Pratical No. 1

Topic : limits & continuity

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{x \to \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2-1}} \right]$

Examine the continuity of following function at given point

$$f(x) = \left.\begin{array}{ll} \frac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \leq \frac{\pi}{2} \\ = \frac{\cos x}{\pi - 2x} & \text{for } \frac{\pi}{2} \leq x < \pi \end{array}\right\} x = \pi/2$$

iii) $$f(x) = \left.\begin{array}{ll} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \leq x < 6 \\ = \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{array}\right\} x = 3 \ \& \ 9$$

Q6 find the value of k so that function $f(x)$ is continous at indicated point

i) $$f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & x < 0 \\ = k & x = 0 \end{array}\right\} \text{at } x = 0$$

ii) $$f(x) = \left.\begin{array}{ll} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x = 0 \end{array}\right\} - \text{at } x = 0$$

iii) $$f(x) = \left.\begin{array}{ll} \frac{\sqrt{3} - \tan x}{\pi - 3x} & x \neq \frac{\pi}{3} \\ = k & x = \frac{\pi}{3} \end{array}\right\} - \text{at } x = \pi/3$$

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Q4 Discuss the continuity of following function in which of these function have removable discontinuity? & redefine the function as so to remove discontinuity

i) $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $x \neq 0$ — at $x=0$
$= 9$ $x = 0$

ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2}$, $x \neq 0$ — at $x=0$
$= \dfrac{\pi}{60}$ $x = 0$

8) if $f(x) = \dfrac{e^{x^2}-\cos x}{x^2}$ for $x \neq 0$ is continous at $x=0$ find $f(0)$

9 $f(x) = \dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continous at $x=\frac{\pi}{2}$ find $f(\frac{\pi}{2})$

Answer

$$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$$

$$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{3a+x}+2\sqrt{x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$\lim_{x\to a} \frac{(\sqrt{a+2x})^2-(\sqrt{3x})^2}{(\sqrt{3a+x})^2-(2\sqrt{x})^2} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$\lim_{x\to a} \frac{a+2x-3x}{3a+x-4x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$\lim_{x\to a} \frac{\cancel{a-x}}{3\cancel{(a-x)}} \times \left(\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right)$$

$$\frac{1}{3} \times \lim_{x\to a} \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \frac{1}{3} \times \left(\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right) = \frac{1}{3}\ \frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{4\sqrt{a}}{6\sqrt{3a}} = \frac{4}{6}\sqrt{\frac{a}{3a}}$$

$$= \frac{2\cancel{4}}{3\cancel{6}\sqrt{3}} = \frac{2}{3\sqrt{3}}$$

2 $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}$

$\lim_{y\to 0}\frac{(\sqrt{a+y})^2-(\sqrt{a})^2}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\quad\boxed{(a+b)(a-b)=a^2-b^2}$

$\lim_{y\to 0}\frac{a+y-a}{(y\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\times(\sqrt{a+y}+\sqrt{a})}$

$=\frac{1}{\sqrt{a+0}\times(\sqrt{a+0}+\sqrt{a})}=\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})}=\frac{1}{2a}$

$=1/2a$ //

3 $\lim_{x\to\frac{\pi}{6}}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

by solving by substitution method

$x\to\pi/6 \quad h\to 0$

$\therefore x-\pi/6\to 0 \quad \therefore h\to 0$

$\therefore x-\frac{\pi}{6}=h\to x=h+\pi/6$

$=\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}(\sin(h+\pi/6))}{\pi-6(h+\frac{\pi}{6})}$

$=\lim_{h\to 0}\frac{(\cos h\cdot\cos(\frac{\pi}{6}))-(\sin h\sin(\pi/6))-\sqrt{3}(\sin h\cdot\cos\frac{\pi}{6}+\cos h\sin\frac{\pi}{6})}{\pi-6h-\pi}$

$=\frac{1}{6}\lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\cos h-\frac{\sin h}{2}-\sqrt{3}(\frac{\sqrt{3}}{2}\sin h+\frac{\cos h}{2})}{h}$

$=-\frac{1}{6}\lim_{h\to 0}\frac{\cancel{\frac{\sqrt{3}}{2}\cos h}-\frac{\sin h}{2}-\frac{3}{2}\sin h-\cancel{\frac{\sqrt{3}}{2}\cos h}}{h}$

$=-\frac{1}{6}\lim_{h\to 0}\frac{-\frac{4\sin h}{2}}{h}=+\frac{2}{6}\lim_{h\to 0}\frac{\sin h}{h}$

$=\frac{1}{3}$ //

4) $\lim_{x\to\infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+3} + \sqrt{x^2-1}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$\lim_{x\to\infty} \frac{(\sqrt{x^2+5})^2 - (\sqrt{x^2-3})^2}{(\sqrt{x^2+3})^2 - (\sqrt{x^2-1})^2} \times \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$\lim_{x\to\infty} \frac{x^2+5 - x^2+3}{x^2+3 - x^2 - 1} \times \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+5} + \sqrt{x^2+3}}$$

$$\lim_{x\to\infty} \frac{\cancel{8}}{\cancel{2}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+5} + \sqrt{x^2+3}}$$

$$4 \times \lim_{x\to\infty} \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+5} + \sqrt{x^2+3}}$$

$$4 \times \lim_{x\to\infty} \frac{\sqrt{1 + 3/x^2} + \sqrt{1 + 1/x^2}}{\sqrt{1 + 5/x^2} + \sqrt{1 + 3/x^2}}$$

$$4 \times \frac{\sqrt{1} + \sqrt{1}}{\sqrt{1} + \sqrt{1}}$$

$$= 4 \times \frac{2}{2}$$

$$= 4 \times 1 = 4$$

5) $F(x) = \dfrac{\sin 2x}{\sqrt{1 - \cos 2x}}$ for $0 < x < \frac{\pi}{2}$

$\quad = \dfrac{\cos x}{\pi - 2x}$ $\quad \frac{\pi}{2} \le x < \pi$

→ to check the continuity at $x = \frac{\pi}{2}$ for the function at $x = \pi/2$

LHL = RHL

LHL → $\lim_{x\to \frac{\pi}{2}^+} \dfrac{\cos x}{\pi - 2x}$

$x - \pi/2 = h$

$h + \pi/2 = x$

$h \to 0 \;\therefore\; x \to 0$

$$\lim_{h\to 0^+} \frac{\cos h + \pi/2}{\pi - 2h - \pi} = \lim_{h\to 0} \frac{+\sin h}{+2h} = \frac{1}{2}$$

RHL → $\lim_{x\to\frac{\pi}{2}^-} \dfrac{\sin 2x}{\sqrt{1 - \cos 2x}} = \lim_{x\to\frac{\pi}{2}} \dfrac{\sin 2x}{\sqrt{2\sin^2 x}} = \dfrac{1}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}} \dfrac{\sin 2x}{\sin x}$

$$= \frac{1}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}} \frac{2\sin x \cdot \cos x}{\sin x} = \frac{2}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}} \cos x$$

$$= \frac{2}{\sqrt{2}} \times 0 = 0$$

LHL ≠ RHL

the function is not continous

4) ii $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x < 6$

$= \frac{x^2-9}{x-3}$ $6 \le x < 9$

for $f(x) = 3$ the check the continuity at $x=3$

LHL = RHL

LHL $= \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^-} \frac{(x+3)(x-3)}{(x-3)}$

$= \lim_{x\to 3^-} x+3 = 3+3 = 6$

LHL = 6

RHL $= \lim_{x\to 3^+} f(x) = \lim_{x\to 3} x+3 = 3+3 = 6$

LHL = RHL

$\therefore$ the function is continous at $x=3$

for $f(x) = 6$ to check the continous at $x=6$

LHL $\to \lim_{x\to 6^-} f(x) = \lim_{x\to 6}(x+3) = 6+3 = 9$

LHL = 9

RHL $\Rightarrow \lim_{x\to 6^+} f(x) = \lim_{x\to 6} \frac{x^2-9}{x-3} = \frac{36-9}{6-9}$

$= \frac{27}{3} = 9$

RHL = 9

LHL = RHL

the function continous at $x=9$

6.) i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$

$= k$ $x = 0$ $\Big\} - x=0$

$\to$ Since $f(x)$ is continous at $x=0$ as given

so $f(x) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos 4x}{x^2}$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2}$

$k = \lim_{x\to 0} 2\left(\frac{\sin 2x}{(2x)}\right)^2 \times 4 = 8 \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2$

$k = 8 \times (1)^2$

$k = 8$

ii $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \ne 0$ at $x=0$

$= k$ $x = 0$

Since the function is continous at $x=0$

$f(x) = \lim_{x\to 0} f(x) = \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$= (1+\tan^2 h)^{1/\tan^2 x}$

