Ball and Beam Control Project Phase 2
ES245: Control Systems

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***Abstract*—The ball and beam system is an example of an unstable dynamic system commonly used in control theory applications. This report aims to design a suitable controller to stabilize the ball's position, ensuring precise control in various operational scenarios.**

***Index Terms*—ball and beam, control systems, PID controller, system dynamics**

I. Introduction

The ball and beam system is an example of an unstable dynamic system commonly used in control theory applications. This system involves a ball that must be positioned on a beam, which a motor can tilt. The objective is to automatically regulate the ball's position by adjusting the angle of the beam in response to its movement. Given the system's inherent instability, effective control techniques are essential to maintain the desired position of the ball, particularly in applications such as aerospace stabilization and process control in industrial settings. This report aims to design a suitable controller that accurately calculates and adjusts the beam's angle to stabilize the ball's position, ensuring precise control in various operational scenarios.

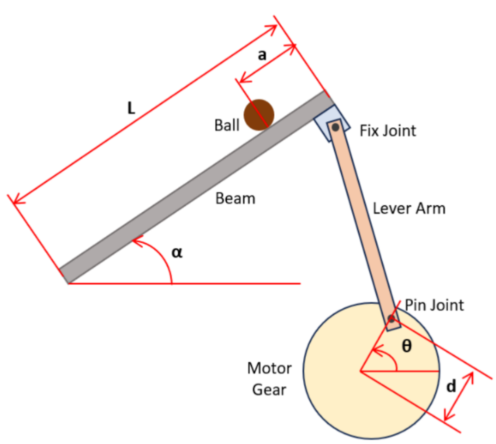


Fig. 1. Physical System

II. Assumptions

We derived the Sytem's transfer function to get the relationship between the ball's position and the servo motor angle. During the derivation, we made some assumptions such as

- Friction between the ball and beam is neglected.
- System transfer function is linearized around the operating point.
- Sensors and actuators are assumed ideal with no delays or noise.
- External disturbances are ignored.
- Actuator response is considered instantaneous.

III. Tasks

Task 6: Root Locus and Bode Plot of Ball-Beam System

1.1 Using MATLAB, plot the following for the system's open-loop transfer function::

- Plot root locus and comment on the system's stability

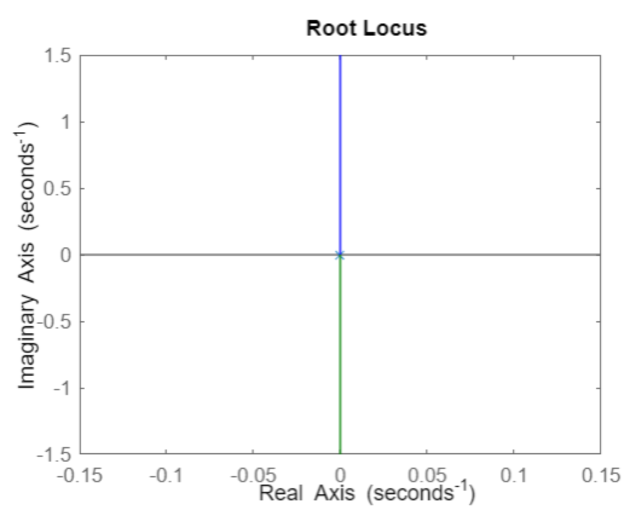


Fig. 2. Root Locus plot of Open-loop TF

The open-loop poles are located on the imaginary axis (at or near the origin), indicating marginal stability in the open-loop system.

- Plot the bode plot and report the phase margin and gain margin of the system

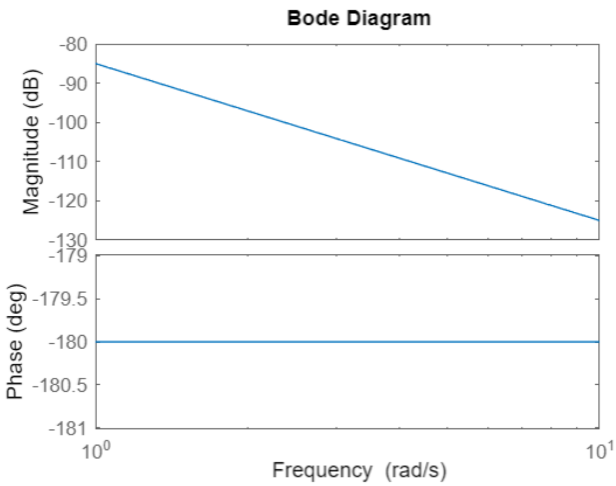


Fig. 3. Bode Diagram of System's Open-loop TF

Using the Bode Diagrams, we got the Phase margin and Gain margin of the Open loop transfer function. We got the
Phase margin = 0.0 deg
Gain margin = 1.0 db

