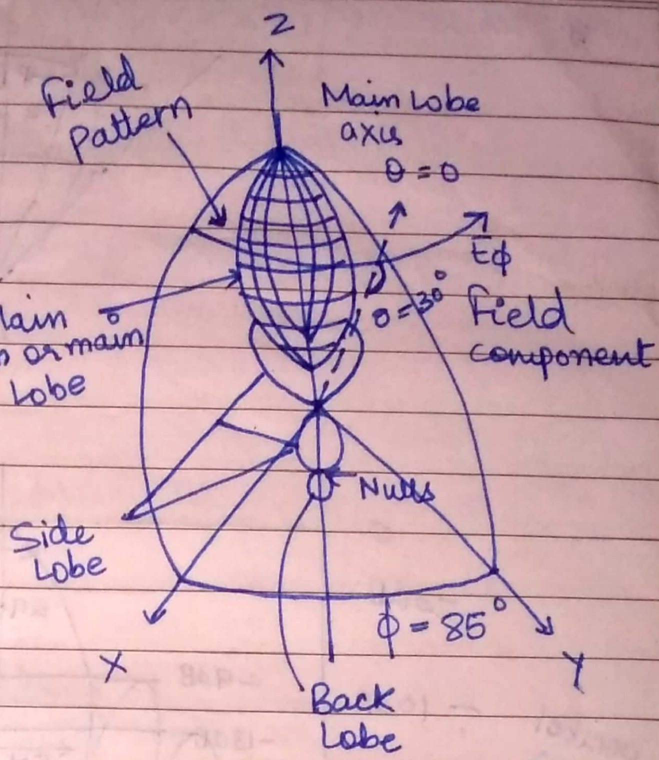
Field Pattern



1) $E_\theta(\theta,\phi)$ V/m
2) $E_\phi(\theta,\phi)$ V/m
3) phases $\delta_\theta(\theta,\phi)$ or $\delta_\phi(\theta,\phi)$ (rad. or degree)

Fig 1: 3D field pattern of a directional Antenna with max. radiation in z-direction at θ=0°

Most of the radiation is contained in a Main beam (or Main lobe).

