SSY285 - Home Assignment M3

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Introduction

In this assignment, Linear state estimation and control of DC-motor with flywheel as given by in the previous assignments according to the figure 1 is done, it consists of an electric motor which drives a flywheel and it is influenced by an external torque. In this assignment we deal with .

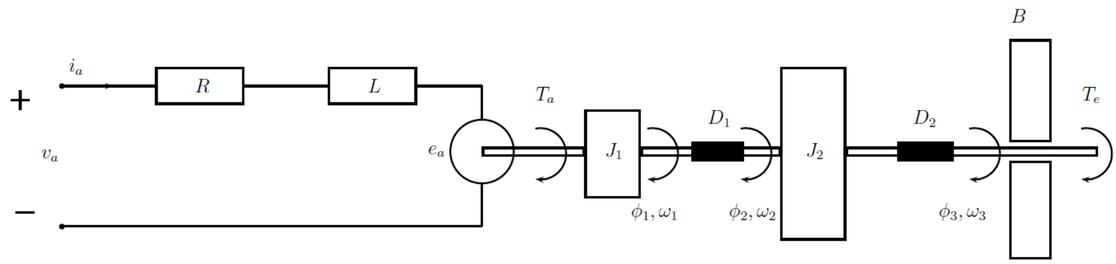

Figure 1: DC motor with flywheel

Various symbols used in this assignment and its respective description is given by the table 1. These symbols would be used in all the tasks of this assignment.
According to the assignment matrices A,B,C and D is to be taken from the assignment 1. Hence, the respective matrices are as follows:

$$A = \begin{bmatrix} -\frac{K_T \cdot K_E}{J_1 \cdot R} & 0 & -\frac{D_1}{J_1} & \frac{D_1}{J_1} & 0 \\ 0 & 0 & \frac{D_1}{J_2} & -(\frac{D_1}{J_2} + \frac{D_2}{J_2}) & \frac{D_2}{J_2} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{D_2}{B} & -\frac{D_2}{B} \end{bmatrix} \tag{1}$$

Table 1: Description of the symbols used in the assignment

Symbol	Description
v_a	External voltage applied to the rotor
i_a	Rotor current
e_a	Induced rotor voltage
L	Rotor Inductance
R	Rotor resistance
K_E	Coefficient related to induced voltage e_a
T_a	Rotor produced torque
K_T	Coefficient related to rotor torque constant driving rotor current i_a
ϕ_1, ϕ_2, ϕ_3	Angles
$\omega_1, \omega_2, \omega_3$	Angular speeds
J_1	Rotor inertia
J_2	Flywheel inertia
D_1, D_2	Torsional springs
B	Linear friction proportional to the angular speed
T_e	External torque applied to flywheel axis

$$B = \begin{bmatrix} \frac{K_T}{J_1 \cdot R} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4}$$

The value for the parameters are given by the table 2.

Table 2: Parameters and its values

Parameter	Value
R	$1\,\Omega$
K_E	$10^{-1}\,\mathrm{V\,s\,rad^{-1}}$
K_T	$10^{-1}\,\mathrm{N\,m\,A^{-1}}$
J_1	$10^{-5}\,\mathrm{kgm^2}$
J_2	$4\times10^{-5}\,\mathrm{kgm^2}$
D_1	$20\,\mathrm{N\,m\,rad^{-1}}$
D_2	$2\,\mathrm{N\,m\,rad^{-1}}$
B	$2\times10^{-3}\,\mathrm{N\,m\,s}$

From the assignment 2, A_d and B_d matrices are considered in this assignment.

$$A_d = e^{A_c T_s} = \begin{bmatrix} -0.0489 & 0.5946 & -782.8328 & 773.7475 & 9.0853 \\ 0.1491 & 0.7749 & 342.5299 & -370.9589 & 28.4290 \\ 4.499\,93\times10^{-4} & 2.3406\times10^{-4} & 0.4010 & 0.5964 & 0.0026 \\ 5.8514\times10^{-5} & 9.1912\times10^{-4} & 0.2076 & 0.7747 & 0.0175 \\ 1.3088\times10^{-5} & 3.5054\times10^{-4} & 0.0585 & 0.5686 & 0.3729 \end{bmatrix} \tag{5}$$

$$B_d = \begin{bmatrix} 4.499 & 1.309 \\ 0.5851 & 8.763 \\ 0.003154 & 0.0002857 \\ 0.0001595 & 0.003211 \\ 2.857\times10^{-5} & 0.3168 \end{bmatrix} \tag{6}$$

