EMT

i) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

1) frequency domain $\nabla^2 \vec{E} = \gamma^2 \vec{E}$

$\nabla^2 \vec{H} = \gamma^2 \vec{H}$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

intrinsic propagation Const

α = Attenuation Const in Nepers per meter

β = phase Const in rad/meter

2) Time domain

$$\nabla^2 \vec{E} = \sigma\mu \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \sigma\mu \frac{\partial \vec{E}}{\partial t} + \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

or $$\nabla^2 \vec{H} = \sigma\mu \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = \sigma\mu \frac{\partial \vec{H}}{\partial t} + \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2}$$

for lossless medium

$\omega\epsilon > \sigma$

$$\gamma = (1+j)\sqrt{\pi f \mu\sigma} = \frac{(\ )}{\text{meter}}$$

Waveguide

X-band (8.00 GHz - 12.00 GHz)

US standard waveguide WR-90

inner width 2.286 cm (0.9 inch)

height (1.016 cm ~0.4 inch)

TE (Transverse electric) mode

or

TM (Transverse magnetic) mode

TE_{mn} or TM_{mn}

⇓

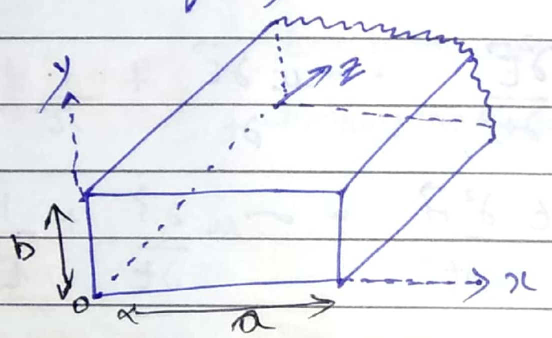
integer m denotes number of half waves electric or magnetic field intensity in x-direction

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_y^2} + \frac{1}{\lambda_c^2}$$

$\lambda_0 = ?$ $\quad$ TE_{10} dominant mode

n ⇒ no. of half wave in y direction if the propagation of the waves is assumed along z-direction

⊛ Solution of Equation in Rectangular waveguide



TE ($E_z = 0$, $E_x = 0$, $E_y \neq 0$
$H_z \neq 0$, $H_x \neq 0$, $H_y = 0$)

TM ($E_z \neq 0$, $E_x \neq 0$, $E_y = 0$
$H_z = 0$, $H_x = 0$, $H_y \neq 0$)

⇒ $\nabla^2 \vec{E} = \gamma^2 \vec{E}$ (1)

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega t)} = \alpha + i\beta$

$$= \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon} = (\alpha + j\beta)$$

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

for free space $\sigma = 0$ i.e, $\gamma^2 = -\omega^2\mu\epsilon = -\frac{\omega^2}{c^2}$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\nabla^2\psi = \gamma^2\psi \quad (3)$$

Helmholtz Equation
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \gamma^2\psi \quad (4)$$

Using method of separation of variable

$$\psi = X(x)\, Y(y)\, Z(z) \quad (5)$$

$X(x)$ is a function of x-coordinate only
$Y(y)$ " " " " y - "
$Z(z)$ " " " " z - "

Using the value of ψ from Eq (5) in Eq (4), dividing the entire eq^n by XYZ, we get

$$\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{-k_x^2} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{-k_y^2} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{-k_z^2} = \gamma^2 \quad (6)$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

or $\frac{\partial^2 X}{\partial x^2} + K_x^2 X = 0$ — (8)

Similarly $\frac{\partial^2 Y}{\partial y^2} + K_y^2 Y = 0$ — (9)

$\frac{\partial^2 Z}{\partial z^2} + K_z^2 Z = 0$ — (10)

The general solution of Eqns (8), (9) & (10)

$X = A \sin(K_x x) + B \cos(K_x x)$ (11)

$Y = C \sin(K_y y) + D \cos(K_y y)$ (12)

$Z = E \sin(K_z z) + F \cos(K_z z)$ (13)

Total Solution

$\psi = XYZ$

$= (A \sin K_x x + B \cos K_x x)(C \sin(K_y y) + D \cos K_y y)(E \sin(K_z z) + F \cos(K_z z))$ — (14)

γ_g = propagation constant in guide

γ = intrinsic propagation constant

Let

$\gamma_g^2 = \gamma^2 + K_x^2 + K_y^2 = \gamma^2 + K_c^2$ (15)

$K_c = (K_x^2 + K_y^2)^{1/2}$ is called the cut off wave no

for a loss less dielectric $\sigma = 0$

$\gamma^2 = -\omega^2 \mu \epsilon$

$$\gamma_g^2 = -\omega^2\mu\epsilon + K_c^2 = -(\omega^2\mu\epsilon - K_c^2)$$

$$\gamma_g = \pm j\sqrt{(\omega^2\mu\epsilon - K_c^2)} \quad -(16)$$

Three Cases

Case I There is no propagation $\omega^2\mu\epsilon = K_c^2$, $\gamma_g = 0$

$$\omega^2\mu\epsilon = K_c^2 = K_x^2 + K_y^2$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{K_x^2 + K_y^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{K_x^2 + K_y^2}$$

$$K_c^2 = \omega^2\mu\epsilon - \beta_g^2$$

$$\beta_g^2 = \omega^2\mu\epsilon - K_c^2$$

$$= \omega^2\mu\epsilon\left[1 - \frac{K_c^2}{\omega^2\mu\epsilon}\right]$$

$$\beta_g = \omega\sqrt{\mu\epsilon}\left[1 - \frac{\omega_c^2}{\omega^2}\right]^{1/2}$$

$$= \omega\sqrt{\mu\epsilon}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \quad \Big| \quad K_c = \frac{\omega_c}{c}$$

Case II

$$\omega^2\mu\epsilon > K_c^2$$

$$\beta_g = \omega\sqrt{\mu\epsilon}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}$$

$$\gamma_g = \pm j\beta_g = \pm j\omega\sqrt{\mu\epsilon}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}$$

Case IV

$\omega^2 \mu \epsilon < K_c^2$ wave will be attenuated

$$\gamma_g = \pm \alpha_g = \pm \omega \sqrt{\mu \epsilon} \left[\left(\frac{f_c}{f}\right)^2 - 1 \right]^{1/2}$$

$$\psi = (A \sin k_x x) + B \cos (k_x x) \, (C \sin k_y y) + D \cos k_y y) \, e^{-\alpha_g z}$$

