Assignment - I

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1. Define Antenna Array. What is meant by the gain of an Antenna? Show that in case of N-Elements linear Arrays the normalized array factor is given by

$$(AF)_n \simeq \left[\frac{\sin \frac{N}{2}\psi}{\frac{N}{2}\psi}\right].$$

2. What is an Antenna Arrays? What are the reasons for using Antenna Arrays? Explain in detail the behaviour of Broadside and End-fire Arrays.

3. Differentiate between Broadside and End-fire Arrays. Derive the expression for the Array factor of linear broadside array of N-elements.

4. What is End-fire Array? Deduce an Expression for the radiation pattern of an Endfire Array with N-vertical dipoles.

EMT

Assignment

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A1. Antenna array is a set of multiple connected antennas which work together as a single antenna, to transmit or receive radio waves.

The radio waves radiated by each individual antenna combine and superpose, to enhance the power radiated in desired directions and cancelling in other directions.

Antenna gain is measure of effectiveness with which the antenna can radiate the power delivered towards target.

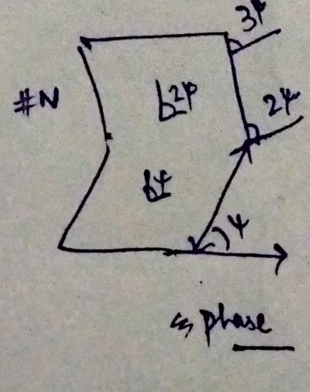
$$G = \frac{4\pi\eta A}{\lambda^2}$$ where η = efficiency, A = aperture area.

Array Factor

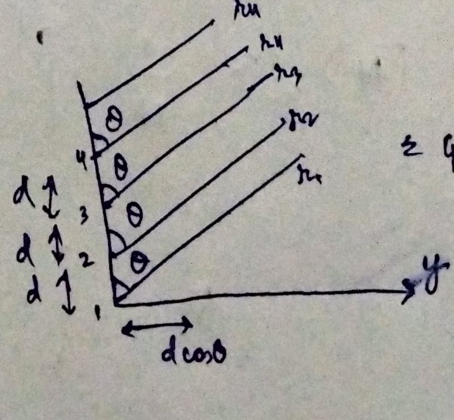
$$(AF) = 1 + e^{j(kd\cos\theta + \beta)} + e^{j2(kd\cos\theta + \beta)} + \cdots + e^{j(N-1)(kd\cos\theta + \beta)} \quad \text{— (1)}$$

$$= \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta + \beta)}$$ where β = progressive wave.

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad \text{— (2)}, \quad \psi = kd\cos\theta + \beta.$$



$\Rightarrow$ phase



$\Rightarrow$ Geometry.

Multiplying by $e^{j\psi}$ on both sides,

$$(AF)\, e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(N-1)\psi} + e^{jN\psi} \quad \text{— (3)}$$

Subtracting (3) from (2),

$$AF(e^{j\psi} - 1) = (e^{jN\psi} - 1)$$

$$AF = \frac{(e^{jN\psi} - 1)}{(e^{j\psi} - 1)} = e^{j\frac{(N-1)}{2}\psi} \left[\frac{e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}}{e^{j\frac{\psi}{2}} - e^{-j\psi/2}} \right]$$

$$AF = e^{j\left(\frac{N-1}{2}\right)\psi} \left(\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin \psi/2} \right) \quad \text{— (4)}$$

Considering reference pt, N = 1,

$$AF = \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}$$ without oscillatory point,

$$AF \approx \frac{\sin \frac{N\psi}{2}}{\psi/2},$$ for small values of ψ, $\sin \psi/2 = \psi/2$.

$$(AF)_{max} = N \left(\frac{N\psi/2}{\psi/2} \right).$$

Normalized Array factor,

$$(AF)_n = \frac{1}{N} \left[\frac{\sin \frac{N\psi}{2}}{\sin \psi/2} \right]$$

$$(AF)_n \approx \frac{1}{N} \left[\frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}} \right]$$

$$\approx \left[\frac{\sin \frac{N\psi}{2}}{\frac{N}{2}\psi} \right].$$

Ans 2. Antenna arrays consist of a set of multiple connected antennas in a properly spaced and phased manner which act as the point elements. resultant wave at any point is the superposition of each point source.