Question A

In this task, we assume that the noise is added to both the inputs. Inputs are external V_a and T_e. Both the input noises are having zero mean, uncorrelated sequences and normally distributed to account for 99.7% of the realisation. The upper bounds are 0.3 V and 0.1 N m (10% of the T_e, i.e 1 N m). The task is to propose a co-variance matrix for the disturbance matrix $w(t)$ and N matrix to be used in

$$x(t+1) = Ax(t) + Bu(t) + Nw(t). \tag{7}$$

Since both the noise is having 99.7% realisation for normal distribution. From standard normal distribution table(Appendix) we can find z number and $z = 2.77$.

$$\text{Standard deviation} = \frac{X - \mu}{z} \tag{8}$$

$$variance = \text{Standard deviation}^2 \tag{9}$$

For both cases mean $\mu = 0$, X = 0.3 V and 0.1 N m. The variance matrix is

$$w(t) = \begin{bmatrix} 0.011728 \\ 0.001303 \end{bmatrix} \tag{10}$$

and since the noises are uncorrelated, the co-variance matrix is given by

$$R_1 = \begin{bmatrix} 0.011728 & 0 \\ 0 & 0.001303 \end{bmatrix} \tag{11}$$

By adding input noises, the equation(7) can be re-modelled as,

$$x(t+1) = Ax(t) + B(u(t) + w(t)) \tag{12}$$

$$x(t+1) = Ax(t) + Bu(t) + Bw(t)) \tag{13}$$

From equation(7) and equation(13), the **N matrix is equal to** B_d.

Question B

Measurement disturbances n_1, n_2 are added to the output. The disturbances are upper bounded by 0.02 rad and 0.01 rad s^{-1}, and are assumed to be discrete time, zero mean uncorrelated white noises (normally distributed, account for 99.7% of the realisation for bounds).The task is to propose a co-variance matrix for the measurement disturbance vector n.

By using the equations(8 and 9), Variance matrix

$$V_2(t) = \begin{bmatrix} 52.1284 \times 10^{-6} \\ 26.0642 \times 10^{-6} \end{bmatrix} \tag{14}$$

For both Variances, mean $\mu = 0$, X $= 0.02\,\mathrm{rad}$ and $0.01\,\mathrm{rad\,s^{-1}}$. Since the noises are uncorrelated, the covariance matrix is given by

$$R_2 = \begin{bmatrix} 52.1284 \times 10^{-6} & 0 \\ 0 & 26.0642 \times 10^{-6} \end{bmatrix} \tag{15}$$

Question C

In this question, cross spectrum of w and n is zero, that is

$$R_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

In this task, discrete time Kalman filter is to be computed. Observer gain matrix L, covariance matrix of the state estimation error P and the eigen values of observer is to be calculated.

The estimation equation is given by

$$\hat{x}(k+1|k) = (A_d - L \cdot C)\hat{x}(k|k-1) + B_d \cdot u(k) + L(y(k) - D \cdot u(k)) \tag{17}$$

Kalman gain matrix L is found by,

$$L = (APC^T + NR_{12})(CPC^T + R_2)^{-1} \tag{18}$$

Where P is the minimisation of error covariance matrix $x(k) - \hat{x}(k|k-1)$ and is given by,

$$P = APA^T + NR_1N^T - (APC^T + NR_{12})(CPC^T + R_2)^{-1}(APC^T + NR_{12})^T \tag{19}$$

By using the equation(18),

$$L = \begin{bmatrix} 0.001 & 0.6970 \\ -0.0294 & 1.6747 \\ 0.0007 & 0.0009 \\ 0.0007 & 0.0018 \\ 0.0005 & 0.0113 \end{bmatrix} \tag{20}$$

