Unit 3

Guided Waves

Introduction

Point-to-point communication and numerous other applications require electromagnetic waves to remain localised in the transverse direction, as they propagate. However, waves of finite transverse size suffer diffraction divergence. They also suffer scattering losses due to particles, refractive index irregularities and other obstacles in their path. Such losses can be overcome by guiding the waves through coaxial cables, metallic pipes, glass fibres and planar guiding systems. The guiding systems have other advantages too. For instance, wave guides loaded with slow wave structures can reduce the phase velocity of microwaves much below the velocity of light in vacuum, facilitating the amplification of these waves by electron beams (via *Cerenkov interaction*) in travelling wave tubes. Guided slow waves, under suitable conditions, can be used to accelerate electrons to ultra-relativistic energies as in a linear accelerator (LINAC). Microwave and optical passive devices like directional couplers, circulators, attenuators, etc. too involve guided propagation.

In this unit, we study guided propagation of electromagnetic waves through various kinds of guiding systems. We begin with the surface plasma wave (SPW), that is guided along the interface between a conductor and a dielectric. Then, we study the propagation of waves through parallel plane guiding system, rectangular waveguide, cylindrical waveguide, sheath helix, rippled wall guiding systems and coaxial cable. The propagation through optical fibres is taken up separately in the chapter on optical communication.

A common feature of all the guiding systems, except the rippled wall configuration, is that in one direction (chosen as z axis in this chapter) electrical and dielectric properties of the medium or guiding structure do not change. Hence, for a monochromatic wave of frequency ω propagating along the z direction, the t, z variations of fields can be taken as

$$\mathbf{E}, \mathbf{H} \sim e^{-i(\omega t - k_z z)}. \tag{1}$$

Then, the only problem is to solve the wave equation in one or two space variables, with proper boundary conditions, to deduce the dispersion relation (k_z as a function of ω) and the transverse variation of field amplitudes (called mode structure).

3.1 Surface Plasma Waves

A surface plasma wave (also known as a surface wave) is a guided electromagnetic wave that propagates along the boundary between a conductor and a dielectric. Its amplitude is maximum on the boundary and falls off, away from it, in either medium.

Consider a conductor-dielectric interface at $x = 0$ (Fig. 3.1). The conductor, characterised by effective relative permittivity ϵ_{eff}, occupies the half space $x < 0$, while the dielectric of relative permittivity ϵ_r lies in $x > 0$. ϵ_{eff} is related to dielectric constant ϵ_L, electrical conductivity σ of the conductor and the frequency ω of the wave as

$$\epsilon_{eff} = \epsilon_L + \frac{i\sigma}{\omega\,\epsilon_0}.$$

We presume t, z variations of fields as given in eq. (1), take y-variation to be zero and look for solutions of wave equation that fall off with $|x|$, as one moves away from the interface. If we choose $\mathbf{E} \parallel \hat{y}$, we do not get such a solution hence we discard this case. We consider $\mathbf{E} = E_x\,\hat{x} + E_z\,\hat{z}$. The procedure is as follows. We solve

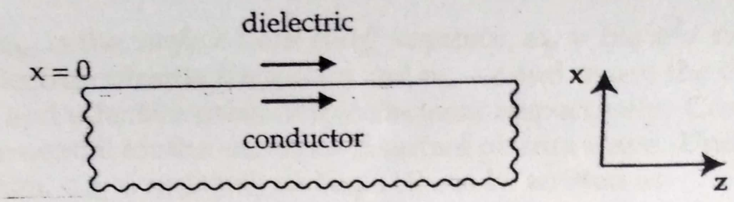

Figure 3.1: Schematic of surface plasma wave propagation over a conductor - dielectric interface

the wave equation for E_z in the two media, deduce E_x, H_y from E_z using the relevant Maxwell's equations and apply the continuity of E_z and H_y across $x = 0$ to obtain the dispersion relation.

