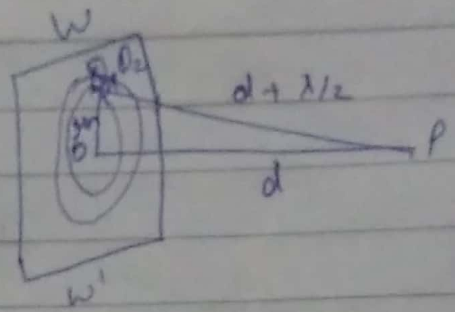
9 - April 19

# Optics

## Fresnel Diffraction




n no. of circles formed of radius $d+\frac{\lambda}{2}$, $d+\frac{2\lambda}{2}$ ...

Ag Angular region b/w two consecutive circle is half Period zone

Radius of half Period zone,

$$r_n = \left[\left(d + \frac{n\lambda}{2}\right)^2 - d^2\right]^{\frac{1}{2}}$$

$$r_n = \left(\cancel{d^2} + \frac{n^2\lambda^2}{4} + \frac{2dn\lambda}{2} - \cancel{d^2}\right)^{1/2}$$

$$r_n = \left(\frac{n^2\lambda^2}{4} + dn\lambda\right)^{1/2}$$

$$= \sqrt{n\lambda d}\left(\frac{n\lambda}{4d} + 1\right)^{1/2}$$

$$\boxed{r_n \approx \sqrt{n\lambda d}}$$

Area of 1st circle, $A_1 = \pi r_1^2$

$A_1 = \pi\lambda d$

Area of nth half Period zone, $A_n = \pi r_n^2 - \pi r_{n+1}^2$

$$\boxed{A_n = \pi\lambda d}$$

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amplitude contribution by 1st half Period zone = ~~$u_1$~~, $u_1$
" " " 2nd " " = ~~$u_2$~~ $u_2$

$\therefore$ amplitude at Point P, $u(P) = u_1 + u_2 + u_3 + ----$

$\because$ Corresponding Points are out of Phase
2nd half Period zone is out of Phase than 1st half

$\therefore$ $u(P) = u_1 - u_2 + u_3 - u_4 ----$

$u_n \propto A_n$
$\propto 1/d$
$\propto \left(\frac{1+\cos\chi}{2}\right)$

$\therefore$ $u_n \propto \frac{A_n}{d}\left(\frac{1+\cos\chi}{2}\right)$

$\because$ $d$ & $\left(\frac{1+\cos\chi}{2}\right)$ are ~~varying~~, increasing & $A_n$ is Constant

$\therefore$ $u(P) = u_1 - u_2 + u_3 - u_4 + u_5 ---$
$= \frac{u_1}{2} + \underbrace{\left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right)}_{\text{Net term is Sligthly (-ve)}} + \underbrace{\left(\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right)}_{-ve} \frac{u_5}{2}$

if $m$ is odd
$u(P) < \frac{u_1}{2} + \frac{u_m}{2}$

if $m$ is even
$u(P) < \frac{u_1}{2} + \frac{u_{m-1}}{2} + \mu_m$

$[\mu = \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6]$

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$$u(P) = u_1 - \frac{u_2}{2} + \frac{u_2}{2} + u_3 - \frac{u_4}{2} + \frac{u_4}{2} + u_5 - \frac{u_6}{2} - \frac{u_6}{2} \ldots$$

$$= u_1 - \frac{u_2}{2} - \left(\frac{u_2}{2} - u_3 + \frac{u_4}{2}\right) - \left(\frac{u_4}{2} - u_5 + \frac{u_6}{2}\right) - \ldots$$

$$u(P) > u_1 - \frac{u_2}{2} - \frac{u_m}{2} \quad (\text{if } m \text{ is even})$$

Rewrite, $> \frac{u_1}{2} - \frac{u_m}{2}$

$$u(P) > u_1 - \frac{u_2}{2} - \frac{1}{2}u_{m-1} + u_m \quad (m \rightarrow \text{odd})$$