① ✓

TE Mode in Rectangular waveguide

$E_z = 0$, $H_z \neq 0$

using helmholtz equation

$$\nabla^2 H_z = \gamma^2 H_z \quad -(1)$$

$$H_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \left(\frac{m\pi x}{a}\right)\right]\left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right)\right] e^{-j k_g z} \quad -(2)$$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$

Using maxwell's equation

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -j\omega \vec{B} \left(\frac{\partial}{\partial t} = j\omega\right) = -j\omega\mu \vec{H} \quad -(3)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + j\omega\epsilon\vec{E} \quad \Big| \quad \vec{D} = \epsilon\vec{E}$$

for lossless dielectric $\sigma = 0$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} \quad -(4)$$

from Eq. (3) in rectangular coordinate

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\,(\hat{i}H_x + \hat{j}H_y + \hat{k}H_z)$$

$$= \hat{i}(H_x) - \hat{j}(H_y) + \hat{k}(H_z)$$

$$= -j\omega\mu\,(\hat{i}H_x + \hat{j}H_y + \hat{k}H_z)$$

Equation on both sides, we get

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad -(5)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad -(6)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad -(7)$$

Similarly using $\nabla \times \vec{H} = j\omega\epsilon \vec{E}$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \quad -(8)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad -(9)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad -(10)$$

$$\frac{\partial}{\partial z} = -j\beta_g, \quad E_z = 0 \qquad \left[\begin{array}{l} H_z = (\,)(\,)e^{-j\beta_g z} \\ \frac{\partial}{\partial z} = -j\beta_g \end{array}\right.$$

from eq (5)

$$\beta_g E_y = -\omega\mu H_x \quad -(11)$$

from eq (6)

$$\beta_g E_x = \omega\mu H_y \quad -(12)$$

from eq. (7)

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad -(13)$$

from eq (8).

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega \epsilon E_x \quad - (14)$$

from eq (9)

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y \quad - (15)$$

from eq - (10)

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad - (16)$$

E_x, E_y, H_x, H_y in terms of H_z

from eq (12)

$$H_y = \beta_g \frac{E_x}{\omega \mu}$$

from eq (14)

$$\frac{\partial H_z}{\partial y} = \ldots\ldots$$

$$E_x = \frac{-j\omega\mu}{(-\beta_g^2 + \omega^2\mu\epsilon)} \frac{\partial H_z}{\partial y}$$

$$= \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} \quad - (17) \quad , \quad k_c^2 = \omega^2\mu\epsilon - \beta_g^2$$

Similarly

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} \quad - (18)$$

$$E_z = 0 \quad - (19)$$

$$H_x = \frac{-j\beta_g}{k_c^2} \frac{\partial H_z}{\partial x} \quad - (20)$$

$$H_y = -j\frac{\beta_g}{k_c^2}\frac{\partial H_z}{\partial y} \quad -(21)$$

$$H_z = Eq.(2) \quad -(22)$$

differentiate Eq.(2) w.r.to x,y and substituting the values in eqs.(17) to (22)

Using the boundaries conditions

$E_x = 0$

$$\frac{\partial H_z}{\partial y}(Eq.17) = 0 \text{ at } y = 0, b$$

$$C_n = 0$$

$E_y = 0$,

$$\frac{\partial H_z}{\partial x}(Eq.17) = 0 \text{ at } x = 0, a$$

$$A_m = 0$$

$$\frac{\partial H_z}{\partial x} = 0$$

$A_m = 0$, $C_n = 0$

from eq.(2)

$$H_z = B_m \cos\left(\frac{m\pi x}{a}\right) D_n \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$= B_m \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

$$H_z = H_{oz} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \quad -(24)$$

amplitude of the wave

Using the values of H_z from Eq.(24) in eqs (17) to (22), we get

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y} = \frac{-j\omega\mu}{k_c^2} H_{oz} \cos\left(\frac{m\pi x}{a}\right)\left(-\sin\left(\frac{n\pi y}{b}\right)\right) \times \frac{n\pi}{b} e^{-j\beta g z}$$

$$= \frac{j\omega\mu}{k_c^2}\frac{n\pi}{b} H_{oz} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cdot e^{-j\beta g z}$$

$$E_x = E_{ox} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g z} \quad -(25)$$

Similarly

$$E_y = E_{oy} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g z} \quad -(26)$$

$$E_z = 0 \quad (27)$$

$$H_x = -\frac{j\beta g}{k_c^2}\frac{\partial H_z}{\partial x} = H_{ox} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g z} \quad -(28)$$

$$H_y = \frac{-j\beta g}{k_c^2}\frac{\partial H_z}{\partial y} = H_{oy} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g z} \quad -(29)$$

$$H_z = H_{oz} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g z} \quad -(30)$$

TE_{mn}

$m = 0, 1, 2, \ldots$

$n = 0, 1, 2, \ldots$

Cutoff wave number $k_c = \left(k_x^2 + k_y^2\right)^{1/2} = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c \sqrt{\mu\epsilon} \quad - (31)$$

a, b

Cutoff frequency, $f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$$= \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}, \quad c = \frac{1}{\sqrt{\mu\epsilon}}$$

Propagation Constant β_g

$$\beta_g = \omega\sqrt{\mu\epsilon}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \quad - (33)$$

group velocity $V_g = \frac{\omega}{\beta_g} = \frac{1}{\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{V_{ph}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$V_{ph} = \frac{1}{\sqrt{\mu\epsilon}}$ Phase velocity in z-direction

Characteristic wave impedance

$$Z_g = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta_g}$$

$$= \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad - (35)$$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$ is the intrinsic impedance

guide wavelength for TE_{mn}

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$\lambda_c = 2a$, $\lambda g =$, $f = \frac{c}{\lambda_0}$

$\lambda = V_{ph}/f$ is the wave length in unbound dielectric medium

- when two or more modes have same cutoff frequency it is called degenerate modes.

- If the mode has lowest cutoff frequency then it is in dominant mode.

