CT 1

Aditya Singh
2K19/EP/005

Q1 $\{(p \to q) \wedge q\} \to p$

ans. c) contingency.

Q2 ans. a) $(p \leftrightarrow q) \leftrightarrow (p \to q) \wedge (q \to p)$.

Q3 ans. a) Mathematical induction

Q4 $a_n = (2.4)\,3^n + 0.6\,(-2^n)$.

Q5 Direct proof is a sequence of statements which are either givens or deductions from previous statements, & whose last statement is the conclusion to be proved.

if $x > y$, $\max(x,y) = x$, $\min(x,y) = y$

$\therefore \max(x,y) + \min(x,y) = x + y$

if $x < y$, $\max(x,y) = y$, $\min(x,y) = x$

$\therefore \max(x,y) + \min(x,y) = y + x$.

Q6 if P is a set of integers such that

i) a is in P, ii) if all integers k, $a \le k \le n$ are in P, then $n+1$ is also in P.

then $P = \{x \in \mathbb{Z} \mid x \ge a\}$ that is, P is the set of all integers greater than or equal to a.

Let $M(n) \Rightarrow$ "You can run n^{th} mile".

Basis Step : $n=1, n=2$,

$M(1)$ and $M(2)$ are true, according to question.

Inductive :

Assume $M(1), M(2) \dots M(K)$ are all true,

then you can run first K miles.

Since, $M(K-1)$ is true, then $M(K+1)$ is true,

as you can run 2 miles more after a specific mile.

Conclusion :

By the principle of strong induction,

$M(n)$ is true for all positive n integers.

Q7. Proof with example.

$a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1, a_1 = 6$.

$\therefore r^2 - 6r + 9 = 0$ has only 3 as root.

Solution format : $\alpha_1 3^n + \alpha_2 n 3^n$,

$a_0 = 1 = \alpha_1$, $a_1 = 6 = \alpha_1 3 + \alpha 3$

$\Rightarrow$ $\boxed{\alpha_1 = 1, \alpha_2 = 1}$

$\therefore$ $a_n = \underline{3^n + n3^n}$