ASSIGNMENT-02

EP-208 COMPUTATIONAL METHODS

(Integration & Ordinary Differential Equations)

1. Evaluate the integral: $I = \int_{y=1}^{1.6} \int_{x=1}^{1.6} \frac{dxdy}{x+y}$ using Simpson's 3/8-Rule and taking six subintervals along x and y –axis.
2. Evaluate $\int_0^1 \int_0^1 e^{x+y} dxdy$ using trapezoidal and Simpson's 1/3 rule. (Take $h = k = 0.50$).

3. The table given below reveals the velocity V of a body during the time t. Find the acceleration of the body at x = 1.1 meter

x(meter)	1.0	1.1	1.2	1.3	1.4
V(meter/sec)	43.1	47.7	52.1	56.4	60.8

4. Find the value of y for x=0.1 by Picard's method, given that
$$\frac{dy}{dx} = \frac{y-x}{y+x} , \qquad y(0) = 1$$
5. Employ Taylor's method to obtain approximate value of y at x=0.2 for the differential equation
$$\frac{dy}{dx} = 2y + 3e^x , \qquad y(0) = 0$$
Compare the numerical solution obtained with exact solution.
6. Evaluate $y(0.1)$ correct to four decimal places using Taylor's series methods if
$$\frac{dy}{dx} = x^2 + y^2 , \qquad y(0) = 1$$
7. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, with initial condition y=1 at x =0; Find y for x=0.1 by Euler's method
8. Solve the following by Euler's modified method:
$$\frac{dy}{dx} = \log(x+y) , \qquad y(1) = 2$$
at x=1.2 and 1.4 with h=0.2.
9. Using Euler's modified method, obtain a solution of the equation
$$\frac{dy}{dx} = x + \sqrt{y}$$
with initial conditions y=1 at x=0, for the range $0 \le x \le 0.6$ in steps of 0.2.

10. Apply fourth-order-Runge-Kutta method to find the approximate value of *y* for *x*=0.2, in steps of 0.1

$$\frac{dy}{dx} = x + y^2$$

Given that *y*=1 where *x*=0.

11. Using Runge-Kutta method of fourth order, find *y* for *x*=0.1 ,0.2 ,0.3 given that

$$\frac{dy}{dx} = xy + y^2, \qquad y(0) = 1$$

12. Given: $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$y(0) = 1$, $y'(0) = 0$, obtain *y* for *x*=0.1,0.2 by any method Runge-Kutta method of fourth order.

13. Apply Picard's method to find the third approximation to the value of y and z, given that

dy/dx=z, dz/dx=x^3(y+z). given y=1, z=1/2 when x=0

EP 208

Assignment - 2

Integration & ODE

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$y \backslash x$	1	1.1	1.2	1.3	1.4	1.5	1.6
1	0.50	0.47	0.45	0.43	0.416	0.40	0.384
1.1	0.47	0.45	0.43	0.416	0.40	0.384	0.370
1.2	0.45	0.43	0.416	0.40	0.384	0.370	0.357
1.3	0.43	0.416	0.40	0.384	0.370	0.357	0.344
1.4	0.416	0.40	0.384	0.370	0.357	0.344	0.333
1.5	0.40	0.384	0.370	0.357	0.344	0.333	0.322
1.6	0.384	0.370	0.357	0.344	0.333	0.322	0.312

$$\int_1^{1.6}\int_1^{1.6} \frac{dx\,dy}{x+y} = \frac{3h}{8}\frac{3k}{8}\left[f(1) + f(1.6) + 2f(1.3) + 3\left[f(1.1) + f(1.2) + f(1.4) + f(1.5)\right]\right]$$

$$= \frac{3h}{8}\left[I_0 + I_6 + 2I_3 + 3\left(I_1 + I_2 + I_4 + I_5 \right)\right]$$

$$I_0 = \frac{3k}{8}\left[f(1,1) + f(1,1.6) + 2f(1,1.3) + 3\{ f(1,1.1) + f(1,1.2) + f(1,1.4) + f(1,1.5)\}\right]$$

$$= \frac{3\times 0.1}{8}\left[0.384 + 0.5 + 0.86 + 5.208 \right] = 0.2607 .$$

$$I_2 = \frac{3K}{8}\left[f(1.2,1) + f(1.2,1.6) + 2f(1.2,1.3) + 3\{ f(1.2,1.1) + f(1.2,1.2) + f(1.2,1.4) + f(1.2,1.5)\}\right]$$

