superconductivity : It is the ability of certain materials to conduct electric current with practically zero resistance.

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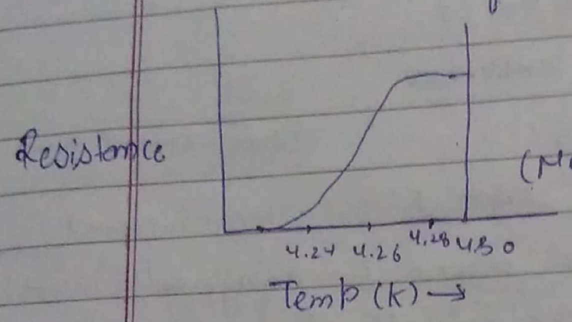
CMP (Friday class)

Superconductivity,

Properties: ① zero resistance
② Perfect demagnetic (have low (≈0) resistance)



(Mercury)

T_c → Transition Temp

T_c – 4.2K

La–Ba–Cu–O → T_c = 34K

* Meissner Effect :- The ~~expeltsion~~ expulsion of magnetic ~~flask~~ flux from a superconducting material when it cooled below the transition temperature ~~when it is~~ in a magnetic field is called Meissner effect

Cooled

→ When we decrease the temp. below T_c then, magnetic flux away from the sample.

→ When we increase the temp. above the T_c then, magnetic flux is Penetrated.

B → magnetic field, $B = \mu_0 (H + M)$

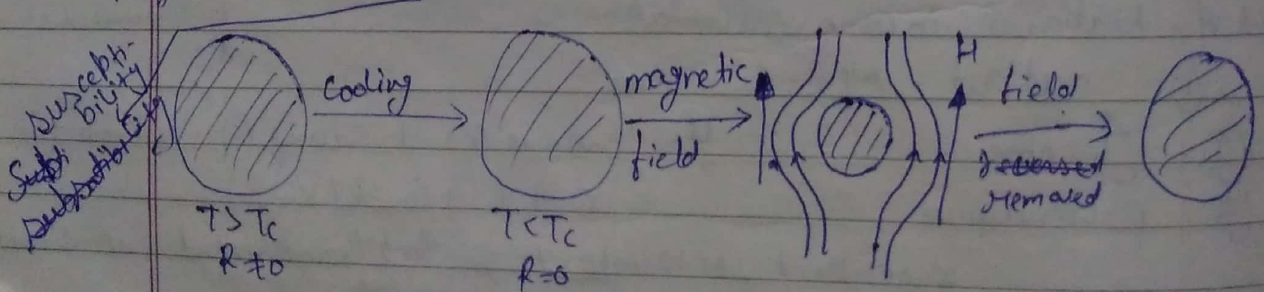
H → strength of M.F. $H = -M$

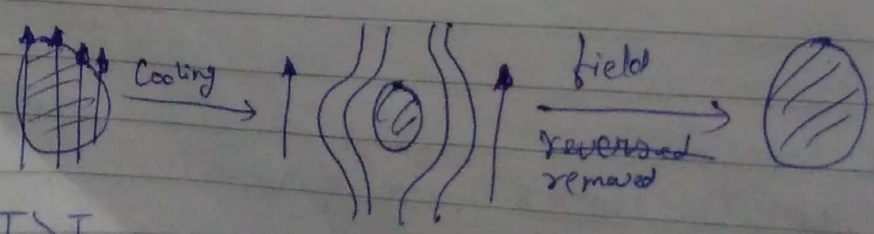
M → Magnetization or intensity of magnetic field.

$$\boxed{\chi = \frac{M}{H} = -1}$$

Susceptibility

A.C.T. Meissner effect,
$(\therefore B = 0)$
so, $\mu_0 \neq 0$
$\therefore H + M = 0$





T > T_c

: Magnetic Behaviour of Superconductor.

$\rho = 0$ resistivity | → The value of magnetic field at which it will destroy superconductivity even temp is below T_c.

