# Lid-Driven Cavity Flow (Project - 4)

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## 1. Problem Statement

### 1.1. Introduction

This project focuses on the computational analysis of fluid flow within a square cavity, where the top boundary moves horizontally at a constant velocity of 1 m/s. Such a flow is called Lid-Driven Cavity Flow, and we need to implement the SIMPLE (Semi-Implicit Method for Pressure-linked Equations) algorithm to solve the 2-D Incompressible Navier-Stokes equations to obtain the steady-state solution. Here is a schematic of the setup:

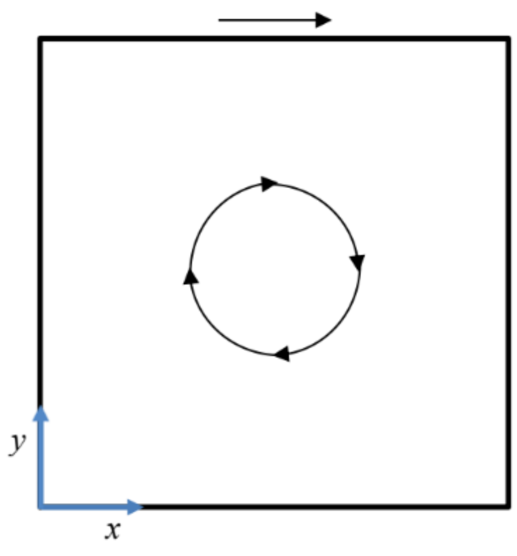


**Figure 1.** Schematic of Lid-Driven Cavity flow

The kinematic viscosity ($\nu$) of the fluid chosen is equal to 0.01 $m^2/s$. The flow is assumed to be incompressible, which means that density is not a function of space or time and is treated as a constant. A staggered mesh approach will also be implemented in this project.

### 1.2. Boundary Conditions

The boundary conditions are as follows:

$$
\begin{aligned}
u(0, y) = 0, \& v(0, y) = 0 \\
u(1, y) = 0, \& v(1, y) = 0 \\
u(x, 0) = 0, \& v(x, 0) = 0 \\
u(x, 1) = 1, \& v(x, 1) = 0
\end{aligned}
$$

As we can see, the top surface of the cavity moves with a horizontal velocity ($u$) of 1 m/s. The no-penetration and no-slip boundary conditions are applicable on all surfaces. Apart from these, we also need the boundary conditions for pressure. These will be derived later in the report.

## 2. Governing Equations

The governing equations for the Lid-Driven cavity flow consist of the continuity equation (Mass conservation) and Navier-Stokes equations (Momentum conservation). Given below are the steady-state incompressible 2-D governing equations:

***Continuity Equation***

$$\int_{CS} (\vec{V} \cdot \hat{n})\, dA = 0 \tag{1}$$

**X-Momentum Equation**

$$\int_{CS} \rho u(\vec{V} \cdot \hat{n})\, dA = -\int_{CS} p\,(\hat{n} \cdot \hat{\imath})\, dA + \int_{CS} (\mu\, \vec{\nabla} u) \cdot \hat{n}\, dA \tag{2}$$

**Y-Momentum Equation**

$$\int_{CS} \rho v(\vec{V} \cdot \hat{n})\, dA = -\int_{CS} p\,(\hat{n} \cdot \hat{\jmath})\, dA + \int_{CS} (\mu\, \vec{\nabla} v) \cdot \hat{n}\, dA \tag{3}$$

## 3. Mesh details

### 3.1. Mesh details

The cavity is divided in both the x and y dimensions. The spatial steps are denoted by $\Delta x \& \Delta y$ respectively. Since the length and height of the square cavity are 1, there will be $\frac{1}{\Delta x} \& \frac{1}{\Delta y}$ cells in the x and y directions, respectively. However, we will be using the staggered grid approach to implement the SIMPLE algorithm. The staggered mesh approach is used to avoid numerical issues like checker-boarding, which can arise when velocity and pressure are stored at the same grid locations. Here is a schematic of the Grid that we will be using.

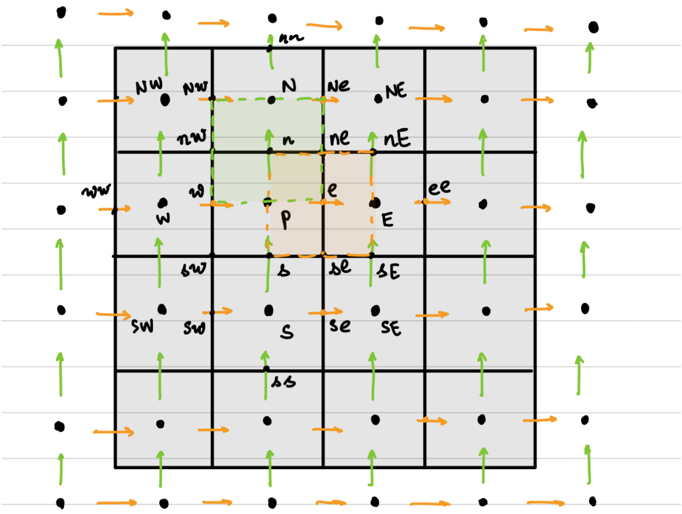


**Figure 2.** Staggered Grid

Here, three different grids are used. The first one is the one with black points, which is for the continuity equation. The orange grid is staggered in the x-direction and is used for the x-momentum equation. Similarly, the green grid is staggered in the y-direction and is used for the y-momentum equation.

**P** refers to the cell centre in the continuity grid. **E, W, N, S** are cell centres of the neighbouring cells to the right, left, top, and bottom of the **P** centred cell, respectively.

The x-momentum cell is centred on the point **e**, which is the face centre between the **P** and **E** continuity cells. The neighbours of this cell are **ee, w, Ne, Se** to the right, left, top and bottom of the **e** centred cell, respectively. These neighbours refer to the cell centres of the staggered grid.

Similarly, the y-momentum cell is centred on the point **n**, which is the face centre between the **N** and **P** continuity cells. The neighbours of this cell are **nE, nW, nn, s** to the right, left, top and bottom of the **n** centred cell, respectively. These neighbours refer to the cell centres of the staggered grid.