$= e^{1}$ $\quad (1+x)^{1/x} = e$

$k = e$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \frac{\pi}{3}$ $\Big\}$ at $x = \pi/3$

$= k \quad x = \frac{\pi}{3}$

Since $f(x)$ is continous function at $x = \pi/3$

$f(x) = \lim\limits_{x \to \frac{\pi}{3}} f(x) = \lim\limits_{x \to \frac{\pi}{3}} \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$

$\therefore h \to 0 \therefore x \to \frac{\pi}{3}$

$\therefore h \to 0 \quad \therefore x - \frac{\pi}{3} \to 0$

$h = x - \frac{\pi}{3} \quad \therefore x = h + \frac{\pi}{3}$

$\therefore k = \lim\limits_{h \to 0} \dfrac{\sqrt{3} - \tan(h + \frac{\pi}{3})}{\pi - 3(h + \frac{\pi}{3})}$

$= \lim\limits_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tanh + \tan(\pi/3)}{1 - \tanh \cdot \tan(\pi/3)}}{-3h}$

$= \lim\limits_{h \to 0} \dfrac{\sqrt{3} - \left(\dfrac{\tanh + \sqrt{3}}{1 - \sqrt{3}\tanh}\right)}{-3h} = -\dfrac{1}{3} \lim\limits_{h \to 0} \dfrac{\sqrt{3} - \left(\dfrac{\tan h}{1 - \sqrt{3}}\right)}{h}$

$= \lim\limits_{h \to 0} \sqrt{3}$

Q4

9 $f(x) = \dfrac{1 - \cos 3x}{x \tan x} \quad x \neq 0$

$= 9 \quad x = 0$

$\Rightarrow$ First to check the continuity of at $x \to 0$

$f(0) = 9$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{1 - \cos 3x}{x \tan x} = \lim\limits_{x \to 0} \dfrac{2 \sin^2 \frac{3x}{2}}{x \tan x}$

$= 2 \lim\limits_{x \to 0} \dfrac{(\sin \frac{3x}{2})^2}{x \tan x} = 2 \lim\limits_{x \to 0} \dfrac{\sin^2 (3x/2)}{x^2 \frac{\tan x}{x}}$

$= 2 \lim\limits_{x \to 0} \dfrac{(\sin \frac{3x}{2})^2}{\frac{9x^2}{4} \frac{\tan x}{x}} \times \dfrac{9}{4} = \dfrac{2 \times 9}{2 \times 4} \lim\limits_{x \to 0} \dfrac{\left(\dfrac{\sin \frac{3x}{2}}{\frac{3x}{2}}\right)^2}{\left(\dfrac{\tan x}{x}\right)}$

$= \dfrac{9}{2} \times \dfrac{1^2}{1} = \dfrac{9}{2} \neq 9 \neq f(0)$

f is not continous

So it is removable discontinous

to redefine the function

$f(x) = \dfrac{1 - \cos 3x}{x \tan x} \quad x \neq 0$

$= 9/2 \quad x = 0$

ii $f(x) = \frac{(e^{3x}-1) \times \sin x^{\circ}}{x^2}$ $x \neq 0$

$= \pi/60$ $x = 0$

+ the function at $x=0$ is
$f(x) = \pi/60$

to function to be continous.
$f(x) = \lim_{x \to 0} f(x)$

$\lim_{x \to 0} f(x) = \frac{(e^{3x}-1) \times \sin x^{\circ}}{x^2}$

$= \lim_{x \to 0} \frac{(e^{3x}-1)}{x^2} \sin x \times \frac{\pi}{180}$

$= \lim_{x \to 0} 3 \times \frac{(e^{3x}-1)}{3x} \lim_{x \to 0} \frac{\sin x}{x} \times \frac{\pi}{180}$

$= 3(1) \times \frac{\pi}{180} = \frac{3\pi}{180} = \frac{\pi}{60}$

$\lim_{x \to 0} f(x) = \frac{\pi}{60} = f(0)$

$\therefore$ the function is continous

3) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ at $x \neq 0$

as the function is continous at $x = 0$

$f(0) = \lim_{x \to 0} f(x)$

So to find the value at $x = 0 \Rightarrow f(0)$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = \lim_{x \to 0} \frac{e^{x^2} - 1 + 1 - \cos x}{x^2}$

$= \lim_{x \to 0} \left(\frac{e^{x^2}-1}{x^2}\right) + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$= \lim_{x \to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$= \lim_{x \to 0} 1 + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{4x^2/4}\right)^2$

$= 1 + \frac{2}{4} \lim_{x \to 0} \left(\frac{\sin x/2}{x/2}\right)^2$

$= 1 + \frac{1}{2}$

$= 3/2 = f(0)$

$f(0) = 3/2$ //

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9 $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

As the function is continous at $x = \pi/2$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to \pi/2} f(x) = \lim_{x \to 0} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$

$= \lim_{x \to \pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{(\sqrt{2})^2 - (\sqrt{1+\sin x})^2}{\cos^2 x \times \sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{2 - 1 - \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1 - \sin x}{(1 - \sin^2 x)} \times \dfrac{1}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{\cancel{(1-\sin x)}}{(1+\sin x) \times \cancel{(1-\sin x)}} \times \lim_{x \to 0} \dfrac{1}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{1}{1+\sin x} \times \lim_{x \to 0} \dfrac{1}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \dfrac{1}{1+\sin\theta} \times \dfrac{1}{\sqrt{2} + \sqrt{1+\sin \pi/2}}$

$\dfrac{1}{2} \times \dfrac{1}{\sqrt{2}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}}$

Practical No. 2

- Topic : Differentiablity

Q1 Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

Q2 If $f(x) = 4x + 1$ $x \le 2$
$= x^2 + 5$ $x > 2$ at $x = 2$
then f is differentiable or not

Q3 If $f(x) = 4x + 7$ $x < 3$
$= x^2 + 3x + 1$ $x \geq 3$ at $x = 3$
then find f is differentiable or Not

Q4 $f(x) = 8x - 5$ $x \le 2$
$= 3x^2 - 4x + 7$ $x > 2$ at $x = 2$
then find f is differentiable or not

P.T.O

Answer

Q1

i $f(x) = \cot x$ $\quad DF(a) = \frac{F(x) - F(a)}{x - a}$

$$DF(a) = \lim_{x\to a} \frac{\cot x - \cot a}{x - a}$$

Using substituting ~~at~~ $x = a + h$;

$h = x - a$

$x \to 0$ & $h \to 0$

$$DF(a) = \lim_{h\to 0} \frac{\cot(a+h) - \cot a}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cos(a+h)}{\sin(a+h)} - \frac{\cos a}{\sin a}}{h}$$

$$\lim_{h\to 0} \frac{\cos(a+h).\sin a - \cos a.\sin(a+h)}{\sin(a+h).\sin a . h}$$

$$\lim_{h\to 0} \frac{\sin(a-h-a)}{h.\sin(a+h).\sin h} = \frac{-\sin h}{\sin(a+h).\sin h.h}$$

$$\lim_{h\to 0} -\frac{\sin h}{h} \times \lim_{h\to 0} \frac{1}{\sin(a+h)\sin a}$$

$$-1 \times \frac{1}{\sin a . \sin a}$$

$$= \frac{-1}{\sin^2 a} = \operatorname{cosec}^2 a$$

function is differentiable at R to R

ii $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) : \lim_{x\to a} \frac{(f(x) - f(a))}{x - a}$$

$$f(x) = Df(a) = \lim_{x\to a} \frac{\operatorname{cosec}(a+h) - \operatorname{cosec} a}{a+h}$$

$x - a = h$ $\quad x \to a$

$x = a + h$ $\quad h \to 0$

$$= \lim_{h\to 0} \frac{\operatorname{cosec}(a+h) - \operatorname{cosec} a}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sin(a+h)} - \frac{1}{\sin a}}{h} =$$

$$= \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{h . \sin(a+h) . \sin a}$$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{a+h+a}{2}\right)\cdot \sin\left(-\frac{h}{2}\right)}{\sin(h+a)\cdot \sin a \times h}$$

$$= 2\lim_{h\to 0} \frac{\cos(a+h/2)}{\sin(a+h)\sin a} \times -\frac{1}{2}\lim_{2h\to 0} \frac{\sin(-h/2)}{(-h/2)}$$

$$= -\frac{2}{2}\lim_{h\to 0} \frac{\cos(a+h/2)}{\sin(a+h)\cdot \sin a} = -\frac{1}{2}\,\frac{\cos a}{\sin a\cdot \sin a}$$

$$= -\cot a\cdot \operatorname{cosec} a$$

f is differentiable at ~~x = 2~~ R to R

iii $\sec(x)$

$$f(x) = \sec x$$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$x - a = h \quad x \to a$

