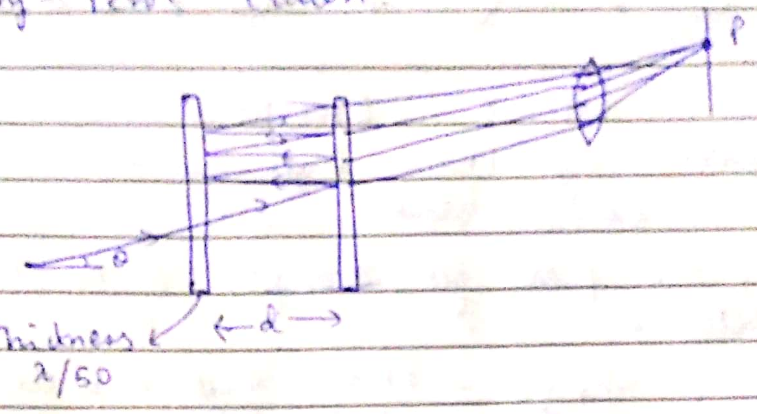
Adding them, we get

$$\boxed{R + T = 1}$$

→ Fabry-Perot Interferometer   Laser ⊛ Pedrotti

If the distance 'd' is fixed, it is called as Fabry-Perot Etalon.



$$T_{max} = 1 \quad ; \quad T_{min} = \frac{1}{1+F}$$

$$\frac{T_{max} - T_{min}}{T_{min}} = \frac{1 - \frac{1}{1+F}}{\frac{1}{1+F}} = F \text{ (coeff of finesse)}$$

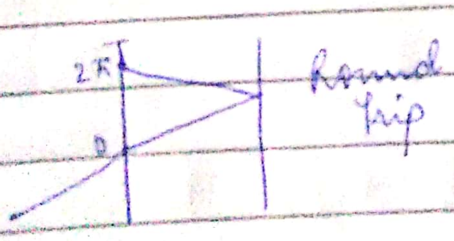
$$\sin \delta/2 = \pm 1$$

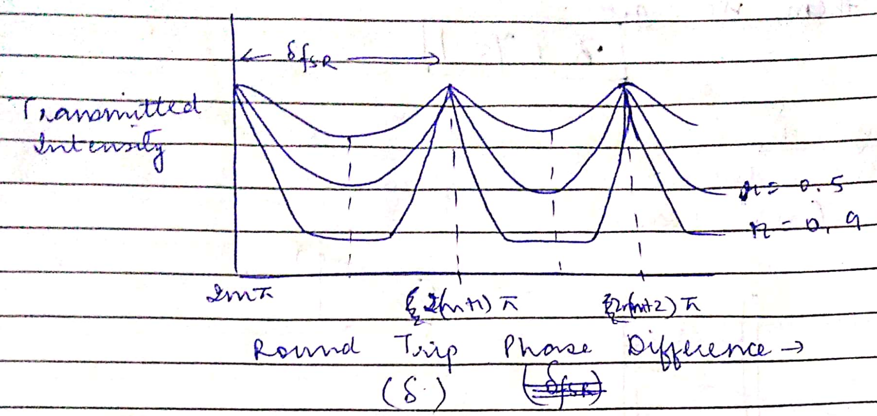
Figure of Merit $\mathcal{F} = \frac{\pi\sqrt{F}}{2} = \frac{\pi r}{1-r^2} = \frac{\pi\sqrt{R}}{1-R}$

Figure of Merit:

It is the ratio of separation b/w 2 transmittance peaks, to the FWHM of the peaks.