## 4. Discretized equations

We will be using the Finite Volume Method to discretize the governing equations. Continuity, x-momentum, and y-momentum equations need to be solved simultaneously. In this section, we shall deal with the Discretization of the equations only for the interior points. This will give us a basic framework of the governing equations, which we will extend to formulate the equations at the boundaries and corners of our domain.

### 4.1. Discretized Continuity equation

The discretized continuity equation can be split into 4 integrals for each face, which can be simplified to the following equation:

$$u_e A_e - u_w A_w + v_n A_n - v_s A_s = 0$$

Here, $A_e, A_w, A_n, \& A_s$ are the face areas of the continuity cell. $A_e = A_w = \Delta y$ and $A_s = A_n = \Delta x$.

Therefore, the discretized continuity equation reduces to:

$$(u_e - u_w)\Delta y + (v_n - v_s)\Delta x = 0 \tag{4}$$

### 4.2. Discretized X-momentum equation

We split the original governing equation into 3 parts. Each integral will be individually discretized, and then all parts will be joined to get the final discretized X-momentum equation.

#### *First Integral*

The term $\int_{CS} u(\vec{V} \cdot \hat{n})\, dA$ can be discretized as:

$$\rho\left[\int_E u_E^n(u_E^{n+1}\, dA) - \int_P u_P^n(u_P^{n+1}\, dA) + \int_{ne} v_{ne}^n(u_{ne}^{n+1}\, dA) - \int_{se} v_{se}^n(u_{se}^{n+1}\, dA)\right]$$

This can further be written as:

$$\dot{m}_E u_E + \dot{m}_P u_P + \dot{m}_{ne} u_{ne} + \dot{m}_{se} u_{se}$$

where,

$$\begin{aligned} \dot{m}_E &= \rho u_E^n \Delta y \\ \dot{m}_P &= \rho u_P^n \Delta y \\ \dot{m}_{ne} &= \rho v_{ne}^n \Delta x \\ \dot{m}_{se} &= \rho v_{se}^n \Delta x \end{aligned}$$

Now, the u velocities can be approximated as a linear function of the velocities of the staggered face centres.

$$\begin{aligned} u_E &= \frac{1}{2}(u_e + u_{ee}) \\ u_P &= \frac{1}{2}(u_e + u_w) \\ u_{ne} &= \frac{1}{2}(u_e + u_{Ne}) \\ u_{se} &= \frac{1}{2}(u_e + u_{Se}) \end{aligned}$$

Substituting these into the equation above and grouping the like terms together, we get:

$$u_e a_e^c + u_{ee} a_{ee}^c + u_w a_e^c + u_{Ne} a_{Ne}^c + u_{Se} a_{Se}^c$$

where, $a_e^c = \frac{\dot{m}_E + \dot{m}_P + \dot{m}_{ne} + \dot{m}_{se}}{2}$, $a_{ee}^c = \frac{\dot{m}_E}{2}$, $a_w^c = \frac{\dot{m}_P}{2}$, $a_{Ne}^c = \frac{\dot{m}_{ne}}{2}$, and $a_{Se}^c = \frac{\dot{m}_{se}}{2}$. We can also observe that the coefficients of the neighbouring cell velocities add up to the coefficient of the cell centre velocity (staggered mesh).

#### *Second Integral*

The term $-\int_{CS} p\,(\hat{n} \cdot \hat{\imath})\, dA$ can be written as:

$$-\int_{CS} p\,(\hat{n} \cdot \hat{\imath})\, dA = (p_P - p_E)\Delta y$$

#### *Third Integral*

The third and final integral for the x-momentum equation is $\int_{CS}(\mu \vec{\nabla} u) \cdot \hat{n}\, dA$, which can be written as:

$$\mu\left[\frac{\partial u}{\partial x}|_E A_E - \frac{\partial u}{\partial x}|_P A_P + \frac{\partial u}{\partial y}|_{ne} A_{ne} - \frac{\partial u}{\partial y}|_{se} A_{se})\right]$$

Here,

$$\begin{aligned} \frac{\partial u}{\partial x}|_E &= \frac{u_{ee} - u_e}{\Delta x} \\ \frac{\partial u}{\partial x}|_P &= \frac{u_e - u_w}{\Delta x} \\ \frac{\partial u}{\partial y}|_{ne} &= \frac{u_{Ne} - u_e}{\Delta y} \\ \frac{\partial u}{\partial y}|_{se} &= \frac{u_e - u_{Se}}{\Delta y} \end{aligned}$$

Substituting these in the above equation and rearranging the terms while also substituting the areas, we get:

$$u_e a_e^v + u_{ee} a_{ee}^v + u_w a_w^v + u_{Ne} a_{Ne}^v + u_{Se} a_{Se}^v$$

where, $a_e^v = -\mu(\frac{2\Delta y}{\Delta x} + \frac{2\Delta x}{\Delta y})$, $a_{ee}^v = a_w^v = \mu\frac{\Delta y}{\Delta x}$, $a_{Ne}^v = a_{Se}^v = \mu\frac{\Delta x}{\Delta y}$.

Now that we have the discretized forms of all three integrals, we can combine them to get the following equation:

$$a_e u_e = \sum_{nb} a_{nb} u_{nb} + (p_P - p_E)\Delta y \tag{5}$$

The summation term adds up all the weighted velocities of the neighbouring staggered cell centres. For the x-momentum equation, the neighbours (nb) are **ee, w, Se, Ne**. Also, the coefficients of the velocities $a_e$ & $a_{nb}$ are given by $(a_e^c - a_e^v)$ & $(a_{nb}^v - a_{nb}^c)$, respectively.

$$a_e = \frac{\dot{m}_E + \dot{m}_P + \dot{m}_{ne} + \dot{m}_{se}}{2} + 2\mu(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y})$$
$$a_{ee} = \mu\frac{\Delta y}{\Delta x} - \frac{\dot{m}_E}{2}$$
$$a_w = \mu\frac{\Delta y}{\Delta x} - \frac{\dot{m}_P}{2}$$
$$a_{Ne} = \mu\frac{\Delta x}{\Delta y} - \frac{\dot{m}_{ne}}{2}$$
$$a_{Se} = \mu\frac{\Delta x}{\Delta y} - \frac{\dot{m}_{se}}{2}$$

Similarly, we can write for the discretized y-momentum equation.

