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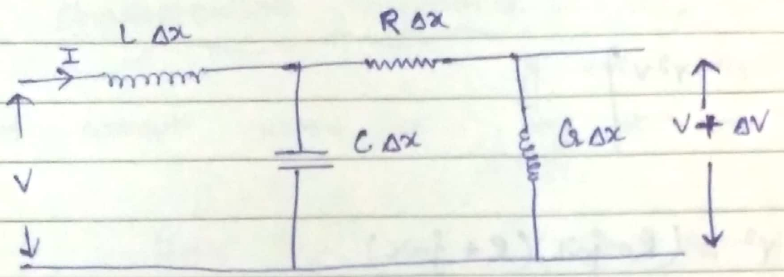
Transmission lines

Transmission line acts as as communication media

e.g. 2 wire line, co-axial lines

- the communication b/w transmitter & receiver through a transmission line is called wire line communication.



Transmission line

R - Resistance

L - inductance

C - capacitance

G - admittance $[G = 1/Z]$

Z - impedance

I - current

ΔV - voltage drop across arms

$$\Delta V = -(R\Delta x + j\omega L\Delta x)I$$

$$\Rightarrow \frac{\Delta V}{\Delta x} = -(R + j\omega L)I \quad \text{—(1)}$$

at $x \to 0$

$$\boxed{\frac{dV}{dx} = -ZI} \quad \text{—(A)} \quad ; \quad \boxed{Z = R + j\omega L}$$

total impedance

Now, $\Delta I = -(G\Delta x + j\omega C\Delta x)V$

$$\frac{\Delta I}{\Delta x} = -(G + j\omega C)V$$

at $x \to 0$

$$\boxed{\frac{dI}{dx} = -YV} \quad \text{—(2)}$$

Y = total admittance

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Now, differentiating (A) w.r.t x

$$\frac{d^2V}{dx^2} = -(R+j\omega L)\frac{dI}{dx}$$

So,

$$\frac{d^2V}{dx^2} = +(R+j\omega L)(G+j\omega C)V$$

$$\left[\frac{d^2V}{dx^2} = \gamma^2 V\right]$$

where

$$\gamma^2 = (R+j\omega L)(R+j\omega C)$$

Similarly in x-dir.

$$\left[\frac{d^2V}{dx^2} = \gamma^2 V\right]$$

* All the eqn(s) are applicable to transient soln of both V and I and are fn of x or in general t.

* Soln of Transmission Line Equations :-

Soln of $\frac{d^2V}{dx^2} = \gamma^2 V$ is

$$V = V_+ e^{-\gamma x} + V_- e^{\gamma x} = V_+ e^{-\alpha x} e^{-j\beta x} + V_- e^{\alpha x} e^{j\beta x}$$

$$[\gamma = \alpha + i\beta]$$

where V_+ and V_- represent complex quantities.

$e^{-j\beta x} \rightarrow$ wave travelling in +ve x dir.

$e^{j\beta x} \rightarrow$ wave going in -ve x dir.

[$\beta x \rightarrow$ electrical length of the line]

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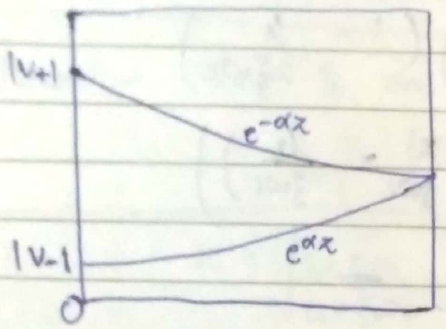
also,

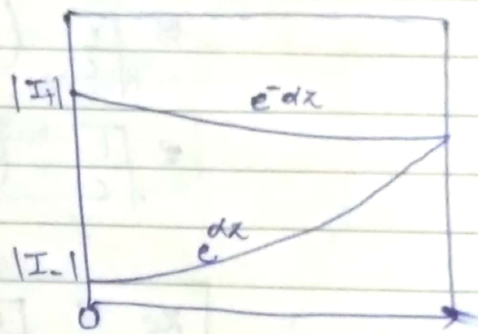
$$I = Y_0 \left[V_+ e^{-\gamma x} - V_- e^{\gamma x}\right]$$

$$\Rightarrow Y_0 \left\{V_+ e^{-\alpha x} e^{-j\beta x} - V_- e^{\alpha x} e^{j\beta x}\right\}$$

Characteristic impedance :- (Z_0) of line is defined as the ratio of +ve transmitting voltage to the line current wave at any pt. on line

$$Z_0 = \frac{V_+}{I_+} = \frac{V_-}{I} \quad \text{and} \quad Z = \frac{1}{Y_0} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$





