Jacobs University Bremen

Natural Science Laboratory
Signals and Systems Lab

Fall Semester 2021

Lab Experiment 1 – Fourier Series and Fourier Transform

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Place of execution	: Research 1, Room 55
Date of execution	: 13 October 2021

Prelab: Fourier Series and Fourier Transform

Problem 1: Decibels

1) Given $x(t) = 2\cos(2\pi 1000t)$

   a) What is the signal amplitude in V_{pp}?
   Ans: The signal amplitude in $V_{pp} = 4V$.

   b) What is the root-mean-square value of the provided signal in V_{rms} ?
   Ans: The root mean square value of the signal is $\frac{V_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

   c) What is the amplitude of the spectral peak in dBV_{rms} ?
   Ans: The amplitude in dB for V_{rms} is: $20 \cdot log_{10}(V_{rms})$ = 3.011 dB

2) For a square wave of 4 V_{pp} and the voltage level changes between −2 V and 2 V,

   a) What is the signal amplitude in V_{rms}?
   Ans: The amplitude of the signal is: V_{rms} = 2

   b) What is the amplitude in dB V_{rms} ?
   Ans: The amplitude of the signal is: $20 \cdot log_{10}(V_{rms}) = 6.02\ dB$

Problem 2: Determination of Fourier Series Coefficients

1. Determine the Fourier series coefficients up to the 5th harmonic of the function
   $f(t) = \frac{2}{T} \cdot t \qquad \frac{-T}{2} < 0 < \frac{T}{2}$
   The function $f(t)$ is odd, as $f(-t) = -f(t)$. So, the Fourier series coefficients a_n = 0.
   Now,

$$b_n = \frac{2}{T} \cdot \int_{\frac{-T}{2}}^{\frac{T}{2}} \left(\frac{2}{T} \cdot t \cdot \sin(nw_0 t)\right) dt$$

$$= \frac{4}{T^2} \int_{\frac{-T}{2}}^{\frac{T}{2}} (t \cdot \sin(nw_0 t))\, dt$$

   Using integration by parts,

$$b_n = \frac{4}{T^2}\left(\left[\frac{-t \cdot \cos(nw_o t)}{nw_o}\right]_{-\frac{T}{2}}^{\frac{T}{2}} + \frac{1}{nw_o}\int_{\frac{-T}{2}}^{\frac{T}{2}} \cos(nw_o t)\, dt\right)$$

$$b_n = \frac{-2 \cdot (-1)^n}{\pi \cdot n}$$

So, the Fourier series becomes,

$$\therefore f(t) = \sum_{k=1}^{\infty} \frac{-2 \cdot (-1)^k}{\pi \cdot k} \sin k\omega_0 t$$

For k = 1, $f(t) = \frac{2}{\pi} \sin \omega_0 t$

For k = 2, $f(t) = \frac{2}{\pi} \sin \omega_0 t + \frac{-2}{2 \cdot \pi} \sin 2 \cdot \omega_0 t$

For k = 3, $f(t) = \frac{2}{\pi} \sin \omega_0 t + \frac{-2}{2 \cdot \pi} \sin 2 \cdot \omega_0 t + \frac{2}{3 \cdot \pi} \sin 3 \cdot \omega_0 t$

For k = 4, $f(t) = \frac{2}{\pi} \sin \omega_0 t + \frac{-2}{2 \cdot \pi} \sin 2 \cdot \omega_0 t + \frac{2}{3\pi} \sin 3 \cdot \omega_0 t + \frac{-2}{4 \cdot \pi} \sin 4 \cdot \omega_0 t$

For k = 5, $f(t) = \frac{2}{\pi} \sin \omega_0 t + \frac{-2}{2\pi} \sin 2 \cdot \omega_0 t + \frac{2}{3\pi} \sin 3 \cdot \omega_0 t + \frac{-2}{4 \cdot \pi} \sin 4 \cdot \omega_0 t + \frac{2}{5 \cdot \pi} \sin 5 \cdot \omega_0 t$

2. Use MATLAB to plot the original function.

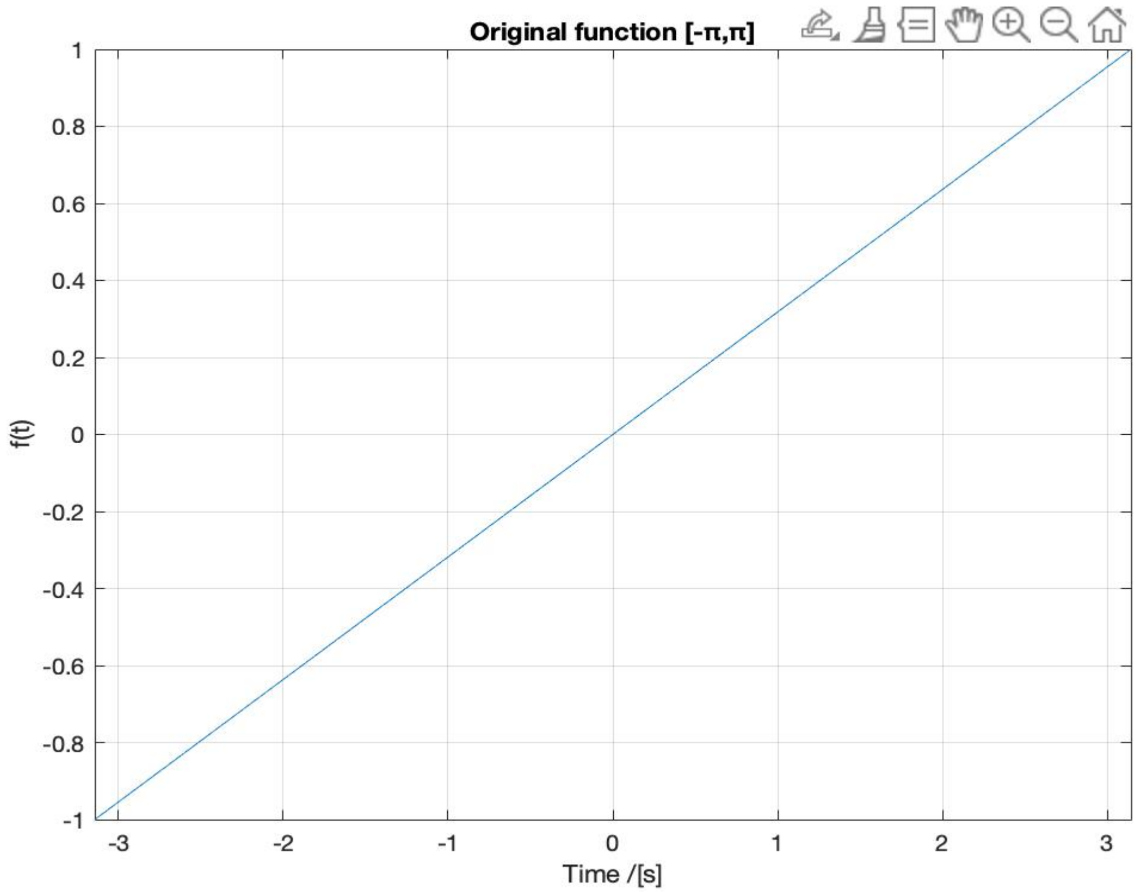

Figure 1: MATLAB plot of given original function

The MATLAB code used is:

```
%Original Function Plot
T = 2*pi;
t = (-T/2:0.0001:T/2);
y = (2/T)*t;
figure(1);
plot(t,y);
xlim([-pi,pi]);
grid on;
xlabel("Time /[s]");
ylabel("f(t)")
title("Original function [-π,π]");
```

3. Use MATLAB to plot function using the calculated coefficients.

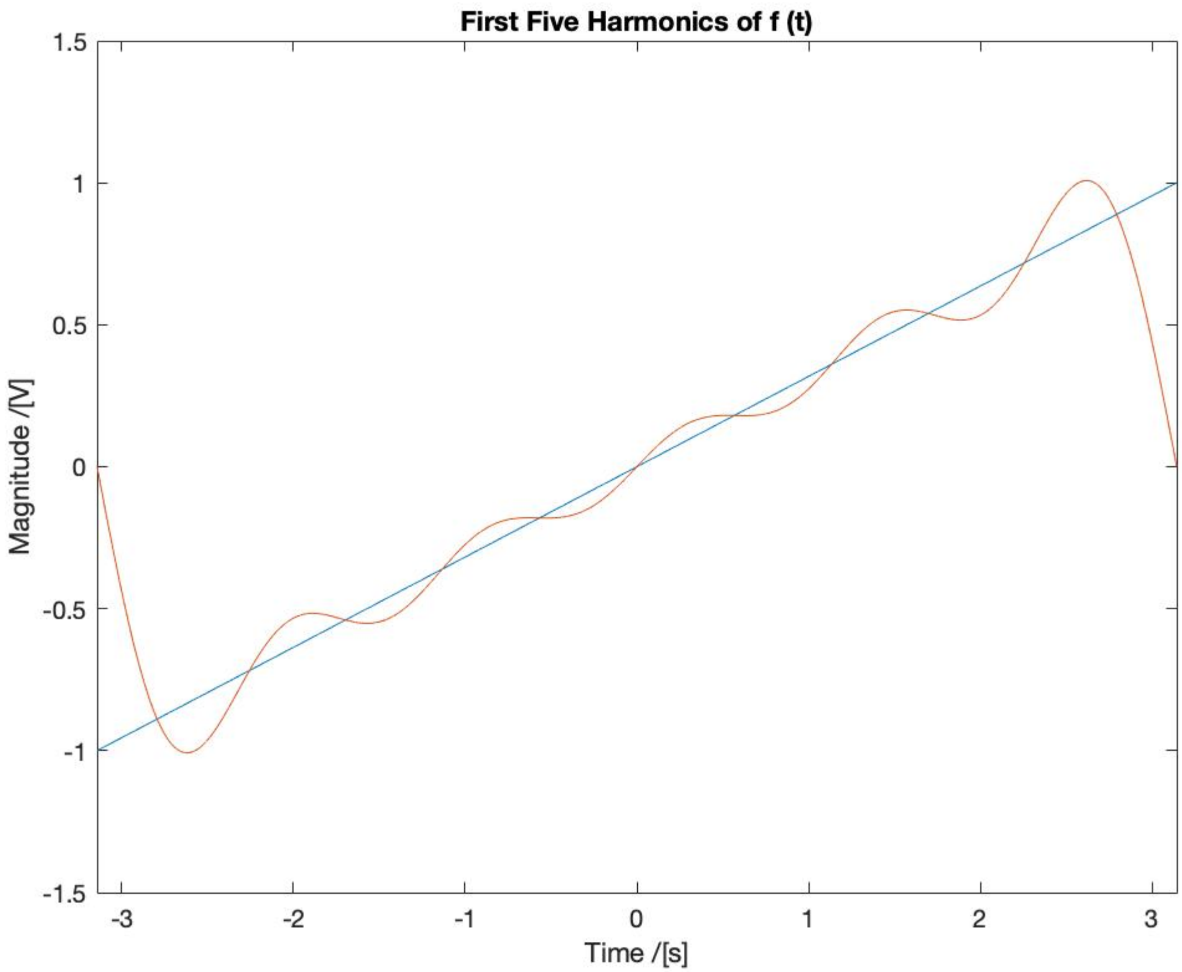

Figure 2: MATLAB plot of function using calculated coefficients

The MATLAB code used is:

```
%First five harmonics plot
f = 0;
for n=1:5
    f = f + ((-2*(-1)^n/(n*pi)))*sin(n*2*pi/T*t);
end
figure(2);
plot(t,y);
hold on;
plot(t,f);
xlim([-pi,pi]);
title('Fourier Series');
xlabel("Time /[s]");
ylabel("Magnitude /[V]")
title("First Five Harmonics of f (t)");
```

Problem 3: FFT of a Square/Rectangular Wave

- Generating a square wave of 1ms period, 2 V_{pp} amplitude, no offset, and duty cycle 50%

