**CSE 564: Reinforcement Learning** **(Due: 13 /09/20)**

# Home-Work #1

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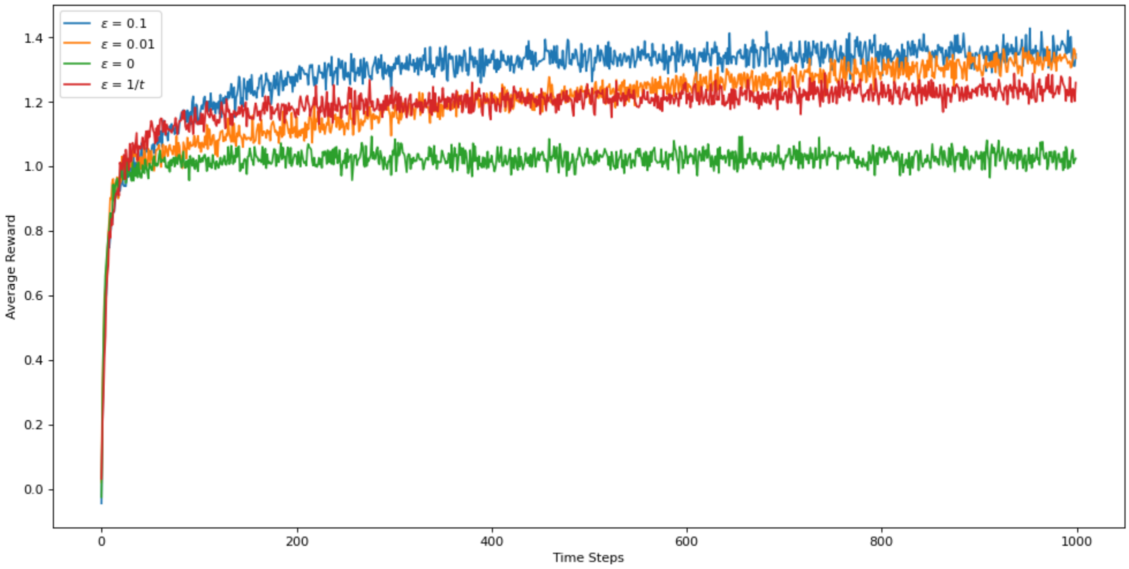
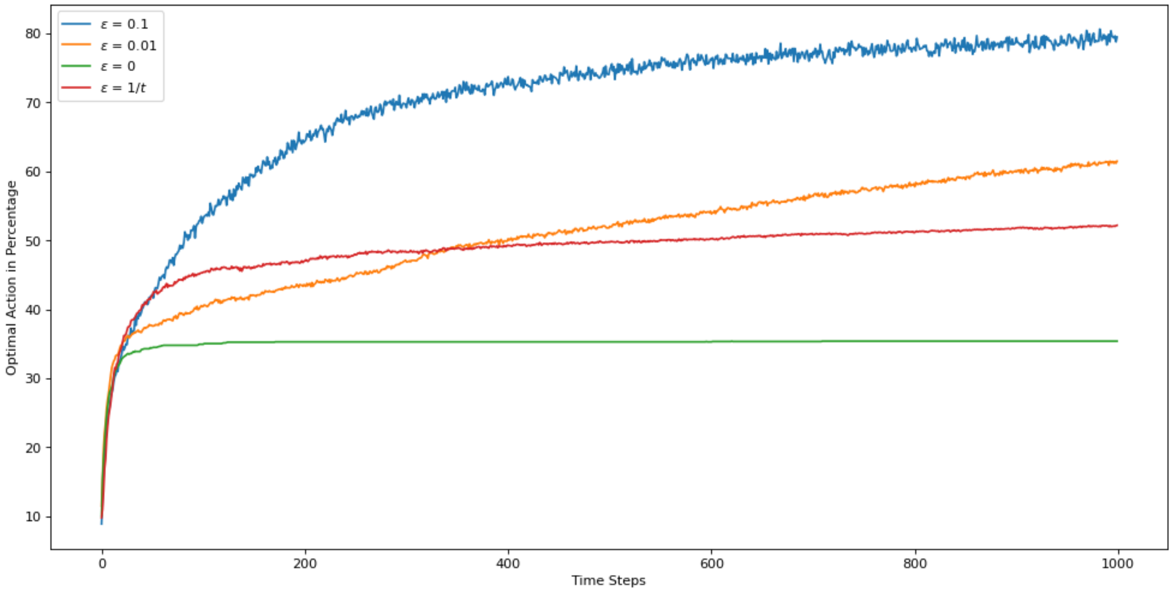
## Problem 1

