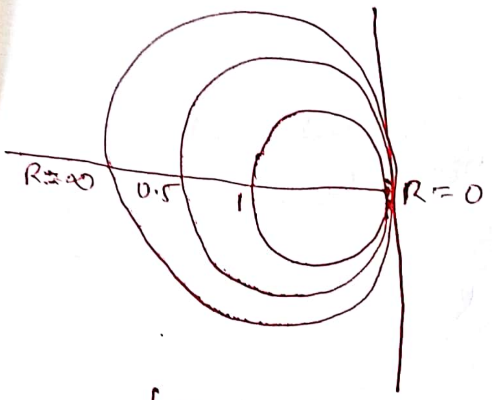
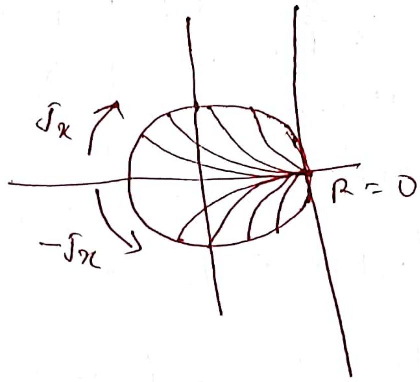


(constant r circles)



constant x circles

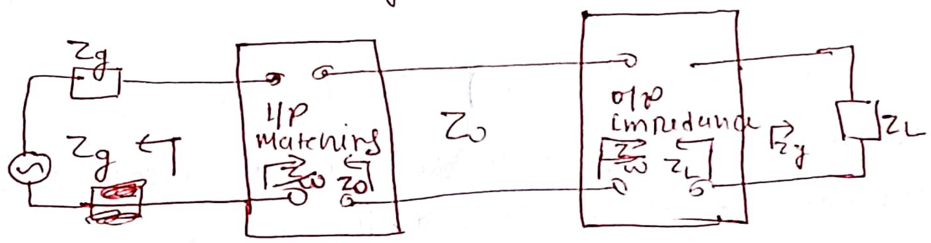
Impedance Matching

It is very desirable with RF transmission lines. Standing wave lead to the increased losses. A properly terminated line (flat line) has SWR of unity and transmit the given power without reflection resulting in maximum transmission efficiency

Matching the transmission line means the equal impedance seen looking at both directions from a given terminal pair for maximum power transfer.

In circuit theory the maximum power transfer require that load impedance is equal to the complex conjugate of the generator impedance whereas in transmission line prob the Matching means terminating the line with characteristic impedan Z_0...

the line so that SWR becomes unity. Fig shows the matched transmission line system.

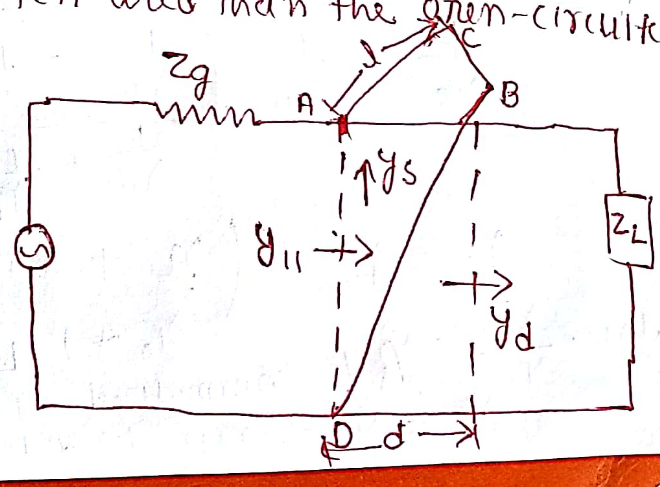


~~Single~~ stub Matching:

The matching problems involve parallel connection on the transmission line, It is necessary to work out the problems with admittance rather than with impedance. The impedance matching involve the following.

1) single stub matching
2) Double stub matching
3) Quarter wave transformer.

1) Single stub matching: Generally a single lumped inductor or capacitor can match the transmission line, It is more common to use the susceptive properties of the short-circuited section of transmission lines. As the short-circuited lines are easier to obtain, It is very often used than the open-circuited lines.



For a lossless line with $Z_g = Y_0$, the maximum power transfer requires $Y_{11} = Y_0$

where, Y_{11} → total admittance of the line and stub looking to the right at the point A-D.

The stub should be located at a point on the line where a real part of the admittance looking towards the load is Y_0

i.e. $y_{11} = y_d \pm y_s = 1$ i.e. $Y_{11} = Y_d \pm Y_s = Y_0$

The stub length is adjustable so that its susceptance cancle out the susceptance of the line at the junction.

if the load is complex impedance then use of smith chart is involved to find the length and position of the chart. if the load impedance is a pure real quantity then following given method of finding length & position of the stub.

