UNIVERSITY OF BRISTOL

September / August 2018 Examination Period

FACULTY OF ENGINEERING

Third Year / M Level Examination for the Degree of
Bachelor of Science / Master of Engineering / Masters of Science

COMS30127 / COMSM2127
Computational Neuroscience

TIME ALLOWED:
2 hours

Answers to COMS30127 / COMSM2127: Computational Neuroscience

Intended Learning Outcomes:

Section A: short questions - answer all questions

Q1. What is the difference between synaptic facilitation and synaptic depression.

Solution: In the first the synapse gets stronger, in the second it gets weaker.

Q2. Are ion channels stochastic or deterministic?

Solution: Stochastic

Q3. The probability of the potassium gate begin open is often written as n^4; in the Hodgkin Huxley model what reason is given for this being a fourth power?

Solution: The potassium gate is composed of four subgates whose individual probabilities of being open are independent.

Q4. Sketch the nullclines for the Fitzhugh-Nagumo model.

Solution: This will show a cubic with positive cubic coefficient crossed by a line with positive slope.

Q5. Neurons cannot fire at arbitrarily high rates because after a spike there is typically a short period of time, usually several milliseconds, where they are resistant to spiking again. What is this time period called?

Solution: refractory period

Q6. What is an axon?

Solution: It is the 'tube' along which the spike travels away from the cell; other words might include neurite, cable, wire.

Q7. Hodgkin and Huxley built a computational model of the action potential dynamics in axons. From which species did they record the electrophysiological data to constrain the model?

Solution: The squid, one mark for 'giant squid'.

Q8. Who famously described a rule for synaptic plasticity as: "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."?

Solution: Donald Hebb.

Q9. Solve the equation

$$3\frac{dv}{dt} = -v$$

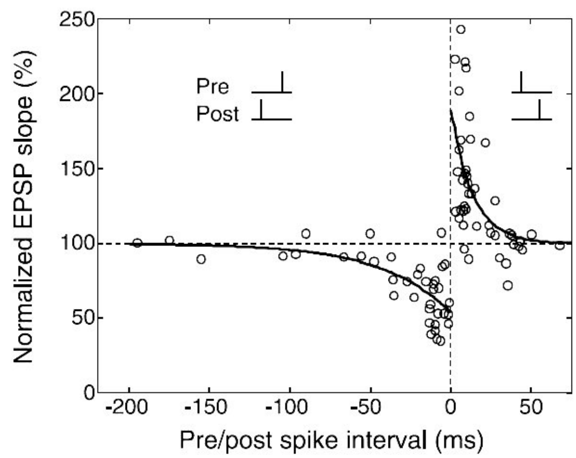
with $v(0) = -1$.

Solution: Solved by ansatz or integrating factor this give $v = -\exp(-t/3)$.

Q10. How many layers does the mammalian neocortex have?

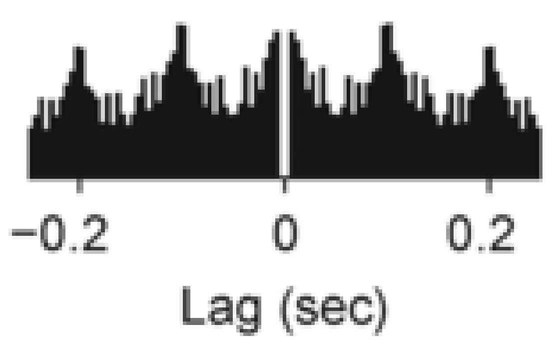
Solution: six.

Q11. The classic spike-timing dependent plasticity learning rules are shown below, taken from Dan, Yang, and Mu-ming Poo. "Spike timing-dependent plasticity of neural circuits." Neuron 44 (2004): 23-30. Explain how this differs from 'neurons that fire together wire together'.



Solution: If the post neuron fires just before the pre neuron this is 'neurons firing together' but it causes depression rather than facilitation.

Q12. This image, taken from Newman, Jonathan P., et al. "Optogenetic feedback control of neural activity." eLife 4 (2015) shows the auto-correllogram for a cell in rat brain that responds to stimulation of a whisker. Speculate on the frequency the whisker is being stimulated at.



Solution: The peaks show a 10Hz oscillation in the neuron so that probably corresponds to the whisker stimulation.