I prepared a 10 arm test-bed with variance = 1, $\epsilon = 0, 0.1, 0.01$ and also a time dependant $\epsilon(t) = 1/t$. The plots are as follows:

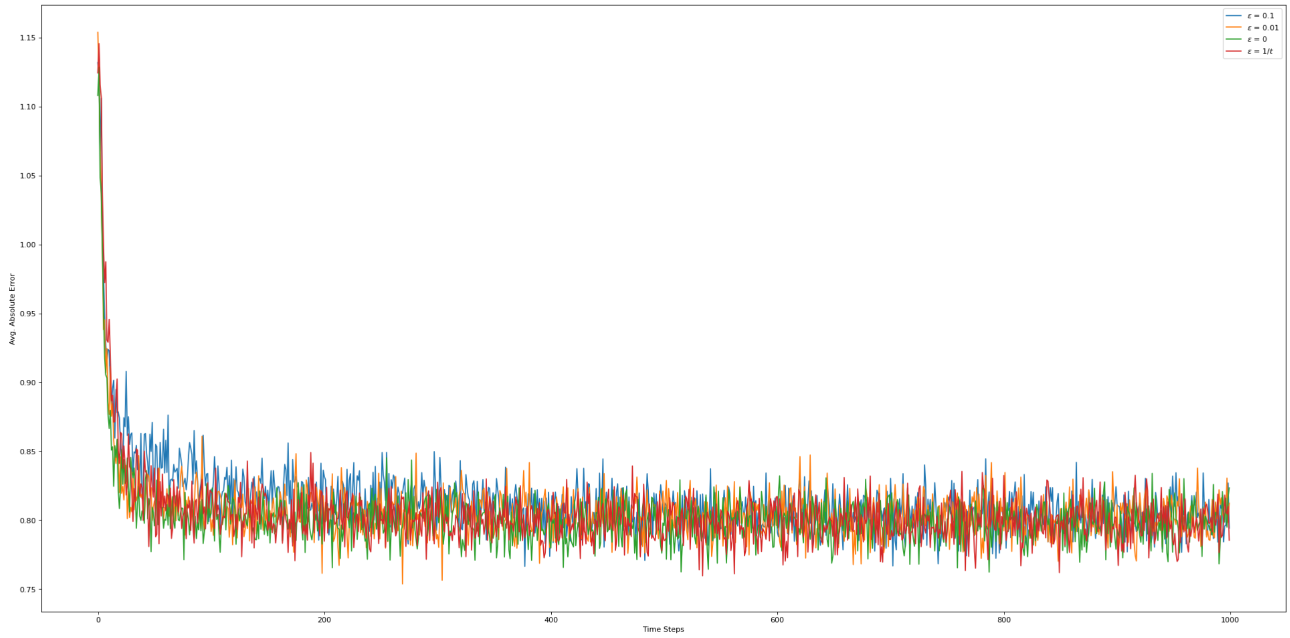
***Average Reward***



***Optimal Action in (%)***



### ***Average Absolute Error***



For each of the graphs, I simulated 2000 independent runs of 1000 time-steps each using the basic $\epsilon$ - greedy method, with varying $\epsilon$ over $[\ 0.1,\ 0.01,\ 1/t\ ]$ and also a greedy option and plot them on various parameters such as **Average Reward**, **Optimal Action** and **Average Absolute Error**.

Each arm was picked with mean = 0 and variance = 1. A unit variance test-bed has a considerable lower amount of difference among rewards and the graph was quite a less noisier.

