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ASSIGNMENT - 2

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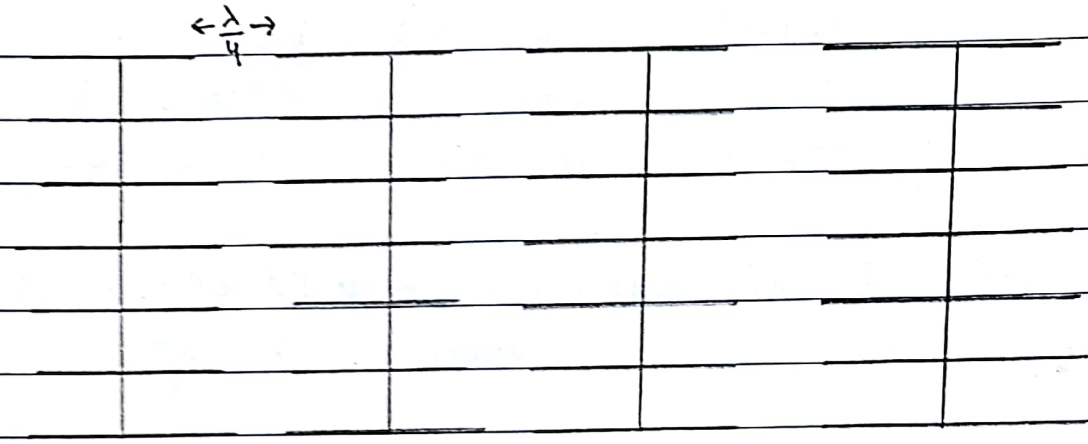
Ans 1

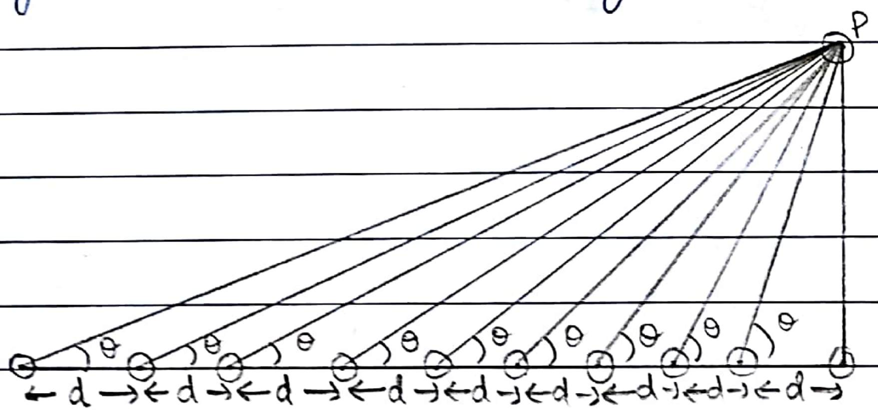
(Top View)



(side view)

Typical antenna lengths in a broadside array are from $2 - 10\lambda$ and typical spacing are $\lambda/2$ or $\lambda/4$ or λ.

★ Consider a linear array of antenna's spaced at distance "d" from each other on the x-axis. Assuming that the electric field due to each antenna is equal in magnitude then considering "n" antenna.



Now as point P is very far from the array thus angles from all array's can be approximated as "θ"
So calculating the path difference of these dipoles with respect to the first dipole we get.

$\delta_1 = 0$ $\quad \delta_2 = d\cos\theta$ $\quad \delta_3 = 2d\cos\theta$ $\quad \delta_4 = 3d\cos\theta$

$\delta_5 = 4d\cos\theta$ thus $\delta_n = (n-1)\,d\cos\theta$.

So, the phase difference $\beta \Rightarrow Kd\cos\theta \Rightarrow K\delta$

where $K = \frac{2\pi}{\lambda}$

where α is some initial phase difference.

