# Optics

## Coherent sources
- same frequency
- maintain constant phase difference

Interference by (Production of coherent waves):
1. Division of Wavefront
2. Division of Amplitude

The 2 beams superimpose

$$y_1 = a \cos \omega t$$
$$y_2 = a \cos(\omega t + \phi)$$
$$y = y_1 + y_2$$
$$= a[\cos \omega t + \cos(\omega t + \phi)]$$
$$= 2a \cos \phi/2 \cos(\omega t + \phi/2)$$

$$\Rightarrow \quad I = 4a^2 \cos^2 \phi/2 \cos^2(\omega t + \phi/2)$$
$$= 4a^2 \cos^2 \phi/2$$

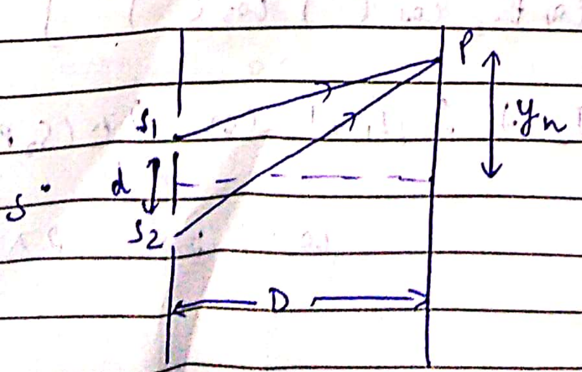
[as $\langle \cos^2(\omega t + \phi/2) \rangle = \frac{1}{2}$ (a constant)]

So, $\cos^2 \phi/2$ will decide the maxima/minima of the pattern.

$\phi = \pm\pi, \pm 3\pi, + \ldots \Rightarrow$ Minima

$\phi = 0, \pm 2\pi, \pm 4\pi, \ldots \Rightarrow$ Maxima

* YDSE



$$(S_2P)^2 - (S_1P)^2 = \left(D^2 + \left(y_n + \frac{d}{2}\right)^2\right) - \left(D^2 + \left(y_n - \frac{d}{2}\right)^2\right)$$

$$= 2 y_n d$$

$$(S_2P + S_1P)(S_2P - S_1P) = 2 y_n d$$

$$\underbrace{S_2P - S_1P}_{\text{Path difference}} = \frac{2 y_n d}{2D}$$

$$\Rightarrow \quad n\lambda = \cancel{2}\ \frac{y_n d}{D}$$

$$(n+1)\lambda = \frac{y_{n+1} d}{D}$$

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}$$

$$\vec{E_1} = \hat{i}\ E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)$$

$$\vec{E_2} = \hat{i}\ E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$= \hat{i}\left[E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right]$$

$$I = k\left[E_{01}^2 \cos^2(\ ) + E_{02}^2 \cos^2(\ ) + 2 E_{01} E_{02} \cos(\ ) \cos(\ )\right]$$

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$$= k\left[I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\frac{2\pi}{\lambda}(S_2P - S_1P)\right]$$

$$= k\left[I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cdot\cos\delta\right]$$

$$\Rightarrow \cos\delta = \pm 1$$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ $\quad$ $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