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→ At zero resistance → when mag. flux varies → can we calculated by ~~maxwell~~ equation

$$\nabla \times E = \frac{-\partial B}{\partial T} \quad \text{(Maxwell equation)}$$

$$\rho = 0$$

$$E = \rho J \qquad J \to \text{current density}$$

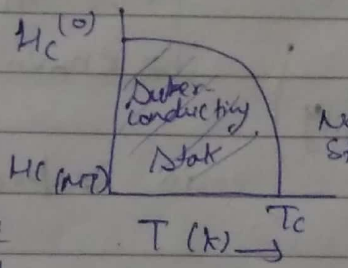
$$\therefore E = 0$$

$$\therefore \frac{\partial B}{\partial T} = 0 \quad \to B \to \text{Constant}$$

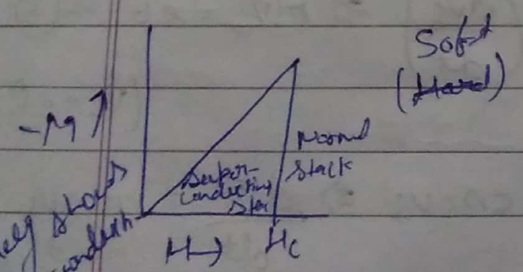
* Critical Magnetic field (H_c):- The tem. at which the magnetic field/resistance caused to zero or superconducting vanishes below the transition temp.

$H_c \to$ Max. Critical strength

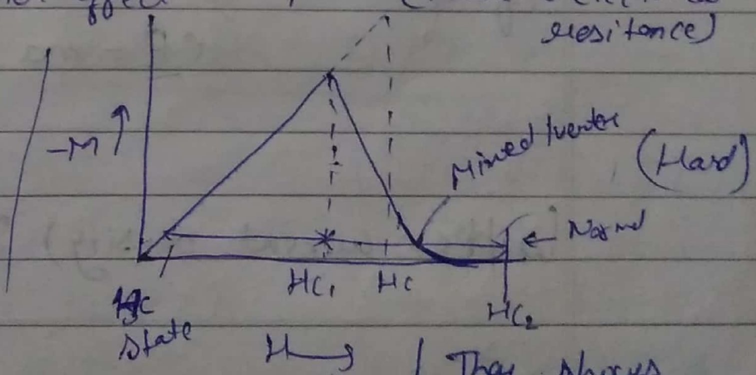
$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



* Superconductor classified in 2 ways:-
① ~~Hard~~ → Completely messiner effect → soft
② ~~Soft~~ → Partially Messiner effect → Hard (have 0 electrical resistance)



They shows superconductivity upto $H_c(T_c)$ and after that they will shows normal conductor



upto H_{c_1} → completely Messiner effect show
" H_{c_1} to H_{c_2} → Penetrating Messiner effect
after H_{c_2} → Normal State (Conductor).

They shows superconductivity even after $H_c(T_c)$ at

Silsbee effect → if H_c is less than 0.1 T, high magnetic field cannot be Produced ~~the~~

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→ Types of superconductor
→ Maxwell's eq.

→ Dynamics of superconductor

At any tem. sum of superconductor and normal conduct. is Total conductor

London Equations (Statement → Book)

n_n → normal current density   no. of normal $\bar{e}$'s

n_s → Super " "   no. of superconduct $\bar{e}$'s

(Total no. of $\bar{e}$) $n = n_n + n_s$ — (1) v_n = velocity of normal $\bar{e}$

$J = J_n + J_s$ v_s = velocity of super electrons

(Total current density) $J = en_nv_n + en_sv_s$ — (2)

in superconductor, there is no resistance,

if voltage is apply

eq. of Motion for superconductivity $\bar{e}$'s

$$eE = ma = m\left(\frac{dv_s}{dt}\right) \Rightarrow m\dot{v}_s = eE \quad \text{— (3)}$$

$$\left(\dot{v}_s = \frac{eE}{m}\right)$$

(supper current density) $J_s = en_sv_s \Rightarrow \frac{dJ_s}{dt} = n_s.e\frac{dv_s}{dt}$

$$\Rightarrow \frac{dJ_s}{dt} = \frac{n_se^2E}{m}$$

1st London eq.

$$\Rightarrow \boxed{\frac{dJ_s}{dt} = \frac{n_se^2E}{m}} \quad \text{— (A)}$$

it express zero resistance Property

Critical

Critical current (I_c) → Minimum current that passes in a sample without destroying its superconductivity.