Field Pattern for Electric field

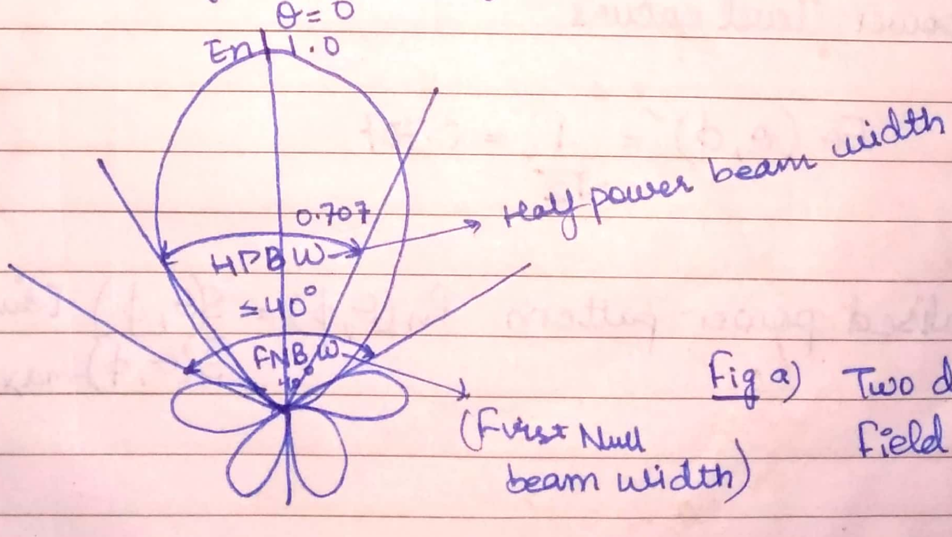


Fig a) Two dimensional field pattern

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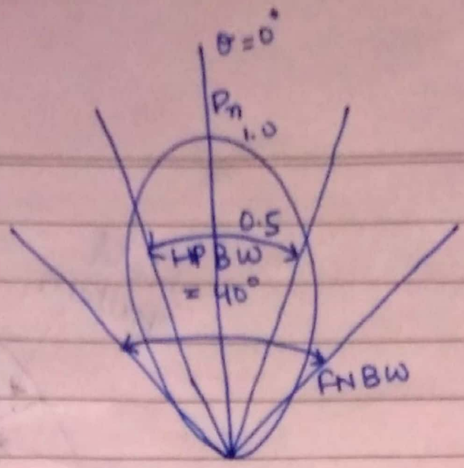
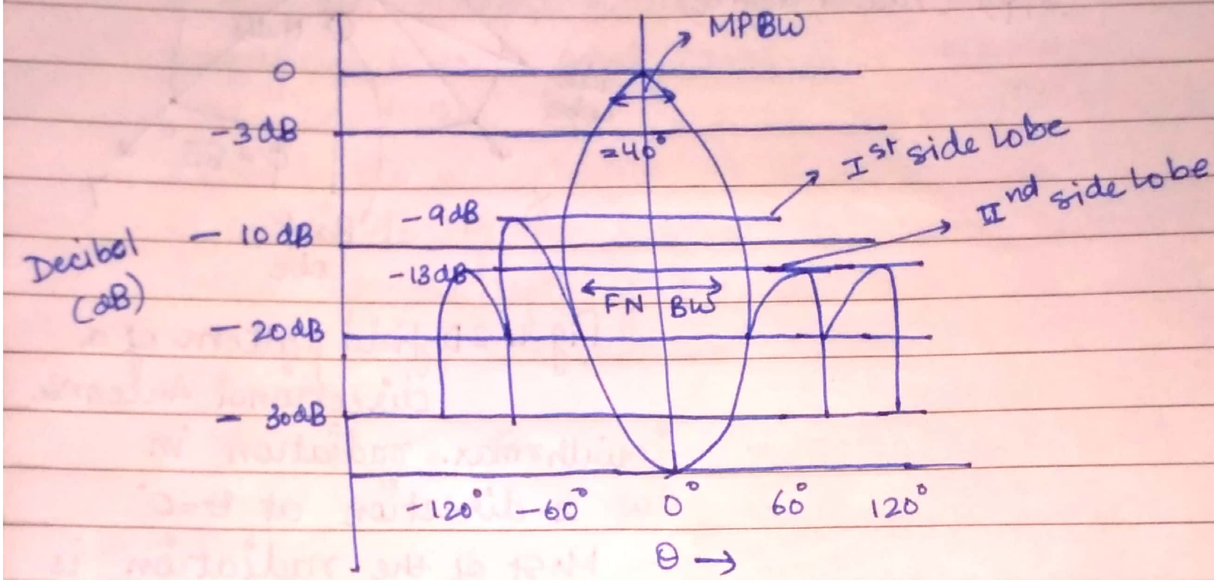


b)

b) Two dimensional power pattern



Normalised Field Pattern $= E_\theta(\theta,\phi)_n = \dfrac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}$ (dimensionless)

Half power level occurs

$$E_\theta(\theta,\phi) = \frac{1}{\sqrt{2}} = 0.707$$

Normalised power pattern $P_n(\theta,\phi) = \dfrac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$ [dimensionless]

$S(\theta,\phi)$ = Poynting Vector = $[E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)]/Z_0$ Watt/m²

$S(\theta,\phi)_{max}$ = maximum value of $S(\theta,\phi)$ W/m²

Z_0 = Intrinsic Impedance of Space = 377 Ω

Decibel ~~Value~~ Level

$$dB = 10 \log_{10} P_n(\theta,\phi)$$

$$P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

Power Pattern $P(\theta,\phi) = \left(\frac{E_\theta^2(\theta,\phi) + E_\phi(\theta,\phi)}{Z_0}\right) r^2$

