Jacobs University Bremen

Natural Science Laboratory
Signals and Systems Lab

Fall Semester 2021

Lab Theory 4 – Sampling

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Prelab: Sampling

Problem 1: The Sampling Theorem

1) Analog signals are usually passed through a low-pass filter prior to sampling. Why is this necessary?
   - High Frequency components in analog signal can cause aliasing (causes different signals to become indistinguishable). In order to prevent this, the high frequency components are cut-off by passing the signals through low pass filter.

2) What is the minimum sampling frequency for a pure sine wave input at 3KHz? Assume that the signal can be completely reconstructed.
   - For the signal to be sampled and reconstructed completely, the sampling frequency should be at least twice the input frequency. Here, the minimum sampling frequency for the pure sine wave should be $2 \cdot 3\ kHz = 6\ kHz$.

3) What is the Nyquist frequency?
   - Nyquist frequency is the minimum frequency above which a signal must be sampled in order to fully recover the original input signal.

4) What are the resulting frequencies for the following input sinusoids $500\ Hz, 2.5\ kHz, 5\ kHz$ and $5.5\ kHz$ if the signals are sampled by a sampling frequency of $5\ kHz$?
   - The resulting frequency (f_{result}) after sampling is calculated by:

$$f_{result} = \left| f_{in} - f_s \cdot nint\left(\frac{f_{in}}{f_s}\right) \right| \tag{1}$$

   where,
   f_{in} is the input frequency
   f_s is the sampling frequency
   $nint(\ )$ is the nearest integer function

   Using equation (1) the resulting frequencies for $f_s = 5\ kHz$ are:

Input frequency (f_{in})	Resulting frequency (f_{result})
$500\ Hz$	$500\ Hz$
$2.5\ kHz$	$2.5\ kHz$
$5\ kHz$	$0\ Hz$
$5.5\ kHz$	$500\ Hz$

5) Mention three frequencies of signal that alias to a $7\ Hz$ signal. The signal is sampled by a constant $30\ Hz$ sampling frequency.
   - The frequencies aliasing should be $7\ Hz$ greater than the multiple of sampling frequency. Thus, the three frequencies are $37\ Hz, 67\ Hz$ and $97\ Hz$.

Problem 2: Impulse Train Sampling and Real Sampling

Consider the circuit shown in figure (1). The input signal $x(t)$ is given by a sine function, with an amplitude of $5\ V$ peak and a frequency of $50\ Hz$. The sampling signal $p(t)$ is represented by a unity impulse train. Use an overall sampling rate of 100 k samples/s) for the whole problem.

