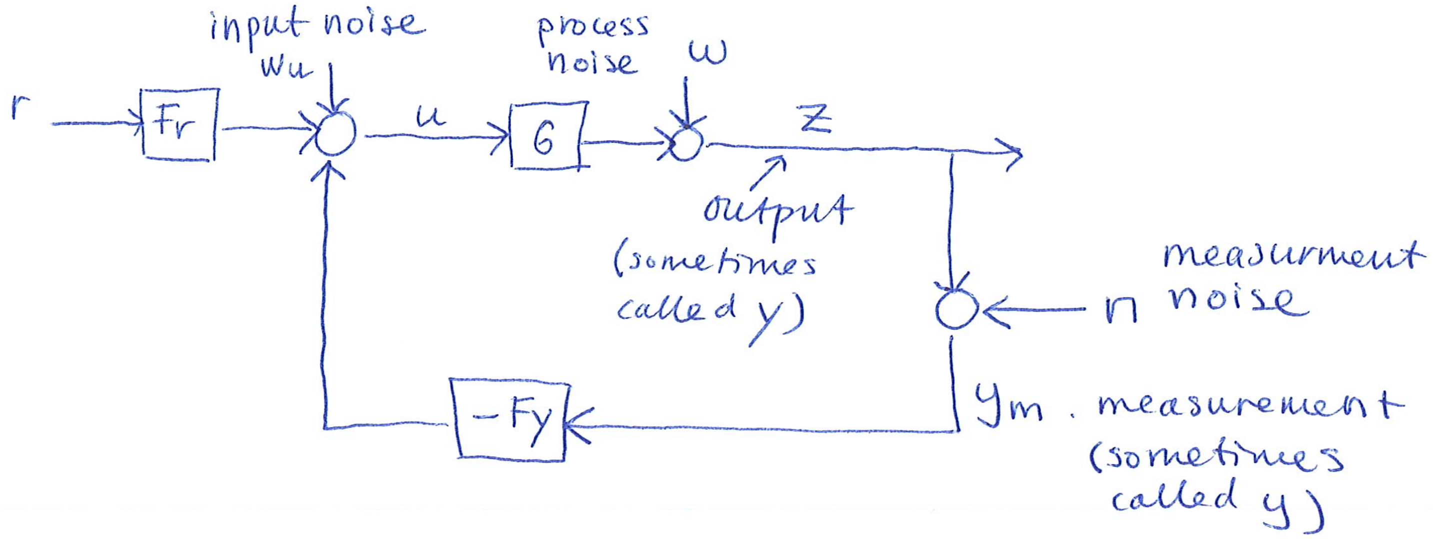
Start point for several of todays excersises:



$$Z = W + G(W_u + F_r \cdot R - F_y(N + Z))$$

$$Z = \underbrace{(I + GF_y)^{-1}}_{= S} \cdot W + (I + GF_y)^{-1} \cdot G \cdot W_u + (I + GF_y)^{-1} GF_r \cdot R - \underbrace{(I + GF_y)^{-1} GF_y}_{= T} \cdot N$$

sensitivity function

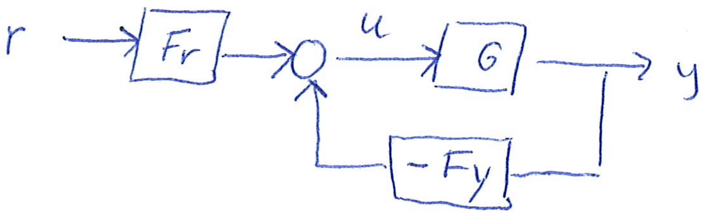
complementary sensitivity function

$$U = W_u + F_r \cdot R - F_y(N + W + G \cdot U)$$

$$U = (I + F_y \cdot G)^{-1} \left[W_u + F_r \cdot R - F_y \cdot N - F_y \cdot W \right]$$

7.01 Given: $G(s) = \frac{s-3}{s+1}$ $\qquad T = \frac{5}{s+5}$

a) Determine $F_r = F_y = F$. Will it work?



Scalar case: $T = \frac{GF_y}{1+GF_y}$ solve for $F_y = F$

$$\Rightarrow F = \frac{T}{G(1-T)} = \frac{\frac{5}{s+5}}{\frac{s-3}{s+1}\left(1 - \frac{5}{s+5}\right)} = \frac{5(s+1)}{(s-3)(s+\cancel{5}-\cancel{5})} = \frac{5(s+1)}{s(s-3)}$$

$Y = G(F_r \cdot R - F_y \cdot Y)$ for $F = F_r = F_y \Rightarrow Y = \underbrace{\frac{GF}{1+GF}}_{T} \cdot R$

T is stable

$\Rightarrow$ input output stable

<u>BUT</u> F is unstable. check internal stability.

