Removal of the current reopens the gate. Thus the system acts as a fast acting relay or switching element and is highly suitable as fast acting memory element in computers.

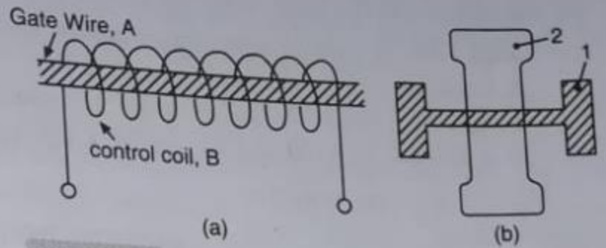


Fig. 42.19: Schematic diagram of a cryotron

The speed of the cryotron switch is dependent on its time constant $\tau = L/R$, where L is the inductance of the control coil B and R is the resistance of the gate coil A in the normal state. Wire wound cryotrons have τ of the order of 10^{-3} s. To reduce the value of τ, R is to be increased and L is to be decreased as far as possible. This objective is achieved by depositing two crosses strips on a substrate and separating them by a thin dielectric layer (Fig. 42.19 b). Strip 1 acts as the gate and is usually made of tin (T_C = 3.7K) and strip 2 acts as the control element usually made of lead (T_C = 7.2 K). Varying the current the strip 2 enables switching the strip 1 from the superconducting state to the normal state and vice versa. Thus opening and closing of the circuit is achieved. Through an appropriate design, the switching can be made faster as the value of τ can be decreased up to 10^{-7} s.

(vii) **MagLev Trains:** The most spectacular application would be the so-called 'MagLev' trains. The coaches of the train do not slide over steel rails but float on a four inch air cushion above the track using superconducting magnets; this eliminates friction and energy loss as heat, allowing the train to reach high speeds of the order of 500 km/hr. Such magnetic levitation trains would make train travel much faster, smoother, and more efficient due to the lack of friction between the tracks and train.

Operation: A typical plan of maglev train is shown in Fig. 42.20. The train has superconducting magnets built into the base of its carriages. An aluminium guide-way is laid on the ground and carries electric current. The repulsion between the two powerful magnetic fields, namely the field produced by the superconductor magnet and the field produced by the electric current in the aluminium guide-way causes magnetic levitation of the train. A levitation of about

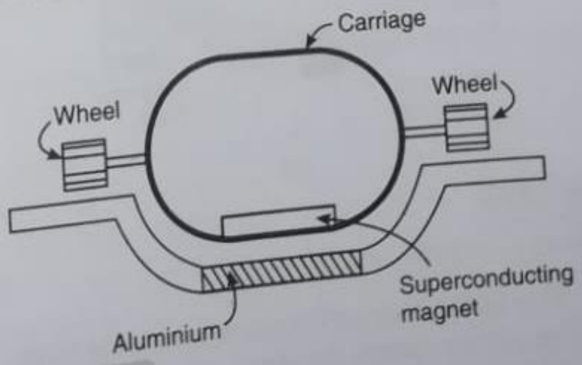


Fig. 42.20: Schematic cross-sectional view of Maglev Vehic

10 to 15 cm is achieved. The train is fitted with retractable wheels, which act in a way similar to the wheels of an airplane. Once the train is levitated in air, the wheels are retracted into the body and the train glides forward on the air cushion. When the train is to be halted, the wheels are drawn out and the train descends slowly onto the guide-way and runs forward till it stops.

(*viii*) **SQUIDS:** A *superconducting quantum interference device* (SQUID) is a device used to measure extremely weak magnetic flux. The heart of a SQUID is a superconducting ring, which contains one or more Josephson junctions. There are two main types of SQUID: DC SQUID and RF (or AC) SQUID.

SQUID is a very sensitive magnetometer, which can measure very weak magnetic fields of the order of 10^{-13} Wb/m^2. The sensitivity of a SQUID to magnetic fields can be enhanced by using a flux transformer (Fig. 42.21). A flux transformer consists of a loop of superconducting material, which is coupled to the SQUID. An external magnetic field produces a persistent super-current in the loop and this current induces a flux in the SQUID. As the loop encloses a much larger area than can a SQUID, the sensitivity of the device gets enhanced.

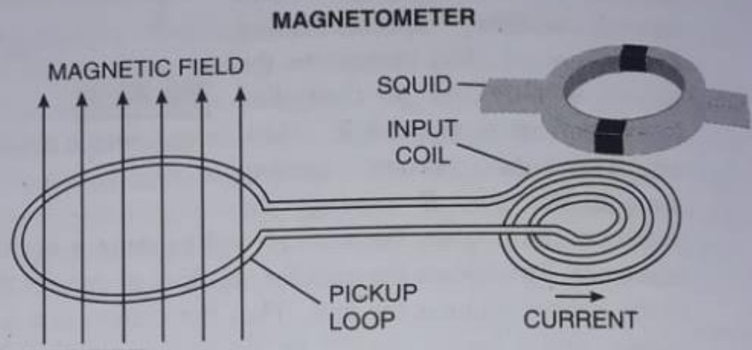


Fig. 42.21:
Magnetometer

Fabrication: SQUIDs are fabricated by depositing a thin niobium layer on an alloy having 10% gold or indium. It acts as the base electrode of the SQUID and the tunnel barrier is oxidized onto this niobium surface. The top electrode is a layer of lead alloy deposited on top of the other two. The entire device is cooled to nearly absolute zero with liquid helium.