### 4.3. Discretized Y-momentum equation

The form of the discretized equation for the y-momentum is almost completely identical to that of the x-momentum. One change is that instead of staggering in the x direction, we solve for the staggered mesh in the y direction. The equation is split into three separate integrals and individually discretized, similar to the x-momentum equation.

***First Integral***

The term $\int_{CS} v(\vec{V}\cdot\hat{n})\,dA$ can be discretized as:

$$\rho\left[\int_N v_N^{n+1}(v_N^n\,dA) - \int_P v_P^{n+1}(v_P^n\,dA) + \int_{nw} v_{nw}^{n+1}(u_{nw}^n\,dA) - \int_{ne} v_{ne}^{n+1}(u_{ne}^n\,dA)\right]$$

This can further be written as:

$$\dot{m}_N v_N + \dot{m}_P v_P + \dot{m}_{nw} v_{nw} + \dot{m}_{ne} v_{ne}$$

where,

$$\dot{m}_N = \rho v_E^n \Delta x$$
$$\dot{m}_P = \rho v_P^n \Delta x$$
$$\dot{m}_{nw} = \rho u_{ne}^n \Delta y$$
$$\dot{m}_{ne} = \rho u_{se}^n \Delta y$$

Now, the v velocities can be approximated as a linear function of the velocities of the staggered face centres.

$$v_N = \frac{1}{2}(v_n + v_{nn})$$
$$v_P = \frac{1}{2}(v_n + v_s)$$
$$v_{nw} = \frac{1}{2}(v_{nW} + v_n)$$
$$v_{ne} = \frac{1}{2}(v_n + v_{nE})$$

Substituting these into the equation above and grouping the like terms together, we get:

$$v_n a_n^c + v_{nn} a_{nn}^c + v_s a_s^c + v_{nW} a_{nW}^c + v_{nE} a_{nE}^c$$

where, $a_n^c = \frac{\dot{m}_N + \dot{m}_P + \dot{m}_{nw} + \dot{m}_{ne}}{2}$, $a_{nn}^c = \frac{\dot{m}_N}{2}$, $a_s^c = \frac{\dot{m}_P}{2}$, $a_{nW}^c = \frac{\dot{m}_{nw}}{2}$, and $a_{nE}^c = \frac{\dot{m}_{ne}}{2}$.

***Second Integral***

The term $-\int_{CS} p\,(\hat{n}\cdot\hat{j})\,dA$ can be written as:

$$-\int_{CS} p\,(\hat{n}\cdot\hat{j})\,dA = (p_P - p_N)\Delta x$$

***Third Integral***

The third and final integral for the y-momentum equation is $\int_{CS}(\mu\,\vec{\nabla}v)\cdot\hat{n}\,dA$, which can be written as:

$$\mu\left[\frac{\partial v}{\partial y}|_N A_N - \frac{\partial v}{\partial y}|_P A_P + \frac{\partial v}{\partial x}|_{nw} A_{nw} - \frac{\partial v}{\partial x}|_{ne} A_{ne})\right]$$

Here,

$$\frac{\partial v}{\partial y}|_N = \frac{v_{nn} - v_n}{\Delta y}$$
$$\frac{\partial v}{\partial y}|_P = \frac{v_n - v_s}{\Delta y}$$
$$\frac{\partial v}{\partial x}|_{nw} = \frac{v_n - v_{nW}}{\Delta x}$$
$$\frac{\partial v}{\partial x}|_{ne} = \frac{v_{nE} - v_n}{\Delta x}$$

Substituting these in the above equation and rearranging the terms while also substituting the areas, we get:

$$v_n a_n^v + v_{nn} a_{nn}^v + v_s a_s^v + v_{nE} a_{nE}^v + v_{nW} a_{nW}^v$$

where, $a_n^v = -\mu(\frac{2\Delta y}{\Delta x} + \frac{2\Delta x}{\Delta y})$, $a_{nn}^v = a_s^v = \mu\frac{\Delta y}{\Delta x}$, $a_{nW}^v = a_{nE}^v = \mu\frac{\Delta x}{\Delta y}$.

Now that we have the discretized forms of all three integrals, we can combine them to get the following equation:

$$a_n v_n = \sum_{nb} a_{nb} v_{nb} + (p_P - p_N)\Delta x \tag{6}$$

The summation term adds up all the weighted velocities of the neighbouring staggered cell centres. For the y-momentum equation, the neighbours (nb) are **nn, s, nW, nE**. Also, the coefficients of the velocities $a_n$ & $a_{nb}$ are given by $(a_n^c - a_n^v)$ & $(a_{nb}^v - a_{nb}^c)$, respectively.

$$a_n = \frac{\dot{m}_N + \dot{m}_P + \dot{m}_{nw} + \dot{m}_{ne}}{2} + 2\mu(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y})$$
$$a_{nn} = \mu\frac{\Delta y}{\Delta x} - \frac{\dot{m}_N}{2}$$
$$a_s = \mu\frac{\Delta y}{\Delta x} - \frac{\dot{m}_P}{2}$$
$$a_{nW} = \mu\frac{\Delta x}{\Delta y} - \frac{\dot{m}_{nw}}{2}$$
$$a_{nE} = \mu\frac{\Delta x}{\Delta y} - \frac{\dot{m}_{ne}}{2}$$

## 5. Discretized equations at Boundaries and Corners

As discussed above, the equations that we have discretized above are valid only for points lying inside our domain. We know the values of velocities at the boundaries, therefore, we can use these velocities to find out the equations at the boundaries. We have four edges (Left wall, Top wall, Right wall, and Bottom wall) and 4 corners (Top Left, Top Right, Bottom Right, and Bottom Left). Each of these parts will have three equations: the continuity equation, the x-momentum equation, and the y-momentum equation. Hence, we end up having 24 equations only for the edges and corners and 3 equations for interior points. Finally, we should have a system of 27 equations that needs to be updated after every iteration of the SIMPLE algorithm.

