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8) Electric and Magnetic Equation

E and M eqtn can derived from maxwell's equation, which in time domain expressed as

$$\nabla \cdot D = \rho_v \quad -(1)$$
$$\nabla \cdot B = 0 \quad -(2)$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$
$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad -(4)$$

E = Eletric field Intensity (V/m)
H = magnetic field Intensity (A/m)
D = Electric flux Density (C/m^2)
B = magnetic flux Density ($webr/m^2$ or Tesla)

1 Tesla = 1 webr/m^2 = 10^4 gauss = 3×10^6 ESU

J = Electric Current density A/m^2
ρ_v = Electric charge Density C/m^3

ϵ = dielectric permitivity (Faraday/m)
μ = magnetic permeablity (Henry/m)
$\epsilon_0 = 8.854 \times 10^{-12}$ [for free space]
$\mu_0 = 4\pi \times 10^{-7}$ H/m

If sinusoidal time function in the form of $e^{j\omega t}$ is assumed, $\partial/\partial t$ can be repaced by $j\omega$. Hence (3) and (4) eqtn

$$\nabla \times E = -j\omega\mu H \quad -(5)$$

$$\nabla \times H = (\sigma + j\omega\epsilon) E \quad -(6)$$

Eqtn (7) and (8) know as Homogeneous vector Helmholtz equation

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Taking curl of Eqtn (3) on b/s

$$\nabla \times (\nabla \times E) = -j\omega\mu(\nabla \times H)$$

Substituting $\nabla \times H$ from eqtn (6)

$$-\nabla^2 E + \nabla(\nabla \cdot E) = -j\omega\mu(\sigma + j\omega\epsilon)E$$

since, In free space the space charge density ($\rho_v = 0$), and in a perfect conduct time-varying * or static field do not exist.

$$\Rightarrow \nabla \cdot D = \rho_v = 0$$

$$\Rightarrow \nabla \cdot E = 0$$

frequency domain wave equation

$$\boxed{+\nabla^2 E = +j\omega\mu(\sigma + j\omega\epsilon)E} \; * \quad — (7)$$

$$\boxed{\nabla^2 E = \gamma^2 E} \; * \quad — (8)$$

where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$

γ = is called intrinsic propagation constant of medium

a = attenuation constant (nepers/m)

b = phase constant (rad/m)

where $(\nabla^2 = \nabla \cdot \nabla)$

If we take curl of eqtn (4) and substitute $\nabla \times E$ by $-j\omega\mu H$ we get

$$\boxed{\nabla^2 H = j\omega\mu(\sigma + j\omega\epsilon)H} \quad (9)$$

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In diffrential form eqtn ⑦ can be

Time domain wave equation
$$\boxed{\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}} \quad -⑩ \approx ⑦$$

If we take curl of eqtn ⑨ and substitute $\nabla \times E$ by $-j\omega\mu H$ we get

$$\left(j\omega = \frac{\partial}{\partial t}\right)$$

$$\boxed{\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}} \quad -⑪ \approx ⑨$$

1) These equation can be manipulated Acc. to question (medium !!)

1. Free space

$(\sigma = 0)$ $(\varepsilon = \varepsilon_0)$ $(\mu = \mu_0)$

2) Good conductors

$(\sigma = \infty)$ $(\varepsilon = \varepsilon_0)$* $(\mu = \mu_r\mu_0)$ or $(\sigma >> \omega\varepsilon)$*
$(\cdot \rho_v = 0)$

3) Lossless Dielectrics

$(\sigma = 0)$ $(\varepsilon = \varepsilon_r\varepsilon_0)$ $(\mu = \mu_r\mu_0)$ or $(\sigma << \omega\varepsilon)$*

4) Lossy Dielectrics

$(\sigma \neq 0)$ $(\varepsilon = \varepsilon_r\varepsilon_0)$ $(\mu = \mu_r\mu_0)$

Generalised form of Maxwell's Equation

Diffrential form	Integral form	Remarks
$\nabla \cdot D = \rho_v$	$\oint D \cdot ds = \oint \rho_v \, dV$	Gauss law
$\nabla \cdot B = 0$	$\oint B \cdot ds = 0$	Non-existence of monopole
$\nabla \times E = \frac{-\partial B}{\partial t}$	$\oint E \cdot dl = -\frac{\partial}{\partial t} \int_s B \cdot ds$	Farraday's law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) ds$	Ampere's Circuit law.

Equation of Continuity

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

Constitutive Relation

$$D = \epsilon E = \epsilon_0 E + P \qquad * \; \epsilon = \epsilon_r \epsilon_0$$

$$B = \mu H = \mu_0 (H + M) \qquad * \; \mu = \mu_r \mu_0$$

$$J = \sigma E + \rho_v u$$

Boundary condition.

① $E_{1t} = E_{2t}$ Tangential component of EF continious across Boundaries.

② $\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$ " " " Eflux density are discontinious accross boundaries.

③ $D_{1n} = D_{2n}$ if $(\rho_v = 0)$ continious

④ $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$ Discontinious.

⑤ $B_{n1} = B_{n2}$ continious

⑥ $\mu_1 H_{n1} = \mu_2 H_{n2}$ discontinious

⑦ $H_{1t} = H_{2t}$ continious if $J_s = 0$

⑧ $\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$ discontinious