If $a > b$ then it is in dominant mode

→ TM modes in Rectangular waveguide

$E_z \neq 0$, $H_z = 0$ TM_{mn}

Helmholtz Equation

$$\nabla^2 E_z = \gamma^2 E_z \quad (1)$$

Solution of Eq.(2)

$$E_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right)\right]\left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right)\right] e^{-j\beta_g z}$$

tangent component of the E-field must vanish at the conducting surface (2)

i) $E_z = 0$ at $x = 0, a$ $B_m = 0$

ii) $E_z = 0$ at $y = 0, b$ $D_n = 0$

T.E

$$H_z = H_{0z} \cos \frac{m\pi}{a} x \cos \frac{n\pi}{b} y \, e^{-j\beta_g z}$$

$$E_z = E_{0z} \sin \frac{m\pi x}{a} \sin \frac{n\pi}{b} y \quad e^{-j\beta_g z} \quad -(3)$$

$$\frac{\partial}{\partial z} = -j\beta_g$$

$m = 1, 2, 3 \ldots$
$n = 1, 2, 3$

except $m = n = 0$

TM_{01}, TM_{10} does not exist in rectangular waveguide

TE_{10} is the dominant mode for $a > b$

$H_z = 0 \qquad \nabla \times \vec{E} = -j\omega\mu\vec{H}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = j\omega\mu \left(\hat{i} H_x + \hat{j} H_y + \hat{k} H_z \right)$$

Equating on both sides we get

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial y} + j\beta_g E_y = -j\omega\mu H_x \quad -(4)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

or

$$j\beta_g E_x + \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (5)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad -(6)$$

$\sim$ly for $\nabla \times \vec{H} = j\omega\epsilon \vec{E}$

$$\beta_g H_y = \omega\epsilon E_x \quad -(7)$$

$$-\beta_g H_x = \omega\epsilon E_y \quad -(8)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad (9)$$

The resultant field equations for TM modes

$$E_x = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial x} \quad -(10)$$

$$E_y = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial y} \quad -(11)$$

$$E_z = E_{0z}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z} \quad -(12)$$

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y} \quad -(13)$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x} \quad -(14)$$

$$H_z = 0 \quad -(15)$$

$$k_c^2 = \omega^2\mu\epsilon - \beta_g^2$$

Using the value of Ez from (12) in Eqs (10), (11), (13) & (14), we obtain

$$E_x = E_{0x}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_g z} \quad -(16)$$

$$E_y = E_{oy} \sin\left(m \frac{\pi x}{a}\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta_g z} \quad (18)$$

$$H_x = H_{ox} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta_g z} - (19)$$

$$H_y = H_{oy} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta_g z} - (20)$$

$$H_z = 0 - (21)$$

Cut off frequency, $f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$ - (22)

Propogation constant

$$\beta_g = \omega\sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} - (23)$$

Guide wavelength

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \lambda = \frac{c}{f} - (24)$$

Group velocity

$$v_g = \frac{v_{ph}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} - (25)$$

Phase velocity $v_{ph} = \frac{1}{\sqrt{\mu\epsilon}}$

Characteristic wave impedance $Z_g = \frac{\beta_g}{\omega\epsilon} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ -(2

$\eta = \sqrt{\frac{\mu}{\epsilon}}$ is the intrinsic impedance of unbound dielectric

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}, \quad c = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$$

$\mu_r = 1$, ϵ_r = finite

Ques An air filled rectangular waveguide of insight dimension 7x 3.5 cm (L x B) operates in the dominant mode TE_{10} mode.

i) find the cutoff frequency — $2.14 \times 10^9 = 2.14$ GHz

ii) Determine the phase velocity of the wave in the guide at frequency of 3.5 GHz

iii) Determine the guided wavelength at the same frequency — 10.84 cm

Sol^n

i) $$f_c = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \qquad c = \frac{1}{\sqrt{\mu\epsilon}}$$

as $m = 1$ $n = 0$ in TE_{10}

as $\mu = \mu_0$
$\epsilon = \epsilon_0$

$$= \frac{3\times10^8}{2}\sqrt{\frac{1}{49}}$$

$$= \frac{3\times10^8}{2\times7}$$

$$= \frac{3}{14}\times10^8$$

$$= 2.14 \text{ GHz}$$

ii) $$V_{ph} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

iii) $$\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda = \frac{c}{v}$$

$$= \frac{3\times10^8}{3.5\times10^9}$$

$$= \frac{3}{35}$$

$$\lambda g = \frac{3/35}{\sqrt{1-\left(\frac{2.14\times10^9}{3.5\times10^9}\right)^2}}$$

$$= \frac{3/35}{\sqrt{1-\left(\frac{2.14}{3.5}\right)^2}}$$

$$= \frac{3/35}{\sqrt{1-0.373}}$$

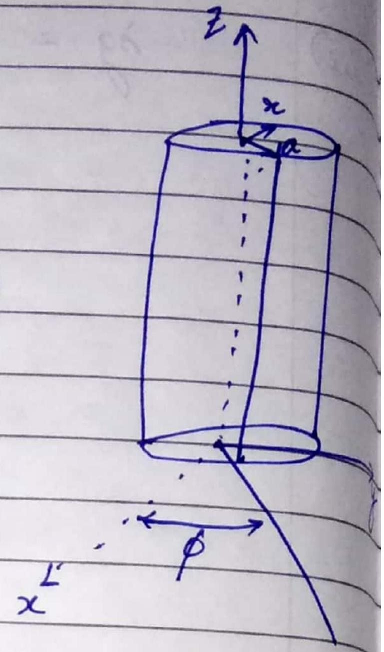
$$= \frac{3}{35\times0.791}$$

$$= \frac{3}{27.685}$$

$$= 10.84\ \text{cm}$$

• Circular Waveguide -

→ Solⁿ of Circular waveguide



Consider Cylindrical Coordinates (r, ϕ, z)

Scalar Helmholtz Equation

$$\nabla^2 \psi = \gamma^2 \psi \quad -(1)$$

scalar operator

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$ = is the intrinsic propagation

α = Attenuation Const in nepers per meter

β = Phase Const in rad. per meter

Eq(1) Can be rewritten

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} = \gamma^2\psi \quad -(2)$$

We want to solve eqⁿ (2) using method of separation of Variable

$$\psi = R(r)\,\Phi(\phi)\,Z(z) \quad -(3)$$

where $R(r)$ is the function of r coordinate only

$\Phi(\phi)$ is " " " ϕ " "

$Z(z)$ " " " " z " "

using the value of ψ from Eq (3) in Eq(2) and divide the resultant by Eq (3)

$$\frac{1}{rR}\frac{\partial}{\partial r}\left(\frac{\partial R}{\partial r}\right) + \frac{1}{r^2\Phi}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \gamma^2 \quad (4)$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \gamma_g^2 \quad -(5)$$

$$\frac{\partial^2 Z}{\partial z^2} - \gamma_g^2 Z = 0 \quad -(6)$$

Sol^n of Eq. (4)

$$Z = Ae^{-\gamma_g z} + Be^{-\gamma_g z} \quad -(7)$$

γ_g = propagation constant in the guide.