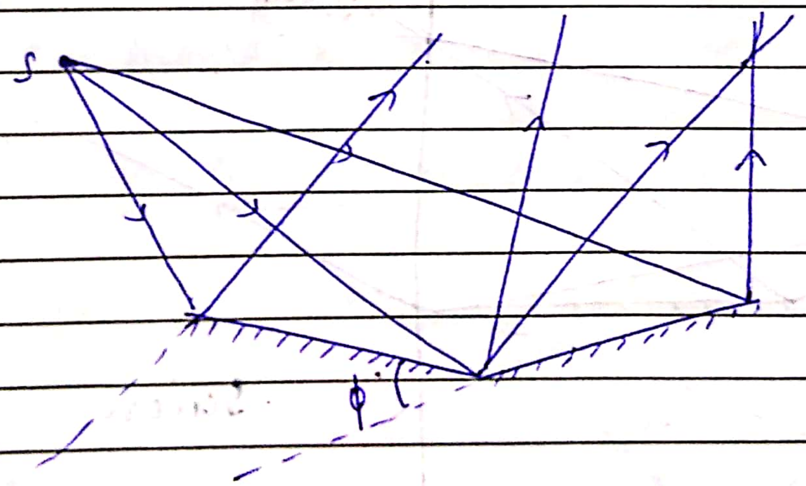
If $I_1 = I_2 = I_0$

$$I = 2I_0(1 + \cos\delta)$$

$$= 4 I_0 \cos^2 \delta/2$$

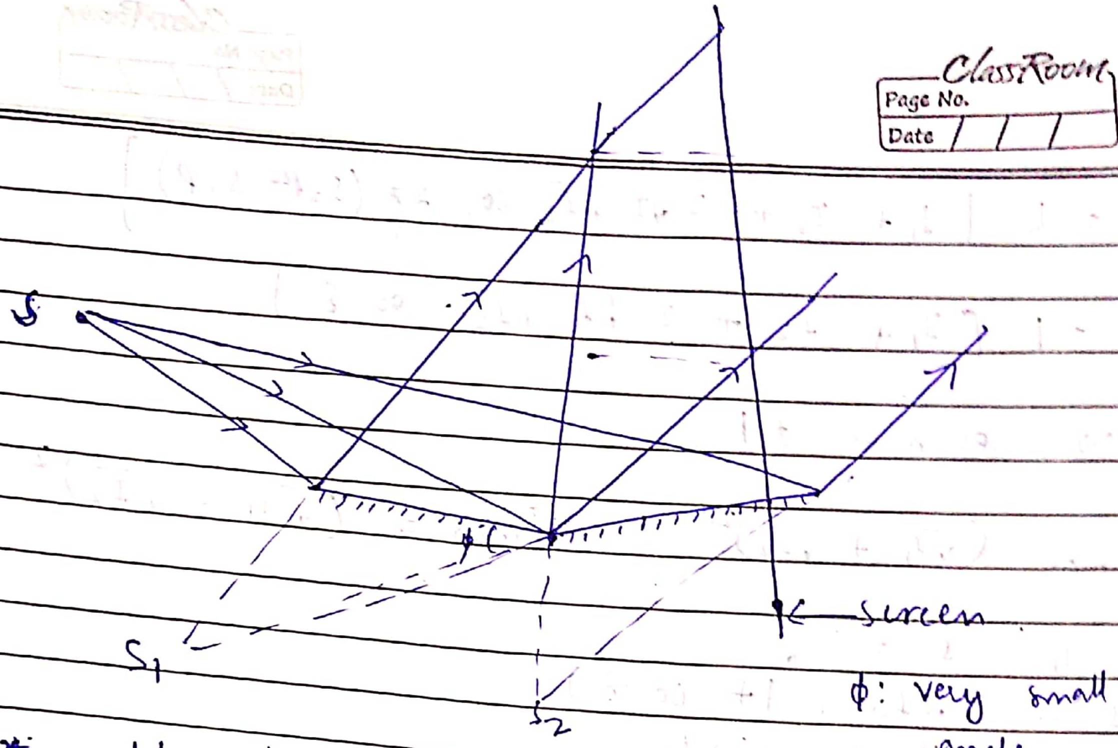
(A) Division of wavefront

* Fresnel's Two Mirror Arrangement (Division of Wavefront)