1.2 Consider the design criteria: less than 5% overshoot and settling time less than 3 seconds within a 2% tolerance band. Complete the following::

- **Root Locus:** Plot the design criteria using MATLAB and recommend the appropriate compensator to achieve the desired performance.
  We designed controllers to achieve specific settling time and max—overshoot values by observing where poles lie relative to the grid. The sgrid function used the desired values of Settling time and overshoot to automatically find the damping ratio and natural frequency value.

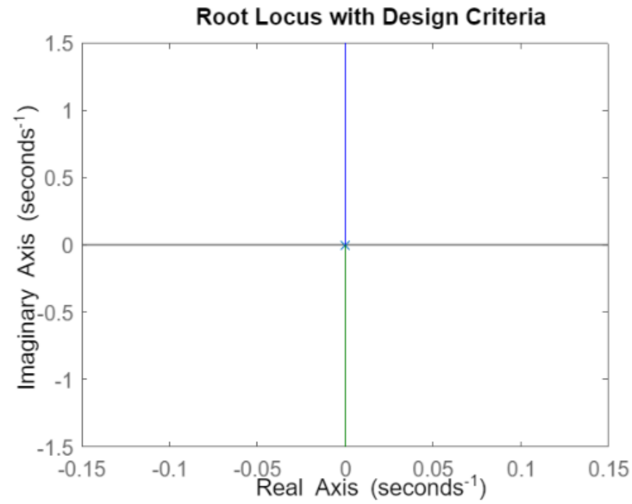
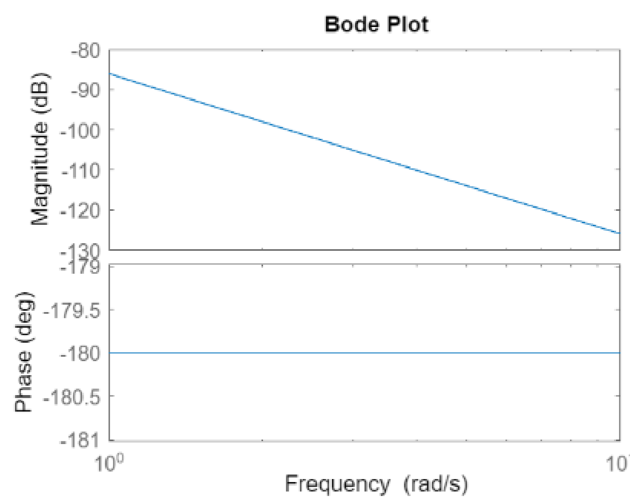


Fig. 4. Bode plots of Open-loop TF

Root locus is plotted with design criteria of 5% overshoot and 3 seconds settling time.

- **Bode Plot:** analyze the plot to recommend the compensator to meet the design criteria.



`Bode plot is plotted to analyze system frequency response.`

Fig. 5. Bode plots of Compensated Open Loop TF

Task 7: Controller Design using Root Locus and Bode Plot Approaches

7.1: Based on the previous recommendations, design a first-order lead/lag compensator to meet the design criteria:

We have to design a Lead Compensator that meets the criteria - Maximum Percent Overshoot of less than 5 percent and settling time of less than 3 seconds for a unit step input.