$$= \frac{3\times 0.1}{8}\left[0.45 + 0.357 + 5.6 \right] = 0.2402 .$$

$$I_2 = \frac{3k}{8}\Big[f(1.1,1) + f(1.1,1.6) + 2f(1.1,1.3) + 3\{ f(1.1,1.1) + f(1.1,1.2) + f(1.1,1.4) + f(1.1,1.5)\}\Big]$$

$$= \frac{3\times 0.1}{8}\left[0.774 + 0.768 + 4.629 \right] = 0.2314 .$$

$$I_3 = \frac{3k}{8}\Big[f(1.3,1) + f(1.3,1.6) + 2f(1.3,1.3) + 3\{ f(1.3,1.1) + f(1.3,1.2) + f(1.3,1.4) + f(1.3,1.5)\}\Big]$$

$$= \frac{3\times 0.1}{8}\left(0.774 + 0.768 + 4.929 \right) = 0.2426 .$$

$$I_4 = \frac{3k}{8}\Big[f(1.4,1) + f(1.4,1.6) + 2f(1.4,1.3) + 3\{ f(1.4,1.1) + f(1.4,1.2) + f(1.4,1.4) + f(1.4,1.5)\}\Big]$$

$$= \frac{3\times 0.1}{8}\left(0.749 + 0.740 + 4.455 \right) = 0.2229 .$$

$$I_5 = \frac{3k}{8}\Big[f(1.5,1) + f(1.5,1.6) + 2f(1.5,1.3) + 3\{ f(1.5,1.1) + f(1.5,1.2) + f(1.5,1.4) + f(1.5,1.5)\}\Big]$$

$$= \frac{3\times 0.1}{8}\left(0.722 + 0.714 + 4.293 \right) = 0.2148 .$$

$$I_6 = \frac{3k}{8}\Big[f(1.6,1) + f(1.6,1.6) + 2f(1.6,1.3) + 3\{ f(1.6,1.1) + f(1.6,1.2) + f(1.5,1.4) + f(1.6,1.5)\}\Big]$$

$$= \frac{3\times 0.1}{8}\left(0.696 + 0.688 + 4.137 \right) = 0.207$$

$$I = \frac{3\times 0.1}{8}\left[0.2607 + 0.2070 + 0.4853 + 2.7279 \right]$$

$$= \underline{0.1380} \quad \text{Ans.}$$

$\boxed{2}$ $\int_0^1\int_0^1 e^{x+y}\,dx\,dy$ $h=k=0.5$

$y \backslash x$	0	0.5	1
0	1	1.64	2.71
0.5	1.64	2.71	4.48
1	2.71	4.48	7.38

#1 Trapezoidal Method

$$\int_a^b\int_c^d f(x,y)\,dx\,dy = \frac{hk}{4}\left(\text{Sum of corner} + 2(\text{sum rem.}) + 4\times\text{interior}\right).$$

$$\int_0^1\int_0^1 e^{x+y}\,dx\,dy = \frac{0.5\times0.5}{4}\Big((1+2.71+2.71+7.38)+2(1.64+1.64+4.48+4.48)+4\times2.71\Big).$$

$$= 3.0762.$$

Simpson's rule.

$$I=\frac{hk}{9}\Big[f(0,0)+4f(0,0.5)+f(0,1)$$
$$+4\{f(0.5,0)+4f(0.5,0.5)+f(0.5,1)\}$$
$$+f(1,0)+4f(1,0.5)+f(1,1)\Big]$$

$$= \frac{0.5\times0.5}{9}\Big[1+4(1.648)+2.71+4(1.6487+4\times2.71+4.4817)$$
$$+2.71+4\times4.48+7.38\Big].$$

$$= 2.9545$$

$\boxed{3}$

$$a = \frac{dv}{dt} = \frac{v\,dv}{dx}$$

$$\cancel{\int dt \neq \int \frac{1}{v}\,dx}$$

$$\cancel{t \neq \int_{1.0}^{1.4} \frac{1}{v}\,dx}$$

$$a(1.1) = V(1.1) \times \left.\frac{dv}{dx}\right]_{x=1.1\,m}$$

using Newton's

forward differentiation

x	V	ΔV	$\Delta^2 V$	$\Delta^3 V$	$\Delta^4 V$
1.0	43.1				
		4.6			
1.1	47.7		-0.2		
		4.4		0.1	
1.2	52.1		-0.1		0.1
		4.3		0.2	
1.3	56.4		0.1		
		4.4			
1.4	60.8				