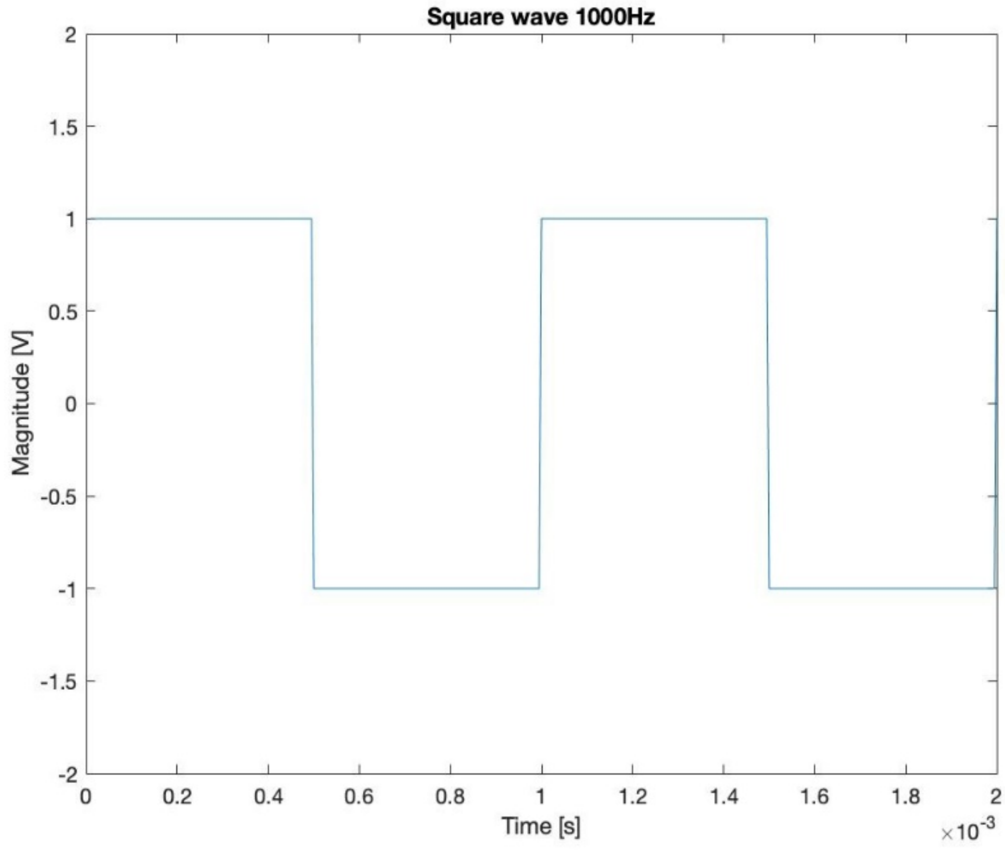

Figure 3: Square wave in time domain

The MATLAB code used is:

```
fs = 200000; % sampling frequency
t = 0.00001:1/fs:0.05;
f = 1000; % frequency of signal
y = square(2*pi*f*t, 50);
plot (t,y);
xlim([0,0.002]);
ylim([-2,2]);
xlabel("Time [s]");
ylabel("Magnitude [V]");
title("Square wave 1000Hz");
```

- Obtaining the FFT spectrum using MATLAB FFT function. Making the FFT length to be the length of the square wave data vector.

  Duty Cycle: 50%

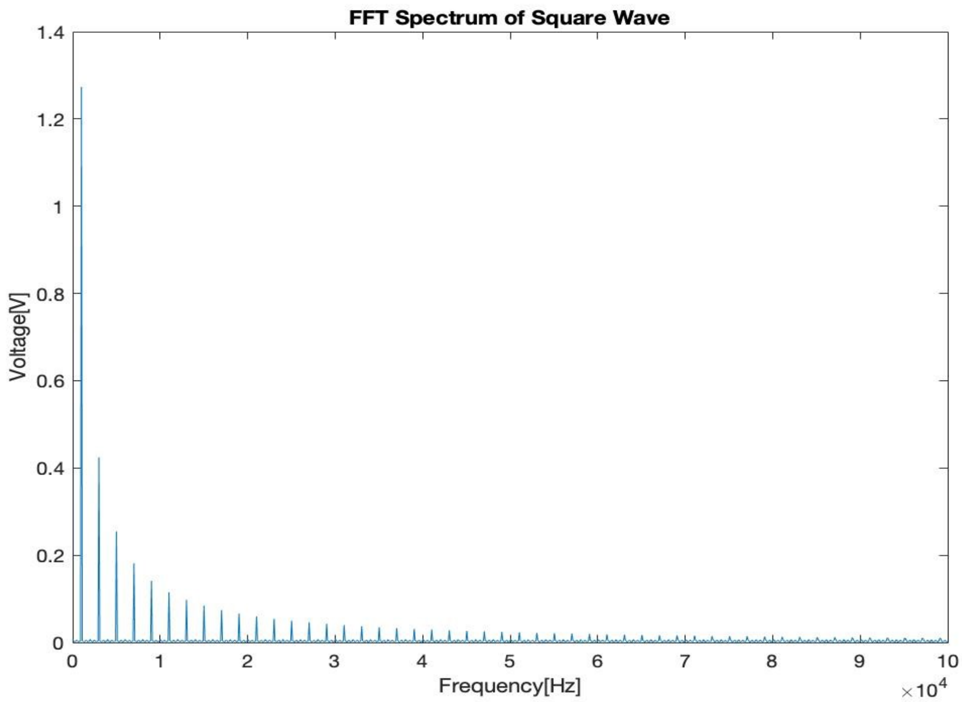

Figure 4: FFT Spectrum using MATLAB for duty cycle 50%

The MATLAB code used is:

```
fs = 200000; % sampling frequency
t = 0:1/fs:0.01;
y = square(2*pi*1000*t, 50);
f_nyq = fs/2;
y_fft = fft(y);
y_fft = 2*abs(y_fft)/length(y);
y_fft = y_fft(1:length(y)/2);
f = linspace(0, f_nyq, length(y)/2);
plot(f, y_fft);
xlabel('Frequency[Hz]');
ylabel('Voltage[V]');
title("FFT Spectrum of Square Wave");
```

- Plot the single-sided amplitude spectrum in dBV_{rms}

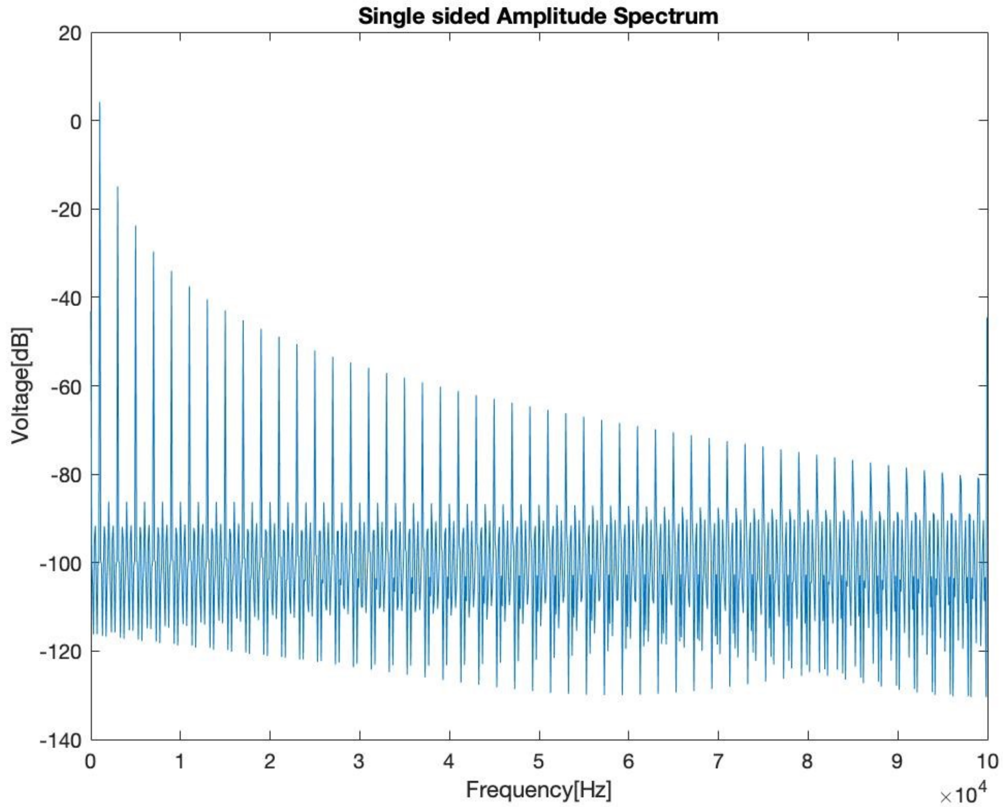

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Figure 5: Single sided amplitude spectrum in dBV_{rms}

The MATLAB code used is:

```
%square wave
fs = 200000; % sampling frequency
t = 0:1/fs:0.01;
y = square(2*pi*1000*t, 50);
%FFT
f_nyq = fs/2;
y_fft = fft(y);
y_fft = 2*abs(y_fft)/length(y);
y_fft = y_fft(1:length(y)/2);
f = fs*(0:(length(y)/2))/length(y);
%Single Sided Spectrum in dB
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
S(2:end-1) = 2*S(2:end-1);
%plot
plot(f, S);
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
title("Single sided Amplitude Spectrum");
```

- Plot the spectrum including only the first four harmonics in dBV_{rms}.

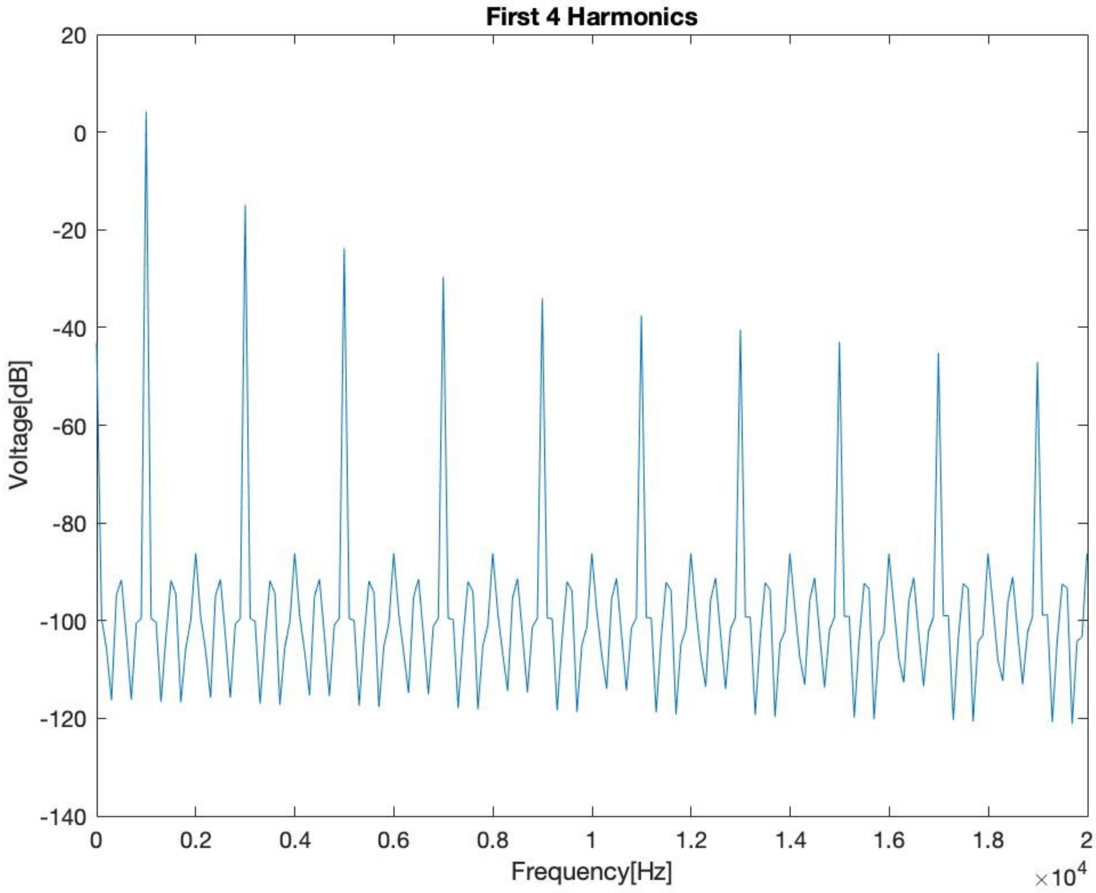


Figure 6: First four harmonics for dBV_{rms}

The MATLAB code used is:

```
%square wave
fs = 200000; % sampling frequency
t = 0:1/fs:0.01;
y = square(2*pi*1000*t, 50);
f_nyq = fs/2;
y_fft = fft(y);
y_fft = 2*abs(y_fft)/length(y);
y_fft = y_fft(1:length(y)/2);
f = fs*(0:(length(y)/2))/length(y);
%Single Sided Spectrum in dB
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
S(2:end-1) = 2*S(2:end-1);
plot(f, S);
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
title("First 4 Harmonics");
xlim([0,20000]);
```

