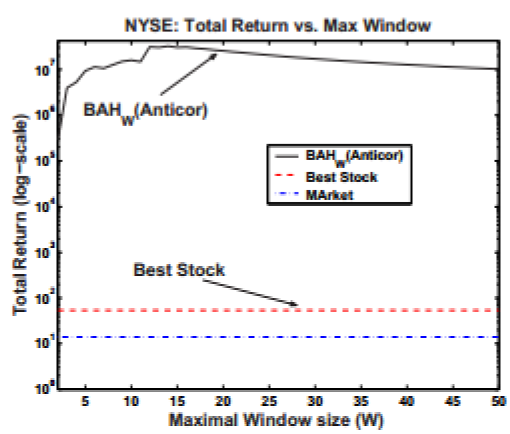
Aditya Pidaparti
CSCI 5991
11/14/2016

Portfolio selection theory began in 1952, with Harry Markowitz developing the CAPM model. [1] It started with a simple premise: maximize return while minimizing risk. His theory has been expanded and developed, and one of the sub-theories to appear is mean reversion theory. It predicts that all stocks will revert to the mean growth rate of the market. As such, stocks that perform above the average are predicted lose that additional growth, and stocks performing poorly are predicted to grow more to return to the mean.

Two algorithms which seek to take advantage of mean reversion theory are Anticor[2] and OLMAR[3]. Anticor takes advantage of market fluctuations through reallocating wealth from stocks performing well to options anti-correlated with the first stock's performance. It finds this correlation by comparing trends over the past *w* days. This is done to select how much information can be considered when deciding allocations, as both minimal and maximal information lead to poor choices. OLMAR takes a similar approach, but uses Passive Aggressive learning by checking if the expected return is high enough to meet a given constraint. If it isn't, then the algorithm will aggressively move towards a new portfolio to meet the condition while still attempting to remain close to the previous portfolio using the same window method.

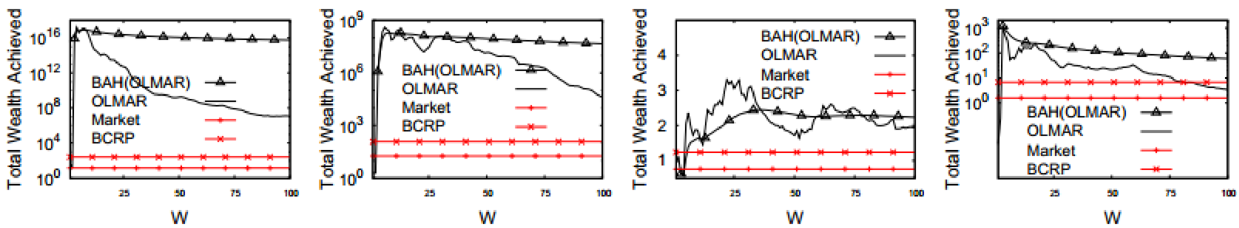
Anticor and OLMAR operate in a similar way, yet have many differences. Both rely on mean reversion theory, and were designed to take advantage of market fluctuations, adopting the 'follow the loser' approach as opposed to trying to predict winning stocks. To minimize their own fluctuations, the two algorithms prevent moving too quickly from the current selection. However, Anticor does this through calculating how much it desires to move from the current portfolio selection and making larger moves costlier, while OLMAR uses Passive Aggressive (PA) learning. To determine if a new portfolio is needed, OLMAR computes the predicted price relatives for the next day and checks if the current portfolio meets the constraint. If not, it will update the portfolio to one which does while minimizing change from the current allocation.

To implement mean reversion, an algorithm can take a single-period or a multi-period approach. In the former, only one day's growth for each stock is considered, and as was shown by the authors of OLMAR[3], this method does not perform well. Instead, both Anticor and OLMAR use a window examining the past *w* days to be able to analyze multiple days. This



limited window is prevent too little or too much data being analyzed, since both algorithms show that the optimal performance (across multiple data sets) is achieved when less than 30 days in the past are considered, as is shown in Figures 1[2] and 2.[3]

Figure 1: Performance of Anticor increases as the W (date range to analyze) increases, followed by a downward trend.