$x = h + a \quad h \to 0$

$$= \lim_{h\to 0} \frac{\sec(a+h) - \sec a}{h} = \lim_{h\to 0} \frac{\frac{1}{\cos(a+h)} - \frac{1}{\cos a}}{h}$$

$$\lim_{h\to 0} \frac{\cos a - \cos(a+h)}{\cos(a+h)\cdot \cos a\cdot h}$$

$$\lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\cdot \sin\left(-\frac{h}{2}\right)}{\cos(a+h)\cdot \cos a\cdot h} = \lim_{h\to 0} \frac{2\sin\left(a+\frac{h}{2}\right)\cdot \sin\left(-\frac{h}{2}\right)}{\cos(a+h)\cos(a)\cdot h}$$

$$= 2\lim_{h\to 0} \frac{\sin(a+h/2)}{\cos(a+h)\cdot \cos a} \cdot -\frac{1}{2}\lim_{h\to 0} \frac{\sin(-h/2)}{-h/2}$$

$$= \frac{\sin a}{\cos a \times \cos a} \times 1$$

$$= \tan a\cdot \sec a$$

$$= \tan a\cdot \sec a$$

the function is differentiable at R to R

2 $f(x) = 4x + 1 \quad x \le 2$
$\quad = x^2 + 5 \quad x > 2$ } at $x = 2$

to check the differentiablity of $f(x)$ the LHD = RHD

$$RHD = df(x^+) = df(2^+) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to 2} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x\to 2} \frac{x^2+5-9}{x-2} = \lim_{x\to 2} \frac{(x^2-4)}{x-2}$$

$$= \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} x+2$$

$$RHD = 4$$

$LHD = Df(a^-) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2} \frac{4x+1-9}{x-2}$

$= \lim_{x \to 2} \frac{(4x-8)}{x-2} = 4\lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{x-2}}$

$= 4$

LHD = RHD

the function is differentiable at $x=2$

$f(x) = \begin{cases} 4x+7 & x<3 \\ x^2+3x+1 & x \geq 3 \end{cases}$ at $x = 3$

to check differentiablity function should LHD = RHD

$RHD = Df(x^+) = Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3} = \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{\cancel{(x-3)}(x+6)}{\cancel{(x-3)}} = \lim_{x \to 3^+} x+6$

$= 9$

$LHD = Df(a^-) = Df(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(3^-)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+1-13}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3}$

$= 4 \lim_{x \to 3^-} \frac{\cancel{x-3}}{\cancel{x-3}}$

$= 4$

LHD ≠ RHD

the function is not differentiable at $x=3$

4.) $f(x) = \begin{cases} 8x-5 & x \leq 2 \\ 3x^2-4x+7 & x>2 \end{cases}$ at $x=2$

to check differentiablity at $x=2$ the function should LHD=RHD

$RHD = Df(x^+) = Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2} = \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2} = \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$$\lim_{x\to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2^+} 3x+2$$

$$= 6+2 = 8$$

$$LHD = Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$\lim_{x\to 2^-} \frac{(8x-5-11)}{(x-2)} = \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$\lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}} = 8$$

LHD = RHD

the function is differentiable

6/12/17

Practical NO. 3

1 Topic : Application of derivative

Q Find the interval in which function is increa[sing] or decresing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q Find the interval in which function concave-u[p]ward or concave-downward

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

* Answers

1. $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

for interval of an increasing function

$f'(x) > 0$

$3x^2 - 5 > 0$

$(\sqrt{3}x + \sqrt{5})(\sqrt{3}x - \sqrt{5}) > 0$

$+ \quad - \quad +$

$-\frac{\sqrt{5}}{\sqrt{3}} \qquad \frac{\sqrt{5}}{\sqrt{3}}$

$x \in \left(-\infty, -\frac{\sqrt{5}}{\sqrt{3}}\right) \cup \left(\frac{\sqrt{5}}{\sqrt{3}}, \infty\right)$

For interval when f is decreasing

$f'(x) < 0$

$3x^2 - 5 < 0$

$(\sqrt{3}x - \sqrt{5})(\sqrt{3}x + \sqrt{5}) < 0$

$+ \quad - \quad +$

$-\frac{\sqrt{5}}{\sqrt{3}} \qquad \frac{\sqrt{5}}{\sqrt{3}}$

$x \in \left(-\frac{\sqrt{5}}{\sqrt{3}}, \frac{\sqrt{5}}{\sqrt{3}}\right)$ is decreasing function

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

for interval of f is increasing function

$f'(x) > 0$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$-\infty \qquad 2 \qquad \infty$

f is increasing when $x \in (2, \infty)$

for f in a decreasing function

$f'(x) < 0$

$2x - 4 < 0$

$x - 2 < 0$

where $x \in (-\infty, 2)$ where function is decreasing

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

for f when it is a increasing function

$f'(x) > 0$

$6x^2 - 2x - 20 > 0$

$3x^2 - x - 10 > 0$

$3x^2 - 6x + 5x - 10 > 0$

$3x(x-2) + 5(x-2) > 0$

$(3x+5)(x-2) > 0$

+ $-\frac{5}{3}$ − 2 +

$x \in (-\infty, -\frac{5}{3}) \cup (2, \infty)$

for f when its decresing function

$f'(x) < 0$

$6x^2 - 2x - 20 < 0$

$3x^2 - x - 10 < 0$

$3x(x-2) + 5(x-2) < 0$

$(3x+5) \cdot (x-2) < 0$

where $x \in (-\frac{5}{3}, 2)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

when f is an incresing function

$f'(x) > 0$

$3x^2 - 27 > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

+ -3 − 3 +

$\therefore$ for incresing function $x \in (-\infty, -3) \cup (3, \infty)$

when f is an decresing function

$f'(x) < 0$

$3x^2 - 27 < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

-3 3

for f is decresing when $f\, x \in (-3, 3)$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$
$f'(x) = -24 - 18x + 6x^2$
$f'(x) = 6x^2 - 18x - 24$

when f is an incresing function
$f'(x) > 0$
$6x^2 - 18x - 24 > 0$
$x^2 - 3x - 4 > 0$
$x^2 - 4x + x - 4 > 0$
$x(x-4) + 1(x-4) > 0$
$(x+1)(x-4) > 0$

$+ \quad - \quad +$
$-1 \quad 4$

f is incresing when $x \in (-\infty, -1) \cup (4, \infty)$

when f is an decresing function
$f(x) < 0$
$6x^2 - 18x - 24 < 0$
$x^2 - 3x - 4 < 0$
$x^2 - 4x + x - 4 < 0$
$x(x-4) + 1(x-4) < 0$
$(x+1)(x-4) < 0$

f is ~~decresing~~ decresing when $x \in (-1, 4)$

Q2
i) $y = 3x^2 - 2x^3$
let $f(x) = 3x^2 - 2x^3$
$f'(x) = 6x - 6x^2$
$f''(x) = 6 - 12x$

To find inter-val of concave upward
the $f''(x) > 0$
$-12x + 6 > 0$
$-2x + 1 > 0$
$x > 1/2$
$\therefore x \in (1/2, \infty)$

• for concave downward
$f''(x) < 0$
$-12x + 6 < 0$
$x < \frac{1}{2}$

$x \in (-\infty, \frac{1}{2})$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
let $f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$
$f'(x) = 4x^3 - 18x^2 + 24x + 5$
$f''(x) = 12x^2 - 36x + 24$

To find interval of concave upward
$f''(x) > 0$
$12x^2 - 36x + 24 > 0$
$x^2 - 3x + 2 > 0$

or $x^2-2x-x+2>0$
$x(x-2)-1(x-2)>0$
$(x-1)(x-2)>0$
$(x-1)(x-2)>0$

$+ \quad - \quad +$ (1, 2)

The interval in Concave upward
$x \in (-\infty, 1) \cup (2, \infty)$

For con cave down ward
$f''(x)<0$
$x^2-2x-x+2<0$
$x(x-2)-1(x-2)<0$
$(x-1)(x-2)<0$

$+ \quad - \quad +$ (1, 2)

the interval in which ~~it~~ to concave down war
$x \in (1, 2)$

Q iii $y = x^3-27x+5$
let $f(x) = x^3-27x+5$
$f'(x) = 3x^2-27$
$f''(x) = 6x$

For con Cave upward ~~of~~ the interval will be $f''(x)>0$
$6x>0$
$x>0$

$x \in (0, \infty)$ in concave upward

for concave downward
$f''(x)<0$
$6x<0$
$x<0$

$x \in (-\infty, 0)$ for concave downward

iv $y = 69-24x-9x^2+2x^3$
let $f(x) = 69-24x-9x^2+2x^3$
$f'(x) = -24-18x+6x^2$
$f''(x) = -18+12x$

to find interval of concave up ward
$f''(x)>0$
$12x-18>0$
$12x>18$
$x>\frac{3}{2}$