- Repeating previous steps for 20% and 33% duty cycles

Duty Cycle: 20%

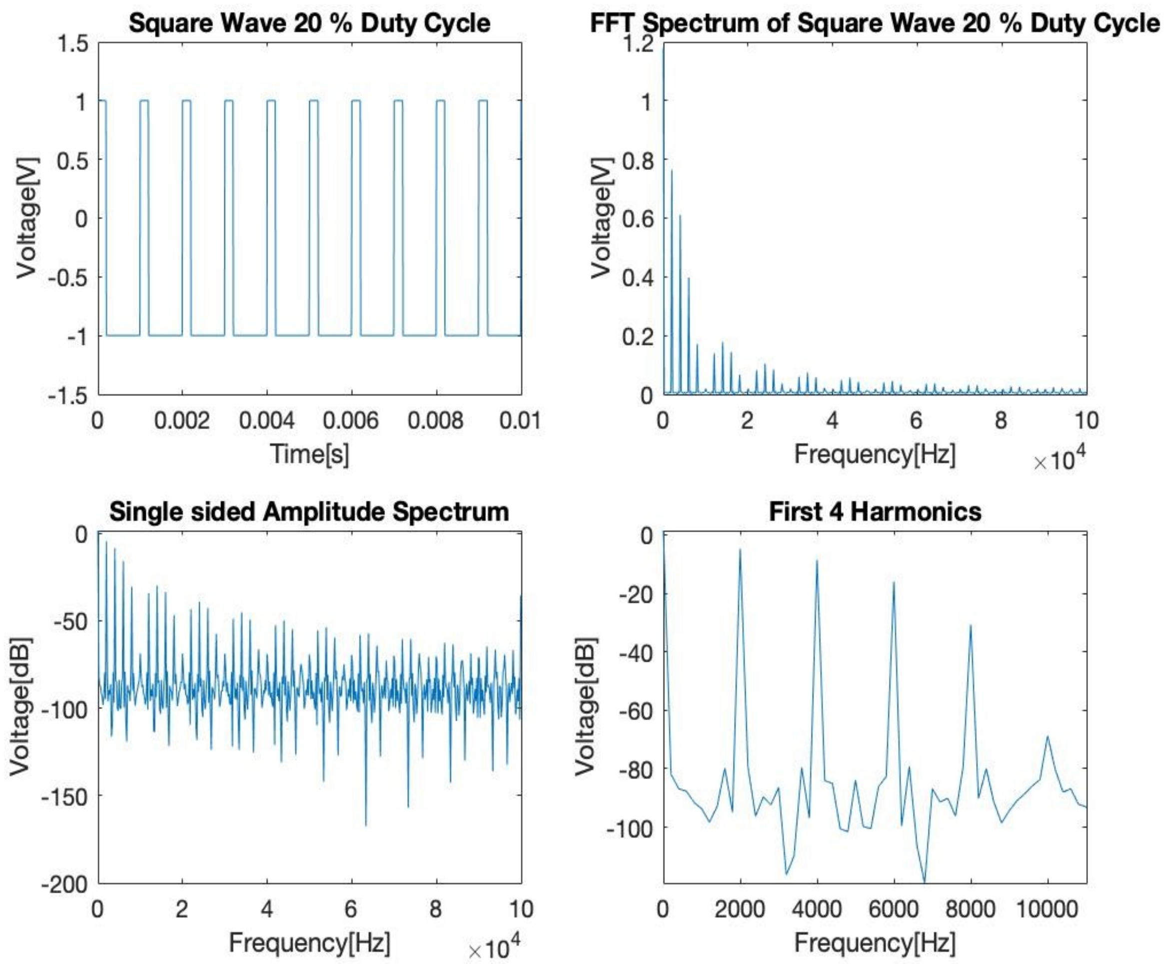


Figure 7: Signal and FFT for 20% duty cycle

The MATLAB code is:

```
%square wave
t = 0:0.00001:0.01;
y = square(2*pi*1000*t, 20); % 20% Duty Cycle
%FFT
fs = 200000; % sampling frequency
f_nyq = fs/2;
y_fft = fft(y);
y_fft = 2*abs(y_fft)/length(y);
y_fft = y_fft(1:length(y)/2);
f = fs*(0:(length(y)/2))/length(y);
%Single Sided Spectrum in dB
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
S(2:end-1) = 2*S(2:end-1);
```

```
subplot(2,2,1);
plot(t, y), axis([0 0.01 -1.5 1.5]);
title("Square Wave 20 % Duty Cycle");
xlabel('Time[s]');
ylabel('Voltage[V]');
subplot(2,2,3);
plot(f, S);
title("Single sided Amplitude Spectrum");
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
subplot(2,2,4);
plot(f, S);
title("First 4 Harmonics");
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
xlim([0 11000]);
f = linspace(0, f_nyq, length(y)/2);
subplot(2,2,2);
plot(f, y_fft);
title("FFT Spectrum of Square Wave 20 % Duty Cycle");
xlabel('Frequency[Hz]');
ylabel('Voltage[V]');
```

Duty Cycle: 33%

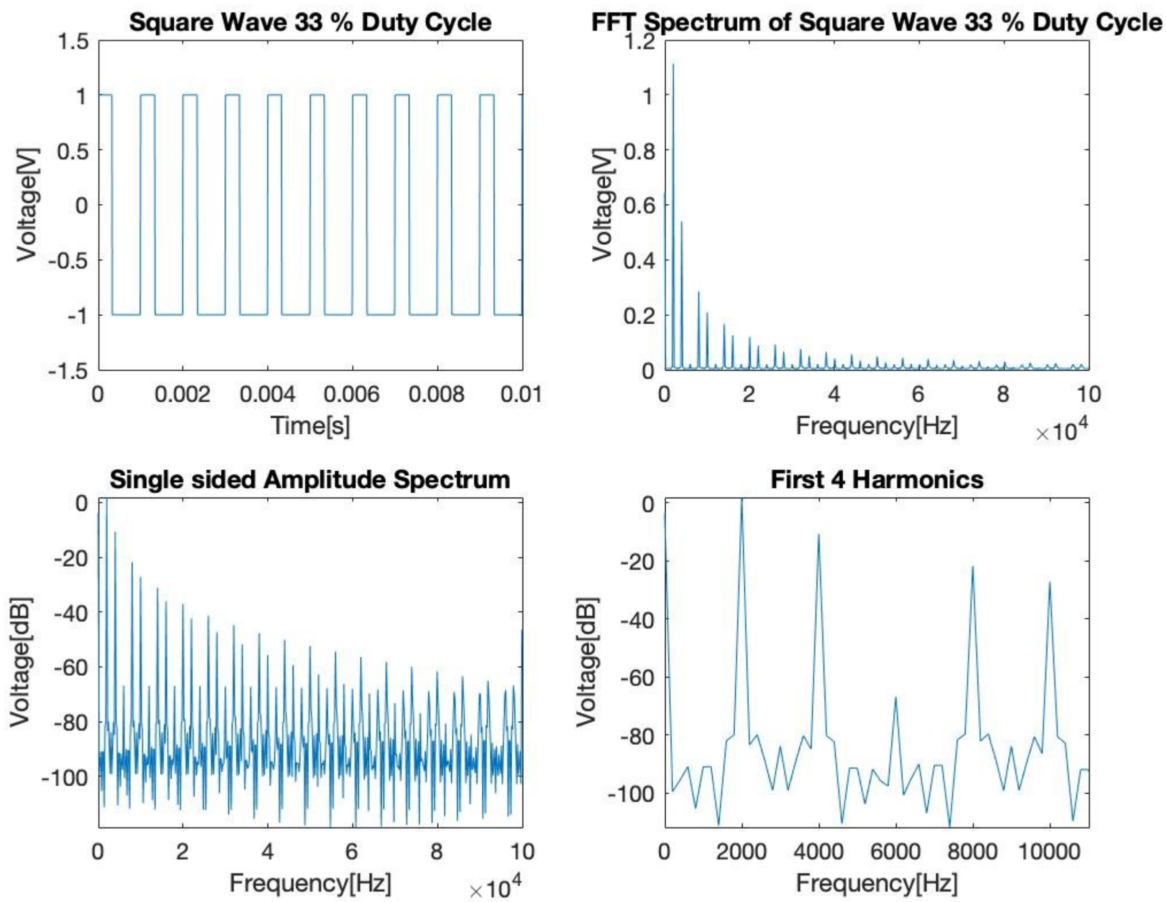


Figure 8: Signal and FFT for 20% duty cycle

The MATLAB code used is:

```
%square wave
t = 0:0.00001:0.01;
y = square(2*pi*1000*t, 33); % 33% Duty Cycle
%FFT
fs = 200000; % sampling frequency
f_nyq = fs/2;
y_fft = fft(y);
y_fft = 2*abs(y_fft)/length(y);
y_fft = y_fft(1:length(y)/2);
f = fs*(0:(length(y)/2))/length(y);
%Single Sided Spectrum in dB
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
S(2:end-1) = 2*S(2:end-1);
%plot
subplot(2,2,1);
plot(t, y), axis([0 0.01 -1.5 1.5]);
title("Square Wave 33 % Duty Cycle");
xlabel('Time[s]');
ylabel('Voltage[V]');
subplot(2,2,3);
plot(f, S);
title("Single sided Amplitude Spectrum");
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
subplot(2,2,4);
plot(f, S);
title("First 4 Harmonics");
xlabel('Frequency[Hz]');
ylabel('Voltage[dB]');
xlim([0 11000]);
f = linspace(0, f_nyq, length(y)/2);
subplot(2,2,2);
plot(f, y_fft);
title("FFT Spectrum of Square Wave 33 % Duty Cycle");
xlabel('Frequency[Hz]');
ylabel('Voltage[V]');
```

- Discussing the changes for smaller pulse width.

  $c_k = \frac{2}{kw_0T} \cdot sin(kw_0T_1) = \frac{1}{k\pi} \cdot \sin\,(kw_0T_1)$

In this equation, T is the period of the signal and T_1 is the width of periodic impulses. If the duty cycle was decreased, the magnitude spectra of square wave reached zero for multiples of w_0. The magnitude of the spectrum is seen to be increased when the pulse width T_1 was lowered. For 20% duty cycle, the magnitude spectrum was the highest and was the lowest for 50%. When the cycle duty was lowered, one part of the input signal came close together. This resulted into additional Fourier series coefficients. Thus, resulting into larger amplitude in the spectrum.

Problem 4: FFT of a Sound Sample

- Plotting the sound sample

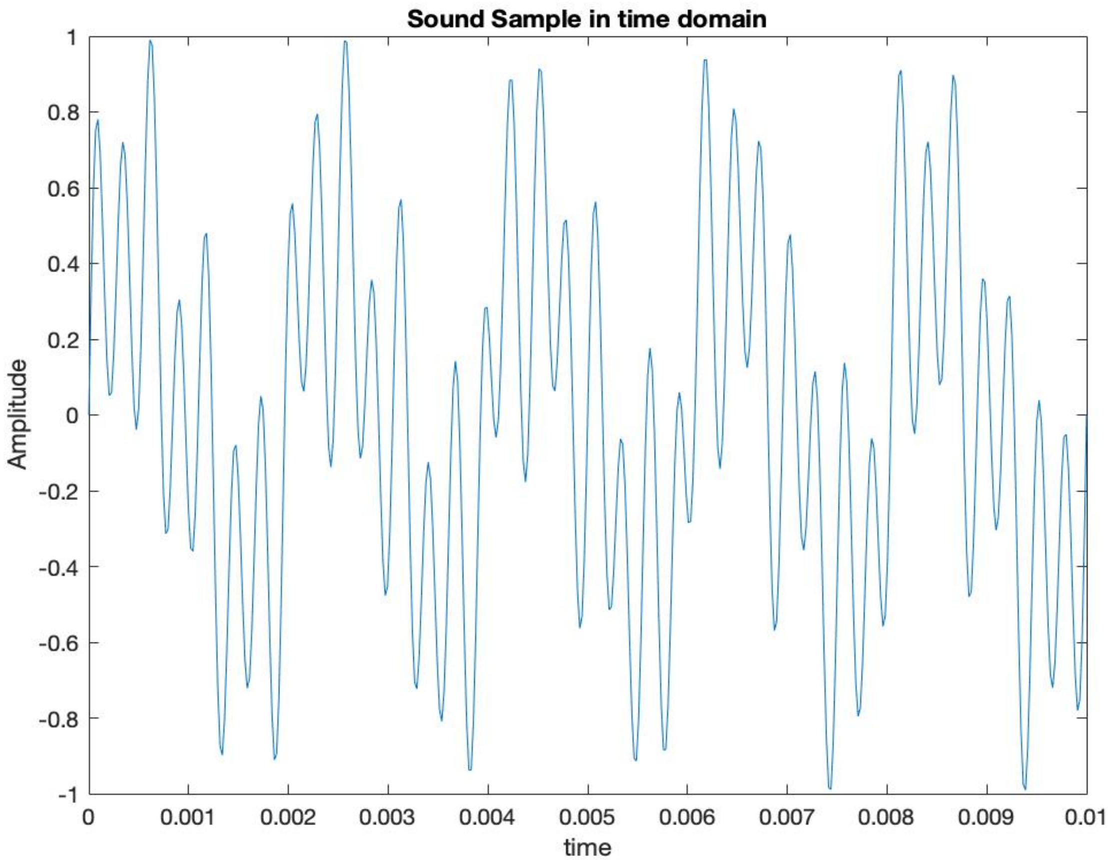


Figure 9: Plot of the sound sample

The MATLAB code used is:

```
[y,Fs] = audioread("sound_sample.wav");
t = 0:1/Fs:(numel(y)-1)/Fs;
plot(t(1:442), y(1:442));
xlabel("time")
ylabel("Amplitude");
title("Sound Sample in time domain");
```