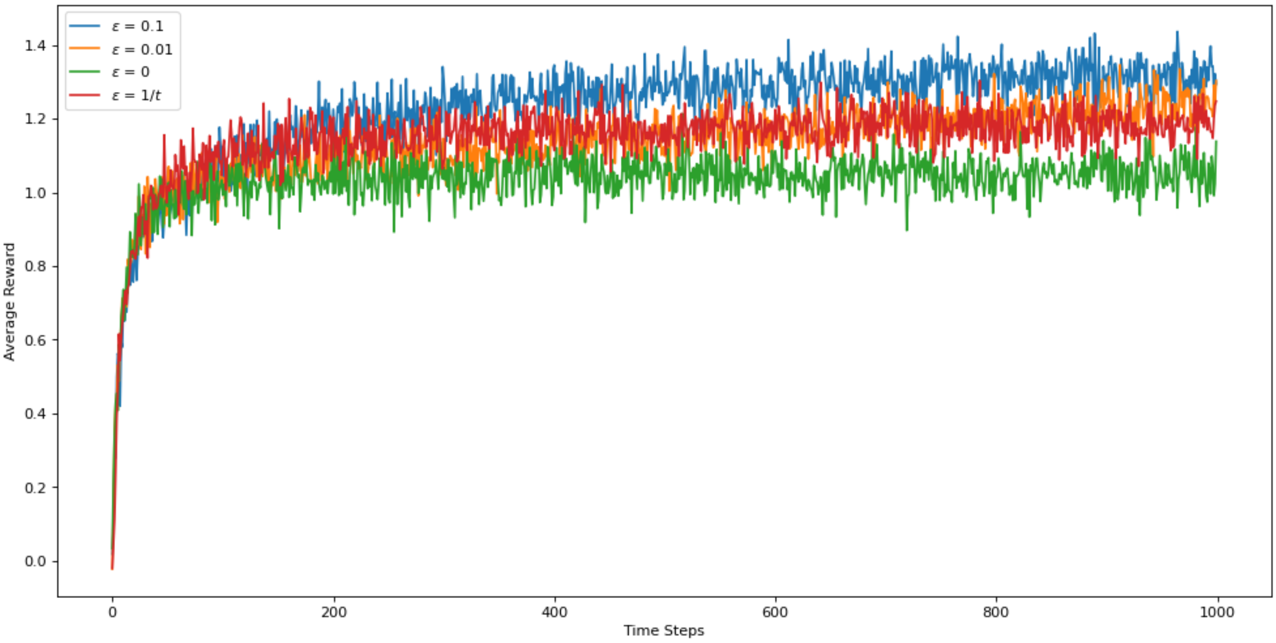
From the graph as we are dealing in a *Practical Sense* (only a 1000 Time Steps) $\epsilon\ =\ 0.1$ tends to perform the best. This is so because of a higher initial exploration rate, $\epsilon\ =\ 0.1$, but clearly the rate of increase for $\epsilon\ =\ 0.01$ is better, i.e. say maybe a greater $t\ =\ 10000$, $\epsilon\ =\ 0.01$ might perform better.

In the long run tho, $\epsilon\ =\ 1/t$ would perform the best as every $\epsilon$ - greedy would have fairly good estimates of the truth, $\epsilon\ =\ 1/t$ would pick the greedy almost all of the times, therefore have the maximum average reward.