$U = F_r \cdot R - F_y \cdot G \cdot U \; \{F_r = F_y = F\} \Rightarrow U = \underbrace{\frac{F}{1+FG}}_{G_{ru}} \cdot R$

$$G_{ru}(s) = \frac{\frac{5(s+1)}{s(s-3)}}{1 + \frac{5(s+1)}{s(s-3)} \cdot \frac{(s-3)}{(s+1)}} = \frac{5(s+1)}{s(s-3)\left(1 + \frac{5}{s}\right)} = \frac{5(s+1)}{(s-3)(s+5)}$$

unstable poles $\Rightarrow$ not internally stable!

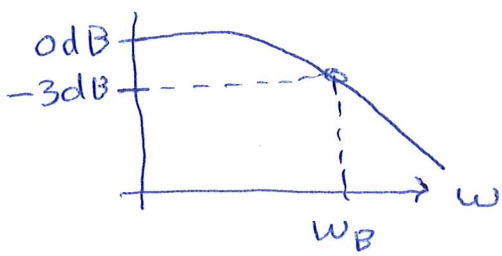
(b) Suggest alternative T. still having bandwidth 5 rad/s but results in a stable system.

Recap: Bandwidth ω_B, frequency when the amplitude has decreased 3 dB from the gain at $\omega = 0$

$$X\,dB = 20 \cdot \log_{10}(|G|) \iff 10^{(X/20)} = |G|$$

$$T(0) = \frac{5}{5} = 1 \Rightarrow 20 \underbrace{\log(1)}_{0} = 0\,dB$$

bode(T)



$$T(\omega \to \infty) = 0 \to -\infty\,dB$$

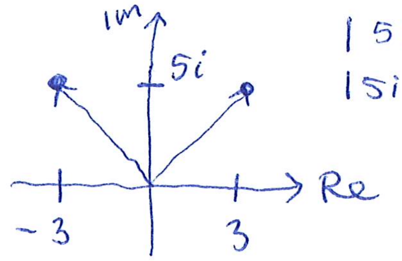
$$-3dB = 10^{-3/20} = \frac{1}{10^{3/20}} \approx \frac{1}{1{,}42} \approx \frac{1}{\sqrt{2}}$$

$$|T(5i)| = \left|\frac{5}{5i+5}\right| = \left|\frac{1}{i+1}\right| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \approx -3dB \Rightarrow \omega_B = 5 \frac{rad}{s}$$

Suggestion new T(s):

* Do not cancel out the zero in (s-3)
* Add (s+3) in denominator to get the same bandwidth

$$T(s) = \frac{5(s-3)}{(s+3)(s+5)}$$



$|5i-3| = |5i+3|$

$$|T(0)| = |-1| = 1$$

$$|T(5i)| = \left|\frac{5(5i-3)}{(5i+3)(5i+5)}\right| = \frac{1}{\sqrt{2}}$$

new controller: $$F = \frac{T}{G(1-T)} = \frac{\dfrac{5\cancel{(s-3)}}{(s+3)(s+5)}}{\dfrac{\cancel{s-3}}{s+1}\left(1 - \dfrac{5(s-3)}{(s+3)(s+5)}\right)} = \frac{5(s+1)}{(s+3)(s+5) - 5(s-3)}$$

$$= \frac{5(s+1)}{s^2+3s+30} \Rightarrow \text{poles in } s = -1{,}5 \pm i\sqrt{27{,}75}$$

All poles in G_{ru} will be in LHP

(c) Sensitivity function $S(s) = \frac{1}{1+GF}$

$$T = \frac{GF}{1+GF}$$

$$S = \frac{1+GF-GF}{1+GF} = 1 - T = 1 - \frac{5(s-3)}{(s+3)(s+5)}$$

$$= \frac{(s+3)(s+5) - 5(s-3)}{(s+3)(s+5)} = \frac{s^2+3s+30}{(s+3)(s+5)}$$

Note: $S(0) = \frac{30}{3 \cdot 5} = 2$

(d) Different F_r & F_y $\Rightarrow$ more degrees of freedom

F_y affects T & S $\Rightarrow$ how process noise & measurement noise affects the output

use F_r to get the desierd bandwidth from input to output

(7.6) (i) Disturbance on output dampen 1000 $\omega < 2$ rad/s

(ii) Stable in spite of model uncertainty

$$\underbrace{|\Delta G|}_{\text{absolute error}} \leq 100|G| \quad \text{for } \omega > 20 \text{ rad/s}$$