By using the equation(19),

$$P = \begin{bmatrix} 0.4022 & 0.0290 & 0.0001 & 0.0000 & 0.0012 \\ 0.0290 & 0.1286 & 0.0001 & 0.0000 & 0.0037 \\ 0.0001 & 0.0001 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0012 & 0.0037 & -0.0000 & 0.0000 & 0.0001 \end{bmatrix} \tag{21}$$

Observer eigen value of matrix $[A - LC]$ is

$$O_{eig} = \begin{bmatrix} 0.1438 + 0.5208i \\ 0.1438 - 0.5208i \\ -0.0001 + 0.0000i \\ -0.6874 + 0.0000i \\ 0.9993 + 0.0000i \end{bmatrix} \tag{22}$$

From the equation(22), values on the last row of the matrix is approximately equal to unity, which signifies that the corresponding state is not reliable.

Question D

The task is to design a discrete time Linear Quadratic Gaussian controller and simulate the closed-loop answer to a step $r_{\omega 2}$ (jumping from an initial value 10 to 100) for the discrete time and noise corrupted system above. Kalman filter gain equation (20) is to be chosen to reconstruct system states. For the LQ controller, we are supposed to choose and use two appropriate dimensional weighting matrices Q_u and Q_x .

LQ controller gain is found by,

$$L_{lq} = Q_u^{-1} B^T S \tag{23}$$

Where S is the unique, positive, semi-definite, symmetric solution to the matrix equation.

$$A^T S + SA + M^T Q_x M - SBQ_u^{-1} B^T S = 0 \tag{24}$$

Where Q_x and Q_u are weighting matrices of the given LQ controller

$$Q_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

$$Q_u = 5 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{26}$$

From the equation(23), the proportional and integral gains are obtained.

$$K_{p-Gain} = L_{lq}(:, 1:5) = \begin{bmatrix} 0.0099 & 0.0943 & -114.3510 & 113.6448 & 0.7028 \\ 0.0135 & 0.1338 & 39.2148 & -42.2025 & 2.9883 \end{bmatrix} \tag{27}$$

$$K_{i-Gain} = L_{lq}(:, 6) = \begin{bmatrix} -0.0305 \\ -0.0563 \end{bmatrix} \tag{28}$$

Figure 2 shows the simulink model for the system with Linear Quadratic integrator controller.