### 5.1. Edges

For edges, care needs to be taken while formulating the equations because the mesh extends beyond the domain. We will see how it is incorporated in the following sections

#### *5.1.1. Left Wall*

For the left wall, we have the following structure of the staggered mesh:

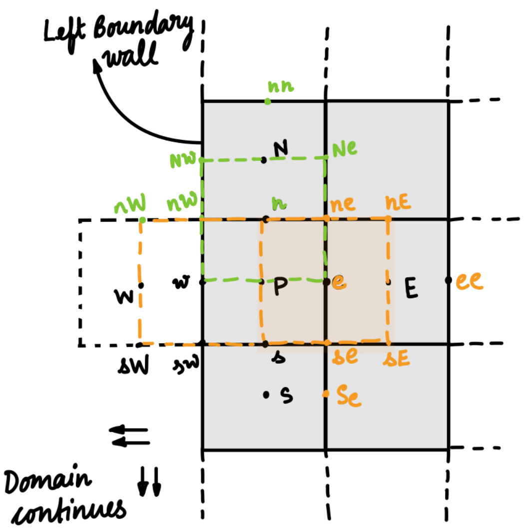


**Figure 3.** Left wall staggered mesh

Writing down the discretized continuity equation for the cell with centre P,

$$u_e A_e - u_w A_w + v_n A_n - v_s A_s = 0$$

Since it is the left boundary, $u_w = 0$ is known. Therefore, the equation reduces to:

$$u_e \Delta y + (v_n - v_s)\Delta x = 0 \tag{7}$$

Now, we need to simplify the x-momentum boundary equation for the left wall. We will be solving for the cell with centre at **e**.

$$a_e u_e = \sum_{nb} a_{nb} u_{nb} + (p_P - p_E)\Delta y$$

Again, since $u_w = 0$, the x-momentum equation becomes:

$$u_e = \frac{1}{a_e}(a_{Ne}u_{Ne} + a_{ee}u_{ee} + a_{Se}u_{Se}) + \frac{\Delta y}{a_e}(p_P - p_E) \tag{8}$$

For the y-momentum equation, we solve for the cell with centre at **n**.

$$a_n v_n = \sum_{nb} a_{nb} v_{nb} + (p_P - p_N)\Delta x$$

Here, we run into a problem. The left neighbour ($nW$) of the y-momentum grid is lying outside the domain. How do we actually set the boundary condition in this case? We do not know the values of velocity outside the domain. Therefore, we make an approximation. Since we know the value of y velocity at the wall (at $nw$) to be zero, we will make an assumption that the boundary velocity is the average of the velocities just inside and outside the domain. Mathematically, this reduces to:

$$\frac{v_{nW} + v_n}{2} = v_{nw} = 0$$
$$\Rightarrow \; v_{nW} = -v_n$$

By substituting this value of $v_{nW}$ into our y-momentum equation:

$$v_n = \frac{1}{(a_{nn} + a_{nW})}(a_{nn}v_{nn} + a_{nE}v_{nE} + a_s v_s + (p_P - p_N)\Delta x) \tag{9}$$

#### *5.1.2. Top Wall*

This edge is slightly different from the other edges because it has a constant x velocity of 1 m/s. The mesh structure looks something like this:

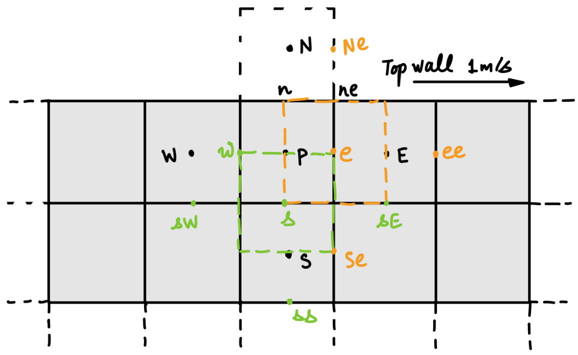


**Figure 4.** Top wall staggered mesh

The continuity equation for the P cell, in this case, reduces to:

$$(u_e - u_w)\Delta y - v_s \Delta x = 0 \tag{10}$$

This is because, for the top boundary, $v_n = 0$ is known.

Similar to what was done for the left wall can be done to the top moving wall as well. Instead of applying it to the y-momentum equation, it will have to be applied to the x-momentum equation. We are solving this for the cell centered at e.

The neighbour cell centred at $Ne$ lies outside the domain. Therefore, the x-velocity at $Ne$ will have to be extrapolated linearly.

$$\frac{u_e + u_{Ne}}{2} = u_{ne} = 1$$
$$\Rightarrow \; u_{Ne} = 2 - u_e$$

Substituting this value in the above equation, we get:

$$u_e = \frac{1}{(a_e + a_{Ne})}(2a_{Ne} + a_{ee}u_{ee} + a_{Se}u_{Se} + a_w u_w) + \frac{\Delta y}{(a_e + a_{Ne})}(p_P - p_E) \tag{11}$$

The y-momentum equation need not be solved at this edge, since for the centre of the y staggered mesh, we already have information about its velocity (boundary conditions).