At microwave frequencies it can be seen that

$R <<< \omega L$ and $G <<< \omega C$

$$\gamma = \sqrt{(R + j\omega L)(R + j\omega C)}$$

$$\sqrt{(j\omega)^2 LC}\ \sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{R}{j\omega C}\right)}$$

$$\approx j\omega\sqrt{LC}\left[\left(1 + \frac{R}{2j\omega L}\right)\left(1 + \frac{R}{2j\omega C}\right)\right]$$

$$\approx j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{R}{j\omega C}\right)\right]$$

$$\gamma = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{C}{L}}\right) + j\omega\sqrt{LC}$$

$$\gamma = \alpha + i\beta$$

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$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\beta = \omega\sqrt{LC}$$

$\|^{ly}$, the characteristic impedance is found to be

$$Z_0 = \sqrt{\frac{R+j\omega L}{R+j\omega C}}$$

$$\Rightarrow \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

$$\approx \sqrt{\frac{L}{C}}\left(1+\frac{R}{2j\omega L}\right)\left(1-\frac{G}{2j\omega C}\right)$$

$$\approx \sqrt{\frac{L}{C}}\left(1+\frac{1}{2}\left(\frac{R}{j\omega L}-\frac{G}{j\omega C}\right)\right)$$

$$\boxed{Z_0 \approx \sqrt{\frac{L}{C}}}$$

So, phase velocity

$$\left[u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}\right]$$

* transmission lines can be characterized into 3 categories :-

1. Loss less line
2. Distortion less line
3. low Less line

* loss less line are one in which at radio frequency microwave frequency, the inductive reactance is much larger than that series resistance & capacitive reactance is much larger than shunt resistance.

$$R = G = 0$$

$$R <<< L\omega \quad \& \quad G <<< \omega C$$

* Distortion line is one in which condition $\frac{R}{L} = \frac{G}{C}$ is satisfied

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

Using condn of ~~loss~~ less loss line

$$Z = \sqrt{\frac{j\omega L}{j\omega C}} \Rightarrow \sqrt{\frac{L}{C}}$$

So, $R_0 = \sqrt{\frac{L}{C}}$

So, $R_0 = \sqrt{\frac{L}{C}}$ & $X_0 = 0$

→ Characteristic impedance of line is purely resistive.

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Distortion less line :-

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R}{G}\left(\frac{1 + R/j\omega L}{1 + G/j\omega C}\right)}$$

in distortion less line,

$$\frac{R}{L} = \frac{G}{C}$$

$$\frac{R}{G} = \frac{L}{C}$$

$$Z = \sqrt{\frac{L}{C}\left(\cancel{\frac{1 + G/j\omega C}{1 + G/j\omega C}}\right)}$$

$$\boxed{Z = \sqrt{\frac{L}{C}}}$$

$$R_0 = \sqrt{\frac{L}{C}} \quad \& \quad X_0 = 0$$

* Distortion less line are loss less line.

* **Low less line :-**

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Rightarrow \sqrt{\frac{j\omega L}{j\omega C}\left(\frac{1 + R/j\omega L}{1 + G/j\omega C}\right)}$$

$$Z_0 = \sqrt{\frac{L}{C}}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{-1/2}$$

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$$Z_0 = \left(\frac{L}{C}\right)^{1/2} \left(1 + \frac{R}{2j\omega L}\right)\left(1 - \frac{G}{2j\omega C}\right)$$

$$\left(\frac{L}{C}\right)^{1/2} + \frac{1}{2}$$

$$\left(\frac{L}{C}\right)^{1/2} \left(1 + \frac{R}{2j\omega L} - \frac{G}{2j\omega C}\right)$$

$$\sqrt{\frac{L}{C}} + \frac{1}{2}\left[\frac{R}{j\omega}\left(\frac{L}{C}\right)^{1/2}\right] - \left[\frac{1}{2}\frac{G}{2j\omega C}\left(\frac{L}{C}\right)^{1/2}\right]$$

$$\sqrt{\frac{L}{C}} + \frac{1}{2j\omega}\left[\frac{R}{L}\sqrt{\frac{L}{C}} - \frac{G}{C}\sqrt{\frac{L}{C}}\right]$$

$$\boxed{R \approx \sqrt{\frac{L}{C}}}$$

$$X_0 = \frac{1}{2\omega}\left(\frac{R}{\sqrt{LC}} - \frac{G\sqrt{L}}{C\sqrt{C}}\right)$$

* phase velocity :-

$$v_p = \omega/\beta$$

propagation constant

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z}$$

$\gamma \rightarrow$ ~~propo~~ propagation const.