Case 1: Taking $M_p = 0.05$ and $t_s = 3\,\text{s}$.
For the design criteria, we design a Lead Compensator:

$$G_c(s) = K_c \frac{s + 1/T}{s + 1/\alpha T}, \quad 0 < \alpha < 1$$

Given:

$$M_p < 5\%, \quad t_s < 3s$$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} < 0.05$$

Taking the natural logarithm on both sides:

$$-\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = \ln(0.05)$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{-\ln(0.05)}{\pi}$$

Squaring both sides:

$$\zeta^2 = \frac{(-\ln(0.05)/\pi)^2}{1+(-\ln(0.05)/\pi)^2}$$

Calculating step-by-step:

$$\zeta^2 = \frac{(-2.9957)^2}{1+(-2.9957)^2}$$

$$\zeta^2 = \frac{8.974}{1+8.974} = \frac{8.974}{9.974} \approx 0.8996$$

$$\zeta \approx 0.6901$$

The settling time criteria:

$$\frac{4}{\zeta\omega_n} < 3$$

For the settling time:

$$t_s = \frac{4}{\zeta\omega_n} < 3 \quad \Longrightarrow \quad \omega_n = \frac{4}{3 \times 0.6901} = 1.932\,\text{rad/s}$$

The closed-loop poles are determined from the characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Substitute $\zeta = 0.6901$ and $\omega_n = 1.932$:

$$s^2 + 2(0.6901)(1.932)s + (1.932)^2 = 0$$

$$s^2 + (2.666)s + 3.732 = 0$$

Solve for the roots of the equation:

$$s = -1.333 \pm 1.382i$$

The phase angle of the dominant pole is calculated as:

$$\text{Angle of } s_1 = -1.333 + 1.382i:$$

$$\theta = \tan^{-1}\left(\frac{1.382}{-1.333}\right) \approx 133.63^\circ$$

For the open-loop transfer function:

$$G(s) = \frac{0.8936}{s^2}$$

Phase calculation for $G(s)$:

$$\Theta = \angle G(s) = \angle\left(\frac{0.8936}{(-1.333+1.382i)^2}\right)$$

$$\Theta = -2 \times 133.63 = -267.26^\circ$$

Phase deficiency:

$$\Phi_d + \Theta = \text{odd multiple of } 180^\circ$$

$$\Phi_d + \Theta = (2n+1)180^\circ$$

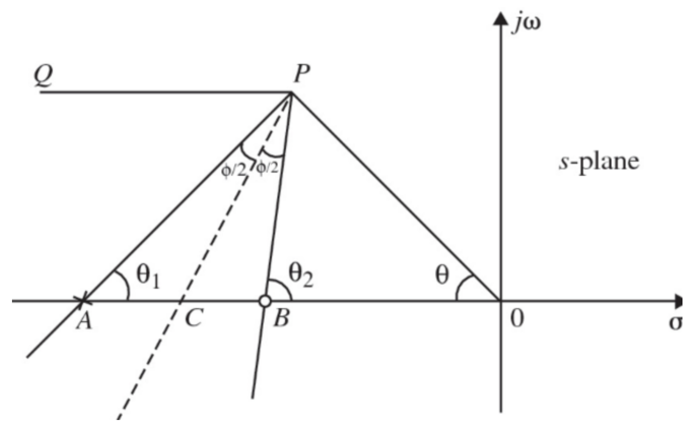
$$\Phi_d + \Theta = -180^\circ$$

$$\phi_d = -180^\circ - 0 = -180^\circ + 267.26^\circ$$

$$\phi_d = 87.26^\circ$$

$$\theta_2 = \frac{\phi_{\text{peak}} + \phi_d}{2} = \frac{110.445^\circ}{2}$$

$$\theta_2 = 23.185^\circ$$



The location of zero will lie on the line passing through the direct pole and making an angle θ_2:

$$\frac{-1.3382}{z+1.33} = \tan(10.445^\circ)$$

$$z + 1.33 = 0.05212 \quad \Rightarrow \quad z = -0.088$$

$$\frac{-1.3382}{p+1.33} = \tan(23.185^\circ)$$

$$p + 1.33 = 0.594 \quad \Rightarrow \quad p = -1.594$$

And the transfer function:

$$G_c(s) = K\frac{(s+0.808)}{(s+4.594)}$$

By the magnitude condition:

$$|T(s)G_c(s)| = 1, \quad s_1 = -1.33 + 1.3382j$$

$$\left|K\frac{(s_1+0.808)}{(s_1+4.594)}\right| = 1$$

Substitute s_1:

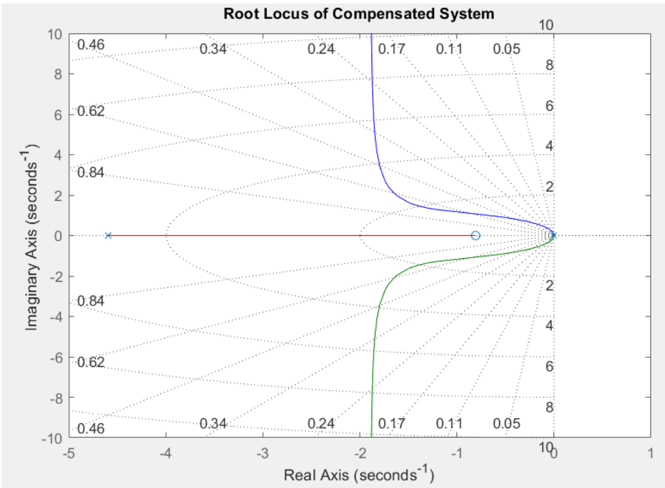
$$K\frac{(5.1+0.808)}{5.1+4.594} = \frac{0.8336}{5.2} = 1$$

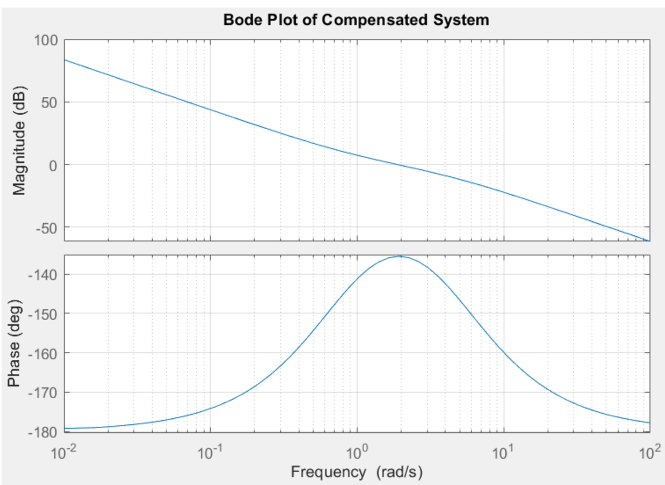
$$K = 9.914$$

Final controller:

$$G_c(s)G(s) = 8.859\frac{(s+0.808)}{s^2(s+4.594)}$$

Plots





7.2 : Comment on both approaches' pole-zero placement strategies and explain how the compensator pole-zero placement affects the root locus and bode plot.

First-Order Lead Compensator: A first-order lead compensator $C(s)$ can be designed using the root locus method. A lead compensator in root locus form is given by:

$$C(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \quad (0<\alpha<1)$$

A phase-lead compensator tends to shift the root locus toward the left in the complex s-plane. This results in an improvement in the system's stability and an increase in its response speed.

To determine the asymptotes of the root locus that lead to the zeros at infinity, the equation to find the intersection of the asymptotes along the real axis is as follows:

$$a = \frac{\sum(\text{poles}) - \sum(\text{zeros})}{\#(\text{poles}) - \#(\text{zeros})}$$

When a lead compensator is added to a system, the value of this intersection will be a larger negative number than it was before. The net number of zeros and poles will be the same (one zero and one pole are added), but the added pole is a larger negative number than the added zero. Thus, the result of a lead compensator is that the asymptotes' intersection is moved further to the left in the complex plane, and the entire root locus is shifted to the left as well. This tends to increase the region of stability and the system's response speed.

Frequency Response Approach: A first-order phase-lead compensator can also be designed using the frequency response method. The compensator's transfer function in this form is:

$$C(j\omega) = K_c\alpha\frac{Tj\omega+1}{\alpha Tj\omega+1} = K_c\frac{j\omega+\frac{1}{T}}{j\omega+\frac{1}{\alpha T}}, \quad (0<\alpha<1)$$

In this approach, the zero at $\frac{1}{T}$ introduces a phase lead in the frequency range where the system has insufficient phase margin, improving stability. The pole at $\frac{1}{\alpha T}$, being at a higher frequency than the zero, ensures that the system gain decreases at higher frequencies. This reduces the impact of noise and disturbances while maintaining robustness.

