3.4 Dynamic system $\frac{d}{dt}\xi = -\xi + u\eta^3$ (I)

$0 = -\eta + u^2 e^{\eta}$ (II)

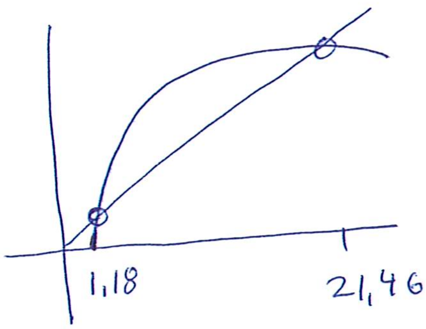
a) stationary operating point (ξ_0, u_0, η_0). $\xi_0 = 1$

$$\begin{cases} 0 = -1 + u_0\eta_0^3 & \Rightarrow u_0 = \frac{1}{\eta_0^3} \\ 0 = -\eta_0 + u_0^2 e^{\eta_0} & \Rightarrow \eta_0 = \left(\frac{1}{\eta_0^3}\right)^2 e^{\eta_0} \end{cases}$$

$\Rightarrow \eta_0^7 = e^{\eta_0}$ $\quad 7 \cdot \ln(\eta_0) = \eta_0$

ln

graphical solution



$\eta_0 = 1{,}18$

$u_0 = \frac{1}{(1{,}18)^3} \approx 0{,}60$

3,4 b) output should be $y = \eta \cdot \xi$.

Input u.

State ξ. Linearize.

$$\frac{d}{dt}(\Delta\xi) = \frac{\partial f}{\partial \xi}\Big|_0 \Delta\xi + \frac{\partial f}{\partial u}\Big|_0 \Delta u \qquad \text{for } f = \frac{\partial \xi}{\partial t}$$

$$\frac{\partial f}{\partial \xi} = -1 \qquad \frac{\partial f}{\partial u} = 1 \cdot \eta_0^3 + \xi_0 3\eta_0^2 \cdot \frac{\partial \eta}{du}\Big|_0$$

$$\frac{d\eta}{du} = \left(\frac{du}{d\eta}\right)^{-1} \qquad \text{from II} \quad u^2 = \frac{\eta}{e^{\eta}} \Rightarrow u = \eta^{0,5} \cdot e^{-\eta \cdot 0,5}$$

$$\frac{du}{d\eta} = 0,5 \cdot \eta^{-0,5} \cdot e^{-\eta \cdot 0,5} + \eta^{0,5} \cdot (-0,5) \cdot e^{-\eta \cdot 0,5} =$$

$$= \frac{0,5}{e^{\eta \cdot 0,5}}\left(\frac{1}{\sqrt{\eta}} - \sqrt{\eta}\right)\Big|_0 = -0,0468 \Rightarrow \frac{d\eta}{du} = -21,3$$

$$\frac{\partial f}{du} = (1 \cdot 1,18^3 + 3 \cdot 0,6 \cdot 1,18^2 \cdot (-21,3)) = -52,4$$

$$\frac{\partial y}{\partial \xi}\Big|_0 = \eta_0 \qquad \frac{\partial y}{du} = \frac{\partial y}{\partial \eta} \cdot \frac{\partial \eta}{du}\Big|_0 = \xi_0 \cdot (-21,3)$$

$$\therefore \begin{cases} \frac{d}{dt}(\Delta\xi) = -1\,\Delta\xi - 52,4\,\Delta u \\ y = 1,18\,\Delta\xi + (-21,3)\,\Delta u \end{cases}$$

c) system stability:

$$G = C(sI - A)^{-1} + D = \frac{1,18}{s+1} + (-21,3)\frac{(s+1)}{(s+1)}$$

$\Rightarrow$ pole in -1 independent of operating point

3.7 $\frac{dz}{dt} = Az + Bu$ $\frac{dy}{dt} = Ay + Bu$

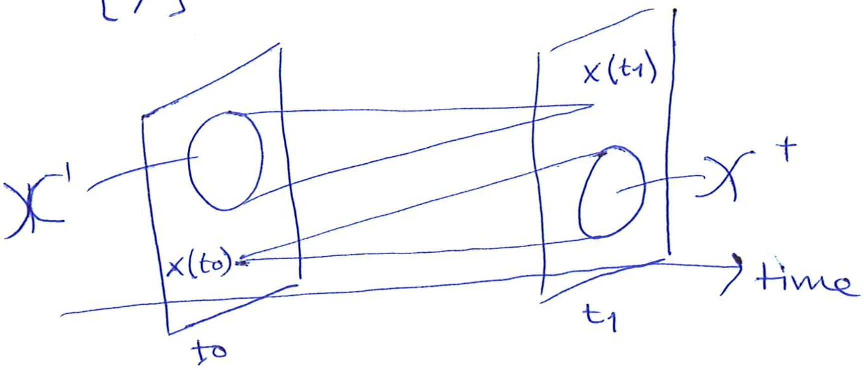
a) y & z same initial conditions $\Rightarrow y = z \ \forall u$

reachability for $x = \begin{bmatrix} z \\ y \end{bmatrix}$

Time-invariant linear continious system
reachable = controllable

We want to control the system, using u, to any state $x^* \in \mathbb{R}^n$

However, z & y are dependent!