(Full Width at Half maximum)





As r increases, the peaks get sharper and the contrast gets better.

fSR : Free spectral range - Phase shift associated with two adjacent peaks

$$\delta_{fSR} = \delta_{m+1} - \delta_m$$
$$= 2\pi$$

F : coeff of finesse

$\mathcal{F}$: figure of meri

$$\Rightarrow \quad T = \frac{1}{1 + F\sin^2\delta/2} = \frac{1}{1 + \frac{4\mathcal{F}^2}{\pi^2}\sin^2\delta/2}$$

for FWHM, Put $T = 1/2$.

$$\frac{1}{2} = \frac{1}{1 + F\sin^2\delta/2}$$

$$1 = F\sin^2\delta/2 \quad \Rightarrow \quad \sin\delta/2 = \sqrt{\frac{1}{F}}$$

OR

$$1 = \frac{4\mathcal{F}^2}{\pi^2}\sin^2\frac{\delta}{2} \quad \cancel{\Rightarrow \sin}$$

NOW, $$\frac{\delta}{2} = 2m\pi + \delta_{1/2}$$

$$\Rightarrow \sin^2\left(\frac{2m\pi + \delta_{1/2}}{2}\right) = \frac{\pi^2}{4\mathcal{F}^2}$$

$$\Rightarrow \sin^2\left(m\pi + \frac{\delta_{1/2}}{2}\right) = \frac{\pi^2}{4\mathcal{F}^2}$$

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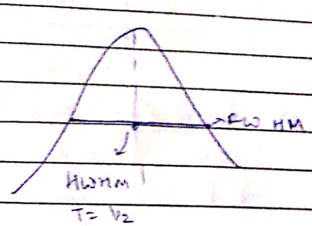
~~sin~~ $\sin^2 \frac{\delta_{1/2}}{2} = \frac{\pi^2}{4\mathcal{F}^2}$ ($\because \delta_{1/2}$ is very small)

$\approx \left(\frac{\delta_{1/2}}{2}\right)^2 = \frac{\pi^2}{4\mathcal{F}^2}$

$\Rightarrow \boxed{\delta_{1/2} = \frac{\pi}{\mathcal{F}}}$



$FWHM = 2\delta_{1/2} = \frac{2\pi}{\mathcal{F}}$

$\boxed{\mathcal{F} = \frac{\delta_{fsR}}{FWHM}}$

Now,

$\delta = 2kd$

$= 2\left(\frac{2\pi}{\lambda}\right) d$

$= 2m\pi, \quad m = 0, \pm 1, \pm 2 \ldots$

$\Rightarrow d_m = \frac{m\lambda}{2} \; ; \; d_{m+1} = (m+1)\frac{\lambda}{2}$

cavity length.

$d_{m+1} - d_m = \frac{\lambda}{2}$

$\Rightarrow$ Now, for diff. wavelengths (no source is perfectly monochromatic):

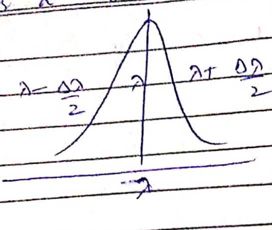
$d_1 = \frac{m\lambda_1}{2} \; ; \; d_2 = \frac{m\lambda_2}{2}$

$d_1 - d_2 = \frac{m}{2}(\lambda_1 - \lambda_2)$

or $\boxed{\Delta\lambda = \frac{2\Delta d}{m}}$

or $\Delta\lambda = \frac{2\Delta d}{\frac{2d_1}{\lambda_1}} \Rightarrow \frac{\Delta\lambda}{\lambda_1} = \frac{\Delta d}{d_1} \; ; \; \boxed{\frac{\Delta\lambda}{\lambda} = \frac{\Delta d}{d}}$

For a continuous source:



So, the formula is modified to

$$\frac{\Delta\lambda}{\lambda} = \frac{2\,\Delta d}{d}.$$

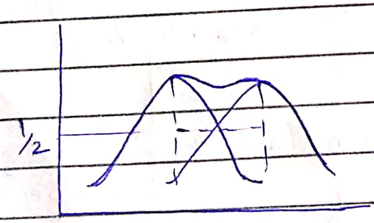
⇒ **Resolving Power**: The minimum $\Delta\lambda$ an instrument can record.

$$R.P. = \frac{\lambda}{\Delta\lambda} = \frac{d}{2\Delta d} \quad \text{—— (1)}$$

Now, $\mathcal{F} = \frac{\delta_{FSR}}{2\delta_{1/2}} = \frac{k\,d_{FSR}}{2k\,d_{1/2}} = \frac{d_{FSR}}{2d_{1/2}}$

$$\Rightarrow 2d_{1/2} = \frac{d_{FSR}}{\mathcal{F}} = \frac{\lambda}{2\mathcal{F}} \quad \text{—— (2)}$$

Rayleigh Criteria: Just resolved point.