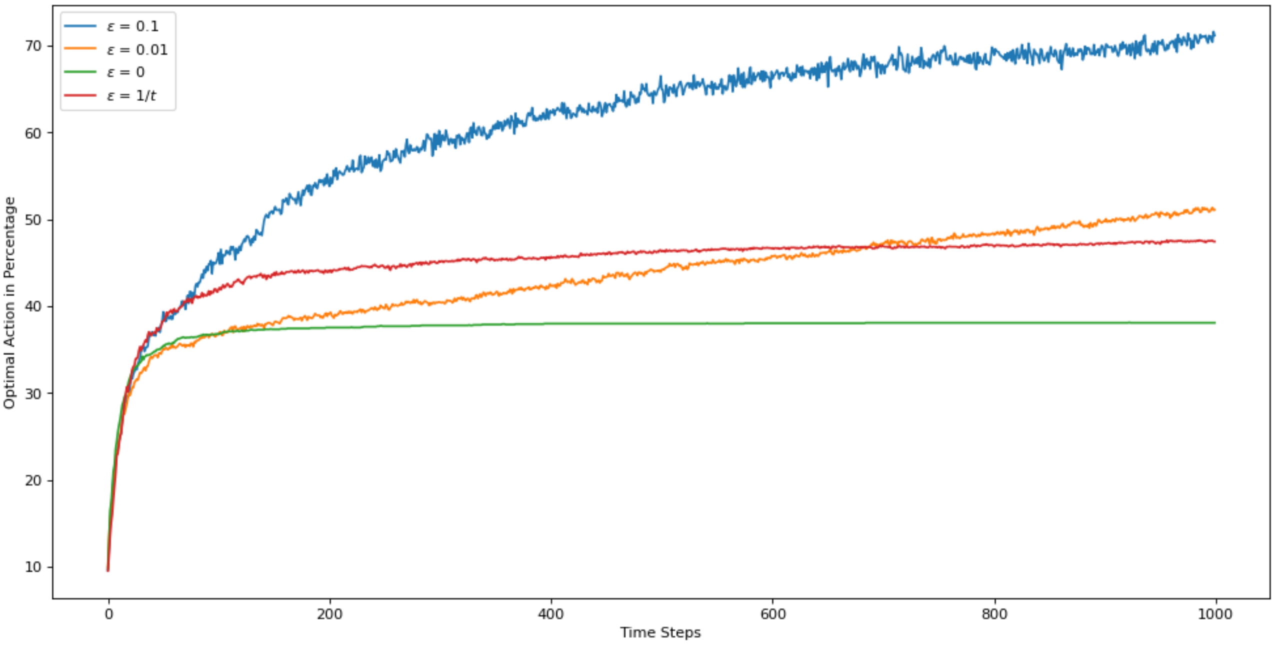
## Problem 2

This time around the variance = 4.

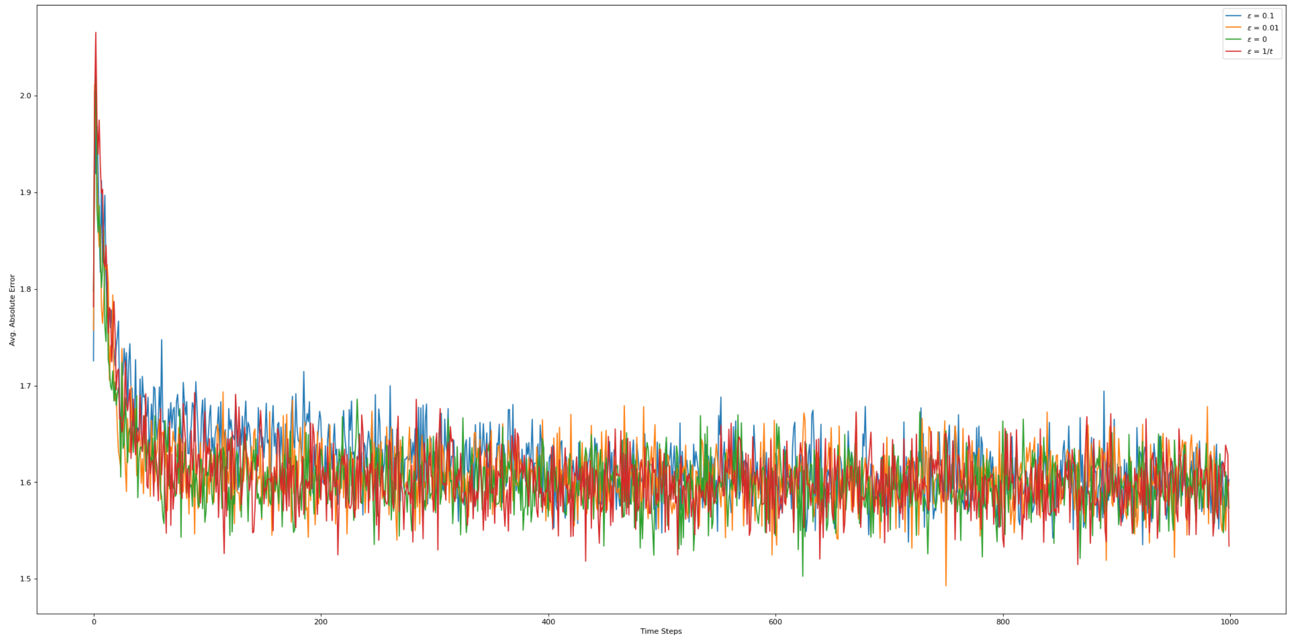
***Average Reward***



***Optimal Action in (%)***



***Average Absolute Error***



For each of the graphs, I simulated 2000 independent runs of 1000 time-steps each using the basic $\epsilon$ - greedy method, with varying $\epsilon$ over $[\ 0.1,\ 0.01,\ 1/t\ ]$ and also a greedy option and plot them on various parameters such as **Average Reward**, **Optimal Action** and **Average Absolute Error**.

Each arm was picked with mean = 0 and variance = 4. A variance $=$ 4, results in quite noisier estimates as compared to a unit variance test bed.

From the graph as we are dealing in a *Practical Sense* (only a 1000 Time Steps) $\epsilon\ =\ 0.1$ tends to perform the best. This is so because of a higher initial exploration rate, $\epsilon\ =\ 0.1$, but clearly the rate of increase for $\epsilon\ =\ 0.01$ is better, i.e. say maybe a greater $t\ =\ 10000$, $\epsilon\ =\ 0.01$ might perform better.