$x \in (\frac{3}{2}, \infty)$ for concave upward

for interval of concave downward
$f''(x)<0$
$12x-18<0$
$x<\frac{3}{2}$

$x \in (-\infty, \frac{3}{2})$ for concave downward

v) $y = 2x^3 + x^2 - 20x + 4$

Let $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

to find interval of concave upward

$f''(x) > 0$

$12x + 2 > 0$

$x > \frac{1}{6}$

$\therefore x \in \left(\frac{1}{6}, \infty\right)$ for concave upward

& for concave downward

$f''(x) < 0$

$12x + 2 < 0$

$x < \frac{-1}{6}$

$\therefore x \in \left(-\infty, -\frac{1}{6}\right)$ for concave downward

Practical No. 4

Q.1.) Find maximum & minimum value of following function

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[2, 3]$

Q2) Find the root of following equations by newton's method

[Take four iteration only] & correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ at $(x_0 = 0)$

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

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i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 2 \times \frac{16}{x^3}$

$f'(x) = 2x - \frac{32}{x^3}$

$\therefore f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x^4 - 32 = 0$

$x^4 - 16 = 0$

$(x^2 - (4)^2)(x^2 + (4)^2) = 0$

$x^2 - (4) = 0 \quad ; x^2 + \cancel{16} = 0$

$x^2 = \cancel{16}\,4 \qquad x^2 = \cancel{16}\,4$

$x = \pm 2 \qquad x = \cancel{\pm}\,2$

$\cancel{x = \pm 4}$

$f''(x) = 2 + \frac{32}{x^4} \times 3$

$= 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4} = 2 + \frac{96}{16} \cdot 6$

$= 2 + 6$

$= 8 > 0$

has minimum value at $x = 2$

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$f(2) = 2^2 + 16/2^2 = 4 + 4 = 8$

The maximum value is 8

$f''(-2) = 2 + 96/x^4 = 2 + \frac{96}{16} = 8 > 0$

There is no minimum value at ~~x~~ because x is an even function at $f''(x)$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

$f'(x) = 15x^4 - 15x^2$

$f'(x) = 0$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$x = 0$ or $x = \pm 1$

$f''(x) = 60x^3 - 30x$

$f''(1) = 60 - 30 = 30 > 0$

$f''(1)$ has maximum value at $x = 1$

$f(1) = 3 - 5 + 3 = 6 - 5$

$= 1$

maximum value is 1

$f''(-1) = 60x^3 - 30x = 60(-1)^3 - 30(-1)$

$= -30$

f has ~~max~~ minimum value at $x = -1$

$f(1) = 3 - 5(-1)^3 + 3(-1)^5 = 3 + 5 - 3$

$= 5$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

Consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6\times(0) - 6 = -6 < 0$

f has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1$

$f(0) = 1$

The value of f at maxima is 1

$f''(2) = 6(2) - 6 = 12 - 6 = 6 > 0$

f has minimum value at $x = 2$

$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = 9 - 12$

$= -3$,,

f has ~~maxim~~ minimum value at $x = 2$

The value of f is -3 at minima

iv] $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

$x = -1$ & $x = 2$

$f''(x) = 12x - 6$

~~$f''(x)$~~ $f''(x) = 12x - 6$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f as ~~maximum~~ maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 13 - 5 = 8$

f as minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

Q2

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

by newton method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{0+1} = x_0 - \frac{f(x_n)}{f'(x_n)} = 0 - \frac{9.5}{-55} = 0.1727$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.1727 - \frac{0.0829}{-55.9467} = 0.1727 + \frac{0.0}{55}$

$x_2 = 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3\times(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -559393$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.1712 + \frac{0.0011}{559393}$

$= 0.1712$

the root of the Equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

$f'(x) = 3x^2 - 4$

$\therefore$ $f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9$

$f(2) = -9$

$\therefore$ $f(3) = 3^3 - 4(3) - 9 = 27 - 12 - 9 = 6$

$f(3) = 6$

let $x_0 = 3$ be initial approximation

$\therefore$ by newton method

$x_{n+1} = x_n - \frac{f x_n}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{6}{23} = 2.7392$

$x_1 = 2.7392$

- $f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$f(x_1) = 0.596$

- $f'(x_1) = 3\times(2.7393)^2 - 4 = 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7392 - \frac{0.5960}{18.5096}$

$X_2 = 2.7071$

Scanned with CamScanner

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$f(x_2) = 0.0102$

• $f'(x_2) = 3(2.7071)^2 - 4 = 17.9851$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7071 - \frac{0.0102}{17.9851} = 2.7016$

$x_3 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9 = 19.7158 - 10.806 -$

$f(x_3) = -0.0901$

$f'(x_3) = 3(2.7015) - 4 = 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943} = 2.7015 + 0.0050$

$x_4 = 2.7065$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

• $f(1) = (1)^3 - 1.8(1)^2 - 10 \times 1 + 17 = 1 - 1.8 - 10 + 17 = 6.$

$\therefore f(1) = 6.2$

• $f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17 = 8 - 7.2 - 20 + 17 = -2$

$\therefore f(2) = -2.2$

Let $x_0 = 2$ be initial approximation

$x_{n+1} = x_n + \frac{f(x_n)}{f'x_n} \therefore x_1 = x_0 + \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2}{5}$

$x_1 = 2 - 0.4230$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577) - 10(1.577) - 10$

$= 3.9219 - 4.4764 - 15.77 + 17$

$f(x_1) = 0.6755$

$f'(x_1) = 3(1.577)^2 - 5.6772 - 10 = -8.2614$

$f'(x_1) = -8.2614$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$x_2 = 1.6592$

$f(x_2) = (1.6592)^3 - (1.8)(1.6592) - 10(1.6592) + 17 = 4.5672 - 4.9553 - 16.5 +$

$f(x_2) = 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10 = 8.2588 - 5.9731 - 10$

$f'(x_2) = -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.0204}{7.7143} = 1.6592 + 0.0026$

$x_3 = 1.6618$

$f(x_3) = [1.6618]^3 - (1.8)(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$f(x_3) = 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10 = 8.2847 - 5.9824 - 10$

$f'(x_3) = -7.6477$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.8618 + \frac{0.0004}{7.6477}$

$x_4 = 1.6618$

root of Equation 1.6618

Practical No. 5

Topic: integration

Q Solve the following Integration

i] $\int \frac{dx}{\sqrt{x^2+2x-3}}$

ii] $\int (4e^{3x}+1)\,dx$

iii] $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv] $\int \frac{x^3+3x+4}{\sqrt{x}} \cdot dx$

v] $\int t^4 \cdot \sin(2t^4) \cdot dt$

vi] $\sqrt{x} \cdot (x^2-1)\,dx$

vii] $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) \cdot dx$

viii] $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x} \cdot \sin 2x\,dx$

x $\int \frac{(x^2-2x)\,dx}{x^3-3x^2+1}$

Answer

1 $\int \frac{dx}{\sqrt{x^2+2x-3}} = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

$\therefore \int \frac{dx}{x^2+a^2} = \log|x+\sqrt{x^2+a^2}|$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

$= \log|x+1+\sqrt{(x+1)^2-(2)^2}| + c$

iii $\int (4e^{3x}+1)\, dx$

$= \int 4e^{3x}. dx + \int dx$

$= 4\int e^{3x}. dx + \int dx$

$= \frac{4}{3} e^{3x} + x + c$

ii $\int (2x^2 - 3\sin x + 5\sqrt{x}) dx = \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= \int 2x^2. dx - \int 3\sin x\, dx + \int 5\sqrt{x}\, dx$

$= 2\frac{x^3}{3} \cancel{+} 3[-\cos x] + \int 5x^{1/2}\, dx$

$= \frac{2}{3}x^3 + 3\cos x + 5 \times \frac{x^{3/2}}{3/2}$

$= \cancel{\frac{2x^3}{3}} + 3\cos x + 10\frac{x^{3/2}}{3} + c$

iv $\int \frac{x^3+3x+4}{\sqrt{x}} . dx$

$= \int \frac{x^3}{\sqrt{x}}. dx + \int \frac{3x}{\sqrt{x}} dx + 4\int \frac{1}{\sqrt{x}} . dx$

$= \int x^{3-\frac{1}{2}} dx + 3\int x^{1-1/2}. dx + 4\int x^{-1/2}. dx$

$= \int x^{5/2}. dx + 3\int x^{1/2}. dx + 4\int x^{-1/2}. dx$

$= \left[\frac{x^{7/2}}{7/2}\right] + 3\left[\frac{x^{3/2}}{3/2}\right] + 4 \times \left[\frac{x^{1/2}}{1/2}\right]$

$= \frac{2x^{7/2}}{7} + \cancel{3}\frac{2x^{3/2}}{\cancel{3}} + 8x^{1/2} = \frac{2x^{7/2}}{7} + 2x^{3/2} + 8x^{1/2}$