Advantages :

- increased signal range.
- high directivity
- interference from a particular set of directions can be cancelled.
- array enables the option to increase sensitivity in a specific direction.
- signal to noise ratio and interference is increased.
- higher gain with reduced power wastage.

In an endfire array, indentical elements are fed equal magnitude current but phase varying progressively along the line.

There is no radiation perpendicular to plane of array due to cancellation as first and third elements are fed out-of-phase current.

Broadside and End-fire have the same physical arrangement. Both are linear and resonant as they consist of resonant elements, having narrower beam and high directionality.

Ans 3. **Broadside Array :**

- array elements are fed with currents of equal magnitudes (amplitudes) and in phase.
- direction of maximum radiation is perpendicular to the direction of array axis.
- phase difference, $\alpha = 0$.
- radiation pattern is bidirectional.

End-fire Array :

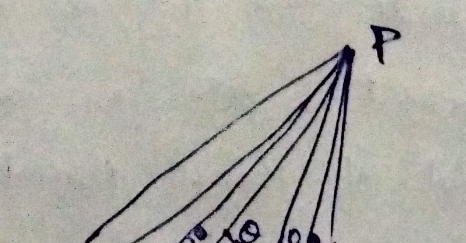
- array elements are fed with currents of equal magnitude and out of phase.
- maxm radiation directions is along the direction of the array's axis.
- phase difference, $\alpha = -\beta d$.
- radiation pattern is unidirectional.

Let a linear array of n-element with spacing b/w consecutive elements being 'd', initial phase diff. is α',

$\therefore$ total phase difference

$$\psi = k d \cos\theta + \alpha$$

$(k = 2\pi/\lambda)$.



Electric field at P,

$$\vec{E} = \sum_{i=1}^{n} E_i = E_1 + E_2 + E_3 + \dots + E_n$$

$$\vec{E} = e^{j(0\psi)} + e^{j(\psi)} + e^{j2\psi} + \dots + e^{j(n-1)\psi} \quad \text{—(1)}$$

Multiplying both sides by $e^{j\psi}$,

$$\vec{E}\,e^{j\psi} = e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi} \quad \text{—(2)}$$

(1) − (2),

$$\vec{E} = \frac{1 - e^{nj\psi}}{1 - e^{j\psi}} = \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left(\frac{e^{-jn\psi/2} - e^{jn\psi/2}}{e^{-j\psi/2} - e^{j\psi/2}} \right)$$

$$\vec{E} = e^{j(n-1)\psi/2} \left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin(\psi/2)} \right]$$

now, $\psi \to 0$, $\vec{E} \to \vec{E}_{max}$.

$$\lim_{\psi \to 0} \vec{E} = \lim_{\psi \to 0} e^{j(n-1)\psi/2} \left(\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right) = n$$

$\underline{E_{max} = n}$.

$$\frac{E}{E_{max}} = AF = \frac{1}{N} \left(\frac{\sin n\psi/2}{\sin \psi/2} \right)$$

for broadside array, max^n radiation is perpendicular to axis of array, $\theta = \pi/2$ or $3\pi/2$.

1st maxima at $\psi = 0$, $\psi = kd\cos\theta + \alpha$,

$\alpha = 0$, $k\cos\theta + \alpha = 0$, $\psi = kd\cos\theta$.

$$(AF)_n = \frac{1}{n} \left[\frac{\sin\left(\frac{nkd\cos\theta}{2}\right)}{\sin\left(\frac{kd\cos\theta}{2}\right)} \right]$$

Ans 4 Endfire array is an antenna array in which a number of identical elements which are fed of phase current of equal magnitude.

Main beam in radiation pattern is along array axis and pattern is unidirectional.

End-fire Arrays are designed to focus the main beam of the array factor along the array axis in either $\theta = 0°$ or $\theta = 180°$.

Max^m value of array factor occurs

when, $\psi = kd\cos\theta + \alpha = 0$,

at $\theta = 0°$, $\cos\theta = 1$, $\theta = 180°$, $\cos\theta = -1$,

$\alpha = -kd$ $\qquad\qquad$ $\alpha = kd$

α must be $\pm kd$,

$\psi = kd(\cos\theta \mp 1)$

$$(AF)_n = \frac{1}{n}\left[\frac{\sin\left(n\frac{kd(\cos\theta \mp 1)}{2}\right)}{\sin\left(\frac{kd(\cos\theta \mp 1)}{2}\right)}\right].$$

normalized array factor.