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When E is 0 ∴ J_s is constant
i.e., after removable of electric field, there is a current
So it express the zero resistance property

Take both side curl in eq. (A)

$$\nabla \times \frac{\partial J_s}{\partial t} = -\frac{n_s e^2}{m} \nabla \times E$$

$$\nabla \times \frac{\partial J_s}{\partial T} = -\frac{n_s e^2}{m} \frac{\partial B}{\partial t}$$

using Maxwell eq
$\nabla \times E = \dot{B} = -\frac{\partial B}{\partial t}$

integrating both sides

$$\nabla \times J_s = -\frac{n_s e^2}{m} B$$

$$\boxed{B = \frac{-m}{n_s e^2} \, \text{curl} \, J_s}$$

2nd eq. London eq.

it explains

When $B = 0$ ∴ J_s is conservative

✱ Derivation for the Penetration Depth

$$\nabla \times B = \mu_0 J_s$$

using
$\vec{a} \times \vec{b} \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

Take curl both sides

$\nabla \times \nabla \times B$
$= (\nabla . B)\nabla - (\nabla . \nabla)B$
$= 0 - \nabla^2 B$

$$\nabla \times \nabla \times B = \mu_0 \nabla \times J_s$$

$$-\nabla^2 B = \mu_0 \nabla \times J_s$$

$$-\nabla^2 B = -\mu_0 \frac{n_s e^2 B}{m}$$

$\left(\nabla(\nabla . B) - \nabla^2 B = 0 - \nabla^2 B \Rightarrow -\nabla^2 B \right)$

$$\nabla^2 B = \mu_0 \frac{n_s e^2 B}{m} = \frac{1}{\lambda^2} B$$

in case of 1-D,

formula for Penetration depth (λ)

If in 1D $\frac{\partial^2 B}{\partial x^2} = \frac{1}{\lambda^2} B$

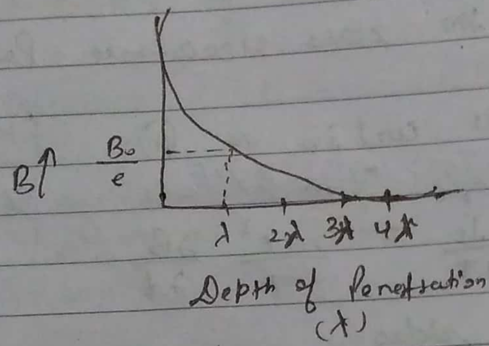
$$\left[\therefore \lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{\frac{1}{2}} \right]$$

$\lambda \rightarrow$ Penetration Depth

(sol^n for 1-D)

$$B_z(x) = B_z(0) \exp\left(-\frac{x}{\lambda}\right)$$

$B_0 \rightarrow$ inital flux

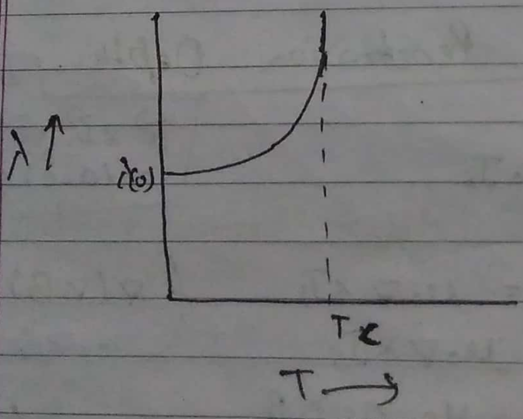


$$\boxed{\text{curl } J_s = \frac{-B}{\mu_0 \lambda^2}}$$

$$\boxed{J_s = \frac{E}{\mu_0 \lambda^2}}$$

How Penetration Depth varies with temperature.

* Effect of temp. on Penetration.



