Density of States

The no. of energy states having energy value between E and $E+dE$ is

$$Z(E)\,dE = \frac{\pi}{2} n^2 dn \rightarrow (1)$$

We know that $E = \dfrac{n^2 h^2}{8mL^2} \rightarrow (2)$

$$\Rightarrow n^2 = \frac{8mL^2E}{h^2} \rightarrow (3)$$

$$\Rightarrow n = \left[\frac{8mL^2E}{h^2}\right]^{1/2} \rightarrow (4)$$

Differentiating n^2 gives

$$2n\,dn = \frac{8mL^2\,dE}{h^2}$$

$$\therefore dn = \left\{\frac{1}{2n}\right\}\left[\frac{8mL^2}{h^2}\right] dE$$

$$= \left[\frac{8mL^2}{h^2}\right]\left\{\frac{1}{2}\right\}\left[\frac{h^2}{8mL^2}\right]^{1/2}\frac{dE}{E^{1/2}}$$

$$\therefore dn = \left\{\frac{1}{2}\right\}\left[\frac{8mL^2}{h^2}\right]^{1/2}\frac{dE}{E^{1/2}} \rightarrow (5)$$

Substituting 'n^2' and dn in eqn (1).

$$Z(E)\,dE = \frac{\pi}{2}\left[\frac{8mL^2E}{h^2}\right] \times \frac{1}{2}\left[\frac{8mL^2}{h^2}\right]^{1/2}\frac{dE}{E^{1/2}}$$

~~According to paulies exclusion principle two electrons with opposite spin can occupy the same state~~

~~$\therefore Z(E)\,dE = \frac{4\pi}{34}\left[2m\right.$~~

(2)

$$\therefore Z(E)\,dE = \frac{\pi}{4}\left[\frac{8mL^2}{h^2}\right]^{3/2} E^{1/2}\,dE .$$

According to paulies Exclusion principle, two electrons of opposite spin can occupy each state.

$$\therefore Z(E)\,dE = 2\times\frac{\pi}{4}\left[\frac{8mL^2}{h^2}\right]^{3/2} E^{1/2}\,dE .$$

$$= \frac{4\pi}{h^3}\,(2m)^{3/2} L^3 E^{1/2}\,dE .$$

Density of states is given by the number of states per unit volume.

$$\therefore Z(E)\,dE = \frac{\frac{4\pi}{h^3}(2m)^{3/2}\cancel{L^3}\,E^{1/2}\,dE}{\cancel{L^3}} .$$

$$\boxed{\therefore Z(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2}\,dE .}$$

Fermi distribution function

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

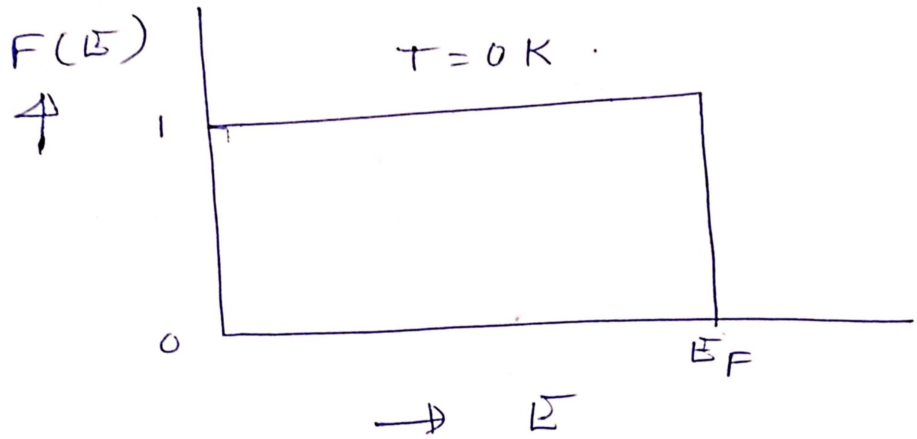
$F(E) \rightarrow$ Fermi distribution function

$E_F \rightarrow$ Fermi energy

$E \rightarrow$ Energy of an energy level occupied by an electron

$K \rightarrow$ Boltzmann constant

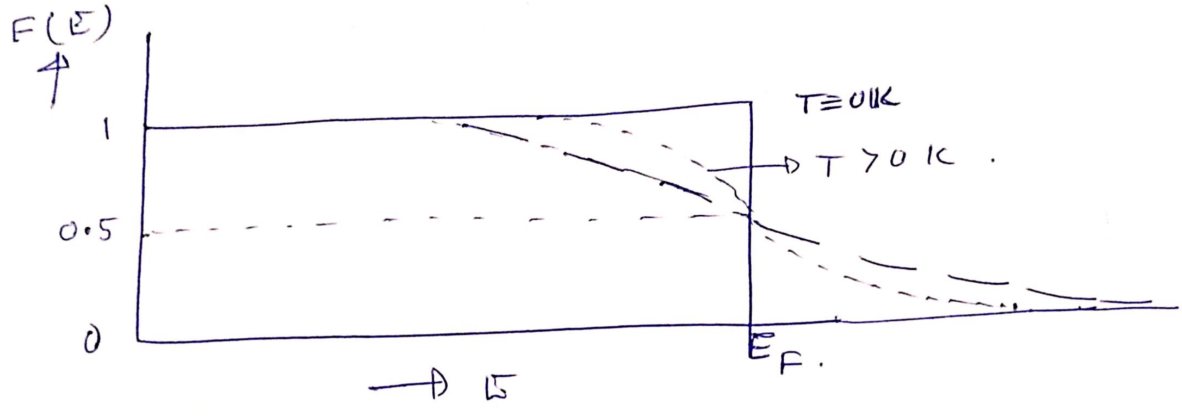
$T \rightarrow$ Absolute temperature



At T = 0 K

(i) For $E < E_F \Rightarrow F(E) = 1$

(ii) For $E > E_F \Rightarrow F(E) = 0$



At any temperature T > 0 K,

If $E = E_F$; then $F(E) = \frac{1}{2}$.

Therefore Fermi energy can be defined as

(i) Max energy of filled states at 0 K.

(ii) Fermi energy is the energy of the state at which the probability of electron occupation is 1/2 at any temperature above 0 K.