ASSIGNMENT-01

EP-208 COMPUTATIONAL METHODS

(ROOT AND INTERPOLATION)

1. Apply five approximations to obtain the root of the equation $x^3 - 5x + 3 = 0$ which lies between 0 and 1 by Bisection Method.

2. Apply five approximations to obtain the root of the equation $x\log_{10}x = 1.2$ which lies between 2 and 5 by Bisection Method.

3. Find a root of the following equations by false position method.

(i) $xe^x = \cos x$ root lies between (0,1)

(ii) $x^3 - x^2 - 2 = 0$ root lies between (1,2)

4. Find the root of $e^x - 4x = 0$ equations by Newtons method.

5. Show that

(i) $$u_1x + u_2x^2 + u_3x^3 + \cdots = \frac{x}{1-x}u_1 + \left(\frac{x}{1-x}\right)^2 \Delta u_1 + \left(\frac{x}{1-x}\right)^3 \Delta^2 u_1$$

(ii) $$u_0 + \frac{u_1x}{1!} + \frac{u_2x^2}{2!} + \frac{u_3x^3}{3!} + \cdots = e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \frac{x^3}{3!}\Delta^3 u_0 + \ldots\right)$$

6. Use Gauss's forward formula to evaluate y_{30}, given that

$y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$ and $y_{37} = 15.5154$

7. Find the value of e^x when (a) $x = 0.644$ using Stirling, Bessel and Everett's formulae (b) $x = 0.638$ using Stirling and Bessel formulae; from the following data values

x	0.61	0.62	0.63	0.64	0.65	0.66	0.67
$y = e^x$	1.840431	1.858928	1.877610	1.896481	1.915541	1.934792	1.954237

8. Use Striling's formula to interpolate the value of $y=e^x$ at x =1.91 from the data

x	1.7	1.8	1.9	2.0	2.1	2.2
$y = e^x$	5.4739	6.0496	6.6859	7.3891	8.1662	9.0250

Aditya Singh
2K19/EP/005

EP-208 (Computational Methods)

Assignment - 1

[1] $f(x) = x^3 - 5x + 3$.

1st iteration :

$f(0) = 3 > 0$ and $f(1) = -1 < 0$.

$\therefore \quad x_0 = \frac{0+1}{2} = 0.5$;

$f(x_0) = f(0.5) = 0.5^3 - 5(0.5) + 3 = 0.625 > 0$.

2nd iteration :

$f(0.5) = 0.625 > 0$ and $f(1) = -1 < 0$.

$\therefore \quad x_1 = \frac{0.5+1}{2} = 0.75$

$f(0.75) = 0.75^3 - 5(0.75) + 3 = -0.3281 < 0$

3rd iteration :

$f(0.5) = 0.625 > 0$ and $f(0.75) = -0.3281 < 0$

$\therefore \quad x_2 = \frac{0.5 + 0.75}{2} = 0.625$.

$f(x_2) = f(0.625) = 0.1191 > 0$.

4th iteration :

$f(0.625) = 0.1191 > 0$ and $f(0.75) = -0.3281 < 0$

$\therefore \quad x_3 = \frac{0.625 + 0.75}{2} = 0.6875$

$f(x_3) = f(0.6875) = -0.1125 < 0$,

5th Iteration :

$f(0.625) = 0.1191 > 0$ and $f(0.6875) = -0.1125 < 0$

$\therefore x_4 = \dfrac{0.625 + 0.6875}{2} = 0.6562$.

2

$f(x) = x\log_{10}(x) - 1.2$.

x	2	3	4	5
$f(x)$	-0.5979	0.2314	1.2082	2.2949

$f(2) = -0.5979 < 0$ and $f(3) = 0.2314 > 0$

$\therefore$ Root lies between 2 & 3.

1st iteration :

$f(2) < 0$ and $f(3) > 0$

$\therefore x_0 = \dfrac{2+3}{2} = 2.5$

$f(x_0) = f(2.5) = 2.5\log(2.5) - 1.2 = -0.2051 < 0$

2nd iteration :

$f(2.5) = -0.2051 < 0$ and $f(3) = 0.2314 > 0$.

$\therefore x_1 = \dfrac{2.5+3}{2} = 2.75$

$f(2.75) = 0.0082 > 0$.

3rd iteration :

$f(2.5) = -0.2051 < 0$ and $f(2.75) = 0.0082 > 0$

$x_2 = \dfrac{2.5 + 2.75}{2} = 2.625$.

$f(x_2) = f(2.625) = -0.0998 < 0$.

4th iteration :

$f(2.625) = -0.0998 < 0$ and $f(2.75) = 0.0082 > 0$

$x_3 = \frac{2.625 + 2.75}{2} = 2.6875$.

$f(x_3) = f(2.6875) = -0.0461 < 0$.