- Plotting the single sided amplitude spectrum:

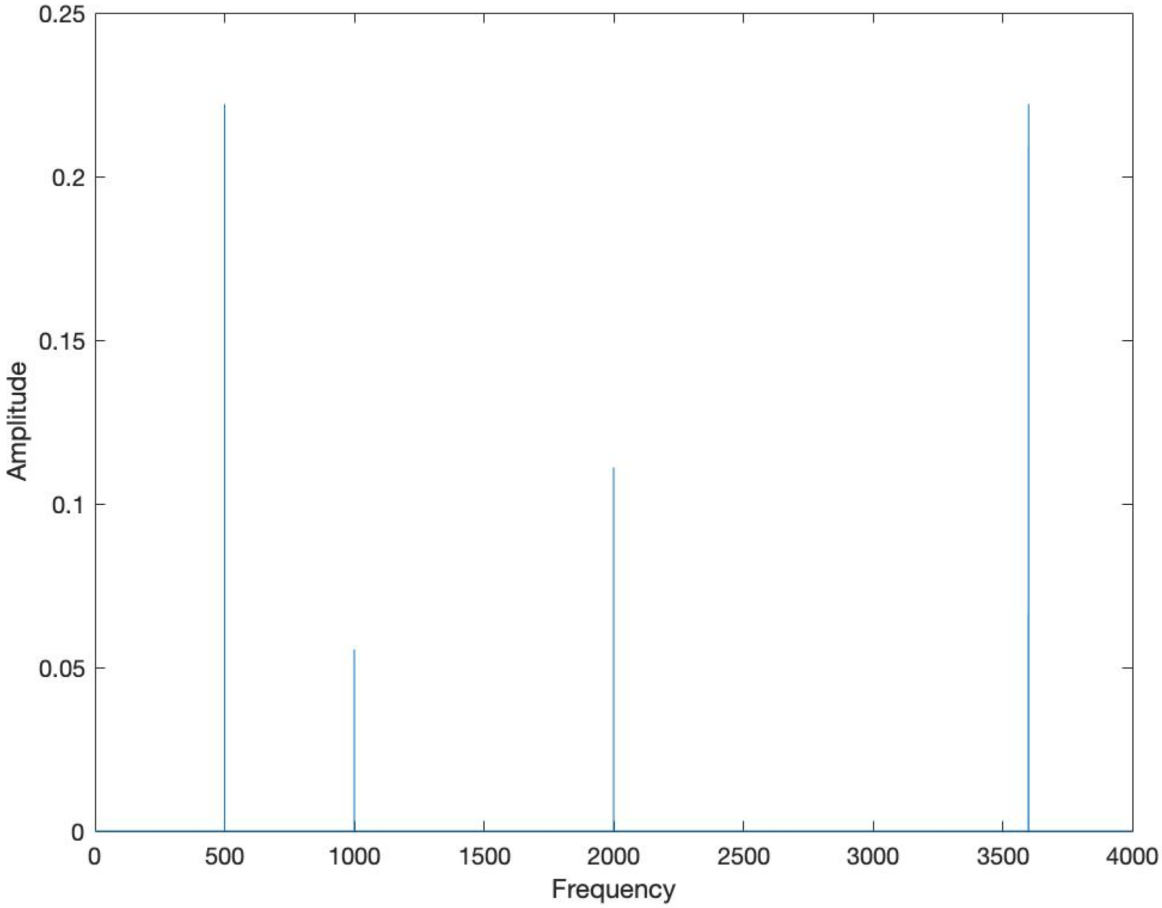


Figure 10: Plot of the single sided amplitude spectrum

The MATLAB code is:

```
[y,Fs] = audioread("sound_sample.wav");
t = 0:1/Fs:(numel(y)-1)/Fs;
Y = fft(y)/numel(y);
p = (0:(numel(y)-1)/2);
fp = Fs*p/numel(y);
plot(fp, abs(Y(1:(numel(y)+1)/2)));
xlim([0,4000])
xlabel("Frequency")
ylabel("Amplitude")
```

- The tones forming this signal are at the points at which half of the FFT spectrum exists. The signal is a multitone at frequencies $500\ Hz$, $1000\ Hz$, $2000\ Hz$, and $3600\ Hz$.

1. Introduction and Theory

A signal is a function that conveys information. It can be represented in time domain as well as in frequency domain, where the frequency domain representation is called the spectrum of the signal. The technique studied during the experiment, Fourier analysis, deals with the decomposition of the signal into its constituent sinusoidal waves, i.e., any time-varying signal can be constructed by superimposing sinusoidal waves of appropriate frequency, amplitude, and phase. The Fourier transform is used to transform a signal from the time domain to the frequency domain. For arbitrary signals, the signal must first be digitized, and a Discrete Fourier Transform is performed. Conversely, the inverse Fourier transform is used to transform a signal from the frequency domain to the time domain.

In this experiment, the Fourier analysis of single and multi-tone sinusoidal waves as well as of square wave is done. The corresponding signals are observed in the time domain as well as the frequency domain in the oscilloscope and later compared with the theoretical wave constricted through MATLAB.

In order for the understanding of the frequency-time domain analysis of signals the concepts of the Fourier Transform and series should be dealt in two parts as:

i) Continuous Time Signals

A continuous-time periodic signal can be described by the sum of basic signals. Some important concepts for these signals are:

- Periodic Signals: A signal is defined as periodic, if for some positive value of T, the signal can be described:

$$f(t) = f(t + nT) \tag{1}$$

The fundamental period is the minimum positive, nonzero value of T for which the above equation is satisfied. The fundamental frequency (ω_0) is then referred by:

$$\omega_0 = \frac{2\pi}{T} \tag{2}$$

- Fourier Series Coefficients: A function $f(t)$ will have the following Fourier coefficients:

$$\alpha_n = \frac{2}{T}\int_0^T f(t) \cdot \cos(n\omega_0 t)\, dt \tag{3}$$

$$\beta_n = \frac{2}{T}\int_0^T f(t) \cdot \sin(n\omega_0 t)\, dt \tag{4}$$

When the series itself is represented as:

$$f(t) = \alpha_0 + \sum_{n=1}^{\infty}(\alpha_n \cos(n\omega_0 t) + \beta_n \sin(n\omega_0 t)) \tag{5}$$

Similarly, the signal can also be represented as a sum of superimposed complex exponential functions as:

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t} \tag{6}$$

Where:

$$c_n = \frac{1}{T}\int_0^T f(t) \cdot e^{-jn\omega_0 t} dt \tag{7}$$

The coefficients of both the representations can be related by the following equation:

$$c_{\pm n} = \frac{\alpha_n \mp j\beta_n}{2} \tag{8}$$

- Fourier Transform: The continuous time Fourier transform is a generalization of the Fourier series. It can be applied to aperiodic signals as well. It only applies to continuous time aperiodic signals. The Fourier transform of a given signal of a function $f(t)$ is defined as:

$$F(j\omega) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-j\omega t} dt \tag{9}$$

The time domain signal can be obtained from Fourier domain through inverse transform which is defined as:

$$f(t) = \int_{-\infty}^{+\infty} F(j\omega) \cdot e^{j\omega t} dt \tag{10}$$

ii) Discrete Time Signals

A discrete signal or discrete-time signal is a time series consisting of a sequence of quantities.

- Periodic Signals: A signal is defined as periodic, with period N when:

$$f[n] = f[n + N] \tag{11}$$

This must hold for all n. The fundamental period is the smallest positive integer N for which the above equation holds. The fundamental frequency (ω_0) is then referred by:

$$\omega_0 = \frac{2\pi}{N} \tag{12}$$

- Fourier Series Coefficients: A function $f(t)$ will have the following Fourier coefficients:

$$a_k = \frac{1}{N}\sum_{n=(N)} a[n] e^{jkw_0 n} \tag{13}$$

When the series itself is represented as:

$$f[n] = \frac{1}{N}\sum_{n=(N)} a_k e^{jkw_0 n} \tag{14}$$

- Fourier Transform: The discrete time Fourier transform converts a time domain sequence resulting from sampling a continuous time signal into an equivalent frequency domain sequence representing the frequency content of the given signal. The Fourier transform of a given signal of a function $f[n]$ is defined as:

$$F[k] = \sum_{n=0}^{N-1} f[n]e^{2\pi kn/N} \quad for\ k = 1,2,3, \dots N-1 \tag{15}$$

The Inverse Discrete Fourier Transform (IDFT) performs the reverse operation and converts a frequency domain sequence into an equivalent time domain sequence:

$$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k]e^{2\pi kn/N} \quad for\ n = 1,2,3, \dots N-1 \tag{16}$$

The transfer of a continuous time signal into a discrete time signal or the transfer from the world of analog signal processing to the world of digital signal processing is sampling. These different types of transforms are used for different cases with the discrete case used mostly in practice.

- Continuous Fourier Transform: The Continuous Fourier transform is used to transform a continuous time signal into the frequency domain. It describes the continuous spectrum of a non-periodic time signal.

- Discrete Fourier Transform: The Discrete Fourier Transform is used in the case where both the time and the frequency variables are discrete.

- Fast Fourier Transform (FFT):
  FFT is a special algorithm which implements the discrete Fourier transform with considerable savings in computational time. FFT is not a different transform from the DFT, but rather just a means of computing the DFT with a considerable reduction in the number of calculations required.

  This function is used both in the oscilloscope during the experiment and in MATLAB during calculation.

In this experiment, the frequency domain representation of different signals is observed through the FFT function usage. These representations are analyzed and compared with the theoretical FFT obtained of the same signal through MATLAB.

2. Execution

2.1 Experimental Setup

Workbench Number 8

- Breadboard
- Signal Generator
- Oscilloscope
- BNC Cable
- Tools from workbench
- Auxiliary Function Generator
- 10k0 and 1000K Resistors

2.1.1 Experimental Part 1 – Overview and Setup

- A sinusoidal wave signal with peak-to-peak voltage of 2 V_{pp}, 0 V offset with a frequency of 500 Hz is generated from signal generator.
- The signal is observed through oscilloscope in time domain.
- The FFT function of the oscilloscope is used to observe the frequency spectrum of the signal.
- Then, a sinusoidal wave with 2 kHz frequency with 0 offset was generated and the value of V_{pp} was altered to have 0 dB spectrum peak.

2.1.2 Experimental Part 1-Execution and Results:

- The required sinusoidal wave having 500 Hz frequency, 2 V_{pp} amplitude and no offset was generated as shown in figure (1) and all the properties were verified using the measure function.

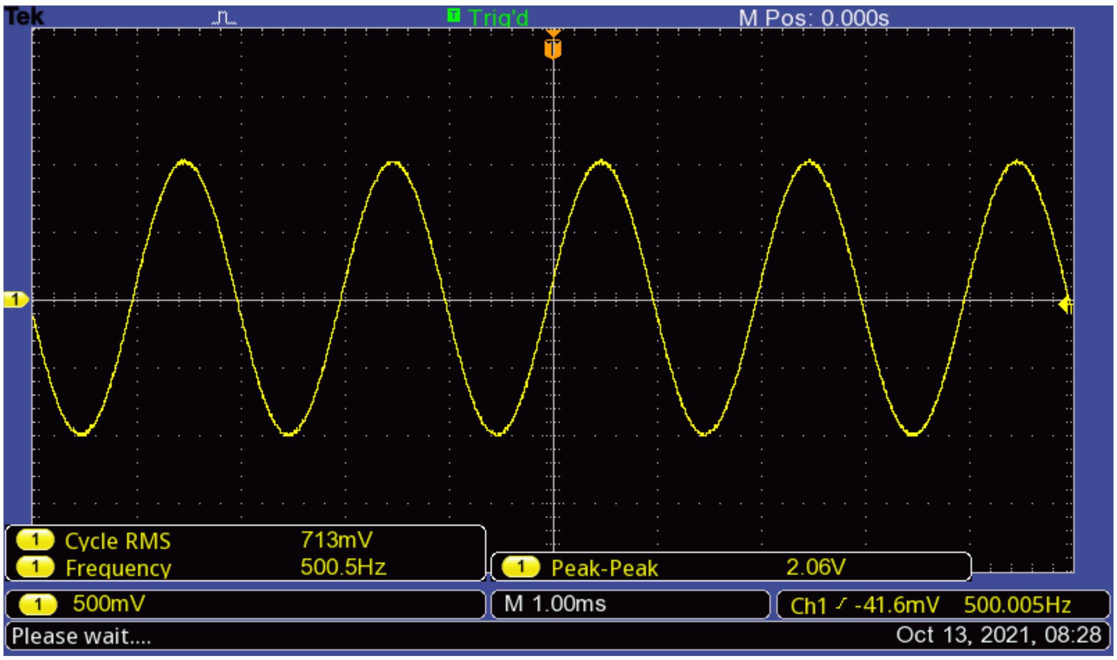


Figure 1: Hardcopy of first sinusoidal wave

- Generating FFT spectrum using the oscilloscope FFT function, as shown in figure (2) and the cursor was used to measure the following properties:
- Sampling Frequency = 495 Hz, spectrum peak = -3.39 dB