$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8x^{1/2}$

v) $\int t^{7}. \sin(2t^4). dt =$

$t^4 = x$

$4t^3\, dt = dx$

$I = \frac{1}{4}\int 4t^3. t^4 . \sin(2t^4). dt$

Substituting the value

$= \frac{1}{4}\int x . \sin 2x . dt$

$= \frac{1}{4}\left[x\int \sin 2x - \int\left[\int \sin 2x . \frac{d(x)}{dx}\right]\right]$

$= \frac{1}{4}\left[-\frac{x\cos 2x}{2} + \int \frac{1}{2}\cos 2x\right]$

$= -\frac{1}{8}\left[\frac{x\cos 2x}{2} + \frac{1}{4}\sin 2x\right]$

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resubstitution the value

$= \frac{1}{8} t^4 \frac{\sin t^4}{2} + \frac{1}{16} \sin t^4$

vi $\int \sqrt{x}(x^2-1)\, dx$

$I = \int \sqrt{x} . x^2 - \sqrt{x} . dx$

$= \int (x^{2+1/2} - x^{1/2}) . dx$

$= \int (x^{5/2} - x^{1/2}) . dx$

$= \frac{2}{7} . x^{7/2} - \frac{2}{3} . x^{3/2} + c$

vii $\int \frac{1}{x^3} . \sin\left(\frac{1}{x^2}\right) . dx$

$\frac{1}{x^2} = t$

$-\frac{2}{x^3} dx = dt$

Substituting the value

$I = \frac{1}{-2} \int \frac{-2}{x^3} . \sin\left(\frac{1}{x^2}\right) . dx = \frac{-1}{2} \int \sin t . dt$

$= -\frac{1}{2} \times - \cos t$

$= \frac{\cos t}{2}$

resubstituting the value

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right)$

viii $\int \frac{\cos x}{\sqrt[3]{\sin x}} . dx = I$

let $\sin x = t$

$\cos x . dx = dt$

substituting the value

$I = \int \frac{dt}{\sqrt[3]{t^2}} = \int t^{-2/3} . dt$

$= \frac{t^{-2/3+1}}{-2/3+1} = \frac{t^{1/3}}{1/3}$

$= 3 (\sin x)^{1/3} = 3 \times \sqrt[3]{\sin x}$

ix. $\int e^{\cos^2 x} . \sin 2x . dx = I$

let $\cos^2 x = t$

substitution $- \sin 2x . dx = dt$

$\therefore I = -1 \int - \sin 2x . e^{\cos^2 x} dx$

$= - \int e^t . dt = - e^t$

resubstitution

$= - e^{\cos^2 x}$

$$x \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \cdot dx$$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \cdot dx$$

substituting $x^3 - 3x^2 + 1 = t$

$\cancel{(3x^2 - 6x)dx = t}$

$\cancel{3(x^2 - 2x) =}$

$(3x^2 - 6x) \cdot dx = dt$

$3(x^2 - 2x) \cdot dx = dt$

$$I = \frac{1}{3} \times \int \frac{3(x^2 - 2x)}{x^3 - 3x^2 + 1} \cdot dx$$

$$= \frac{1}{3} \times \int \frac{dt}{t} = \frac{\log|t|}{3}$$

resubstitution

$$= \frac{\log|x^3 - 3x^2 + 1|}{3}$$

03/01/2019

Practical NO. 6

Topic : Application of integration & numerical integration

Q1) Find length of following curve :

1) $x = \sin t$, $y = 1 - \cos t$ · $t \in [0, 2\pi]$

2) $y = \sqrt{4 - x^2}$. where $x \in [-2, 2]$

3) $y = x^{3/2}$ in $x \in [0, 4]$

4) $x = \sin t$, $y = 3\cos t$ where $t \in [0, 2\pi]$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2) Using Simpose rule solve the following

1) $\int_0^0 e^{x^2} \cdot dx$ with $n = 4$

2) $\int_0^4 x^2 \cdot dx$ with $n = 4$

3) $\int_0^{\pi/3} \sqrt{\sin x} \cdot dx$ with $n = 6$

Q1

1] $x = t\sin t$, $y = 1-\cos t$ where $t \in [0, 2\pi]$

For length of curve $l = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \cdot dt$

$x = \sin t$ $\quad y = 1-\cos t$

$\frac{dx}{dt} = \frac{d}{dt}(\sin t)$, $\frac{dy}{dt} = \frac{d}{dt}(1-\cos t)$

$\frac{dx}{dt} = \cancel{\cos t}$ $1-\cos t$, $\frac{dy}{dt} = \sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt = \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t} \cdot dt$

$= \int_0^{2\pi} \sqrt{1} \cdot dt = [x]_0^{2\pi} = 2\pi$

length of the curve is $= 2\pi$

$y = \sqrt{4-x^2}$ where $x \in [-2, 2]$

For length of curve $l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\frac{dy}{dx} = \frac{d\sqrt{4-x^2}}{dx} = \frac{d}{dx}(4-x^2)^{1/2} = \frac{1 \times 2x}{2\sqrt{4-x^2}}$

$= \frac{x}{\sqrt{4-x^2}}$

$l = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_{-2}^{2} \sqrt{1 + \left(\frac{x}{\sqrt{4-x^2}}\right)} \cdot dx$

$\int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}} \cdot dx = \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{(4-x^2)}} \cdot dx$

$2\int_{-2}^{2} \frac{dx}{\sqrt{(4-x^2)}} = 2\int_{-2}^{2} \frac{dx}{\sqrt{(2)^2 - x^2}}$

$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2} = \left[\sin^{-1}\left(\frac{2}{2}\right) - \sin^{-1}\left(\frac{-2}{2}\right)\right]$

$= \sin^{-1}(1) - \sin^{-1}(-1) = 2\left(\frac{\pi}{2} - \frac{\pi}{2}\right)$

$= 2\pi$

Q3 $y = x^{3/2}$ where $[0, 4]$

For length of curve $= l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\frac{dy}{dx} = \frac{3}{2} \times x^{1/2}$

$\therefore\ l = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \int_0^4 \sqrt{1 + \left(\frac{3}{2}(x)^{1/2}\right)^2} \cdot dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4}x} = \frac{1}{2}\int_0^4 \sqrt{(4 + 9x)}$

$= \frac{1}{2} \times \frac{1}{9}\left[\frac{(4+9x)^{3/2}}{3/2}\right]_0^4 = \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{2}\left[(4+9(4))^{3/2} - (4+9(0))^{3/2}\right] = \frac{1}{27}\left(\sqrt{(40)^3} - \sqrt{(4)^3}\right)$

$\frac{1}{27}\left(\sqrt{64000} - \sqrt{64}\right) = \frac{1}{27}\left(40\sqrt{40} - 8\right)$

$\frac{8}{27}\left(5\sqrt{40} - 1\right) = \frac{\left(10\sqrt{10} - 1\right)}{27} \times 8$

length of Curve is $= \frac{80\sqrt{10}}{27} - \frac{8}{27}$

4 $x = 3\sin t$, $y = 3\cos t$ when $t \in [0, 2\pi]$

For length of curve $= \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \cdot dt$

$x = 3\sin t$ $\quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\quad y = -3\sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \cdot dt = \int_0^{2\pi} \sqrt{(3\sin t)^2 + (3\cos t)^2}$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}$

$= \int_0^{2\pi} \sqrt{9}$

$= 6\pi$

5) $x = t\cdot\sin t$ $\quad y = 1-\cos t$

$\frac{dx}{dt} = \sin t - \cos t$. $\quad y = \sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \int_0^{2\pi} \sqrt{(\sin t - t\cdot\cos t)^2 + (\sin t)^2} = \int_0^{2\pi} \sqrt{\sin^2 t - 2\cdot\sin t\cdot\cos t + \sin^2 t}$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \cdot dt = \int_0^{2\pi} \sqrt{2 - 2\cos t} \cdot dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \int_0^{2\pi} \sqrt{2\cdot 2\sin^2 t/2}$

$= \int_0^{2\pi} \sqrt{4\cdot\sin^2 t/2} = \int_0^{2\pi} 2\cdot\sin t/2 = \left[-4\cos t/2\right]_0^{2\pi}$

$= -4\cos\pi - -4\cos 0 = 4 + 4 = 8$,

length of curve is 8

Q2

i) $\int_0^2 e^{x^2}\cdot dx$ with $n=4$

$n=4$, $a=0$, $b=2$, $\therefore h = \frac{b-a}{n} = \frac{2}{4} = 1/2$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$\int_0^2 e^{x^2}\cdot dx = \frac{0.5}{3}\left[(1+54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$