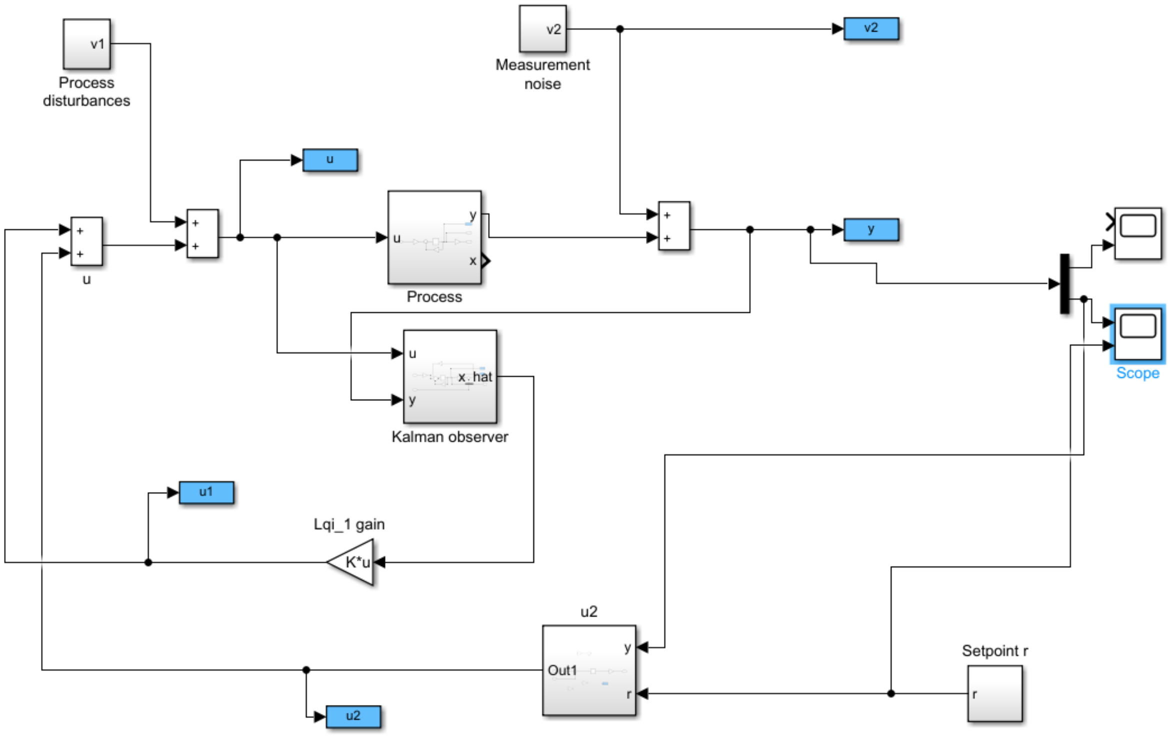

Figure 2: Simulation Model: Closed loop system using LQI controller

Initially, the speed reference step $r_{\omega 2}$ is jumped from an initial value 10 to 100 at time 10 s. From the figure 3, the system with LQI controller is following the reference step input and reaching steady state at time 25.67 s.

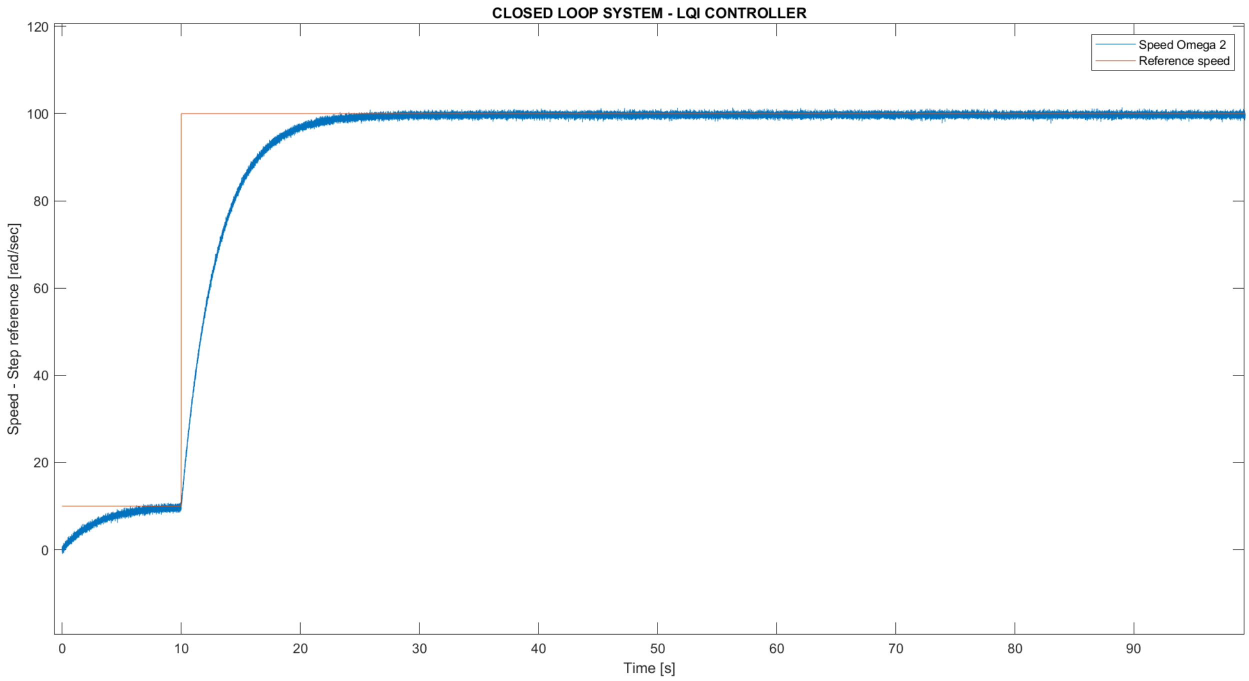

Figure 3: Output result: Using LQI controller, the speed is following its reference value

