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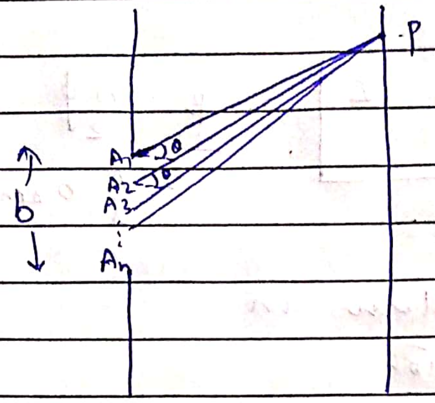
Diffraction

Say 'N' no. of point sources comprising the slit $A_1 A_2 \ldots A_n$



$$b = (n-1)\Delta$$

when $n \to \infty$; $\Delta \to 0$

As n is large,

$$b \approx n\Delta$$

Δ: distance b/w A_1 & A_2, A_2 & A_3...

$$E = a\cos\omega t + a\cos(\omega t - \phi) + a\cos(\omega t - 3\phi) + \ldots$$

amplitude of electric field

$$E = a\left[\cos\omega t + \cos(\omega t - \phi) + \cos(\omega t - 3\phi) + \ldots\right]$$

b/w A_1 & A_2, path diff = $\Delta \sin\theta$

$$\phi = \frac{2\pi}{\lambda}\Delta\sin\theta$$

So, $$E = a\frac{\sin n\phi/2}{\sin\phi/2}\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right]$$

$$\Rightarrow E = E_0\cos\left(\omega t - \tfrac{1}{2}(n-1)\phi\right)$$

Now, $$\phi = \frac{2\pi}{\lambda}\Delta\sin\theta$$

$$n\phi = \frac{2\pi}{\lambda}n\Delta\sin\theta = \frac{2\pi}{\lambda}b\sin\theta$$

As Δ is very small, so is $\phi/2$ and we can approximate $\sin\phi/2$ as $\phi/2$.

$$E_0 = a\frac{\sin n\phi/2}{\phi/2}\times\frac{n}{n} = na\frac{\sin\beta}{\beta}$$

where $\beta = \frac{n\phi}{2}$

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$\Rightarrow E_0 = A \frac{\sin\beta}{\beta}$

So, $E = A \frac{\sin\beta}{\beta} \cos\left[\omega t - (n-1)\frac{\phi}{2}\right]$

$I = \langle E^2 \rangle = \boxed{I_0 \frac{\sin^2\beta}{\beta^2} = I}$ $\beta = \frac{n\phi}{2} = \frac{\pi}{\lambda} b \sin\theta$

$\langle \cos \text{ term} \rangle = \text{Constant} (\tfrac{1}{2})$

contains 't'

Intensity of diffraction pattern in single slit diffraction

$\beta = \frac{n\phi}{2}$, $\frac{\pi b \sin\theta}{\lambda} = \pm m\pi$ (minima condition)

$\Rightarrow b\sin\theta = \pm m\lambda$; $\sin\theta = 0$ for minima

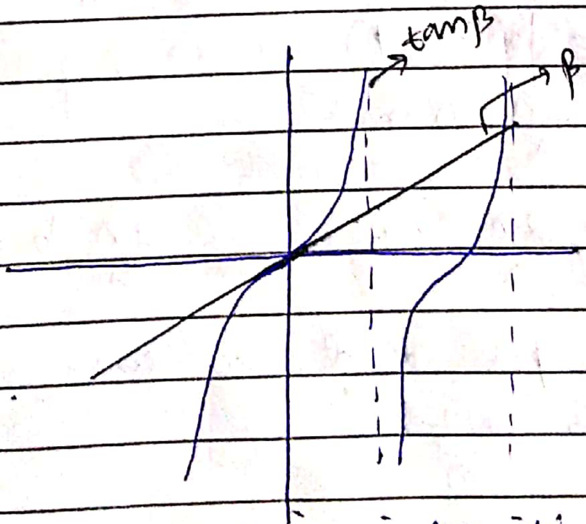
Principal maxima
$\boxed{\beta = 0,\ I = I_0}$

For maxima, $\frac{dI}{d\beta} = I_0\left[\frac{2\sin\beta\cos\beta\cdot\beta^2 - 2\beta\sin^2\beta}{\beta^4}\right] = 0$

gave the condition of minima

$\Rightarrow \frac{\sin\beta}{I_0}\left[\beta\cos\beta - \sin\beta\right] = 0$

$\Rightarrow \underline{\beta = \tan\beta} \rightarrow$ maxima



The points of intersection of the two graphs (β and $\tan\beta$) gives the points of maxima

