* Errors:-

→ Max. permissible error:-

→ It is denoted as $(\lambda_{exp} - \lambda_{true}) = "\Delta\lambda"$

Now the range will be,

$$[(\lambda_{exp} - \Delta\lambda) \text{ to } (\lambda_{exp} + \Delta\lambda)]$$

If

$$u = u(x_1, x_2, x_3, \ldots\ldots x_n)$$

error involved in measuring x_1 is dx_1

llly, " " " " x_2 is dx_2

⋮

and so on.

$$u + du = u(x_1 + dx_1, x_2 + dx_2, \ldots x_n + dx_n)$$

$$\cancel{u} + du = \cancel{u(x_1, x_2, \ldots x_n)} + \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} dx_i + \underbrace{\sum_{i=a} + \sum_{i=b}}_{\text{higher order terms}}$$

$$\boxed{du = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} dx_i}$$

where (du) is max. permissible error in (u).

Q) Let $u = \frac{xy}{z}$.

$x = 1$, $dx = 0.1$ | 0.1
$y = 1$, $dy = 0.1$ | 0.1
$z = 2$, $dz = 0.01$ | 0.1

$\log u = \log x + \log y - \log z$ | $u = 0.5 = 1/2$

$\log u = \log x + \log y + \log(\cancel{1}z)$

being we have add the errors.

$$\frac{du}{u} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$\frac{du}{u} = \frac{0.1}{1} + \frac{0.1}{1} + \frac{0.01}{2}$$

$$\frac{du}{u} = 0.2005 \quad \text{--- (1)} \quad \Rightarrow \boxed{du = 0.1025}$$

In 2nd case:-

$$\frac{du}{u} = 0.25$$

$$du = (0.25)(1/2)$$

$$\boxed{du = 0.125}$$

So, $u = (0.5 \pm 0.1025)$
$= (0.5 \pm 0.125)$

Q) $(e+d)\sin\theta = n\lambda$

$$\lambda = \frac{(e+d)\sin\theta}{n}$$

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(c) Let $u = \frac{xy}{z}$.

$x = 1$	, $dx = 0.1$	0.1
$y = 1$	, $dy = 0.1$	0.1
$z = 2$	, $dz = 0.01$	0.1

$\log u = \log x + \log y - \log z$ | $u = 0.5 = 1/2$

$\log u = \log x + \log y \; (+) \; \log(z)$

being we have add the errors.

$$\frac{du}{u} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$\frac{du}{u} = \frac{0.1}{1} + \frac{0.1}{1} + \frac{0.01}{2}$$

$$\frac{du}{u} = 0.205 \quad — \; (1) \quad \Rightarrow \boxed{du = 0.1025}$$

In 2nd case:

$$\frac{du}{u} = 0.25$$

$$du = (0.25)(1/2)$$

$$\boxed{du = 0.125}$$

So, $u = (0.5 \pm 0.1025)$

$= (0.5 \pm 0.125)$

d) $(e+d)\sin\theta = n\lambda$

$$\lambda = \frac{(e+d)\sin\theta}{n}$$

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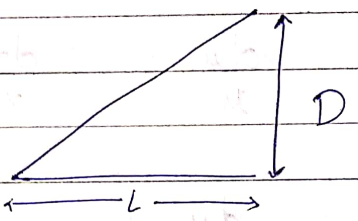
$$\log(\lambda) = \log(e+d) + \log(\sin\theta) + \log(n).$$

$$\frac{d\lambda}{\lambda} = 0 + \frac{\cos\theta}{\sin\theta} . d\theta + 0$$

$$\boxed{d\lambda = \lambda \cot\theta . d\theta}$$

2)

$$D = \frac{\lambda L}{2x} = \frac{\lambda L N}{2(x_2 - x_1)}$$



$$\log D = \log(\lambda) + \log(L) + \log(N) + \log(2) + \log(x_2 - x_1)$$

$$\frac{dD}{D} = \underbrace{\frac{d\lambda}{\lambda}}_{\text{Given}} + \frac{dL}{L} + \frac{dN}{N} + 0 + \frac{\cancel{d(x_2-x_1)}\,(dx_2 + dx_1)}{(x_2 - x_1)}$$

$$= 0 + \frac{dL}{L} + \frac{dN}{N} + \frac{2dx_1}{x_2 - x_1}$$

where $dN = 1$ (chone ... of ...