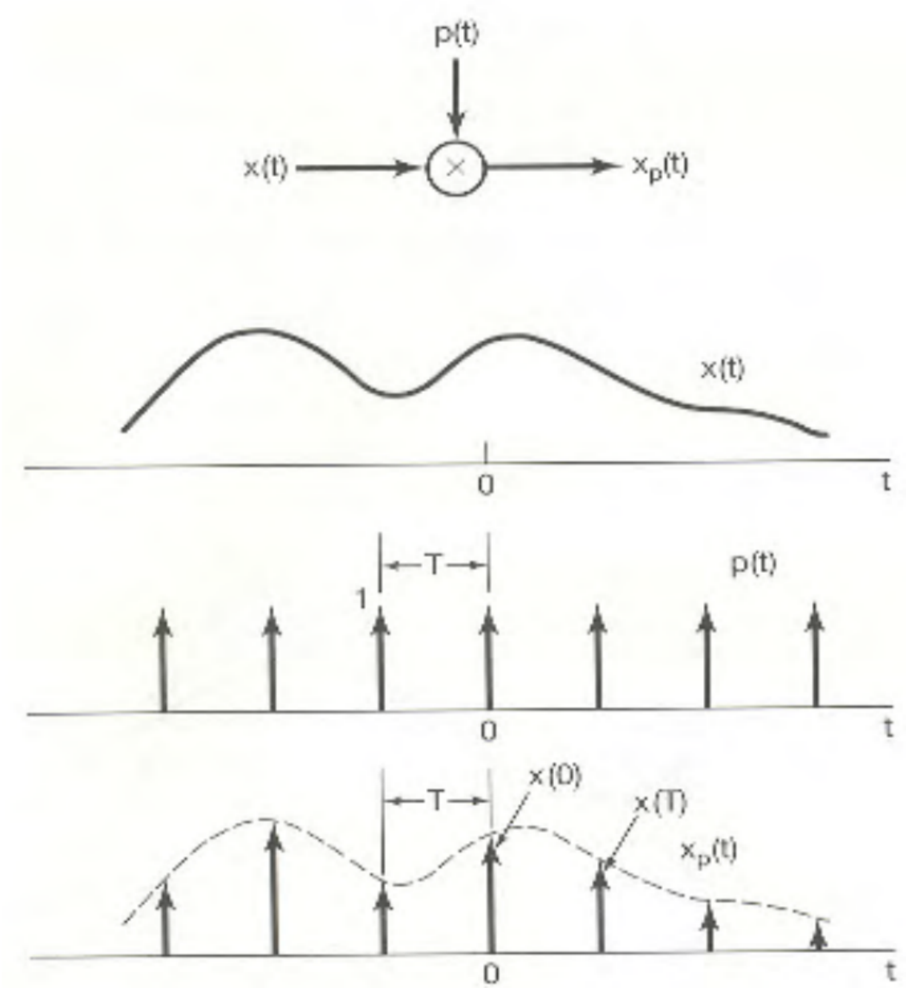

Figure 1: Schematic illustration of Impulse Train sampling

1) Carry out simulations for the following cases:
   (a) Under Sampling (use 48 Hz)
   (b) Nyquist Sampling
   (c) Over Sampling (use 1000 Hz)

   Use the command subplot to visualize the continuous signal x(t), the sampling signal p(t) and the result for each of these cases.

a) Under Sampling (use 48 Hz)

