Implementation of Backpropagation and Training a Palindrome Network

Aditya Pande	22M2108
Balbir Singh	22M0747
Tarun Bisht	23D0386
Vivek Kumar Trivedi	22N0457

Problem Statement

- **Input**: 10-bit String (of numbers)

- **Output**: 1 if Palindrome, 0 otherwise

Architecture and Hyperparameter details

INPUT LAYER   HIDDEN LAYER   OUTPUT LAYER

INPUT LAYER

No of neurons : 10
Input layer is **FULLY CONNECTED** to next layer

HIDDEN LAYER

No of neurons : 2
Activation : Sigmoid σ

OUTPUT LAYER

No of neurons : 1
Threshold : 0.5 for predicting 1
Activation : Sigmoid σ
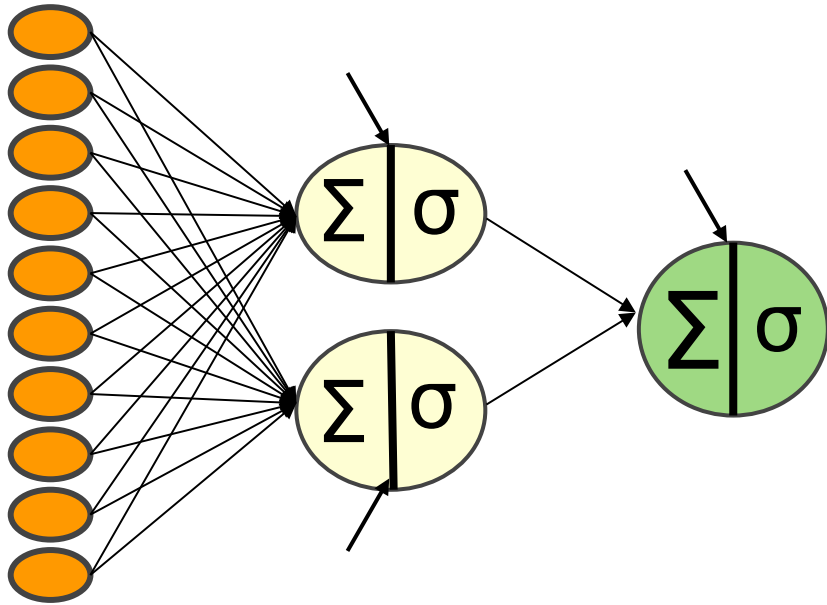
BIAS IS USED IN HIDDEN AS WELL AS OUTPUT LAYER

HYPERPARAMETER

- Learning Rate: 0.1
- Batch Size: 32
- Momentum **β:** 0.9
- OverSampling: duplicate 20 times Minor class

Overall performance

Accuracy (Total and class-wise)