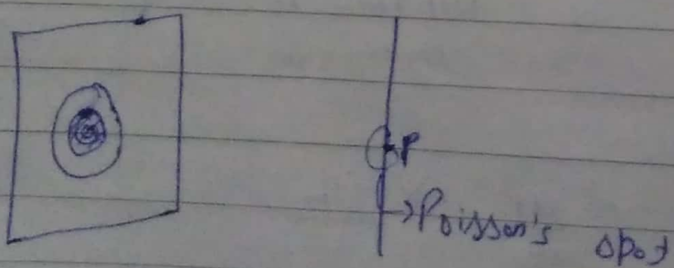
Rewrite, $> \frac{u_1}{2} - \frac{1}{2}u_{m-1} + u_m$

$\because u_1 >>> u_m$

$$\therefore \frac{u_1}{2} - \frac{u_m}{2} < u(P) < \frac{u_1}{2} + \frac{u_m}{2}$$

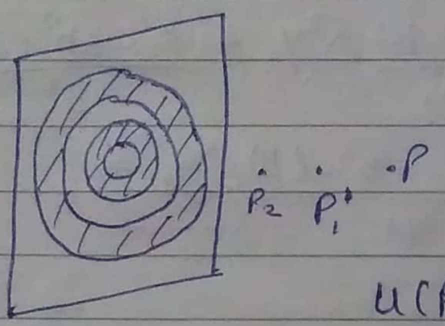
$$\therefore \boxed{u(P) = \frac{u_1}{2}}$$

→ In case of circular apperature at centre point there is a bright fringe.



## * Zone Plate

$$u(P) = u_1 - \underset{=0}{u_2} + u_3 - \underset{=0}{u_4} + u_5 - -$$



$$\therefore \; u(P) = u_1 + u_3 + u_5 - -$$

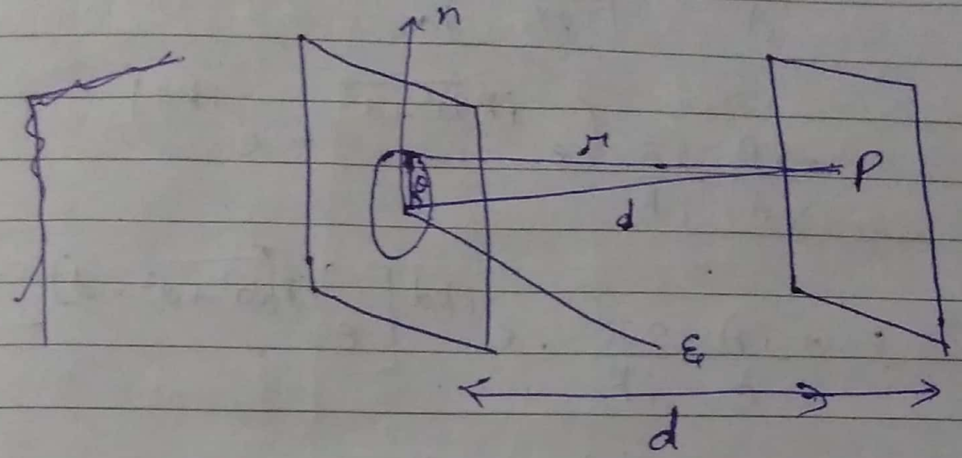
$$u(P_1) = u_1 - u_2 + \underset{=0}{(u_3 - u_4)} + u_5 - u_6$$

$$u(P_1) = u_1 - u_2 + u_5 - u_6$$

$$u(P_2) = \underbrace{u_1 - u_2 + u_3}_{} - \underbrace{u_4 + u_5 - u_6}_{=0} + u_7 - u_8 + u_9$$

# Optics

Brute Force Method




$$\frac{A e^{ikr}}{r}\, d\xi\, d\eta$$

$$u(P) = C \iint_A \frac{A e^{ikr}}{r}\, d\xi\, d\eta \qquad \left(C = \frac{-i}{\lambda}\right)$$