from eq^n (4)

$$\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} - (\gamma^2 - \gamma_g^2)r^2 = 0 \quad -(8)$$

$$\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} = -n^2$$

or

$$\frac{\partial^2\Phi}{\partial\phi^2} + n^2\Phi = 0 \quad -(9)$$

$$\Phi = A_n \sin(n\phi) + B_n \cos(n\phi) \quad -(10)$$

$$r\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) - n^2R - (\gamma^2 - \gamma_g^2)r^2R = 0$$

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or

$$r\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right)+\left[(K_c r)^2-n^2\right]R=0 \quad -(11)$$

where

$$K_c^2+\gamma^2+\gamma_g^2 \Rightarrow K_c^2=-(\gamma^2-\gamma_g^2) \quad -(12)$$

$$r^2\frac{\partial^2 R}{\partial r^2}+r\frac{\partial R}{\partial r}+\left[(K_c r)^2-n^2\right]R=0 \quad -(13)$$

for lossless medium $\sigma=0$, $\gamma^2=-\omega^2\mu\epsilon$

$\gamma_g=j\beta_g \quad \therefore \gamma_g^2=-\beta_g^2$

from Eq. (12)

$$K_c^2-\omega^2\mu\epsilon=-\beta_g^2$$

or

$$\beta_g=\pm\sqrt{\omega^2\mu\epsilon-K_c^2}$$

Sol^n of Eq. (13)

$$R=C_n J_n(K_c r)+D_n Y_n(K_c r)$$

$\Downarrow$

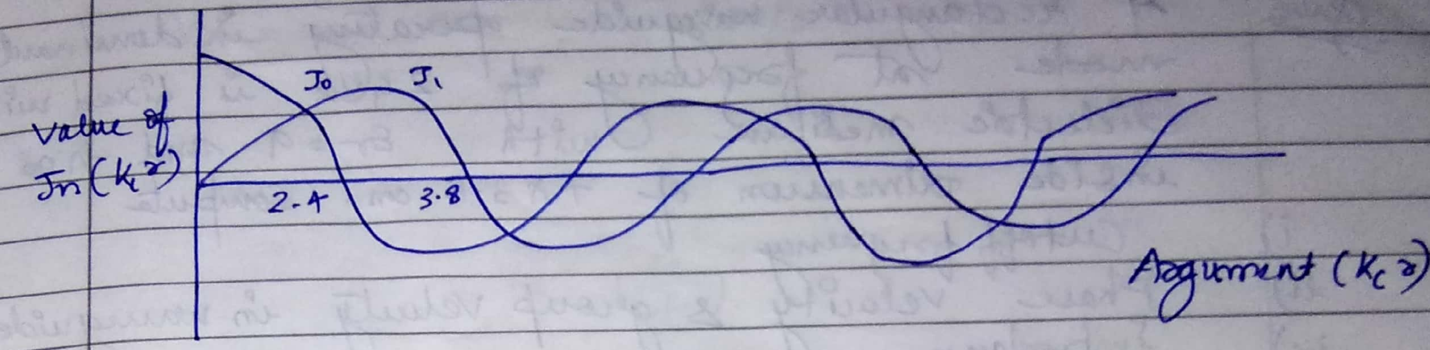
$\cos(K_c r)$ for $r<a$

Y_n $\sin(K_c r)$ for $r>a$

General Sol^n

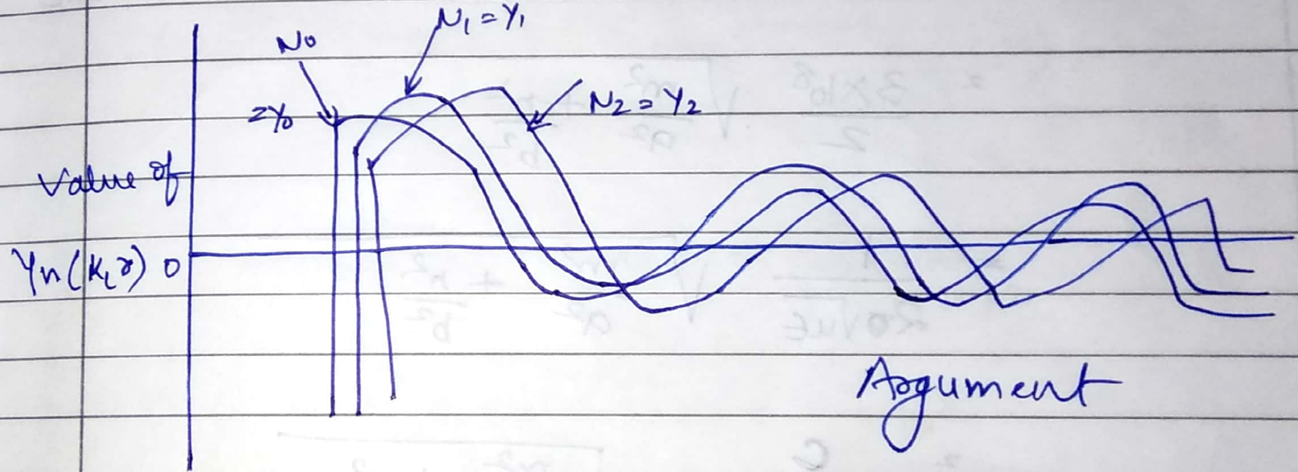
$$\psi=R\,\Phi\,Z$$

$$=\left[C_n J_n(K_c r)+D_n Y_n(K_c r)\right]\times\left[A_n\sin(n\phi)+B_n\cos(n\phi)\right]e^{\pm j\beta_g z}$$



$$\psi = C_n J_n(K_c r) \left[A_n \sin(n\phi) + B_n \cos(n\phi) \right] e^{-j\beta_g z}$$

at $r = 0$, $K_c r = 0$



$$A_n \sin(n\phi) + B_n \cos(n\phi) = \sqrt{A_n^2 + B_n^2} \cos\left[n\phi + \tan^{-1}\frac{A_n}{B_n}\right]$$

$$= F_n \cos(n\phi) \quad [n\phi \gg \phi]$$

$$\psi = C_n J_n(K_c r) F_n \cos(n\phi) e^{-j\beta_g z}$$

$$= C_n F_n J_n(K_c r) \cos(n\phi) e^{-j\beta_g z}$$

$$\psi = \psi_0 J_n(K_c r) \cos(n\phi) e^{-j\beta_g z}$$

Ques A rectangular waveguide operating in dominant mode at frequency of 2 GHz is fixed with dielectric medium with $\varepsilon_r = 9$ and has inside dimension of 7×3.5 cm. Compute