$\beta_1 = 0$ $\quad \beta_2 = Kd\cos\theta + \alpha$ $\quad \beta_3 = 2Kd\cos\theta + 2\alpha$

$\beta_4 = 3Kd\cos\theta + 3\alpha$ thus $\beta_n = (n-1)Kd\cos\theta + (n-1)\alpha$

Now let $Kd\cos\theta + \alpha = \psi$ then

$\beta_1 = 0$ $\quad \beta_2 = \psi$ $\quad \beta_3 = 2\psi$ $\quad \beta_4 = 3\psi$ $\quad \beta_5 = 4\psi$

thus $\beta_n = (n-1)\psi$

Now,

Net $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4} + \dots + \vec{E_n}$

$$\vec{E} = e^{-j0\psi} + e^{-j\psi} + e^{-j1\psi} + e^{-j2\psi} + \dots + e^{-j(n-1)\psi} \quad -(1)$$

Now multiply (1) by $e^{-j\psi}$

$$\vec{E}e^{-j\psi} = e^{-j\psi} + e^{-2j\psi} + e^{-3j\psi} + \dots + e^{-jn(\psi)} \quad - (2)$$

Now (1) - (2)

$$\vec{E} - \vec{E} e^{-j\psi} = 1 - e^{-jn(\psi)}$$

$$\vec{E} = \frac{1 - e^{-jn\psi}}{1 - e^{-j\psi}} = \frac{e^{-\frac{jn\psi}{2}}}{e^{-\frac{j\psi}{2}}} \left[\frac{e^{\frac{j\psi n}{2}} - e^{-\frac{j\psi n}{2}}}{e^{\frac{j\psi}{2}} - e^{-\frac{j\psi}{2}}} \right]$$

$$= e^{-j\left(\frac{n-1}{2}\right)\psi} \left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \right]$$

If reference is taken from point (1) then $e^{-j\left(\frac{n-1}{2}\right)\psi} = 1$

Now normalizing the electric field by dividing by $\vec{E}_{max}$

$E = E_{max}$ at $\psi \to 0$

so $Kd\cos\theta + \beta$

so

$$E = \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad \text{as } \psi \to 0 \; ; \; \vec{E} \to n$$

thus $E_{max} = n$

Now $$E_n = \frac{E}{E_{max}} = \frac{1}{n} \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad - (3)$$

the term in (3) is called array factor we can say that A_f is maximum in direction when $\psi = 0$ i.e all sources are in phase when $\psi = 0$

★ Now for Broadside Array's the sources are in phase and $d = \lambda/2$ thus,

$$\psi = \frac{2\pi}{\lambda} d\cos\theta + \alpha \quad \text{as } \alpha = 0 \text{ \& } d = \lambda/2$$

$$\psi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} \cos\theta + 0 = \pi\cos\theta$$

$$\vec{E}_n = \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

for maximum's $\psi = 0$ thus $\pi\cos\theta = 0$

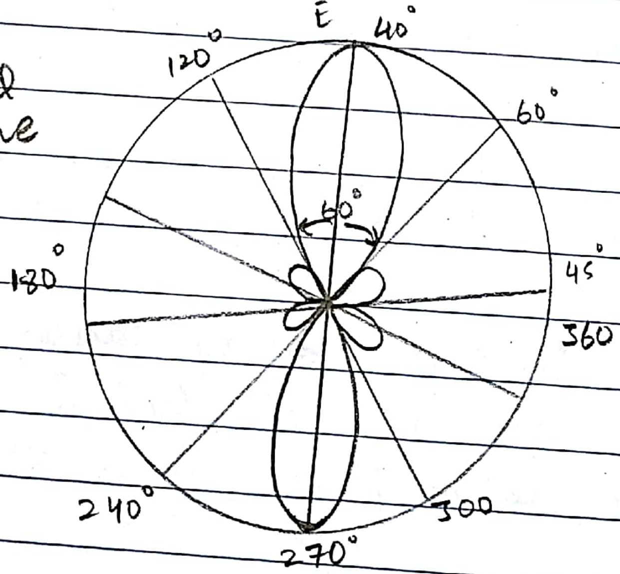
so $\cos\theta = 0$

$$\theta = \frac{(2m+1)}{2}\pi \qquad \theta \in (0, 2\pi)$$

$$\theta = \frac{\pi}{2} \qquad \theta = \frac{3\pi}{2} \quad \text{maximum}$$

for $n = 4$ Null $=$ $E = 0$ $\sin(2\psi) = 0$ so

vertical plane



$$2\psi = n\pi$$

$$2\pi\cos\theta = m\pi$$

$$\cos\theta = m/2$$

as m

$$-a < m < 2$$

thus

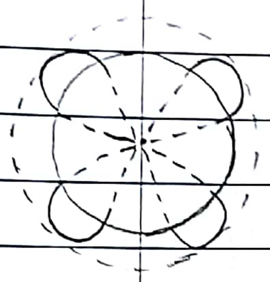
$$\theta \to \pm\frac{\pi}{4}, \pm\frac{\pi}{3}, 0$$