(area element in circular aperture) $= \frac{-i/k}{2\pi} = -\frac{iA}{\lambda}\int_0^{2\pi}\int_0^{a} \frac{e^{ikr}}{r}\, \rho\, d\rho\, d\phi$

$$\rho^2 + d^2 = r^2$$
$$\rho\, d\rho = r\, dr$$

$$\therefore\ = \frac{-iA}{\lambda}\int_0^{2\pi}\int_0^{a} \frac{e^{ikr}}{r}\cdot r\, dr\, d\phi$$

$$= \frac{-iA}{\lambda}\int_0^{2\pi}\int_0^{a} e^{ikr}\, dr\, d\phi$$

$$= -\frac{iA}{\lambda}\int_0^{2\pi} d\phi \int_0^{a} e^{ikr}\, dr \,.$$

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$$= \frac{-iA}{d} 2\pi \int_{d}^{\sqrt{a^2+d^2}} e^{ikr} dr$$

$$= \frac{-iA}{d} 2\pi . \left[\frac{e^{ikr}}{ik}\right]_{a}^{\sqrt{a^2+d^2}}$$

$$= \frac{-iA}{d} \frac{2\pi}{ik} \left(e^{ik\sqrt{a^2+d^2}} - e^{ikd}\right)$$

$$= \frac{-iA}{d} \frac{2\pi}{ik} . e^{ikd} \left[e^{ik[\sqrt{a^2+d^2}-d]} - 1\right]$$

$$= -Ae^{ikd} \left[e^{ik(\sqrt{a^2+d^2}-d)} - 1\right]$$

$$= Ae^{ikd} \left[1 - e^{ik(\sqrt{a^2+d^2}-d)}\right]$$

Put $k(\sqrt{a^2+d^2} - d) = p\pi$

$$= Ae^{ikd} \left[1 - e^{ip\pi}\right]$$

$$= Ae^{ikd} e^{\frac{ip\pi}{2}} \left[e^{-\frac{ip\pi}{2}} - e^{ip\pi/2}\right]$$

$$= Ae^{ikd} . e^{\frac{ip\pi}{2}} . \left(2\sin \frac{p\pi}{2}\right)$$

$$= 2i\, e^{\frac{ip\pi}{2}} \left(2\sin \frac{p\pi}{2}\right)$$

$$\therefore \boxed{I = 4I_0 \sin^2 \frac{p\pi}{2}}$$

Fresnel No. $(p) = \frac{k}{\pi}\left[\sqrt{a^2+d^2} - d\right]$

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if $P \to$ odd, intensity is maximum
$P \to$ even, " " minimum

if d is much larger than a

$$\therefore \frac{k}{\pi}\left[d\left(1+\frac{a^2}{2d^2}\right)-d\right]$$

$$= \frac{k}{\pi} \frac{da^2}{2d}$$

$$\boxed{P = \frac{ka^2}{\pi 2d}}$$

$u_0(P) = u_1(P) + u_2(P)$ (Bamlette's Principle)

$u_2(P) = u_0(P) - u_1(P)$

$-u_2(P) = u_0(P)\left[1 - 1 + e^{iP\pi}\right]$

$\cancel{-}u_2(P) = \cancel{-}u_0(P)(e^{iP\pi})$

Poison spot $\to$ Bright fringe is at centre.

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Optics

Laser

$$u(P) = \frac{-iA}{\lambda} \iint \frac{e^{ikr}}{r} \, d\xi \, d\eta$$

(distance), $r = \left[(x-\xi)^2 + (y-\eta)^2 + z^2\right]^{1/2}$

$$r = z\left[1 + \frac{(x-\xi)^2}{z^2} + \frac{(y-\eta)^2}{z^2}\right]^{1/2}$$

neglecting high order terms

$$r \approx z + \frac{(x-\xi)^2}{2z} + \frac{(y-\eta)^2}{2z}$$

$\because |x-\xi| \ll z$

$|y-\eta| \ll z$

$$u(P) \approx \frac{-i}{\lambda} \iint_A \frac{A(\xi,\eta)}{z} e^{ik\left[z + \frac{(x-\xi)^2}{2z} + \frac{(y-\eta)^2}{2z}\right]} d\xi \, d\eta$$

$$\boxed{u(P) = \frac{-ie^{ikz}}{\lambda z} \iint A(\xi,\eta) e^{ik\left[\frac{(x-\xi)^2}{2z} + \frac{(y-\eta)^2}{2z}\right]} d\xi \, d\eta}$$

$$A(\xi,\eta) = A$$

$$u(P) = \frac{-iAe^{ikz}}{\lambda z} \iint e^{ik\frac{(x-\xi)^2}{2z}} d\xi \int e^{ik\frac{(y-\eta)^2}{2z}} d\eta$$