(a) NYSE (O) (b) NYSE (N) (c) DJA (d) TSE

Figure 2: Performance of the OLMAR algorithm across multiple datasets. Notice the same pattern in datasets a, b, and d as in Figure 1.

Both Figure 1 and 2 don't refer to the direct application of Anticor or OLMAR to the market. Those results have far greater variance with respect to a window frame, so instead both algorithms have been also implemented upon themselves in a layered approach. Using a buy-and-hold (BAH) strategy, the authors of Anticor ran it for window frames of 3 to 30 days on datasets to establish 'experts'. Then, Anticor was run again on those experts, and wealth was allocated based on the upper-level Anticor algorithm considering the past 3 to 50 days in BAH. OLMAR was also implemented in a similar fashion, except OLMAR was applied directly and then BAH was layered onto it. The results of these methods are below in Tables 2 and 3. The reason for taking this approach was to eliminate fluctuations in the portfolio results, which did smooth out the portfolio's gains as is shown below.

Two major differences between Anticor and OLMAR are the computational complexity and the heuristic nature of Anticor. Anticor calculates its recommendations as a function of $O(n^3)$, while OLMAR can make predictions in $O(n)$, where n is the number of days. As shown in Table 1, this results in serious delays.

Methods	NYSE (O)	NYSE (N)	DJA	TSE
B^{NN}	4.93E+04	3.39E+04	1.28E+03	1.32E+03
CORN	8.78E+03	1.03E+04	172	1.59E+03
Anticor	2.57E+03	1.93E+03	175	2.15E+03
PAMR	8	7	0.5	2
CWMR	123	68	9	162
OLMAR	4.0	3.3	0.3	0.7

(c) Computational Time (seconds)

Table 1: The computation time for several algorithms across multiple datasets. Note that Anticor is approximately 3 orders of magnitude larger in computing time than OLMAR.

In addition, OLMAR has the added advantage of minimized heuristics. As OLMAR's authors point out[3], OLMAR only requires one level of decision on the size of the window frame, while Anticor requires both the inner window size to be selected and the outer window size.

Anticor and OLMAR both show potential, as shown in Tables 2[2] and 3[3], however, the two use the same strategy of 'follow-the-loser', which is not ubiquitous throughout algorithmic trading. While these two tend to invest in stocks that are performing poorly, two other algorithms, Exponentiated Gradient (EG)[4] and Online Lazy Updates (OLU)[5] use 'follow-the-winner' approaches, attempting to find the best portfolio in hindsight. All four algorithms utilize multiplicative updates to calculate end return and try to minimize portfolio change from day to day, but EG and OLU are fundamentally different, as they are 'follow-the-winner' algorithms, hoping to find the best possible portfolio. EG doesn't account for any extenuating factors, [illegible]t OLU attempts to increase its [illegible]bility by accounting for transaction costs of trading. It does so through 'lazily' updating the portfolio, or minimizing loss [illegible] less frequent trading. OLMAR and Anticor both are noted to still be successful with very small transaction costs, but fail with larger ones and do not directly account for that loss EG is shown in Tables 2 and 3 for comparison:

Algorithm	NYSE	TSE	SP500	DJIA	NYSE^{-1}	TSE^{-1}	SP500^{-1}	DJIA^{-1}
MARKET (U-BAH)	14.49	1.61	1.34	0.76	0.11	1.67	0.87	1.43
BEST STOCK	54.14	6.27	3.77	1.18	0.32	**37.64**	1.65	2.77
CBAL*	250.59	6.77	4.06	1.23	2.86	**58.61**	1.91	2.97
U-CBAL	27.07	1.59	1.64	0.81	0.22	1.18	1.09	1.53
ANTI^1	**17,059,811.56**	**26.77**	**5.56**	**1.59**	**246.22**	7.12	**6.61**	**3.67**
ANTI^2	**238,820,058.10**	**39.07**	**5.88**	**2.28**	**1383.78**	7.27	**9.69**	**4.60**
LZ	79.78	1.32	1.67	0.89	5.41	4.80	1.20	1.83
EG	27.08	1.59	1.64	0.81	0.22	1.19	1.09	1.53
UNIVERSAL	26.99	1.59	1.62	0.80	0.22	1.19	1.07	1.53

Table 2: The performance of Anticor. Anti[1] denotes the direct application of Anticor on the market, and Anti[2] denotes the layered application discussed above. This data was tested on four different datasets, and then the datasets inverse. Note the success of Anticor across datasets, especially the layered variant.