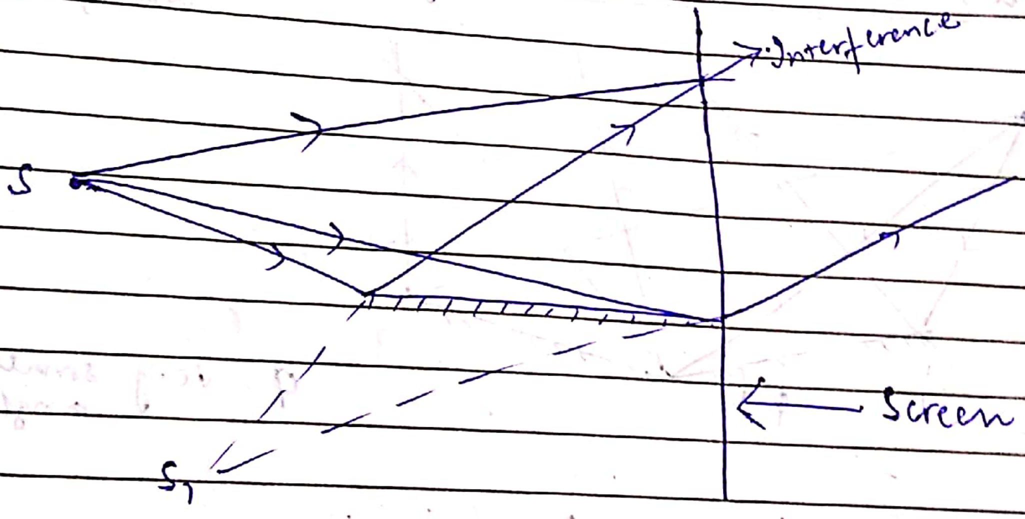



$\phi$ : very small angle

- Interference pattern is obtained in the region where two wavefronts overlap.
- we have 2 virtual sources in this method.
- central maxima is bright fringe, as phase change of $\pi$ occurs for both



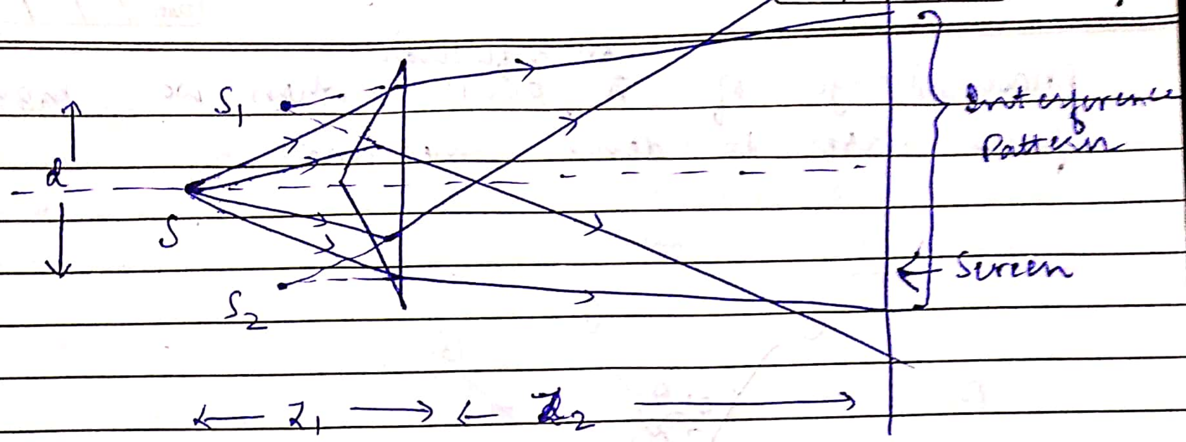

* Lloyd's Mirror Method




- division of wavefront
- one real & one virtual source
- one source has a phase change and other does not.