$$\boxed{\frac{dD}{D} = \frac{dL}{L} + \frac{1}{N} + \frac{2dx_1}{x_2 - x_1}}$$

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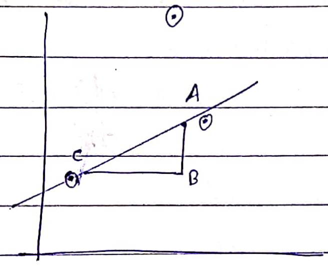
* Calculation of (μ) in Graphs:-

max. per. error

eg 1:-

$$\mu = A + \frac{B}{\lambda^2}$$



(μ) in A = A ± L.C of y-axis

(μ) in B = B ± $\frac{\text{L.C of AB}}{\text{L.C of BC}}$

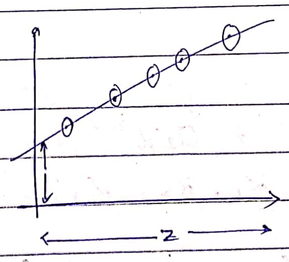
* Angle of divergence for laser beam: (θ):-

$$\theta = \frac{\lambda}{\pi\, \omega(0)}$$ → spot size in cavity.

$$\log \theta = \underbrace{\log(\lambda)}_{\to 0} + \underbrace{\log(\pi)}_{\to 0} + \log(\omega(0))$$

$$\boxed{\frac{d\theta}{\theta} = \frac{d\,\omega(0)}{\omega(0)}}$$

where error in $\omega(0)$ is L.C of y-axis



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(2)

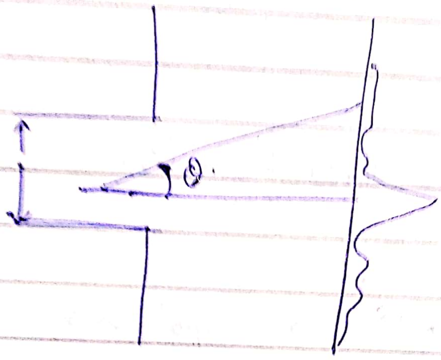
$$I = A^2 \frac{\sin^2\alpha}{\alpha^2}$$

$$\frac{dI}{d\alpha} = A^2 \frac{(\alpha^2)(\sin 2\alpha) - 2\alpha\sin^2\alpha}{\alpha^4} = 0$$

$$\alpha \sin 2\alpha - \sin^2\alpha = 0$$

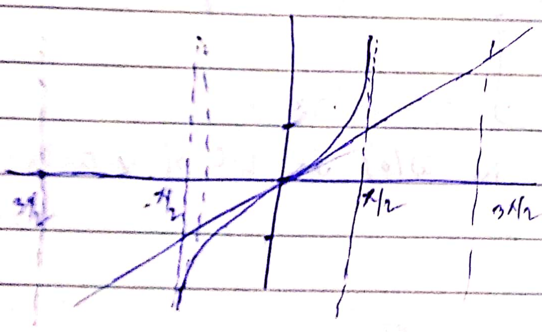
$$\sin^2\alpha - \alpha\sin 2\alpha = 0$$

$$\boxed{\alpha = \tan\alpha}$$



$$\alpha = \frac{\pi e \sin\theta}{\lambda}$$

$y = \alpha$ $\qquad$ $y = \tan\alpha$.



$\alpha = 0$	$y_1 = 0$	$y_2 = 0$
$\alpha = \pi/4$	$y_1 = \pi/4$	$y_2 = 1$
$\alpha = \pi/2$	$y_1 = \pi/2$	$y_2 = \infty$
$\alpha = 3\pi/2$	$y_1 = 3\pi/2$	$y_2 = -\infty$

$\Rightarrow$ $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots\cdots a_{n-1} x + a_n = 0$

where, $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$ are the roots of the eq^n.

$(x-\alpha_1)(x-\alpha_2)\ldots\ldots(x-\alpha_n) = 0$

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$$\sum \alpha_p = \frac{-a_1}{a_0}$$

$$\sum \alpha_1 \alpha_2 = a_2/a_0$$

$$\sum \alpha_1 \alpha_2 \alpha_3 = -a_3/a_0$$

$$\vdots$$

$$\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n = (-1)^n \, (a_n/a_0)$$

\# complex roots $= n-(p+q)$

=> To find the max no. of +ve roots of an eq^n:

$$f(x) = 2x^7 - x^5 + 4x^3 - 5$$

$$+ \quad - \quad + \quad -$$

$p = 3$ (becoz 3 times sign changes)

=> To find the max no. of -ve roots of an eq^n:

replace, $x \to (-x)$

$$f(x) = -2x^7 + x^5 - 4x^3 - 5$$

$$- \quad + \quad - \quad -$$

$q = 2$ (becoz 2 times sign changes.

=> To find the max no. of complex roots:

$= n - (p+q) = 7-(5) = 2.$

$n \hookrightarrow$ max power.

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Q) $3x^3 - 4x^2 + x + 88 = 0$

$\alpha = 2 + \sqrt{7}i$
$\beta = 2 - \sqrt{7}i$
$\gamma = ?$

A,
$\alpha + \beta + \gamma = -\left(\frac{-4}{3}\right) = \frac{4}{3}.$

$4 + \gamma = \frac{4}{3}.$

$\boxed{\gamma = -8/3}$

Q) $x^3 + 3x^2 + 4x + 2 = 0$ AP Series.