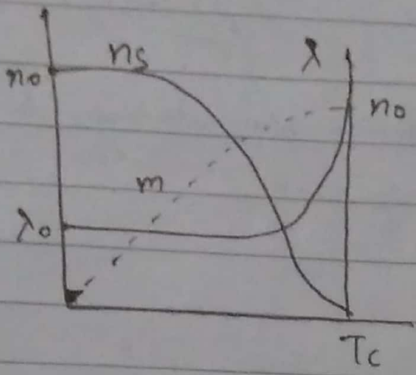
$T_c \rightarrow$ Critical Tempertaue
$\rightarrow$ Penetration is infinite at T_c

$$\lambda(T) = \lambda(0)\left(1 - \frac{T^4}{T_c^4}\right)^{-\frac{1}{2}}$$

at $T = T_c$, Normal conductor is convert into Superconductor.

at low temp, it is independent of temp.

When co $W = \frac{n_s}{n_0} = \left(1 - \frac{T^4}{T_c^4}\right)$



∴ no is ↑ing
Penetration depth increases rapidly and approches infinity as the temp. approches the T_c

When λ is below Tc
Normal co

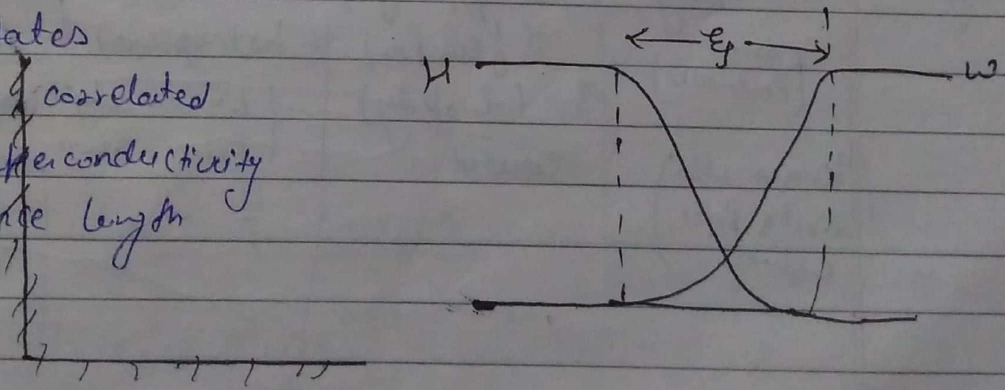
* Characteristic of superconductor.
Related to ~~enge~~ energy gap &

VF & ~~enge~~ energy gap

Coherence length :-

$$\xi_0 = \frac{2\hbar V_F}{\pi \Delta}$$

The max. distance up to which the states of pair ē are correlated to produced superconductivity is called coherence length



It is a distance within which the order parameter is changes from its max value in the bulks superconductor region to zero in the normal region. The distance is known as coherence length.

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* Application of Supercondutor
(1) ~~stuttdtz~~ SQUIDS
(2) High TC Superconductor
(3) Magnetic levaition
(4) Cryotron
~~Application~~

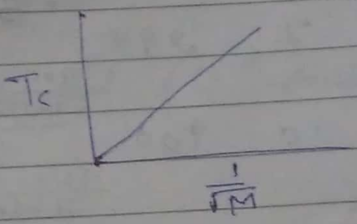
$T_c \to$ Transtition temp.
High $T_c \to$ high transtition

(1) SQUIDS → Superconductor Quantum

Isotope effects:

$$T_c \propto M^{-\frac{1}{2}}$$

$$T_c \propto \frac{1}{\sqrt{M}}$$



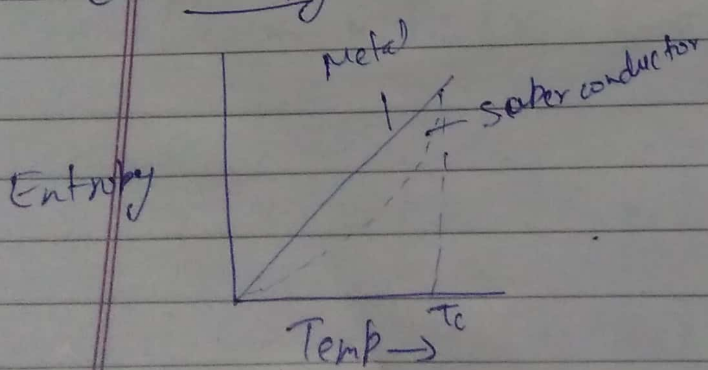
$T_c M^{1/2} = \text{Constant}$, This is isotope effect

