Knowledge Distillation

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Outline

Overview of Knowledge Distillation

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- These large models are slow and take a lot of storage space, but are greatly accurate in what they do.
- What do we need to do if we need a highly accurate model on an edge device with limited memory, storage space, and computational power?
- Usage of new techniques such as Knowledge Distillation come into play.

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- **Response Based Knowledge:**
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  - Take the output from the last layer of the teacher and try to mimic this output using a smaller model.
- **Offline Distillation:**
  - Take a pre-trained teacher and distill its knowledge to the student.

Algorithms

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- **Quantized :** Here we use a high precision teacher, quantized on feature maps and which transfers knowledge to a quantized student.

Loss Function

- MNIST data we used the softmax cross-entropy, or the log-loss with softmax activation:

$$l(y, \mathbf{f}(x)) = -f_y(x) + \log \left[\sum_{y' \in [L]} e^{f_{y'}(x)} \right]$$

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- For the CIFAR-10 dataset, we've used a combination of two losses: cls_loss (cross loss) and div_loss (for distillation).

$$\text{cls\_loss} = -\sum_{x \in X} a(x) \log(s(x)) \quad \text{div\_loss} = KL\_div(s, t)$$

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Where a is the one-shot true probability distribution of training data, s is the distribution obtained by applying softmax to the student logits, and t is the distribution obtained by applying softmax to the teacher logits.

Bayes' Knows Best

Lemma 1: $Var_{S\sim\mathbb{P}^N}[\hat{R}_*(f;S)] \leq Var_{S\sim\mathbb{P}^N}[\hat{R}(f;S)]$ i.e., Bayes's risk is lesser than empirical risk.

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We observe that for population risk:

$$R(\mathbf{f}) = \mathbb{E}_x[p^*(x)^T l(f(x))]$$

where $p^*(x) = [\mathbb{P}(y|x)]_{y\in[L]}$ is the Bayes class probability distribution over the labels.

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Proof:

$$\begin{aligned}
Var_{S\sim\mathbb{P}^N}[\hat{R}_*(\mathbf{f};S)] &= \frac{1}{N} Var[\mathbb{E}_{y|x} l(y,\mathbf{f}(x))] \\
&= \frac{1}{N}\mathbb{E}_x[\mathbb{E}_{y|x}[l(y,\mathbf{f}(x))]^2] - \frac{1}{N}\mathbb{E}_x[\mathbb{E}_{y|x}[l(y,\mathbf{f}(x))]]^2 \\
&\leq \frac{1}{N}\mathbb{E}_x[\mathbb{E}_{y|x}[l(y,\mathbf{f}(x))^2]] - \frac{1}{N}\mathbb{E}_x[\mathbb{E}_{y|x}[l(y,\mathbf{f}(x))]]^2 \\
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&= Var_{S\sim\mathbb{P}^N}[\hat{R}(\mathbf{f};S)]
\end{aligned}
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Here equality holds iff
$(\forall x \in X)(\forall y, y' \in support(p^*(x)))\; l(y,f(x)) = l(y',f(x))$

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\begin{aligned}
\Delta :&= \tilde{R}(\mathbf{f}; S) - R(\mathbf{f}) \\
\mathbb{E}(\Delta^2) &= Var(\Delta) + E(\Delta)^2 \\
\mathbb{E}(\Delta) &= \mathbb{E}_x[(p^t(x) - p^*(x))^t l(\mathbf{f}(x))] \\
&\leq \mathbb{E}_x[||(p^t(x) - p^*(x))^t||_2 \cdot ||l(\mathbf{f}(x))||_2] \\
&\leq c \cdot \mathbb{E}_x[||p^t(x) - p^*(x))^t||_2]
\end{aligned}
$$

Bias Variance Bound:

Also,

$$Var(\Delta) = Var(\tilde{R}(\mathbf{f}; S)) = \frac{1}{N} Var[p^t(x)^T l(\mathbf{f}(x))]$$

Thus

$$\mathbb{E}(\Delta^2) \leq \frac{1}{N} Var[p^t(x)^T l(\mathbf{f}(x))] + \mathcal{O}(||\mathbb{E}[p^t(x)] - p^*(x))||_2^2 + Var[p^t(x)])$$

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No, it doesn't have to be the same. To train on a dataset unseen by the teacher, the student model can use cls_loss of the new dataset to account for it, and the div_loss for the teacher model to learn from the teacher, and produce weights for itself.