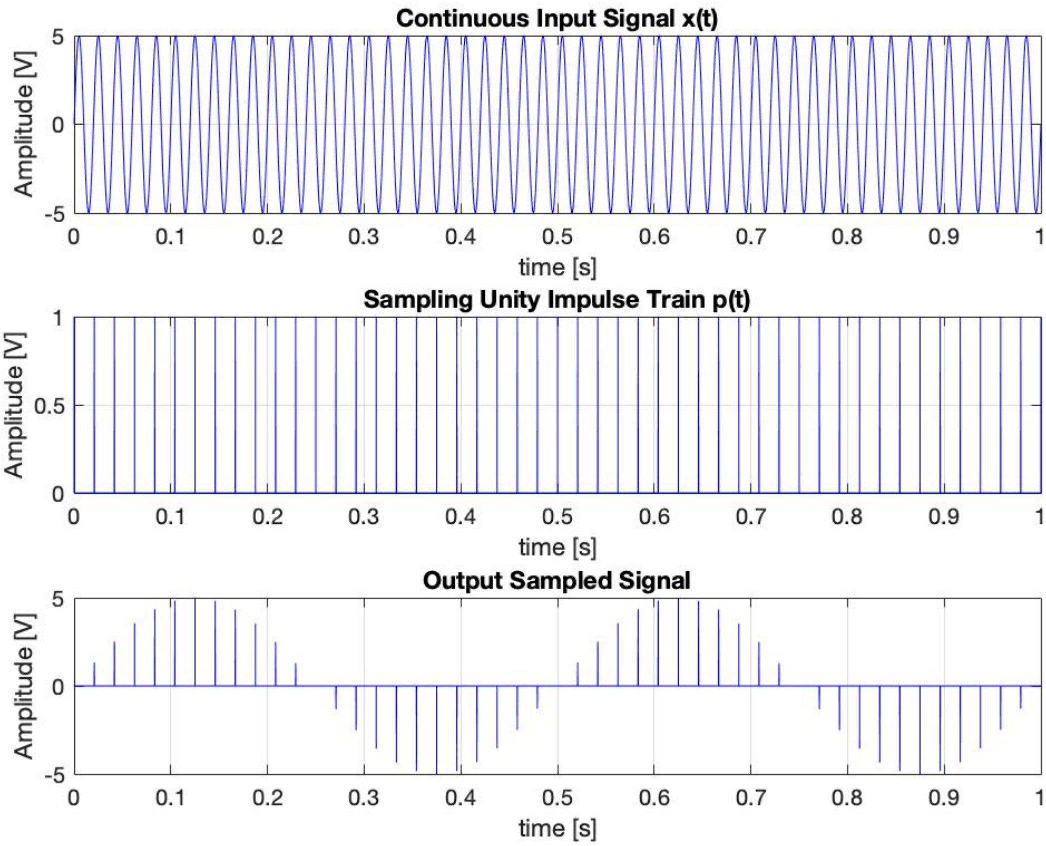

Figure 2: Plot for Under Sampling by Impulse Train

The MATLAB code used is:

```
% Under Sampling
fs=100000;
f=48;
t=0:1/fs:1;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

p=(1+square(2*pi*t*f,0.1))/2;
subplot(3,1,2);
plot(t,p,'b');
title('Sampling Unity Impulse Train p(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;
```

```
output = x.*p;
subplot(3,1,3);
plot(t,output,'b');
xlim([0,1])
ylim([-5,5]);
title('Output Sampled Signal');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;
```