$$\gamma = \alpha + j\beta$$

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① loss less line

$$R = G = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta$$

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

loss line has zero attenuation constant $\alpha = 0$

② low Less line :-

$R <<< L\omega$ & $G <<< C\omega$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\sqrt{(j\omega L)(j\omega C)}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

$$j\omega\sqrt{LC} + \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$

$$j\omega\sqrt{LC} + \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$

$$\alpha = \frac{R}{2Z_0} + \frac{GZ_0}{2} \qquad \beta = \omega\sqrt{LC}$$

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$$\gamma = \sqrt{ZY} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\boxed{v_p = \frac{\omega}{\beta} = 1/\sqrt{LC}}$$

iii) distortion line :-

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$j\omega\sqrt{LC}\left(1 + \frac{G}{j\omega C}\right) \qquad \left[\text{as } \frac{R}{L} = \frac{G}{C}\right]$$

$$j\omega\sqrt{LC}\left(\frac{j\omega C + G}{j\omega C}\right) \qquad G\sqrt{\frac{C}{L}}$$

$$\cancel{\sqrt{\tfrac{C}{L}}\; j\omega C / \sqrt{LC}} + \cancel{\frac{G\sqrt{LC}}{L}}$$

$$G\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\boxed{\alpha = G\sqrt{\frac{C}{L}} \quad \& \quad \beta = \omega\sqrt{LC}}$$

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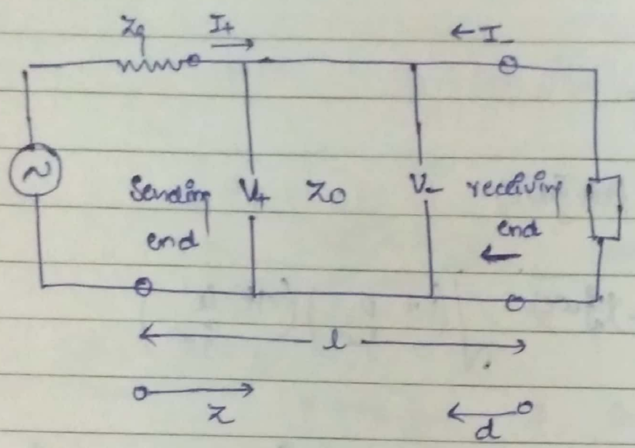
* Reflection coefficient :-

Soln of transmission line contains 2 components : one travelling in the +ve z direction & the other travelling in the -ve z dir. If the load impedance is equal to the line characteristic impedance, reflected travelling wave does not exist.



Now,

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$

$$I = I_+ e^{-\gamma z} + I_- e^{\gamma z}$$

Now, current can be expressed in terms of voltage

$$I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_- e^{\gamma z}}{Z_0}$$

If the line has length l, then

$$V_l = V_+ e^{-\gamma l} + V_- e^{+\gamma l}$$

$$I_l = \frac{V_+ e^{-\gamma l}}{Z_0} - \frac{V_- e^{+\gamma l}}{Z_0}$$

$\Rightarrow$ Ratio of the voltage to the current at the receiving end is the load impedance i.e.

$$\boxed{Z_l = \frac{V_l}{I_l} = Z_0 \frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}}}$$

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Now,

Reflection coefficient, Γ (gamma) is defined as

$$\boxed{\Gamma = \frac{\text{reflected voltage or current}}{\text{incident voltage or current}}}$$

$$\left[\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{I_{ref}}{I_{inc}} \right]$$

$$\boxed{\Gamma = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

usually, it is a complex quantity.

So,

$$\boxed{\Gamma_L = |\Gamma_L| e^{j\theta_L}}$$

→ magnitude & never greater than unity

θ → phase angle b/w incident & reflected voltages at the receiving end

general soln is

$$\boxed{\Gamma = \frac{V_- e^{-\gamma z}}{V_+ e^{\gamma z}}}$$

$$\Gamma = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} \Rightarrow \frac{V_-}{V_+} e^{2\gamma l}$$

$$Z_L = \frac{V_L}{I_L} = Z_0 \left(\frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}} \right)$$

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$$Z_L = Z_0 \frac{V_+ e^{-\gamma l}}{V_+ e^{\gamma l}} \frac{\left(1 + \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}\right)}{\left(1 - \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}\right)}$$

$$\boxed{Z_L = Z_0 \left(\frac{1+\Gamma_L}{1-\Gamma_L}\right)}$$

$$Z_L(1-\Gamma_L) = Z_0(1+\Gamma_L)$$

$$Z_L - \Gamma_L Z_L = Z_0 + \Gamma_L Z_0$$

$$Z_L - Z_0 = \Gamma_L (Z_L + Z_0)$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

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