~~The third and~~ fourth Maxwell's equations, on replacing $\partial/\partial t$ with $-i\omega$, can be written as

$$\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}, \tag{2}$$

$$\nabla \times \mathbf{H} = i\omega \epsilon_0 \epsilon'_{eff} \mathbf{E}, \tag{3}$$

where $\epsilon'_{eff}=\epsilon_{eff}$ for $x < 0$ and $\epsilon'_{eff}=\epsilon_r$ for $x > 0$. Taking curl of eq. (2), using eq. (3) and the identity $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E}$, we get

$$\nabla^2\mathbf{E} - \nabla(\nabla.\mathbf{E}) + \frac{\omega^2}{c^2}\epsilon'_{eff}\mathbf{E} = 0. \tag{4}$$

$\nabla.$ of this equation, in each of the two media, gives $\epsilon'_{eff}\ \nabla.\ \mathbf{E} = 0$, i.e., $\nabla.\ \mathbf{E} = 0$. Hence eq. (4) becomes

$$\nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\epsilon'_{eff}\mathbf{E} = 0. \tag{5}$$

Using $\partial/\partial z = ik_z$ to write $\nabla^2 = \partial^2/\partial x^2 - k_z^2$ in eq. (5), we obtain for E_z,

$$\frac{\partial^2 E_z}{\partial x^2} - \alpha^2 E_z = 0, \tag{6}$$

$$\text{where} \quad \alpha^2 = \alpha_1^2 \equiv k_z^2 - \omega^2\frac{\epsilon_r}{c^2} \quad \text{for } x > 0,$$

$$\alpha^2 = \alpha_2^2 \equiv k_z^2 - \omega^2\frac{\epsilon_{eff}}{c^2} \quad \text{for } x < 0. \tag{7}$$

The well-behaved solution of eq. (6), that vanishes at $x \to \pm\infty$, is

$$E_z = Ae^{-\alpha_1 x}e^{-i(\omega t - k_z z)}, \quad x > 0$$
$$= A'e^{\alpha_2 x}e^{-i(\omega t - k_z z)}, \quad x < 0. \qquad (8)$$

Demanding the continuity of E_z at $x = 0$, we get $A' = A$. Using eq. (8) in $\nabla . \mathbf{E} = 0$, we obtain

$$E_x = \left(i\frac{k_z}{\alpha_1}\right) Ae^{-\alpha_1 x}e^{-i(\omega t - k_z z)}, \quad x > 0$$
$$= -\left(i\frac{k_z}{\alpha_2}\right) Ae^{\alpha_2 x}e^{-i(\omega t - k_z z)}, \quad x < 0. \qquad (9)$$

Substituting for E_x and E_z in eq. (2), we get the magnetic field of the wave as

$$H_y = i\left(\omega\frac{\epsilon_r}{\mu_0 c^2 \alpha_1}\right) Ae^{-\alpha_1 x}e^{-i(\omega t - k_z z)}, \quad x > 0$$
$$= -i\left(\omega\frac{\epsilon_{eff}}{\mu_0 c^2 \alpha_2}\right) Ae^{\alpha_2 x}e^{-i(\omega t - k_z z)}, \quad x < 0. \qquad (10)$$

Applying the continuity of H_y (or $\epsilon'_{eff}\, E_x$) at $x = 0$, we get $\epsilon_{eff}/\epsilon_r = -\alpha_2/\alpha_1$ or

$$k_z^2 = \frac{\omega^2}{c^2}\frac{\epsilon_r \epsilon_{eff}}{\epsilon_r + \epsilon_{eff}}. \qquad (11)$$

This is the dispersion relation for surface plasma wave. If ϵ_r and ϵ_{eff} are real, k_z^2 is real. The decay rates of wave amplitude in the two media are α_1 and α_2 respectively, given by

$$\alpha_1 = \frac{(\omega\ \epsilon_r\ /c)}{[-(\epsilon_r + \epsilon_{eff})]^{\frac{1}{2}}}$$
$$\alpha_2 = \frac{(\omega/c)}{[-(\epsilon_r + \epsilon_{eff})/\ \epsilon_{eff}^2]^{\frac{1}{2}}}. \qquad (12)$$

For α_1, α_2 to be real one must have $\epsilon_r + \epsilon_{eff} < 0$. In a conductor, if collisions are ignored, $\sigma = -n_0 e^2/\ mi\omega$, $\epsilon_{eff} = \epsilon_L - \omega_p^2/\omega^2$. This condition gives

$$\omega < \omega_{sc}; \quad \omega_{sc} = \frac{\omega_p}{(\epsilon_r + \epsilon_L)^{\frac{1}{2}}} \qquad (13)$$

where ω_{sc} is the *surface wave cutoff frequency*, $\omega_p = (n_0 e^2 / m \epsilon_0)^{1/2}$ is the electron plasma frequency and n_0, $-e$ and m are the density, charge and effective mass of free electrons respectively. Condition (13) is essential for the existence of surface plasma wave. Under this condition, ϵ_{eff} is negative and eq. (11) can be written as

$$k_z^2 = \frac{\omega^2 \epsilon_r}{c^2} \frac{\omega_p^2/\omega^2 - \epsilon_L}{\omega_p^2/\omega^2 - (\epsilon_L + \epsilon_r)}. \tag{14}$$