* Fresnel's Biprism Method / Expt

- 0.5° angle on ~~sides~~ base of prism
- 179° angle of prism



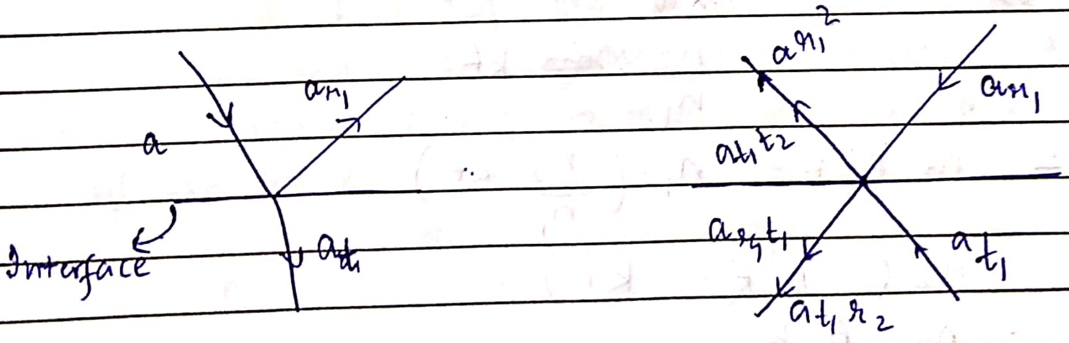

- 2 virtual sources $S_1$ and $S_2$

- $\beta = \frac{\lambda D}{d} = \frac{\lambda(Z_1+Z_2)}{d}$

- division of wavefront
- 'd' should be small to maintain coherence.

## (B) Division of Amplitude

**Principle of optical Reversibility :**




From principle of conservation of energy:

$a r_1^2 + a t_1 t_2 = a \Rightarrow \boxed{r_1^2 = 1 - t_1 t_2}$

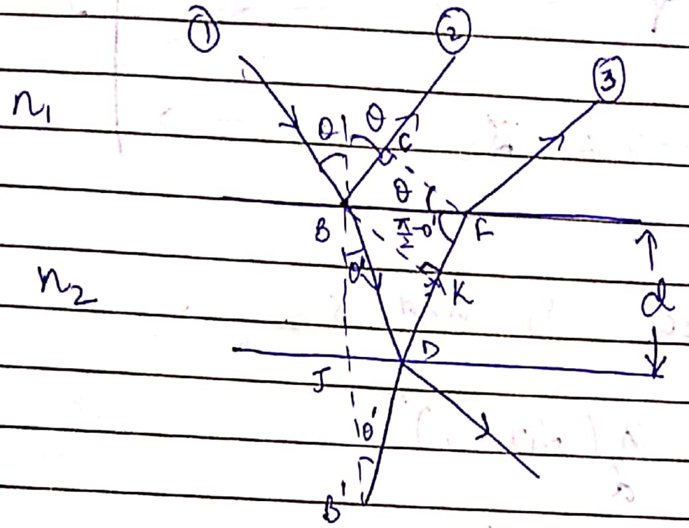
$a r_1 t_1 + a t_1 r_2 = 0 \Rightarrow \boxed{r_1 = -r_1}$

Phase change of $\pi$ occurs on reflection

Stoke's Relations

Phase change of $\pi$ occurs on reflection when we move from rarer to denser medium.

<u>Cosine Law</u>



$BD = B'D$
$BJ = B'J$

$$\Delta = n_2(BD + DF) - n_1 BC$$
$$= n_2(B'D + DF) - n_1 BC$$
$$= n_2 B'F - n_1 BC$$

$$BC = BF \sin\theta = \frac{KF}{\sin\theta'} \sin\theta$$
$$= \frac{n_2}{n_1} KF$$

so, $$\Delta = n_2 B'F - n_1\left(\frac{n_2}{n_1} KF\right)$$
$$= n_2(B'F - KF)$$
$$= n_2 B'K$$

$\Rightarrow$ $$\boxed{\Delta = 2n_2 d \cos\theta'}$$

$\Rightarrow$ Reflection Index $= \dfrac{n_2 - n_1}{n_2 + n_1}$ Transmission Index $= \dfrac{2n_1}{n_2 + n_1}$