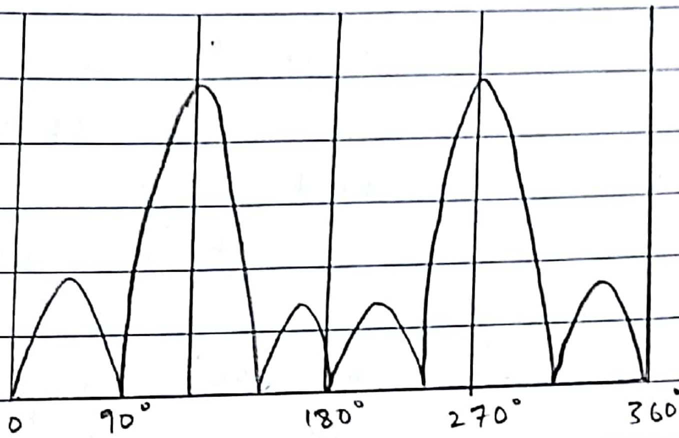
Now

$$FNBW = 2\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

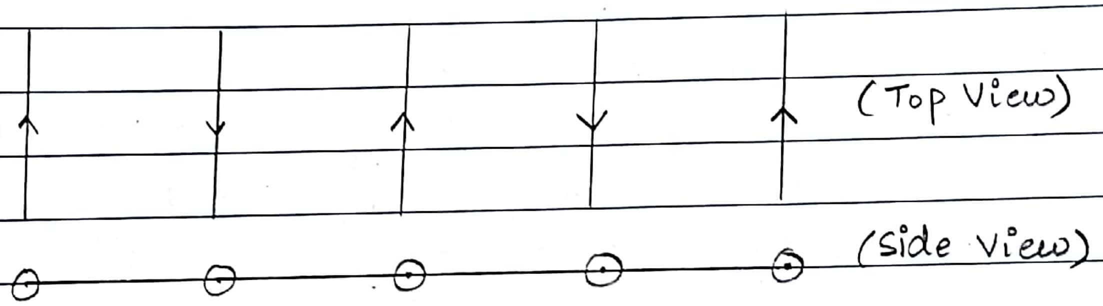
FNBW = $\pi/3$

Horizontal Plane





Ans 2



☆ There is no radiation perpendicular to take plane of the array because of cancellation by consecutive elements usually the spacing in $\lambda/4$ or $3\lambda/4$ this diverts energy to the main lobe of the beam thus minor losses are avoided and directivity is increased, the beam becomes narrower as elements increases.

☆ Consider a linear array of antenna spaced at an antenna of distance "d" from each other on the x-axis. Assuming "n" elements and that the

electric field due to each other antenna is equ
magnitude then.
Now at point P is far from the array thus all angles can be approximated to "θ" thus.
the path difference δ w.r.t mode 1 is
$\Rightarrow \delta_1 = 0$ to $\delta_2 = d\cos\theta$ $\Rightarrow \delta_3 = 2d\cos\theta \dots$ $\delta_n = (n-1)d\cos\theta$

thus corresponding phase differences are with initial phase being "α" then

$\beta_1 = \alpha$ $\beta_2 = kd\cos\theta + \alpha$ $\beta_3 = 2(kd\cos\theta + \alpha) \dots$

$\beta_n = (n-1)[kd\cos\theta + \alpha]$

where $k = \frac{2\pi}{\lambda}$

Now let $\psi = kd\cos\theta + \alpha$

$$\vec{E} = \vec{E_1} + \vec{E_2} + \dots \vec{E_n}$$

$$\vec{E} = [e^{-0j\psi} + e^{-j\psi} + e^{-2j\psi} + \dots + e^{-nj\psi}) E_0 \quad -(1)$$