### 5.1.3. Right Wall

This edge is very similar to the left edge, as almost all the conditions are the same on the right and left walls. Here is the mesh structure for the right edge:

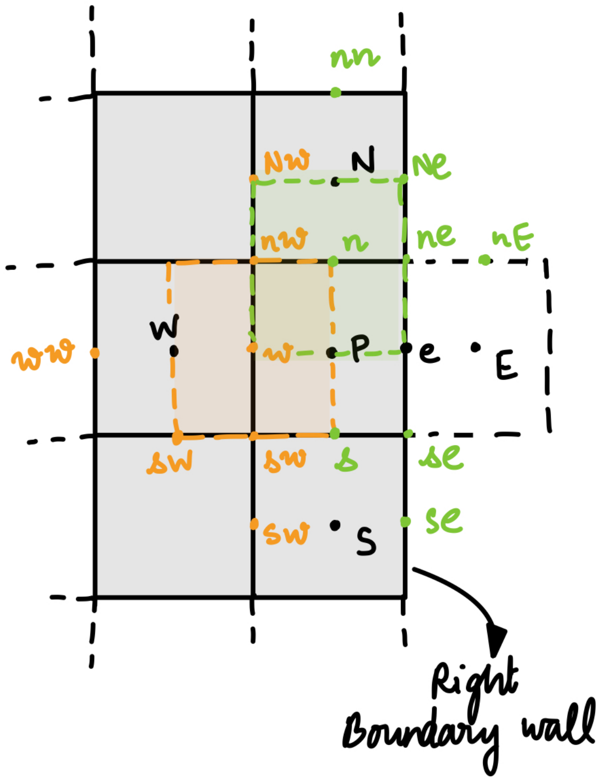


**Figure 5.** Right wall staggered mesh

The continuity equation for the P cell after substituting $u_e = 0$ is given by:

$$-u_w \Delta y + (v_n - v_s)\Delta x = 0 \tag{12}$$

Similar to the top edge, the x-momentum equation does not need to be solved for the staggered cell centred at point **e**. This is because we already know the x velocity at this point to be 0.

Now, moving on to the y-momentum equation, one of its neighbouring cell centres ($nE$) of the cell centred at $n$ lies outside the domain. Therefore, linear interpolation needs to be applied here as well.

$$\frac{v_{nE} + v_n}{2} = v_{ne} = 0$$
$$\Rightarrow \; v_{nE} = -v_n$$

Substituting this velocity into the y-momentum equation, we get:

$$v_n = \frac{1}{(a_n + a_{nE})}(a_{nn}v_{nn} + a_s v_s + a_{nW}v_{nW} + (p_P - p_N)\Delta x) \tag{13}$$

### 5.1.4. Bottom Wall

This surface is also similar to the first surface. Here is the mesh structure for the bottom part:

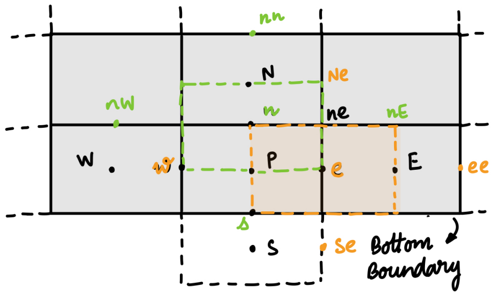


**Figure 6.** Bottom wall staggered mesh

We write the continuity equation for the cell centred at point **P**. Since $v_s$ is equal to zero, the continuity equation becomes:

$$(u_e - u_w)\Delta y + v_n \Delta x = 0 \tag{14}$$

The x-momentum equation will be solved for the cell centred at point **e**. Since we do not know the value of x velocity at point $Se$ (neighbour cell of x-staggered mesh), we shall interpolate the velocity linearly.

$$\frac{u_e + u_{Se}}{2} = u_{se} = 0$$
$$\Rightarrow \; u_{Se} = -u_e$$

This results in the following x-momentum equation:

$$u_e = \frac{1}{(a_e + a_{Se})}(a_{ee}u_{ee} + a_{Ne}u_{Ne} + a_w u_w + (p_P - p_E)\Delta y) \tag{15}$$

Since $v_s$ is equal to zero for the y-momentum staggered grid (centred at point $n$), the y-momentum equation becomes:

$$v_n = \frac{1}{a_n}(a_{nn}v_{nn} + a_{nE}v_{nE} + a_{nW}v_{nW} + (p_P - p_N)\Delta x) \tag{16}$$

## 5.2. Corners

Now that we are done with discretizing the equations for the edges, we need to discretize the equations individually for all four corners. This is because a corner can be thought of as an intersection of two edges. Hence, it needs to be carefully handled. For example, if we look at the cell centre at the top left boundary ($P$), it is simply an intersection of the top edge and the left edge boundary. Therefore, instead of having one known quantity, we will have two known quantities that have to be shifted to the RHS. This is true for all other corners, as well.

# 6. Understanding the SIMPLE algorithm

SIMPLE stands for Semi-Implicit Method for Pressure Linked Equations. It involves the following steps:

***Step 1: Guessing the Pressure Field*** $(p^*)$***, x - velocity field*** $u_e$***, and y - velocity field*** $(v_n)$

In this step, we initialize the pressure values for all the points. This is referred to as $p^*$. Along with these, we also guess the values of $u_e \& v_n$ for all the staggered points.

***Step 2: Solve for predicted velocities***

Using the momentum equations and guessed field values, we shall solve for the predicted velocities $u^*, v^*$. These are the equations:

$$u_e^* = \frac{1}{a_e}\sum_{nb} a_{nb}u_{nb} + (p_P - p_E)\frac{\Delta y}{a_e}$$
$$v_n^* = \frac{1}{a_n}\sum_{nb} a_{nb}v_{nb} + (p_P - p_N)\frac{\Delta x}{a_n}$$

***Define corrections***

$$u_e = u_e^* + u_e'$$
$$v_n = v_n^* + v_n'$$
$$p_P = p_P^* + p_P'$$

***Calculate the corrections***

Since the velocities need to satisfy both the momentum and the continuity equations simultaneously, we convert the continuity equations into the pressure Poisson equation. This equation is given by:

$$a_P p_P' = \sum_{nb} a_{nb} p_{nb}' + b \tag{17}$$

where,

$$b = (u_e^* - u_w^*)\Delta y + (v_n^* - v_s^*)\Delta x$$

The term "b" can be thought of as the residual of the continuity equation. As the prediction of velocity gets closer to the real value, b keeps getting smaller. Therefore, we will use this as our convergence criteria.