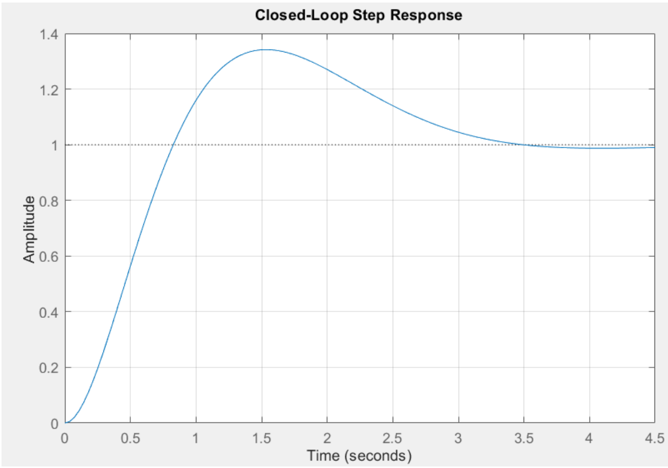
Effects on Bode Plot:

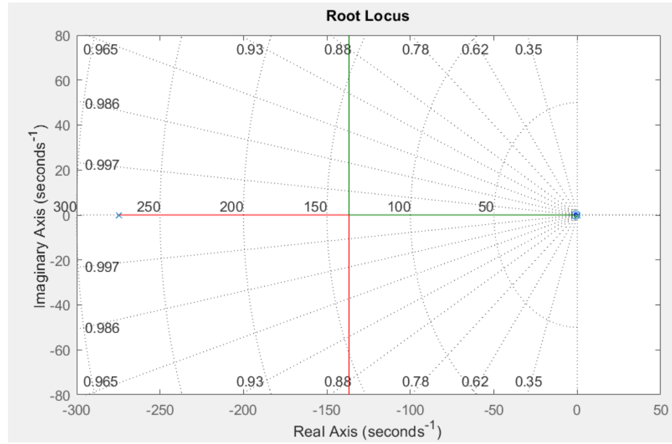
1) **Phase Lead:** The zero provides a positive phase shift, improving the system's phase margin and transient response.
2) **Gain Roll-Off:** The pole limits the system gain at high frequencies, ensuring better noise attenuation and system robustness.

These effects enhance the system's stability, response speed, and resilience to high-frequency disturbances.

7.3: Plot the system's closed-loop response (with the compensator) for a step input. Verify that the system satisfies the design criteria using both the root locus and bode plot approaches.

From Task 7.1, we can plot the closed loop response with a unit step input.





We observe that the initial system performance does not meet the design criteria:

- Settling Time: 3.22 seconds
- Overshoot: 34.18%

To address this, we refine the design parameters through a trial-and-error approach. The updated specifications are:

- Maximum Overshoot (M_p): 0.14 (14%)
- Settling Time (t_s): 0.45 seconds

Damping ratio (η): 0.5305
Natural frequency (ω_n): 16.7555
Open-loop transfer function:

$$TF(s) = \frac{0.8936}{s^2}$$

System poles: $-8.8889 + 14.2033i$, $-8.8889 - 14.2033i$
Desired root: $-8.8889 + 14.2033i$
Phase angle of desired root (deg): 122.0397
Open-loop TF phase at desired root (deg): 115.9206
Additional phase angle (deg): 115.9206
Compensator zero: -1.0223
Compensator pole: -274.6194
Lead compensator transfer function:

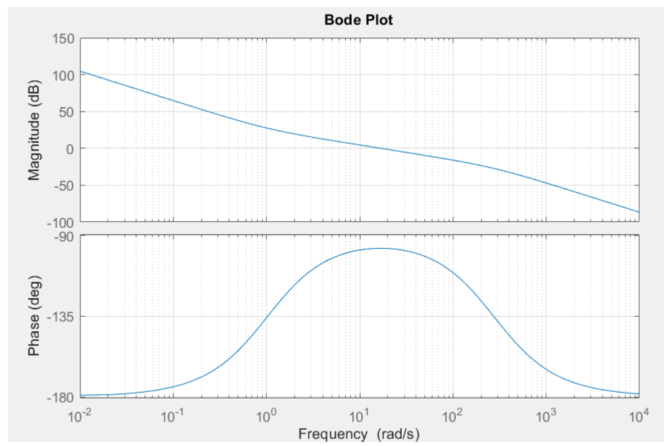
$$Comp(s) = \frac{s + 1.0223}{s + 274.6194}$$