- Total = 100%

- Class0 Non- Palindrome (NP)= 100%

- Class1 Palindrome(P) = 100%

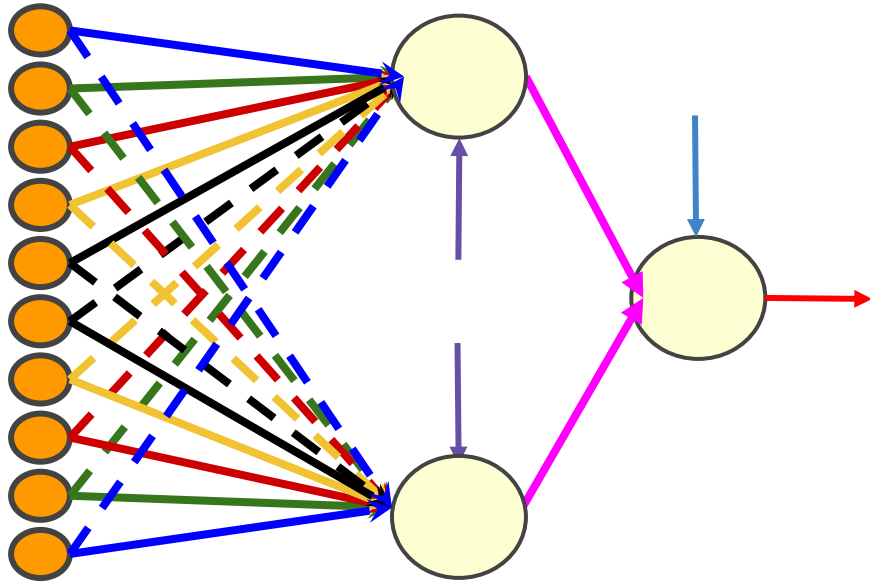## Classification Report

		Precision	Recall	f1-score		support
Class0: Non - Palindrome		1.00	1.00	1.00		992
Class1: Palindrome		1.00	1.00	1.00		32
Accuracy	**Total**				1.00	1024
	Class 0: NP				1.00	992
	Class 1: P				1.00	32

Confusion Matrix

	True Value	
Predicted Value	**1**	**0**
1	True Positive **32**	False Positive **0**
0	False Negative **0**	True Negative **992**

Interpretability of middle layer

W_o Anti Symmetry around axis

$+w11 +w12 +w13 +w14 +w15 \quad -w15 \ -w14 \ -w13 \ -w12 \ -w11$

b_o Same value for both neurons

W_1 Both weights are the same

PALINDROME			
Zup	b_o	Aup	similar
Zlow	b_o	Alo	similar
NON PALINDROME			
Aup	Close to O or 1		
Alow	Close to 1 or O		
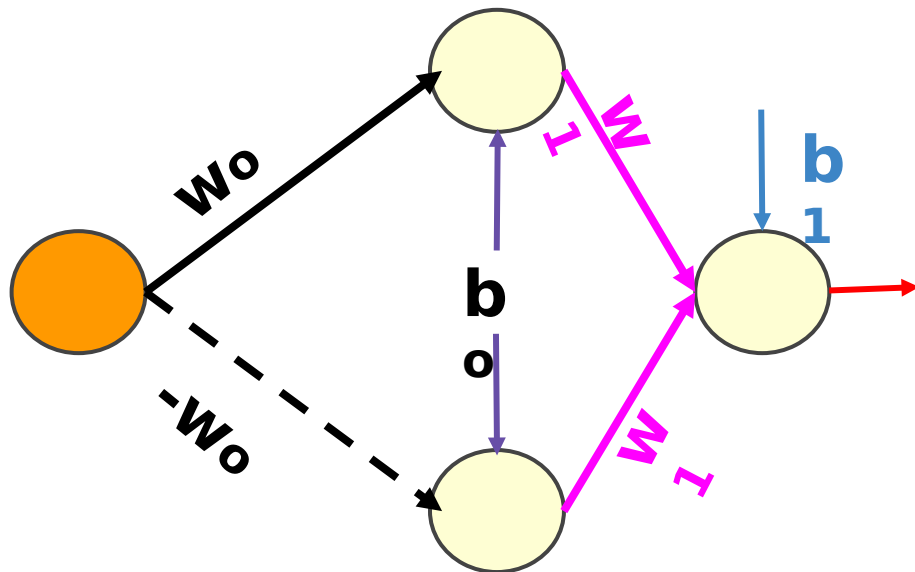
Learnings