$\mathcal{X}^-$ controllability all set of initial states $x(t_0)$ controllable to give final state $x(t_1)$

$\mathcal{X}^+$ reachability the set of all final states $x(t_1)$ reachable starting from a given state $x(t_0)$

OBS Time-invariant, continious, linear system $\Rightarrow \mathcal{X}^- = \mathcal{X}^+$

b) Show that controllability matrix does not have full rank.

ex. $\begin{cases} y = 10\,x(t) & \leftarrow \text{time invariant} \\ y = t \cdot x(t) & \leftarrow \text{NOT time-invariant} \end{cases}$

$x = \begin{bmatrix} z \\ y \end{bmatrix}$ $\dot{x} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}}_{\tilde{A}} x + \underbrace{\begin{bmatrix} B \\ B \end{bmatrix}}_{\tilde{B}} u$

$S = [B \ \ AB \ \dots \ A^{n-1}B]$ full rank $\Rightarrow$ controllable

$\tilde{A}\tilde{B} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} B \\ B \end{bmatrix} = \begin{bmatrix} AB \\ AB \end{bmatrix}$ $\tilde{A}(\tilde{A}\tilde{B}) = \begin{bmatrix} AAB \\ AAB \end{bmatrix}$

$\Rightarrow S_x = \begin{bmatrix} B & AB & AAB & \\ B & AB & AAB & \dots \end{bmatrix}$ $\leftarrow$ Top half is equivalent to the bottom half

$\Rightarrow S_x$ will never have full rank!

3.9 Assum (based on what we have given)

volume V is constant. (Q in & Q out)

$\Rightarrow Q + R = F$

Energy balance: Accumulated ~~heat~~ energy = Flow in − Flow out + generated energy

(1) $\left[\frac{J}{s}\right]$ $\frac{d}{dt}\left(V \cdot T_1 \cdot C_p \cdot \rho\right) = Q C_p \rho T_0 + R C_p \rho T_2 - F \rho C_p \cdot T_1 + \theta P$

$\left[\frac{J}{s}\right]$ $\left[m^3\right]\left[K\right]\left[\frac{J}{kgK}\right]\left[\frac{kg}{m^3}\right]$

$$\Rightarrow \frac{d}{dt} T_1 = \frac{1}{V}\left(Q T_0 + R T_2 - F T_1\right) + \frac{\theta}{V \rho C_p} \cdot p(t)$$

(2) $$\frac{d}{dt} T_2 = \frac{1}{V}\left(F T_1 - R T_2 - Q T_2\right) + \frac{(1-\theta)}{V \rho C_p} p(t)$$

Given: states: T_1 & T_2 input: $p(t)$ disturbance: T_0

$$\frac{d}{dt}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -F/V & R/V \\ F/V & -(R+Q)/V \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \frac{1}{V\rho C_p}\begin{bmatrix} \theta \\ 1-\theta \end{bmatrix} p(t) + \begin{bmatrix} Q/V \\ 0 \end{bmatrix} T_0$$

b) Condition number of controllability matrix S.

$S = \begin{bmatrix} B & AB \end{bmatrix}$ $V = 1$ $R = Q = 1 \Rightarrow F = 2$ $\Rightarrow A = \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix}$ $B = \frac{1}{\rho C_p}\begin{bmatrix} \theta \\ 1-\theta \end{bmatrix}$

$$S = \frac{1}{\rho C_p}\begin{bmatrix} \theta & 1-3\theta \\ 1-\theta & 4\theta - 2 \end{bmatrix}$$

$$AB = \frac{1}{\rho C_p}\begin{bmatrix} -2\theta + 1 - \theta \\ 2\theta - 2 + 2\theta \end{bmatrix}$$

condition number $= \sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$ for λ_i = eigenvalue of $S \cdot S^T$

if $\lambda_{min} \to 0$ condition nr gets large

close to instability

3o11

Determine the pole & zero polynomials of the system.

Lowest order of state required?

chapter 3.3 Poles & Zeros page 48.

theorem 3.5 poles $p(s)$ = least common denominator of all minors of $G(s)$

pole polynomial

~~minor~~ of A : determinant of a square sub-matrix of A

$$G(s) = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad \underbrace{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}_{iii} \qquad a,b,c,d,e,f \; \text{ã} \; \frac{\alpha}{s+2}$$

(i: columns a, d and b, e; ii: columns b, e and c, f)

2x2 minors:

i) $$\begin{vmatrix} \frac{1}{s+2} & \frac{-1}{s+2} \\ \frac{1}{s+2} & \frac{s+1}{s+2} \end{vmatrix} = \frac{s+1}{(s+2)^2} - \frac{-1}{(s+2)^2} = \frac{s+1+1}{(s+2)^2} = \frac{1}{s+2}$$

ii) $$\frac{-1}{(s+2)^2} - \frac{(s+1)}{(s+2)^2} = \frac{-1}{s+2}$$

iii) $$\frac{1}{(s+2)^2} - \frac{1}{(s+2)}^2 = 0$$

Least common denominator

$p(s) = s+2$

$\Rightarrow$ pole in -2

theorem 3.6. Zeros.

Look at ~~the~~ the greatest common division for the numerators of maximal minors.

Here : 1 $\Rightarrow$ no zero.

page 51.

"Total nr. of poles is equal to the order of minimum state representation"

$\Rightarrow$ 1 state needed.