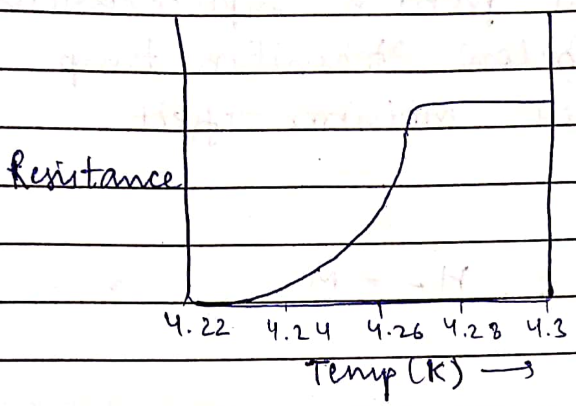
Superconductors



The temperature at which material exhibits zero resistance is called transition temperature. (TC)

- They are very difficult & expensive to fabricate due to low temp.
- In 1986, Bednerz and Muller ~~described~~ discovered a material with high TC

La - Ba - Cu - O ⟶ $T_c = 34K$

Year of discovery	Material	T_c (K)
1911	Hg	4.2 K
1913	Pb	6.2 K
1930	Nb	9.25 K
1940	NbN	15 K
1950	V_3Si	17
1954	Nb_3Sn	18
1960	Nb - Ti	10
1964	$SrTiO_3$	0.7
1970	$Nb_3(Al_2Ge)$	20.7
1977	Nb_3Ge	23
1986	$La_{1.85}Ba_{0.15}CuO_4$	34 Nobel Prize
1987	$YBa_2Cu_3O_7$	90
1988	Bi cuprates	105
1988	Thalium cuprates	125

⇒ Meissner Effect

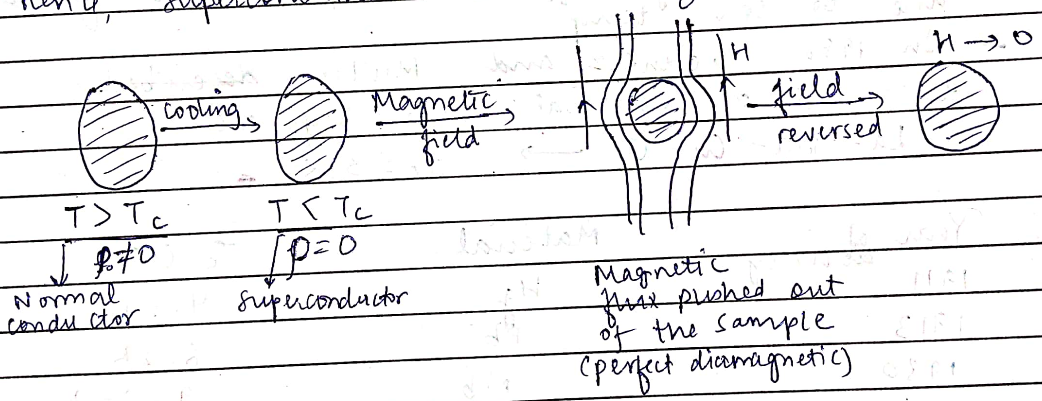
The expulsion of magnetic flux from a superconducting material when it's cooled below transition temp. in a magnetic field is called Meissner Effect.

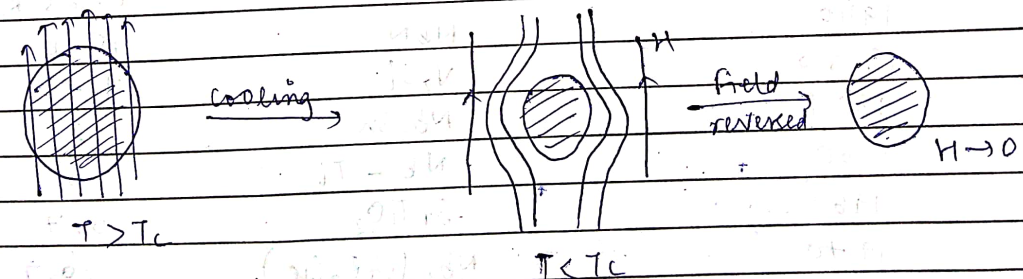
Now, $B = \mu_0(H + M)$

In a superconducting material, $H = -M$

$$\boxed{\chi = \frac{M}{H} = -1}$$

Hence, superconductors are diamagnetic.