In the long run tho, $\epsilon\ =\ 1/t$ would perform the best as every $\epsilon$ - greedy would have fairly good estimates of the truth, $\epsilon\ =\ 1/t$ would pick the greedy almost all of the times, therefore have the maximum average reward.

## Problem 3 (Exercise 2.3)

In the Hand-Written PDF.

## Problem 4

In the Hand-Written PDF.

# RL - HW - 1

S: At time $t = \infty$ ie in the long run all epsilons except $\varepsilon = 0$ ie a greedy policy will all have fairly good estimates of the Retrn ie $Q^*(a)$

The value of the cumulative ~~rewards~~ average rewards would then depend on simply the number of times the greedy action is picked.

That is

$\varepsilon = 0.01$

Using the formula $(1-\varepsilon) + \frac{\varepsilon}{|A|}$

$\Rightarrow$ 0.991 . ie. 99.1% of the times this policy picks the greedy (optimal arm).

$\varepsilon = 0.1$

Similarly as above

$\Rightarrow$ 0.91 ie 91% of the times picks the greedy (optimal arm)

$\underline{\varepsilon = 1/t}$

At $t \to \infty$ we $\underline{\varepsilon = 0}$. Now

this means in theory At $t = \infty$ $\varepsilon = 1/t$ will always pick the optimal (100%) Arm. But practically it is not neccessary for it to perfom best in say ($t = 1000$) time-steps

ie. $\varepsilon = 1/t$ in the long run performs the best.

Ques 4

Part 1

a

While taking the Sample mean to estimate '$Q_n$', even if $Q_1$ defines how the first arm is picked, but for $Q_2$, and subsequent $Q_n$ (s) the choice is independent of $Q_1$ and depends solely on the reward achieved from that arm.

Part 1 (a)

We know that the sample mean ($Q_n$)

$$Q_n = \frac{R_1 + R_2 + \cdots R_{n-1}}{n-1}$$

where 'n' represents the number of times arm has been selected.

$$\Rightarrow \quad Q_n = \frac{1}{n-1}\sum_{i=1}^{n-1} R_i \qquad \text{—— (i)}$$

$$= \frac{1}{n-1}\left(R_{n-1} + \sum_{i=1}^{n-2} R_i\right) \quad \text{(separating } R_{n-1}\text{)}$$

$$= \frac{1}{n-1}\left(R_{n-1} + \frac{(n-2)}{(n-2)}\sum_{i=1}^{n-2} R_i\right) \quad \text{( Using eqn (i) )}$$

i.e.

$$\left[\, Q_n = \frac{1}{n-1}\sum_{i=1}^{n-1} R_i \,\right]$$

$$= \frac{1}{n-1}\left(R_{n-1} + (n-2)\, Q_{n-1}\right)$$

$$= \frac{1}{n-1}\left(R_{n-1} + (n-1)\, Q_{n-1} - Q_{n-1}\right)$$

$$\boxed{Q_n = Q_{n-1} + \frac{1}{n-1}\left(R_{n-1} - Q_{n-1}\right)}$$

$\therefore$ Hence no dependance on $\underline{Q_1}$.

On the other hand using a $\underline{\text{constant step size}}$ '$\underline{\alpha}$' $\rightarrow$

(b) We know that for a testbed with constant step-size 'α' we have, $n$ is the no. of times picked,

$$Q_{n+1} = Q_n + \alpha\,[R_n - Q_n]$$

$$\Rightarrow Q_{n+1} = Q_n + \alpha\,[R_n - Q_n] \longrightarrow \text{(ii)}$$

$$\Rightarrow Q_{n+1} \pm \alpha R_n + (1-\alpha) Q_n$$ ↳ (Expanding)

Again using (ii) to expand $Q_n$.

$$\Rightarrow Q_{n+1} = \alpha R_n + (1-\alpha)(\alpha R_{n-1} + (1-\alpha) Q_{n-1}].$$

$\Rightarrow$ We can now expand the above equation till $n$ reaches 1. i.e.

$$\Rightarrow \quad = (1-\alpha)^n Q_1 + \sum_{i=1}^{n} \alpha (1-\alpha)^{n-i} R_i.$$

$$\Rightarrow \boxed{Q_{n+1} = (1-\alpha)^n Q_1 + \sum_{i=1}^{n} \alpha (1-\alpha)^{n-i} R_i} \quad \text{— (iii)}.$$

In this case $Q_{n+1}$ clearly depends on $Q_1$.