Standard expression, $\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} = \sqrt{\frac{\pi}{\alpha}} \exp\left(\frac{\beta^2}{4\alpha}\right)$

Let $(x-\xi)^2 = x^2$

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$$u(P) = \int_{-\alpha}^{\alpha} e^{\frac{ikx^2}{2z}}\, d\xi \int_{-\alpha}^{\alpha} e^{\frac{iky^2}{2z}}\, d\eta \qquad \left(\alpha = \frac{-ik}{2z}\right)$$

integrating,

$$= \frac{-iAe^{ikz}}{\lambda z} \cdot \frac{\pi (2z)}{-ik}$$

$$u(P) = -Ae^{ikz}$$

Laser Beam Divergence

Phase front at $z=0$

$$A(\xi,\eta) = a \exp\left[-\frac{\xi^2+\eta^2}{w_0^2}\right]$$

$$u(P) = \frac{-i^k e^{ikz}}{\lambda z} \int_{-\alpha}^{\alpha} \exp\left[\frac{ik}{2z}(x-\xi)^2 - \frac{\xi^2}{w_0^2}\right] d\xi$$

$$\int_{-\alpha}^{\alpha} \exp\left[\frac{ik}{2z}(y-\eta)^2 - \frac{\eta^2}{w_0^2}\right] d\eta$$

$$u(P) = \frac{-i^k e^{ikz}}{\lambda z} \int_{-\alpha}^{\alpha} \exp\left[\frac{ik}{2z}(x^2+\xi^2-2x\xi) - \frac{\xi^2}{w_0^2}\right] d\xi$$

$$\int_{-\alpha}^{\alpha} \exp\left[\frac{ik}{2z}(y^2+\eta^2-2y\eta) - \frac{\eta^2}{w_0^2}\right] d\eta$$

$$= \frac{-iae^{ikz}}{\lambda z} \cdot e^{\frac{ik}{2z}(x^2+y^2)} \int_{-\alpha}^{\alpha} e^{-\alpha\xi^2+\beta_1\xi}\, d\xi \int_{-\alpha}^{\alpha} e^{-\alpha\eta^2+\beta_2\eta}\, d\eta$$

$$\alpha = \frac{1}{w_0^2} - \frac{ik}{2z} = \frac{-ik}{2z}(1+i\gamma)$$

$$\beta_1 = \frac{-ikx}{2z}, \quad \beta_2 = \frac{-iky}{2z}$$

$$\gamma = \frac{\lambda z}{\pi w_0^2}$$

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$$= \frac{-iae^{ikz}}{\lambda z}\, e^{\frac{ik}{2z}(x^2+y^2)} \frac{\pi}{\frac{-ik}{2z}(1+i\gamma)} \exp\left[\frac{\beta_1^2}{4\alpha}\right]\exp\left[\frac{\beta_2^2}{4\alpha}\right]$$

$$= \frac{-iae^{ikz}}{\lambda z}\, e^{\frac{ik}{2z}(x^2+y^2)} \frac{\pi}{\frac{-ik}{2z}(1+i\gamma)} \exp\left[\frac{\frac{-ikx}{2z}}{4\alpha}\right] \exp\left[\frac{\frac{-iky}{2z}}{4\alpha}\right]$$

$$u(P) = \frac{a}{1+i\gamma} \exp\left[\frac{-(x^2+y^2)}{w^2(z)}\right] e^{i\phi z}$$

$$w(z) = w_0(1+\gamma^2)^{1/2} = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{1/2}$$

$$\phi = kz + \frac{k}{2z}(x^2+y^2) - \frac{k(x^2+y^2)}{2z(1+\gamma^2)}$$

as distance ↑ing, Divergen is also ↑ing.