if there is a change in 199.5 to 203.4 mass then, there is change in Transtition temp. at 4.185K to 4.146

* Thermal Property.

(1) Entropy (randomness)

As temp. ↑ing, ē starts Vibration at random motion That's randomness is known as Entropy.



Entropy is decrease below Transition temperature
S.C has more order that Metal

Superconductivity affects ē's mainly, so the lattice vibration part remains unaffected. That means the lattice part is same in both normal & superconducting states.

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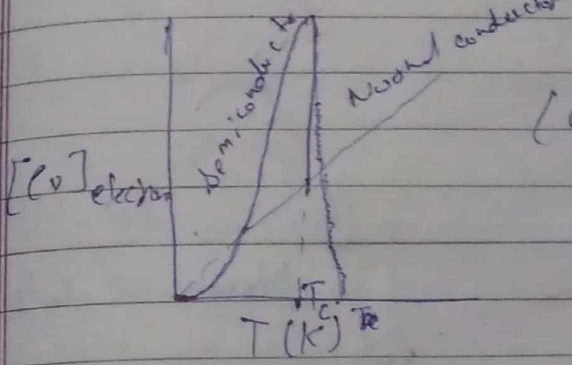
(2) Specific Heat : release from 2 source, 1st is lattice & 2nd is electronic)

Shows a Jump at T_c

$$C_n(T) = \alpha T + BT^3$$ — lattice contribution

αT — electronic contribution.

$\therefore BT^3$ is same in both normal & superconducting states.



$$(C_{el})_s = A\exp\left(\frac{-\Delta}{k_BT}\right)$$

$\Delta = b k_B T_c$, b → constant

$$C = C_{lattice} + C_{electron}$$

el → electron

(Normal electron) $C_N = (C_{lattice})_N + (C_{el})_N$

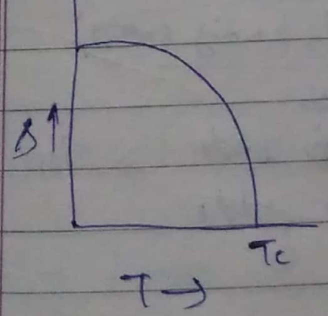
~~s → superconductor~~ $C_s = (C_{lattice})_s + (C_{el})_s$

s → specific heat

$$(C_{lattice})_s = (C_{lattice})_N$$

$$C_N - C_s = (C_{el})_N - (C_{el})_s$$

(3) Energy Gap



For superconductor, ~~Enger~~ Energy gap vary with temp.

When E.G = 0, supercond. convert into normal conductor.

Numerical based on
Isotop eq.
Penetration form magnetic field
London eq.

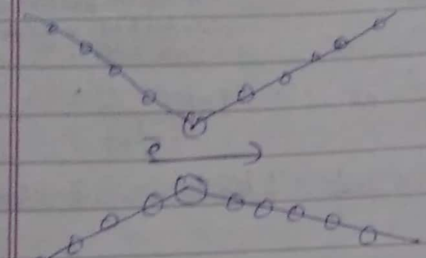
E.G exists at firmy gas

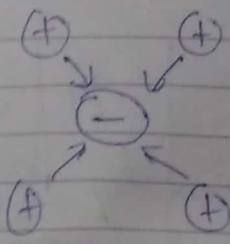
E.G. has no affect on special ē. that carry current in a superconductor. superconductor contains normal ē's as well and these ē's are affected by E.G.