Now, $\frac{\Delta\lambda_{min}}{\lambda} = \frac{\Delta d_{min}}{d} = \frac{2d_{1/2}}{d}$

$\Rightarrow \frac{\Delta\lambda_{min}}{\lambda} = \frac{\lambda}{2\mathcal{F}d}$ from (2)

$\Rightarrow \frac{\lambda}{\Delta\lambda} = \frac{2d}{\lambda}\mathcal{F} \Rightarrow \boxed{R.P. = m\mathcal{F}}$ From (1)

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* Coherence

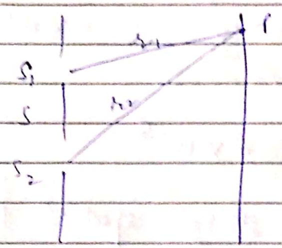
$E = A\cos(kx - \omega t + \phi)$

Coherence Time τ_c: Time for which the wave maintains a constant phase difference, i.e., remains coherent. Also called as temporal time.

Coherent Length: $L = c\tau_c$

change in phase difference occurs because of the very process of production of light.

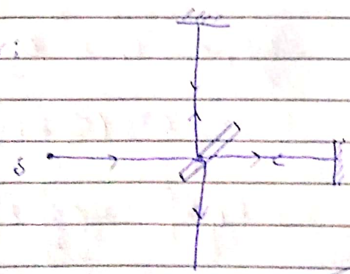
YDSE:



$\frac{r_2 - r_1}{c} << \tau_c$

when $\frac{r_2 - r_1}{c}$ becomes $> \tau_c$, the interference pattern washes off.

Michelson Interferometer:



$\frac{2d}{c} << \tau_c$

Now as discussed

$2d = \lambda^2 / \Delta\lambda$

So fringes wash off for $2d > \lambda^2/\Delta\lambda$

or for $\Delta\lambda \geq \lambda^2/2d$

Hence $2d$ is the coherence length of the source (L).

Hence, $\boxed{\Delta\lambda = \frac{\lambda^2}{L}}$ $\Delta\lambda$ spectral length of the source

$\Rightarrow \Delta\lambda = \frac{\lambda^2}{c\tau_c} \Rightarrow \boxed{\Delta\lambda \propto \frac{1}{\tau_c}}$

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$\nu = \frac{c}{\lambda}$

$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$

$= \frac{c}{\lambda^2} \cdot \frac{\lambda^2}{c\,\tau_c}$

$\Rightarrow \boxed{\Delta\nu = \frac{1}{\tau_c}}$ $\Delta\nu$: spectral length in terms of frequency.

Q Yellow light, $\lambda = 5890$ Å. Find freq. when $\tau_c = 10^{-10}$ s

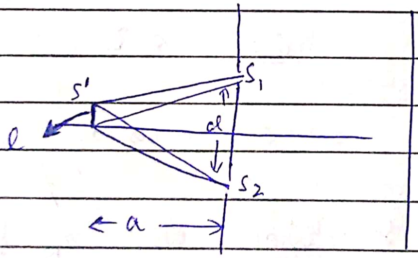
Ans $\nu = \frac{c}{\lambda} = \frac{5 \times 10^4}{10^{-10}} = 5 \times 10^{+14}$ Hz.