b) Nyquist Sampling

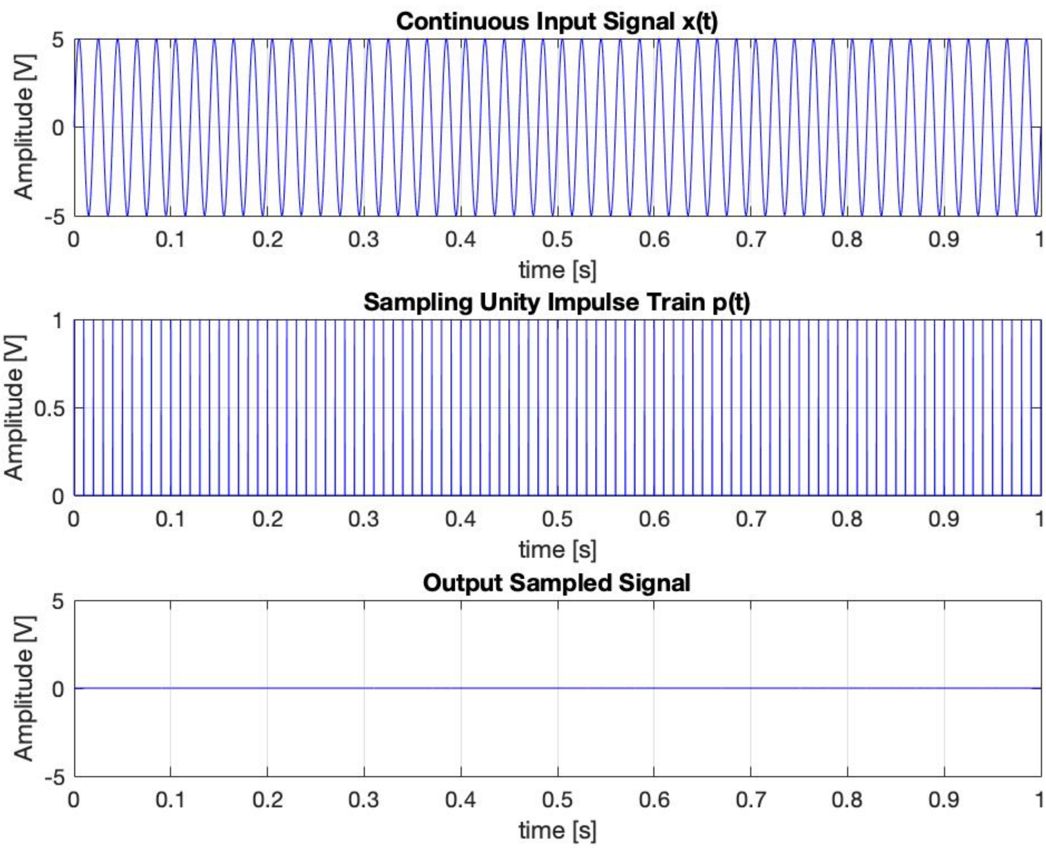

Figure 3: Plot for Nyquist Sampling by Impulse Train

The MATLAB code used is:

```
% Nyquist Sampling
fs=100000;
f = 100;
t=0:1/fs:1;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
```

```
hold on;

p=(1+square(2*pi*t*f,0.1))/2;
subplot(3,1,2);
plot(t,p,'b');
title('Sampling Unity Impulse Train p(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

output = x.*p;
subplot(3,1,3);
plot(t,output,'b');
xlim([0,1])
ylim([-5,5]);
title('Output Sampled Signal');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;
```

c) Over Sampling (use 1000 Hz)

