Problem Setup

Given a 4:1 multiplexer:

- Select lines: a and b
- Inputs:

$$i_0 = c$$
$$i_1 = d$$
$$i_2 = \overline{c}$$
$$i_3 = \overline{c} \cdot \overline{d}$$

Step 1: MUX Output Expression

$$f(a,b,c,d) = \overline{a}\,\overline{b}\,c + \overline{a}b\,d + a\,\overline{b}\,\overline{c} + ab\,\overline{c}\,\overline{d}$$

Step 2: Expand Terms to Include All Variables

Term 1: $\overline{a}\,\overline{b}\,c = \overline{a}\,\overline{b}\,c(d + \overline{d}) = \overline{a}\,\overline{b}\,c\,d + \overline{a}\,\overline{b}\,c\,\overline{d}$
Term 2: $\overline{a}b\,d = \overline{a}b\,d(c + \overline{c}) = \overline{a}b\,d\,c + \overline{a}b\,d\,\overline{c}$
Term 3: $a\,\overline{b}\,\overline{c} = a\,\overline{b}\,\overline{c}(d + \overline{d}) = a\,\overline{b}\,\overline{c}\,d + a\,\overline{b}\,\overline{c}\,\overline{d}$
Term 4: $ab\,\overline{c}\,\overline{d}$ already includes all variables

Step 3: List All Resulting Minterms

Use variable order: a, b, c, d (MSB to LSB):

- $\overline{a}\,\overline{b}\,c\,d \to 0011 = m_3$
- $\overline{a}\,\overline{b}\,c\,\overline{d} \to 0010 = m_2$
- $\overline{a}b\,d\,c \to 0111 = m_7$
- $\overline{a}b\,d\,\overline{c} \to 0101 = m_5$
- $a\,\overline{b}\,\overline{c}\,d \to 1001 = m_9$
- $a\,\overline{b}\,\overline{c}\,\overline{d} \to 1000 = m_8$
- $a\,b\,\overline{c}\,\overline{d} \to 1100 = m_{12}$

Final Answer: Sum of Minterms

$$\boxed{f(a,b,c,d) = \sum m(2,3,5,7,8,9,12)}$$