(1) $* \, e^{-j\psi}$

$$\vec{E} - \vec{E} e^{-j\psi} = 1 - e^{-nj\psi}$$

$$\vec{E} = \frac{1 - e^{-nj\psi}}{1 - e^{j\psi}} = e^{-j\frac{(n-1)}{2}\psi} \quad \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

Now in phase is taken with respect to 1st node

then $e^{-j\frac{(n-1)}{2}\psi} = 1$

thus $\vec{E} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}$

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Now $\vec{E_n} = \frac{\vec{E}}{\vec{E_{max}}}$

Now as $\psi \to 0$ $\vec{E} \to \vec{E_{max}}$

so $n \to \vec{E_{max}}$

$\vec{E_{max}} = n$

So $\vec{E_n} = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)}$

Now for Endfire antennae (ordinary) the lobe in maximum when $\theta = 0°$ so,

or $\theta = 180°$ $\psi = kd\cos\theta + \alpha$

$\psi = kd + \alpha$

Now $E = E_{max}$ when $\psi \to 0$

$kd + \alpha = 0$

$\frac{2\pi}{\lambda} d = -\alpha$

so $d = -\delta$

$\delta = -d_r$

Now if $d = \lambda/4$ $\alpha = -\pi/2$ so the elements are out of phase by 90".

Now for increased directivity Endfire Antenna we use

$\delta = -\left(d + \frac{n}{\pi}\right)$ for increased directivity

So $\psi = d(\cos\theta - 1) + \frac{\pi}{n}$

* for $n = 6$ maxima is at 0° and 180°

$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right) \Rightarrow \theta = 0$

Nulls are at $\theta = 180^\circ$

$\vec{E} = 0$ so $\sin(n\psi/2) = 0$ $\theta = 90^\circ$

$\sin\left(\frac{n\psi}{2}\right) = 0$ $\theta = 270^\circ$

$\frac{n\psi}{2} = m\pi$

$3\psi = m\pi$

$3\left(\frac{2\pi}{\cancel{\lambda}} \times \frac{\lambda\cos\pi}{4\,2}\right) = m\pi$

$\cos\theta \frac{3\pi}{2} + \frac{3\pi}{2} = m\pi$

$\frac{3\cancel{\pi}}{2}\cos\theta = m\cancel{\pi} + \frac{3\cancel{\pi}}{2}$

$-\frac{3}{2} + \frac{3}{2}\cos\theta \Rightarrow \frac{2}{3}m - 1 = \cos\theta$

$\theta = 0^\circ$ ×

$\frac{3}{\cancel{2}}\cos\theta = \frac{2m+3}{\cancel{2}}$

$\cos\theta = \frac{2m-3}{2}$ $m = 0 \to \theta \to 70.53^\circ$

$\theta_n = \cos^{-1}\left(1 - \frac{4m}{6}\right)$ $m = 1$ $\theta \to$

$= \pm 70.53$

$$3\psi = m\pi$$

$$3\left(kd\cos\theta + \frac{\pi}{2}\right) = m\pi$$

$$3\left(\frac{2\cancel{\pi}}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{4}\cos\theta + \frac{\cancel{\pi}}{2}\right) = m\cancel{\pi}$$

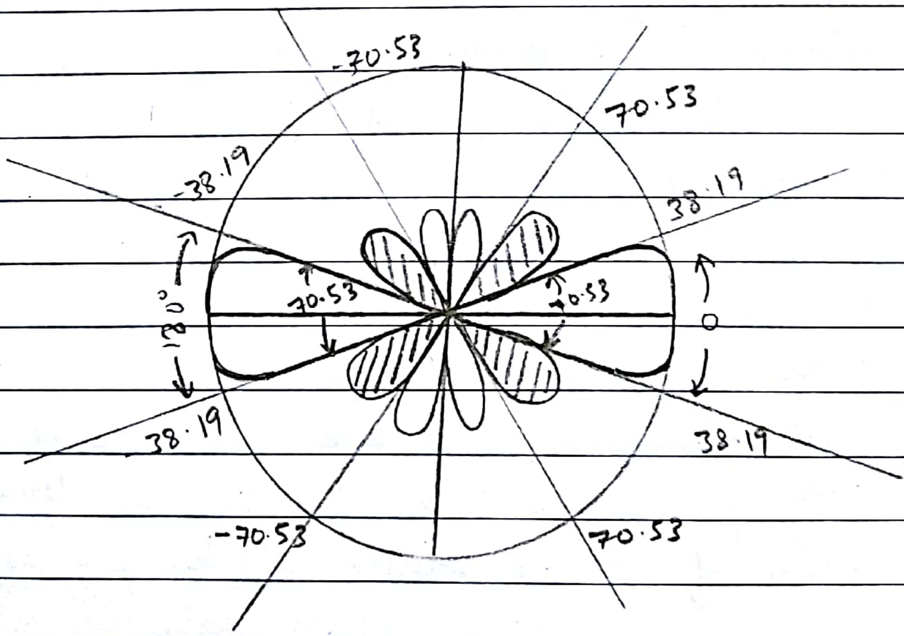
$$\frac{1}{2}\cos\theta + \frac{1}{2} = \frac{m}{3}$$

