

Dielectrics

→ Dielectric is a material that can sustain an electric field but doesn't conduct electric current.

OR

It is a non-conducting/insulating substance that resists passage of electric current.

OR

It is an insulator, a substance that is highly resistant to flow of electric current. The layers of substances are commonly inserted into capacitors to improve their performance.

$$\Rightarrow Q = CV$$

$C = \epsilon_0 K \times A$ : capacitance of air-filled capacitor  
K: dielectric constant

$$Q_0 = \epsilon_0 V$$

$$Q = \frac{C \cdot V}{K}$$

$$Q = \epsilon_0 C V \Rightarrow Q = \epsilon_0 k A V$$

$$V = \frac{V_0}{K}, \text{ and } E = \frac{E_0}{K}$$

$E = \epsilon_0 \epsilon_r$ : absolute permittivity

$\epsilon_0$ : permittivity of free space

$\epsilon_r$ : relative permittivity

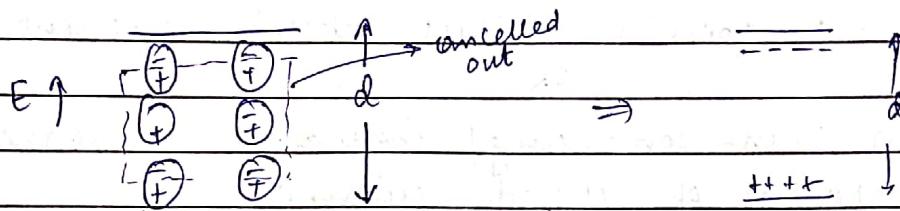
$$C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{\epsilon_0 A}{d}$$

$$\Rightarrow \text{Polarisation, } P = \frac{\sum \mu}{\text{vol}} \leftarrow \begin{array}{l} \text{summation of all} \\ \text{dipole moments} \end{array}$$

$$P = \frac{\langle \mu \rangle N}{\text{vol}} \rightarrow \begin{array}{l} \text{No. of dipoles} \\ \text{Avg } \mu \text{ of dipole} \\ \text{moments} \end{array}$$

$\Rightarrow P$  is a vector quantity, and its magnitude is equivalent to surface charge density.

Proof:



$$\frac{P_2}{V_s} = \frac{\sum_s \mu}{A(d)} = \frac{\sum_s q}{A(d)} \Rightarrow \boxed{P = \frac{Q}{A} \text{ surface charge density}}$$

$\Rightarrow$  Relation b/w  $P$  and  $E$ :

$$\vec{P} \propto \vec{E} \Rightarrow \boxed{\vec{P} = \epsilon_0 \chi \vec{E}}$$

$\chi$ : susceptibility = polarization per unit electric field

This formula is valid for low electric fields.

$\Rightarrow$  Relation b/w polarization & electrical displacement ( $D$ ):

Inside the material, the electric field strength  $\vec{E}$  was replaced by  $\vec{D}$ , called as electric displacement or electric flux density.

$$\boxed{\vec{D} = \epsilon_0 \epsilon_r \vec{E}}$$

$D_0$  is electric flux caused by external field  $\vec{E}$ , then

$$D = D_0 + P$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

$$= \epsilon_0$$

$$\Rightarrow$$

$$\boxed{\epsilon_r = 1 + \chi}$$

dielectric constant

Hence, the dielectric constant ( $k$ ) cannot be less than 1, as  $\chi$  can be a min. of zero.

$k < 10 \rightarrow$  low  $k$  material

$k > 10 \rightarrow$  high  $k$  material

## ① Ionic susceptibility & polarisation

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### → orientational / dipolar Polarisation.

We have a material with built in dipoles that are independent of each other (have random directions, and cannot sustain identical orientation),

i.e., they can rotate freely, in sharp contrast to ionic polarization.

For e.g., water that form natural dipoles because  $O^{2-}$  and  $H^+$  have different centre of charges and each molecule carries a dipole moment which can be drawn as a vector of constant length.

If we only draw a vector sum of dipole moment, then we get a 2-D picture. Here, the sum of all dipole moments will be zero; hence the polarization will also be zero.

Now, if we introduce electric field  $E$ . The dipoles would have tendency to turn into the direction of field because that would lower their energy, and hence the orientation of all dipoles shift a little, so that an average orientation in field direction results in the overall polarization. (This is called orientational polarization).

Consider a polar molecule like  $H_2O$ , which carries a permanent dipole moment,  $\mu = qd$ .