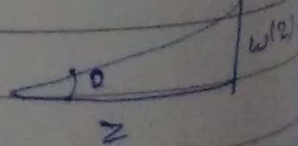
$$I = \frac{I_0}{1+\gamma^2} \exp\left[-2\,\frac{x^2+y^2}{w^2(z)}\right]$$

∵ $w_0$ is very small, So neglected it

$$w(z) \approx \cancel{w_0}\frac{\lambda z}{\pi w_0^2} = \frac{\lambda z}{\pi w_0}$$

Divergence angle, $\tan\theta \approx \frac{w(z)}{z}$

$$\approx \frac{\lambda z}{\pi w_0} \sim \frac{\lambda}{\pi w_0}$$



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* Fresenel diffraction using fresenel integrals

Two fresenel integrals are,

(1) cos, $C(z) = \int_0^{z} \cos \frac{1}{2}(\pi u^2)\, du$

(2) sin, $S(z) = \int_0^{z} \sin\left(\frac{1}{2}\pi u^2\right) du$

$$C(-z) = -C(z)$$
$$S(-z) = -S(z)$$

$$\int_{-\alpha}^{\alpha} e^{\frac{i\pi u^2}{2}}\, du = \sqrt{\frac{\pi}{\frac{-i\pi}{2}}} = \sqrt{\frac{2}{-i}} \times \frac{\sqrt{i}}{\sqrt{i}} = \sqrt{2}\sqrt{i} = \cancel{\sqrt{2}}\, e^{i\pi/2}$$

$$e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$

$$= \sqrt{2}\, e^{i\pi/4}$$
$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$
$$= 1 + i$$

$$\int_0^{\infty} e^{i\pi u^2}\, du = \frac{1+i}{2}$$

$$\int_0^{\infty} \cos\left(\frac{\pi u^2}{2}\right) du + i\int_0^{\infty} \sin \frac{\pi u^2}{2}\, du = \frac{1+i}{2}$$

$$\int_0^{\infty} \cos \frac{\pi u^2}{2}\, du = \frac{1}{2}, \quad \int_0^{\infty} \sin \frac{\pi u^2}{2}\, du = \frac{1}{2}$$

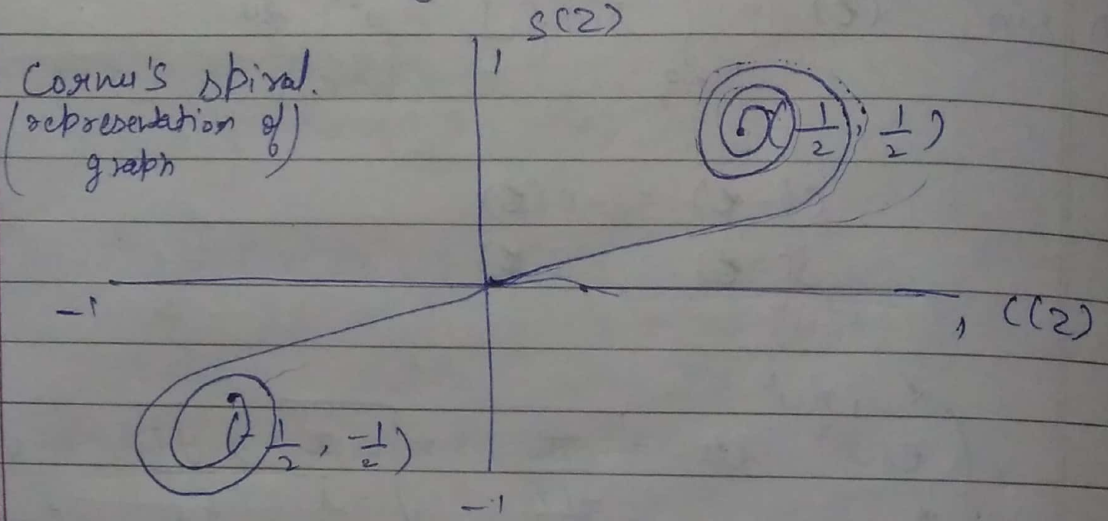
$$\boxed{C(\infty) = \frac{1}{2}} \qquad \boxed{S(\infty) = \frac{1}{2}}$$

$$C(-\alpha) = -C(\alpha) = \frac{-1}{2}$$

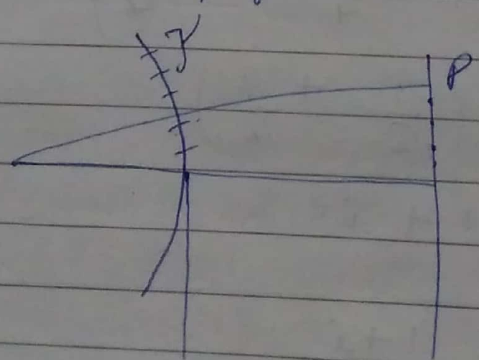
$$S(-\alpha) = -S(\alpha) = \frac{-1}{2}$$