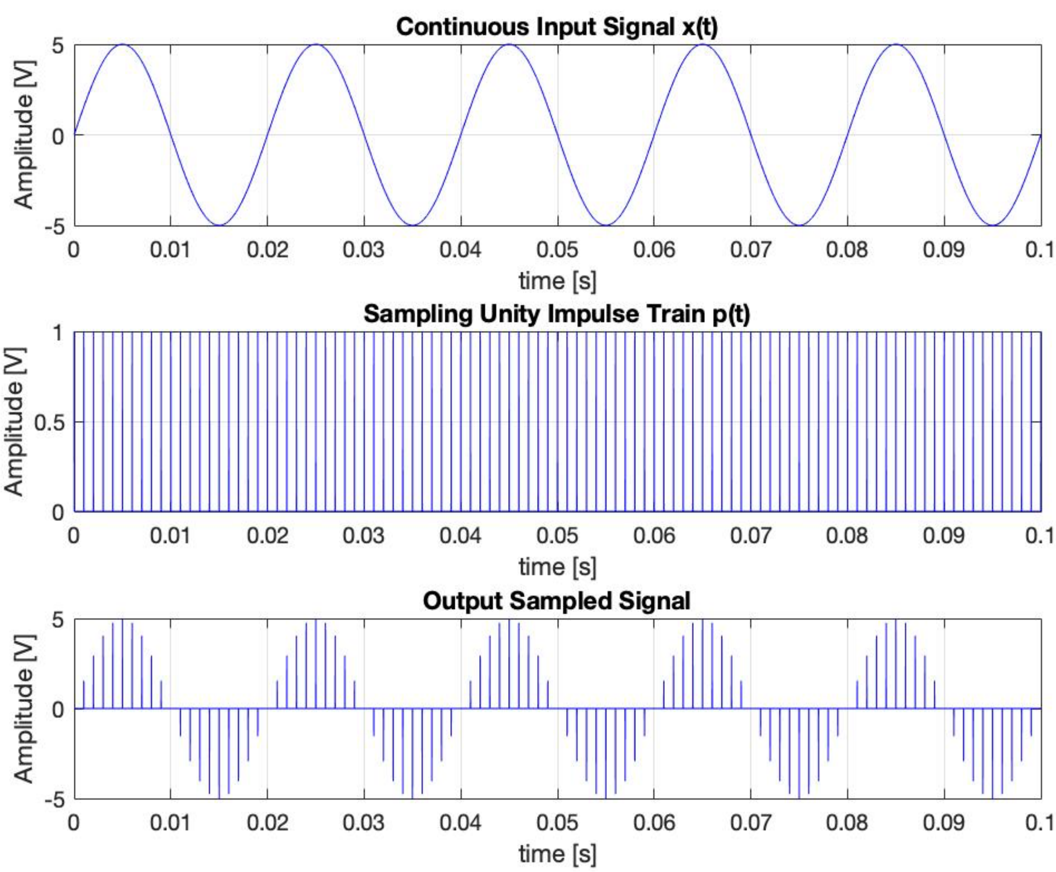

Figure 4: Plot for Over Sampling by Impulse Train

The MATLAB code used is:

```
% Over Sampling
fs=100000;
f = 1000;
t=0:1/fs:0.1;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

p=(1+square(2*pi*t*f,0.1))/2;
subplot(3,1,2);
plot(t,p,'b');
title('Sampling Unity Impulse Train p(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

output = x.*p;
subplot(3,1,3);
plot(t,output,'b');
xlim([0,0.1])
ylim([-5,5]);
title('Output Sampled Signal');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;
```

2) The signal $x(t)$ should be sampled by a rectangular pulse train. Modify the sampling function $p(t)$, so that the width of the sampling pulse is 50% of the sampling period.

   Carry out simulations for the following cases:
   (a) Under Sampling
   (b) Nyquist Sampling
   (c) Over Sampling