$\Delta\nu = 10^{10}$ Hz

$\boxed{\frac{\Delta\nu}{\nu}} = \frac{10^{10}}{5 \times 10^{14}} = 2 \times 10^{-5}$

Monochromaticity of the source spectral Purity

Now Consider an extended source:



$S'S_2 - S'S_1 = \Delta$

$S'S_2 = \left[a^2 + \left(\frac{d}{2} + l\right)^2\right]^{1/2}$

$S'S_1 = \left[a^2 + \left(\frac{d}{2} - l\right)^2\right]^{1/2}$

$S'S_2 - S'S_1 = a + \frac{1}{2a}\left(\frac{d}{2} + l\right)^2 - \left[a + \frac{1}{2a}\left(\frac{d}{2} - l\right)^2\right]$

(Using Binomial expansion).

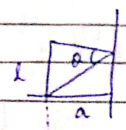
$= \frac{1}{2a}\left[\left(\frac{d}{2} + l\right)^2 - \left(\frac{d}{2} - l\right)^2\right]$

$$= \frac{1}{2a}\left(dl + dl\right) = \frac{dl}{a} = \Delta$$

When $\frac{ld}{a} \approx \frac{\lambda}{2}$, coherence is disturbed and pattern is washed off

$$\Rightarrow l = \frac{\lambda a}{2d} \approx \boxed{\frac{\lambda a}{d} = l}$$

due to corresponding points



as θ is very small.

Now $\theta = \frac{l}{a}$

$$\Rightarrow d = \frac{\lambda}{2\theta} \approx \frac{\lambda}{\theta} \Rightarrow \boxed{d = \frac{\lambda}{\theta}}$$ limiting value (l_w)

l_w : lateral coherence width $= \lambda/\theta$

max^m value to observe interference pattern.

$$\psi(x=0, t) = a e^{i\omega_0 t} \qquad |t| < \tau_c/2$$
$$= 0 \qquad |t| > \tau_c/2$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)\, e^{-i\omega t}\, dt$$

$$\psi(x=0, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega)\, e^{i\omega t}\, d\omega$$

$$A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x=0, t)\, e^{-i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\frac{\tau_c}{2}}^{\frac{\tau_c}{2}} a\, e^{i\omega_0 t} \cdot e^{-i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int a\, e^{it(\omega_0 - \omega)}\, dt.$$

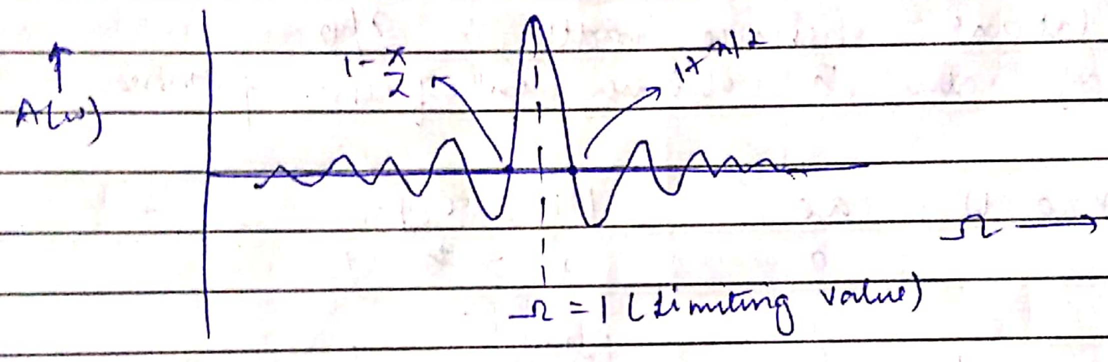
$$= \frac{a}{\sqrt{2\pi}} \left[\frac{e^{it(\omega_0 - \omega)}}{i(\omega_0 - \omega)}\right]_{-\tau_c/2}^{\tau_c/2}$$

$$= \frac{a}{\sqrt{2\pi}} \left[\frac{e^{i\frac{\tau_c}{2}(\omega_0-\omega)} - e^{-i\frac{\tau_c}{2}(\omega_0-\omega)}}{i(\omega_0-\omega)} \right]$$

$$A(\omega) = \sqrt{\frac{2}{\pi}}\, a \, \frac{\sin \frac{1}{2}(\omega-\omega_0)\tau_c}{(\omega_0-\omega)} \qquad \left(\because \frac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta \right)$$

