COMS30127 / COMSM2127 sample paper - DRAFT

Rubrik

This paper contains *two* parts.
The first section contains *15* short questions.
Each question is worth *two marks* and all should be attempted.
The second section contains *three* long questions.
Each long question is worth *20 marks*.
The best *two* long question answers will be used for assessment.
The maximum for this paper is *70 marks*.
This is a two hour exam.

Section A: short questions - answer all questions

1. Neurophysiologists typically distinguish between two cell types in primary visual cortex: simple cells and complex cells. What is the key characteristic that distinguishes these two cell types?

   Solution: Complex cell responses are spatially invariant to the stimulus location, simple cell responses are not.

2. Hebb's rule is often paraphrased as 'neurons that fire together wire together'; why is this no longer considered accurate?

   Solution: It ignores the temporal structure; spike timing effects are now considered important.

3. What are the principal cells of the cerebellar cortex?

   Solution: Purkinje cells.

4. Neurons cannot fire at arbitrarily high rates because after a spike there is typically a short period of time, usually several milliseconds, where they are resistant to spiking again. What is this time period called?

   Solution: The refractory period.

5. Which channel is typically responsible for spike rate adaptation?

   Solution: Slow potassium.

6. The two principal forms of aphasia are expressive aphasia and fluent aphasia, one is distinguished by the inability to find words, the other by

the inability to understand language. These are associated with lesions in which brain areas.

Solution: Broca's area and Wernicke's area.

7. In the Hodgkin-Huxley model of the squid giant axon, which ion is responsible for the initial rise in the voltage during a spike?

   Solution: Sodium.

8. What is the typical scale of voltage differences in the brain: microvolts, millivolts or volts?

   Solution: millivolts.

9. For a spike train with spike times $\{t_1, t_2, \ldots, t_n\}$ evoked by stimulus $s(t)$ define the spike triggered average.

   Solution:
   $$S(\tau) = \frac{1}{n}\sum_i s(t_i - \tau)$$
   .

10. What is d' used for in describing a neuronal response?

    Solution: It measures the difference between two sets of multi-trial responses.

11. Which part of the hippocampus is thought to act as a Hopfield network?

    Solution: CA3

12. Sketch the hippocampus and labelling CA3, CA1 and dentate gyrus.

    Solution: Should show two interlocking horseshoe shapes, the smaller is the dentate gyrus, the larger contains CA3 and CA1, CA3 is at the end that interlocks with the dentate gyrus.

13. Solve the equation
    $$3\frac{dv}{dt} = -v$$
    with $v(0) = 1$.

    Solved by ansatz or integrating factor this give $v = \exp(-t/3)$.

14. What is meant by spike rate adaptation.

    Solution: When ongoing stimulation causes a neuron to fire the firing rate often drops even if the stimulation does not.

15. Define the energy of a pattern in the Hopfield network and briefly describe how this is related to pattern storage and completion.

    Solution: The energy is given by

    $$E = -\frac{1}{2}\sum_{ij} x_i w_{ij} x_j$$

    and stored patterns correspond to local minima.

Section B: long questions - answer two questions

1. This question is about numerical methods.

   (a) Explain how the Euler method for integrating the differential equation
   $$\frac{dv}{dt} = F(v)$$
   is derived from the Taylor expansion. [7 marks]

   (b) Although the error in the Euler method is $O(\delta t^2)$ and therefore small, there is a problem with it accumalating: if the error is always positive, for example, it grows as t increases. Explain why this might be less of a problem when integrating the leaky integrate-and-fire neuron? [5 marks]

   (c) In the second order Runge-Kutta algorithm we define

   $$\begin{aligned} k_1 &= F(v)\delta t \\ k_2 &= F(v + k_1/2)\delta t \end{aligned}$$

   and approximate
   $$v(t + \delta_t) = v(t) + k_2$$
   Show using the Taylor expansion that this is accurate to $O(\delta t^3)$. [8 marks]

   Solution: a) and c) see notes; b) because of the reset.

2. This question is about synapses.

   (a) Let each synapse's weight w_i be plastic, and change according to a Hebbian rule: $w(t+1) = w(t) + \frac{1}{8}x_i y$ where x_i is the ith presynaptic neuron's firing rate, $y = \sum_i w_i x_i$ is the postsynaptic neuron's firing rate, and t indexes time. Is this plasticity rule stable or unstable? Explain your answer. [3 marks]

(b) Now consider a case where there are just two input neurons with firing rates $x_1 = 1$ and $x_2 = 3$, and respective synaptic weights initially both equal to one half, $w_1 = w_2 = \frac{1}{2}$. Compute the synaptic weights after one update. [4 marks]

(c) Let's assume that we always want the sum of the weights to remain $w_1 + w_2 = 1$. This can be achieved by renormalising the synaptic strengths after each update, either by subtracting some fixed amount from each weight, or by dividing both weights by some fixed amount. Normalise the updated synaptic weights from the previous question, first using subtractive normalisation, then compare it with the result obtained with divisive normalisation. Report the resulting weight values for each case. Which method (subtractive or divisive normalisation) leads to a greater difference between w_1 and w_2? [5 marks]