   Use the same sampling rates and the same plot setup as before

a) Under Sampling

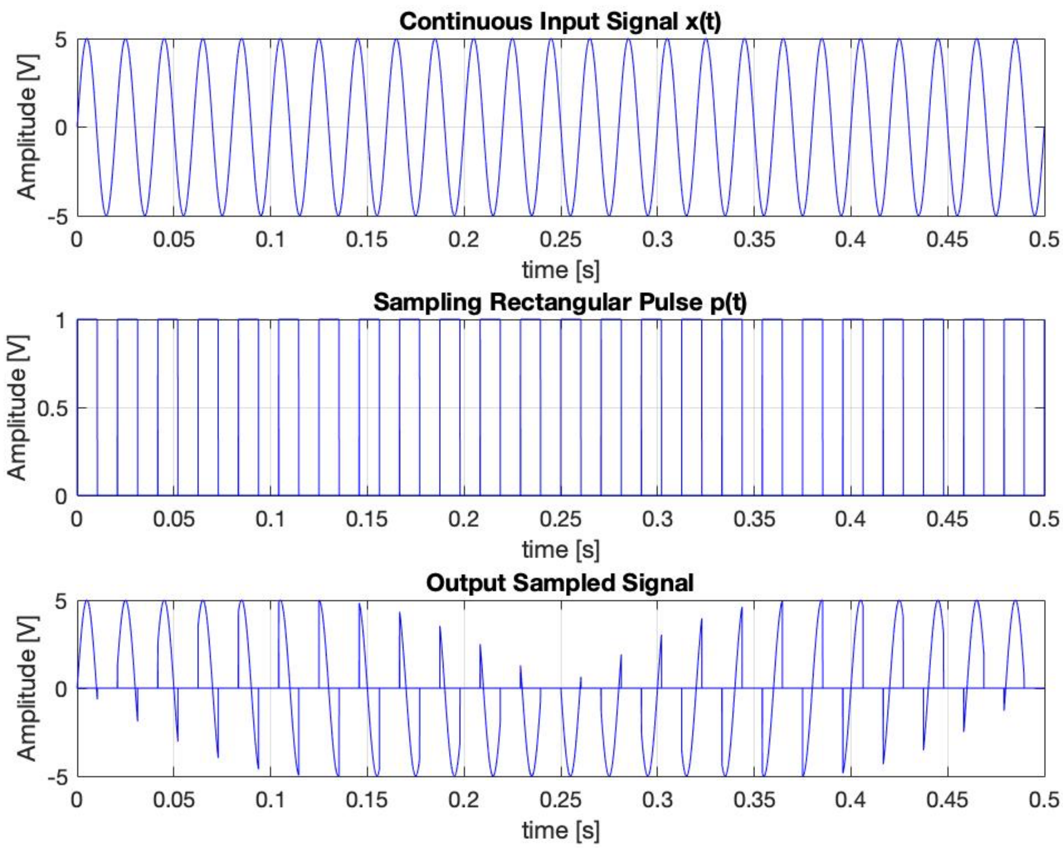

Figure 5: Plot for Under Sampling by Rectangular Pulse Train

The MATLAB code used is:

```
% Under Sampling
fs=100000;
f=48;
t=0:1/fs:0.5;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

rec=[0.1 50];
for train = 1: length(rec)
    for rate = 1: length(f)

        p = max(square(2*pi*f(rate)*t,rec(train)),0);
        subplot(3,1,2);
        plot(t,p,'b');
        title('Sampling Rectangular Pulse p(t)');
```

```
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;

        output = x.*p;
        subplot(3,1,3);
        plot(t,output,'b');
        xlim([0,0.5])
        ylim([-5,5]);
        title('Output Sampled Signal');
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;
    end
end
```