Consider $\Omega = \omega/\omega_0$; $\alpha = \frac{1}{2}\omega_0 \tau_c$

$$A(\omega) = \sqrt{\frac{2}{\pi}}\, \frac{a}{\omega_0} \, \frac{\sin[\alpha(\Omega-1)]}{(\Omega-1)}$$



Zeroes of the func: $\alpha(\Omega-1) = \pm\pi$

$$\Rightarrow \Omega - 1 = \pm\frac{\pi}{\alpha} \Rightarrow \Omega = 1 \pm \frac{\pi}{\alpha}$$

Now, $\Omega = \omega/\omega_0$

$$\Delta\Omega = \frac{\Delta\omega}{\omega_0} \approx \frac{\pi}{\alpha} \qquad \left\{ \begin{array}{l} \text{Change in } \omega \text{ wrt} \\ \omega_0 \text{ is } \frac{\pi}{\alpha} \ (\Omega = 1 \text{ to} \\ \Omega = 1 + \frac{\pi}{\alpha}) \end{array} \right\}$$

$$\Delta\omega = \frac{\omega_0 \pi}{\alpha} = \frac{\omega_0 \pi}{\frac{1}{2}\omega_0 \tau_c} = \frac{2\pi}{\tau_c}$$

$$\Rightarrow 2\pi\,\Delta\nu = \frac{2\pi}{\tau_c} \Rightarrow \boxed{\Delta\nu = \frac{1}{\tau_c}}$$

Frequency spread corresponding to temporal time τ_c

(*) $\Psi = \Psi_1(P,t) + \Psi_2(P,t)$

$$|\Psi|^2 = \Psi_1^*\Psi_1 + \Psi_2^*\Psi_2 + \Psi_1^*\Psi_2 + \Psi_1\Psi_2^*$$

$$= |\Psi_1|^2 + |\Psi_2|^2 + 2\,\mathrm{Re}(\Psi_1^*\Psi_2) \quad (1)$$

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from ① & ② $\gamma_{12} = \frac{\psi_1^*(P,t)\,\psi_2(P,t)}{\sqrt{I_1 I_2}}$

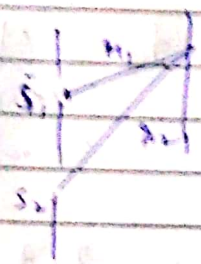
$= \frac{\psi_1^*(P,t)\,\psi_2(P,t)}{(\psi_1(P,t)^2\,\psi_2(P,t)^2)^{1/2}}$

$I_1 = \langle|\psi_1|^2\rangle$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,Re(\gamma_{12})$ ②

γ_{12} : complex degree of coherence

$$\gamma_{12} = \frac{\langle \psi_1^*(P,t)\,\psi_2(P,t)\rangle}{\langle|\psi_1(P,t)|^2\rangle\langle|\psi_2(P,t)|^2\rangle^{1/2}}$$



$$\psi_1(P,t) = \psi\left(S_1, t - \frac{r_1}{c}\right)$$

$$\psi_2(P,t) = \psi\left(S_2, t - \frac{r_2}{c}\right)$$

$$\gamma_{12} = \frac{\langle \psi^*\left(S_1, t - \frac{r_1}{c}\right)\psi\left(S_2, t - \frac{r_2}{c}\right)\rangle}{\langle|\psi(S_1, t - \frac{r_1}{c})|^2\rangle\langle|\psi(S_2, t - \frac{r_2}{c})|^2\rangle^{1/2}}$$

Now $\tau = \frac{r_2 - r_1}{c}$;

γ_{12} : measure of fringe contrast

$$\gamma_{12} = \frac{\langle \psi^*(S_1, t+\tau)\,\psi(S_2, t)\rangle}{\langle \psi(S_1, t+\tau)^2\,\psi(S_2, t)^2\rangle^{1/2}}$$