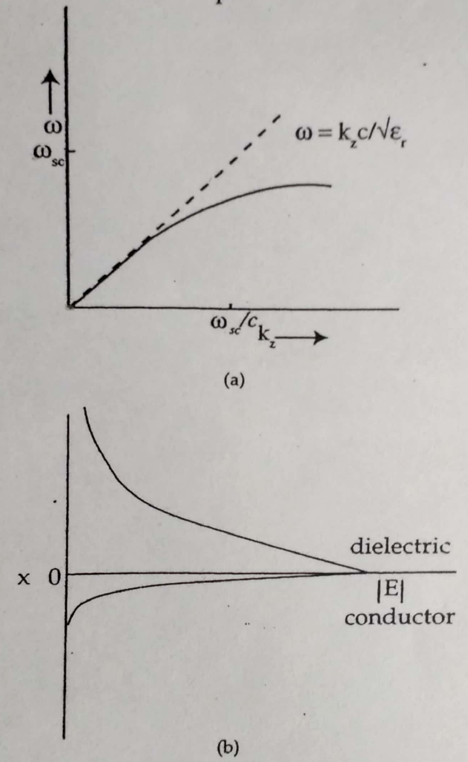

Figure 3.2: (a) Surface plasma wave dispersion relation
(b) amplitude of surface plasme wave as a function of distance from the interface.

We have plotted the variation of ω with k_z in fig. 3.2 . At low frequencies $\omega << \omega_p/(\epsilon_L + \epsilon_r)^{1/2}$, ω varies linearly with k_z and the phase velocity of the wave is $c/\sqrt{\epsilon_r}$. In this case $\alpha_1 = \omega^2 \epsilon_r / c\,\omega_p$, $\alpha_2 = \omega_p \epsilon_r / c$, i.e. $\alpha_1 << \alpha_2$, and the wave is localised in a thin layer of thickness $\sim \alpha_2^{-1}$ in the conductor. However, it extends to a larger width $\sim \alpha_1^{-1}$ in the dielectric. As k_z increases, ω increases less than linearly and asymptotically approaches the value $\omega_{sc} = \omega_p/(\epsilon_L + \epsilon_r)^{1/2}$ as $k_z \rightarrow \infty$. Near this cutoff frequency, the surface plasma wave is strongly localised near the interface.

At optical frequencies, surface plasma waves are usually excited over metals by a laser, using attenuated total reflection configuration (Fig. 3.3). In this scheme, a gold film of thickness $d \leq 0.1\ \mu m$ is deposited over a prism of refractive index η_g. Laser is launched through the other face of the prism onto the glass-metal interface, at angle of incidence θ_i. Usually, 99% of laser light gets reflected back.

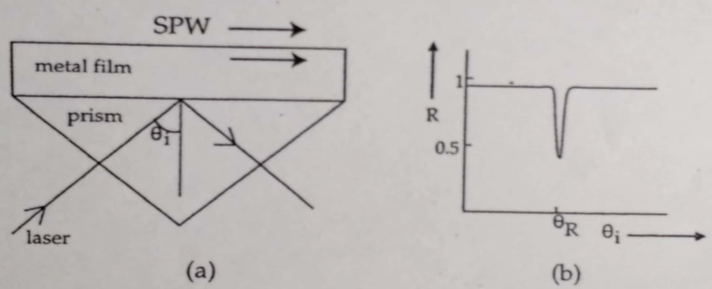

Figure 3.3: (a) Schematic of attenuated total reflection (ATR) geometry for the excitation of a surface plasma wave.
(b) Intensity reflection coefficient as a function of angle of incidence.

However, at a specific angle $\theta_i = \theta_R = \sin^{-1}(c\,k_z / \omega\,\eta_g)$ where k_z is given by eq. (14), the reflectivity drops down drastically below 50%. At this angle, surface wave is excited at the metal-free space boundary, as the component of laser wave vector along the, interface equals the wave vector of the surface plasma wave (SPW) and laser energy tunnels through the metal to excite the SPW. If there is small contamination or deposition of a film over the metal surface, the SPW resonance angle θ_R changes significantly. For this reason, SPW