After finding the pressure corrections at all points, we now find the velocity corrections at all points. This is done by using the equation:

$$a_e u_e' = (p_P' - p_E')\Delta y \tag{18}$$
$$a_n v_n' = (p_P' - p_N')\Delta x \tag{19}$$

***Updating values***

Now that we have both the predictions and corrections, we shall calculate the final values of variables for this particular iteration.

$$u_e = u_e^* + u_e'$$
$$v_n = v_n^* + v_n'$$
$$p_P = p_P^* + p_P'$$

These are the updated values of all the necessary variables. Since b still does not tend to 0, we update these values to be our guess values as we did initially. In this way, we iterate until convergence of b is achieved.

## 7. Results and Discussions

In this section, we shall explain the results of our simulations. We will also compare these to existing validated results, which will ensure our solution is not far away from the expected results.

For our case, the kinematic viscosity is assumed to be $\nu = 0.01 m^2/s$. The velocity of the lid is $1m/s$, and the dimension of the top lid is $L = 1m$. Therefore, the Reynolds number ($R_e$) can be found using:

$$R_e = \frac{L \times u_{top}}{\nu}$$

Which gives us a Reynolds number of 100.

### Expected Results

We need to plot the variation of x-velocity, y-velocity, and pressure along the x = 0.5 and y = 0.5 centerlines for the finest grid case. Looking at already established results (From a research paper):

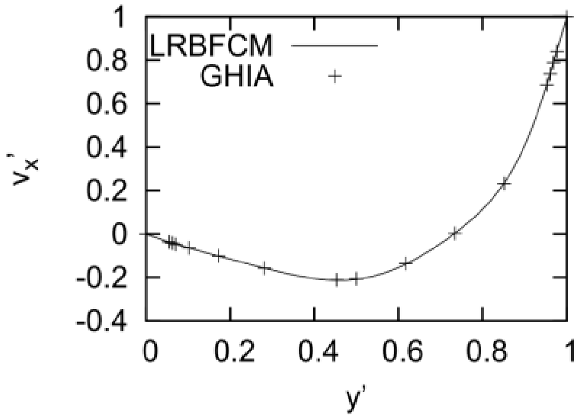


**Figure 7.** x - velocity at x = 0.5 centerline

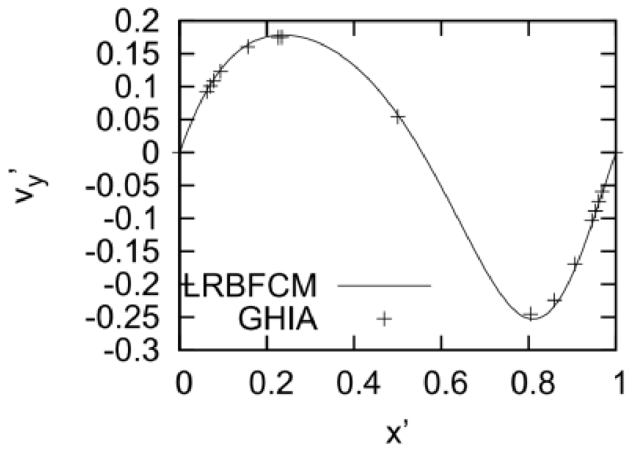


**Figure 8.** y - velocity at y = 0.5 centerline

### Our Simulation Results

***Mesh 1:*** (50 × 50) ***grid***

Here are the results of our simulation.

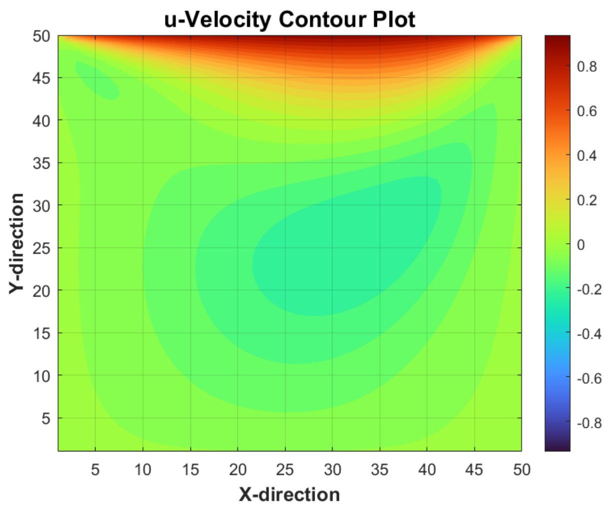


**Figure 9.** x - velocity contour plot

The contour plot of the x-velocity component (u-velocity) indicates the flow behavior influenced by the top lid's motion. The x-velocity reaches its maximum near the top boundary due to the lid's constant motion, which imposes a no-slip condition and drags the fluid along in the x-direction. As the fluid approaches the stationary right boundary, it decelerates and redirects, forming a closed recirculatory pattern.

This recirculation creates a vortex within the cavity, with its center approximately at the geometric center of the domain.

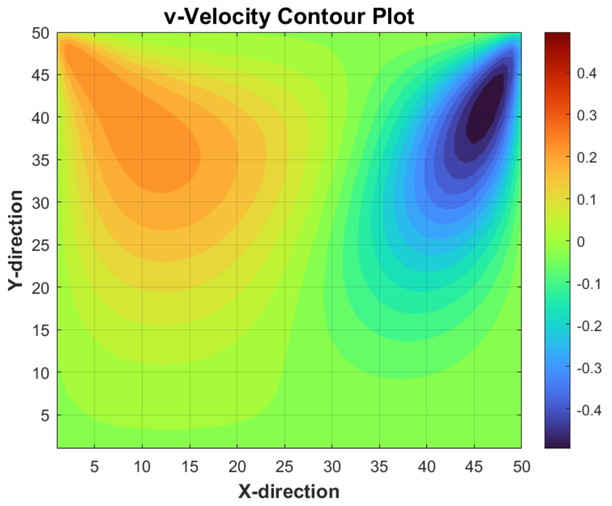


**Figure 10.** y - velocity contour plot

The maximum and minimum values of the y-velocity occur near the left and right boundaries, respectively. This is due to the rotational vortex formed by the lid's motion and the fluid's interaction with the stationary side walls.

As the lid moves, it induces circulation in the cavity, leading to upward flow near the left boundary (positive y-velocity) and downward flow near the right boundary (negative y-velocity). The zero y-velocity region in the center represents the stagnation zone of the vortex where the upward and downward motions balance each other.

