UNIVERSITY OF BRISTOL

September / August 2018 Examination Period

FACULTY OF ENGINEERING

Third Year / M Level Examination for the Degree of
Bachelor of Science / Master of Engineering / Masters of Science

COMS30127 / COMSM2127
Computational Neuroscience

TIME ALLOWED:
2 hours

Answers to COMS30127 / COMSM2127: Computational Neuroscience

Intended Learning Outcomes:

Section A: short questions - answer all questions

Q1. What is the difference between synaptic facilitation and synaptic depression.

Solution: In the first the synapse gets stronger, in the second it gets weaker.

Q2. Are ion channels stochastic or deterministic?

Solution: Deterministic

Q3. The probability of the sodium gate begin open is often written as n^4; why is this a fourth power?

Solution: The sodium gate is composed of four subgates whose individual probabilities of being open are independent.

Q4. Sketch the nullclines for the Fitzhugh-Nagumo model.

Solution: This will show a cubic with positive cubic coefficient crossed by a line with positive slope.

Q5. Neurons cannot fire at arbitrarily high rates because after a spike there is typically a short period of time, usually several milliseconds, where they are resistant to spiking again. What is this time period called?

Solution: refractory period

Q6. What is an axon?

Solution: It is the '[]tube' along which the spike travels away from the cell.

Q7. Hodgkin and Huxley built a computational model of the action potential dynamics in axons. From which species did they record the electrophysiological data to constrain the model.

Solution: The squid, one mark for 'giant squid'.

Q8. Who famously described a rule for synaptic plasticity as: "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."?

Solution: Donald Hebb.

Q9. Solve the equation

$$3\frac{dv}{dt} = -v$$

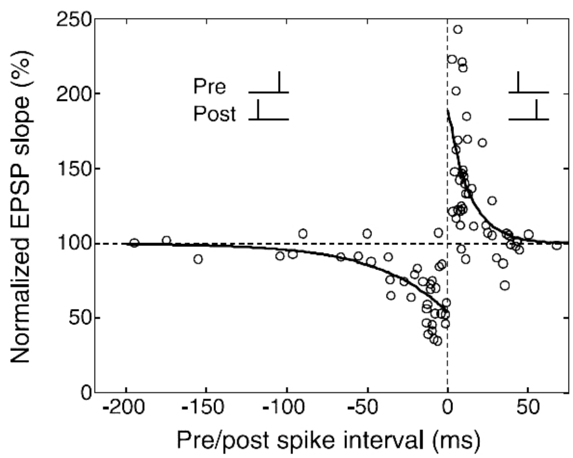
with $v(0) = -1$.

Solution: Solved by ansatz or integrating factor this give $v = -\exp(-t/3)$.

Q10. How many layers has the mammalian neocortex

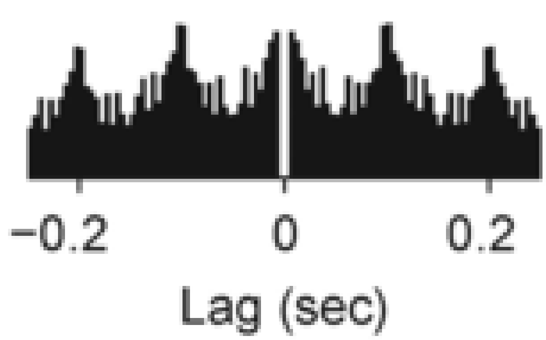
Solution: six.

Q11. The classic spike-timing dependent plasticity learning rules are shown below, taken from Dan, Yang, and Mu-ming Poo. "Spike timing-dependent plasticity of neural circuits." Neuron 44 (2004): 23-30. Explain how this differs from 'neurons that fire together wire together'.



Solution: If the post neuron fires just before the pre neuron this is 'neurons firing together' but it causes depression rather than facilitation.

Q12. This image, taken from Newman, Jonathan P., et al. "Optogenetic feedback control of neural activity." eLife 4 (2015) shows the auto-correllogram for a cell in rat brain that responds to stimulation of a whisker. Speculate on the frequency the whisker is being stimulated at.



Solution: The peaks show a 10Hz oscillation in the neuron so that probably corresponds to the whisker stimulation.

Q13. Which brain region is primarily effected in Parkinson's diseases?

Solution: Basal ganlia (substantia nigra also good)

Q14. The Euler update for

$$\frac{dy}{dt} = F(y)$$

with time step δt and $y(n\delta t) = y_n$ is

$$y_{n+1} = y_n + \delta t F(y)$$

what is the highest order error?

Solution: $d^2y/dt^2 \delta t^2/2$.

Q15. Define the energy of a pattern in the Hopfield network and briefly describe how this is related to pattern storage and completion.

Solution: The energy is given by

$$E = -\frac{1}{2}\sum_{ij} x_i w_{ij} x_j$$

and stored patterns correspond to local minima.

Section B: long questions - answer two questions

Q1. This question is about integrate-and-fire neurons.

(a) The voltage in the integrate-and-fire neuron satisfies:

$$\tau_m \frac{dV}{dt} = E_l - V + R_m I_e$$

However this is not the whole model; what must be added to give the integrate and fire model? *[4 marks]*

(b) Relate the membrane constant τ_m to electrical properties of the cell membrane.