We know for a short circuited line

$$Z_s = Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

where $\gamma = \alpha + j\beta$

$\tanh(i\theta) = i\tan\theta$

$$\therefore Z_{in} = Z_s = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

Converting impedance into Admittance

$$Y_{in} = \frac{1}{Z_{in}}, \quad Y_s = \frac{1}{Z_s}, \quad Y_0 = \frac{1}{Z_0}, \quad Y_L = \frac{1}{Z_L}$$

$$\frac{1}{Y_s} = \frac{1}{Y_0} \left(\frac{\frac{1}{Y_L} + j\frac{1}{Y_0}\tan(\beta l)}{\frac{1}{Y_0} + \frac{1}{Y_L}\tan(\beta l)} \right)$$

$$Y_s = Y_0 \left(\frac{Y_L + jY_0 \tan(\beta l)}{Y_0 + jY_L \tan(\beta l)} \right)$$

when $\frac{Y_s}{Y_0} = y_s \rightarrow$ normalized input admittance

$\frac{Y_L}{Y_0} = y_l$ — normalized load admittance

$$\frac{Y_s}{Y_0} = \frac{Y_0\left(\frac{Y_L}{Y_0} + J\tan\beta l\right)}{Y_0\left(1 + J\frac{Y_L}{Y_0}\tan\beta l\right)}$$

$$\boxed{y_s = \frac{y_l + J\tan\beta l}{1 + Jy_l\tan\beta l}}$$

$$y_s = \frac{y_l + J\tan\beta l}{1 + Jy_l\tan\beta l} \times \frac{1 - Jy_l\tan\beta l}{1 - Jy_l\tan\beta l}$$

$$g_s + Jb_s = \frac{y_l - Jy_l^2\tan\beta l + J\tan\beta l + y_l\tan^2\beta l}{1 + y_l^2\tan^2\beta l}$$

$$g_s = \frac{y_l(1 + \tan^2\beta l)}{1 + y_l^2\tan^2\beta l} \quad \text{——— (A)}$$

$$b_s = \frac{\tan\beta l(1 - y_l^2)}{1 + y_l^2\tan^2(\beta l)} \quad \text{——— (B)}$$

For no reflection, the required condition is

$y_s = 1 + J0$, i.e. $g_s = 1$ and $b_s = 0$

i.e. the position of the stub should be placed where it makes $g_s = 0$

$$1 = y_l\frac{(1 + \tan^2\beta l_s)}{1 + y_l^2\tan^2\beta l_s}$$

$$1 + y_l^2\tan^2\beta l_s = y_l + y_l\tan^2\beta l_s$$

$$(1 - y_l) = y_l\tan^2\beta l_s(1 - y_l)$$

$$\tan^2\beta l_s = \frac{1}{y_l} \qquad \therefore l_s = \frac{1}{\beta}\tan^{-1}\left(\frac{1}{y_l}\right) \quad \text{where } y_l = \frac{Y_L}{Y_0}$$

$$\tan\beta l_s = \sqrt{\frac{1}{y_l}} \qquad = \frac{1}{\beta}\tan^{-1}\left(\frac{Y_0}{Y_L}\right) \quad \therefore l_s = \frac{1}{\beta}\tan^{-1}\left(\frac{Z_L}{Z_0}\right)$$

where Z_L = Load impedance,
Z_0 = characteristics impedance of line.

Smith Chart :-

Graphical method of solving transmission lines. It is a graphical indication of impedance of transmission lines as one move along the lines.

It consists of group of concentric circle (contains resistance & conductance circle)

The chart comprise of the value of the impedance & admittance in the normalised form

$$z_l = \frac{Z_l}{Z_0} = \frac{\text{load impedance}}{\text{characteristic impedance}}$$

$$\text{Normalised admittance} = \frac{1}{z_l} = Y_l$$

The smith chart is constructed within a unit circle

$$\therefore \rho = \frac{Z_l - Z_0}{Z_l + Z_0} \Rightarrow \frac{z_l - 1}{z_l + 1} = |\rho_L| e^{j\theta_l}$$

$$\Rightarrow \rho_r + j\rho_i$$

$$z_l = \frac{Z_l}{Z_0} = r + jx$$

$$\rho_r + j\rho_i = \frac{z_l - 1}{z_l + 1} = \frac{z - 1}{z + 1}$$

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Where

$$z_L = r + jx = \frac{(r+jx)-1}{(r+jx)+1}$$

$$r + jx = \frac{(1+\rho_r) + j\rho_i}{(1-\rho_r) - j\rho_i}$$

$$\Rightarrow \frac{(1+\rho_r) + j\rho_i}{(1-\rho_r) - j\rho_i} \times \frac{(1-\rho_r) + j\rho_i}{(1-\rho_r) + j\rho_i}$$

$$r + jx = \frac{1 - \rho_r^2 - \rho_i^2 + 2j\rho_i}{(1-\rho_r)^2 + \rho_i^2}$$

$$r = \frac{1 - \rho_r^2 - \rho_i^2}{(1-\rho_r)^2 + \rho_i^2}$$

$$x = \frac{2\rho_i}{(1-\rho_r)^2 + \rho_i^2}$$

Rearranging we get

$$\left[\rho_r - \left(\frac{r}{1+r}\right)\right]^2 + \rho_i^2 = \left(\frac{1}{1+r}\right)^2 \quad \text{— (A)}$$

