ME 605 | Computational Fluid Dynamics

Project 2

Due: 11:59 pm, 24 September 2024

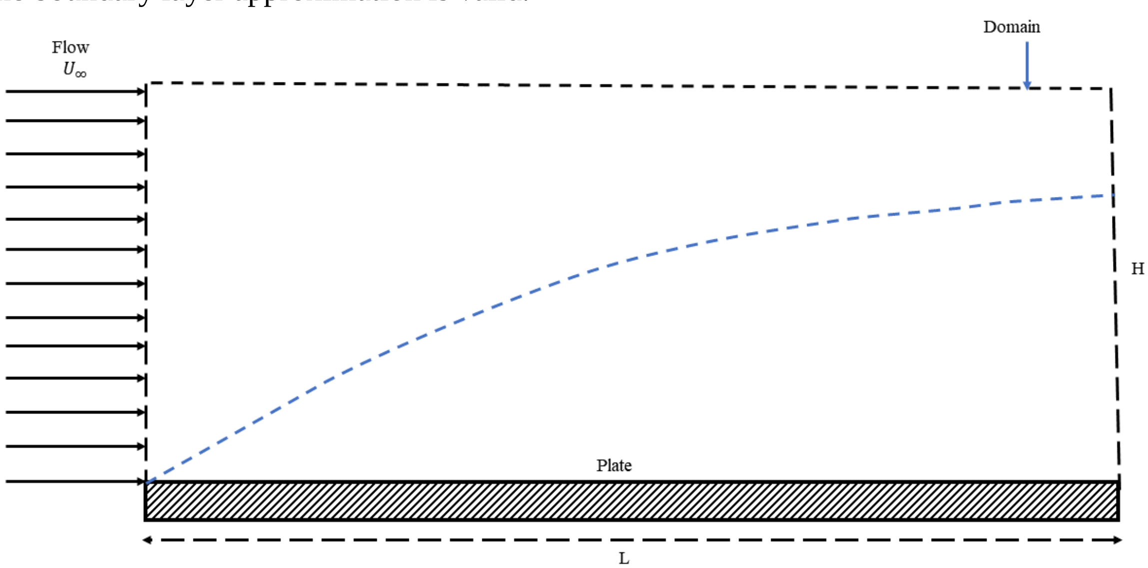
Instructions

1. You can choose any programming language of your choice.
2. Do not use any in-built or intrinsic functions. You are expected to write your computer program (including for solving the system of algebraic equations)
3. You are permitted to work in groups of a maximum of two students. The responsibility of forming groups lies with the students. Each group is expected to submit the project report and the code (one submission per group). Discussion among students (even across groups) is permitted.
4. Your report must consist of (1) a problem statement, (2) mesh details and approach for discretization, (3) derivation and presentation of the final form of the discretized equations, (4) solution methodology, (5) results and discussion; (6) concluding remarks. Note that an in-depth analysis and discussion of results is required.
5. The report must be prepared using WORD or LaTeX. Handwritten reports will not be accepted.
6. Submit the project report and the code in Google Classroom.

Project Statement

Computational Solution of Viscous Flow over a Flat Plate

Consider the flow of a viscous fluid over a flat plate of length L, as shown below. The fluid enters the simulation domain of height H at a uniform velocity, U_∞. Assume that the flow is steady, laminar, and incompressible. Further, assume that the boundary layer approximation is valid.



The resulting governing equations for this specific problem are given below:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

where ν is the kinematic viscosity of the fluid. You are required to write a computer program to solve the above boundary layer equations for this problem for a Reynolds number $Re_L = 10^4$, where

$$Re_L = \frac{U_\infty L}{\nu}$$

The plate length (L), kinematic viscosity (ν), and free-stream velocity (U_∞) should be chosen such that $Re_L = 10^4$. Use the finite difference method for discretization of the PDEs and solve the PDEs computationally using the following schemes:

(1) Euler Explicit scheme. You will have to choose Δx and Δy carefully, as the explicit scheme is not always stable.
(2) Euler Implicit scheme.
(3) Crank-Nicolson scheme.

For each of the above three schemes, you are required to:

1. Compute the x-velocity (u) and y-velocity (v) fields. fields. Show velocity fields as contour plots. Further, plot the normalized x-velocity (F') as a function of similarity variable (η) as a line plot and compare with the Blasius solution. The variables are defined below:

   $$F'(\eta) = \frac{u}{U_\infty} \text{ and } \eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

   Note that you can vary the similarity variable (η) by varying y coordinate at a specific x-location or by varying x coordinate at a specific y location. You may pick one of the two options to show the comparison between your simulation predictions with the Blasius solution.

2. Compute the boundary layer thickness. Plot the variation of boundary layer thickness with x coordinate. How does your simulation result compare with the Blasius flat plate boundary layer solution given below?

   $$\frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}}$$