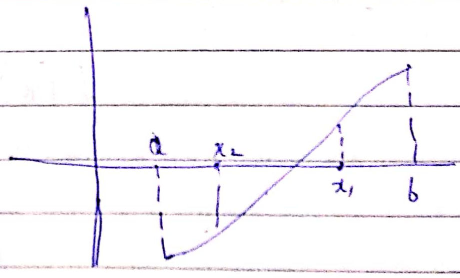
$x^3 + x^2 + 2x^2 + 2x + 2 = 0$

X ———— x ———— x ———— x ———— x ———— X

* Bisection Method:-

If $f(x) = 0$, $[a, b]$

$f(a)\,f(b) < 0$



Method:-

$f(a) = -ve$ $f(b) = +ve$

$x_1 = \frac{(a+b)}{2}$; $f(x_1) = +ve$

$x_2 = \frac{(a + x_1)}{2}$; $f(x_2) = -ve$

$x_3 = \frac{(x_1 + x_2)}{2}$, $f(x_3) = -ve$

$x_4 = \frac{(x_1 + x_3)}{}$, $f(x_4) =$

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→ Size of chord is given by : $\frac{(b-a)}{2^n}$

where $n \to$ no. of iterations.

→ If $\frac{b-a}{2^n} \leq \epsilon$ — (Accuracy), then the chord is in the limits

$$\frac{b-a}{2^n} \leq \epsilon$$

$$b-a \leq \epsilon(2^n).$$

Taking log both sides

$$\log(b-a) \leq \log\epsilon + n\log(2)$$

$$n\log 2 \geq \log(b-a) - \log\epsilon$$

$$\boxed{n \geq \frac{\log(b-a) - \log\epsilon}{\log(2)}}$$

Q)

$f(x) = x^3 - 4x - 9 \qquad [2, 3]$

$f(2) = -9 \to -ve, \quad f(3) = 6 \to +ve$

So, Ist approx. root $x_1 = \left(\frac{2+3}{2}\right) = 2.5, \; f(2.5) = -3.375$

2nd approx root $x_2 = \left(\frac{3+2.5}{2}\right) = 2.75, \; f(2.75) = 0.7969$

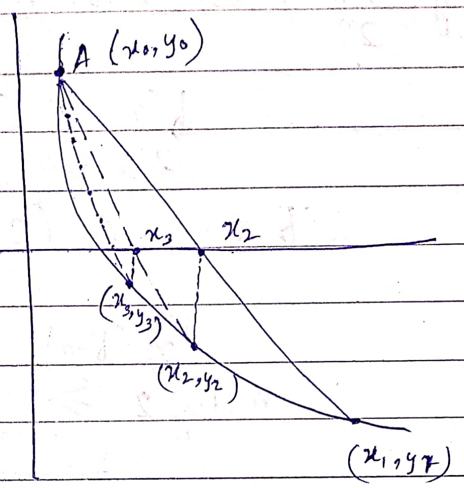
3rd approx. root $x_3 = \left(\frac{2.5+2.75}{2}\right) = 2.625, \; f(2.625) = -1.4121$

4th approx root $x_4 = \left(\frac{2.75+2.625}{2}\right) = 2.6875$

* Method of False Position :-
(Regular-falsi method)

If $f(x) = 0$, $[a, b]$, then $f(a)\,f(b) < 0$

$f(x_0) > 0$, $f(x_1) < 0$

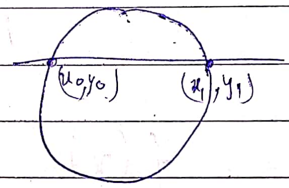


Method:- $(y - y_0) = \left(\dfrac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)$

$x = x_2$ $\quad y = 0$

$$0 - y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x_2 - x_0)$$

$$(x_2 - x_0) = -\left(\frac{x_1 - x_0}{y_1 - y_0}\right) y_0$$



first order approx. ← $\boxed{x_2 = x_0 - \dfrac{(x_1 - x_0)}{(y_1 - y_0)}\, y_0}$

If $y_0 = +ve$, $y_1 = -ve$ and if $y_2 = -ve$
then range will be $[x_0 \;\; x_2]$
↳ should be written in ring order

Now for (x_3), $x_0 \rightarrow x_0$ and
$x_1 \longrightarrow x_2$.

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$$x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_0) \quad , \quad f(x_2) = +ve.$$

x_0	x_1
$f(x_0)$	$f(x_1)$
$(+ve)$	$(-ve)$.

→ Now the nex root will lie b/w (x_1) and (x_2)

If $(x_1 \neq x_2) \Rightarrow$ So, $x_0 \leftarrow x_1$
$x_1 \leftarrow x_2$. and vice versa, if $(x_1 > x_2)$

So, $$x_3 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_0)$$

①) $x^3 - 2x - 5 = 0$ $[2, 3]$.