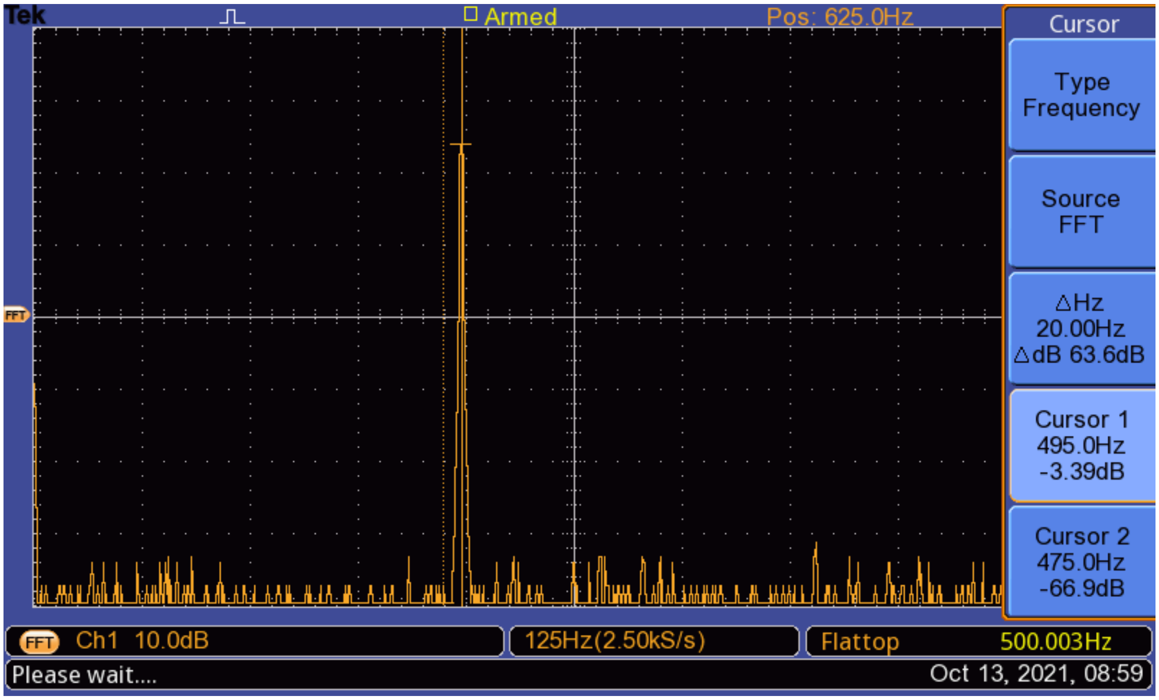


Figure 2: FFT spectrum of figure 1

- Then the value of the amplitude was changed until it had 0 dB spectrum peak, and the spectrums generated in time and frequency domain are shown in figures (3) and (4) respectively.

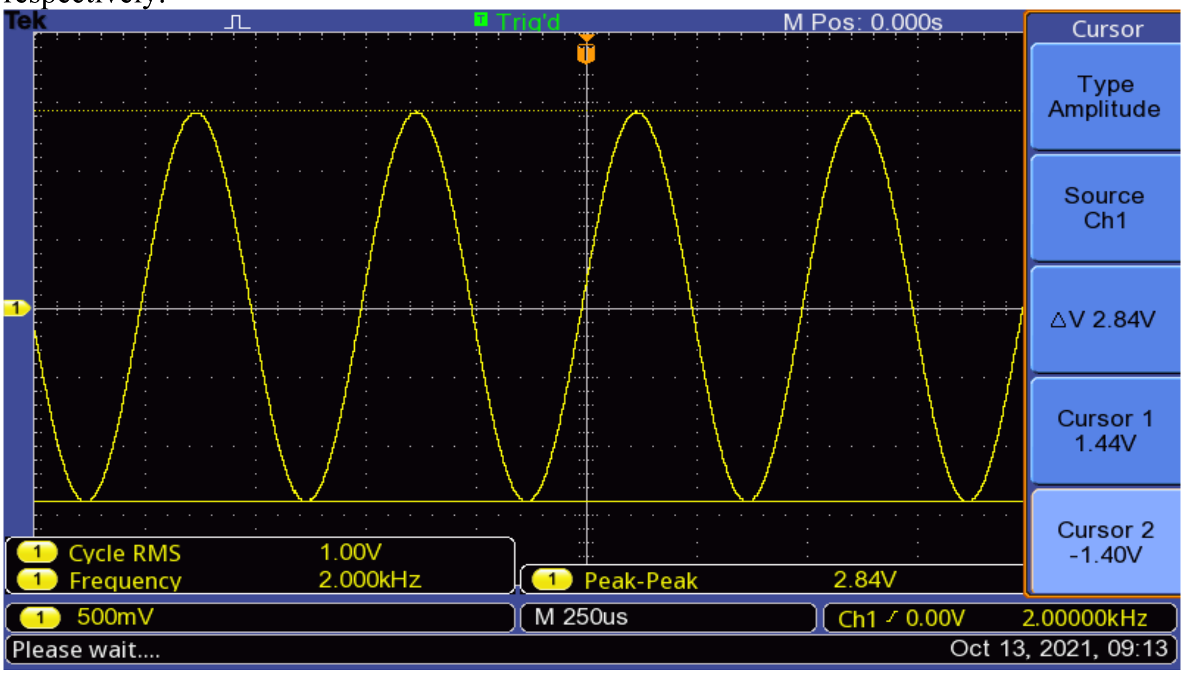


Figure 3: Spectrum in time domain for 0 dB spectrum peak

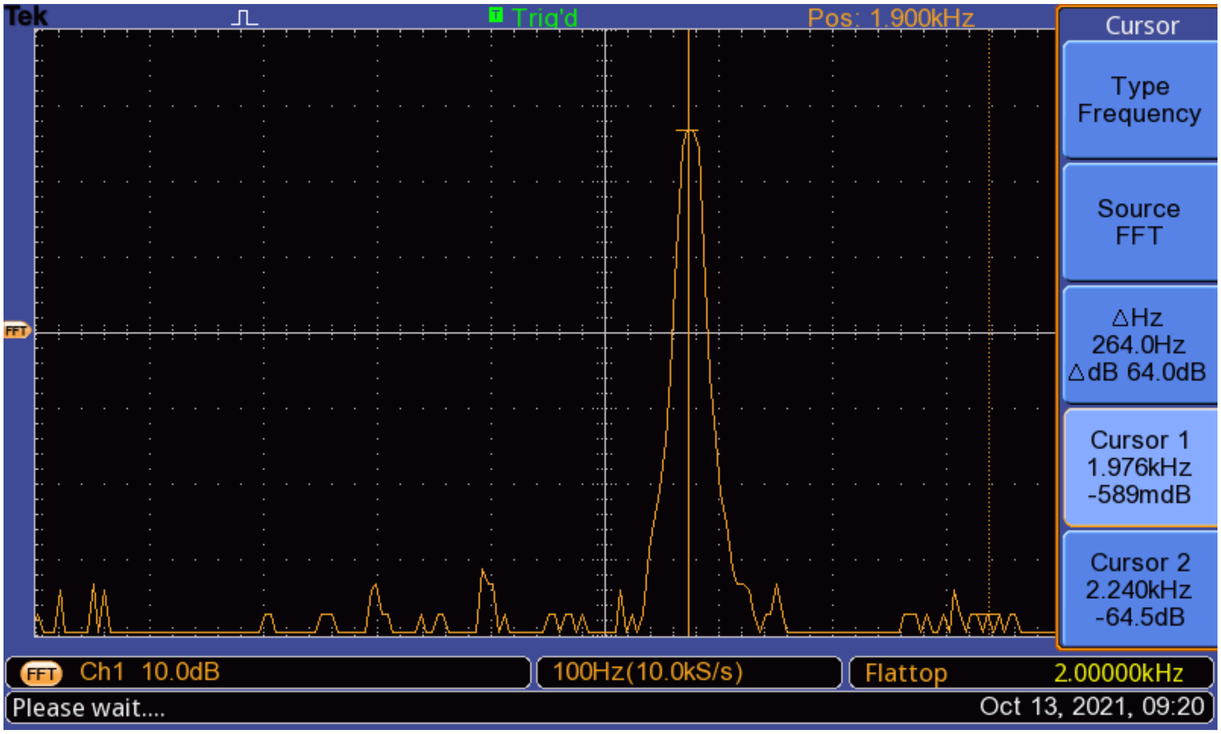


Figure 4: Spectrum in frequency domain for 0 dB spectrum peak

- The value of the amplitude is 1.42 V calculated using both measure function and cursors. Moreover, for the frequency spectrum, the frequency is 1.976 Hz and spectrum peak is -589 mdB using the cursor. A spectrum peak exactly equal to 0 dB could not be generated due to systematic error in the signal generator, and V_{RMS} not being exactly equal to 1 V.

2.2.1 Experimental Part 2: Overview and setup:

- A square wave having 1 ms period, 2 V_{pp} amplitude and no offset was generated using the function generator and all properties were verified using measure function.
- A Fast Fourier Transform (FFT) was obtained using FFT zoom control in the oscilloscope and properties of fundamental and first four harmonics were measured using cursors.
- The FFT spectrum for 20% duty cycles was obtained and the properties of fundamental and first four harmonics were taken.
- Hard copies were taken for each of the steps.

2.2.2 Experimental Part 2: Execution and Results:

- The required square wave having 1 ms period, 2 V_{pp} amplitude and no offset was generated as shown in figure (5) and all the properties were verified using the measure function.

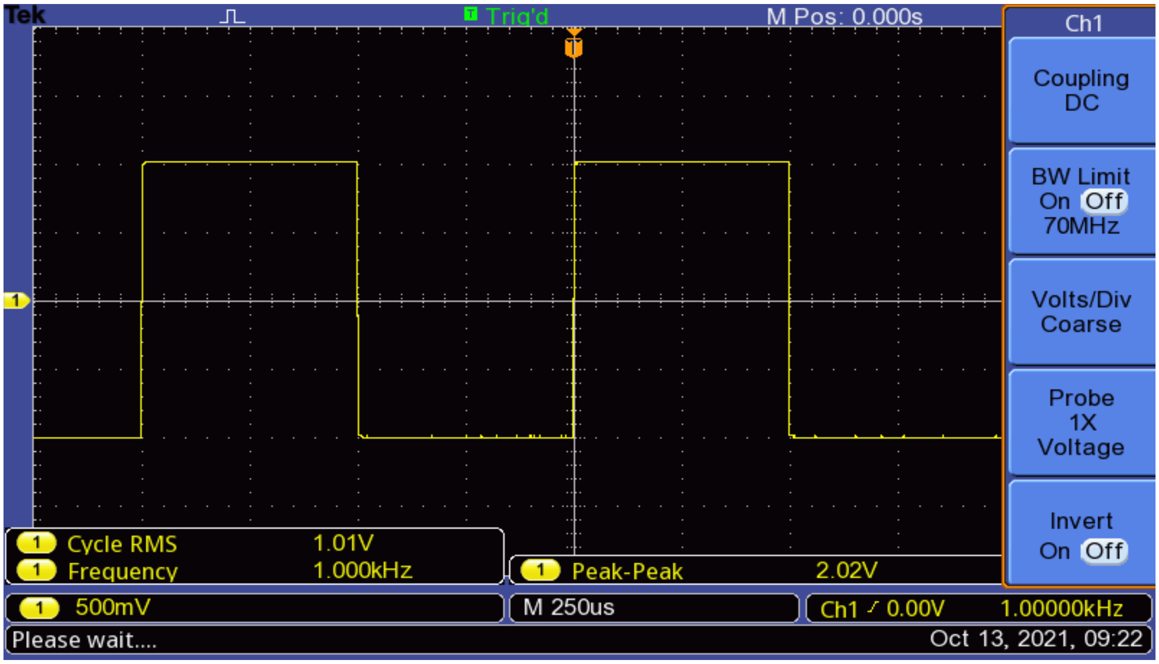


Fig 5: Hardcopy of the square wave

With the measure function, the frequency is 1.0 kHz, V_{pp} is 2.02 V and there is no offset, which verifies the given conditions.

- Generating FFT spectrum using the oscilloscope FFT function, as shown in figure (6) and the cursor was used to measure the following properties. The FFT zoom control was used instead of time base (sec/div) as zoom control provides a factor up to 10 which makes it easier to determine the amplitudes and frequencies for the cursor.