∴ Potential Energy of dipole is given by  $- \mu E \cos \theta = U$   
But, in thermal eqm, the relative no. of molecules with P.E 'U' is prop<sup>n</sup> to  $\exp(-\frac{U}{KT})$ .

Let  $N(\theta)$  be the no. of dipoles per unit solid angle at  $\theta$ :



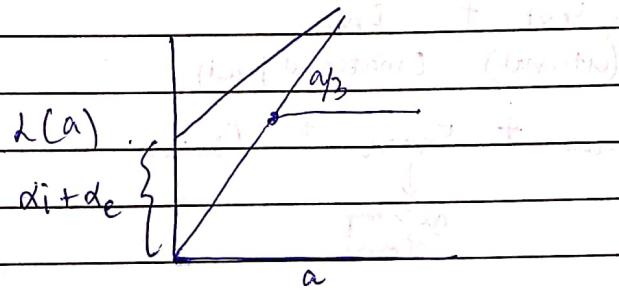
(i) when  $a \gg 1$ 

$$N_s \langle \mu \rangle = N_s \mu \lambda(a)$$

No. of dipoles  
in a certain angle  
(orientation)  $\rightarrow P_o = P_s \lambda(a)$ .

$$a = \frac{\mu E}{K_B T}$$

At condition of high electric field & high temp  
 $\lambda(a) = 1$  and  $P_o = P_s$

(ii)  $a \ll 1$ 

$$\lambda(a) = \coth h(a) - \frac{1}{a}$$

$$= \left[ \frac{1}{a} + \frac{a}{3} + \frac{a^3}{45} + \dots \right] - \frac{1}{a}$$

$$\Rightarrow \lambda(a) \approx \frac{a}{3}$$

$$\text{Now, } P_o = P_s \cdot \frac{a}{3} = N \mu \cdot \frac{\mu E}{3 K T}$$

Orientation  
Polarisation

$$P_o = \frac{N \mu^2 E}{3 K T}$$

$$\Rightarrow P_o = N \alpha_o E ; \quad \alpha_o = \frac{\mu^2}{3 K T}$$

$$\text{Now, } P = P_e + P_i + P_o \rightarrow \text{orientation (Temp. dependent)}$$

$$\text{Temp. independent} \quad P = N \alpha_e E + N \alpha_i E + \frac{N \mu^2 E}{3 K T}$$

$$\therefore \alpha_o = \alpha_e + \alpha_i + \alpha_o$$

\* When dielectric material is placed in external electric field, it is polarised, creating electric dipoles.

Each dipole sets electric field in the vicinity.

Hence, the net electric field at any point within the dielectric material is given by the sum of external field and the field due to all dipoles surrounding that point. This net field is called internal/local/Intrinsic field.

$$E_{\text{local}} = E_{\text{ext}} + E_m$$

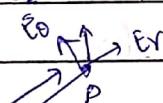
(external) (material field)

$$E_m = E_{\text{polarisation}} + E_{\text{near.}} + E_{\text{lattice}}$$

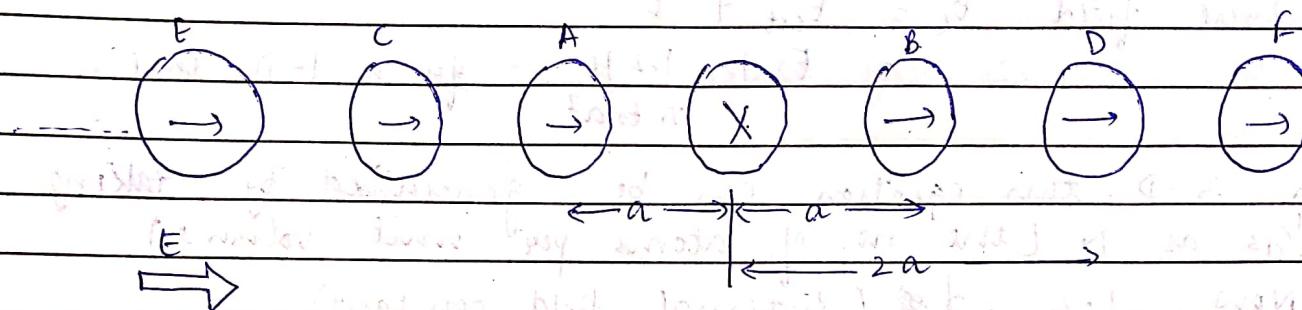
↓  
nearby atoms

$$\Rightarrow E_m = \frac{P}{3\epsilon_0}$$

Let us consider the dipoles with charge  $+q$  and  $-q$ .



