ME 605 | Computational Fluid Dynamics

Project 1

Due: 11:59 pm, September 8, 2024

Instructions

1. You can choose any programming language of your choice.
2. Do not use any in-built or intrinsic functions. You are expected to write your own computer program (including for solving the system of algebraic equations)
3. You are permitted to work in groups of maximum two students. The responsibility of forming groups lies with the students. Each group is expected to submit the project report and the code (one submission per group). Discussion among students (even across groups) is permitted.
4. Your report must consist of (1) problem statement, (2) mesh details and approach for discretization; (3) derivation and presentation of the final form of the discretized equations; (4) solution methodology, (5) results and discussion; (6) concluding remarks. Note that an in-depth analysis and discussion of results is required.
5. Report must be prepared using WORD or LaTeX. Handwritten reports will not be accepted.
6. Submit the project report and the code in Google classroom.

Project Statement: Computer Solution of Elliptic PDEs

You are required to write a computer program and solve the following 2D steady-state diffusion equation using the finite difference method:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = S_\phi$$

on a square domain of unit length. The boundary conditions are as follows:

$$\phi(0, y) = 500 \exp(-50[1 + y^2])$$

$$\phi(1, y) = 100(1 - y) + 500 \exp(-50y^2)$$

$$\phi(x, 0) = 100x + 500 \exp(-50[1 - x]^2)$$

$$\phi(x, 1) = 500 \exp(-50\{[1 - x]^2 + 1\})$$

The source term is given by:

$$S_\phi = 50000 \exp(-50\{[1 - x]^2 + y^2\})(100\{[1 - x]^2 + y^2\} - 2)$$

For reference, the analytical solution is given below:

$$\phi(x, y) = 500 \exp(-50\{[1-x]^2 + y^2\}) + 100x(1-y)$$

(1) Solve the 2D steady-state diffusion equation using Gaussian elimination method for the following grids: 21, 41, and 81 grid points in each direction. Show the computed ϕ field as a contour plot for the finest grid. Plot the CPU run time vs total number of grid points and discuss the trend and comment on the computational efficiency of the Gauss elimination method.

(2) Solve the same 2D steady-state diffusion equation using the Gauss-Seidel iterative method for the following three grids: 41, 81, and 161 grid points in each direction. Show the computed ϕ field as a contour plot for the finest grid. Plot the residual vs number of iterations for the three grids in the same plot. Compare and discuss the dependence of the convergence rate on the total number of grid points. Plot the variation of CPU run time with total number of grid points for the Gauss-Seidel iterative method and discuss the trend. How do the CPU run times of the Gauss-Seidel iterative method compare with those of Gauss elimination method?

(3) Solve the same 2D steady-state diffusion equation using the line-by-line method (row sweep) for the following three grids: 41, 81, and 161 grid points in each direction. Please note that you will need to use the TDMA solver to solve the resulting system of linear equations because of the tridiagonal structure of the resulting coefficient matrices. Show the computed ϕ field as a contour plot for the finest grid. Plot residual vs number of iterations for the line-by-line method and for the Gauss-Seidel method for the 161×161 grid (both in the same plot). Compare and discuss the trends. Plot the variation of CPU run time with total number of grid points for the line-by-line method. How does the CPU run times and the obtained scaling compare with those obtained for the Gauss elimination method and point-wise iterative method?

(4) Solve the same 2D steady-state diffusion equation using the Alternating Direction Implicit (ADI) method for the 41×81 grid and plot the residual vs number of iterations for row-wise sweep, column-wise sweep, and ADI method in the same plot. Discuss the trends.