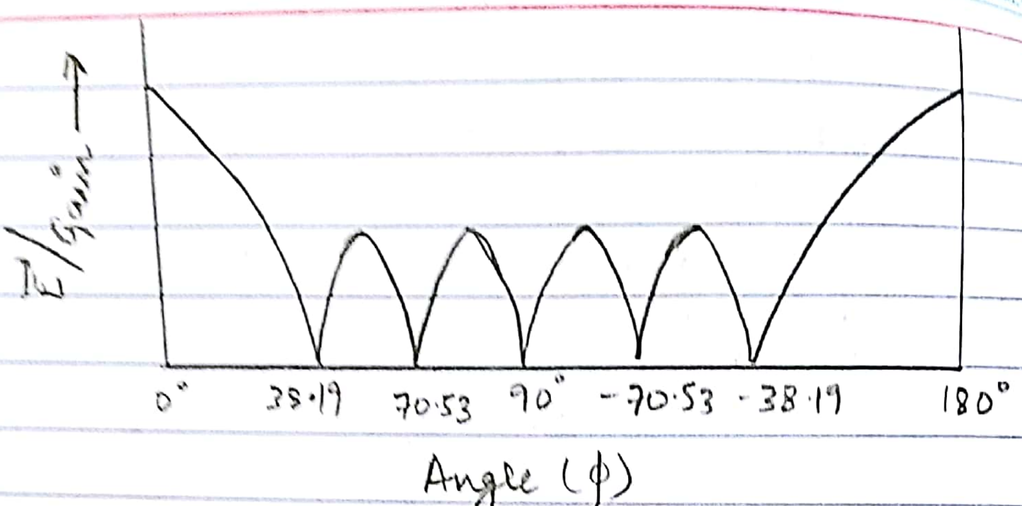
$$\frac{1}{2}\cos\theta = \frac{m}{3} \mp \frac{1}{2}$$

$$\cos\theta = \frac{2}{3}m \mp 1 \qquad m = 1$$

$$\cos\theta = \pm\frac{1}{3} \Rightarrow \theta = \pm 70.53^\circ$$

$m = 2$ $\qquad \cos\theta = -\frac{4}{3} \pm 1 = \frac{-1}{3} = 70.53^\circ$





$$FNBW = \frac{2\lambda}{Nd} = \frac{2 \times \lambda}{6 \times \lambda} \times 4 = \frac{8}{6} rad = \frac{8}{6} \times \frac{180}{\pi} = 76.38^\circ$$

thus first null is at 38.19°

Ans 3 Antenna array's are used because in some application are need to have a narrow beam for large distance communication.

☆ It is acheived using three ways :-

i) Increasing size of antenna

ii) Using Antenna Array's.

☆ They narrow the beam and increases the gain of the arrangement

☆ Usually all elements used are identical to each other. If arrangement in 1 Axis is then linear array then if in 2D plane it is planar array.