(c) Derive a formula for the interspike interval for this neuron when there is a constant current large enough to cause spiking. *[7 marks]*

(d) Neurons often show 'spike rate adaptation': if the neuron is persistently stimulated its firing rate falls. This is often modelled in the integrate-and-fire model by adding a slow potassium current, so

$$\tau_m \frac{dV}{dt} = E_l - V + R_m g(E_K - V) + R_m I_e$$

where

$$\tau_K \frac{dg}{dt} = -g$$

and

$$g \to g + a$$

whenever there is a spike; a a parameter describing the model and E_K is the reversal potential of potassium. How does this model spike rate adaptation. What might g correspond to in the cell. *[5 marks]*

Solution: a) It needs the reset: if $V > V_T$ then $V = V_R$. [2 for the idea, 3 for the two equations]
b) $\tau_m = C_m/R_m$ where C_m is the capacitance of the membrane and R_m is the resistance.

c)
In the model

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e$$

which we can solve from our study of odes, it gives

$$V(t) = E_L + R_m I_e + [V(0) - E_L - R_m I_e]e^{-t/\tau_m}$$

[1 marks] so if the neuron has spiked and is reset at time $t = 0$ and reaches threshold at time $t = T$, assume $V_R = E_L$ we have [2 marks]

$$V_T = E_L + R_m I_e - R_m I_e e^{-T/\tau_m}$$

[1 mark] so

$$e^{-T/\tau_m} = \frac{E_L + R_m I_e - V_T}{R_m I_e}$$

[1 mark] Taking the log of both sides we get

$$T = \tau_m \log\left[\frac{R_m I_e}{E_L + R_m I_e - V_T}\right]$$

[2 marks]

d) When the neuron spikes g goes up, since E_K is equal to or near the leak potential this drives down V slowing spiking, provided the g doesn't have time to recover. [3 marks, 1 mark for garbled version] g is the conductance of a slow potassium channel.

Q2. This question is about rate models. Consider a neuron that varies its mean firing rate $\bar{r}(s)$ as a logarithmic function of the scalar intensity of stimulus s: $\bar{r}(s) = \ln(s)$. Imagine we are trying to detect changes in the stimulus intensity from the neuron's firing rate alone. Due to the noisiness of the firing, assume our detector can only reliably detect firing rate changes of σ or larger, $\Delta r_{min} = \sigma$.

(a) Using the identity $\frac{d}{dx}\ln(x) = \frac{1}{x}$, calculate the smallest detectable change in stimulus intensity Δs_{min}. *[5 marks]*

(b) Sketch a plot with s on the x-axis and $\frac{\Delta s_{min}}{s}$ on the y-axis. *[5 marks]*

(c) This result is related to a psychophysical phenomenon known as Weber's law. Does Weber's law state that humans perceive relative or absolute changes in the intensity of sensory stimuli? Explain how Weber's law follows from the previous result in parts (a) and (b). *[5 marks]*

(d) A more realistic scenario would be where the firing rate noise scales with the square root of r, so that smallest firing rate change our detector can resolve is $\Delta r_{min} = \eta r^{1/2}$. What would the smallest detectable change in stimulus intensity be now? *[5 marks]*

Solution: a) $\Delta s_{min} = \sigma s$ [5 marks for correct answer, 2 marks if correctly begin by taking the derivative dr/ds but make subsequent errors when rearranging terms].
b) Should be a plot with straight horizontal line [5 marks]. [2 marks for an attempt if they correctly plot Δs_{min} vs s instead, which would be an increasing straight line].
c) Weber's law implies humans sense relative changes [3 marks], since $\Delta s_{min}/s$ = constant [2 marks. Also allow answers that invoke the logarithmic relationship between stimulus and response, students do not necessarily need to refer to part b to get full marks].
d) $\Delta s_{min} = \eta s\sqrt{\log s}$ [5 marks. 2 marks if correctly begin by taking the derivative dr/ds but make subsequent errors when rearranging terms].

Q3. This question is about the analysis of spike trains.

(a) What is the spike triggered average? *[3 marks]*

(b) What is the auto-correllogram? *[3 marks]*

(c) What is the peri-stimulus time histogram? *[3 marks]*

(d) Explain when you would use each of these approaches to neuroscientific data? *[3 marks]*

(e) What is a tuning curve? *[3 marks]*

(f) It is believed that a cricket has four neurons encoding the direction air is flowing past its body. Each of the these neurons is associated with a direction $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$, these are unit vectors points at 45°, 135°, −135° and −45° to the axis of the cricket. If the neurons have rectified linear response, so the firing rate of neuron i when the air is flowing in direction $\mathbf{v}$ is given

$$r_i = [\mathbf{v} \cdot \mathbf{c_i}]_+$$

with

$$[x]_+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Sketch the tuning curves of the four neurons.

Solution: a) The STA is

$$f(\tau) = \frac{1}{n}\sum_i s(t_i - \tau)$$

where $s(t)$ is the stimulus at time t and t_i are spike times. [3 for correct, 1 for attempt]
b) Discretize time with time step h and the put in the ith bin the number of spike pairs t_1 and t_2 such that $(t_2 - t_1)/\delta t$ rounds to i. [3 for correct, 1 for attempt]
c) The ith bin contains the number of spikes, averaged over trials, at a time in $[i\delta t, (i+1)\delta t)$ after the stimulus. [3 for correct, 1 for attempt]
d) STA, what features in a stimulus cause spikes; auto-correllogram, neuronal oscillations; PSTH, the effect of a stimulus on spiking. [1 each]
e) If you have a stimulus parameterized by a single parameter, say θ then the tuning curve is a graph of firing rate against θ. [3 for correct, 1 for attempt]
f) This should look like four copies of cosine from $-\pi/2$ to π_2 centred on the four directions.[5 for correct, 2 for any attempt at graphing firing rate against angle].

END OF PAPER