b) Nyquist Sampling

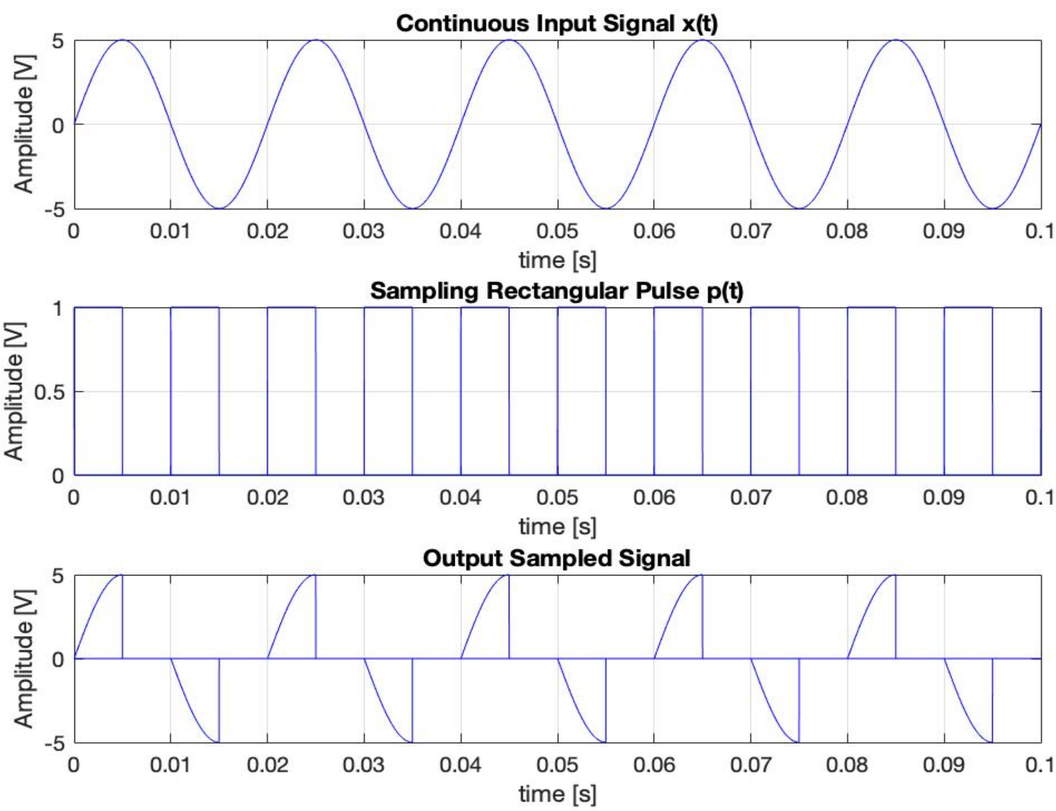

Figure 6: Plot for Nyquist Sampling by Rectangular Pulse Train

The MATLAB code used is:

```
% Nyquist Sampling
fs=100000;
f=100;
t=0:1/fs:0.1;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
```

```
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;
rec=[0.1 50];
for train = 1: length(rec)
    for rate = 1: length(f)

        p = max(square(2*pi*f(rate)*t,rec(train)),0);
        subplot(3,1,2);
        plot(t,p,'b');
        title('Sampling Rectangular Pulse p(t)');
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;

        output = x.*p;
        subplot(3,1,3);
        plot(t,output, 'b');
        xlim([0,0.1])
        ylim([-5,5]);
        title('Output Sampled Signal');
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;
    end
end
```