$f(x_0) = f(2) = -1$ $f(x_1) = f(3) = 16$

$x_0 = 2$ $x_1 = 3$.

I[st] approx. root $\Rightarrow x_2 = x_0 - \frac{(x_1 - x_0)}{(y_1 - y_0)} y_0 = 2 - \frac{(3-2)}{(16+1)}(-1)$.

$x_2 = 2.0588$, $f(x_2) = f(2.0588) = -0.3908$

Now next root will be b/w $[(2.0588)$ and $(3)]$
$\hookrightarrow (-ve)$ $\hookrightarrow (+ve)$

$x_0 = 2.0588$

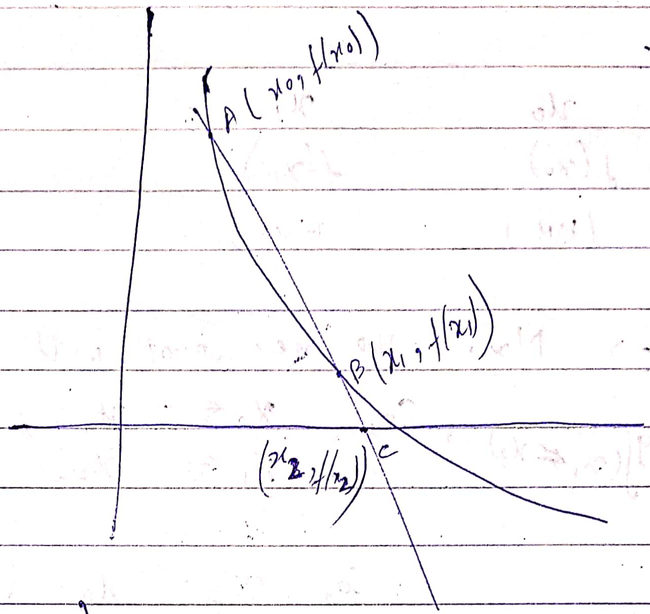
$x_1 = 3$.

2[nd] approx. root $x_3 = (2.0588) - \frac{(3 - 2.0588)}{(16 + 0.3908)} (-0.3908)$

$= 2.0813$, $f(2.0813) =$

* Secant Method:-

A : $(x_0, f(x_0))$
B : $(x_1, f(x_1))$



$$(y - y_1) = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_1)$$

$x \longrightarrow x_2 \quad y \Longrightarrow 0$

$$0 - y_1 = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x_2 - x_1).$$

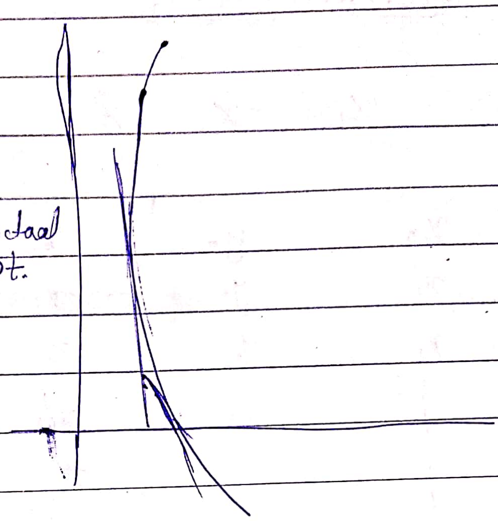
$$(x_2 - x_1) = -\left(\frac{x_1 - x_0}{y_1 - y_0}\right) y_1$$

$$x_2 = x_1 - \frac{(x_1 - x_0)}{(y_1 - y_0)} y_1$$

$$\boxed{x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_1)}$$

$$\boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} f(x_n)}$$

* Newton - Raphson Method:-



n^{th} approx root = x_n

$\cancel{\Delta x_n}$

$\Delta x_n \rightarrow$ diff b/w n^{th} app. root and actual root.

Actual root = $x_n + \Delta x_n$

$$f(x_n + \Delta x_n) = 0.$$

$$f(x_n) + \Delta x_n f'(x_n) + 0 = 0$$

$$\Delta x_n = -\frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Q) $f(x) = x^3 - 5x + 1$ $\qquad x_0 = 0.5$

$$f'(x) = 3x^2 - 5.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{[x_n^3 - 5x_n + 1]}{[3x_n^2 - 5]}$$

$$= \frac{3x_n^3 - \cancel{5x_n} - x_n^3 + \cancel{5x_n} - 1}{3x_n^2 - 5}$$

$$\boxed{x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 - 5}}$$

* INTERPOLATION :-

$x_{i+1} - x_i = h$

x_0	y_0	x_0	y_0	
x_1	y_1	x_1	y_1	$\Delta y_0 = y_1 - y_0 = \nabla y_1 = \delta y_{1/2}$
x_2	y_2	x_2	y_2	$\Delta y_1 = y_2 - y_1 = \nabla y_2 = \delta y_{3/2}$
$\vdots$	$\vdots$	x_3	y_3	$\Delta y_2 = y_3 - y_2 = \nabla y_3 = \delta y_{5/2}$
		x_4	y_4	$\Delta y_3 = y_4 - y_3 = \nabla y_4 = \delta y_{7/2}$.
x_n	y_n.			