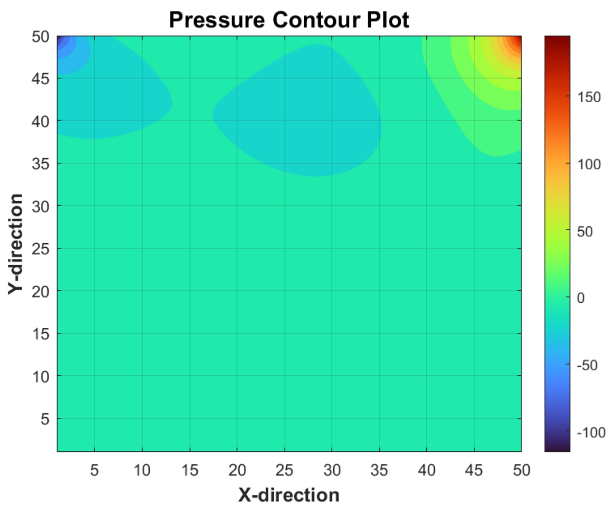


**Figure 11.** pressure contour plot

We can clearly see in the Pressure contour plot that high-pressure regions are observed near the top-right corner, where the fluid is forced against the stationary boundary by the moving lid. Similarly, low-pressure regions appear near the top-left corner, where the flow is pulled away due to the vortex's counterclockwise circulation.

The pressure gradients drive the fluid motion within the cavity, balancing the inertial and viscous forces. The near-uniform pressure field in the central region corresponds to the relatively undisturbed flow in the vortex core.

***Mesh 2:*** $(100 \times 100)$ ***grid***

Here are the contour plots for the second mesh size:

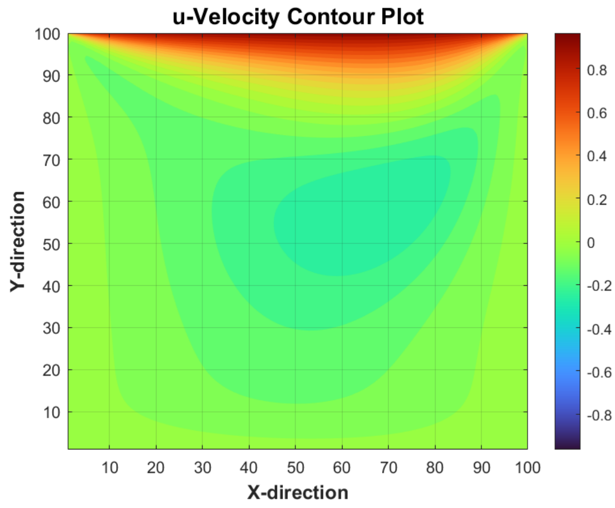


**Figure 12.** x - velocity contour plot

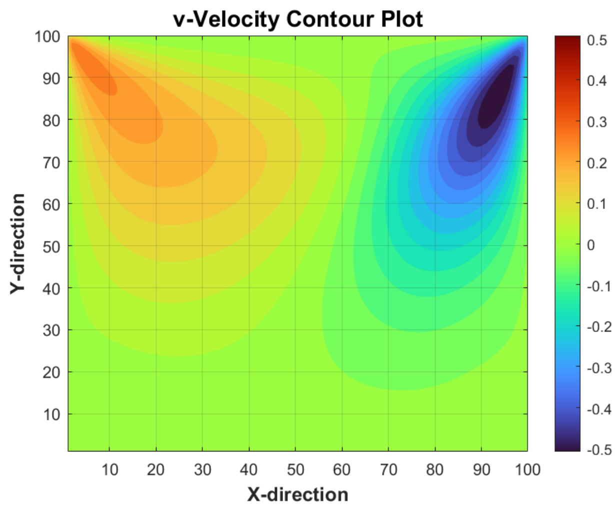


**Figure 13.** y - velocity contour plot

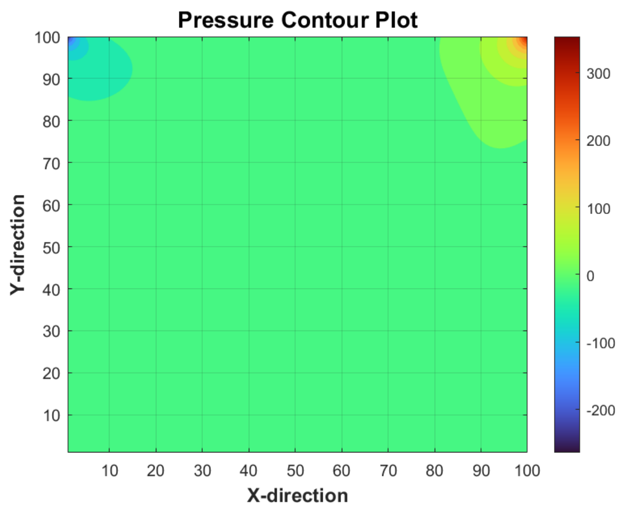


**Figure 14.** pressure contour plot

***Graphical plots***

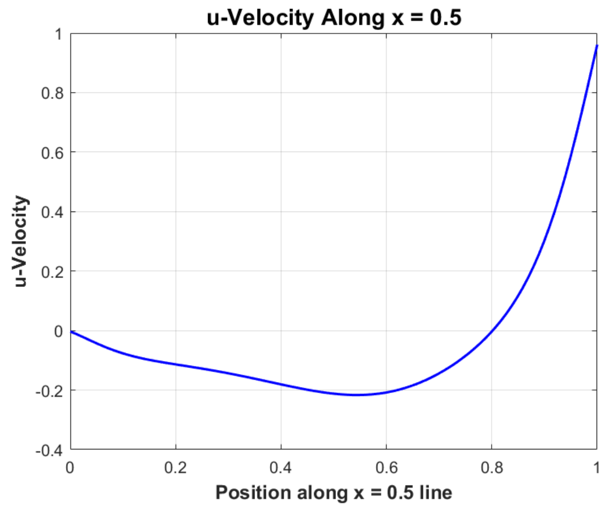


**Figure 15.** x - velocity along x = 0.5 centerline

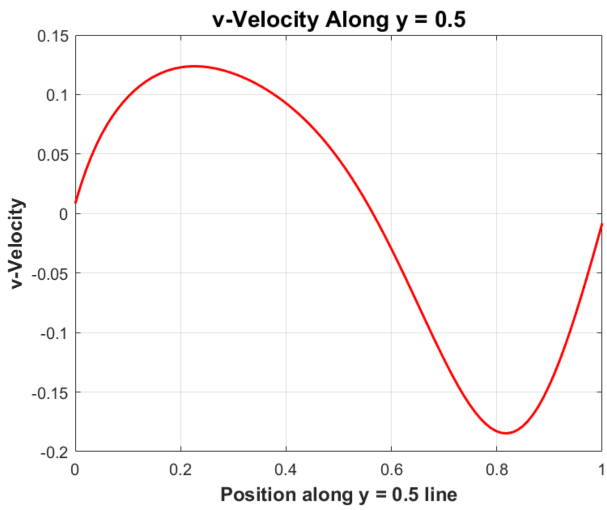


**Figure 16.** y - velocity along y = 0.5 centerline

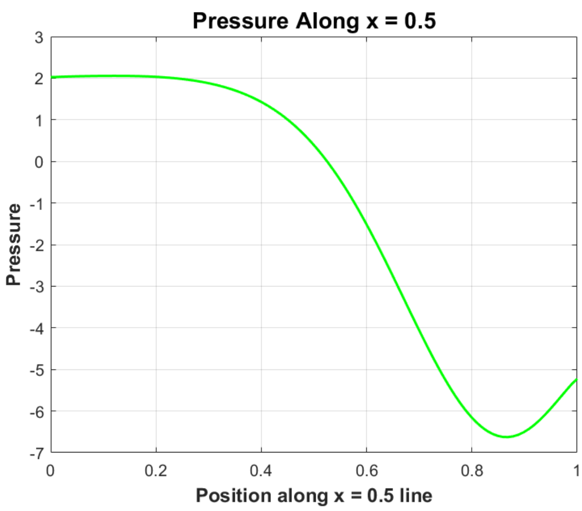


**Figure 17.** Pressure along x = 0.5 centerline

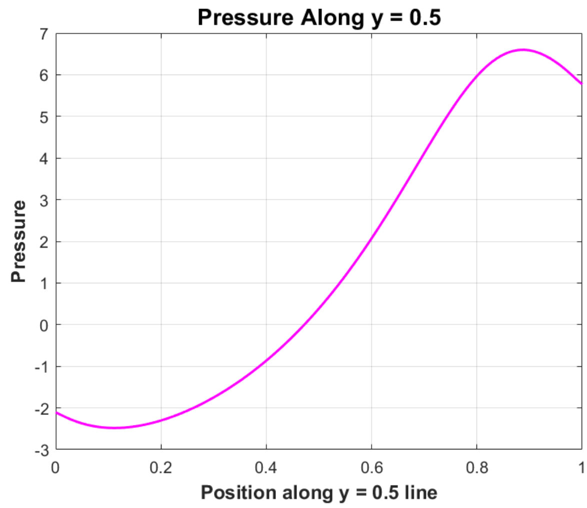


**Figure 18.** Pressure along y = 0.5 centerline

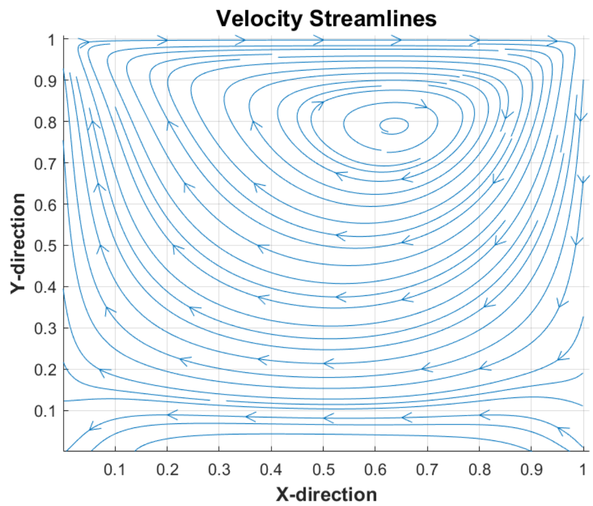


**Figure 19.** Velocity streamlines for $200 \times 200$ grid

## 8. Conclusions

The implementation of the SIMPLE algorithm to solve the lid-driven cavity flow problem has been successfully completed. To ensure the reliability of our results, we first establish grid independence by comparing the contour plots across all simulated cases. It is evident that the contour plots remain consistent, with minimal changes observed between different grid resolutions. Due to time constraints and computational complexity, we were unable to run the simulation for the third and final grid size of $200 \times 200$. However, the above results do show that mesh independence is achieved.

Additionally, when comparing our results to those reported in the referenced paper, the centerline velocity profiles show good agreement. However, it was noted that increasing the number of grid points significantly increased the computational time required for the algorithm to converge.

## 9. Acknowledements

We would like to thank Prof. Dilip Srinivas Sundaram for giving us the opportunity to work on this project. I would also like to thank the TAs of this course who helped me solve the bugs in my code in the tutorial sessions.

## 10. References

[1] Santhosh Kumar, D., Suresh Kumar, K., and Kumar Das, M. (2009). "A Fine Grid Solution for a Lid-Driven Cavity Flow Using Multigrid Method." *Engineering Applications of Computational Fluid Mechanics*, **3**(3), 336–354. https://doi.org/10.1080/19942060.2009.11015275.

[2] K. Mramor, R. Vertnik, and B. Šarler, "Low and Intermediate Re Solution of Lid Driven Cavity Problem by Local Radial Basis Function Collocation Method," *Computers, Materials & Continua*, vol. 36, no. 1, pp. 1–20, 2013. https://cdn.techscience.cn/files/cmc/2013/v36n1/cmc.2013.036.001.pdf.