$h = x_0 - x$

$= 1.1 - 1.0 = \underline{0.1}$

$$\left.\frac{dv}{dx}\right|_{x=1.1} = \frac{1}{h}\left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0\right)$$

$$= \frac{1}{0.1}\left(4.4 + \frac{0.1}{2} + \frac{0.2}{3} - \frac{1}{4}\times 0\right) = 45.16667$$

$$\therefore \quad a(1.1) = V(1.1) \times \left.\frac{dv}{dx}\right|_{x=1.1}$$

$$= 47.7 \times 45.16667$$

$$= 2154.4502 \ m/s^2$$

$\boxed{4}$ $y = 1 + \int_0^x \frac{y-x}{y+x}\,dx.$

put $y=1$,

$$y_1 = 1 + \int_0^x \frac{y-x}{y+x}\,dx = 1 + \int_0^x \left(-1 + \frac{2}{1+x}\right)dx$$

$$= 1 + \left[-x + 2\log(1+x)\right]_0^x$$

$$= 1 - x + 2\log(1+x)$$

put $y = 1 - x + 2\log(1+x)$

$$y_2 = 1 + \int_0^x \frac{1-x+2\log(1+x) - x}{1-x+2\log(1+x)+x}\,dx$$

$$= 1 + \int_0^x \left[1 - \frac{2x}{1+2\log(1+x)}\right]dx.$$

taking $x = 0.1$,

$$y(0.1) = 1 - (0.1) + 2\log(1+0.1)$$

$$= \underline{0.9828}.$$

$\boxed{5}$

$y'' = 2y' + 3e^x$, $\quad y''(0) = 2y'(0) + 3 = 9$

$y''' = 2y'' + 3e^x$, $\quad y'''(0) = 21$

$y^{iv} = 2y''' + 3e^x$, $\quad y^{iv}(0) = 45.$

Taylor series,

$$y(x) = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \dots$$

$= 0 + 3x + \frac{9}{2}x^2 + \frac{21}{6}x^3 + \frac{45}{24}x^4 + \cdots$

$= 3x + \frac{9}{2}x^2 + \frac{21}{6}x^3 + \frac{15}{8}x^4 + \cdots$

Now, $y(0.2) = 3(0.2) + 4.5(0.2)^2 + 3.5(0.2)^3 \cdots$

$= 0.811$ — ⓐ.

for exact soluth,

$\frac{dy}{dx} - 2y = 3e^x$.

$ye^{-2x} = \int 3e^x e^{-2x}\,dx + c$

$= -3e^{-x} + c$

$y = -3e^x + ce^{2x}$

at $y(0) = 0$, ~~c=0~~

$\boxed{c = 3}$ $\Rightarrow$ $y = 3(e^{2x} - e^x)$.

when $x = 0.2$, $y = 3(e^{0.4} - e^{0.2}) = 0.8112$ — ⓑ.

Comparing ⓐ and ⓑ,

$= b - a \Rightarrow 0.8112 - 0.8110$

$= 0.0002$

$\boxed{6}$ $\frac{dy}{dx} = x^2+y^2$; $y(0) = 1$.

$y' = x^2+y^2$, $y'(0) = 1$

$y'' = 2x+2yy'$, $y''(0) = 2$

$y''' = 2+2y'^2+2yy''$, $y'''(0) = 8$

$y^{iv} = 6y'y''+2yy'''$, $y^{iv}(0) = 28$.