$= S_r(\theta,\phi), r^2$

power per steradian or radiation Intensity

Intrinsic Impedance

θ - comp of electric field V/m

Component of Electric field V/m

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Q) An antenna has a field pattern given by
$E(\theta) = \cos^2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find the Half power beam width (HPBW) 66°

Q An antenna has a field pattern given by
$E(\theta) = \cos\theta \cos 2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find

1) HPBW 41°
2) Beam width between first Nulls (FNBW) 90°

Ans 1

$E_\theta = \cos^2\theta = 0.707$

$\theta = 32.77$

Bandwidth $= 2 \times 32.77 \Rightarrow 65^\circ$

Ans 2

$2\cos^3\theta - \cos\theta - 0.707 = 0$

$\cos\theta = x$

$2x^3 - x - 0.707 = 0$

$x(2x^2 - 1) = 0.707$

$x = 0.707$ $\qquad 2x^2 = 1.707$

$\theta = \frac{1}{2}\cos^{-1}\frac{1}{\sqrt{2}\cos\theta'}$

$\theta' = 0$

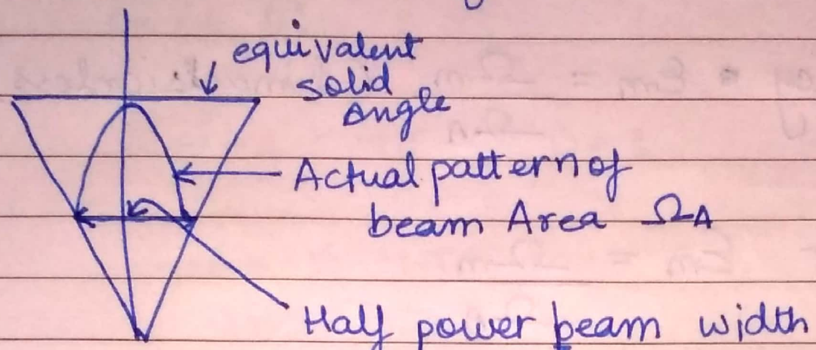
$\theta = ? = 22.5^\circ$

$\theta' = 22.5^\circ, \theta = ?$

Beam Area (or Beam Solid Angle)

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

$d\Omega$ = Solid Angle. (expressed in Steradian (Sr))



$$\text{Beam Area } \Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta\, d\phi$$

$$= \iint_{4\pi} P_n(\theta, \phi)\, d\Omega \ (Sr)$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

$$\text{Power radiation} = P(\theta, \phi)\, \Omega_A \text{ Watt}$$

$$\text{Beam Area} \simeq \Omega_A \cong \theta_{HP}\, \phi_{HP}$$

θ_{HP} and ϕ_{HP} : half power beam width in two planes minor lobe being neglected.

Radiation Intensity

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{max}} = \frac{S(\theta, \phi)}{S(\theta, \phi)_{max}} \quad \text{[dimensionless]}$$

Beam Efficiency

Total beam Area = Main beam Area + Minor beam Area

$$\Omega_A = \Omega_M + \Omega_m$$

Beam efficiency $\varepsilon_M = \frac{\Omega_M}{\Omega_A}$ (dimensionless)

Stray factor $\varepsilon_m = \frac{\Omega_m}{\Omega_A}$

$$\varepsilon_M + \varepsilon_m = 1$$

Directivity D and Gain G

$$\underset{\text{Directivity}}{D} = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}}$$

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\theta=0}^{2\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$\text{Directivity } D = \frac{P(\theta,\phi)_{max}}{(1/4\pi)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

$$= \frac{1}{(1/4\pi)\iint_{4\pi}\left[\frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}\right]d\Omega} = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A}$$

$\vec{E} \times \vec{H} = \bar{S}$

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$P_n(\theta,\phi)\, d\Omega = \dfrac{P(\theta,\phi)}{P(\theta,\phi)_{max}}$ = normalised power pattern