i) Cutoff frequency

ii) Phase velocity & group velocity in waveguide

iii) impedance

Sol^n

i)

$$f_c = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$= \frac{3\times10^8}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$\mu = \mu_0\mu_r$

$$= \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$\varepsilon = \varepsilon_0\varepsilon_r$

$$= \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$\mu_r = 1$

$$= \frac{c}{2\times3}\sqrt{\frac{1}{} + 0}$$

$$= \frac{3\times10^8}{6}\sqrt{\frac{1}{49}}$$

$$\boxed{f_c = \frac{2.14}{3}\ \text{GHz}}$$

ii) $V_{ph} = \frac{1}{\sqrt{\mu\epsilon}} = 10^8 m/s$

iii) $V_g = \frac{V_{ph}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$= \frac{10^8}{\sqrt{1-\left(\frac{2.173}{2}\right)^2}}$

$= \frac{10^8}{\sqrt{1-\left(\frac{0.506}{4}\right)}}$

$= \frac{10^8}{\sqrt{1-0.12}}$

$= \frac{10^8}{\sqrt{0.88}}$

$= \frac{10^8}{0.93}$

$= 1.075 \times 10^8$

iii) Guide Wavelength

$\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ $\lambda = \frac{\cancel{v/c}}{f}$

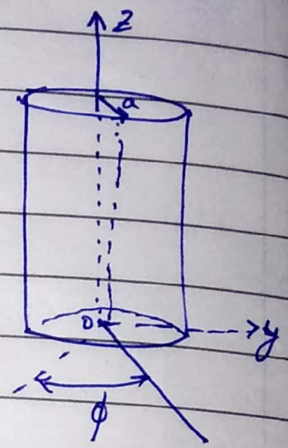
$\lambda g = \frac{10^8}{2\times10^9 \times 1.8697}$ $\lambda g = \frac{0.0802}{3} m$

- TE modes in circular waveguide <u>OR</u>

Propagation of TE wave in a circular waveguide.

TE_{mn} are characterized by $E_Z = 0$
$H_Z = 0$

helmholtz Equation for circular waveguide



$$\nabla^2 H_Z = \gamma^2 H_Z \quad -(1)$$

Soln of eq(1)

$$H_Z = H_{oz} J_n(K_c r) \cos(n\phi) e^{-j\beta_g z} \quad -(2)$$

loss less dielectric Maxwell's Eqn

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega\mu\vec{H} \quad (3)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = j\omega\vec{D} = j\omega\epsilon\vec{E} \quad (4)$$

In ~~circular~~ cylendrical coordinates

$$\frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{vmatrix}$$

$$\frac{\hat{r}}{r}[\quad] - \hat{\phi}[\quad] + \hat{z}[\quad] = j\omega\mu(\hat{r}H_r + \hat{\phi}H_\phi + \hat{z}H_z)$$

Equating the coefficients of $\hat{r}, \hat{\phi}, \hat{z}$, we obtain

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_r \quad -(5)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi \quad -(6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_\phi) - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z \quad -(7)$$

Similarly

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_r \quad -(8)$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E_\phi \quad -(9)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r H_\phi) - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\epsilon H_z \quad -(10)$$

$$K_c^2 = \gamma^2 + \omega^2\mu\epsilon \qquad \sim e^{-j\beta_g z}$$

$$\frac{\partial}{\partial z} = -j\beta_g$$

$$= (-j\beta_g)^2 + \omega^2\mu\epsilon = -\beta_g^2 + \omega^2\mu\epsilon$$

$$K_c = \omega^2\mu\epsilon - \beta_g^2$$

field Components in Cylindrical coordinates

$$E_r = -j\frac{\beta_g}{K_c^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{K_c^2}\frac{1}{r}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = -j\frac{\beta_g}{K_c^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial r}$$

$$H_r = \frac{-j\beta_g}{K_c^2}\frac{\partial H_z}{\partial r} + \frac{j\omega\epsilon}{K_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$H_\phi = \frac{-j\beta}{K_c^2}\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial r}$$

for TE wave $E_z = 0$

$$E_r = -j\frac{\omega\mu}{K_c^2}\frac{1}{r}\frac{\partial H_z}{\partial \phi} \quad -(11)$$

$$E_\phi = \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial r} \quad -(12)$$

$$E_z = 0 \quad -(13)$$

$$H_r = \frac{-j\beta_g}{K_c^2}\frac{\partial H_z}{\partial r} \quad -(14)$$

$$H_\phi = -j\frac{\beta_g}{K_c^2}$$

from eq(17)

$$\left.\frac{\partial H_z}{\partial r}\right|_{r=a} = 0$$

from

$$\frac{\partial}{\partial r}\left[H_{oz} J_n(k_c r) \cos(n\phi) e^{-j\beta g z}\right] = 0$$

or

$$H_{oz} k_c J_n'(k_c r) \cos(n\phi) e^{-j\beta g z} = 0$$

at $r = a$

$J_n'(k_c a) = 0 \Rightarrow k_c a = x_{np}$ (x_{np} is the zero of the Bessel fn)

$$k_c = \frac{x_{np}}{a}$$

zeros of $J_n'(k_c a)'$

P	n = 0	1	2	3
1	3.832	1.841	3.054	4.201
2	7.016	5.331	6.706	8.015
3				

TE_{11} $\quad k_c a = x_{np} = x_{11}$

$$k_c = \frac{x_{11}}{a} = \frac{1.841}{a}$$

$$k_c = \frac{x_{np}}{a} \quad -(19)$$

field Components in case of Cylindrical waveguide

$$E_r = \frac{j\omega\mu}{K_c^2} \frac{1}{r} n H_{0z} J_n\left(\frac{x_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g z}$$

$$= E_{0r} J_n\left(\frac{x_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g z} \quad -(20)$$

Similarly

$$E_\phi = E_{0\phi} J_n'\left(\frac{x_n p r}{a}\right) \cos(n\phi) e^{-j\beta_g z} \quad (21)$$

$$E_z = 0 \quad (22)$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J_n'\left(\frac{x_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g z} \quad -(23)$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n\left(\frac{x_n p r}{a}\right) \sin(n\phi) e^{-j\beta_g z} \quad -(24)$$

$$H_z = H_{0z} J_n\left(\frac{x_n p r}{a}\right) \cos(n\phi) e^{-j\beta_g z} \quad -(25)$$