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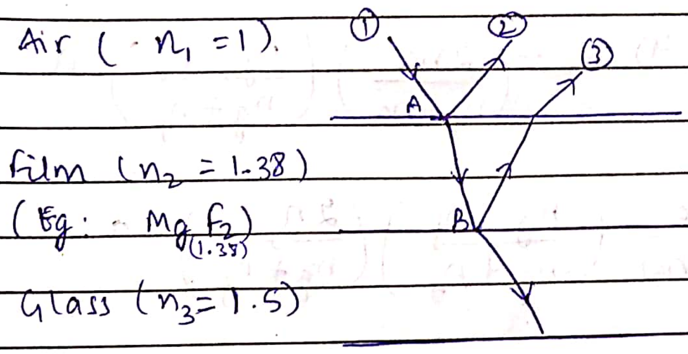
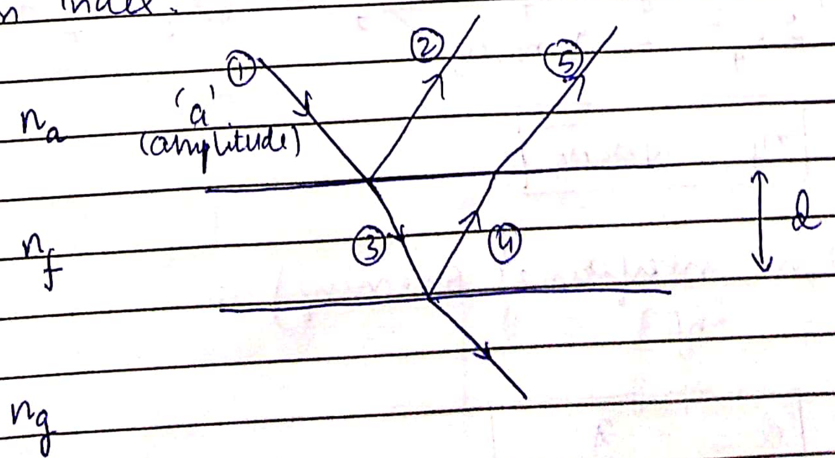
<u>Blooming</u> – Creating a non-reflecting film by introducing a thin film between two media and hence, reducing the loss due to reflection (reflection index) → (upto 90% practically)




Both ② & ③ undergo a phase change of $\pi$ at A & B respectively. So, there is no phase difference and the path difference b/w ② & ③ can be maintained as a multiple of $\lambda/2$ by monitoring the thickness & material of the film, and hence resulting in destructive interference, creating a non-reflecting film.

Illy, the path diff can be maintained as a multiple of $\lambda$ to result in constructive interference, to ~~sa~~ given shine / desired colour to the substance due to high reflection index.



Phase change of $\pi$

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Amplitude of reflected wave (2) = $-\dfrac{n_f - n_a}{n_f + n_a} \times (a)$

Refracted wave (3) = $\dfrac{2n_a}{n_f + n_a} \times a$

$$(4) = -\left(\frac{2n_a}{n_f+n_a}\right)\left(\frac{n_g - n_f}{n_g + n_f}\right) a$$

$$(5) = -\frac{2n_a}{(n_f+n_a)}\left(\frac{n_g-n_f}{n_g+n_f}\right)\left(\frac{2n_f}{n_g+n_f}\right) a$$

For constructive interference, (2) = (5)

$$\Rightarrow \left(\frac{n_f - n_a}{n_f+n_a}\right) = \left[\frac{4\, n_a n_f}{(n_f+n_a)^2}\right]\left(\frac{n_g - n_f}{n_g+n_f}\right)$$

when $n_a = 1$ & $n_f = 1.4$

$$\frac{4\, n_a n_f}{(n_f+n_a)^2} = 0.97 \approx 1$$

$$\Rightarrow \frac{n_f - n_a}{n_f + n_a} = \frac{n_g - n_f}{n_g + n_f}$$

$$\Rightarrow \cancel{n_g n_f} - n_a n_g + n_f^2 - \cancel{n_a n_f} = \cancel{n_f n_g} - n_f^2 + n_a n_g - \cancel{n_a n_f}$$

$$\Rightarrow 2n_f^2 = 2n_a n_g$$

$$\Rightarrow \boxed{n_f = \sqrt{n_a n_g}}$$

For destructive interference (Blooming) :