Consider a dielectric material placed in external field  $E_0$  and also assuming an array of equidistant dipoles within the dielectric material which are aligned in the direction of field, as shown.



Let us find the local field at  $X$ , due to all dipoles in the array.

$$E_{xA} = E_x + E_0 \quad (\text{Field on } X \text{ due to dipole } A).$$

Here ( $r = a$  and  $\theta = 0^\circ + \beta - \alpha$ )

$$E_{xA} = \frac{2\mu \cos(0)}{4\pi \epsilon_0 a^3} = \frac{\mu}{2\pi \epsilon_0 a^3} \cdot 2 E_{xA}$$

Similarly,  $E_{xB} = \frac{2\mu \cos(-180)}{4\pi \epsilon_0 (-a)^3} = -\frac{\mu}{2\pi \epsilon_0 a^3}$

Hence, the total field at  $X$  due to equidistant dipoles A & B;  $E_1 = E_{xA} + E_{xB} = \mu$

$$E_1 = \frac{\mu}{2\pi \epsilon_0 a^3}$$

Similarly, the total field at  $X$  due to C & D dipoles,

$$E_2 = E_{xC} + E_{xD} = \mu$$

$$E_2 = E_{xE} + E_{xF} = \mu$$

Now, Total  $E' = E_1 + E_2 + E_3 + \dots$  with  $n$  terms

$$\text{Total field} = \frac{\mu}{\pi \epsilon_0 a^3} \left[ \frac{1}{1} + \frac{1}{2^3} + \frac{1}{3^3} + \dots \right]$$

$$\text{Total field} = \frac{\mu}{\pi \epsilon_0 a^3} \sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{3\pi^2 \mu}{4a^3}$$

Since  $\sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1.2$ , then

$$E' = \frac{1.2 \mu}{\pi a^3}$$

$$\therefore \text{Local field } E_L = E_{\text{ext}} + E' \\ = E + \frac{1.2 \mu}{\pi a^3} \text{ for a 1-D case.}$$

for 3-D, this equation can be generalized by taking  $\frac{1}{a^3}$  as  $N$  (the no. of atoms per unit volume)

$$\text{Now } 1.2 = \gamma \quad (\text{internal field constant})$$

$$E_L = E + \frac{dN\mu}{E_0 a^3}$$

$$E_L = E + \frac{dP}{E_0 a^3} \quad (\because P = N\mu z, N \propto E)$$

$$P = N \alpha e \left( E + \frac{P_0}{3\epsilon_0} \right)$$

$$\Rightarrow P = \frac{N \alpha e E}{\left( 1 - \frac{N \alpha e}{3\epsilon_0} \right)}$$

N: No. of dipoles per volume  
E: applied electric field

Polarisation is related to applied field strength and is given by

$$D = \epsilon_0 E + P$$

$$\Rightarrow \epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$\Rightarrow P = \epsilon_0 (\epsilon_r - 1) E$$

Equating the two given equations

$$\frac{N \alpha e}{\left( 1 - \frac{N \alpha e}{3\epsilon_0} \right)} = \epsilon_0 (\epsilon_r - 1)$$

$$\Rightarrow \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N \alpha e}{3\epsilon_0}$$

Clausing-Mossotti Relation  
(relation b/w dielectric constant and polarisability)

If  $n = \text{refractive index}$ , then

$$n^2 = \epsilon_r$$

$$\Rightarrow \frac{n^2 - 1}{n^2 + 2} = \frac{N \alpha e}{3\epsilon_0}$$

Dorenta-Lorentz equation  
in Optics

### \* Electronic Polarisation in a.c. field

In presence of alternating field  $E = E_0 e^{i\omega t}$ , the electron cloud would execute SHM, given by

$$m \frac{d^2 r}{dt^2} + 2b \frac{dr}{dt} + f_r = -e E_0 \exp(i\omega t) \quad (1)$$

This is the eqn of motion of a particle following electronic polarisation in a.c. field.

$m\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + f(x) = 0$  is the equation

for damped oscillations.