$\boxed{\Delta y_n = y_{n+1} - y_n.}$ forward opt.

$\Delta y_n + y_n = y_{n+1}$

$\Delta y_{-1} + y_{-1} = y_0$

$\boxed{\nabla y_{n+1} = y_{n+1} - y_n}$ backward opt.

$\boxed{\delta y_{(2n+1/2)} = y_{n+1} - y_n}$ Central diff.

or $\boxed{\delta y_n = y_{n+1/2} - y_{n-1/2}}$

$\boxed{E y_n = y_{n+1}.}$ Shifting opt.

$\boxed{E^{-1} y_n = y_{n-1}}$

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$\Delta y_n = y_{n+1} - y_n$

$\Delta y_n = (E-1) y_n$.

$\boxed{\Delta = E - 1}$ —— (1)

$\nabla y_n = y_n - y_{n-1}$. $= \cancel{y_{n+1} - y_n}$

$\nabla y_n = (y_n - E^{-1} y_n) = (1 - E^{-1}) y_n$

$\boxed{\nabla = 1 - E^{-1}}$ —— (2).

$\delta y_n = y_{n+1/2} - y_{n-1/2}$.

$= \left(E^{1/2} - (E^{-1})^{1/2}\right) y_n$

$$\boxed{\delta y_n = \sqrt{E} - \frac{1}{\sqrt{E}}}$$ —— (3)

(1)

$\Delta^n y_{-1} = \Delta^n y_0 - \Delta^{n+1} y_0 + \Delta^{n+2} y_0 - \Delta^{n+3} y_0 + \ldots$

(1) $\Delta^n y_{-2} = \Delta^n y_0 - 2\Delta^{n+1} y_0 + 3\Delta^{n+2} y_0 - 4\Delta^{n+3} y_0 \ldots$

Sol^n 1 :- $\cancel{\Delta^n y_{-1+n}} = \cancel{E^{-(1+n)} y_n}$ As $\Rightarrow$ $\left.\begin{array}{l} y_{-2} = E^{-2} y_0 \\ y_{-1} = E^{-1} y_0 \end{array}\right\}$

$y_{-1} = E^{-1} y_0$

$\Delta^n y_{-1} = \Delta^n (E^{-1} y_0)$

$= \Delta^n (\Delta + 1)^{-1} y_0$ (by binomial)

Hence proved $\Delta^n y_{-1} = \Delta^n y_0 - \Delta^{n+1} y_0 + \Delta^{n+2} y_0 - \Delta^{n+3} y_0 + \ldots$

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As, $y_{-2} = E^{-2} y_0$. As, $E = \Delta + 1$.

$y_{-2} = (\Delta + 1)^{-2} y_0$.

after expanding.

$y_{-2} = y_0 - 2\Delta y_0 + 3\Delta^2 y_0 - 4\Delta^3 y_0 + \ldots$

multipling by (Δ^n) both sides.

$$\boxed{\Delta^n y_{-2} = \Delta^n y_0 - 2\Delta^{n+1} y_0 + 3\Delta^{n+2} y_0 - 4\Delta^{n+3} y_0 + \ldots}$$

Hence proved

* Newton's Forward Method:

$$x_i = x_0 + ih \qquad (i = 0, 1, 2, 3 \ldots)$$

(any value near starting) x

(any value from bottom) x

x_0	y_0
x_1	y_1
$\vdots$	$\vdots$
x_{n-1}	y_{n-1}
x_n	y_n

$$x = x_0 + ph$$

$$y(x) = ?$$

$$y(x) = y(x_0 + ph)$$

$$= E^p y(x_0)$$

$$= (1+\Delta)^p y_0$$

$$= \left(1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \ldots\right) y_0$$

$$\boxed{y(x) = y_0 + y_0 p\Delta + y_0 \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!} y_0 \Delta^3 + \ldots}$$

* Newton's Backward Method:

$$x = x_0 + ph.$$

$$y(x) = ??$$

$$y(x) = y(x_n + ph)$$

$$= E^p y(x_n)$$

$$= (1-\nabla)^{-p} y_n$$

$$= 1 + p\nabla + \frac{p(p+1)}{2!}\nabla^2 + \frac{p(p+1)(p+2)}{3!}\nabla^3 + \ldots) y_n$$

$$\boxed{y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots}$$

Q)

x	y (1×10^3)	Δ	Δ^2	Δ^3	Δ^4
1951	46				
		20			
1961	66		-5		
		15		+2	
1971	81		-3		-3
		12		-1	
1981	93		-4		
		8			
1991	101				