NON PALINDROME REGION	
Aout	**w1 + b1**

$\sigma(\mathbf{b_o})$

PALINDROME REGION	
Aout	**2 x w1 x $\sigma(\mathbf{b_o})$ + b1**

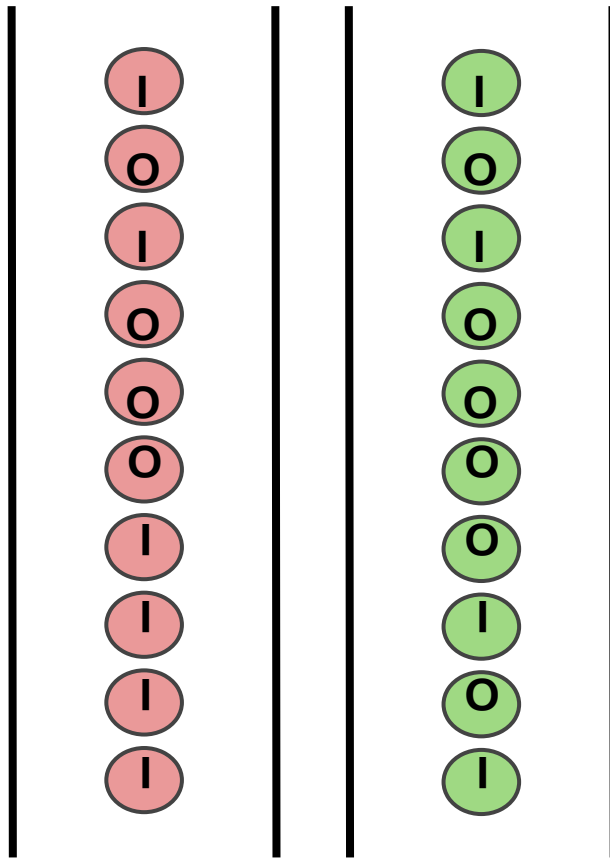NOTE : $\sigma(\mathbf{b_o}) > 0.5$
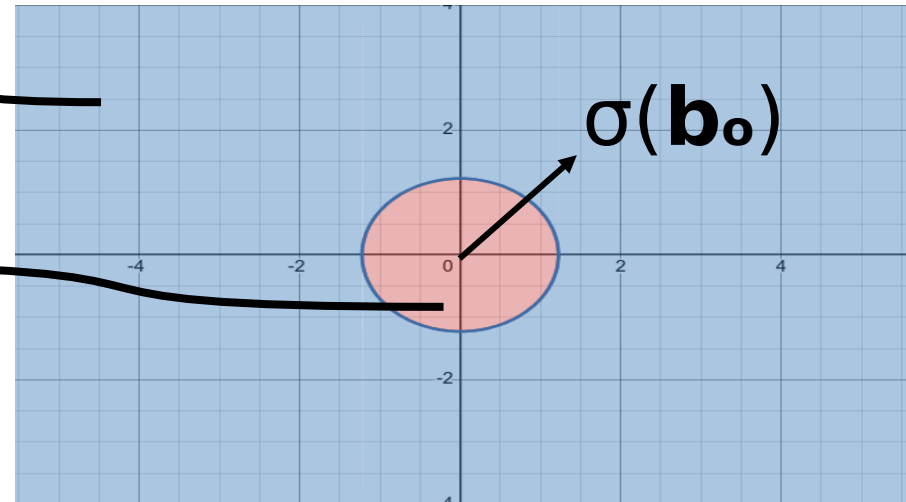
OUTER NETWORK JUST learns this decision boundary as above

- Two neuron in hidden layer is required if activation is sigmoid and no skip connection allowed.

- One neuron is sufficient in case of mod or square activation in hidden layer OR with skip connections

- As data is imbalance, oversampling minority class improves performance.

- Weighted BCE loss did not improved performance.

Evaluation Scheme

- Correct implementation of BP from scratch: 10 marks (show the code parts that implement weight change rules)
- Theory of BP clarity: 10
- Overall Performance: accuracy >=90: 10 marks; 80-89: 9; 70-79: 8; 60-69: 7; 50-59: 40-49: 5; 30-39: 4. And so on
- Interpretability of middle layer: 10
- Demo -10

APPENDIX: Input representation

- Each input x is a 10 x 1 numpy array
- Each entry of the input array form the input of the first (input) layer of the neural network
- The function **generate_combinations** generates all the possible 10 bit lists

```python
def generate_combinations(length):
    if length <= 0:
        return [[]]
    result = [[0], [1]]
    for _ in range(length - 1):
        result = [comb + [0] for comb in result] + [comb + [1] for comb in result]
    return result
```

Example Non -Palindrome	Example Palindrome

Function to generate all 10 bit strings

PLEASE NOTE : This slide is not part of the presentation. This is a REFERENCE slide