We can also see that if 'α' is smaller then the term $\underline{(1-\alpha)}$ is bigger which in turn keeps the coefficient of $Q_1$ larger as compared to a higher 'α'.

i.e. $\underline{Q_1 \text{ has}}$ a higher dependance for smaller 'α'

Part 2

We can update our constant step-size 'α' with another value $w_n$ s.t. $\boxed{w_n = w_{n-1} + \alpha(1 - w_{n-1})}$.

ie $\boxed{\alpha_{new} = \alpha / w_n}$

$$\Rightarrow Q_{n+1} = (1-\alpha)^n Q_1 + \sum_{i=1}^{n} R_i \alpha (1-\alpha)^{n-i}.$$

$\alpha_{new}$ for $Q_1 = \alpha / w_1$.

Now using the formulas for $w_1$ we have $\boxed{w_1 = \alpha}$

$$\Rightarrow Q_{n+1} = \cancel{(0) Q_1} + \sum_{i=1}^{n} R_i \left(\frac{\alpha}{w_i}\right)\left(1-\frac{\alpha}{w_i}\right)^{n-i}$$

i.e. independent of $\underline{Q_1}$.

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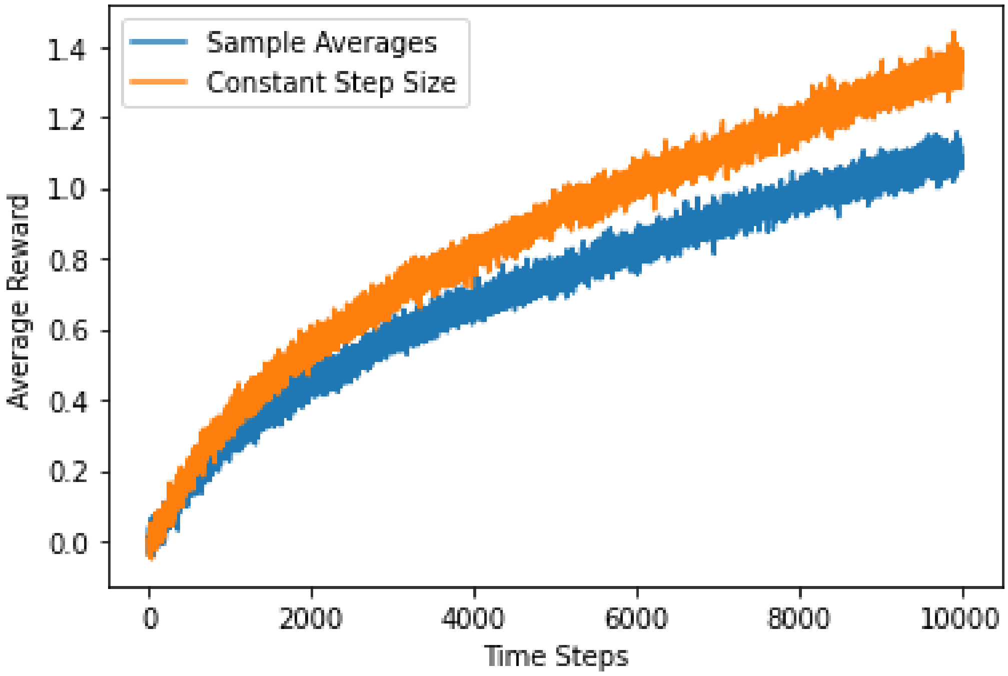
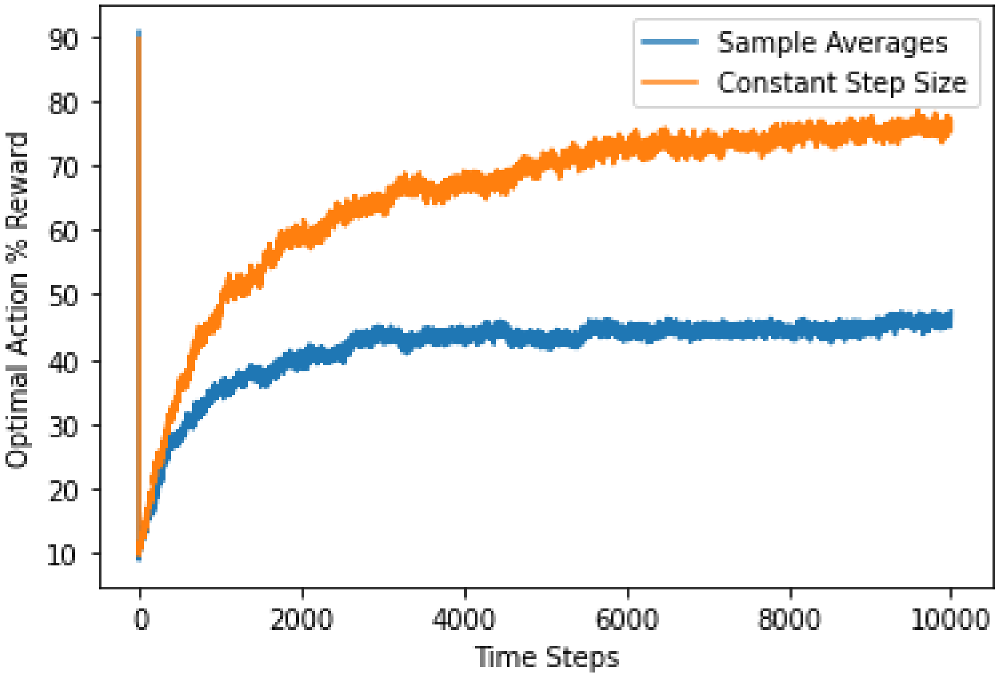
**Problem 5**