Now, as intensity from S_1 & S_2 is same (temporal coherence)

$$\gamma_{12} = \frac{\langle \psi^*(t+\tau)\,\psi(t)\rangle}{\langle|\psi(t)|\rangle^2}$$

As it is constant $(\sqrt{I_1 I_2})$ we can replace $t+\tau$ with t

Now let's say $\psi(t) = A(t)e^{-i[\omega t + \phi(t)]}$

$\psi^*(t+\tau)\,\psi(t) = A^2 e^{i\omega\tau}$

$|\psi(t)|^2 = A^2 = \psi^*(t)\,\psi(t)$

$\Rightarrow \gamma_{12} = e^{i\omega\tau}$

$Re(\gamma_{12}) = \cos\omega\tau$

From ② $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\omega\tau}$

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(Visibility) $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$= \dfrac{2\sqrt{I_1 I_2}}{I_1 + I_2}$

Now, $\gamma_{12} = \underset{\text{Real}}{|\gamma_{12}|}\, \underset{\text{Imaginary}}{e^{i(\omega\tau + \phi)}}$; $\omega\tau + \phi = \alpha$

$= |\gamma_{12}|\, e^{i\alpha}$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, |\gamma_{12}| \cos\alpha$

Max value $\cos\alpha = 1$ $\quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\, |\gamma_{12}|$

Min value $\cos\alpha = -1$ $\quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}\, |\gamma_{12}|$

So, $V = \dfrac{4\sqrt{I_1 I_2}\, |\gamma_{12}|}{2(I_1 + I_2)}$

$= \dfrac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma_{12}| \Rightarrow V \propto |\gamma_{12}|$

Visibility is directly related to the complex degree of coherence.

when $|\gamma_{12}| > 0.88$, contrast is good.

⇒ Interferogram:

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$

$I = I_0(1 + \cos\delta)$; where $\delta = \dfrac{2\pi}{\lambda}\Delta = \dfrac{2\pi\nu}{c}$

From freq range ν to $\nu + d\nu$

$I_t(\nu)\, d\nu = I(\nu)\, d\nu \left[1 + \cos\left(\dfrac{2\pi\nu\Delta}{c}\right)\right]$

Total intensity of the 2 sources when superimposed ⇒ Transmitted intensity which we observe

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$$I_t(\Delta) = \int_0^{\infty} I_t(\nu)\, d\nu$$

$$I_T = \int_0^{\infty} I(\nu)\, d\nu = \frac{I_t(0)}{2}$$

I_T : total intensity of a source

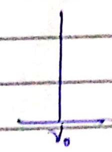
$$\underset{\text{Normalised Transmission}}{\gamma(\Delta)} = \frac{I_t(\Delta) - I_T}{I_T}$$

$$= \frac{1}{I_T} \int_0^{\infty} I(\nu) \cos\left(\frac{2\pi \nu \Delta}{c}\right) d\nu \quad \text{——} (3)$$

<u>Monochromatic source</u> :

$$I(\nu)\, d\nu = I_0\, \underbrace{\delta(\nu - \nu_0)}_{\text{Dirac Delta function}}\, d\nu$$

$$\gamma(\Delta) = \frac{I_0 \int_0^{\infty} \delta(\nu - \nu_0) \cos \frac{2\pi\nu\Delta}{c}\, d\nu}{\int_0^{\infty} I_0\, \delta(\nu - \nu_0)\, d\nu}$$



The function is zero for all values other than $\nu = \nu_0$
At $\nu = \nu_0$; $\delta(\nu - \nu_0) = 1$

$$\gamma(\Delta) = \frac{I_0 \cos \frac{2\pi\nu_0\Delta}{c}}{I_0}$$

$$\Rightarrow \quad \gamma(\Delta) = \cos \frac{2\pi\nu_0\Delta}{c} \quad \text{for all values of } \Delta.$$

If the source is perfectly monochromatic, the coherence is infinite (temporal time is infinite)