APPENDIX: Theory of BP

$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = f(z_j^l)$$

$$a^l = f(w^l \cdot a^{l-1} + b^l)$$
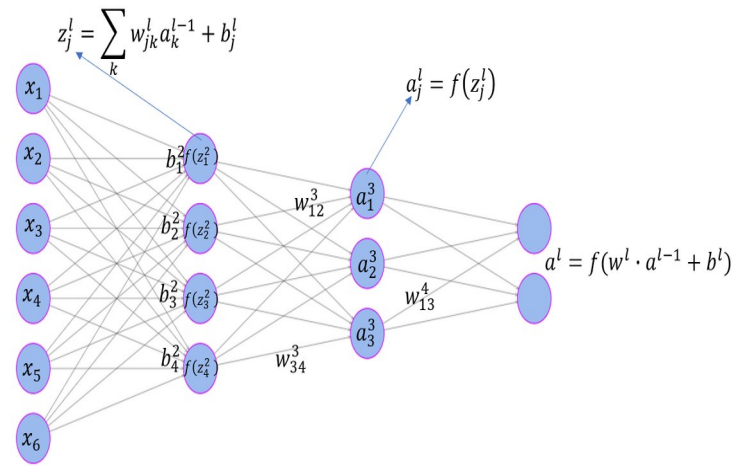
$a^1 = [x_1, x_2 \ldots \ldots \ldots x_n]^T$

$a^l = [a_1^l, a_2^l, \ldots \ldots \ldots \ldots a_n^l]^T$

$b^l = [b_1^l, b_2^l, \ldots \ldots \ldots \ldots b_n^l]^T$

$w^l = [w_{ij}^l]$
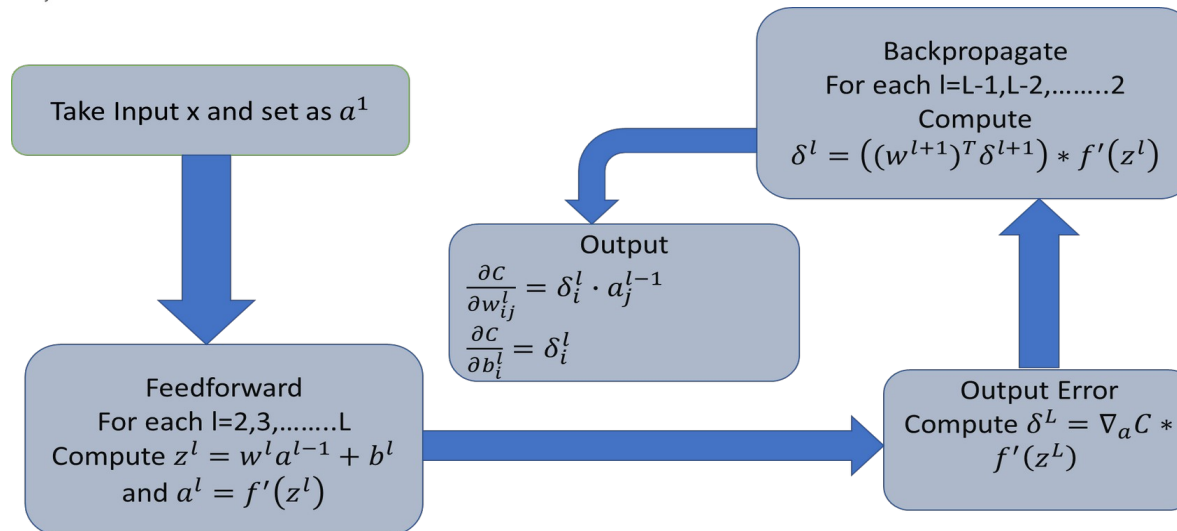
Let's define cost function as $C(a^L, y)$

Now we have to find gradient of C with respect to w_{ij}^l and b_j^l

$$\frac{\partial C}{\partial w_{ij}^L} = \left(\frac{\partial C}{\partial a_i^L} \cdot \frac{\partial a_i^L}{\partial z_i^L} \right) a_j^{L-1} = \delta_i^L \cdot a_j^{L-1}$$