Meissner Effect

⇒ Maxwell's Equation : $\nabla \times E = -\frac{\partial B}{\partial t}$

Now, $E = \rho J$ and $\rho = 0$ (in superconductor)

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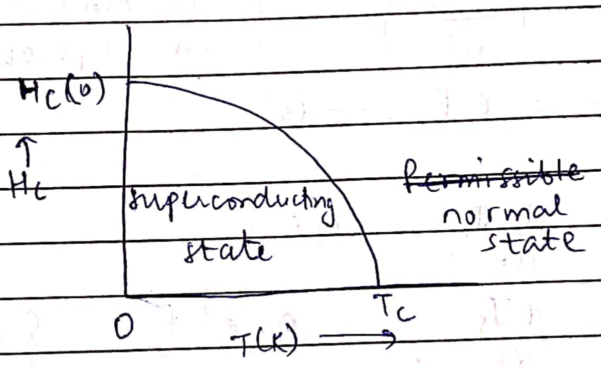
$\Rightarrow \frac{\partial B}{\partial t} = 0 \Rightarrow \boxed{B = \text{constant}}$

Hence, Meissner Effect contradicts the Maxwell's Equation, as B is changed in Meissner Effect.

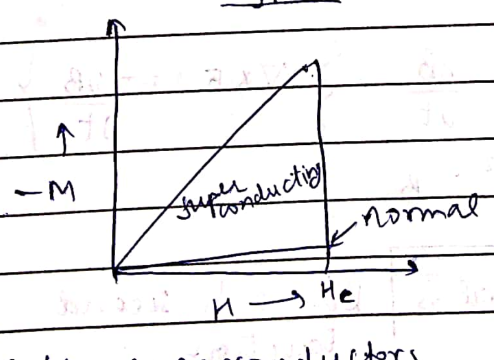
$\Rightarrow$ The value of magnetic field at which the ~~threshold~~ or threshold superconducting vanishes is called critical field (H_c)



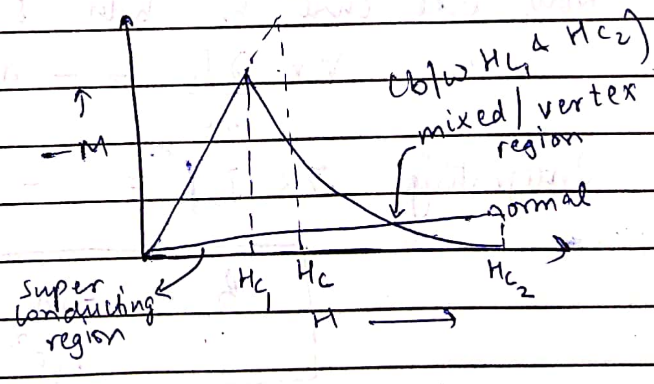
$$H_c(t) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$\Rightarrow$ Types of superconductors :

Type-I



Type-II



- soft superconductors
- completely exhibit Meissner effect
- give away superconductivity on applying small magnetic field.

- hard superconductors
- partially exhibit Meissner effect
- perfect Meissner upto H_{c_1}
- $H_{c_1} \rightarrow H_{c_2}$ magnetic flux increases beyond $H_{c_2} \rightarrow$ normal state

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* London's Equations :

⇒ In 1935, Fritz London & Heinz London, gave the following postulates :

$$\underset{\text{total } e^-}{n} = \underset{\text{normal } e^-}{n_n} + n_s \rightarrow \text{super } e^- \quad — (1)$$

$$J = \underset{\text{normal } e^- \text{ density}}{e n_n V_n} + e n_s V_s \rightarrow \text{super } e^- \text{ density} \quad — (2)$$

$$eE = ma = m\left(\frac{dV_s}{dt}\right) \quad \left\{\because \text{no resistance for } e^- \text{ in superconductors}\right\}$$

$$\Rightarrow m \dot{V_s} = eE \quad — (3)$$

$$J_s = e n_s V_s \Rightarrow \frac{dJ_s}{dt} = n_s e \dot{V_s}$$

$$\Rightarrow \frac{dJ_s}{dt} = \frac{n_s e^2 E}{m} \quad \left\{\because \dot{V_s} = \frac{eE}{m}\right\}$$

$$\Rightarrow \boxed{\frac{dJ_s}{dt} = \frac{n_s e^2 E}{m}}$$ London's first Equation

Hence, current density is constant even when $E = 0$; it proves zero resistance property of superconductors.