$$\frac{\lambda}{2} = 2n_f d$$

$$\Rightarrow \boxed{d = \frac{\lambda}{4n_f}}$$

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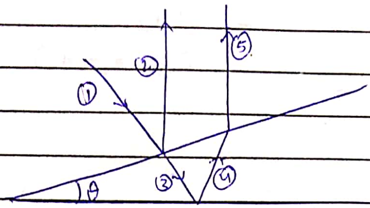
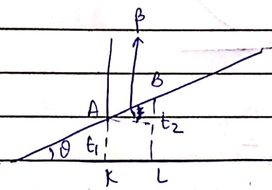
## * Wedge-shaped film




$$\Delta = 2\mu t \cos r \quad [= m\lambda \text{ (dark fringe)}]$$

For normal incidence : $\cos r = 1$




K : $m$th dark fringe

L : $(m+1)$th dark fringe

$$2\mu t_1 = m\lambda$$

$$2\mu t_2 = (m+1)\lambda$$

β : fringe width

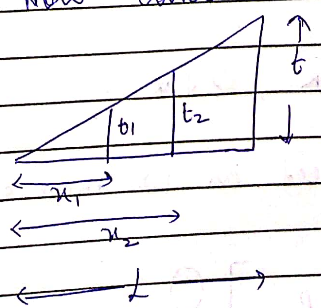
$$\frac{BC}{\beta} = \tan\theta \Rightarrow BC = \beta\tan\theta$$

$$2\mu\beta \cdot \tan\theta = \lambda$$

$$2\mu\beta\theta \approx \lambda$$

$$\boxed{\beta \approx \frac{\lambda}{2\mu\theta}}$$

Now consider :




$$2\mu x_1\theta = m\lambda$$

$$2\mu x_2\theta = (m+N)\lambda$$

$$2\mu(x_2 - x_1)\theta = N\lambda$$

$$\boxed{\theta = \frac{N\lambda}{2\mu(x_2 - x_1)}}$$

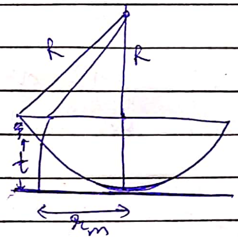
Ex

Thickness of spacer, $t = l \tan\theta$

$$\approx l\theta$$

$$= \frac{lN\lambda}{2\mu(x_2 - x_1)}$$

* Newton's Rings




$$R^2 = (R-t)^2 + r_m^2$$

$$= R^2 + t^2 - 2Rt + r_m^2$$

$$\Rightarrow r_m^2 = 2Rt - t^2$$

$$\Rightarrow \boxed{r_m \approx 2Rt} \quad (t << R)$$

$$2\mu t = m\lambda$$

$$2t = m\lambda \Rightarrow t = \frac{m\lambda}{2}$$

$$r_m^2 \approx m\lambda R$$

$$r_m \approx \sqrt{m\lambda R}$$

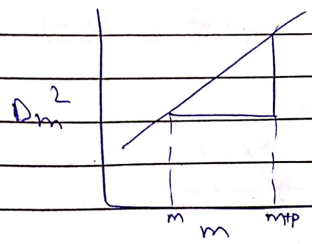
So the consecutive fringes are:

$\sqrt{\lambda R}$, $1.414\sqrt{\lambda R}$, $1.732\sqrt{\lambda R}$, $2\sqrt{\lambda R}$

So, the fringes become closely spaced as we move out.

Now diameter of fringe $D_m^2 = 4m\lambda R$

$D_{m+p}^2 = 4(m+p)\lambda R$ ] (1)




So, we have

$$\frac{D_{m+p}^2 - D_m^2}{p} = \text{slope}$$

from (1)

$$\Rightarrow \frac{D_{m+p}^2 - D_m^2}{p} = 4\lambda R.$$

$$\Rightarrow \boxed{\frac{\text{Slope}}{4R} = \lambda}$$

When $\mu \neq 1$

$$r_m^2 = \frac{m\lambda R}{\mu}$$

The radius of the fringes will decrease further.