Z_g = wave impedance $= \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r}$, $n = 0, 1, 2, 3$

$p = 1, 2, 3, \ldots$

mode propagation constant

$$\beta_g^2 + K_c^2 = \omega^2 \mu \epsilon$$

$$\beta_g = \sqrt{\omega^2 \mu \epsilon - K_c^2} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{x_n p}{a}\right)^2} \quad -(26)$$

The cutoff wave no. of a mode

$\beta_g = 0$

$$K_c = \frac{x_{np}}{a} = \omega\sqrt{\mu\epsilon} \quad -(27)$$

$$k_c = \frac{x_n p}{a} = \omega_c \sqrt{\mu\epsilon}$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \frac{x_{np}}{a} \Bigg| f_c = \frac{c}{2\pi a} x_{np}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0\mu_r\epsilon_r}}$$

Phase velocity

$$V_{ph} = \frac{\omega}{\beta g} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Guide wavelength and wave impedance

$\Downarrow$

$$\lambda g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TE} = \frac{\omega\mu}{\beta g} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

Ques For TE_{11} mode is propagating through a Circular waveguide. readium 5 cm & guide Contain an air dielectric

Determine :-

i) Cutoff fequency

ii) guided wavelength and operating frequency in gHz

iii) Determine wave impedance (Z_g) in the guide

Solⁿ

$$f_c = \frac{c}{2\pi a} x_{np} \qquad \begin{matrix} n=1 \\ p=1 \end{matrix}$$

$$= \frac{3\times10^8 \times 1.842}{2\times3.14\times5\times10^{-2}} = 1.7599 \text{ GHz}$$

ii) $\lambda = \frac{c}{f}$ $\quad [f = 3GHz]$

$= 10^{-1}m = 10cm$

$$\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{fc}{f}\right)^2}} = \frac{10}{\sqrt{1-\left(\frac{1.75}{3}\right)^2}} = 12.311 \text{ cm}$$

iii) $$Zg = \frac{\eta}{\sqrt{1-\left(\frac{fc}{f}\right)^2}} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{1-\left(\frac{1.75}{3}\right)^2}}$$

$$= \frac{376.8}{0.8122}$$

$$= 463.9 \text{ ohm}$$

(*) TM waves in Circular waveguide

OR

Propagation of TM waves in circular waveguide

TM_{np}

$H_z = 0$

$E_z \neq 0$

Helmholtz Equation in Circular waveguide

$$\nabla^2 E_z = \gamma^2 E_z \quad -(1)$$

Soln of Eq (1) Can be written as

$$E_z = E_{0z} \, J_n(k_c r) \cos(n\phi) \, e^{-j\beta g z} \quad -(2)$$

at the boundary of the cylinder i.e, at $r = a$, $E_z = 0$

$$J_n(K_c a) = 0 \Rightarrow K_c a = \chi_{np} \quad (n = 0, 1, 2, 3, \ldots \quad p = 1, 2, 3, \ldots)$$

Table

p \ n =	0	1	2	3
1	2.405	3.832	5.136	6.380
2				
3				

$$\frac{\partial}{\partial z} = -j\beta_g$$

field Equations for Cylendrical waveguide

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \qquad \nabla \times \vec{H} = j\omega\epsilon\vec{E}$$

$$E_r = \frac{-j\beta_g}{K_c^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{K_c^2}\frac{1}{r}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = \frac{-j\beta_g}{K_c^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial r}$$

$$H_r = \frac{-j\beta_g}{K_c^2}\frac{\partial H_z}{\partial r} + \frac{j\omega\epsilon}{K_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$H_\phi = \frac{-j\beta_g}{K_c^2}\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial r}$$

for TM waves $H_z = 0$

$$E_r = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial r}, \quad E_\phi = -\frac{j\beta_g}{k_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi} \quad (A)$$

$$E_z = E_{0z} J_n(k_c r)\cos(n\phi)\, e^{-j\beta_g z}$$

$$H_r = \frac{j\omega\epsilon}{k_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$H_\phi = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial r}$$

$$k_c^2 = \omega^2\mu\epsilon - \beta_g^2$$

or

$$\beta_g^2 = \omega^2\mu\epsilon - k_c^2 = \omega^2\mu\epsilon - \left(\frac{X_{np}}{a}\right)^2$$

Using the values of E_z. Eq(A) can be written

$$E_r = \frac{-j\beta_g}{k_c^2}\frac{\partial E_z}{\partial r} = E_{0r} J_n'\left(\frac{X_{np}}{a} r\right)\cos(n\phi)\, e^{-j\beta_g z}$$

$$E_\phi = -\frac{j\beta_g}{k_c^2}\frac{1}{r}\frac{\partial E_z}{\partial \phi} = E_{0\phi} J_n\left(\frac{X_{np}}{a} r\right)\sin(n\phi)\, e^{-j\beta_g z}$$

$$E_z = E_{0z} J_n\left(\frac{X_{np}}{a} r\right)\cos(n\phi)\, e^{-j\beta_g z}$$

$$H_r = \frac{E_{0\phi}}{Z_g} J_n\left(\frac{X_{np}}{a} r\right)\sin(n\phi)\, e^{-j\beta_g z}$$

where

$$Z_g = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \frac{\beta_g}{\omega\epsilon}$$

$$H_\phi = H_{0\phi}\, J_n'\left(\frac{X_{np}}{a} r\right) \cos(n\phi)\, e^{-j\beta_g z}$$

$$H_z = 0$$

$$\beta_g = \sqrt{\omega^2 \mu\epsilon - k_c^2} = \sqrt{\omega^2\mu\epsilon - \left(\frac{X_{np}}{a}\right)^2}$$

mode Propagation constant

Cutoff wave number $k_c = \frac{X_{np}}{a} = \omega_c\sqrt{\mu\epsilon} = 2\pi f_c \sqrt{\mu\epsilon}$

$f_c = ?$

$$f_c = \frac{X_{np}}{2\pi a}\frac{1}{\sqrt{\mu\epsilon}}$$

Phase Velocity, $v_p = \frac{\omega}{\beta_g} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

Guide wavelength, $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

wave impedance $Z_g = \frac{\beta_g}{\omega\epsilon} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

Ques Calculate the ratio of the cross-section of a circular waveguide to that of the rectangular one, if each is to have the cutoff wavelength for its dominant mode.