(d) Now consider a new plasticity rule (similar to the "BCM" rule), where $\frac{dw_i}{dt} = x_i(y - \theta)$ where θ is a threshold that determines whether the synapse increases or decreases in strength. The threshold itself changes on a slow timescale, according to $\frac{d\theta}{dt} = y^2 - \theta$. What is the steady state value of the threshold in terms of y? [3 marks]

(e) Now consider there are just two input neurons with firing rates $x_1 = 3$ and $x_2 = 5$. If the synaptic weights change according to the BCM-like plasticity rule from the previous question, what is the eventual steady-state value of w_1 as a function of the steady-state value of w_2? [5 marks]

Solution

(a) Unstable. Δw is always ≥ 0, so the weights grow without bound. The rule encourages positive feedback since increasing w will increase y, which further increases w. [3 marks, one mark for correctly stating 'unstable', two more mark for a correct justification.]

(b) In this case $y(t) = (1/2 \times 1) + (1/2 \times 3) = 2$. $w_{1new} = \frac{3}{4}$ and $w_{2new} = \frac{5}{4}$ [4 marks, two marks for each correct answer].

(c) From the answer to the previous part, we have $w_1 + w_2 = 3/4 + 5/4 = 2$. To do subtractive scaling we need to subtract a total of 1 away, which when split equally between the two synapses becomes 1/2 each. So subtractive scaling gives $w_{1new} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ and $w_{2new} = \frac{5}{4} - \frac{1}{2} = \frac{3}{4}$. To do divisive scaling we need to divide both weights by 2. So divisive scaling gives $w_{1new} = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ and $w_{2new} = \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$. Subtractive normalisation results in the bigger difference between the weights (1/2 vs 1/4). [5 marks, 2 marks for each pair of correct weight values, and one further mark for a correct statement that subtractive normalisation gives a greater difference. If the student carried forward a mistake from the previous question, do not penalise them.]

(d) To find the steady-state threshold we set, $\frac{d\theta}{dt} = 0$, then rearrange for θ, giving $\theta_\infty = y^2$. [3 marks for correct answer.]

(e) To find the steady-state weights we set $\frac{dw_i}{dt} = 0$, and insert the steady-state value for θ from the previous answer. We then get $0 = x_i y(1 - y)$. For this to hold either y or $1 - y$ has to equal zero, we choose the latter which implies $y = 1$. Now we have $1 = 3w_1 + 5w_2$. Rearranging gives $w_{1\infty} = (1 - 5w_{2\infty})/3$. [5 marks for correct answer.]

3. This question is about the Hodgkin-Huxley equation.

(a) Sketch a spike; label the up swing and down swing and indicate what aspect of the current dynamics are responsible for each. [4 marks]

(b) Write down the Hodgkin-Huxley equation; include the sodium and potassium channels. [4 marks]

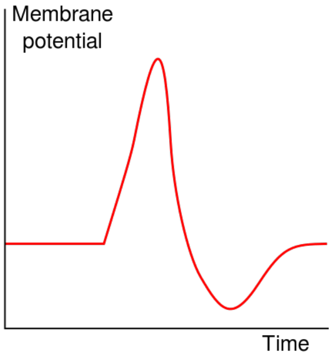
(c) Which axon is this an equation for? [2 marks]

(d) Write down the equation for n, m and h both in terms of the transition probabilities and in terms of the time constant and asymptotic values. [5 marks]

(e) Sketch the asymptotic values for n, m and h indicating what role these play in the spike. [5 marks]

Solution:

The sketch shows a spike like in this picture from wikipedia:



with the up swing, the up bit being due to sodium influx, the down bit is due to the sodium influx being switched off and potassium ions leaving the cell.

The Hodgkin Huxley eqn is

$$C_m \frac{dV}{dt} = g_l(E_l - V) + \bar{g}_{\mathrm{Na}} m^3 h(E_{\mathrm{Na}} - V) + \bar{g}_{\mathrm{K}} n^4 (E_{\mathrm{K}} - V) \tag{1}$$

and this is the equation for the squid giant axon. For each of these eqns:

$$\frac{dl}{dt} = \alpha_l(1 - l) - \beta_l l \tag{2}$$

where l is standing for n, m or h and α_l and β_l are the transition probabilities. This can be rewriten as

$$\frac{dl}{dt} = \alpha_l - (\alpha_l + \beta_l)l \tag{3}$$

or

$$\frac{1}{\alpha_l + \beta_l}\frac{dl}{dt} = \frac{\alpha_l}{\alpha_l + \beta_l} - l \tag{4}$$

so

$$\tau_l = \frac{1}{\alpha_l + \beta_l} \tag{5}$$

and

$$l_\infty = \frac{\alpha_l}{\alpha_l + \beta_l} \tag{6}$$

are the time constant and asymptotic value respectively with

$$\tau_l \frac{dl}{dt} = l_\infty - l \tag{7}$$

The asymptotic values of n, m and h are given in, for example `cdn.comsol.com/wordpress/2016/11/asymptotic-values-Hodgkin-Huxley.png`

and we see that as voltage increases m and n switch on, but h switches off; the difference in the timescale for m and n is what allows the sodium to switch on first.