Q13. Which brain region is primarily effected in Parkinson's diseases?

Solution: Basal ganlia (substantia nigra also good)

Q14. The Euler update for

$$\frac{dy}{dt} = F(y)$$

with time step δt and $y(n\delta t) = y_n$ is

$$y_{n+1} = y_n + \delta t F(y)$$

what is the highest order error?

Solution: $d^2y/dt^2\delta t^2/2$.

Q15. Define the energy of a pattern in the Hopfield network and briefly describe how this is related to pattern storage and completion.

Solution: The energy is given by

$$E = -\frac{1}{2}\sum_{ij} x_i w_{ij} x_j$$

and stored patterns correspond to local minima.

Section B: long questions - answer two questions

Q1. This question is about integrate-and-fire neurons.

(a) The voltage in the integrate-and-fire neuron satisfies:

$$\tau_m \frac{dV}{dt} = E_l - V + R_m I_e$$

However this is not the whole model; what must be added to give the integrate and fire model? *[4 marks]*

(b) Relate the membrane time constant τ_m to electrical properties of the cell membrane. *[4 marks]*

(c) Derive a formula for the interspike interval for this neuron when there is a constant current large enough to cause spiking. *[7 marks]*

(d) Neurons often show 'spike rate adaptation': if the neuron is persistently stimulated its firing rate falls. This is often modelled in the integrate-and-fire model by adding a slow potassium current, so

$$\tau_m \frac{dV}{dt} = E_l - V + R_m g(E_K - V) + R_m I_e$$

where

$$\tau_K \frac{dg}{dt} = -g$$

and

$$g \to g + a$$

whenever there is a spike; a a parameter describing the model and E_K is the reversal potential of potassium. How does this model spike rate adaptation? What might g correspond to in the cell? *[5 marks]*

Solution: a) It needs the reset: if $V > V_T$ then $V = V_R$. [2 for the idea, 3 for the two equations]
b) $\tau_m = C_m R_m$ where C_m is the capacitance of the membrane and R_m is the resistance.

c)
In the model

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e$$

which we can solve from our study of odes, it gives

$$V(t) = E_L + R_m I_e + [V(0) - E_L - R_m I_e]e^{-t/\tau_m}$$

[1 marks] so if the neuron has spiked and is reset at time $t = 0$ and reaches threshold at time $t = T$, assume $V_R = E_L$ we have [2 marks]

$$V_T = E_L + R_m I_e - R_m I_e e^{-T/\tau_m}$$

[1 mark] so

$$e^{-T/\tau_m} = \frac{E_L + R_m I_e - V_T}{R_m I_e}$$

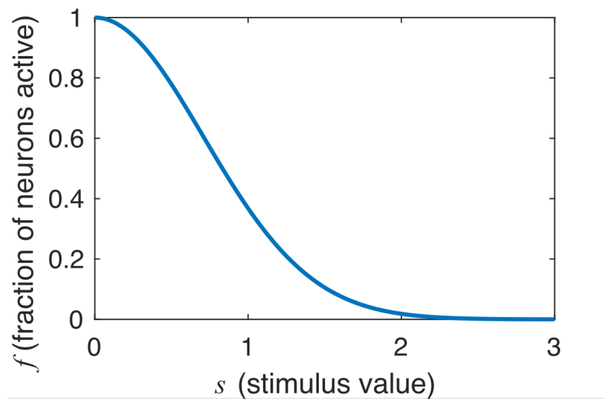
[1 mark] Taking the log of both sides we get

$$T = \tau_m \log\left[\frac{R_m I_e}{E_L + R_m I_e - V_T}\right]$$

[2 marks]

d) When the neuron spikes g goes up, since E_K is equal to or near the leak potential this drives down V slowing spiking, provided the g doesn't have time to recover. [3 marks, 1 mark for garbled version] g is the conductance of a slow potassium channel.

Q2. This question is about sparse coding.