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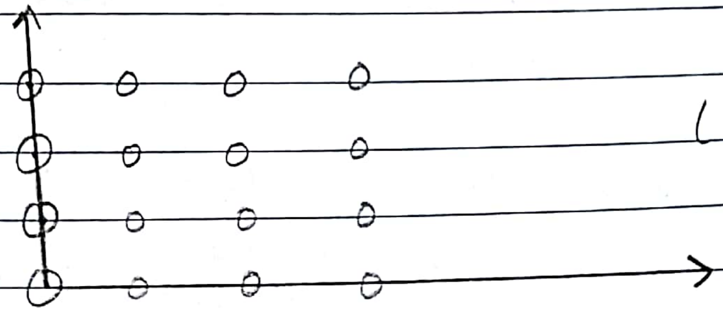
(Linear Array)



(Planar Array)

★ Now for an antenna array of "n" elements the Electric field is given by

$$\vec{E} = \sin\left(\frac{n\psi}{2}\right) \Big/ \sin\left(\frac{\psi}{2}\right) = A_f \Rightarrow \text{Array Factor}$$

$$\vec{E} = \vec{E_i} \times A_f$$

Now if "E_i" is normalised

$$\vec{E} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

$\alpha \rightarrow$ Initial phase Difference

★ <u>Broad side Array</u> :-

That antenna array in which the elements are in phase with each other and the radiation direction in perpendicular to the plane of the array is known as Broadside Array.

Now as radiation direction in a Broadside array is perpendicular to the plane of the array thus.

Major lobe is in direction 90° and 270° thus for as $\alpha = 0$ (All elements in phase)

$\psi = Kd\cos\theta$.

Now for maximas $\psi = 0$ as $\psi \to 0$

$$\lim \frac{\sin(N\psi/2)}{\sin(\psi/2)} = 1 * N = E_{max}$$

$$Kd\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90°, 270° \qquad \theta \in (0°, 360°)$$

for minima if $\sin(N\psi/2) \to 0$ but $\psi \neq 0$

thus $E = 0$ as

$$\frac{N\psi}{2} = m\pi$$

$$\frac{Kd\cos\theta}{2} = \frac{m\pi}{N}$$

where $m \neq 0, N, 2N, \ldots$

$$\frac{2\cancel{\pi}}{\lambda \times \cancel{2}} d\cos\theta = \frac{m}{N}\frac{\cancel{\pi}}{\cancel{d}}$$

$K = \frac{2\pi}{\lambda}$

$$\cos\theta = \frac{m\lambda}{Nd}$$

and usually $d = \lambda/2$

$$\cos^{-1}\left(\frac{m\lambda}{Nd}\right) = \theta_{null}$$

$$\cos^{-1}\left(\frac{2m}{N}\right) = \theta_{null} \quad \text{— (1)}$$

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Now from (1) we can tell that first major lobe is at 90° and second major lobe is at 270° and between two major lobes there are (N-1) nulls including the first nulls i.e there are (N-2) side lobes between each pair of major lobes.

The side lobes becomes smaller & smaller as N increases.

☆ **Endfire Array**: That antenna array in which the elements are at phase with each other and radiation is parallel to the plane of the array is known as a Endfire antenna.

Now as radiation direction in an Endfire antenna is parallel at the plane of the array then θ_{max} is 0° or 180°. Thus now putting in Eqn for ψ.

$$\left.\begin{aligned}\psi &= Kd\cos\theta \pm \alpha \\ \psi &= Kd \pm \alpha\end{aligned}\right] \text{ at } \theta = \theta_{max}$$

Now for maximum $\psi \to 0$ so $E = \lim_{\psi \to 0} \frac{\sin(N\psi/2)}{\sin(\psi/2)} = N = E_{max}$

$$\text{So} \quad Kd + \alpha = 0$$

$$Kd = \mp \alpha$$

$$\alpha = \frac{2\pi d}{\lambda}$$

So $d_r = d$ $\quad$ d_r is path difference

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Now as they are out of plane thus $\alpha = \pm kd$

$$\psi = kd\cos\theta + kd$$

$$\psi = kd(\cos\theta + 1)$$

Now for minimas

$$\sin\left(\frac{N\psi}{2}\right) = 0$$

$$\frac{N\psi}{2} = m\pi$$

$$\psi = \frac{2m\pi}{N}$$

$$kd(\cos\theta \pm 1) = \frac{2m\pi}{N}$$

$$\frac{2\pi}{\lambda} d(\cos\theta \pm 1) = \frac{2m\pi}{N}$$

$$(\cos\theta \pm 1) = \frac{m}{N} * \left(\frac{d}{\lambda}\right)^{-1}$$

$$\cos\theta \pm 1 = \frac{m\lambda}{dN}$$

$$\cos\theta = \frac{m\lambda}{dN} \pm N$$

$$\theta_{null} = \cos^{-1}\left(\frac{m\lambda}{dN} \pm 1\right) \quad \text{as } \cos\theta = \cos(-\theta)$$