$C(0) = 0, \quad S(0) = 0$

$\therefore$ graph move from 1 corner to another and move symmetrically.



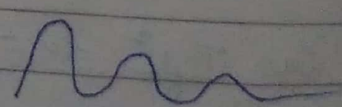

Half Period zones




UP + I ~~[illegible]~~ Maxima

+ I + - II Minima

For Half Period zone,



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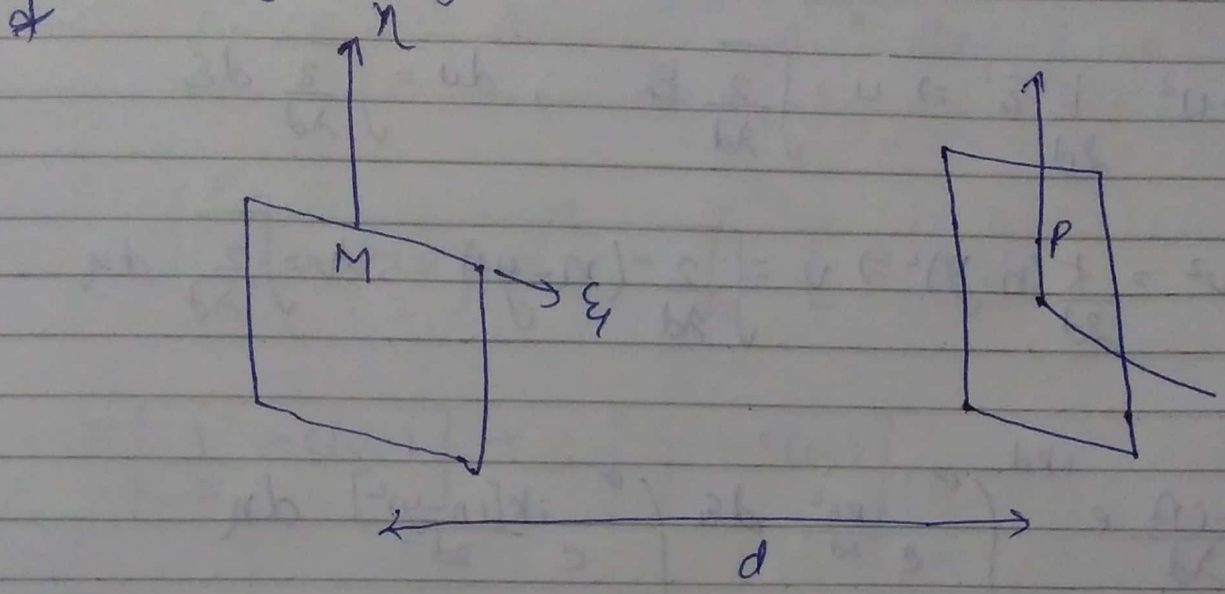
## Optics

Straight edge



$$u(P) = \frac{-i}{\lambda} \iint A\, e^{\frac{ikr}{r}}\, d\xi\, d\eta$$

$$r = MP = \xi^2 + \left[(\eta - y)^2 + d^2\right]^{1/2}$$

$$\approx d + \frac{\xi^2 + (\eta - y^2)}{2d}$$

$$u(P) = \frac{-iA}{\lambda d} \int_{-\infty}^{\infty}\int_{0}^{\infty} e^{ik\left[d + \frac{\xi^2 + (\eta - y)^2}{2d}\right]}\, d\xi\, d\eta$$