(i) $\Rightarrow$ $$Z = S\, W + S \cdot G \cdot W_u + S \cdot G \cdot F_r \cdot R - T \cdot N$$

$\uparrow$ disturbance on output

$$|S(i\omega)| < 1/1000 \quad \forall\, \omega < 2 \text{ rad/s}$$

Robustness chapt 6.3

eq 6.29 $$|T(i\omega)| < \frac{1}{|\Delta_G(i\omega)|} \quad \forall\, \omega \Rightarrow \text{stable syst}$$

here Δ_G is <u>relative</u> model error

scalar case.

$$\Rightarrow |\Delta_G| = \frac{|\Delta G|}{|G|} \leq 100$$

$$S = \frac{1}{1+GF} \qquad |S(i\omega)| = \left|\frac{1}{1+G(i\omega)F(i\omega)}\right| < \frac{1}{1000} \quad \text{for } \omega < 2$$

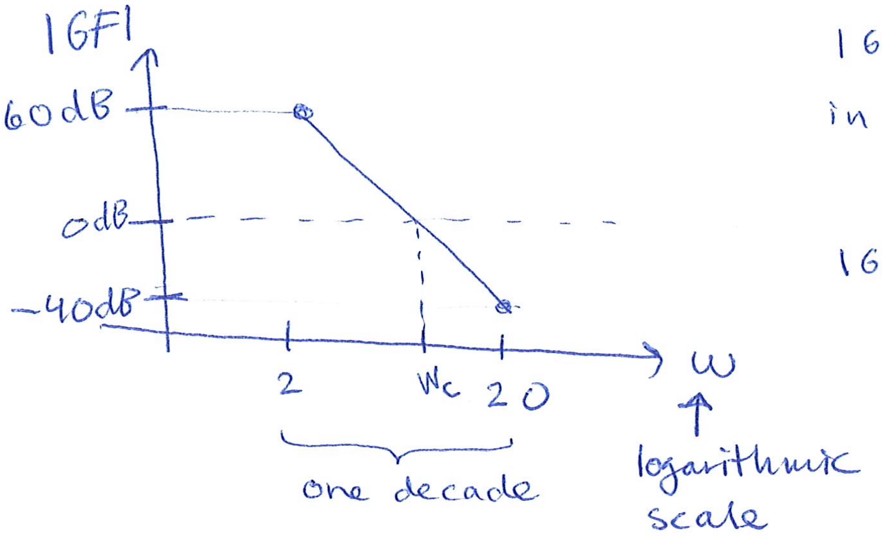
$$|G(i\omega)F(i\omega)| \gg 1 \Rightarrow \frac{1}{|GF|} < \frac{1}{1000} \Rightarrow |GF| > 1000 \ \forall\, \omega < 2$$

stability requirers: $$|T| < \frac{1}{|\Delta_G|} < \frac{1}{100} \quad \forall\, \omega > 20 \text{ rad/s}$$

$$|T| = \left|\frac{GF}{1+GF}\right| < \frac{1}{100} \Rightarrow GF \text{ is "small" compared to } 1 \Rightarrow$$

$$|T| \approx |GF| < \frac{1}{100} \quad \forall\, \omega > 20 \text{ rad}$$

Look at the loop gain $|GF|$:



$|GF| > 1000 \;\; \forall \, \omega < 2$

in dB: $20 \cdot \underbrace{\log(10^3)}_{3} = 60\,dB$

$|GF| < 10^{-2} \;\; \forall \, \omega > 20$

$20 \cdot \underbrace{\log(10^{-2})}_{-2} = -40\,dB$

"Amplitude & phase are coupled" page 177-179

if the slope of the amplitude curve is $-\alpha$

$|GF|$ decrease with $20 \cdot \alpha$ dB/decade

$\Rightarrow$ phase loss at least $-\alpha \, \pi/2$

here 100 dB loss in 1 decade $\Rightarrow$ $\alpha = 100/20 = 5$
(2 → 20 dB)

phase loss of $-5 \cdot \pi/2 = -2{,}5\,\pi$

* stability requiers positive phase margin
$\Rightarrow$ $\arg(GF) > -\pi$ at w_c where $|GF|_{w_c} = 1$