So, RHS in ① indicates external force (forced osc.).

$$2b = \frac{\mu_0 e^2 w_0^2}{6\pi m c} \quad \begin{matrix} \text{Damping} \\ \text{Factor} \end{matrix}$$

$w_0$ : natural frequency       $m$ : mass of particle

$c$ : speed of light

$$f = \frac{e^2}{4\pi\epsilon_0 R^2} \quad \begin{matrix} \text{Force} \\ \text{constant} \end{matrix}$$

$$\text{Now, } \frac{d^2x}{dt^2} + \frac{2b}{m} \frac{dx}{dt} + \frac{f}{m} x = -\frac{e E_0}{m} \exp(iwt)$$

Solution of this equation can be written as:

$$x = A \exp(iwt)$$

$$A i^2 w^2 \exp(iwt) + \frac{2b}{m} A i w \exp(iwt) + \frac{f}{m} A \exp(iwt) = -\frac{e E_0}{m} \exp(iwt)$$

$$\mu_e = \frac{e^2 E_0}{m} \exp(i\omega t)$$

$$(w_0^2 - w^2) + \frac{2ib\omega}{m}$$

Now:  $\mu_e = \alpha_e E$   
 $\Rightarrow \alpha_e = \frac{\mu_e}{E}$

$$\Rightarrow \alpha_e = \frac{e^2/m}{(w_0^2 - w^2) + i(2b\omega/m)}$$

As  $\alpha_e$  contains both imaginary and real parts:

$$\begin{aligned} \alpha_e &= \frac{e^2/m}{(w_0^2 - w^2) + i(2b\omega/m)} \\ &= \frac{e^2(w_0^2 - w^2)}{(w_0^2 - w^2)^2 + 4b^2\omega^2/m^2} + i \frac{e^2(2b\omega)}{m^2} \end{aligned}$$

Comparing it with  $\alpha_e = \alpha_e' + i\alpha_e''$

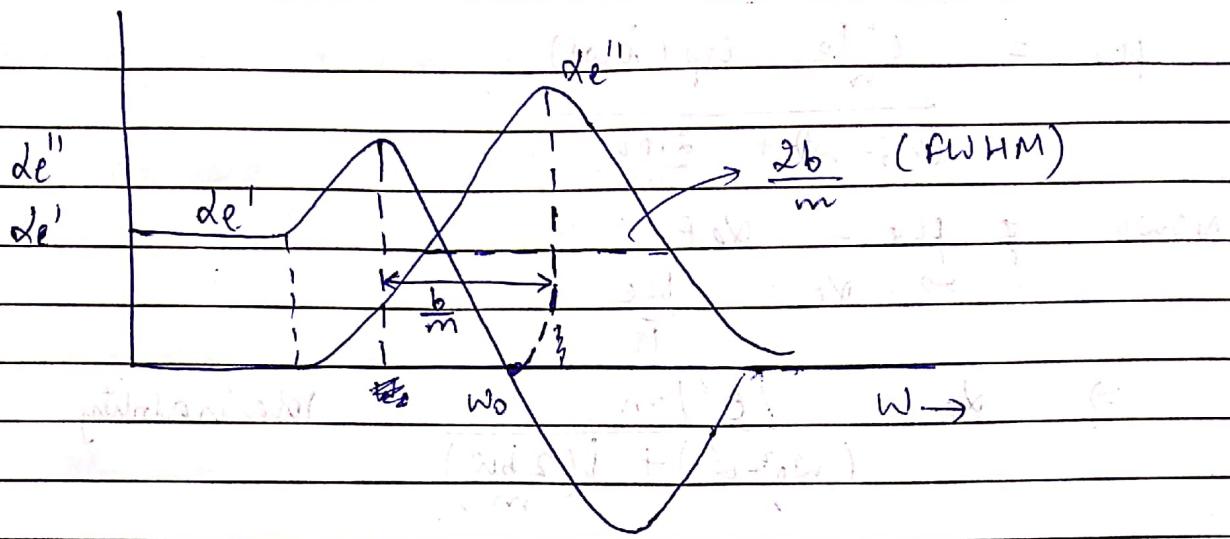
$$\alpha_e' = \frac{e^2}{m} \left[ \frac{(w_0^2 - w^2)}{(w_0^2 - w^2)^2 + 4b^2\omega^2/m^2} \right] \quad \begin{matrix} \text{real part} \\ \text{Imaginary part} \end{matrix}$$

$$\alpha_e'' = \frac{e^2}{m^2} \left[ \frac{2b\omega}{(w_0^2 - w^2)^2 + 4b^2\omega^2/m^2} \right] \quad \begin{matrix} \text{Imaginary part} \\ \text{Real part} \end{matrix}$$

Consider the graph in the book:

① When  $\omega = 0$ ,  $\alpha_e' = \frac{e^2}{m} \left( \frac{1}{w_0^2} \right)$