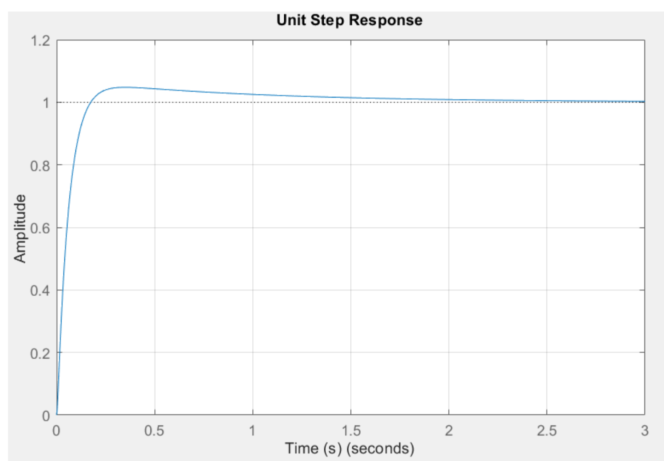
Compensator gain (K_c): 5149.3
Total system with compensator:

$$G_c(s)G(s) = \frac{4601.4s + 4704.0}{s^3 + 274.6194s^2}$$

Closed-loop transfer function:

$$Closed\_Loop(s) = \frac{4601.4s + 4704.0}{s^3 + 274.6194s^2 + 4601.4s + 4704.0}$$





Settling time (s): 2.9711
Overshoot (%): 4.7674

IV. Final Setup Images

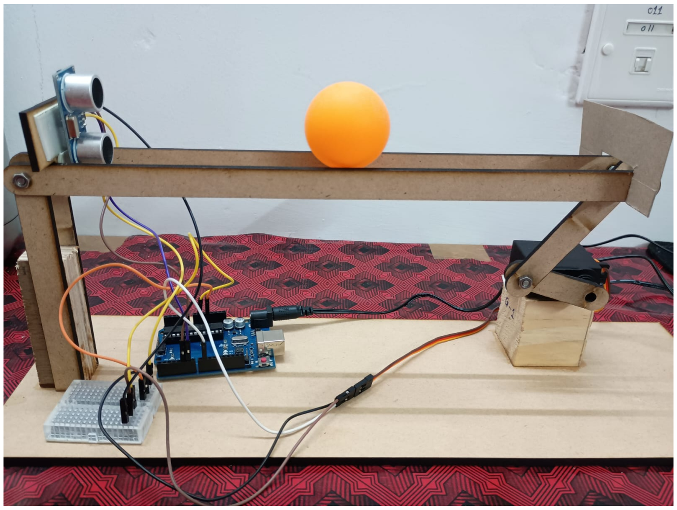


Fig. 6. Final Setup

V. Challenges Faced

- The initial system failed to balance the ball at the desired location, even after tuning the *Ki* and *Kd* values.
- We replaced the original servo motor with a 180-degree servo motor, requiring further tuning of *Kp*, *Ki*, and *Kd*.
- The ultrasonic sensor occasionally provided inaccurate or ”garbage” values at certain positions.
- Code adjustments were made to filter erroneous sensor readings and ensure stable beam balancing.

VI. Key Findings

- **PID Tuning:** Tuning *Kp*, *Ki*, and *Kd* with a 180-degree servo improved stability.
- **Pole-Zero Placement:** Effective compensator design reduced overshoot and settling time.
- **Sensor Accuracy:** Filtering ultrasonic sensor errors was essential for precise control.
- **Closed-Loop Performance:** System met design goals with less than 5% overshoot and settling time under 3 seconds.
- **Practical Success:** PID controller maintained stability and responded well to disturbances.
- **Code Optimization:** Adjustments ensured reliable real-time control.

VII. Future Scope of Work

- Incorporate nonlinear dynamics for greater accuracy in real-world scenarios.
- Model and account for friction between the ball and beam to improve performance.
- Optimize the system to improve response time.
- Enhance the controller to balance the ball faster than before.

VIII. Acknowledgment

The authors would like to thank IIT Gandhinagar Mechanical Lab for the resources and guidance provided to complete this project. We want to extend our heartfelt gratitude towards Professor Vineet and PHD Rajdeep Singh for their invaluable help and guidance.

References

[1] ”Ball and Beam System,” Control Tutorials for MATLAB and Simulink, University of Michigan, accessed October 20, 2024. [Online]. Available: https://ctms.engin.umich.edu/CTMS/index.php?example=BallBeam&section=SimulinkSimscape