The points come out as 1.43π, 0.246π

The consecutive intensities come out as
I_0, $0.0496\, I_0$, $0.0168\, I_0$, $0.0083\, I_0$ ...
(4.96 % of ~~initial~~ central maxima) (1.68 %) (0.83 %)

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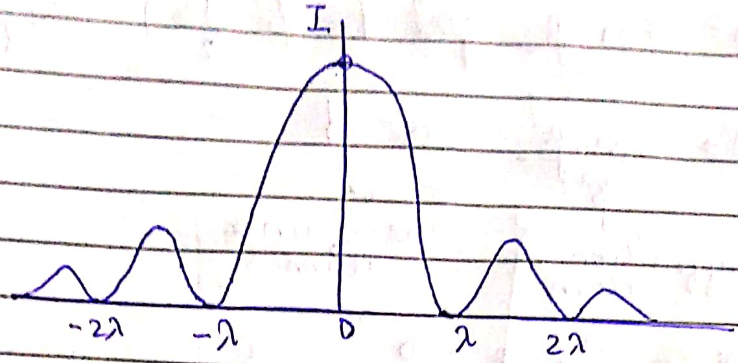
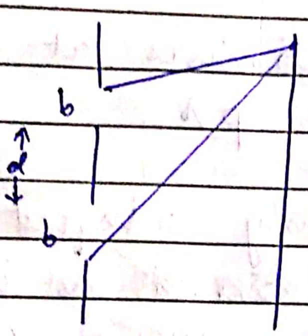


⇒ Double slit Diffraction



$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$\beta = \frac{n\phi}{2} \quad \because \frac{(n-1)}{2}\phi \approx \frac{n\phi}{2}$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \phi_1)$$

$$\phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

Final superposition occurs by superimposition of the two intensities. (add entire pattern instead of individual terms like interference)

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1)\right]$$

$$= 2A\frac{\sin\beta}{\beta}\cos\left(\frac{2\omega t - 2\beta - \phi_1}{2}\right)\cos\left(\frac{\phi_1}{2}\right)$$

$$= 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{\phi_1}{2}\right)$$

$$\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta$$

Now $I = \langle E^2 \rangle$

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time avg value of cos term

$$I = 4A^2 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma \left(\frac{1}{2}\right)$$

as it contains 't' (optical frequencies)

$$= I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{Diffraction term}} \underbrace{\cos^2\gamma}_{\text{Interference term.}}$$

1) Principal maxima : $\beta = 0$
2) Diffraction Minima : $b\sin\theta = m\lambda$
3) Interference minima : $\cos^2\gamma = 0 \Rightarrow \gamma = (2n+1)\frac{\pi}{2}$

4) Interference maxima : When γ maximises at
$$d\sin\theta = p\lambda$$

When ② & ④ are simultaneously satisfied, then interference ~~pattern~~ maximum will not be observed as it should've, and is called the missing order.

Missing order : $\frac{b}{d} = \frac{m}{p}$

$$\Rightarrow d = \left(\frac{p}{m}\right) b = kb$$; k will be an integer as an integer maxima is suppressed

So, whenever this is satisfied, missing order occurs. The maxima is suppressed.

When $d = b$, it effectively behaves as single slit.

$\Rightarrow$ N- slit Diffraction

$$E = \frac{A\sin\beta}{\beta}\left[\cos(\omega t-\beta) + \cos(\omega t-\beta-\phi_1) + \cos(\omega t-\beta-2\phi_1) + \dots + \cos\left[\omega t-\beta-(N-1)\phi_1\right]\right]$$

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$$E = A \frac{\sin\beta}{\beta} \frac{\sin N\gamma}{\sin\gamma} \cos\left[\omega t - \beta - \frac{1}{2}(N-1)\phi_1\right]$$

where $\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$

$$I = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{Diffraction}} \underbrace{\frac{\sin^2 N\gamma}{\sin^2\gamma}}_{\text{Interference}}$$

Interference minima : $N\gamma = p\pi$

$\Rightarrow \gamma = \pm \frac{p\pi}{N}$

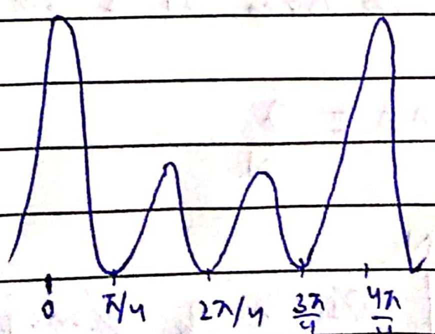
The interference minima will not be observed at $p = 0, \pm N, \pm 2N \ldots$
At these limiting values, interference maxima is observed.

$$\lim_{\gamma \to m\pi} \frac{\sin^2 N\gamma}{\sin^2\gamma} = \pm N \; ; \; I = I_0 \frac{\sin^2\beta}{\beta^2} N^2$$

For all values of $p < N$ (integers) there will be a minima for all p, and secondary maximas in between.