$$u(P) = -\frac{iA}{\lambda d}\int_{-\infty}^{\infty}$$

$$\cdot\ \frac{1}{2}\pi u^2 = \frac{k}{2d}\varepsilon_1^2 \Rightarrow u = \sqrt{\frac{2}{\lambda d}}\,\varepsilon_1 \quad , \ du = \sqrt{\frac{2}{\lambda d}}\,d\varepsilon_1$$

$$\frac{1}{2}\pi v^2 = \frac{k}{2d}(n-y)^2 \Rightarrow v = \sqrt{\frac{2}{\lambda d}}\,(n-y) \quad , dv = \sqrt{\frac{2}{\lambda d}}\,dn$$

$$u(P) = \frac{-iA}{\lambda d}e^{ikd}\int_{-\infty}^{\infty} e^{\frac{ik\varepsilon_1^2}{2d}}\,d\varepsilon_1 \int_{0}^{\infty} e^{\frac{ik[(n-y)^2]}{2d}}\,dn$$

$$u(P) = \frac{iA}{\cancel{\lambda d}}e^{ikd}\ \frac{\cancel{\lambda d}}{2} \Rightarrow \frac{iAe^{ikd}}{2}$$

$$u(P) = \frac{-iU_0}{2}\int_{-\infty}^{\infty} e^{\frac{i\pi u^2}{2}}\,du \int_{v_0} e^{\frac{i\pi v^2}{2}}\,dv \quad [U_0 = Ae^{ikd}]$$

$$n = 0 \quad , v = -\sqrt{\frac{2}{\lambda d}}\,y = v_0$$

$$\int_{-\infty}^{\infty} e^{\frac{i\pi u^2}{2}} du = 2\left[C(\alpha) + iS(\alpha)\right]$$

$$\int_{v_0}^{\infty} e^{\frac{i\pi v^2}{2}} dv = \int_{0}^{\infty} e^{\frac{i\pi v^2}{2}} dv - \int_{0}^{v_0} e^{\frac{i\pi v^2}{2}} dv$$

$$\int_0^{\infty}\left[\cos\left(\frac{\pi v^2}{2}\right) dv\right] - \int_0^{v_0}\left[\quad\right]$$

$$= \left[\frac{1}{2} - C(v_0)\right] + i\left[\frac{1}{2} - S(v_0)\right]$$

$$u(P) = \frac{i u_0}{2}(1+i)\left[\left(\frac{1}{2} - C(v_0)\right) + i\left(\frac{1}{2} - S(v_0)\right)\right]$$

$$u(P) = \frac{u_0}{2}(1-i)\left[\left(\frac{1}{2} - C(v_0)\right) + i\left(\frac{1}{2} - S(v_0)\right)\right]$$

Deep down in geomatrical shadow region, intensity is 0

$y = -\infty$, $v_0 = +\infty$, $C(v_0) = S(v_0) = \frac{1}{2}$

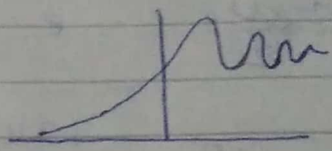
$\therefore \frac{1}{2} - \frac{1}{2} = 0$

$\therefore u(P) = 0$

$\therefore$ I also 0

A) edge, of

$y = 0 \quad , v_0 = 0$

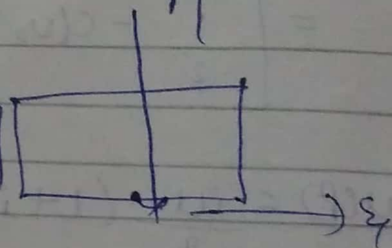


$$u(P) = \frac{u_0}{2}(1-i)\left[\frac{1}{2} + \frac{1}{2}i\right] \qquad C(v_0) = S(v_0) = 0$$

$$u(P) = \frac{u_0}{2}(1-i)\left(\frac{1+i}{2}\right) \Rightarrow \frac{u_0}{4}(1-i^2) \Rightarrow \cancel{\frac{u_0}{4}} \quad \cancel{I = \frac{I_0}{4}}$$

$$\Rightarrow \frac{u_0}{4} \cdot 2 \Rightarrow \frac{u_0}{2} \quad \therefore I = \frac{I_0}{4}$$