* BCS Theory: (Bardeen, Cooper, Schrieffer): It divides into 3 Parts

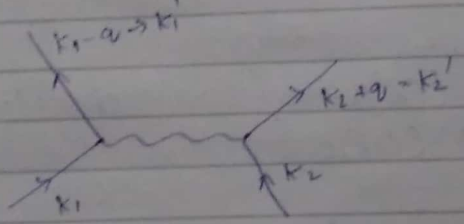
① Electron ~~full on~~ phonon and electron interaction.

phonon: Quantize vibrated energy





Book
- phenon - e interaction
- isotope effect: Mass affect the -



$$k_1 + k_2 = k_1' + k_2'$$

Phono energy dominate the e energy e's are interacted

② Cooper Plate → (Certain cooper Plate & Binding energy)

energy of e in Pair state is less than the energy of e in free state

Temp. > T_c e doesn't make Pair so there is no energy gap

Binding energy (Bond energy): energy require to ~~separate the~~ break the cooper Plate

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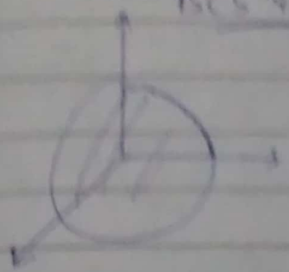
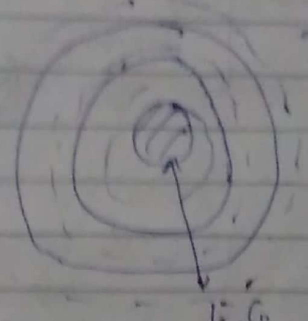
$\xi_0 = \frac{\hbar v_F}{E_g}$ $E_g \rightarrow$ Binding energy
$v_F \rightarrow$ Fermi velocity

** On Fermi level thermal energy going to next state, there is no Fermi...

BCS Ground State

Superconductor

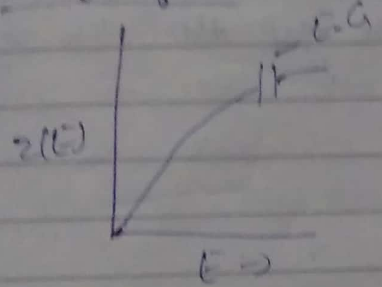




if any state ?

Below the Fermi surface all the state is occupying by e's

density of state



* To ~~Flux~~ Flux quantization

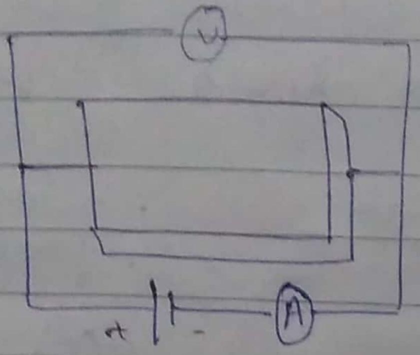
flux passing through rings is quantization flux.

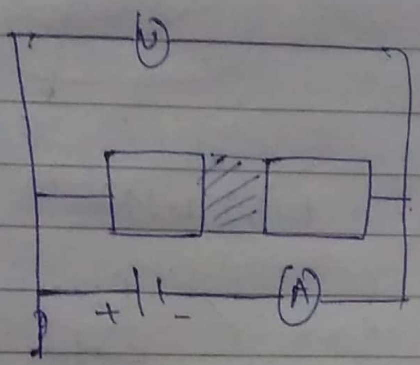
$$\Phi = n\Phi_0 = \frac{nh}{2e}$$

$$\Phi_0 = \frac{h}{2e} = 2.068 \times 10^{-15} \text{ weber}$$

unit of flux is fluxoid.

* Josephson Tunneling.





Current is not flow the b/w the superconductor

if there is no voltage, then DC is flowing b/w the semiconductors
If emitts from the gap then this is AC

$$f = \frac{2eV}{h}$$

→ DC Josephson effect
A DC flows across the juction of 2 superconductor saprated by thin insulater wire in the absence of any magnetic field called DC Josephson effect.

→ A DC voltage Applied across the juction of 2 superconductors saparated by a thin insulator wire causes Rf