5th iteratn :

$f(2.6875) = -0.0461 < 0$ and $f(2.75) = 0.0082 > 0$

$x_4 = \frac{2.6875 + 2.75}{2} = \underline{2.7188}$.

$\boxed{3}$ i) $f(x) = xe^x - \cos x$

$f(0) = -1 < 0$ and $f(1) = 2.178 > 0$

$\therefore$ Root lies b/w $x_0 = 0$ and $x_1 = 1$,

$x_2 = x_0 - f(x_0) \frac{x_1 - x_0}{f(x_1) - f(x_0)}$.

$x_2 = 0 - (-1) \frac{1 - 0}{2.178 - (-1)} = 0.3147$.

$f(x_2) = f(0.3147) = 0.3147e^{0.3147} - \cos(0.3147) = -0.5199$

$f(0.3147) < 0$ and $f(1) > 0$,

$\therefore$ $x_0 = 0.3147$, $x_1 = 1$.

$x_3 = 0.3147 - (-0.5199) \frac{1 - 0.3147}{2.178 - (-0.5199)} = 0.4467$

$f(x_3) = f(0.4467) = -0.2035 < 0$.

$f(0.4467) <$ $f(1) > 0$.
x_0 x_1

$$x_4 = 0.4467 - (-0.2035) \frac{1 - 0.4467}{2.178 - (-0.2035)} = 0.494$$

$f(0.494) = -0.0708$.

$f(0.494) < 0$ and $f(1) > 0$
x_0 x_1

$$x_5 = 0.494 - (-0.0708) \frac{1 - 0.494}{2.178 - (-0.0708)} = 0.5099$$

$f(x_5) = -0.0236$.

$f(0.5099) < 0$ and $f(1) > 0$
x_0 x_1

$x_6 = 0.5152$, $f(x_6) = -0.0078 < 0$.

$f(0.5152) < 0$, $f(1) > 0$.
x_0 x_1

$x_7 = -0.0025$.

$x_8 = 0.5175$

$x_9 = 0.5177$

Ans = 0.5177.

ii) $f(x) = x^3 - x^2 - 2$,

$f(1) = -2 < 0$ and $f(2) = 2 > 0$.

$\therefore$ Root lies b/w $x_0 = 1.5$ and $x_1 = 2$.

$$x_2 = x_0 - f(x_0) \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 1-(-2)\frac{2-1}{2-(-2)} = 1.5$$

$$f(x_2) = f(1.5) = -0.875 .$$

$$\underset{x_0}{f(1.5)} < 0 , \quad \underset{x_1}{f(2)} = 2 > 0$$

$$x_3 = 1.5-(-0.875)\frac{2-1.5}{2-(-0.875)} = 1.6522$$

$$\underset{x_0}{f(1.6522)} = -0.2198 < 0 , \quad \underset{x_1}{f(2)} = 2 > 0 .$$

$$x_4 = 1.6522-(-0.2198)\frac{2-1.6522}{2-(-0.2198)} = 1.6866$$

$$\underset{x_0}{f(1.6866)} = -0.0468 < 0 , \quad \underset{x_1}{f(2)} = 2 > 0$$

$$x_5 = 1.6866-(-0.0468)\frac{2-1.6866}{2-(-0.0468)} = 1.6938 .$$

$$\underset{x_0}{f(1.6938)} = -0.0096 < 0 , \quad \underset{x_1}{f(2)} = 2 > 0$$

$$x_6 = 1.6938-(-0.0096)\frac{2-1.6938}{2-(-0.0096)} = 1.6952 .$$

$$f(1.6952) = -0.002 < 0 , \quad f(2) = 2 > 0$$

$$x_7 = 1.6952-(-0.002)\frac{2-1.6952}{2-(-0.002)}$$

$$= \underline{1.6955} .$$

4 $f(x) = e^x - 4x$, $f'(x) = e^x - 4$.

$x=0$, $f(0) = 1 > 0$

$x=1$, $f(1) = -1.2817 < 0$

$\therefore$ Root lies between 0 and 1,

$x_0 = \frac{0+1}{2} = 0.5$

1st iterath :

$f(x_0) = f(0.5) = e^{0.5} - 4(0.5) = -0.3513$

$f'(x_0) = e^{0.5} - 4 = -2.3513$.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.3506$.

2nd iterath :

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 0.3506 - \frac{0.0175}{-2.5801}$

$x_2 = 0.3574$

3rd iterath :

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 0.3574 - \frac{0}{'''}$.

$x_3 = 0.3574$.