$$u(P) = \frac{-iA}{\lambda d}\int_{-\infty}^{\infty} d\xi \int_{-b/2}^{b/2} d\eta \left[\exp\left[ik\left\{d + \frac{\xi^2 + (\eta - y)^2}{2d}\right\}\right]\right]$$




$$u(P) = \frac{-i}{2}u_0\int_{-\infty}^{\infty}\exp\left(\frac{i\pi u^2}{2}\right)du \int_{-(v_2+v_1)}^{-(v_2-v_1)}\exp\left(\frac{1}{2}\pi v^2\right)dv$$

$$v_1 = \sqrt{\frac{2}{\lambda d}}\,\frac{b}{a} \quad , \quad v_2 = \sqrt{\frac{2}{\lambda d}}\,y$$

$$\eta = \frac{-b}{2} \quad , \quad v = \sqrt{\frac{2}{\lambda d}}\,(\eta - y)$$

$n = b/2$ , $v = \sqrt{\frac{2}{\lambda d}}\left(-\frac{b}{2} - y\right)$

$n = b/2$ , $v = \sqrt{\frac{2}{\lambda d}}\left(\frac{b}{2} - y\right)$

$\cancel{u(P) = -\frac{iA}{\lambda d}\int_{-\infty}^{\infty} d\xi}$

$$u(P) = \frac{-i}{2} u_0 \int_{-\infty}^{\infty} \exp\left(\frac{i\pi u^2}{2}\right) du \int_0^{-(v_2-v_1)} \cos\frac{1}{2}(\pi v^2)\, dv + i(\quad) - \int_0^{-(v_2+v_1)} (\quad)$$

$$u(P) = \frac{1-i}{2} u_0 \left[\left\{C(v_2+v_1) - C(v_2-v_1)\right\} + i\left\{S(v_2+v_1) - S(v_2-v_1)\right\}\right]$$

$$I(P) = \frac{I_0}{2}\left[\left\{C(v_2+v_1) - C(v_2-v_1)\right\}^2 + \left\{S(v_2+v_1) - S(v_2-v_1)\right\}^2\right]$$

$$= \frac{I_0}{2}\left[(x_2-x_1)^2 + (y_2-y_1)^2\right]$$

$v_1 = \sqrt{\frac{2}{\lambda d}}\,\frac{b}{2}$ fixed.

$v_2 = \sqrt{\frac{2}{\lambda d}}\, y \Rightarrow \sqrt{\frac{2d}{\lambda}}\,\frac{y}{d} \Rightarrow \sqrt{\frac{2d}{\lambda}}\,\phi$

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if d is large then, $V_2$ is large, then it is in Limit of fraunhofer differaction.

$$C(V) = \int_0^V \cos \frac{\pi v^2}{2} \, dv$$

$$= \int_0^{\infty} \qquad - \int_V^{\infty} \frac{1}{\pi v} \cos \frac{\pi v^2}{2} \pi v \, dv$$

$$= \frac{1}{2} - \frac{1}{\pi v} \sin \frac{\pi v^2}{2} \int_V^{\infty} + \int_V^{\infty} \frac{1}{\pi v^2} \sin \frac{\pi v^2}{2} \, dv$$

$$\simeq \frac{1}{2} + \frac{1}{\pi v} \sin \frac{\pi v^2}{2}$$

Similary

$$S(V) \approx \frac{1}{2} - \frac{1}{\pi v} \cos \frac{\pi v^2}{2}$$

if $V_2 \to$ large $\qquad V_1 \to$ small

$$C(V_2 + V_1) - C(V_2 - V_1) \approx \left[ \frac{1}{2} + \frac{1}{\pi V_2} \sin \frac{\pi (V_2 - V_1)^2}{2} \right] - \left[ \frac{1}{2} + \frac{1}{\pi V_2} \sin \pi \frac{(V_2 - V_1)^2}{2} \right]$$

After solving

$$= \frac{2}{\pi v_2} \cos \frac{\pi}{2}(v_2^2 + v_1^2) \sin(\pi v_1 v_2)$$

$$= \frac{2}{\pi v_2} \sin \frac{\pi}{2}(v_2^2 + v_1^2) \sin \frac{\pi}{2}(v_2^2 + v_1^2) \sin \pi v_1 v_2$$

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