$$\frac{\partial C}{\partial b_i^L} = \left(\frac{\partial C}{\partial a_k^L} \cdot \frac{\partial a_k^L}{\partial z_i^L} \right) = \delta_i^L$$
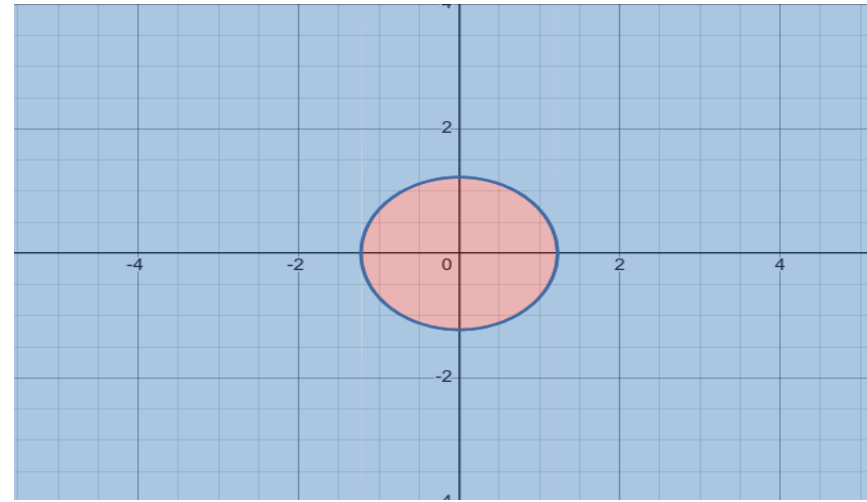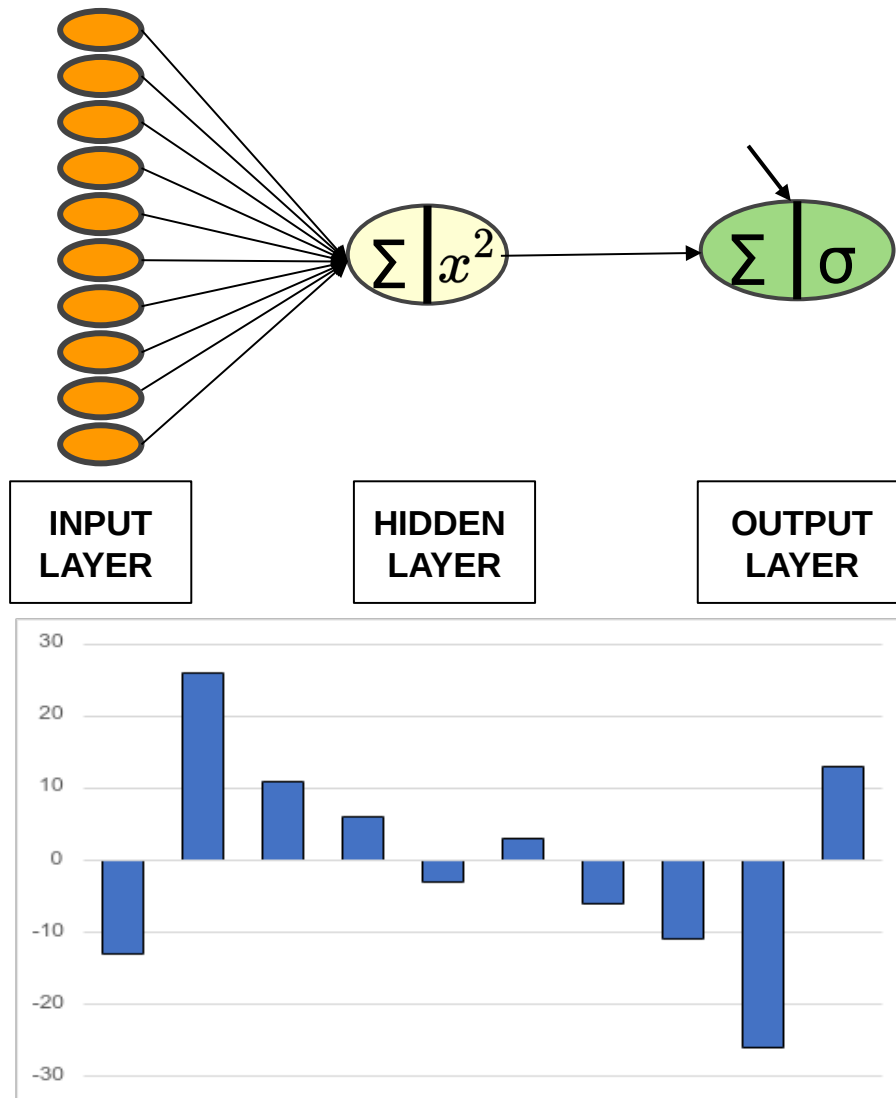
$$\frac{\partial C}{\partial w_{ij}^l} = \left(\sum_k \frac{\partial C}{\partial z_k^{l+1}} \cdot \frac{\partial z_k^{l+1}}{\partial a_i^l} \cdot \frac{\partial a_i^l}{\partial z_i^l} \right) a_j^{l-1} = \left(\sum_k \delta_k^{l+1} w_{ki}^{l+1} f'(z_i^l) \right) a_j^{l-1} = \delta_i^l \cdot a_j^{L-1}$$