Solⁿ

Cutoff wavelength of rectangular waveguide

$\lambda = \frac{c}{v}$

$$\lambda = \frac{c}{\frac{c}{2}\sqrt{\left(\frac{m^2}{a^2}\right)+\left(\frac{n^2}{b^2}\right)}}$$

$m = 1$
$n = 0$

for dominant mode $m=1, n=0$

$$\lambda = \frac{2c\times a}{c\times m} = \frac{2a}{m} = 2a$$

Cut off wavelength for Circular waveguide.

$$f_c = \frac{X_{np}}{2\pi a} \cdot \frac{1}{\sqrt{\mu\epsilon}}$$

$$f_c = \frac{c}{2\pi a} \cdot X_{np}$$

$$\lambda = \frac{c}{\frac{c}{2\pi a} X_{np}}$$

$$\lambda_c = \frac{2\pi a}{X_{np}}$$

$$\frac{\cancel{\lambda_{circular}}}{\cancel{\lambda_{rectang}}} = \frac{2\pi a_1}{}$$

$$\lambda_{circular} = \frac{2\pi a_c}{X_{np}}$$

$$\lambda_{rect.} = 2a_r$$

$$\lambda_{cir.} = \lambda_{rect.}$$

$$\frac{\cancel{2}\pi a_c}{X_{np}} = \cancel{2}a_r$$

$$\frac{a_c}{a_r} = \frac{X_{np}}{\pi} = \frac{1.842}{3.14}$$

$$= 0.586$$

or $\lambda_c = \frac{2\pi}{K_c} = \frac{2\pi a}{K_c a} = \frac{2\pi a}{1.84}$

$K_c a = x_{11} = 1.84$

$$\boxed{\lambda_c = 3.41a}$$

$A_c = \pi a^2$

Rectangular

$\lambda_c' = \lambda_c$

$\lambda_c' = 2a'$

$2a' = 3.41a \Rightarrow a' = 1.705a$

$A_r = a' \times b' = a' \times \frac{a'}{2} = \frac{a'^2}{2} = \frac{(1.705a)^2}{2}$

$\frac{A_c}{A_r} = \frac{\pi a^2 \times 2}{(1.705)^2 a^2} = \frac{2\pi}{(1.705)^2} = 2.161$

⊛ TEM modes in Circular Waveguide

OR

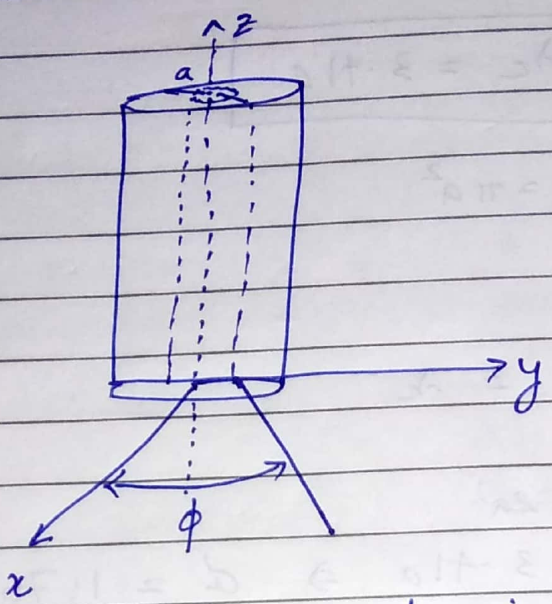
Propagation of TEM modes in Circular waveguides

TEM (Transverse electric and magnetic)

TEM are characterized by $E_z = 0 = H_z$

1. This means that electric and magnetic field vector are completely transverse to the direction of wave propagation.

2. This mode can't exist in hollow waveguide, since it requires two conductors such co-axial transmission line and two open wire line.



Properties of TEM modes in lossless medium

1. Its cutoff frequency is zero.
2. Its transmission line is two conductors system.
3. Its wave impedance is the impedance in an unbounded dielectric
4. Its propagation constant is constant in an unbounded dielectric.
5. Its phase velocity is the velocity of light in an unbounded dielectric

Ques An air fill circular waveguide has a radius of 2cm and is to carry at frequency of 10 GHz. find all TE_{np} and TM_{np} mode for energy transmission is possible.

Sol^n $K_{ca} = \frac{1}{c} \times 2\pi \times a f_c = \frac{2\pi \times (10^4 \times 10)}{3\times 10^{8}} \times 2\times 10^{-2} = 4.18$

Energy is true for modes which have K_{ca} less than K_{ca}

TE01 (3.832)   TM01 (2.405)

TE11 (1.84)   TM11 (3.83)

TE21 (3.05)

Ques An air fill rectangular waveguide 8×4 cm. suppose a wave at frequency $f = 1.5 f_c$. find the cutoff frequency and ratio of the guide velocity to the velocity in free space for the dominant mode

Sol^n

$$f_c = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$= \frac{3\times 10^8}{2}\sqrt{\frac{1}{64}}$$

$$= \frac{c}{2\sqrt{\mu_r \varepsilon_r}}\sqrt{\frac{m^2}{a^2}}$$

$$= \frac{c}{2\times 1} \times \frac{1}{8\times 10^{-2}} = \frac{3\times 10^{10}}{16}$$

$$= 1.875\ GHz$$

$$V_{ph} = \frac{1}{\sqrt{\mu\varepsilon}} = 3\times 10^8$$

$$V_g = \frac{V_{ph}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times 10^8}{\sqrt{1-\left(\frac{f_c}{1.5 f_c}\right)^2}}$$

$$= \frac{3\times 10^8}{\sqrt{1-\left(\frac{1}{1.5}\right)^2}} = \frac{3\times 10^8}{\sqrt{1-0.44}}$$

$$= \frac{3\times 10^8}{0.74} = 4.05\times 10^8$$

$$\frac{V_{ph}}{V_g} = \frac{3\times10^8}{4.05\times10^8}$$

$$\boxed{\frac{V_{ph}}{V_g} = \frac{3}{4.05} = 0.74}$$

(*) Power transmission in Rectangular waveguides

Transmitted Power

$$P_{tr} = \oint_s \vec{P}\cdot d\vec{s} = \oint_s (\vec{E}\times\vec{H})\,d\vec{s} = \frac{1}{2}\oint_s (\vec{E}\times\vec{H}^*)\cdot d\vec{s}$$