Methods	NYSE (O)	NYSE (N)	DJA	TSE
Market	14.50	18.06	0.76	1.61
Best-stock	54.14	83.51	1.19	6.28
BCRP	250.60	120.32	1.24	6.78
UP	26.68	31.49	0.81	1.60
EG	27.09	31.00	0.81	1.59
ONS	109.19	21.59	1.53	1.62
B^K	1.08E+09	4.64E+03	0.68	1.62
B^{NN}	3.35E+11	6.80E+04	0.88	2.27
CORN	1.48E+13	5.37E+05	0.84	3.56
Anticor	2.41E+08	6.21E+06	2.29	39.36
PAMR	5.14E+15	1.25E+06	0.68	264.86
CWMR	6.49E+15	1.41E+06	0.68	332.62
OLMAR	**3.68E+16**	**2.54E+08**	2.06	**424.80**
BAH(OLMAR)	2.27E+16	1.41E+08	**2.38**	172.11
MAX(OLMAR)	1.62E+17	3.95E+08	3.30	1.18E+03

Table 3: The performance of Anticor, OLMAR, and BAH(OLMAR). Note that a direct application of OLMAR tends to perform better across datasets than the BAH method, which is the opposite of how Anticor behaved. However, in either case OLMAR outperformed Anticor in all datasets.

Anticor and OLMAR also have fail cases and thus room for improvement. When they are implemented, cases such as bankruptcy (stock goes to 0), inclusion of other securities, and lack of risk analysis all present issues. In the case of bankruptcy, as both cases involve a ‘follow-the-loser’ approach there is a chance that all wealth will be invested into a company that does not survive. However, in Table 2, the NYSE dataset contained stocks that all gained value over the period of the dataset (22 years). Thus, the inverse of NYSE, all stocks lost value. However, we still see a multiplicative gain in Anticor. This is due to both algorithms taking advantage of volatility in the market. However, in the case of investment in multiple types of securities, these algorithms may not be good choices. The decreased volatility of a bond market, for example, can mean a ‘follow-the-winner’ approach will have a better result. Bonds were setup in the market to have low fluctuation rates, whereas ‘follow-the-loser’ approaches rely on stock volatility to make gains. Finally, risk analysis in Anticor and OLMAR is nonexistent, but there are potential modifications. Implementation points in Anticor could include market volatility as a cost multiplier when calculating how far to move from the previous day’s portfolio. This would enable Anticor to make more risky portfolios costlier to move to. In OLMAR, a maximum risk could be placed as a second constraint on the Passive Aggressive learning it undertakes. Placing risk hear makes OLMAR immediately turn away from stocks that are too volatile, but doesn’t interfere with the algorithm’s mathematics, which should preserve OLMAR as much as possible.

OLMAR and Anticor prove to be successful, with their issues. Through exploitation of the mean reversion theory, both show multiplicative gain across many datasets. OLMAR proves itself to be a more successful version of Anticor, addressing many of the shortcomings and providing an even better method to use on the datasets.

References

1. Markowitz, Harry. "Portfolio selection." *The journal of finance* 7.1 (1952): 77-91.
2. Borodin, Allan, Ran El-Yaniv, and Vincent Gogan. "Can we learn to beat the best stock." Journal of Artificial Intelligence Research 21 (2004): 579-594.
3. Li, Bin, and Steven CH Hoi. "On-line portfolio selection with moving average reversion." *arXiv preprint arXiv:1206.4626* (2012).
4. Helmbold, David P., et al. "On-Line Portfolio Selection Using Multiplicative Updates." *Mathematical Finance* 8.4 (1998): 325-347.
5. Das, Puja, Nicholas Johnson, and Arindam Banerjee. "Online Lazy Updates for Portfolio Selection with Transaction Costs." *AAAI*. 2013.