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No, that may not be the case. Suppose the data has different contexts with respect to the teacher and student. In that case, misleading feedback from the div_loss term will lead to generalization failure and noise amplification from the train data, which gives us garbage results.

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- Worked on two datasets: MNIST and CIFAR-10.
- Aim was to have a student with an inference time that is 10-100 times faster than the teacher.

MNIST Dataset

- Contains data on handwritten digits, of 10 classes, where each data is represented as a grayscale image of size 28×28.

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- Varying the student's structure, epochs, temperature, and batch size.
- Throughout, the student will be trained on 3 epochs. Each student will have a single dense, fully connected hidden layer, of variable sizes.

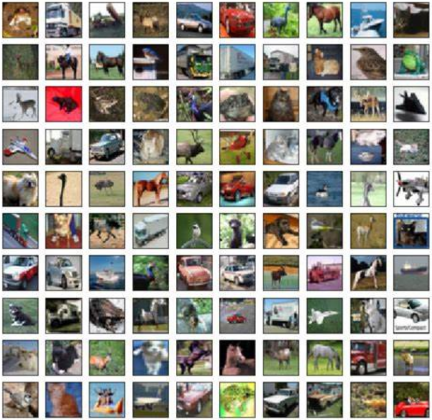
MNIST Dataset

Table: Experiments on MNIST dataset

Student Layer Size	**Temperature**	**No. of Epochs**	**Batch Size**	**Accuracy**	**Training Time** (seconds)	**Inference Time Ratio** (Teacher/Student)
50	3.5	3	32	95.97%	30.998	**315.332**
300	3.5	3	16	97.24%	95.715	138.753
300	3.5	3	64	97.12%	48.421	129.564
300	3.5	2	32	96.17%	**25.108**	126.174
300	10	3	32	96.87%	64.573	123.326
300	1	3	32	96.98%	65.171	123.264
300	3.5	3	32	**97.26%**	64.854	119.127
300	3.5	5	32	97.23%	247.926	116.654
600	3.5	3	32	97.21%	103.010	38.005

CIFAR-10

- It contains data on 10 classes namely: airplanes, cars, birds, cats, deer, dogs, frogs, horses, ships, and trucks



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- We'll be varying the student's structure, blocks per layer, the stride of the Base Block, and loss function.
- Here we have a feature-based knowledge, offline distillation using a Quantized algorithm.

CIFAR-10

Table: Experiments on CIFAR-10 dataset

Experiment	**Accuracy**	**Training Time** (seconds)	**Inference Time Ratio** (Teacher / Student)
Baseline	79.3%	400	6.58
Blocks = [1, 1, 1, 1]	79.1%	275	8.53
Blocks = [2, 2, 0, 0]	78.6%	370	7.81
Blocks = [1, 1, 0, 0]	79.4%	340	9.85
Blocks = [1, 1, 1, 1], Stride = 2	77.6%	280	**11.60**
Blocks = [1, 1, 1, 1], Stride = 4	**79.9%**	405	9.94
Ideal with no KD loss	75.8%	330	10.84
Ideal on reduced train set	61.2%	**60**	10.33
Ideal on reduced train set with no KD loss	56.0%	**60**	9.79

CIFAR-10 - Notation:

- *Ideal* here refers to the student model with blocks $[1, 1, 1, 1]$ and stride $= 2$ since it had the greatest Inference Time Ratio without compromising much on accuracy

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- *Ideal* here refers to the student model with blocks $[1, 1, 1, 1]$ and stride $= 2$ since it had the greatest Inference Time Ratio without compromising much on accuracy
- Reduced Train Set refers to the CIFAR-10 training set but with only the first 500 images of each class (instead of 5000).

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- Teacher model can be used to produce labels in the absence of labeled data, as seen in the MNIST experiment.
- Teacher can also be used to make up for the lack of training data, as shown in the experiment with the reduced CIFAR-10 dataset.

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