5 (i) $u_1x + u_2x^2 + u_3x^3 + \dots = \frac{x}{1-x}u_1 + \left(\frac{x}{1-x}\right)^2 \Delta u_1 + \dots$

LHS ,

$u_1x + u_2x^2 + u_3x^3 + \dots = x(1 + xE + E^2x^2 + \dots)u_1$

$= x\frac{1}{1-xE}u_1$

$= x\left[\frac{1}{1-x(1+\Delta)}\right]u_1$

$$= x\left[\frac{1}{(1-x)-x\Delta}\right] u_1$$

$$= \frac{x}{(1-x)}\left[1-\frac{x\Delta}{(1-x)}\right]^{-1} u_1$$

$$= \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \cdots$$

$$= \underline{RHS}.$$

ii)

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \cdots$$

$$= u_0 + \frac{E^1 u_0 x}{1!} + \frac{E^2 u_0 x^2}{2!} + \frac{E^3 u_0 x^3}{3!} + \cdots$$

$$= u_0\left[1 + \frac{E^1 x}{1!} + \frac{E^2 x^2}{2!} + \frac{E^3 x^3}{3!} + \cdots\right]$$

$$\because \left(e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$

$$u_0 e^{Ex} = u_0 e^{(1+\Delta)x} = u_0 e^x e^{\Delta x}$$

$$= u_0 e^x\left(1 + \frac{\Delta x}{1!} + \frac{x^2\Delta^2}{2!} + \frac{x^3\Delta^3}{3!} + \cdots\right)$$

$$= e^x\left(u_0 + \frac{u_0\Delta x}{1!} + \frac{u_0 x^2\Delta^2}{2!} + \frac{u_0\Delta^3 x^3}{3!} + \cdots\right)$$

$$= \underline{\underline{RHS}}.$$

$\boxed{6}$ $h = 25 - 21 = 4$.

Taking $x_0 = 29$, $p = \dfrac{x - x_0}{h} = \dfrac{x-29}{4}$.

x	$p = \frac{x-29}{4}$	y	Δy	$\Delta^2 y$	$\Delta^2 y$	$\Delta^4 y$
21	-2	18.4708				
			-0.6564			
25	1	17.8144		-0.051		
			-0.7074		-0.0054	
29	0	17.107		-0.0564		-0.0022
			-0.7638		-0076	
33	1	16.3432		-0.064		
			-0.8278			
37	2	15.5154				

for $x = 30$, $p = \dfrac{x - x_0}{h} = \dfrac{1}{4} = 0.25$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \cdots$$

$$y_{0.25} = 17.107 + (0.25)(-0.7638) + \frac{(0.25)(0.25-1)}{2}(0.0564) +$$

$$\frac{(0.25+1)0.25(0.25-1)}{6}.(-0.0076) + \frac{(0.25+1)0.25(0.25-1)(0.25-2)}{24} \times (0.0022)$$

$$= 17.107 + (-0.1909) + 0.0052875 + 0.0002968 75$$

$$+ 0.0000375977$$

$$= 16.9216.$$

$\therefore y(30) = \underline{16.9216}$.

7

$h = 0.62 - 0.61 = 0.01$

Taking $x_0 = 0.64$, $p = \frac{x - x_0}{h} = \frac{x - 0.64}{0.01}$

x	$p = \frac{x-0.64}{0.01}$	y	Δy	$\Delta^2 y$	$\Delta^3 y$ $\times 10^{-6}$	$\Delta^4 y$ $\times 10^{-6}$	$\Delta^5 y$ $\times 10^{-6}$	$\Delta^6 y$ $\times 10^{-6}$
0.61	-3	1.840431						
			0.018497					
0.62	-2	1.858928		0.000185				
			0.018682		4			
0.63	-1	1.87761		0.000189		-4		
			0.018871		0		6	
0.64	0	1.896481		0.000189		2		-7
			0.01906		2		-1	
0.65	1	1.915541		0.000191		1		
			0.019351		3			
0.66	2	1.934892		0.000194				
			0.019445					
0.67	3	1.954237						

a) 0.638, $p = \frac{x - x_0}{h} = \frac{0.638 - 0.64}{0.01} = -0.2$

Stirling's Formula

$$y_p = y_0 + p.\frac{\Delta y_0 + \Delta y_{-1}}{2} + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2 - 1^2)}{3!}.\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \dots$$

$$y_{-0.2} = 1.896481 + (-0.2).\frac{(0.01906 + 0.018871)}{2} + \left(\frac{0.04}{2}\right).(0.000189)$$

$$+ \frac{(-0.2)(0.04-1)}{6}.\left(\frac{0.000002}{2}\right) + \frac{(0.04)(0.04-1)}{24}.(0.000002)$$