Now, take curl on both sides

$$\nabla \times \frac{\partial J_s}{\partial t} = -\frac{n_s e^2}{m}\frac{\partial B}{\partial t} \quad \left\{\because \nabla \times E = -\frac{\partial B}{\partial t}\right\}$$

Integrating $\nabla \times J_s = -\frac{n_s e^2}{m} B$

$$\boxed{B = -\frac{m}{n_s e^2} \text{ curl } J_s}$$ London's second Equation

It describes Meissner's ~~Equation~~ Effect. For a diamagnetic substance, $B = 0$ and hence $\nabla \times J_s = 0$

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* Penetration Depth

⇒ Consider $\nabla \times B = \mu_0 J_s$

$$\Rightarrow \nabla \times \nabla \times B = \mu_0 \nabla \times J_s$$

$$\left\{ \nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B = -\nabla^2 B \right\}$$

$$\Rightarrow -\nabla^2 B = \mu_0 \left(-\frac{n_s e^2 B}{m} \right)$$

$$\Rightarrow \nabla^2 B = \frac{1}{\lambda^2} B \; ; \text{ where } \boxed{\lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2}}$$

$$\Rightarrow \boxed{\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{\lambda^2} B_z}$$

λ: penetration depth.

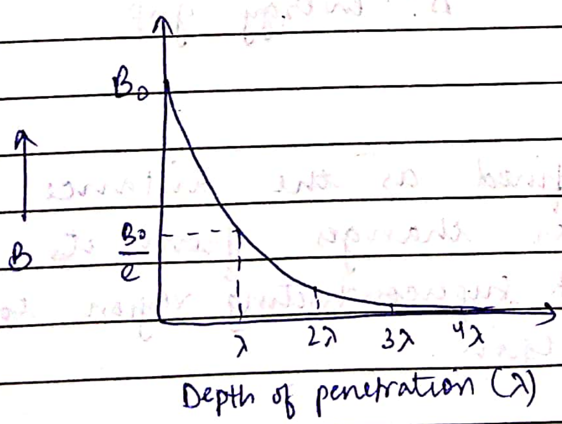
The solution of this diff equation is:

$$B_z(x) = B_z(0) \exp\left(-\frac{x}{\lambda} \right)$$

So, when $x = \lambda$ (penetration depth)

$$\boxed{B_z(x) = \frac{B_z(0)}{e}}$$

The distance at which the magnetic field falls to 1/e of its initial value is called penetration depth.



London's Equations in terms of λ:

$$\boxed{\begin{aligned} \text{curl } J_s &= -\frac{B}{\mu_0 \lambda^2} \\ J_s &= \frac{E}{\mu_0 \lambda^2} \end{aligned}}$$

⇒ Effect on temperature on λ:

$$\lambda(T) = \lambda(0) \left(1 - \frac{T^4}{T_c^4} \right)^{-1/2}$$

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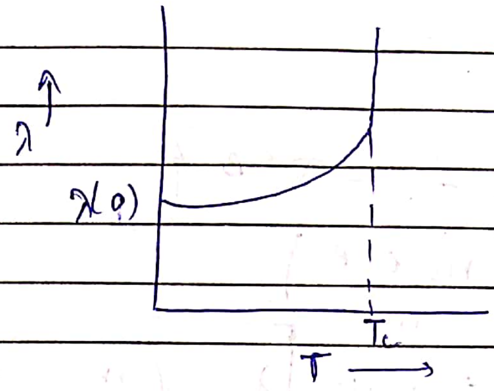
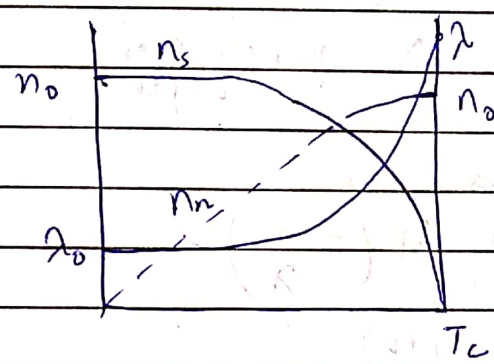


At $T < T_c$, it acts as a superconductor and hence finite λ.

At $T \geq T_c$, magnetic flux is constant everywhere.



$$\omega = \frac{n_s}{n_0} = \left[1 - \frac{T^4}{T_c^4}\right]$$

ω: order parameter

n_0: Total e^- density

* Coherence Length

→ proposed by Pippard in 1953.

$$\boxed{\xi = \frac{2 \hbar V_F}{\pi \Delta}}$$

ξ: xi (Coherence Length)

Δ: energy gap

Coherence Length is defined as the distance within which the order parameter changes from its max^m value in the bulk superconducting region to zero in the normal region.

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