Taylor series,

$$y_1 = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \cdots$$

$$= 1 + (0.1)\cdot 1 + \frac{(0.1)^2}{2!}(2) + \frac{(0.1)^3}{3!}(8) + \cdots$$

$$= 1 + 0.1 + 0.01 + 0.0013 + 0 + \sim$$

$$y(0.1) = \underline{1.1115}$$

$\boxed{7}$ $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

euler method,

$$y_1 = y_0 + xf(x_0, y_0)$$

$$= 1 + (0.1)f(0, 1)$$

$$y(0.1) = 1 + (0.1)(1)$$

$$= \underline{1.1}$$

$\boxed{8}$ $\frac{dy}{dx} = \log(x+y)$, $y(1) = 2$, $h = 0.2$

Modified Euler,

$$y_{m+1} = y_m + hf\left(x_m + \frac{1}{2}h,\ y_m + \frac{1}{2}hf(x_m, y_m)\right)$$

$$y_1 = y_0 + hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}hf(x_0, y_0)\right)$$

$$= y_0 + hf(1.1,\ 2.0477)$$

$$y(0.2) = 2 + 0.2(0.488) = \underline{2.0996}$$

Now, $f(x_1, y_1) = f(1.2, 2.0996) = 0.5185$

$$y_2 = y_1 + hf\left(x_1 + \frac{1}{2}h,\ y_1 + \frac{1}{2}hf(x_1, y_1)\right)$$

$$= 2.0996 + 0.2(0.538)$$

$$y(1.4) = \underline{2.2072}$$

$\boxed{9}$ $\frac{dy}{dx} = x + \sqrt{y}$, $y(0) = 1$, $h = 0.2$.

$f(x_0, y_0) = f(0, 1) = 1$.

$x_0 + \frac{1}{2}h = 0 + \frac{0.2}{2} = 0.1$

$y_0 + \frac{1}{2}hf(x_0, y_0) = 1 + \frac{0.2}{2} \times 1 = 1.1$

$$y_1 = y_0 + hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}hf(x_0, y_0)\right)$$

$$y(0.2) = 1 + 0.2 \times 1.1488 = \underline{1.2298}$$

$f(x_1, y_1) = f(0.2, 1.2289) = 1.3089$

$x_1 + \frac{1}{2}h = 0.2 + \frac{0.2}{2} = 0.3$

$y_1 + \frac{1}{2}hf(x_1, y_1) = 1.2298 + \frac{0.2}{2} \times 1.3089 = 1.3607$

$y_2 = y_1 + hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}hf(x_1, y_1)\right)$

$= 1.2298 + 0.2 \times 1.4665.$

$y(0.4) = \boxed{1.5231}.$

$f(x_2, y_2) = f(0.4, 1.5231) = 1.6341$

$f(x_2 + \frac{1}{2}h, y_2 + \frac{h}{2}f(x_2, y_2) = f(0.5, 1.6865)$

$= 1.7986$

$y_3 = y_2 + h \times 1.7986$

$y(0.6) = 1.531 + 0.2 \times 1.7986 = \boxed{1.8828}.$

10 $K_1 = hf(x_0, y_0) = 0.1 f(0,1) = 0.1$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = (0.1)(1.1525) = 0.1153$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = 0.1(1.1686) = 0.1169$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.1(1.3474) = 0.1347$

$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= 1 + \frac{1}{6}(0.1 + 2(0.1153) + 2(0.1169) + 0.1347))$

$y(0.1) = \boxed{1.1165}$

$$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 1.1165 + \frac{1}{6}\left(0.1347 + 2(0.1551) + 2(0.1576) + 0.1823\right)$$

$$\boxed{y(0.2) = 1.2736}$$

$\boxed{11}$

$$\frac{dy}{dx} = xy + y^2, \quad y(0) = 1, \quad h = 0.1$$

$$K_1 = hf(x_0, y_0) = 0.1(1) = 0.1$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.1(1.155) = 0.1155$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = 0.1(1.1717) = 0.1172$$

$$K_4 = hf(x_0 + h, y_0 + K_3) = 0.1(1.358) = 0.136.$$

$$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$\boxed{y(0.1) = 1.1169}$$

Now,

$$K_1 = hf(x_1, y_1) = 0.1(1.3591) = 0.1359$$

$$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) = 0.1(1.5816) = 0.1582$$

$$K_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right) = 0.161$$

$$K_4 = hf(x_1 + h, y_1 + K_3) = 0.1888$$

$$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$\boxed{y(0.2) = 1.2774}$$

"y,
$$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= 1.2774 + \frac{1}{6}(0.1887 + 2(0.2225) + 2(0.2275) + 0.2716)$$
$$\boxed{y(0.3) = 1.5041}$$

[12]

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

Let $z = \frac{dy}{dx} \Rightarrow \frac{dz}{dx} = \frac{d^2y}{dx^2}$

$$\frac{dz}{dx} = f(x, y, z) \qquad \left.\begin{aligned} z' &= -xz - y \\ z &= y' \end{aligned}\right\} \quad \begin{aligned} y(0) &= 1, \\ z(0) &= 1. \end{aligned}$$