Cal. the (y) in year (1955) and (1985)

For 1955:- $h = 10$, $x = 1955$ (by forward method)

$x = x_0 + ph$

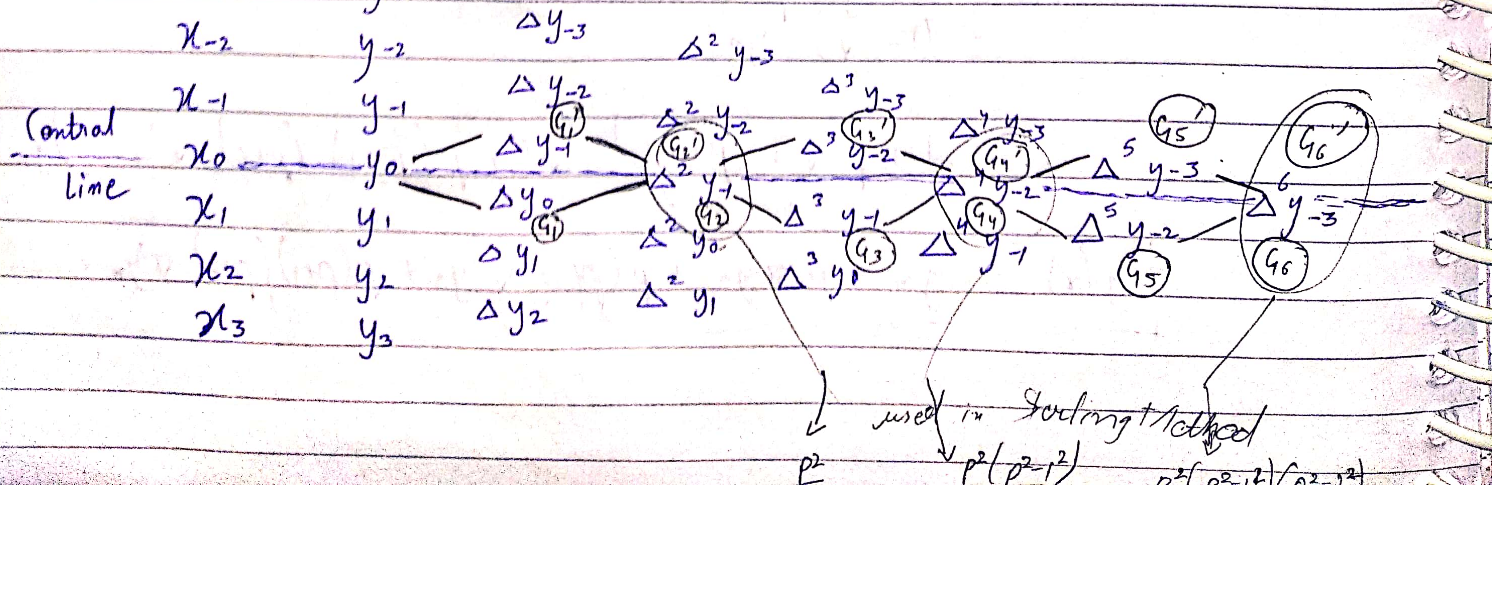
$p = \dfrac{x - x_0}{h} = \dfrac{1955 - 1951}{10} = 0.4$

For 1985, $x_n = 1991$ $h = 10$ (by backward method).

$x = x_n + ph$

$p = \dfrac{x - x_n}{h} = \dfrac{1985 - 1991}{10} = -0.6$

* Gauss Forward Method :-



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$$\left\{\begin{array}{l}\Delta^n y_{-1} = \Delta^n y_0 - \Delta^{n+1} y_0 + \Delta^{n+2} y_0 - \cdots \\ \Delta^n y_{-2} = \Delta^n y_0 - 2\Delta^{n+1} y_0 + 3\Delta^{n+2} y_0 + \cdots\end{array}\right\}$$

$$y(x) = y_0 + G_1 \Delta y_0 + G_2 \Delta^2 y_{-1} \left[\Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 - \Delta^5 y_0 + \cdots\right]$$

$$+ G_3 \Delta^3 y_{-1} \left[\Delta^3 y_0 - \Delta^4 y_0 + \Delta^5 y_0 - \Delta^6 y_0 + \cdots\right]$$

$$+ G_4 \Delta^4 y_{-2} \left[\Delta^4 y_0 - \Delta^5 y_0 + \Delta^6 y_0 - \Delta^7 y_0 + \cdots\right]$$

$$+ G_5 \Delta^5 y_{-2} \left[\Delta^5 y_0 - \Delta^6 y_0 + \Delta^7 y_0 - \Delta^8 y_0 + \cdots\right]$$

$$+ G_6 \Delta^6 y_{-3}$$

By backward method:-

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

By comparing the coefficients

$\Delta y_0 :$ $G_1 = p$

$\Delta^2 y_0 :$ $G_2 = \frac{p(p-1)}{2!}$

$\Delta^3 y_0 :$ $-G_2 + G_3 = \frac{p(p-1)(p-2)}{3!}$

$$G_3 = \frac{p(p-1)(p-2)}{3!} + \frac{p(p-1)}{2!} = \frac{p(p-1)}{3!}\left[(p-2) + 3\right]$$

$$G_2 - G_3 + G_4 = \frac{p(p-1)(p-2)(p-3)}{4!} = \boxed{\frac{p(p-1)(p+1)}{3!} = G_4}$$