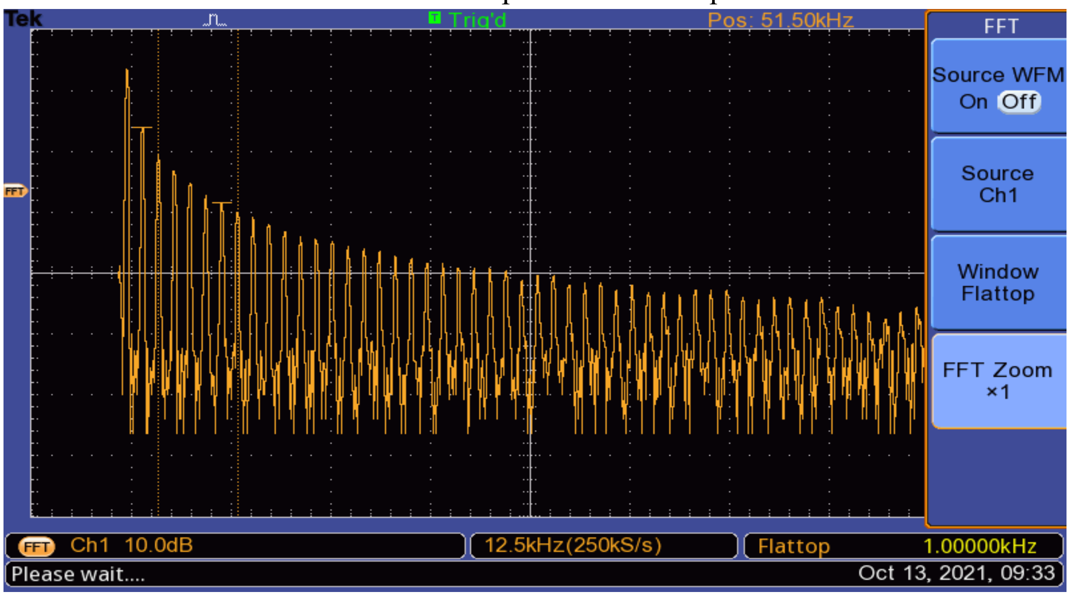


Figure 6: FFT spectrum of the square wave in figure 5 in x1 zoom

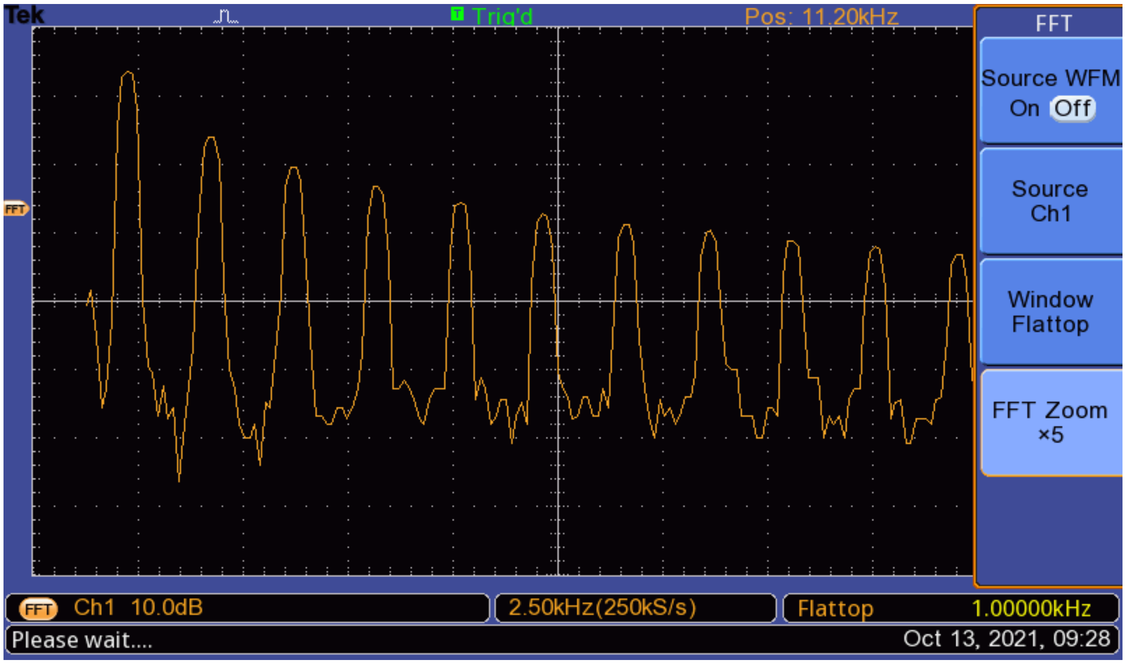


Figure 7: Figure 5 in x5 zoom.

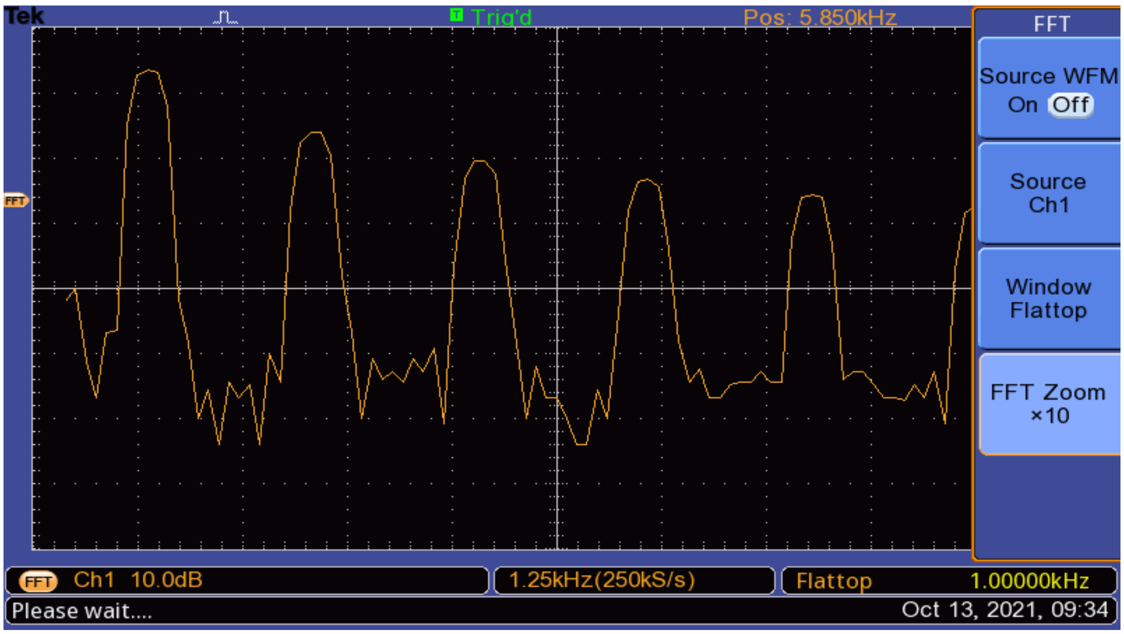


Figure 8: Figure 5 in x10 zoom

- The parameters of fundamental and first four harmonics measured using the cursor are:

Harmonics	Frequency [kHz]	Amplitude [dB]
Fundamental	0.900	-0.969
First	2.90	-10.5
Second	4.9	-14.9
Third	7.0	-17.7
Fourth	9.0	-20.1

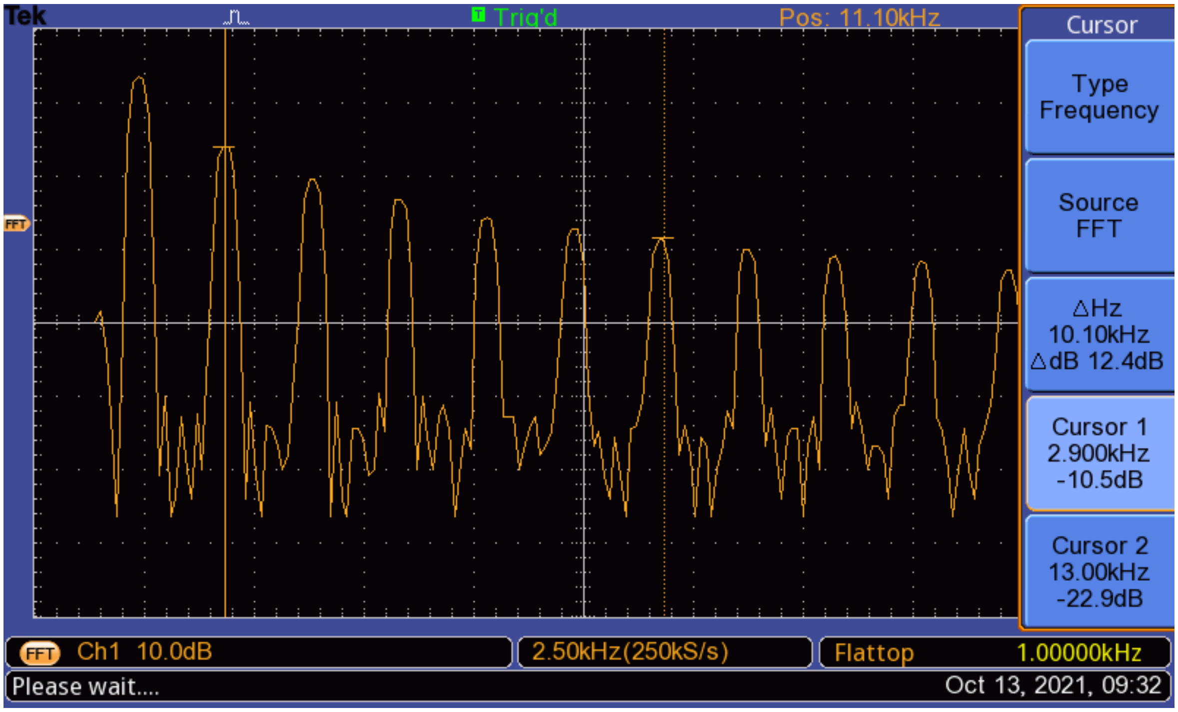


Figure 9: Sample measurement of frequency and spectrum peak for first harmonic

- Obtaining the FFT spectrum for 20% duty cycles and determining the amplitudes of fundamental and first five harmonics.