$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$

$= 1.1779$

2 $\int_0^4 x^2 . dx$

$a = 0 \quad b = 4 \quad n = 4$

$\therefore \quad h = \frac{a-b}{n} = \frac{4}{4} = 1$

x	0	1	2	3	4
Y	0	$\cancel{1}$	4	$\cancel{9}$	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 . dx = \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{1}{3}\left[16 + 40 + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2 . dx = 21.533$$

iii $\int_0^{\pi/3} \sqrt{\sin x} \, . dx$ where $n = 6$

$a = 0 \quad b = \pi/3 \quad n = 6$

$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
Y	0	0.4166	0.58	0.70	0.80	0.8727	0.98
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\cancel{\int_0^{\pi/3} \sqrt{\sin x} \approx \frac{\pi/18}{3} \times [12.1163}$

$$\int_0^{\pi/3} \sqrt{\sin x} \, . dx = h/3\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.9360 + 4[0.4167 + 0.7071) + 2(0.5848 + 8017\right.$$

$$= \pi/54 \times (1.3473 + 4(1.999) + 2(1.3865))$$

$$= \pi/54\,(1.3473 + 7.996 + 2.773)$$

$$\cancel{= \frac{\pi}{54} + 12.1163}$$

$$\int_0^{\pi/3} \sqrt{\sin x} \, . dx = 7049$$

Practical No. 7

Topic :- Differential Equation

Q1 Solve the following differential equation

i $x.\frac{dy}{dx} + y = e^x$

ii $e^x.\frac{dy}{dx} + 2e^x.y = 1$

iii $x.\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

iv $x.\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

v $e^{2x}.\frac{dy}{dx} + 2e^{2x}.y = 2x$

vi $\sec^2 x.\tan y.dx + \sec^2 y.\tan x.dy = 0$

7) $\frac{dy}{dx} = \sin(x-y+1)$

8) $\frac{dy}{dx} = \frac{2y + 2x + 3y - 1}{6x + 4y + 6}$

1) $x.\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

$P(x) = 1/x \qquad Q(x) = \frac{e^x}{x}$

$I.f = e^{\int 1/x \cdot dx} = e^{\ln x} = x$

$I.f = x$

$y.(I.f) = \int Q(x).(I.f).dx + c$

$y.x = \int \frac{e^x}{\cancel{x}}.\cancel{x}.dx + c$

$= \int e^x.dx$

$y.x = e^x + c$

ii $e^x.\frac{dy}{dx} + 2e^x.y = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$Q(x) = e^{-x} \qquad P(x) = 2$

$$IF = e^{\int 2 \cdot dx} = e^{2x}$$

$$IF = e^{2x}$$

$$y \cdot (I.F) = \int Q(x) \cdot (IF) \cdot dx$$

$$y \cdot e^{2x} = \int e^{-x} \cdot e^{2x} \cdot dx + c$$

$$y \cdot e^{2x} = \int e^{-x+2x} \cdot dx$$

$$y \cdot e^{2x} = \int e^{x} \cdot dx$$

$$y \cdot e^{2x} = e^{x} + c$$

iii) $x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$x \cdot \frac{dy}{dx} + 2y = \frac{\cos x}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$$Q(x) = \frac{\cos x}{x^2} \qquad P(x) = \frac{2}{x}$$

$$IF = e^{\int P(x) \cdot dx} = e^{\int \frac{2}{x} \cdot dx} = e^{2\ln x} = e^{\ln(x^2)}$$

$$IF = x^2$$

$$y \cdot (IF) = \int Q(x)\,(IF)\,dx$$

$$y \cdot x^2 = \int \frac{\cos x}{x^2} \cdot x^2 \cdot dx$$

$$y \cdot x^2 = \int \cos x \cdot dx$$

$$y \cdot x^2 = \sin x + c$$

iv) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$

$$P(x) = 3/x \qquad Q(x) = \frac{\sin x}{x^3}$$

$$I.f = e^{\int P(x) \cdot dx} = e^{\int \frac{3}{x} \cdot dx} = e^{3\ln x} = e^{\ln(x^3)}$$

$$I.f = x^3$$

$$y \cdot (I.f) = \int Q(x)(If) \cdot dx$$

$$y \cdot x^3 = \int \frac{\sin x}{x^3} \cdot x^3 \cdot dx$$

$$y \cdot x^3 = \int \sin x \cdot dx$$

$$\therefore \; y \cdot x^3 = -\cos x + c$$

$$e^{2x}.\frac{dy}{dx} + 2e^{2x}.y = 2x$$

$$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}}$

$$y.\ IF = e^{\int P(x).dx} = e^{\int 2.dx} = e^{2x}$$

$$y.(IF) = \int Q(x).(IF)\,dx$$

$$y.e^{2x} = \int \frac{2x}{e^{2x}}.e^{2x}.dx$$

$$y.e^{2x} = \int 2x.dx$$

$$y.e^{2x} = \cancel{2} \times \frac{x^2}{\cancel{2}} + C$$

$$y.e^{2x} = x^2 + C$$

$$\sec^2 x.\tan y.dx + \sec^2 y\tan x.dy = 0$$

$$\sec^2 x.\tan y.\,dx = -\sec^2 y.\tan x\,dy$$

$$\frac{\sec^2 x.\,dx}{\tan x} = -\frac{\sec^2 y}{\tan y}.dy$$

Integrating both side

$$\int \frac{\sec^2 x}{\tan x}.dx = -\int \frac{\sec^2 y}{\tan y}.dy$$

$$\log|\tan x| = -\log|\tan y| + C$$

$$\log|\tan x| + \log|\tan y| = c$$

$$\log|\tan x.\tan y| = c$$

$$\tan x.\tan y = e^{c}$$

vii] $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $(x-y+1) = v$

differentiation on both side

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

Substituting

$$1 - \frac{dv}{dx} = \sin^2 v$$

33. $\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\sec^2 v . dv = dx$

Integrating on Both Side

$\int \sec^2 v . dv = \int dx$

$\tan v = x + C$

$\tan(x-y+1) = x + C$

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$

$2x + 3y = v$

differentiating on both sid

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$ $\therefore$ $\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

Substituting

$\frac{1}{3} \times \left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} - 2 = \frac{v-1}{v+2}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$\frac{dv}{dx} = \frac{3v+3}{v+2}$

$\cancel{\frac{dv}{dx}}$ $\frac{3(v+1)}{v+2}$

$\int \frac{v+2}{v+1} dv = \int 3 dx$

~~v +~~ log|x+1|

$$\int \frac{v+1}{v}\,dv + \int \frac{1}{v+1} \cdot dv = 3x$$

$$v + \log|v| = 3x + c$$

$$2x + 3y + \log|2x+3y| = 3x + c$$

$$3y = x - \log|2x + 3y + 1|$$

AK 10/01/20

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Practical No. 8

Topic: Euler's Method

* Using Euler's method find the following:

1) $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 2$, $h = 0.2$ find $y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ for $h = 0.5$ & $h = 0.25$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ find $y(1.2)$ which $h = 0.2$

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* Answer

1) $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y_0 = 2$ $x_0 = 0$

$\therefore h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h.f(x_n, f_n)$

n	x_n	y_n	$f(x_n, f_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

By Euler's formula

$y(2) = 9.2831$

2. $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$ $y_0 = 0$ $x_0 = 2$ $h = 0.2$

using Euler iteration formula

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1605	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

By Euler's formula

$y(1) = 1.2942$

3 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$ $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2			
2	0.4			
3	0.6			
4	0.8			
5	1			

$\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2$ $x_0 = 1$

where $h = 0.5$

using Euler's iteration formula

$y_{n+1} = y_n + h.f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	~~1~~	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

by Euler's formula

$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's formula

$y(2) = 8.9048$

$\frac{dy}{dx} = \sqrt{x.y} + 2$ $y_0 = 1$, $x_0 = 1$ $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h.f(x_n, y_n)$

n	x	y	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	~~1.6~~		

by Euler formula

A+ 13/01/2020

$y(1.2) = 1.6$

Practical No. 9

Topic: Limits & Partial order derivative

Q1] Evaluate following limit

i] $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$ (ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii] $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

Q2] Find f_x, f_y for each following

i $f(x,y) = xye^{x^2+y^2}$ ii] $f(y,x) = e^x \cdot \cos y$

iii $f(x,y) = x^3 \cdot y^2 - 3x^2 \cdot y + y^3 + 1$

3] using defination find value f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

4] Find all second order partial derivative of f also verify $f_{xy} = f_{yx}$

[i] $f(x,y)$ $\frac{y^2-xy}{x^2}$ ii $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

iii $f(x,y) = \sin(x,y) + e^{x+y}$

] ~~Find all second order~~ Partial derivative of find linearization of $f(x,y)$ at given point

) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(x,y) = \log x + \log y$ at $(1,1)$

Q1

i $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

$x \to -4$; $y \to -1$

$\lim_{(x,y)\to(-4,-1)} \frac{(-4)^3 - 3\times(-1) + (-1)^2 - 1}{-4\times -1 + 5}$