Consider $N = 4$, $\cancel{p}$ $\gamma = \frac{\pm p\pi}{4}$

Minimas at $\gamma = \pi/4, 2\pi/4, 3\pi/4$
Principal maximas at $\gamma = 0, \pi, \ldots$



$(n-1) \rightarrow$ minimas
$(n-2) \rightarrow$ secondary maximas

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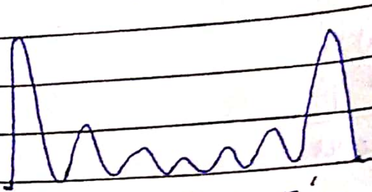
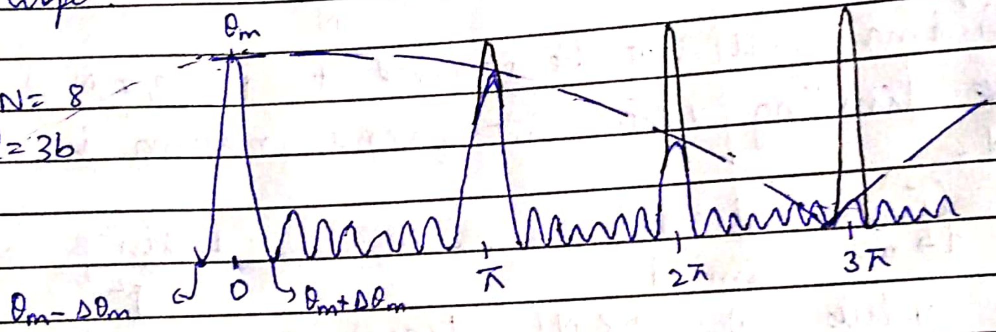
As N increases, the principal maxima is sharper, suppressing the secondary maxima further due to redistribution of energy.

N = 7



Now, due to diffraction, the intensity decreases from 1st to second maximum and so on, due to an envelope.

For N = 8
d = 3b



FWHM = $\Delta\theta_m$

At maxima, $d \sin\theta_m = m\lambda$

$$d \sin(\theta_m + \Delta\theta_m) = m\lambda \pm \lambda/N$$

$$d\left[\sin\theta_m \cos\Delta\theta_m \pm \sin\Delta\theta_m \cos\theta_m\right] = m\lambda \pm \frac{\lambda}{N}$$

$$\Rightarrow m\lambda \cos\Delta\theta_m \pm d \sin\Delta\theta_m \cos\theta_m = m\lambda \pm \frac{\lambda}{N}$$

As $\Delta\theta_m$ is very small, $\cos\Delta\theta_m \simeq 1$ & $\sin\Delta\theta_m \simeq \Delta\theta_m$

$$\Rightarrow m\lambda \pm d\,\Delta\theta_m \cos\theta_m = m\lambda \pm \frac{\lambda}{N}$$

$$\Rightarrow d \cos\theta_m \Delta\theta_m = \lambda/N$$

$$\Rightarrow \boxed{\Delta\theta_m = \frac{\lambda}{N d \cos\theta_m}}$$ width of principal maxima.

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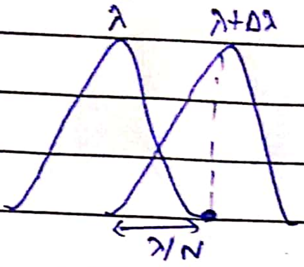
As 'N' and 'd' increase, the principal maxima becomes sharper.

<u>Resolving Power</u>:



$d\sin\theta = m(\lambda + \Delta\lambda)$ — ①
~~(Just resolved~~ maxima of $(\lambda+\Delta\lambda)$.

$d\sin\theta = m\lambda + \dfrac{\lambda}{N}$ — ②
minima of λ.

Both these equations have to be simultaneously satisfied due to just resolved condition.

① = ②

$$m\,\Delta\lambda = \frac{\lambda}{N} \Rightarrow \boxed{\frac{\lambda}{\Delta\lambda} = mN}$$

The formula suggests that as we increase 'N', the resolution can be increased to infinity. → dist b/w 2 slits
But from $D = Nd$, as we increase N, 'd' decreases, and 'd' limits the order of diffraction maxima.

$d\sin\theta = m\lambda$ → fixed

As we decrease 'd', the values of 'm' satisfying the equation reduces, hence the order of diffraction decreases. So, if the maxima are not observed, resolution has no meaning.

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