* Gauss Backward Method:-

⋮

Solve in the same manner.

$G_1' = p$

$G_2' = \frac{p(p+1)}{2!}$

$G_3' = \frac{p(p+1)(p-1)}{3!}$

$G_4' = \frac{p(p+1)(p-1)(p+2)}{4!}$

Comparison

$G_1 = p$

$G_2 = \frac{p(p-1)}{2!}$

$G_3 = \frac{p(p-1)(p+1)}{3!}$

$G_4 = \frac{p(p-1)(p+1)(p-2)}{4!}$

* Sterlin Method:-

$$\left(\frac{G_2 + G_2'}{2}\right) = \frac{1}{2}\left[\frac{p(p-1)}{2!} + \frac{p(p+1)}{2!}\right] = \frac{p^2}{2}$$

$$\frac{G_4 + G_4'}{2} = \frac{1}{2}\left[\frac{p(p-1)(p+1)}{4!}\{(p-2)+(p+2)\}\right] = \frac{p^2(p^2-1^2)}{4!}$$

$$\frac{G_6+G_6'}{2} = \frac{p^2(p^2-1^2)(p^2-2^2)}{6!}$$

* Bessel's Method:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
	E_0	Δy_{-1}	E_2	$\Delta^3 y_{-2}$	E_4	$\Delta^5 y_{-3}$	E_6
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$ B_3		$\Delta^5 y_{-2}$	
x_1	y_1	B_1	$\Delta^2 y_0$ B_2		$\Delta^4 y_{-1}$ B_4		
	F_0		F_2	$\Delta^3 y_0$	F_4		
		Δy_1					
x_2	y_2		$\Delta^2 y_1$				
		Δy_2					
x_3	y_3						

$$y(x) = \frac{y_0+y_1}{2} + B_1\Delta y_0 + \frac{B_2}{2}\left(\Delta^2 y_0 + \Delta^2 y_{-1}\right) + B_3\Delta^3 y_{-1} + \frac{B_4}{2}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right)$$

$$= \left(\frac{y_0+y_0+\Delta y_0}{2}\right) + B_1\Delta y_0 + \frac{B_2}{2}\left(\Delta^2 y_0 + \Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 \ldots\right) + B_3\left(\Delta^3 y_0 - \Delta^4 y_0 + \Delta^5 y_0 + \ldots\right) + \frac{B_4}{2}\left(\left(\Delta^4 y_0 - 2\Delta^5 y_0 + 3\Delta^6 y_0 - \ldots\right) + \left(\Delta^4 y_0 - \Delta^5 y_0 + \Delta^6 y_0 + \ldots\right)\right)$$

$$= y_0 + \left(B_1 + \frac{1}{2}\right) y_0 + \ldots\ldots$$

As Newton's Forward Method:-

$$\left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots \right]$$

After comparing the coefficients.

Δy_0 : $B_1 + \frac{1}{2} = p$, $B_1 = p - \frac{1}{2}$.

$\Delta^2 y_0$: $B_2 = \frac{p(p-1)}{2!}$, $B_2 = \frac{p(p-1)}{2!}$

$\Delta^3 y_0$: $-\frac{B_2}{2} + B_3 = \frac{p(p-1)(p-2)}{3!}$

* Everett's Method:-

$$y(x) = E_0 y_0 + E_2 \Delta^2 y_{-1} + E_4 \Delta^4 y_{-2} +$$

$$F_0 y_1 + F_2 \Delta^2 y_0 + F_4 \Delta^4 y_{-1}$$

$$y(x) = E_0 y_0 + E_2\{\Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 + \dots\} + E_4\{\Delta^4 y_0 - 2\Delta^5 y_0$$

$$+ 3\Delta^6 y_0 + \dots\} +$$

$$F_0(y_0 + \Delta y_0) + F_2(\Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 + \dots) + F_4(\Delta^4 y_0$$

$$- \Delta^5 y_0 + \Delta^5 y_0 - \Delta^6 y_0 + \dots)$$

By comparing the coefficients

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As Newton's Forward Method:-

$$\left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots \right]$$

After comparing the coefficients.

$\Delta y_0 :$ $B_1 + \frac{1}{2} = p$, $B_1 = p - 1/2$.