c) Over Sampling

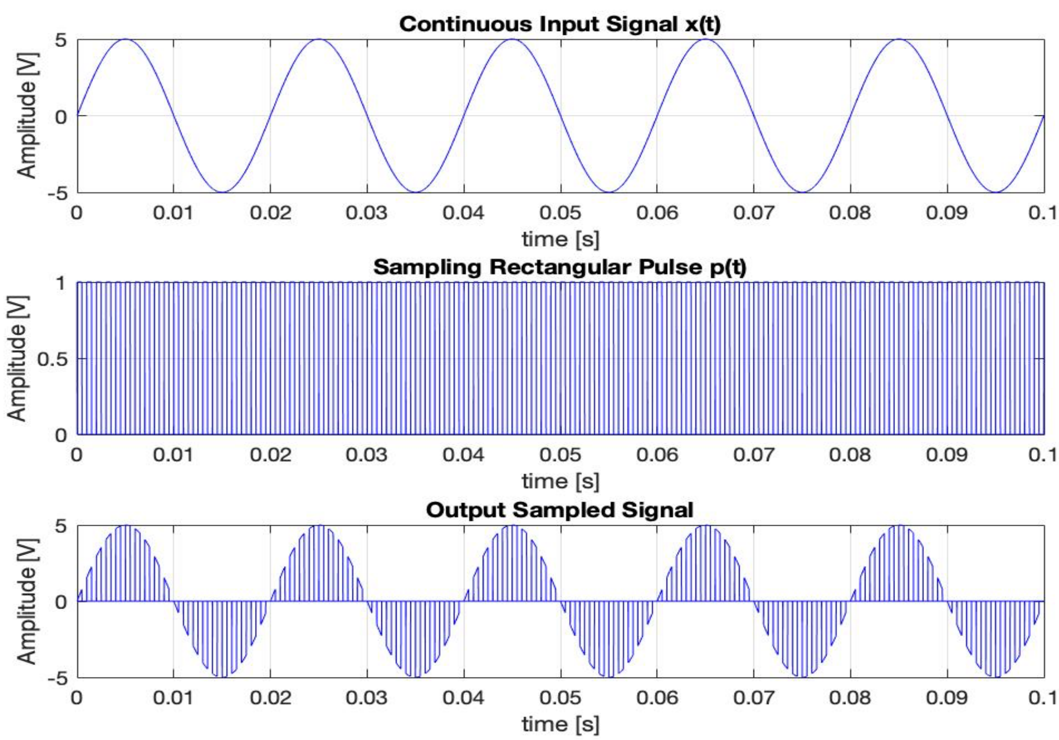

Figure 7: Plot for Over Sampling by Rectangular Pulse Train

The MATLAB code used is:

```
% Over Sampling
fs=100000;
f=1000;
t=0:1/fs:0.1;
w = 2*pi*50;
x = 5*sin(w*t);
subplot(3,1,1);
plot(t,x,'b');
title('Continuous Input Signal x(t)');
xlabel('time [s]'); ylabel('Amplitude [V]');
grid on;
hold on;

rec=[0.1 50];
for train = 1: length(rec)
    for rate = 1: length(f)

        p = max(square(2*pi*f(rate)*t,rec(train)),0);
        subplot(3,1,2);
        plot(t,p,'b');
        title('Sampling Rectangular Pulse p(t)');
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;

        output = x.*p;
        subplot(3,1,3);
        plot(t,output, 'b');
        xlim([0,0.1])
        ylim([-5,5]);
        title('Output Sampled Signal');
        xlabel('time [s]'); ylabel('Amplitude [V]');
        grid on;
        hold on;
    end
end
```

Problem 3: Sampling using a Sampling Bridge

Modify the circuit in figure (8) in such a way that a single sampling source can be used to sample the input signal.

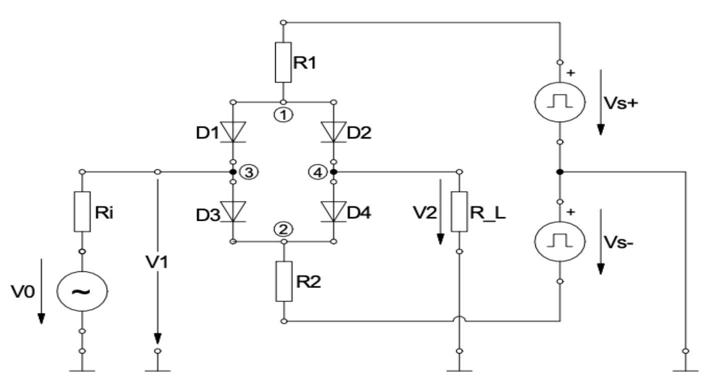

Figure 8: High speed Sampling circuit based on four diodes

1) Sketch the Modified Circuit.

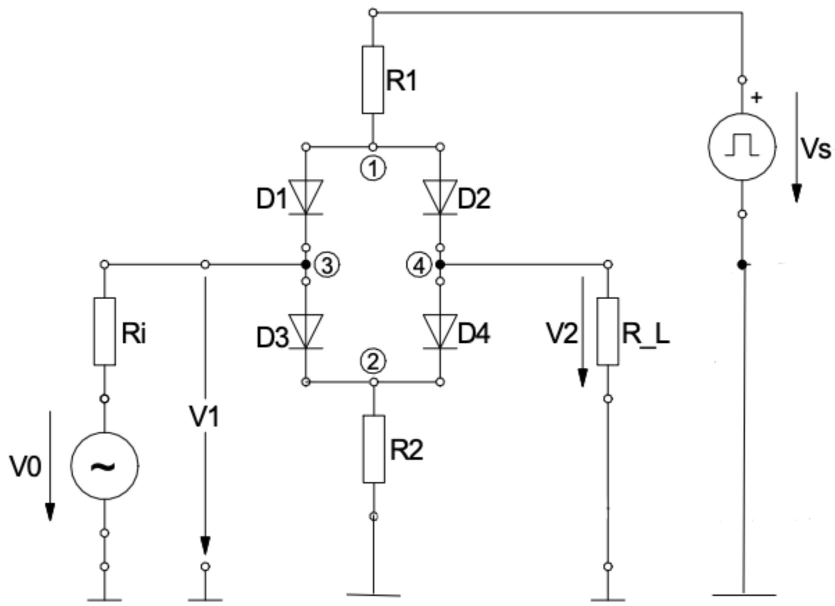

Figure 9: Modified Circuit

2) Explain the operation of the modified circuit.

The only difference with the circuit from the manual is the ground. For the original circuit, the signal changes around 0V and for the modified circuit around an offset which is $\frac{V_1}{2}$. The input signal must have its reference ground around this offset. In general, the circuit works only with positive voltages.