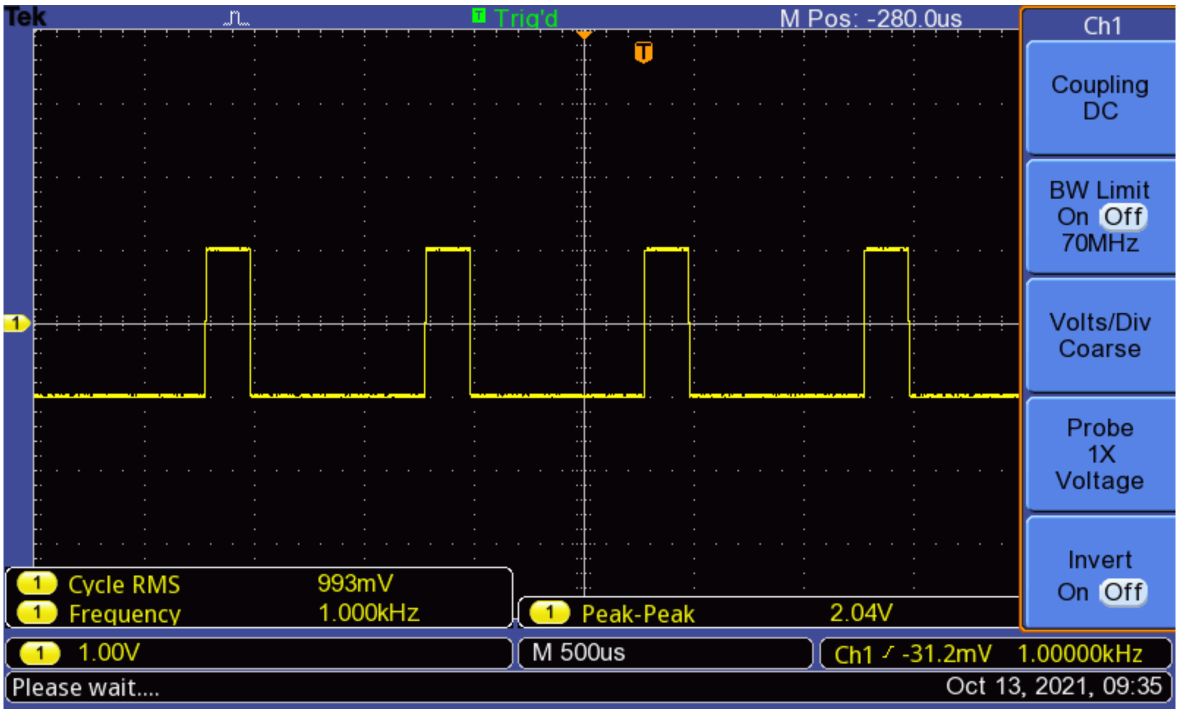


Figure 10: Square wave for 20% duty cycles

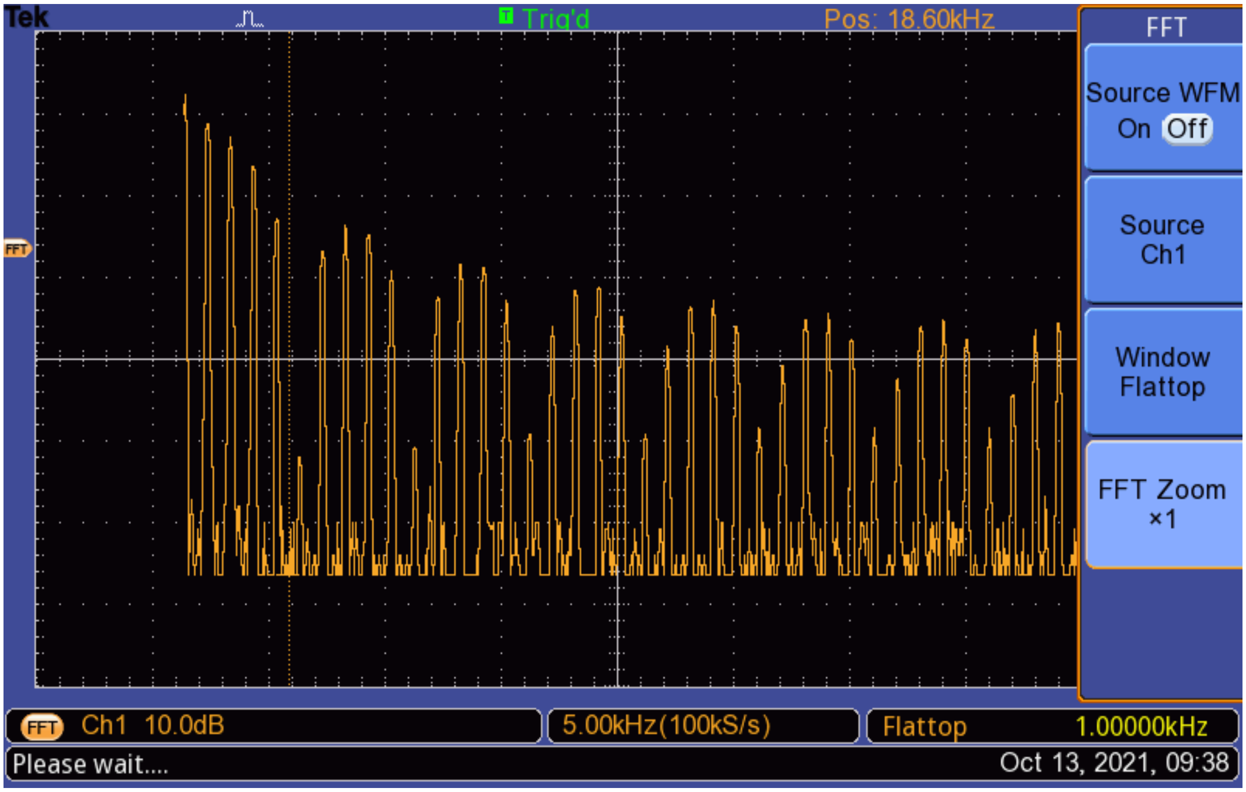


Figure 11: FFT spectrum of figure 10 in x1 zoom

- The parameters for fundamental and first five harmonics measured in x5 zoom are:

Harmonics	Frequency [kHz]	Spectrum Peak [dB]
Fundamental	0.960	-5.37
First	1.96	-7.37
Second	2.96	-10.9
Third	3.960	-18.9
Fourth	4.96	-41.3
Fifth	5.92	-21.3

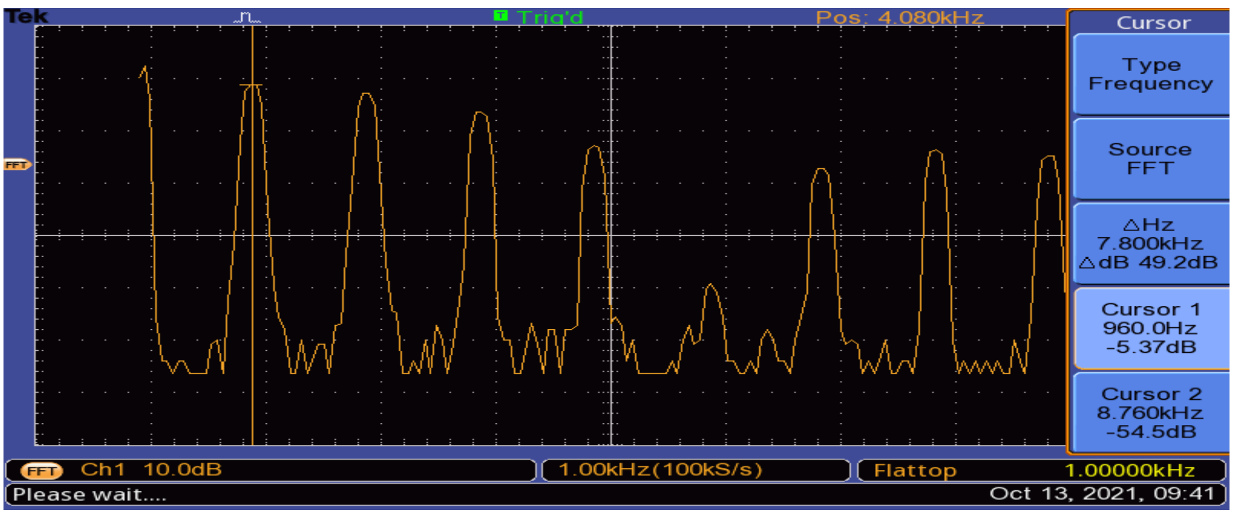


Figure 12: Sample measurement of frequency and spectrum peak for fundamental harmonic

2.3.1 Experimental Part 3: Overview and setup:

- The signal from the sine output of the auxiliary signal generator was combined with a 2 V_{pp}, 10 kHz sinusoidal wave from the Agilent signal generator using the circuit in figure 12.
- Hard copies were taken for the signals in time and frequency domains.

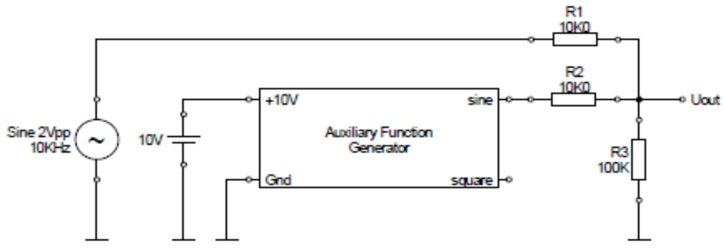


Figure 13: Circuit for Part 3

2.3.2 Experimental Part 3 - Execution and Results:

- After combining the sine output of the auxiliary signal generator with a 2 V_{pp}, 10 kHz sinusoidal wave, the following spectrum was obtained:

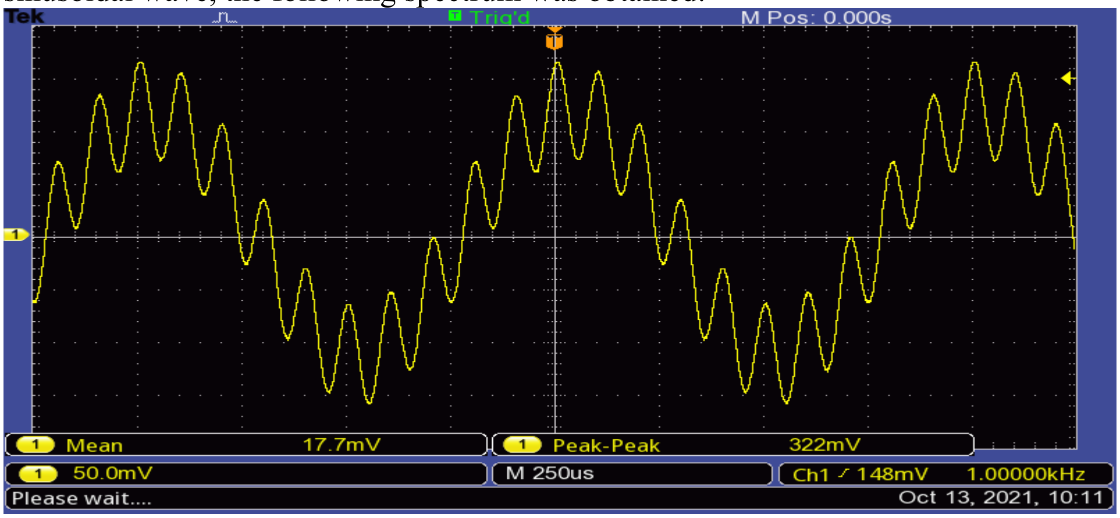


Figure 14: Spectrum generated for part 3 in time domain

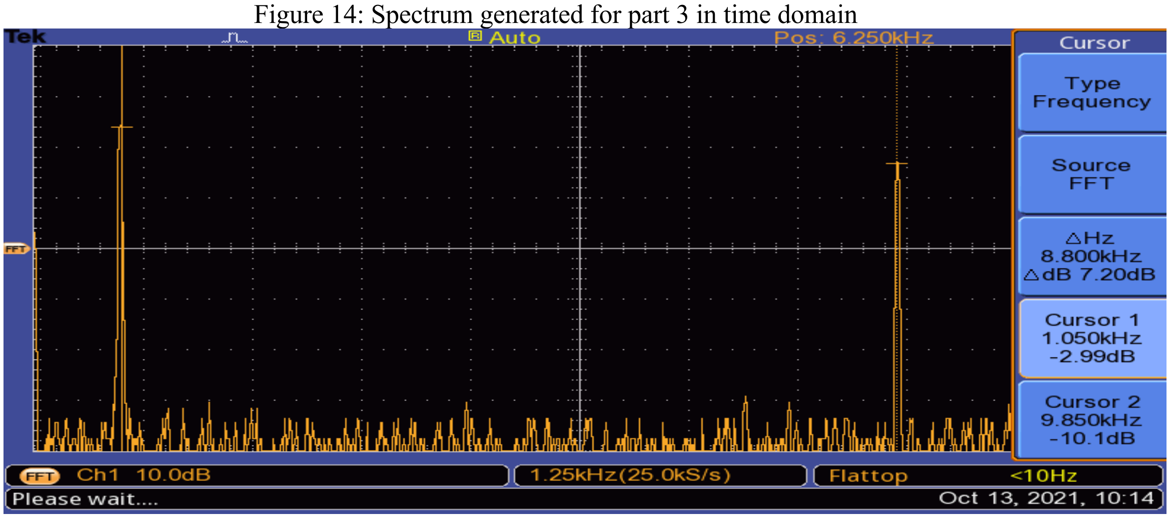
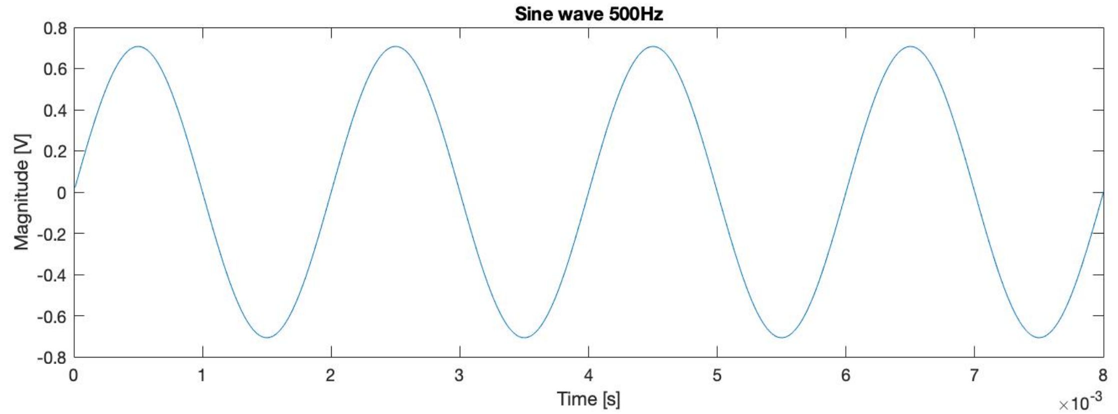


Figure 15: FFT spectrum for the signal for part 3

3 Evaluation

3.1 Evaluation Experiment Part 1

- According to the TDS-200 series extension modules the reference value of the oscilloscope for $0\ dB = 1\ V_{RMS}$.
  So, for $0\ dB$ the $V_{pp} = 2.828\ V$. From oscilloscope the $V_{pp} = 2.84\ V$ for the value of almost $0\ dB$.
- Using MATLAB to plot a sinusoidal wave having $500\ Hz$ frequency and $2\ V_{pp}$ amplitude in time and frequency domain:



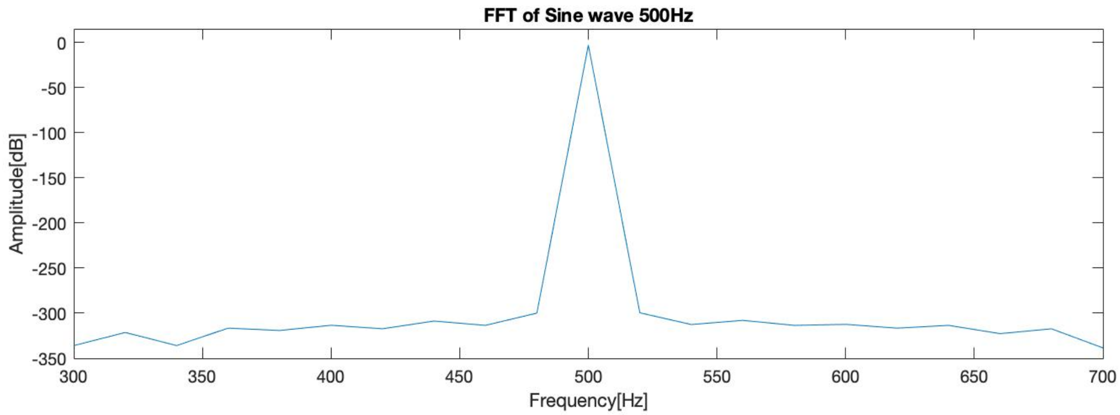


Figure 16: MATLAB plot of the FFT spectra for 500 Hz sine wave

The MATLAB code used is:

```
fs = 100000; % sampling frequency
t = 0.00001:1/fs:0.05;
f = 500; % frequency of signal
A = sqrt(2)/2 ; %Amplitude 2Vpp = 0.707 Vrms
y = A*sin(2*pi*f*t);
subplot(2,1,1);
plot (t,y);
xlim([0,0.008]);
xlabel("Time [s]");
ylabel("Magnitude [V]");
```

```
title("Sine wave");
%FFT and Single Sided Spectrum in dB
f_nyq = fs/2;
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
f=linspace(0,fnyq,length(S));
subplot(2,1,2);
plot(f, S);
xlabel('Frequency[Hz]');
ylabel('Amplitude[dB]');
xlim([300,700]);
ylim([-350,15]);
```

- The measurement from the oscilloscope is consistent with the calculation.

	Measured (Oscilloscope)	Calculated (MATLAB)
Frequency [Hz]	495.0	500
Amplitude [dB]	-3.39	-3.01

- Using MATLAB to plot a sinusoidal wave having 2 kHz frequency and $0dB$ spectrum peak:

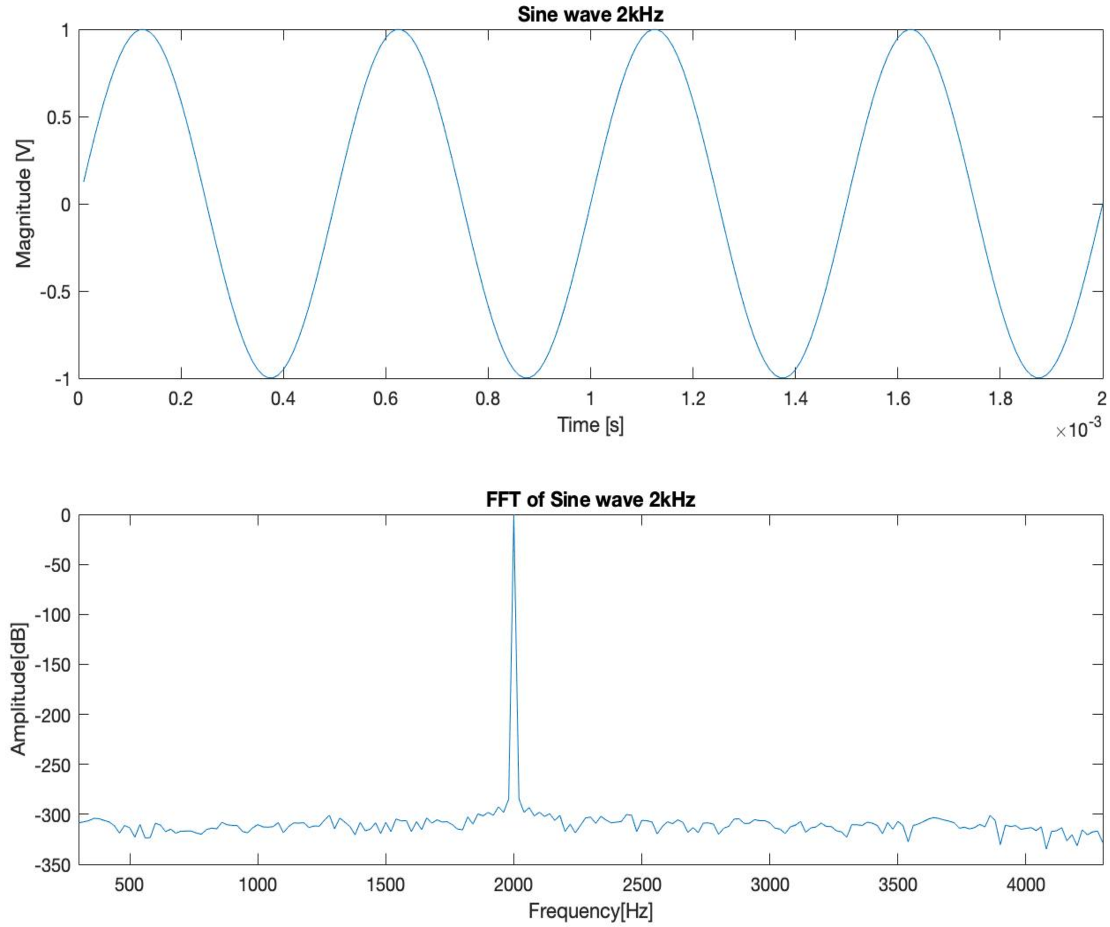


Figure 17: MATLAB plot of the FFT spectra for 0dB spectrum peak

The MATLAB code used is:

```
fs = 100000; % sampling frequency
t = 0.00001:1/fs:0.05;
f = 2000; % frequency of signal
A = 1 ; %Amplitude for 0 dB = 1 Vrms
y = A*sin(2*pi*f*t);
subplot(2,1,1);
plot (t,y);
xlim([0,0.002]);
xlabel("Time [s]");
ylabel("Magnitude [V]");
title("Sine wave 2kHz");
%FFT and Single Sided Spectrum in dB
f_nyq = fs/2;
S = abs(2*fft(y)/length(y));
S = db(S);
S = S(1:length(y)/2+1);
f=linspace(0,fnyq,length(S));
subplot(2,1,2);
plot(f, S);
xlim([300,4300]);
ylim([-350,0]);
xlabel('Frequency[Hz]');
ylabel('Amplitude[dB]');
title("FFT of Sine wave 2kHz");
```

- The measurement from the oscilloscope is consistent with the calculation.

	Measured (Oscilloscope)	Calculated (MATLAB)
Frequency [kHz]	1.976	2
Amplitude [dB]	-0.589	0

In the measurement, a perfect 0 dB measurement could not be achieved due to limitation from the equipment.

- Comparison of measured values and results from MATLAB:

It can be seen from the plot that the FFT spectra is fairly similar between MATLAB and oscilloscope in terms of peaks. The plot from the calculation looks smoother. Also, during the peaks the spectra from the calculation have steeper slope and pointed top compared to the spectra seen from the oscilloscope.
Similarly, 0 dB spectra peak was harder to obtain during measurement as a small change in V_{pp} would lead to the decibel value increasing fast and skipping 0 dB. Thus, the value had to be settled to the nearest achievable value to 0 dB which was $(-589\ mdB)$.
Similarly, the noise and error caused from the generator also adds to the reasons for this inaccuracy. Also, in the oscilloscope another source of magnitude error could be the interpolation when using Zoom mode to expand the signal.

3.2 Evaluation Experiment Part 2:

- The frequency scale can be expanded in order to accurately measure the frequency components with the time base (sec/div) control.
  The resolution for the FTT can be set with the time base control. By changing the time base, the sampling rate can be changed, which affects the way the signal is viewed. As the time base is increased to expand the signal, the measured bandwidth decreases and vice versa. This concludes the inverse proportionality between the time base and bandwidth.

- Effect of changing duty cycles on FFT:

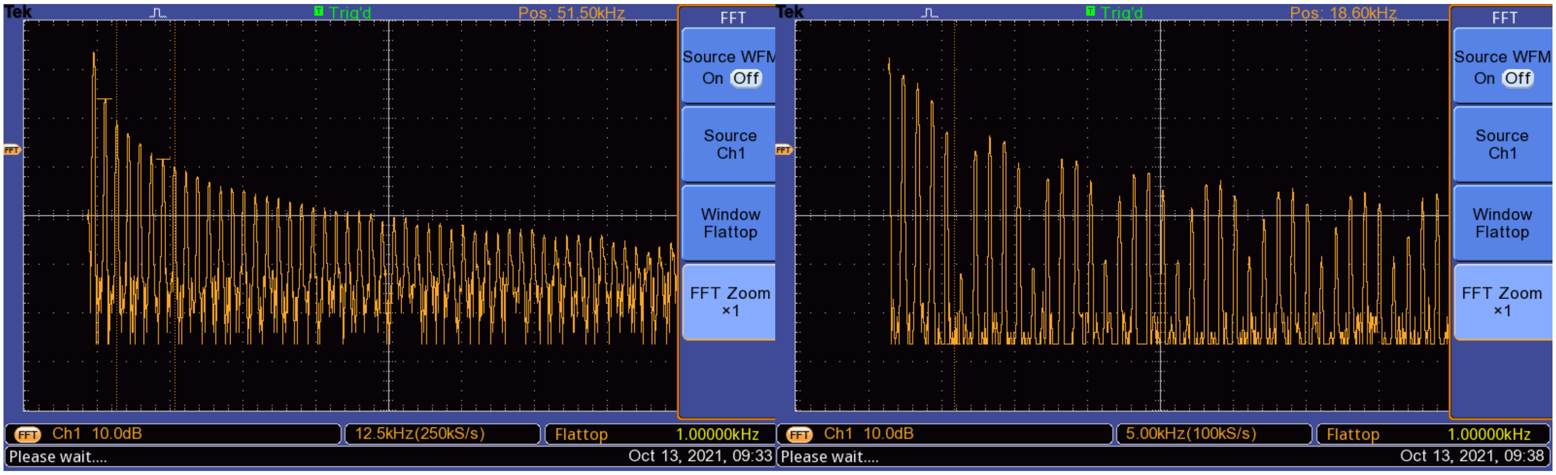


Figure 18: FFT spectrum for duty cycle 50% and 20%

As shown in figure, decreasing the duty cycle reduces the dc offset value and vice versa. Moreover, decreasing the duty cycle reduces the low-frequency spectral content of the spectrum, but does not affect the high-frequency content. Furthermore, the waveform of 50% duty cycle carries more energy than the waveform of 20% duty cycle.

3.3. Evaluation Experiment Part 3:

- The FFT of sinusoidal wave spectrum is an impulse and when another sinusoidal is added with the sinusoid from Auxiliary signal generator, FFT of each of them also got added which gave two impulses, as shown in figure 15. Thus, adding functions in time domain would add them in Fourier domain, which proves the linearity property of Fourier Transform.
  This can also be verified from the equation of Fourier transform in equation (9) which is an integral, thus following linear property.

4 Conclusion

The experiments conducted, focused on the concept and understanding of the analysis of signal in Frequency or Fourier Domain. The series of experiments dealt with the case of sinusoid signal and square wave of which the Fast Fourier Transform is performed and the properties are observed.

The FFT of sinusoid and square waves for different values of frequencies were generated using oscilloscope and signal generator. The FFT spectrum were studied for different values of peak-to-peak voltages for the sinusoid signal and different duty cycles values for the square wave. In the final part, the signals from the signal generator and auxiliary signal generators were combined to observe the linearity in Fourier transform. The measurements were verified using through the calculated plots generated from MATLAB.

In the experiment, it was observed that 0 dB spectrum peak could not be achieved in practice due to systematic errors in the instruments. It was also known that a sinusoid in time domain has an impulse in the Fourier domain. In case of a square wave in time domain, it shows different harmonics with decreasing amplitude in the frequency domain, commonly known as $sinc()$ function. Furthermore, changing the duty cycle changes the dc offset value of the FFT spectrum. In the end, linearity property of Fourier Transform was tested and the results were verified.

All in all, the series of experiments fulfilled its objective of making us understand the analysis of a signal with various characteristics like rms values and duty cycles changed through Fourier analysis. The properties were visually acknowledged through their plots seen during the experiment in the oscilloscope and the theoretical plots through MATLAB.

5 References

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2. Alan V. Oppenheim, Alan S. Willsky, with S. Hamid, Signals and Systems

3. Fast Fourier Transform URL: https://www.nti-audio.com/en/support/know-how/fast-fourier-transform-fft

4. Charles K. Alexander, Fundamentals of Electric Circuits