$$(D_m^2)_\mu = \frac{4 m \lambda R}{\mu}$$

(*) Michelsen Interferometer

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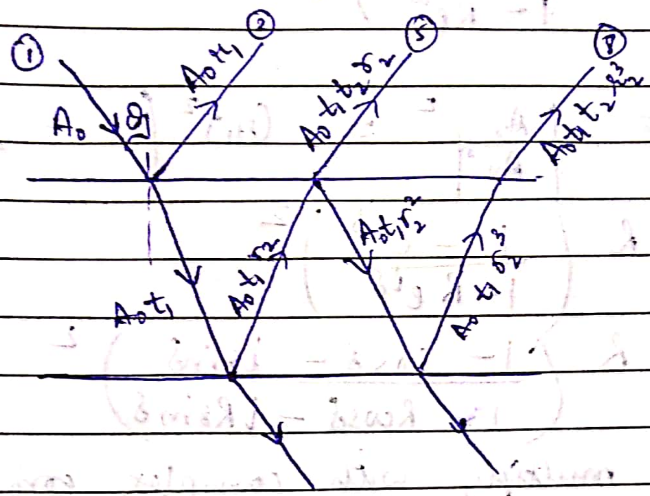
## * Multiple Beam Interference




Phase diff. corresponding to path diff. b/w 2 & 5 $\rightarrow \delta$, 5 & 8 $\rightarrow \delta$, 2 & 8 $\rightarrow 2\delta$

$$\Delta = 2nd\cos\theta$$

$$\delta = \frac{2\pi}{\lambda}(2nd\cos\theta)$$

NOW, $A_r = A_0 r_1 + A_0 t_1 t_2 r_2 e^{i\delta} + A_0 t_1 t_2 r_2^3 e^{2i\delta} + \dots$

$$= A_0\left[r_1 + t_1 t_2 r_2 e^{i\delta} + t_1 t_2 r_2^3 e^{2i\delta} + t_1 t_2 r_2^5 e^{3i\delta} + \dots\right]$$

$$= A_0\left[r_1 + t_1 t_2 r_2 e^{i\delta}\left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \dots\right)\right]$$

$$= A_0\left[r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right]$$

From Stokes' Law: $r_2 = -r_1$

$r_1 r_2 = r^2 = R$

$T = t_1 t_2$

$R + T = 1$

$$= A_0 r_1\left[1 - \frac{t_1 t_2 e^{i\delta}}{1 - r^2 e^{i\delta}}\right]$$

$$\frac{A_r}{A_0} = r_1\left(1 - \frac{\varepsilon(1-R)e^{i\delta}}{1-Re^{i\delta}}\right)$$

~~Total~~

Reflectance $\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = (r_1)^2\left(1 - \frac{(1-R)e^{i\delta}}{1-Re^{i\delta}}\right)^2$

$$= R\left(\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right)^2$$

$$= R\left(\frac{1-\cos\delta - i\sin\delta}{1-R\cos\delta - iR\sin\delta}\right)^2$$

for complex nos., multiply with complex conjugate to calculate square:

$$\mathcal{R} = R\cdot\left(\frac{(1-\cos\delta)^2 + \sin^2\delta}{(1-R\cos\delta)^2 + R^2\sin^2\delta}\right)$$

$$= R\,\frac{1+\cos^2\delta - 2\cos\delta + \sin^2\delta}{1+R^2\sin^2\delta + R^2\cos^2\delta - 2R\cos\delta}$$

$$= \frac{R\,2(1-\cos\delta)}{1+R^2-2R\cos\delta}$$

$$= \frac{4R\sin^2\delta/2}{(1-R)^2 + 4R\sin^2\delta/2}$$

Now, Coefficient of Finesse, $f = \frac{4R}{(1-R)^2}$ ⇒ indicates bright and dark contrast

So, $\boxed{\mathcal{R} = \frac{f\sin^2\delta/2}{1+f\sin^2\delta/2}}$

IIly Total transmittance $\mathcal{T} = \left|\frac{A_t}{A_0}\right|^2 = \frac{1}{1+f\sin^2\delta/2}$

Adding them, we get $\boxed{x + y = 1}$