# EXERCISE 2.1

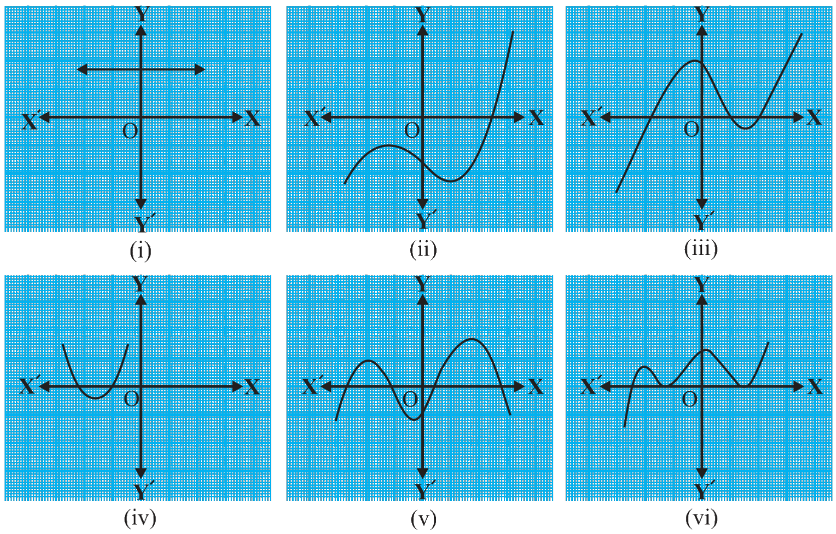


Figure 1: Illustration related to polynomials

## 2.3 Relationship between Zeroes and Coefficients of a Polynomial

You have already seen that the zero of a linear polynomial $ax + b$ is $-\frac{b}{a}$. We will now try to answer the question raised in Section 2.1 regarding the relationship between zeroes and coefficients of a quadratic polynomial. For this, let us take a quadratic polynomial, say $p(x) = 2x^2 - 8x + 6$. In Class IX, you have learnt how to factorise quadratic polynomials by splitting the middle term. So, here we need to split the middle term '–8x' as a sum of two terms, whose product is $6 \times 2x^2 = 12x^2$. So, we write,

**Factorising:**

$$\begin{aligned} 2x^2 - 8x + 6 &= 2x^2 - 6x - 2x + 6 \\ &= 2x(x-3) - 2(x-3) \\ &= (2x-2)(x-3) = 2(x-1)(x-3) \end{aligned}$$

So, the value of $p(x) = 2x^2 - 8x + 6$ is zero when $x - 1 = 0$ or $x - 3 = 0$, i.e., when $x = 1$ or $x = 3$. So, the zeroes of $2x^2 - 8x + 6$ are 1 and 3. Observe that:

$$\text{Sum of zeroes} = 1 + 3 = 4 = \tfrac{-(-8)}{2} = \tfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 1 \cdot 3 = 3 = \tfrac{6}{2} = \tfrac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Let us take one more quadratic polynomial, say, $p(x) = 3x^2 + 5x - 2$. By the method of splitting the middle term,

$$\begin{aligned} 3x^2 + 5x - 2 &= 3x^2 + 6x - x - 2 \\ &= 3x(x+2) - 1(x+2) \\ &= (3x-1)(x+2) \end{aligned}$$

Hence, the value of $3x^2 + 5x - 2$ is zero when either $3x - 1 = 0$ or $x + 2 = 0$, i.e., when $x = \frac{1}{3}$ or $x = -2$. So, the zeroes of $3x^2 + 5x - 2$ are $\frac{1}{3}$ and $-2$. Observe that:

$$\text{Sum of zeroes} = \tfrac{1}{3} + (-2) = -\tfrac{5}{3} = \tfrac{-5}{3} = \tfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \tfrac{1}{3} \cdot (-2) = -\tfrac{2}{3} = \tfrac{\text{Constant term}}{\text{Coefficient of } x^2}$$

In general, if $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then $x - \alpha$ and $x - \beta$ are the factors of $p(x)$. Therefore,

$$\begin{aligned} ax^2 + bx + c &= k(x-\alpha)(x-\beta), \quad \text{where } k \text{ is a constant} \\ &= k[x^2 - (\alpha+\beta)x + \alpha\beta] \\ &= kx^2 - k(\alpha+\beta)x + k\alpha\beta \end{aligned}$$

Comparing the coefficients of $x^2, x$, and constant terms on both sides, we get:

$$a = k, \quad b = -k(\alpha+\beta), \quad c = k\alpha\beta$$

This gives:

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

i.e.,

$$\text{Sum of zeroes } = \alpha + \beta = \frac{-b}{a} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes } = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Let us consider an example.