Appendix A

Code to calculate controllability, observability, stabilizability, detectability, state transition matrix $e^{A_c t}$, B_d and to know if the discrete system is stable.

```
%% Assignment 3

R = 1; %Ohms
K_E = 0.1; %Vsrad^-1
K_T = 0.1; %NmA^-1
J1 = 10^(-5) ;%kgm^2
J2 = 4*10^(-5) ;%kgm^2
D1 = 20 ; % Nmrad^-1
D2 = 2 ; % Nmrad^-1
B = 2*10^-3 ; %Nms

A = [(-K_T*K_E/(J1*R)) 0 (-D1/J1) D1/J1 0;
    0 0 D1/J2 -(D1+D2)/J2 D2/J2;
    1 0 0 0 0;
    0 1 0 0 0;
    0 0 0 D2/B -D2/B];

B_d = [(K_T/(J1*R)) 0 ;
```

```
    0 0;
    0 0;
    0 0;
    0 1/B];

C = [0 0 0 1 0;
     0 1 0 0 0];
D = zeros(2,2);

t = 1e-3;
h=0.001;
sys = ss(A,B_d,C,D);
D_sys = c2d(sys,t);
A_dis = D_sys.A ;
B_dis = D_sys.B ;

D_sys = ss(A_dis,[B_dis B_dis],C,[D D],t);



R1 = [0.011728 0;0 0.001303];

R2 = [52.1284E-6 0;0 26.0642E-6];

% R1 = [(0.3/3)^2 0; 0 (0.1/3)^2];
% R2 = [(0.02/3)^2 0; 0 (0.01/3)^2];
R12 = zeros(2,2);

% R = [R1 R12;R12' R2];
%% LQI

A_lqi = [A_dis zeros(5,1);-C(2,:) 1];
B_lqi = [B_dis;zeros(1,2)];
C_lqi = [C zeros(2,1)]
N = B_dis;

% Kalman filter
% [K,P,Z,E] = dlqe(A_dis,N,C,R1,R2);% To be checked if this is to be reverted
    back
[KEST,K,P] = kalman(D_sys,R1,R2,R12)

O_eig = eig(A_dis - (K*C));

Poles = zeros(5,1);
for n = 1:1:5
    Poles(n,1) = abs(O_eig(n,1));
end

```

```
Qx = diag([1 1 1 1 1 1]);
Qu = 5*eye(2);

% LQ solution
% [L,S,Lambda] = dlqr(A_lqi,B_lqi,Qx,Qu)
[L,S,Lambda] = dlqr(A_lqi,B_lqi,Qx,Qu)
% Reference feed forward
% K_FF = inv(C_lqi*inv(eye(7)-A_lqi+B_lqi*L)*B_lqi)
%K_FF = ones(2,2);
K_lqi_1 = L(:,1:5);
K_lqi_2 = L(:,6);


% sim('LQG_5by5_2workspace')
sim('LQG_2')




% % Variances = var(x)
% I=find(tout<10);
% subplot(311), plot(tout(I),x(I,2),tout(I),x_hat(I,2),tout(I),r(I,1))
%     ylabel('x_1, r and x_{e1}')
% %     title(['Var(x_1)=',num2str(Variances(1)),'    and    Var(x_2)=',
    num2str(Variances(2))])
%     axis([0 max(tout(I)) -5 5])
% subplot(312), plot(tout(I),x(I,4),tout(I),x_hat(I,4),tout(I),r(I,1))
%     ylabel('x_2')
%     ylabel('x_2, r and x_{e2}')
%     axis([0 max(tout(I)) -5 5])
% subplot(313), plot(tout(I),u(I,:)), ylabel('u')
%     xlabel('time')
%     axis([0 max(tout(I)) min(min(u(I,:)))*1.1 max(max(u(I,:)))*1.1])
% %    axis([0 max(tout(I)) -15 100])


figure(1)
plot(tout,y)
```

Appendix B

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997