$$\frac{\partial C}{\partial b_i^L} = \left(\sum_k \frac{\partial C}{\partial z_k^{l+1}} \cdot \frac{\partial z_k^{l+1}}{\partial a_i^l} \cdot \frac{\partial a_i^l}{\partial z_i^l} \right) = \left(\sum_k \delta_k^{l+1} w_{ki}^{l+1} f'(z_i^l) \right) = \delta_i^L$$
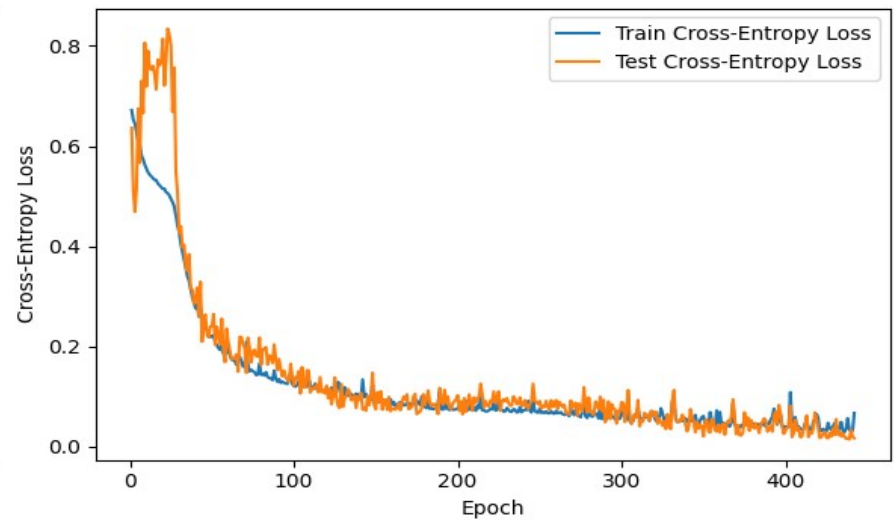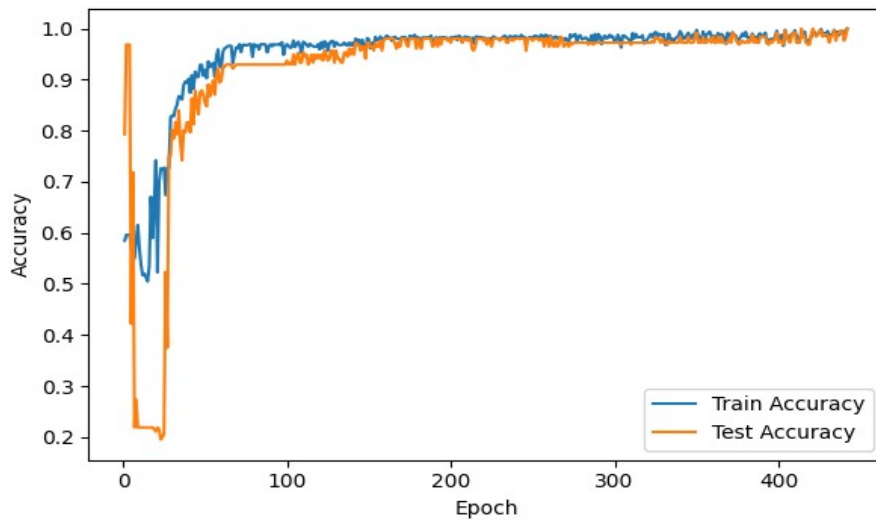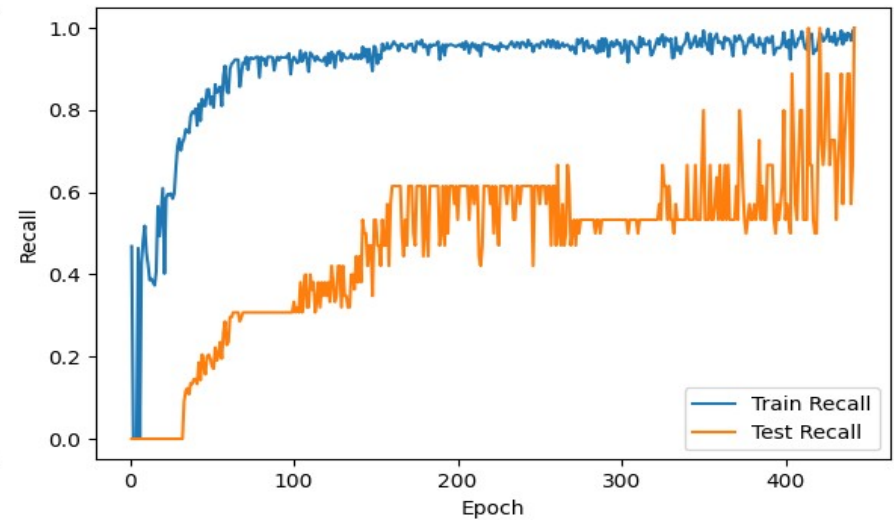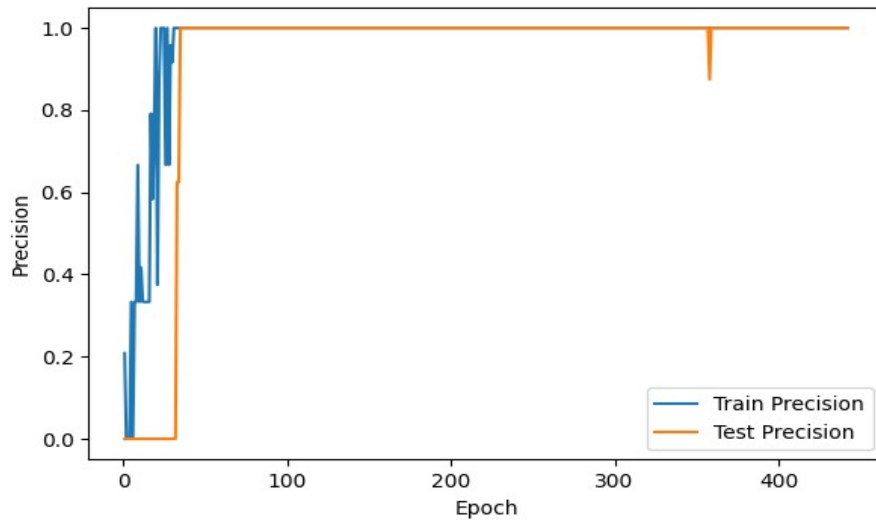
Take Input x and set as a^1

Feedforward
For each l=2,3,........L
Compute $z^l = w^l a^{l-1} + b^l$
and $a^l = f'(z^l)$

Output
$$\frac{\partial C}{\partial w_{ij}^l} = \delta_i^l \cdot a_j^{l-1}$$
$$\frac{\partial C}{\partial b_i^l} = \delta_i^l$$

Backpropagate
For each l=L-1,L-2,........2
Compute
$$\delta^l = \left((w^{l+1})^T \delta^{l+1} \right) * f'(z^l)$$

Output Error
Compute $\delta^L = \nabla_a C * f'(z^L)$

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APPENDIX: One Neuron hidden layer (square activation)

INPUT LAYER

HIDDEN LAYER

OUTPUT LAYER

APPENDIX: Plots