$y(0.638) = \underline{1.892692}$

Bessel's ~~formula~~ :

$$y_p = \frac{y_0+y_1}{2} + \left(p-\frac{1}{2}\right)\Delta y_0 + \frac{p(p-1)}{2!}\cdot\frac{\Delta^2 y_{-1}+\Delta^2 y_0}{2} + \frac{\left(p-\frac{1}{2}\right)p(p-1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{(p+1)p(p-1)(p-2)}{4!}\frac{\Delta^4 y_{-2}+\Delta^4 y_{-1}}{2} + \cdots$$

$$y_{-0.2} = \frac{1.896481 - 1.915541}{2} + \left(-0.2 - \frac{1}{2}\right)(0.01906)$$

$$+ \frac{-0.2(-0.2-1)}{2}\left(\frac{0.000189+0.000191}{2}\right) + \frac{\left(-0.2-\frac{1}{2}\right)0.2(-0.2-1)}{6} \times (0.000002)$$

$$y(0.638) = \underline{1.892692}.$$

Everett's formula ; $\quad q = 1-p = 1-(-0.2) = 1.2$.

$$y_p = qy_0 + \frac{q(q^2-1^2)}{3!}\Delta^2 y_{-1} + \frac{q(q^2-1)(q^2-2^2)}{5!}\Delta^4 y_{-2} +$$

$$\frac{q(q^2-1^2)(q^2-2^2)(q^2-3^2)}{7!}\Delta^6 y_{-3} + \cdots$$

$$y_{-0.2} = (1.2)(1.896481) + \frac{(1.2)(1.44-1)}{6}\cdot(0.000189)$$

$$+ \frac{(1.2)(1.44-1)(1.44-4)}{120}\cdot(2\times10^{-6}) + \frac{(1.2)(0.44)(1.44-4)(1.44-9)}{5040} \times (-7\times10^{-6})$$

$$y(0.638) = \underline{1.892692}.$$

b) $x = 0.644$

$$p = \frac{x - x_0}{h} = \frac{0.644 - 0.64}{0.01} = 0.4$$

Stirling's Formula:

$$y_{0.4} = 1.896481 + (0.4)\frac{(0.01906 - 0.018871)}{2} + \frac{0.16}{2}\cdot(0.000189)$$

$$+ \frac{(0.4)(0.16-1)}{6}\left(\frac{2\times10^{-6}}{2}\right) + \frac{(0.16)(0.16-1)}{24}\cdot(2\times10^{-6}) + \cdots$$

$$y_{0.4} = 1.898481 + 0.0075862 + 0.00001512 + \cancel{56x} \cdots$$

$$y(0.644) = \underline{1.904082}.$$

Bessel's Formula:

$$y_{0.4} = \frac{1.896481 + 1.915541}{2} + \left(0.4 - \frac{1}{2}\right)(0.01906) + \frac{(0.4)(0.4-1)}{2}$$

$$\left(\frac{0.000189 + 0.000191}{2}\right) + \frac{\left(0.4 - \frac{1}{2}\right)0.4(0.4-1)}{6}(2\times10^{-6}) + \cdots$$

$$y(0.644) = \underline{1.904082}.$$

Everett's Formula: $q = 1 - p = 1 - 0.4 = 0.6$.

$$y_{0.4} = (0.6)(1.896481) + \frac{(0.6)(0.36-1)}{6}\cdot(0.000189)$$

$$+ \frac{(0.6)(0.36-1)(0.36-4)}{120}\times(2\times10^{-6}) + \frac{0.6(0.36-1)(0.36-4)(0.36-9)}{5040}\times(-7\times10^{-6}).$$

$$+ \cdots$$

$$y(0.644) = \underline{1.904082}.$$

$\boxed{8}$ $h = 1.8 - 1.7 = 0.1$

Taking $x_0 = 1.9$, $p = \frac{x - x_0}{h} = \frac{x - 1.9}{0.1}$

x	$p = \frac{x-1.9}{0.1}$	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.7	-2	5.4739					
			0.5757				
1.8	-1	6.0496		0.0606			
			0.6363		0.0063		
1.9	0	6.6859		0.0669		0.0007	
			0.7032		0.007		0.0001
2.0	1	7.3891		0.0739		0.0008	
			0.7771		0.0078		
2.1	2	8.1662		0.0817			
			0.8588				
2.2	3	9.025					

for $x = 1.91$, $p = \frac{1.91 - 1.9}{0.1} = \underline{0.1}$.

Sterling formula is,

$$y_p = y_0 + p.\frac{\Delta y_0 + \Delta y_{-1}}{2} + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}.\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}$$
$$+ \frac{p^2(p^2-1^2)}{4!}\Delta^4 y_{-2} + \frac{p(p^2-1)(p^2-2^2)}{5!}.\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}.$$

$$y_{0.1} = 6.6859 + (0.1)\left(\frac{0.7032 + 0.6363}{2}\right) + \left(\frac{0.01}{2}\right)(0.0669)$$
$$+ \frac{(0.1)(0.01-1)}{6}\left(\frac{0.007 + 0.0063}{2}\right) + \frac{0.01(0.01-1)}{24}(0.0007)$$

$$y(1.91) = \underline{6.7531}$$