***Average Reward***



***Optimal Action***

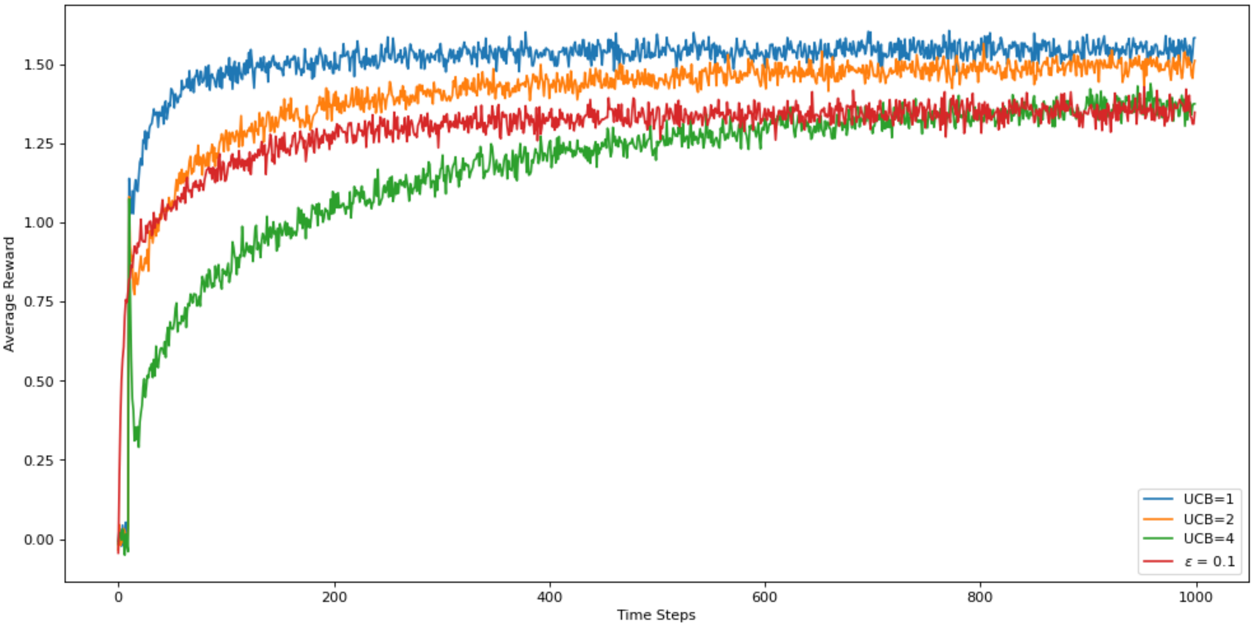


In a Non-Stationary bandit problem, Constant Step Size tend to perform generally quite a lot better as they never fully converge and tend to readjust themselves according to their most recent updates, a quality desirable in a Non - Stationary setting.

Here I ran 200 simulations over 10000 time-steps to better understand the performance difference of both the policies, *Sample Averages* and *Constant Step Size*.