Smaller the beam area, larger will be directivity D

$P_n(\theta,\phi) = 1$

$\Omega_A = 4\pi$ & $D = 1$

$\Omega_A \leq 4\pi$
and $D \geq 1$

Gain $G = KD \leftarrow$ Directivity

$K =$ efficiency factor $= \dfrac{G}{D}$

Effective Area $A = \dfrac{\text{Power}}{\text{Poynting Vector}} = \dfrac{P}{S}$ $= E \times H$

Significance of Area

For a lossless Antenna

$P = \dfrac{V^2}{4R_r}$ $\leftarrow$ Radiation resistance

$A_{em} = \dfrac{V^2}{4R_r S}$

$\uparrow$
Max. effective Aperture

A_e = effective aperture
A_p = Physical aperture

Aperture efficiency = $\varepsilon_{ap} = \frac{A_e}{A_p}$

Power $P = \frac{E_a^2}{Z_0} A_e$ Watt

Z_0 — intrinsic ~~Aperture~~ impedance ($377\ \Omega$)

Aperture beam Area

$$\lambda^2 = A_e \Omega_A$$

Ω_A = beam area (Sr)

Directivity

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{D\lambda^2}{4\pi} = 0.0796\lambda^2 \quad (D = 1)$$

Directivity from pattern

$$D = \frac{P(\theta, \phi)_{max}}{P(\theta, \phi)_{av}} \quad \text{(dimensionless)}$$

$$D = \frac{4\pi A_e}{\lambda^2}$$

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Q) Determine the length of antenna operating at freq. 500 KHz

$$\text{length of antenna} = \frac{C}{f} \times 0.95$$

↓ Velocity vector

$$= 570\ m$$

Pu's

Q A halfwave dipole antenna is capable of reading 1KW & has a 2.15 dB gain over an isotropic antenna. How much power must be delivered to isotropic antenna to match the field strength directional antenna.

$2.15 = 10 \log\left(\frac{P_i}{P_o}\right)$ 1.64 KW

Q Calculate the max. effective aperture of an antenna operating at $\lambda = 2m$ & directivity of 500.

Ans

$$A_{em} = \frac{D\lambda^2}{4\pi} \Rightarrow$$

$$=$$

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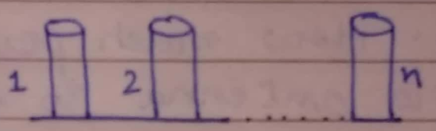
Gain $= 4\pi \dfrac{\text{Radiation Intensity}}{\text{Input power}}$

Gain = Efficiency × Directivity

$G = \eta D$

$$\boxed{\eta = \frac{G}{D}}$$

Antenna Arrays



1) Geometrical configuration of the overall Array (eg. linear, circular, rectangular & spherical etc).
2) The relative displacement between the elements.
3) The excitation amplitude of the individual element.
4) The excitation phase of the individual elements.
5) The relative pattern of the individual elements.