$= \frac{-64+3+1-1}{4+5} = \frac{-61}{9}$

$= -\frac{61}{9}$ //

ii $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$x \to 2$ $y \to 0$

$= \frac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)} = \frac{1(4-8)}{2}$

$= -\frac{4}{2} = -2$

iii $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz} = \lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)\times\cancel{(x-yz)}}{x^2\cancel{(x-yz)}}$

$x \to 1$ $y \to 1$ $z \to 1$

$\lim_{(x,y,z)\to(1,1,1)} \frac{1+1\times1}{(1)^2} = 2$ //

$= 2$ /

Q2 $f(x,y) = x\,y\,e^{x^2+y^2}$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x \cdot y e^{x^2+y^2})$

$= e^x \cdot y \cdot 2x$

$f(x,y) = e^{x^2+y^2} \cdot y \cdot 2x$

$f_y = \frac{\partial (f(x,y))}{\partial y} = \frac{\partial}{\partial y}\, x \cdot y e^{x^2+y^2}$

$= x \cdot e^{x^2+y^2} \cdot 2y$

$f_y = 2yx e^{x^2+y^2}$

ii) $f(x,y) = e^x \cdot \cos y$

$f(x) = \frac{\partial f(x,y)}{\partial x} = \frac{\partial e^x \cdot \cos y}{\partial x} = e^x \cdot \cos$

$f(x) = e^x \cdot \cos y$

$f(y) = \frac{\partial f(x,y)}{\partial y} = \frac{\partial (e^x \cdot \cos y)}{\partial y} = -e^x \cdot \sin$

$f_y = -e^x \cdot \sin y$

$f(x,y) = x^3 \cdot y^2 - 3x^2 y + y^3 + 1$

$f(x) = \frac{\partial (f(x,y))}{\partial x} = \frac{\partial}{\partial x}(x^3 \cdot y^2 - 3x^2 \cdot y + y^3 + 1)$

$f(x) = 3x^2 \cdot y^2 - 6xy$

$f(y) = \frac{\partial (f(x,y))}{\partial y} = \frac{\partial}{\partial y}(x^3 \cdot y^2 - 3x^2 \cdot y + y^3 + 1)$

$f(y) = 2x^3 y - 3x^2 + 3y^2$

Q3

i $f(x,y) = \frac{2x}{1+y^2}$

$f(x) = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right) = \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{1+y^2}$

$\frac{(1+y^2) \times 2 - 2x \times 0}{\cancel{(1+y^2)^2}} = \frac{\cancel{(1+y^2)} \times 2}{\cancel{(1+y^2)}(1+y^2)}$

$f(x) = \frac{2}{1+y^2}$

$f(x)$ at 0,0 $= \frac{2}{1} = 2$ //

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right) = \frac{(1+y^2)\frac{\partial(2x)}{\partial y} - 2x \cdot \frac{\partial}{\partial y}(1+y^2)}{(1+y^2)^2}$$

$$\frac{1+y^2 \times 0 - 2x \cdot 2y}{1+y^2}$$

$$f(y) = \frac{4xy}{1+y^2}$$

$$f(y) \text{ at } (0,0) = \frac{4\times(0\times 0)}{1+0} = 0$$

Q4 i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f(x) = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{x^4}$$

$$= \frac{-x^2 \cdot y - (y^2 - xy)\cdot 2x}{x^4} = \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f(y) = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2xy^2 - 2x^2 y}{x^4}\right) = \frac{x^4 \frac{\partial}{\partial x}(-x^2 y - 2xy^2 - 2x^2 y) - x^2 y - 2xy^2 - 2x^2 y}{(x^4)^2}$$

$$\frac{x^4(2xy - 2y^2 + 4xy) - 4x^3(x^2 y - 2xy + 2x^2 y)}{x^{6}}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right) = \frac{2-0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial(xy)}\left(\frac{-x^2 y - 2xy + 2x^2 y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right) = \frac{x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x} x^2}{x^{2^2}}$$

$$= -x^2 - 4xy + 2x^2$$

$$\therefore f_{xy} = f_{yx}$$

ii) $f(x,y) = x^3 + 3x^2 y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial}{\partial x} x^3 + 3x^2 y^2 - \log(x^2+1)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{\partial}{\partial y} x^3 + 3x^2 y^2 - \log(x^2+1) = 6x^2 y$$

$$f_{xx} = 6x + 6y^2 - \frac{(x^2+1)\frac{\partial(2x)}{\partial x} - 2x\frac{\partial}{\partial x}(x^2+1)}{(x^2+1)^2}$$

$$= 6x + 6y^2 - \frac{2(x^2+1) - 4x^2}{(x^2+1)^2}$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2y) = 6x^2$$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right) = 12xy$$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2y) = 12xy$$

$\therefore$ $f_{xy} = f_{yx}$

$f(xy) = \sin(xy) + e^{x+y}$

$\therefore$ $f(x) = y\cos(xy) + e^{x+y}\cdot 1$; $f_y = x\cos(xy) + e^{x+y}$

$\cancel{f(x) = y\cdot\cos(xy) + e^{x+y}}$; $= x\cdot\cos(xy) + e^{x+y}$

$$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y}) = -y\sin(xy)\cdot y + e^{x+y}$$

$$= -y^2\sin(xy) + e^{x+y}$$

$$f_{yy} = \frac{\partial}{\partial y}(x\cos(x.y) + e^{x+y}) = -x^2\sin y + e^{x+y}$$

$$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y}) = -y^2\cdot\sin(xy) + \cos(xy) + e^{x+y}$$

$$f_{yx} = \frac{\partial}{\partial y}(x\cos(x.y) + \cancel{\cos(xy)})e^{x+y}$$

$$= -x^2\sin xy + \cos(xy) + e^{x+y}$$

$f(xy) \neq f(yx)$

Q5 $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{2}$

$f(x) = \frac{2x}{2\sqrt{x^2+y^2}}$ $\quad$ $f_y = \frac{2y}{2\sqrt{x^2+y^2}}$

$f(x) = \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$f(x)$ at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad$ $f(y) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f(y)(a,b)(y-b)$$

$$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}} = \sqrt{2} + \frac{(x-1+y-1)}{\sqrt{2}}$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$$

$= \sqrt{2} + \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}} y - \frac{x}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii $f(x,y) = 1 - x \cdot y \sin x$ at $(\frac{\pi}{2}, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \pi/2$

$f(x) = 0 - 1 + y \cos x$ $\quad f_y = 0 - 0 + \sin x$

$f(x)$ at $(\frac{\pi}{2}, 0) = -1 + 0$
$= -1$

f_y at $(\frac{\pi}{2}, 0) = \sin \frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(x-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + y$

$\cancel{= 1 - \frac{\pi}{2} - x + \pi/2} = 1 - x + y$

$= 1 - x + y$

iii $f(x,y) = \log x + \log y$
$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f_y(a,b)(y-b)$
$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

AR
24/01/2021

35

Practical NO. 10

1 Topic : Directional derivative, Gradient vector & maxima, minima, tangent & normal vector

Q1 Find the directional derivative of following at given point & direction of given vector

i $f(x,y) = 2x + 2y - 3$, $a = (1,-1)$, $u = 3i\ j$

ii $f(x,y) = y^2 - 4x - 3$, $a = (3,4)$, $u = i + 5j$

iii $f(x,y) = 2x + 3y$, $a = (1,2)$, $u = 3i + 4j$

Q2 Find gradient vector for following function at given point

i $f(x,y) = x^4 + y^x$, $a = (1,1)$

ii $f(x,y) = (\tan^{-1}x) \cdot y^2$ $a = (1,-1)$

iii $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

Q3 Find the equation of tangent & normal to each of following curve at of point

i $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

ii $x^2 + y^2 - 2x + 3y + 2$ at $(2,-2)$

Q4 Find equation of tangent and normal line of following surface

1) $x^2 + 2yz + 3y + xz = 7$ at $(2,1,0)$

2) $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q5 Find maxima & minima

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4z$

ii $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

iii $f(x,y) = 2x^4 + 3x^2y - y^2$

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Q1. ii

i $f(x,y) = x + 2y - 3$; at $a = (1,-1)$

here ~~unit~~ vector $u = 3\hat{i} - \hat{j}$

$|u| = \sqrt{9+1} = \sqrt{10}$

unit vector along $u = \frac{u}{|u|} = \frac{1}{\sqrt{10}}(3\hat{i} - \hat{j})$

$= \frac{3\hat{i}}{\sqrt{10}} - \frac{1}{\sqrt{10}}\hat{j}$

$u = \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)\right)$

$= f\left((1,-1) + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right)\right) = f\left(\left(1 + \frac{3h}{\sqrt{10}}\right), -\left(1 + \frac{h}{\sqrt{10}}\right)\right)$