This is the static case. ( $D.C.$ )



② When  $\omega = \omega_0$ ,  $\alpha''_e = 0$  (at resonance)

$$\alpha''_e = \frac{e^2}{m} \quad (\text{maximum})$$

③ When  $\omega < \omega_0$ ; Both  $\alpha'_e$  &  $\alpha''_e$  are positive

④ When  $\omega > \omega_0$ ;  $\alpha'_e$  is negative  
 $\alpha''_e$  is positive

$$\text{Now, } \alpha = \alpha_e^0 + \alpha'_e + \alpha''_e$$

$$= \left( \frac{e^2}{m\omega_0^2} + \frac{e^2}{\omega_0^2} \left[ \frac{1}{M^+} + \frac{1}{M^-} \right] + \frac{\mu^2}{3kT} \right)$$

∴ Under the condition of static field.

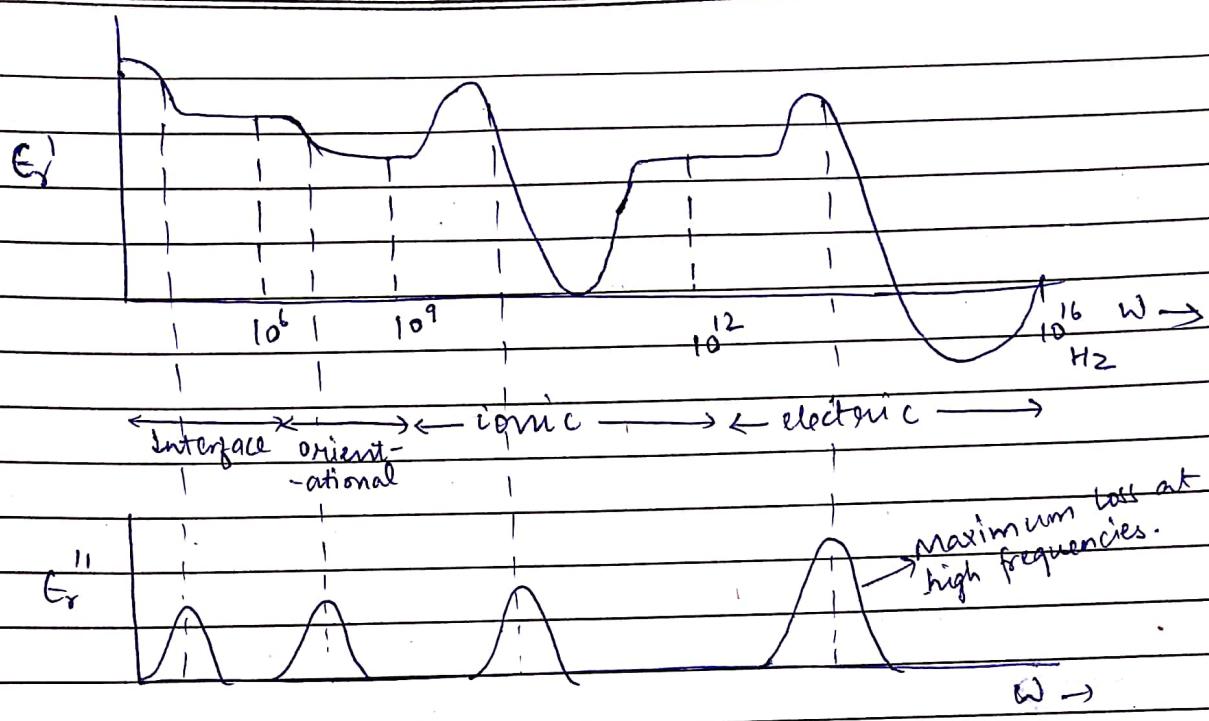
$$\alpha = \frac{e^2/m}{(\omega_0^2 - \omega^2) + 2ib\omega} + \frac{e^2/M \text{ (diss.)}}{(\omega_0^2 - \omega^2) + 2ib\omega} + \frac{\alpha_e^0(0)}{1 + i\omega\tau}$$

Under dynamic/ AC field

where  $M$ : reduced mass (approx. w.r.t. air/water)  
 $\tau$ : relaxation time ( $\sim 10^{-10} \text{ sec}$ )

Plot b/w dielectric constants & frequency:

$$\epsilon_r = \epsilon_r' - \frac{i\epsilon_r''}{\text{losses}}$$



Loss in dielectric are due to imaginary parts.