$$\theta_{null} = \cos^{-1}\left(\mp 1 + \frac{m\lambda}{dN}\right) \quad -(2)$$

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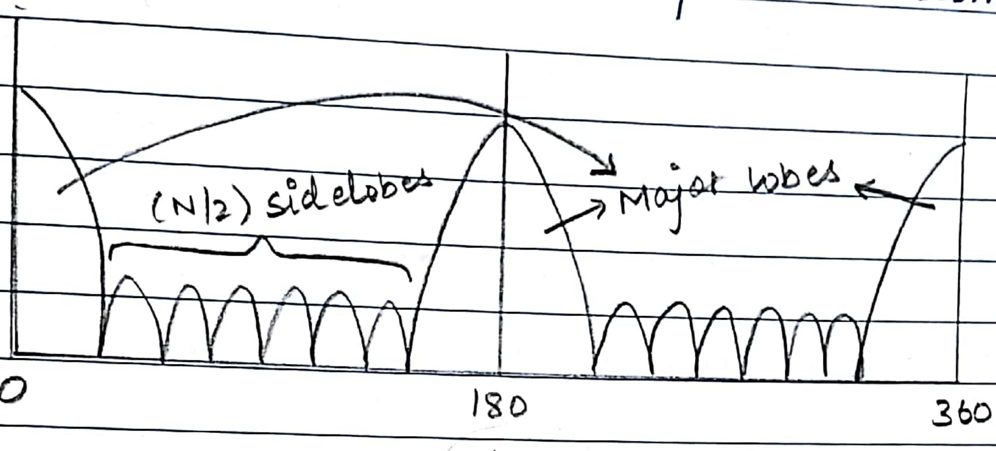
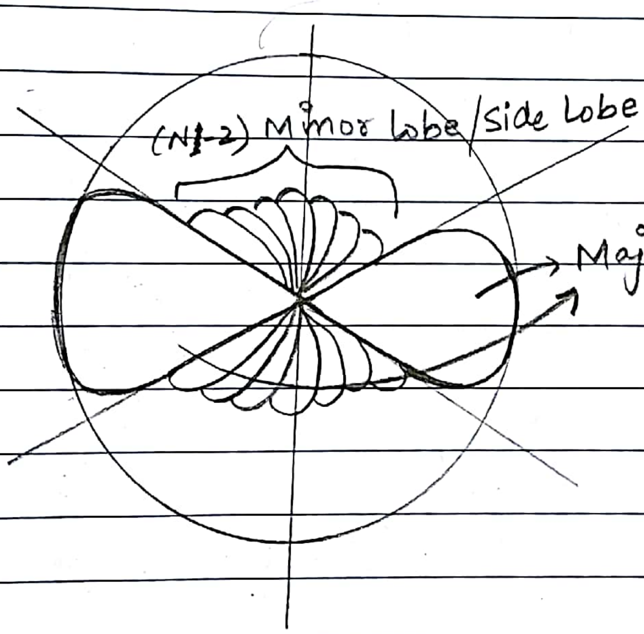
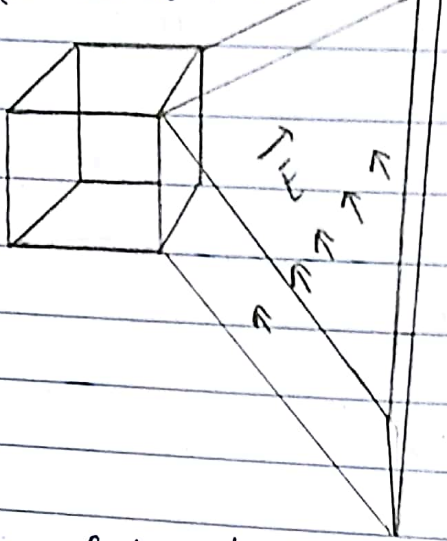
Now from (2) we see that the number of side lobes is equal to (N-2) and number of null's is equal to (N-1) thus the radiation pattern becomes.