(a) What does the term 'sparse coding' mean? *[2 marks]*

(b) Consider a population of N binary neurons, where each neuron can either be in one of two discrete states: ON or OFF. What is the total number of possible binary patterns that can be jointly represented these N neurons? *[2 marks]*

(c) Now consider a case where the circuit ensures that only 2 neurons are ever simultaneously ON, while the remaining $N - 2$ neurons are OFF. How many possible binary patterns are allowed in this scheme? *[1 mark]*

(d) Consider a population of neurons where the fraction of ON neurons f depended on the value of some stimulus s according to

$$f(s) = \exp(-s^2/a) \tag{1}$$

where a is a parameter (plotted in the above figure). A measure of the *absolute* sensitivity of the neural population activity to changes in the stimulus is the magnitude of the derivative of f with respect to s, $|\frac{df}{ds}|$. What is $|\frac{df}{ds}|$ as a function of s? *[3 marks]*

(e) Evaluate $|\frac{df}{ds}|$ for $s = a$ and for $s = 2a$. Assuming $a > 1$, is the absolute sensitivity higher for $s = a$ or $s = 2a$? *[5 marks]*

(f) A measure of the *relative* sensitivity of the population activity to the stimulus is $\frac{|\frac{df}{ds}|}{f}$. Evaluate this measure for $s = a$ and for $s = 2a$. Is the relative sensitivity higher for $s = a$ or $s = 2a$? *[5 marks]*

(g) Do sparse codes convey greater absolute or relative sensitivity to changes in the stimulus? *[2 marks]*

Solution: a) Sparse coding refers to a neural coding scheme where most neurons are typically silent (or each neuron is silent most of the time). [2 marks]
b) Total number of possible binary patterns is 2^N. [2 marks]
c) Number of possible patterns with 2 ON neurons is $\binom{N}{2} = N(N-1)/2 = (N^2 - N)/2$. [1 mark]
d) $|\frac{df}{ds}| = \frac{2s}{a}\exp(-s^2/a)$ [3 marks]
e) $|\frac{df}{ds}|(s = a) = \frac{2}{e^a}$ and $|\frac{df}{ds}|(s = 2a) = \frac{4}{e^4 a}$. [2 marks each answer] The absolute sensitivity is greater for the former $s = a$ case. [1 mark]
f) $\frac{|\frac{df}{ds}|}{f}(s = a) = 2$ and $\frac{|\frac{df}{ds}|}{f}(s = 2a) = 4$. [2 marks each answer] The relative sensitivity is greater for the latter $s = 2a$ case. [1 mark]
g) Sparse codes are more sensitive to relative changes in stimulus than absolute changes. [2 marks]

Q3. This question is about the analysis of spike trains. In the first three parts and in part (e) the answer should include mathematical formulae.

(a) What is the spike triggered average? *[3 marks]*

(b) What is the auto-correllogram? *[3 marks]*

(c) What is the peri-stimulus time histogram? *[3 marks]*

(d) Explain when you would use each of these approaches to the analysis of neuro-scientific data? *[3 marks]*

(e) What is a tuning curve? *[3 marks]*

(f) It is believed that a cricket has four neurons encoding the direction air is flowing past its body. Each of the these neurons is associated with a direction $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$, these are unit vectors points at 45°, 135°, $-135°$ and $-45°$ to the axis of the cricket. If the neurons have rectified linear response, so the firing rate of neuron i when the air is flowing in direction $\mathbf{v}$ is given by

$$r_i = [\mathbf{v} \cdot \mathbf{c_i}]_+$$

with

$$[x]_+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Sketch the tuning curves of the four neurons. *[5 marks]*

Solution: a) The STA is

$$f(\tau) = \frac{1}{n}\sum_i s(t_i - \tau)$$

where $s(t)$ is the stimulus at time t and t_i are spike times. [3 for correct, 1 for attempt]
b) Discretize time with time step h and the put in the ith bin the number of spike pairs t_1 and t_2 such that $(t_2 - t_1)/\delta t$ rounds to i. [3 for correct, 1 for attempt]
c) The ith bin contains the number of spikes, averaged over trials, at a time in $[i\delta t, (i+1)\delta t)$ after the stimulus. [3 for correct, 1 for attempt]
d) STA, what features in a stimulus cause spikes; auto-correllogram, neuronal oscillations; PSTH, the effect of a stimulus on spiking. [1 each]
e) If you have a stimulus parameterized by a single parameter, say θ then the tuning curve is a graph of firing rate against θ. [3 for correct, 1 for attempt]
f) This should look like four copies of cosine from $-\pi/2$ to π_2 centred on the four directions.[5 for correct, 2 for any attempt at graphing firing rate against angle].

END OF PAPER