Identity $\vec{E}\times\vec{H} = Re\,\vec{E}\times Re\,\vec{H} = \frac{1}{2}Re\left[\vec{E}\times\vec{H} + \vec{E}\times\vec{H}^*\right]$

$$Re\,\vec{E}\cdot Re\,\vec{H} = \frac{1}{2}Re\left[\vec{E}\cdot\vec{H} + \vec{E}\cdot\vec{H}^*\right]$$

for a lossless dielectric, the time average power flow

$$P_{tr} = \frac{1}{2Z}\int_a |E|^2 da = \frac{Z}{2}\int |H|^2 da$$

$$H = \frac{E}{Z}$$

where $Z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

for TM_{mn} wave impedance

$$Z = \eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$= \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$P_{tr} = \frac{1}{2\eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\,dy \quad -(1)$$

for TE_{mn}

$$Z = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$P_{tr} = \frac{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}{2\eta} \int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\,dy$$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$ is the intrinsic impedance

(7) Power losses in Rectangular waveguides

Two types of losses

1) losses in the dielectric
2) losses in the guide waves

for dielectric $\sigma << \omega\epsilon$

$$\alpha = j\omega\sqrt{\mu\epsilon}\left(-j\frac{\sigma}{2\omega\epsilon}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \eta\frac{\sigma}{2} \quad -(1)$$

η is the intrinsic impedance

The attenuation for TE mode

$$\alpha_g = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{2}\sigma\sqrt{1-\left(\frac{f_c}{f}\right)^2} \text{ for TM mode}$$

$$|E| = |E_{0z}|\, e^{-\alpha_g z}$$

$$|H| = |H_{0z}|\, e^{-\alpha_g z} \quad , \quad \gamma = \alpha + j\beta$$

$$\gamma = \alpha$$

$|E_{0z}|$ and $|H_{0z}|$ are the amplitude of the electric and magnetic fields

$$P = \vec{E} \times \vec{H}$$

$$P_{tr} = (P_{tr} + P_{loss})\, e^{-2\alpha_g z}$$

for $P_{loss} << P_{tr}$, $2\alpha_g z << 1$

$$e^{2\alpha_g z} = \frac{(P_{tr} + P_{loss})}{P_{tr}} = 1 + \frac{P_{loss}}{P_{tr}}$$

or

$$\cancel{1} + \frac{P_{loss}}{P_{tr}} = e^{2\alpha_g z} = \cancel{1} + 2\alpha_g z$$

$$\alpha_g = \frac{P_{loss}}{2 P_{tr}}$$

If there is surface resistance

$$R_s = \frac{\rho}{\delta} = \frac{1}{\sigma\delta} = \frac{\alpha_g}{\sigma} = \sqrt{\frac{\pi f \mu}{\sigma}} \quad \Omega/\text{sq. m}$$

ρ = resistivity of the conducting wall in ohm-meter

δ = skin depth or depth of penetration in m

$$P_h = \frac{R_s}{2} \int_s |H_t|^2 ds \quad \text{watt per unit length}$$

H_t is the tangential component of the magnetic intensity

$$P_{tr} = \frac{1}{2Z_g}\int_a |E|^2 da = \frac{Z_g}{2}\int_a |H|^2 da$$

$$\alpha_g = \frac{R_s \int_s |H_t|^2 ds}{2Z_g \int_a |H|^2 da}$$

* Power Transmission in circular waveguides

$$\vec{P} = \vec{E} \times \vec{H}$$

Power transmitted through a circular waveguides

$$P_{tr} = \oint_s \vec{P} \cdot d\vec{s} = \oint \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s}$$

$$= \frac{1}{2Z_g}\int_a |E|^2 da = \frac{Z_g}{2}\int_a |H|^2 da, \quad Z = \frac{E}{H}$$

for a loss less dielectric, time average power transmitted circular waveguide.

$$P_{tr} = \frac{1}{2Z_g}\int_0^{2\pi}\int_0^a \left[|E_r|^2 + |E_\phi|^2\right] r\,dr\,d\phi \quad (1)$$

$$= \frac{Z_g}{2}\int_0^{2\pi}\int_0^a \left[|H_r|^2 + |H_\phi|^2\right] r\,dr\,d\phi \quad -(2)$$

$$Z = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r}$$ is the wave impedance in the guide

a = radius of the circular waveguide

for TE_{np} mode

$$Z_g = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$P_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r\,dr\,d\phi$$

for TM_{np} mode

$$P_{tr} = \frac{1}{2\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r\,dr\,d\phi.$$

$$Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

for a coaxial line

the average power transmitted

$$P_{tr} = \frac{1}{2\eta} \int_0^{2\pi} \int_0^a \left[|E_r|^2 + |E_\phi|^2 \right] r\,dr\,d\phi$$

If current carried by the centre conductor of a coaxial line

$$I_z = I_0 e^{-j\beta_g z}$$

from Ampere's law $\oint \vec{H} \cdot d\vec{l} = I = I_0 e^{-j\beta_g z} = 2\pi r H_\phi$

$$H_\phi = \frac{I_0}{2\pi r} e^{-j\beta_g z}$$

$$P_{tr} = \frac{1}{2\eta} \int_0^{2\pi} \int_0^b |\eta H_\phi|^2 r\,dr\,d\phi$$

$$= \frac{1}{2\eta} \int_0^{2\pi} \int_0^b \eta^2 \frac{I_0^2}{4\pi^2 r^2} \left| e^{-j\beta_g z} \right|^2 r\,dr\,d\phi$$

$$P_{tr} = \frac{\eta I_0^2}{8\pi^2} \int_0^{2\pi} \int_0^{b} \frac{1}{r}\, dr\, d\phi$$

$$= \frac{\eta I_0^2 \times 2\pi}{8\pi^2} \int_a^b \frac{1}{r}\, dr = \frac{\eta I_0^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

Potential rise from the outer Conductor

$$V_r = -\int_b^a E_r\, dr = -\int_b^a \eta H_\phi\, dr = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right) I$$

Characteristic impedance of a coaxial line

$$Z_0 = \frac{V}{I} = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

Power transmitted

$$P_{tr} = \frac{1}{2\eta} \int_0^{2\pi} \int_a^b |\eta H_\phi|^2 r\, dr\, d\phi$$

$$= \frac{\eta I_0^2}{4\pi} \ln\left(\frac{b}{a}\right) = \frac{\eta I_0 \ln\left(\frac{b}{a}\right)}{2\pi \times 2} I_0$$

$$= \frac{1}{2} V_0 I_0$$

$$P_L = 2\alpha P_{tr}$$

$$\alpha = \frac{P_L}{2P_{tr}} = \frac{\text{Power loss per unit length}}{2 \text{ Power transmitted}}$$

for a lossless conductor attenuation constant $= \dfrac{R_s \int_s |H_t|^2 ds}{2\eta \int_a |H|^2 da}$

$$\alpha = \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$