Two-Elements Arrays

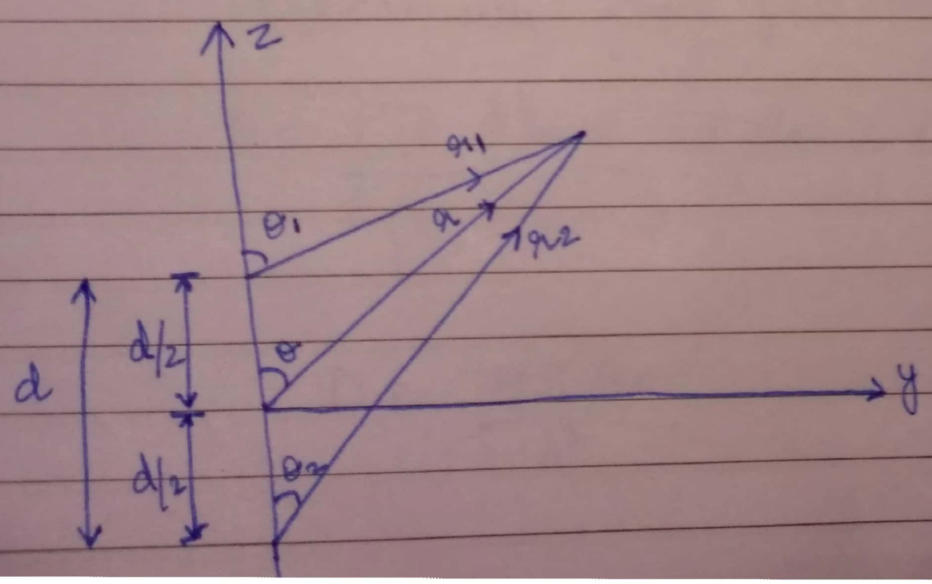


Fig: Two Infinite dipoles.

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Total Electric field

$$E_t = E_1 + E_2 = \hat{a}_\theta j\eta \frac{KI_0 l}{4\pi}\left[\frac{e^{-j\left(Kr_1 - \frac{\beta}{2}\right)}}{r_1}\cos\theta_1 + \frac{e^{-j\left(Kr_2 + \frac{\beta}{2}\right)}}{r_2}\cos\theta_2\right]$$

β = the difference in phase excitation between the elements.

Let $\theta_1 \simeq \theta_2 \simeq \theta$

$$\left.\begin{aligned} r_1 &\simeq r - d/_2 \cos\theta \\ r_2 &\simeq r + d/_2 \cos\theta \end{aligned}\right\} \text{for phase variation}$$

$r_1 \simeq r_2 \simeq r$ for Amplitude Variation

Eq. (1) can be rewritten

$$E_t = \hat{a}_\theta j\eta \frac{KI_0 l}{4\pi r} e^{-jKr}\cos\theta\left[e^{\frac{j(Kd\cos\theta+\beta)}{2}} + e^{\frac{-j(Kd\cos\theta+\beta)}{2}}\right]$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$E_t = \hat{a}_\theta j\eta \frac{KI_0 l}{4\pi r} e^{-jKr}\cos\theta \; \underbrace{2\cos\left[\frac{1}{2}(Kd\cos\theta + \beta)\right]}_{AF} \quad -(2)$$

η = intrinsic impedance of the medium

Af (Array factor) = $2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]$

This can be rewritten for normalized form

$$(Af)_n = \cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]$$

$$E(\text{total}) = \left[E \text{ (single element at reference point)}\right] \times \text{Array factor}$$

Each array has its own Array factor. This array factor, in general, is the function of the number of elements their geometrical arrangement, their relative magnitude their relative phases & their spacing.

Q) Calculate the gain of antenna with a circular aperture of diameter 3 meters & freq of 5GHz.

Ans) 24625 ⇒ gain

Ans

$$D = \frac{4\pi A_{em}}{\lambda^2} = \frac{4\pi}{\lambda^2}(\pi \times r^2)$$

$$= \frac{4\times\pi}{\left(\frac{3\times10^8}{5\times10^9}\right)^2} \times (\pi \times (1.5)^2)$$

$G = \eta D$

$\Downarrow$

1

8) The rad^n resistance of antenna is $72\,\Omega$ & loss resistance is $8\,\Omega$. What is directivity if power gain is 16.