The schematic of a two-junction SQUID [direct current (DC) SQUID] is shown in Fig. 42.22 (a). It consists of two Joseph junctions arranged in parallel so that electrons tunneling through the junctions demonstrate quantum interference.

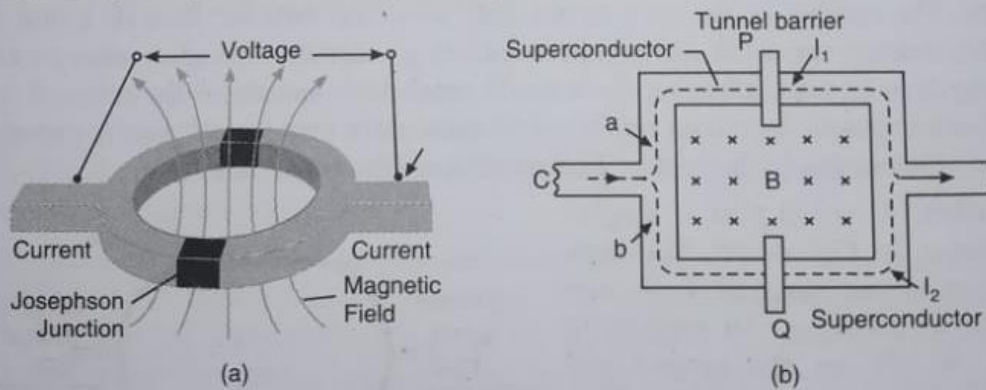


Fig. 42.22:
Working of two-junction SQUID

Working: A dc supercurrent is applied to the SQUID (Fig. 42.22 b). This current, known as *bias* current, enters the device through the arm *A*. It is divided along two paths *a* and *b* and again merge into one and leaves through the arm *B*. *P* and *Q* are the Josephson junctions an the insulating layers at P and Q are of different thickness. I_1 and I_2 are the currents tunneling through the junctions P and Q respectively. In a superconductor, a single wave function describes all the Cooper pairs. The wave function experiences a phase shift at the junctions P and Q. Let the phase difference between points A and B taken on a path through junction P be δ_a and the phase difference between points A and B taken on a path through junction Q be δ_b. In the absence of magnetic field these two phases are equal. That is, $\delta_b - \delta_a$.

When a magnetic field is applied perpendicular to the loop, the flux passes through the loop, and changes the quantum mechanical phase difference across each of the two junctions. The wave functions at the two Josephson junctions interfere with each other. In other words, the supercurrents flowing along the paths a and b interfere. Hence, the device is named SQUID. The interference closely resembles the optical interference observed with Young's double slit. In the case of light, the phase difference between light waves is due to the difference in optical path lengths. In case of super-current interference, the waves are the de Broglie waves of Cooper pairs, and the phase difference is caused by the applied magnetic field. According to Josephson's theory, the phase difference between the reunited currents is directly proportional to the magnetic flux, Φ, through the ring. It can be shown that the total current through two parallel Josephson junctions is given by

$$I_T = 2(I_0 \sin \delta_0) \cos \frac{e\Phi}{hc} \qquad ...(42.17)$$

The above relation indicates that a progressive increase or decrease of the magnetic flux, causes the current to oscillate between a maximum and a minimum value. Maxima in the current occur whenever the magnetic flux increases by one flux quantum. Thus, the period of these oscillations is one flux quantum Φ_0.

$$\Phi_0 = \frac{h}{2e} = 2.06 \times 10^{-15} \text{ webers}$$

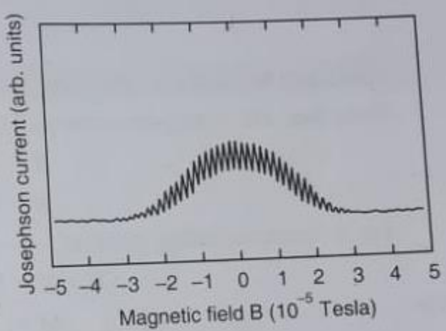


Fig. 42.23:

Variation of the current as a function of the magnetic flux applied.

Fig. 42.23 shows the variation of the current through a pair of Josephson junctions as a function of the magnetic flux applied.

In practice, instead of the current, we measure the voltage across the SQUID, which also oscillates with the changing magnetic field. Thus, the SQUID is a flux-to-voltage transducer, converting a tiny change in magnetic flux into voltage. The flux Φ is related to the magnetic field B through the relation

$$\Phi = BA$$

where A is the area of the ring. With the help of this relation, flux measurements are converted into magnetic field measurements.

Applications of Squids: SQUIDs are used to measure very small magnetic fields. Since the current is sensitive to very small changes in the magnetic field, the SQUID acts as a very sensitive magnetometer. As ordinary magnetometers, SQUIDs are capable of measuring magnetic fluctuations of the order of 10^{-13} T. Because of their extreme sensitivity, SQUIDs find applications in many fields, engineering, medicine and many other fields. For example geologists use them for measuring rock magnetism and continental drift. Physical processes such as muscular or neural activity, in humans create magnetic fields as small as a thousan billionth of a tesla. Human heart generates magnetic fields of about 10^{-14} Wb/m^2 and the human brain generates magnetic fields of about 10^{-15} Wb/m^2. SQUIDs can detect these feeble fields and an array of SQUIDs is used in *magnetoencephalography* (MEG), the process of brain imaging. The SQUIDs are also used in nondestructive testing. In testing for corrosion of aluminum sheets riveted together in aircraft, the SQUID measures the influence of the aircraft skin on an applied oscillating magnetic field; a change in electrical conductivity reveals the defects.