Class Test-01

Sept 03, 2021

Question 1. $\{(p \to q) \wedge q\} \to p$ is

(a) Tautology

(b) Contradiction

(c) Contingency

(d) None of the above.

Question 2. To prove a theorem that is a biconditional statement, that is a satement of the form $p \leftrightarrow q$, we show that $p \to q$ and $q \to p$ are both true. The validity of this approach is based on the tautology

(a) $(p \leftrightarrow q) \leftrightarrow (p \to q) \wedge (q \to p)$

(b) $(p \leftrightarrow q) \leftrightarrow (p \to q) \vee (q \to p)$

(c) $(p \leftrightarrow q) \to (p \to q) \wedge (q \to p)$

(d) $(p \leftrightarrow q) \to (p \to q) \vee (q \to p)$.

Question 3. A sequence satisfying the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k},$$

where $c_1, c_2, \cdots c_k \in \mathbb{R}$ and $c_k \neq 0$ with initial conditions

$$a_0 = C_0, a_1 = C_1, \cdots, a_{k-1} = C_{k-1}$$

is uniquely determined using

(a) Mathematical Induction

(b) Strong Induction

(c) Proof by Contraposition

(d) None of the above.

Question 4. What is the solution of the recurrence relation

$$a_n = a_{n-1} + 6a_{n-2}, \quad \text{for} \quad n \geq 2,$$

with $a_0 = 3$ and $a_1 = 6$. (write answer only).

Question 5. What is a direct proof? Prove that if x and y are real numbers, then

$$\max\{x, y\} + \min\{x, y\} = x + y.$$

Question 6. What is the principle of strong induction? Use strong induction to show that if you can run one mile or two miles, and if you can always run two more miles once you have run a specified number of miles, then you can run any number of miles.

Question 7. Let c_1 and c_2 be real numbers. Suppose that $r^2 - c_1 r - c_2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if and only if $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for $n = 0, 1, 2, \cdots$ where α_1 and α_2 are constants.