$D = 17.78$

Ans

$$G = \eta D$$

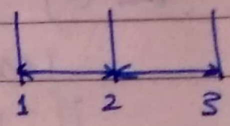
$$D = \frac{G}{\eta} \Rightarrow \frac{16 \times 72}{8} \Rightarrow 144$$

$$\eta = \frac{R_r}{R_r + R_l}$$

$$\Rightarrow \frac{\cancel{72}\,9}{\cancel{80}\,10}$$

$$D = \frac{16 \times 10}{9} \Rightarrow \frac{160}{9} = 17.78$$

N- Elements Linear Array Uniform Amplitude and Spacing



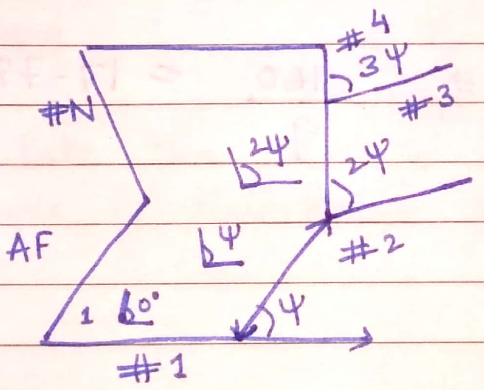
$\beta \rightarrow$ progressive phase

$$AF = 1 + e^{j(kd\cos\theta + \beta)} + e^{j2(kd\cos\theta + \beta)} + e^{j3(kd\cos\theta + \beta)} + \cdots + e^{j(N-1)(kd\cos\theta + \beta)}$$

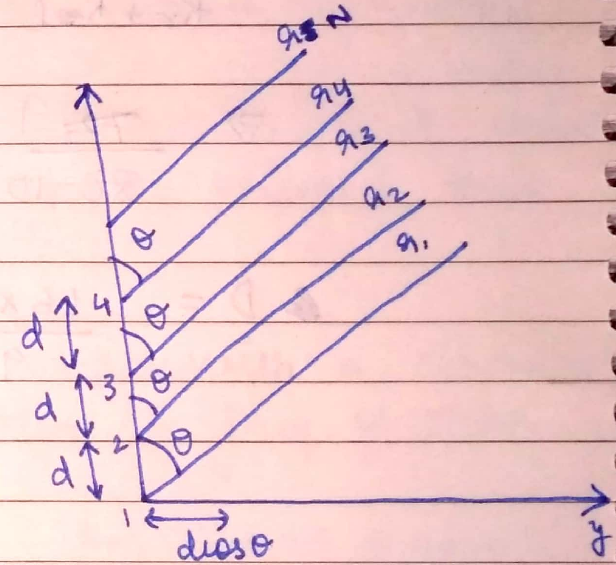
$$= \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta + \beta)} \quad (1)$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} , \quad (2)$$

where $\psi = kd\cos\theta + \beta$



b) phase diagram



(a) Geometry

Fig : Far field geometry and phase diagram of N- elements Array.

Multiplying Eq (2) by $e^{j\psi}$ on both sides , we get

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$$(AF)\, e^{j\omega} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(N-1)\psi} + e^{j\psi N} \quad (3)$$

~~Substituting~~ Substracting Eq (2) from Eq (3) we get

$$AF\,(e^{j\omega} - 1) = (-1 + e^{jN\psi})$$

$$AF = \frac{(e^{jN\psi} - 1)}{(e^{j\omega} - 1)} = e^{j\frac{(N-1)}{2}\psi} \frac{\left[e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}\right]}{\left[e^{j\frac{1}{2}\psi} - e^{-j\frac{1}{2}\psi}\right]}$$

$$AF = e^{j\left(\frac{N-1}{2}\right)\psi} \left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\frac{\psi}{2}}\right] \quad — (5)$$

consider reference point $(N=1)$

$$AF = \frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}} \quad (6)$$ without oscillatory point

for small values of ψ

$$AF \simeq \frac{\sin\frac{N\psi}{2}}{\frac{\psi}{2}} \quad (7), \qquad \sin\frac{\psi}{2} \simeq \frac{\psi}{2}$$

The maximum value of Eq. (6) or Eq. (7) is N

from Eq (6) $(AF)_{max} = \dfrac{\frac{N\psi}{2}}{\psi/2} = N$

From Eq. (7) $(AF)_{max} = \dfrac{N\psi/2}{\psi/2} = N$

So normalize array factor

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}}\right] \quad - (8)$$

or

$$(AF)_n \simeq \left[\frac{\sin\frac{N\psi}{2}}{\frac{N\psi}{2}}\right] \quad - (9)$$