$\Delta^2 y_0 :$ $B_2 = \frac{p(p-1)}{2!}$, $B_2 = \frac{p(p-1)}{2!}$

$\Delta^3 y_0 :$ $-\frac{B_2}{2} + B_3 = \frac{p(p-1)(p-2)}{3!}$

* Everett's Method:-

$$y(x) = E_0 y_0 + E_2 \Delta^2 y_{-1} + E_4 \Delta^4 y_{-2} +$$
$$F_0 y_1 + F_2 \Delta^2 y_0 + F_4 \Delta^4 y_{-1}$$

$$y(x) = E_0 y_0 + E_2 \{ \Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 + \cdots \} + E_4 \{ \Delta^4 y_0 - 2\Delta^5 y_0$$
$$+ 3\Delta^6 y_0 + \cdots \} +$$

$$F_0 (y_0 + \Delta y_0) + F_2 (\Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 + \cdots) + F_4 (\Delta^4 y_0$$
$$- \Delta^5 y_0 + \Delta^5 y_0 - \Delta^6 y_0 + \cdots)$$

By comparing the coefficients

$y_0: \quad E_0 + F_0 = 1$

$\Delta y_0: \quad F_0 = p, \quad E_0 = (1-p) = q. \quad - (2)$

$\Delta^2 y_0: \quad E_2 + F_2 = \frac{p(p-1)}{2!} \quad - (3)$

$\Delta^3 y_0: \quad -E_2 = \frac{p(p-1)(p-2)}{3!}$

$$E_2 = -\frac{p(p-1)(p-2)}{3!} = -(1-q)(1-q-1)(1-q-2)$$

$$= \frac{q(1-q)(-1-q)}{3!} = \frac{q(q-1)(q+1)}{3!}$$

$$E_2 = \frac{q(q^2-1^2)}{3!} \quad - (1)$$

from eq (3)

$$F_2 = \frac{p(p-1)}{2!} - \frac{q(q^2-1^2)}{3!}$$

$$= \frac{(1-q)(-q)}{2!} - \cancel{\frac{q(q^2-1^2)}{3!}}$$

$$\boxed{F_2 = \frac{p(p^2-1^2)}{3!}}$$

$$\boxed{F_4 = \frac{p(p^2-1^2)(p^2-2^2)}{5!}}$$

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* Newton Finite divided difference Method:- and Lagrangian's Formula:-

$$y(x) = a_0(x-x_1)(x-x_2)\cdots(x-x_n) + a_1(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n) + a_2(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)$$

$x = x_0$ $\qquad\qquad$ $y = y_0$

So,

$$y_0 = a_0(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)$$

$$a_0 = \frac{y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)}$$

$x = x_1$ $\qquad\qquad$ $y = y_1$

$$a_1 = \frac{y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)}$$

$$\boxed{y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_0)\cdots(x_1-x_0)}y_1 + \cdots}$$

* Finite divided Difference:-

$$[x_0 \; x_1] = \frac{y_1-y_0}{x_1-x_0}$$

$$[x_2 \; x_3] = \frac{y_3-y_2}{x_3-x_2}$$

$\Big\} \rightarrow$ first divided difference

$$[x_0 \quad x_1 \quad x_2] = \left[\frac{[x_1 \quad x_2] - [x_0 \quad x_1]}{(x_2 - x_0)} \right] \rightarrow 2^{nd} \text{ divided difference}$$

$$[x \quad x_0] = \frac{y_0 - y}{x_0 - x}.$$

$$y_0 - y = (x_0 - x)\,[x \quad x_0]$$

$$y_0 + (x - x_0)\,[x \quad x_0] = y \qquad \text{— (1)}$$

$$[x \quad x_0 \quad x_1] = \frac{[x_0 \quad x_1] - [x \quad x_0]}{(x_1 - x)}$$

$$[x_0 \quad x_1] - [x \quad x_0] = (x_1 - x)\,[x \quad x_0 \quad x_1]$$

$$[x \quad x_0] = [x_0 \quad x_1] + (x - x_1)\,[x \quad x_0 \quad x_1] \text{ — (2)}$$

$$\boxed{\begin{aligned} y(x) = {} & y_0 + (x - x_0)[x_0 \quad x_1] + (x - x_0)(x - x_1)[x_0 \quad x_1 \quad x_2] \\ & + (x - x_0)(x - x_1)(x - x_2) \; 3^{rd} \text{ ofd.} \end{aligned}}$$

Q)

x	y	I^{st} dd	2^{nd} dd	3^{rd} d.d	4^{th}
4	48				
		52			
5	100		97-52/7-4 =15	1	
		194/2 = 97			0
7	294		202-97/5 = 21	1	
		202			0
10	900		27	1	
		310			
11	1210		33		
		818/2 = 409			
13	2028				5^{th} dd

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$-x = 8 \qquad y(8) = ?$