$f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right) = 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3 = 1 + \frac{3h}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}}$

ii

$f(a+hu) = \frac{h}{\sqrt{10}} - 4$

a:

~~$f(a) = f(2,3)$~~

$f(a) = f(1,-1) = 1 + 2(-1) - 3 = -4$

$f'(a) = \lim_{h \to 0} \frac{f(a+bh) - f(a)}{h} = \frac{\frac{h}{\sqrt{10}} - 4 + 4}{h}$

$= \frac{1}{\sqrt{10}}$

2 $f(x,y) = y^2 - 4x - 3$ $\quad a = (3,4)$

$|u| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$ $\quad u = \hat{i} + 5\hat{j}$

unit vector along u is $= \frac{u}{|u|} = \frac{1}{\sqrt{26}}(\hat{i} + 5\hat{j})$

$= \frac{\hat{i}}{\sqrt{26}} + \frac{5\hat{j}}{\sqrt{26}}$

$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$

$= f\left((3,4) + \left(\frac{h}{\sqrt{26}}, \frac{5h}{\sqrt{26}}\right)\right)$

$f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

putting into $f(x,y)$

$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) - 3$

$= 16 + \frac{36h}{\sqrt{26}} + \frac{25h^2}{26} - 12 - \frac{4h}{\sqrt{26}} - 3$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 1$

$$f(a) = f(3,4) = \cancel{-12+16+3}\;\overset{16-12-3}{} = +1$$

$$f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h} = \frac{25h^2}{\sqrt{26}}.$$

$$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 1 - 1}{h}$$

$$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} - 0}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}} = \lim_{h\to 0} \left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$$

$$= \frac{36}{\sqrt{26}}$$

$f(x,y) = 2x + 3y \qquad a = 1, 2$

$u = 3\hat{i} + 4\hat{j}$ is not unit vector

$|u| = \sqrt{9+16} = \sqrt{25} = 5$

unit vector along $u = \frac{u}{|u|} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$$\hat{u} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a+hu) = f\left((1,2), h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$$

$$f(a+hu) = f\left((1,2) + \left(\frac{3h}{5}, \frac{4}{5}h\right)\right)$$

$$f\left(1+\frac{3h}{5}, 2+\frac{4h}{5}\right)$$

Putting in $f(x,y)$

$$= 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = 8 + \frac{18h}{5}$$

$f(a) = f(1,2) = 2 + 6 = 8,$

$$f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h} = \lim_{h\to 0} \frac{\cancel{8} + \frac{18\cancel{h}}{5} - \cancel{8}}{\cancel{h}}$$

$$f(a) = \frac{18}{5}$$

Q2 1

i $f(x,y) = x^y + y^x$, $a = 1,1$

$f(x) = y.x^{y-1} + y^x \log y$

$f(y) = x^y.\log x + x.y^{x-1}$

$\nabla f(x,y) = f_x((f_x, f_y) = (y.x^{y-1} + y^x \log y, y.\log x + x.y^{x-1})$

$\nabla f(1,1) = \left(1(1)^{1-1} + (1)^1.\log 1, 1\log 1 + 1.1^{(1-1)}\right)$

$(1\times 1^0 + 0, 0 + 1^0)$

$\nabla f(1,1) = (1,1)$

ii $F(x,y) = (\tan^{-1}x)(y)^2$ at $a = 1, -1$

$F(x) = \frac{y^2}{1+x^2}$

$F(y) = 2\tan^{-1}x \cdot y$

$\nabla F(x,y) = (f_x, f_y) = \left(\frac{y^2}{1+x^2}, 2\tan^{-1}x \cdot y\right)$

$= \left(\frac{1}{2}, 2 \times \frac{\pi}{4} \cdot 1\right) = \left(\frac{1}{2}, \frac{\pi}{2}\right)$

$\nabla f(x,y) = \left(\frac{1}{2}, \frac{\pi}{2}\right)$

iii $f(x,y,z) = x \cdot y \cdot z - e^{x+y-z}$ $a = (1, -1, 0)$

$f(x) = yz - e^{x+y-z}$

$f(y) = x \cdot z - e^{x+y-z}$

$f(z) = x \cdot y + e^{x+y-z}$

$\nabla F(x,y,z) = (f_x, f_y, f_z)$

$\nabla f(1,-1,0) = yz - e^{x+y-z}, xz - e^{x+y-z}, xy + e^{x+y-z}$

$= (-1) \cdot 0 - e^{1-1-0}, 1(0) - e^{1-1-0}, 1(1) + e^{1-1-0}$

$= (-1, -1, 2)$

Q3
1 $x^2 \cdot \cos y + e^{xy} = 2$ at $(1, 0)$

$F(x,y) = x^2 \cdot \cos y + e^{xy} - 2$

$f_x = 2x \cos y + y \cdot e^{xy}$

$f_y = -\sin y \cdot x^2 + x \cdot e^{xy}$

$(x_0, y_0) = (1, 0)$

f_x at $(1,0) = 2(1)\cos(0) + 0$
$= 2$

F_y at $(1,0) = (1)^2 \sin(0) + 1$
$= 1$

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

now
for eq normal Equation

$bx + ay + d = 0$

$x + 2y + d = 0$

at $(1, 0)$

$1 + 2(0) + d = 0$

$d = -1$

Normal equation $x + 2y - 1 = 0$

ii

a) $x^2+y^2-2x+3y+2=0$

$f(x,y) = x^2+y^2-2x+3y+2$

$f_x = 2x-2$ $\quad f_x$ at $(2,-2) = 2\times(2)-2 =$

$f_y = 2y+3$ $\quad f_y$ at $(2,-2) = 2(-2)+3 =$

Eq of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

iii $2x-4-y-2=0$

$2x-y-6=0$

Eq of normal

$bx+ay+d=0$

$-x+2y+d=0$

at $(2,-2)$

$-2+(-4)+d=0$

$d=6$

$-x+2y+6=0$

Q4

Ex, $x^2-2yz+3y+xz=7$

$f_x = 2x+z$

$f(x)$ at $(2,1,0) = 2\times 2 = 4$

$f_y = -2z+3$

f_y at $(2,1,0) = 3$

$f_z = 0-2y+x$

f_z at $(2,1,0) = -2+2 = 0$

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)=0$

$4x-8+3y-3=0$

$4x+3y-11=0 \rightarrow$ Equation of tangent

for Equation of normal

$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$

iii $3xyz-x-y+z=-4$ at $(1,-1,2)$

$f(x,y,z) = 3xyz-x-y+z+4$

$f_x = 3yz-1$

f_x at $(1,-1,2) = 3(-1)(2)-1$

$= -7$

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03 $f_y = 3xz - 0 - 1 = 3xz - 1$

f_y at $(1,-1,2) = 3(1)(2) - 1$
$= 5$

$f_z = 3xy + 1$
f_z at $(1,-1,2) = 3(1)(-1) + 1$
$= -2$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$7x + 3y - 2z + 16 = 0$

Equation of normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q5

i $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 6 - 3y = 6x - 3y + 6$ -1

$f_y = 2y - 3x + 0 - 4$
$= 2y - 3x - 4$ -2

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ -3

$f_y = 0$
$f_y = 0$
$2y - 3y - 4 = 0$
$2y - 3y = 4$ -4

multiply eq 3 by 2 & subract Eq 4 from 3

$4x - 2y = -4$
$2y + 3x = 4$
$7x = 0$

Substituting value of x

$2(0) - y = -2$
$-y = -2$
$y = 2$

critical point at $(0,2)$

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:3

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9 = 3 > 0$

$r > 0$ and $rt - s^2 > 0$

f has minimum at (0, 2)

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 4 + 0 + 0 - 8$

$= -4$

ii $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y = 3x^2 - 2y$

now $f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 6y = 0$

multiply in 1 by 2 & sub 2 from 1

$12x^2 + 18y = 0$

$12x^2 - 8y = 0$

$y = 0$

Subraction 3 in 2

$3x^2 - 2(0) = 0$

$3x^2 = 0$

$x = 0$

critical point are (0, 0)

now $r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = 1 - 2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

at (0, 0)

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 1(0) = 0$

$r = 0$ & $rt - s^2 = 0$

nothing can be said

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iii $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f(x) = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad 2x + 2 = 0$

$2(x+1) = 0$

$x = -1$

$f_y = 0$

$-2y + 8 = 0$

$-2(y-4) = 0$

$y = 4$

Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4$

f has minimum at $(-1, 4)$

$f(x,y)$ at $(-1, 4)$

$x^2 - y^2 + 2x + 8y - 70 = 0$

$(-1)^2 - (4)^2 + (-2) - 8(4) - 70 = 0$

$= 1 - 16 - 2 + 32 - 70$

$= -88 + 33$

$= -55$