To find the nulls of the array

$$\sin\frac{N\psi}{2} = 0 \Rightarrow \left.\frac{N\psi}{2}\right|_{\theta=\theta_n} = \pm n\pi$$

$$\Rightarrow \theta_n = \cos\left[\frac{\lambda}{2d\pi}\left(-\beta \pm \frac{2n\pi}{N}\right)\right] \quad - (10)$$

$$\psi = \pm\frac{2n\pi}{N}, \quad \psi = kd\cos\theta + \beta$$

$$n \neq N, 2N, 3N \ldots\ldots$$

$$\left.\frac{N\psi}{2}\right|_{\theta=\theta_n} = \pm n\pi, \quad \psi = \pm\frac{2n\pi}{N}$$

$$\psi = kd\cos\theta + \beta \Rightarrow \cos\theta = (-\beta + \psi)\frac{1}{kd}$$

$$\cos\theta = \frac{1}{(2\pi/\lambda)d}\left[-\beta \pm \frac{2n\pi}{N}\right]$$

$$\theta = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n\pi}{N}\right)\right]$$

The maximum value of Eq (8)

$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta + \beta)\Big|_{\theta=\theta_m} = \pm m\pi$$

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right] \quad -[11]$$

$m = 0, 1, 2 \ldots$

The array factor of Eq (9) has one maximum and occurs when $m = 0$ in Eq (11)

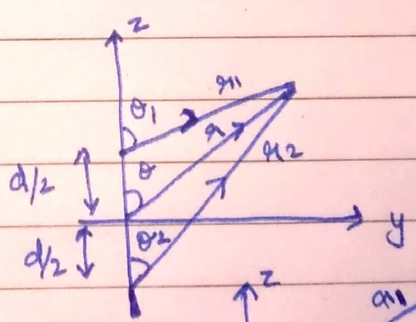
$$\theta_m = \cos^{-1}\left[\frac{\lambda\beta}{2\pi d}\right], \quad \cos(-\theta) = \cos\theta$$

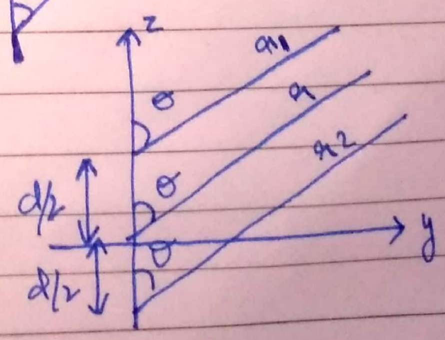
Q Given the array of figs (a) & (b) find the nulls of the total field when $d = \lambda/4$ and

a) $\beta = 0$

b) $\beta = +\pi/2$

c) $\beta = -\pi/2$





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Ans) $E_t = \cos\theta \cos\left[\frac{\cancel{1}}{\cancel{2}} \left(\frac{kd\cos\theta + \beta}{2}\right)\right]$

$\beta = 0$

$$\frac{kd}{2} = \frac{\cancel{2}\pi}{\cancel{\lambda}} \frac{\cancel{\lambda}}{\cancel{4}2} = \frac{\pi}{\cancel{2}4}$$

$E_{tn} = \cos\theta \cos\left(\frac{\pi}{4}\cos\theta\right) = 0$

for nulls

so $\Rightarrow$ $\cos\theta = 0$
$\theta = \pi/2$

$$\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0 = \cos\frac{\pi}{2}$$

$$\frac{\cancel{\pi}}{\cancel{4}2}\cos\theta_n = \frac{\cancel{\pi}}{\cancel{2}}$$

$\cos\theta_n = 2 \Rightarrow \theta_n = \cos^{-1}(2)$