&

$$(\rho_r - 1)^2 + \left(\rho_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad \text{— (B)}$$

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Eqn (A) and (B) are identical to the eqn of circle $[(x-g)^2 + (y-h)^2 = a^2]$ (with radius a) Centre (g, h), centre of constant resistance circle is $\left(\frac{r}{1+r}, 0\right)$ with radius $\left(\frac{1}{1+r}\right)$.

Characteristic of Smith chart :-

→ the constant 'r' & 'x' loci of 2 families of orthogonal circles in the chart the
→ The constant 'r' & 'x' circle all passes through the pts. $\Gamma_r \Rightarrow 1$ & $\Gamma_i = 0$

→ the upper half of the diagram represents $+jx$ and lower half represents $-jx$.

→ for admittance, the constant 'r' circle will becomes constant (g) circle & constant (x) circles will becomes constant susceptive (b) circle,

→ The distance around the chart is $\lambda/2$.

→ At a point of $Z_{min} = \frac{1}{SWR}$, there is a V_{min} on the line.

→ At a point of $Z_{max} = SWR$, V_{max} on the line

→ the distance are given in wavelengths towards the generator at also towards the load

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→ As the normalised admittance (Y) is a reciprocal of the normalised impedance, the corresponding quantities in the admittance chart out 180° out of phase with those in the impedance chart

→ the normalised impedance & admittance repeated for every $\lambda/2$.

→ the outer most circle $R=0$ the one

→ the axis of the chart is the that will be passing through the centre the chart.

→

Applications

1. To find SWR.
2. To find ~~voltage~~ location of V_{max} & V_{min} from load
3. To find unknown load impedance from SWR or reflection coefficient.
4. To find length & position of stub used in transmission impedance matching for perfect (i.e. $Z_L=1$)

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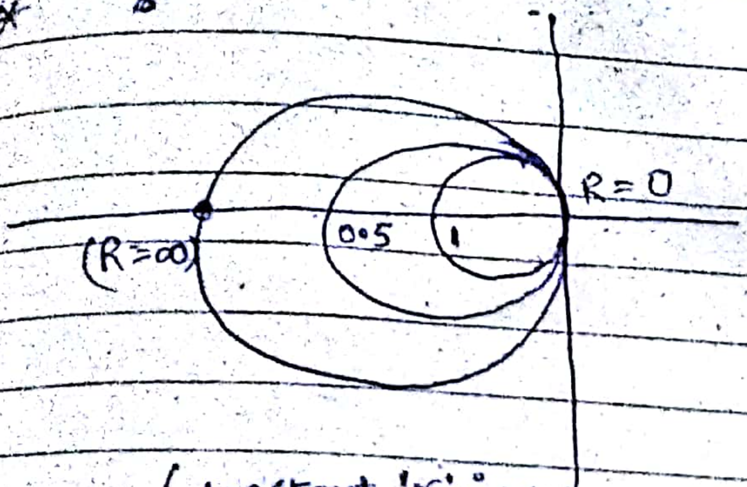
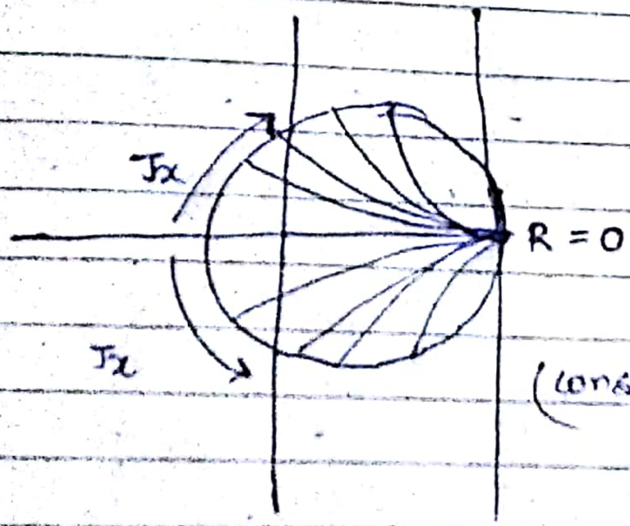


(constant 'r' circle)



(constant 'x' circle)