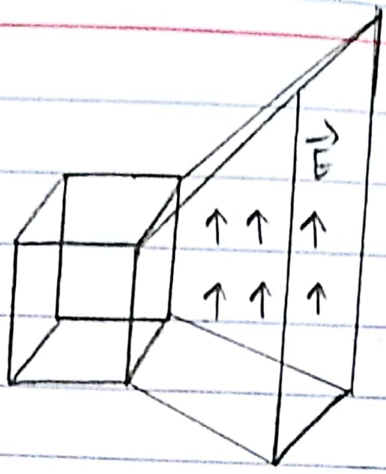




Ans 4 Assuming waveguide (Rectangular) operates in TE_{10} mode the only plane corresponding to the $\vec{E}$ plane is flared it is called sector $\vec{E}$ plane horn antenna similarly for Sectoral H plane horn antenna only H plane wall's are flared at out. When both are flared ~~at~~ out it is known as pyramidal horn

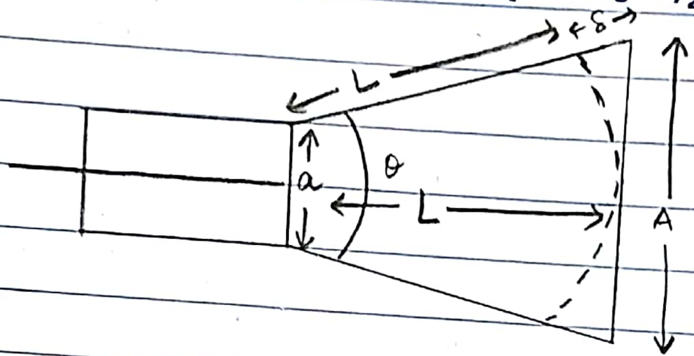
(Sectoral H plane)





(Sectoral E plane)

consider horn antenna as shown in figure



Now as $a << A$ thus

$$\left(\frac{A}{2}\right)^2 + L^2 = (L+\delta)^2$$

$$\frac{A^2}{4} + \cancel{L^2} = \cancel{L^2} + 2L\delta + \delta^2$$

$$\frac{A^2}{4} = 2L\delta + \delta^2 \qquad \text{as } \delta << L$$

$$\frac{A^2}{4} = 2L\delta$$

$$\text{as } L = \frac{A^2}{8\delta}$$

Now directivity is given by

$$G = \frac{4\pi A_e}{\lambda^2} \times \eta \qquad G = \eta D$$

$$D = \frac{4\pi A_e}{\lambda^2}$$

Now $D \propto A_e = A \times B$

$D \propto$ Area

thus more the area of the flaring higher the directivity

where L is flare length
and θ is flare angle

now in $\vec{E}$ plane and $\vec{H}$ plane S_e and S_h are

$$S_e = \frac{A^2}{8L} \qquad S_h = \frac{B^2}{8L}$$

where A and B are dimensionless of flare

Now $\tan\frac{\theta}{2} = \frac{A}{2L}$ so $\frac{\theta}{2} = \tan^{-1}\left(\frac{A}{2L}\right) = 2\tan^{-1}\left(\frac{A}{2L}\right)$

Now S_e must not be more than 0.25λ
and S_h must not be more than 0.4λ or Antenna will not function properly

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The optimum horn angle θ_0 and S_0 are

$$S_0 = \frac{L}{\cos(\theta/2)} - L \quad \text{for any angle } \theta$$

$$L = \frac{S_0 \cos(\theta/2)}{1 - \cos(\theta/2)} \quad \text{optimum length.}$$

Now directivity is given by

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \varepsilon A_p}{\lambda^2}$$

$\varepsilon \rightarrow$ Aperture Efficiency

for rectangular horn $A_p = A \times B$

so $$D = \frac{4\pi \varepsilon AB}{\lambda^2}$$

Now $$HPBW_E = \frac{56^\circ \lambda}{A}$$

$$HPBW_H = \frac{67^\circ \lambda}{B}$$

* Applications of horn Antenna are :-

i) feed for Reflector Antenna's
ii) short range radar systems
iii) Gain standards.