## Problem 6

***Average Reward***



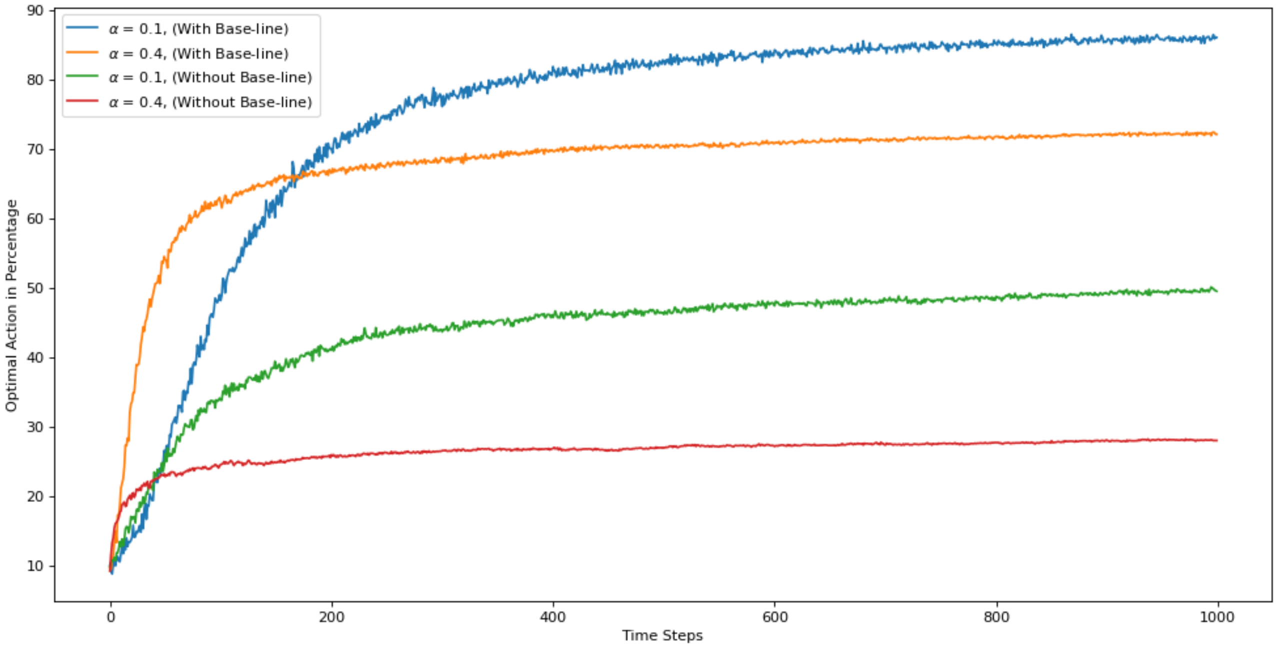
The Upper Confidence Bound algorithm initially favours exploration over exploitation and in the later stages gives more priority to exploitation. UCB forces the selection of arms that have not been selected even once in the first $k$ time steps for a $k$ armed bandit system. As a result in our case of a 10 arm bandit system, the first 10 selections (although in a random order) are always the 10 different arms of the 10 armed bandit. As a result we can see for the first 10 time steps the average reward is considerably low and at times even drops below 0.

But at the $11^{th}$ time-step as each arm has been picked exactly once, which mathematical terms means that the value $c\sqrt{\frac{\log t}{N_t(a)}}$ is same for every arm $a$, hence the system now picks the arm which at that stage is optimal i.e. on the basis of $Q_t(a)$ only. As this happens for every one of the 2000 runs, the average reward at the $11^{th}$ time step is quite considerably better than the previous 10 time steps and thus a **significant spike** is observed.

After the $11^{th}$ time step the reward would again go down due to the uncertainty increased by the now unequal Confidence Bound Term $c\sqrt{\frac{\log t}{N_t(a)}}$. The average reward value finally rises when the algorithm has a lot more certainty about the optimal arms.

## Problem 7

### *Optimal Action*



The simulation was carried over for 2000 runs and 1000 time-steps. For the plots :

- The mean for the test-bed was +4 and the variance was 1
- $\overline{R_t}$ was calculated using incremental updates, i.e. $\overline{R}_{t-1}$
- $H_t(a)$ was calculated using the gradient update rules (from Book)
- $\pi_t(a)$ was calculated using the soft-max distribution

The Optimal Action performs quite better for $\alpha = 0.1$, with baseline as compared to the similar $\alpha$ without the base line i.e. $\overline{R_t} = 0$.

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