Runge-Kutta 4th order,

$j_1 = hf(x_0, y_0, z_0 = 0.1(f(0,1,0) = -0.1$
$k_1 = hg(x_0, y_0, z_0) = 0.1g(0,1,0) = 0.$

$j_2 = hf(x_0 + h/2, y_0 + k_1/2, z_0 + j_1/2) = -0.0998$
$k_2 = hg(x_0 + h/2, y_0 + k_1/2, z_0 + j_1/2) = -0.005$

$j_3 = hf(x_0 + h/2, y_0 + k_2/2, z_0 + j_2/2) = -0.0995$
$k_3 = hf(x_0 + h/2, y_0 + k_2/2, z_0 + j_2/2) = -0.0005$

$j_4 = hf(x_0 + h, y_0 + k_3, z_0 + j_3) = -0.09005$
$k_4 = hf(x_0 + h, y_0 + k_3, z_0 + j_3) = -0.00005$

$$j = \frac{1}{6}(j_1 + 2j_2 + 2j_3 + j_4) = -0.0994$$
$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = -0.0018416$$

$$z(0.1) = z(0) + J = 0 - (-0.0994)$$
$$= \underline{0.0994}$$

$$\underline{y(0.1)} = y(0) + K = 1 + (-0.0018416)$$
$$= \underline{0.99815}$$

Ily,

$$J' = \frac{1}{6}(j_1' + 2j_2' + 2j_3' + j_4') = 0.6118$$
$$K' = \frac{1}{6}(k_1' + 2k_2' + 2k_3' + k_4') = -0.02528.$$

$$z(0.2) = z(0.1) + J' = 0.0994 - 0.6118$$
$$= \underline{-0.5124}$$

$$\underline{y(0.2)} = y(0.1) + K' = 0.99815 - 0.02528$$
$$= \underline{0.99562}$$

$\boxed{13}$ $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, $y(0) = 1$, $z(0) = \frac{1}{2}$.

Let,

$$y_{n+1} = y_n + \int_{x_0}^{x} f(x, y_n, z_n)\,dx$$
$$z_{n+1} = z_n + \int_{x_0}^{x} g(x, y_n, z_n)\,dx.$$

first approx

$$y_1 = y_0 + \int_{x_0}^{x_0} f(x, y_0, z_0)\,dx$$
$$= 1 + \int_0^x (\tfrac{1}{2})\,dx = 1 + \frac{x}{2}.$$

$$z_1 = z_0 + \int_{x_0}^{x} g(x, y_0, z_0)\,dx$$
$$= \tfrac{1}{2} + \int_0^x \frac{3x^3}{2}\,dx = \frac{1}{2} + \frac{3}{8}x^4.$$

second approx

$$y_2 = y_0 + \int_{x_0}^{x} \left(\frac{1}{2} + (3/8) x^4 \right) dx$$

$$= 1 + \left[\int_0^x \left(\frac{x}{2} + \frac{3}{40} x^5 \right) \right]_0^x = 1 + \frac{x}{2} + \frac{3x^5}{40}$$

$$z_2 = z_0 + \int_0^x \left(\frac{3}{8} x^7 + \frac{x^4}{2} + \frac{3x}{2} \right)$$

$$= 1/2 + \frac{3x^8}{64} + \frac{x^5}{10} + \frac{3}{8} x^4 + \ldots$$

third approx

$$y_3 = y_0 + \int_{x_0}^{x} \left(3/64 \, x^8 + 1/10 \, x^5 + 3/8 \, x^4 + \frac{1}{2} \right) dx.$$

$$y_3 = 1/192 \, x^9 + 1/60 \, x^6 + 3/40 \, x^5 + 1/2 \, x + 1.$$

$$z_3 = z_0 + \int_{x_0}^{x} \left(3/64 \, x^{11} + \frac{7}{40} x^8 + \frac{3}{8} x^7 + \frac{1}{2} x^4 + \frac{3}{2} x^3 \right) dx$$

$$= 1/2 + \frac{1}{256} x^{12} + \frac{7}{360} x^9 + \frac{3}{64} x^8 + \frac{1}{10} x^5 + \frac{3}{8} x^4.$$

END