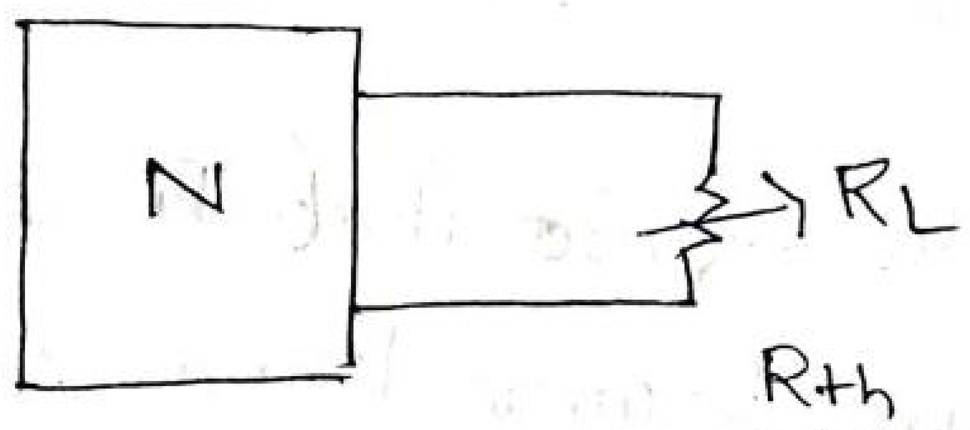
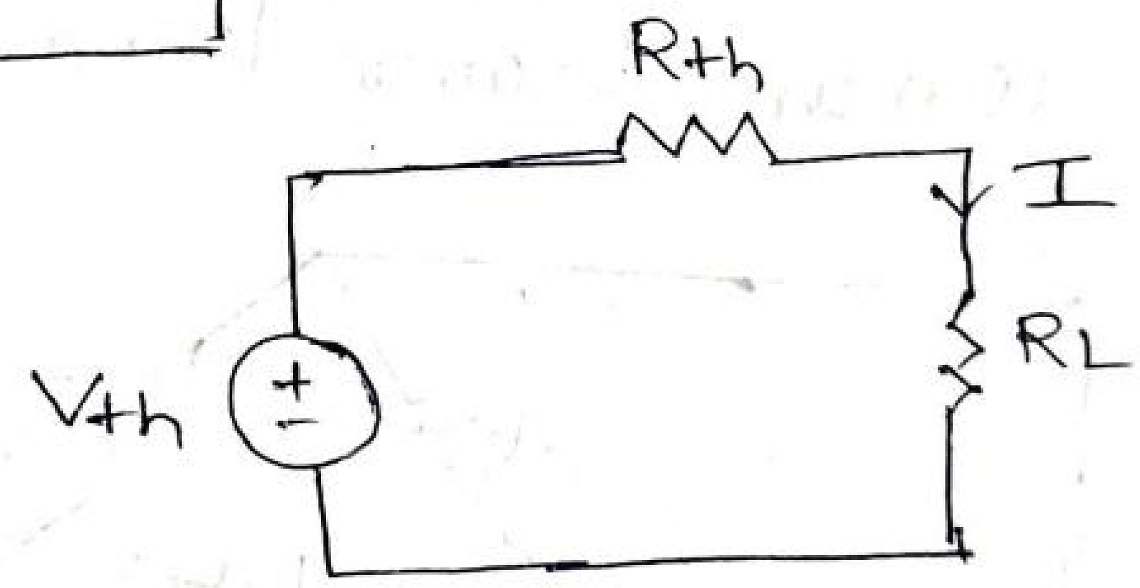
Lec. 09. 26/02/20

Maximum Power Transfer Theorem.

The output obtained form a network is maximum when load resistance is equal to internal resistance of the network as seen from the load.

According to Therenin theorem, every network can be represented ~~having~~ by ~~by~~ single voltage source having effective internal resistance R_{th} shown in Fig.





$$I = \frac{V_{th}}{R_{th}+R_L}$$

Output power, $P = I^2 R_L$

$$P = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$

Power will be maximum $\frac{d}{dR_L} P = 0$

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{(R_{th}+R_L)^2 \, 1 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^2} \right] = 0$$

$$R_{Th} + R_L - 2R_L = 0$$

$$R_{Th} - R_L = 0$$

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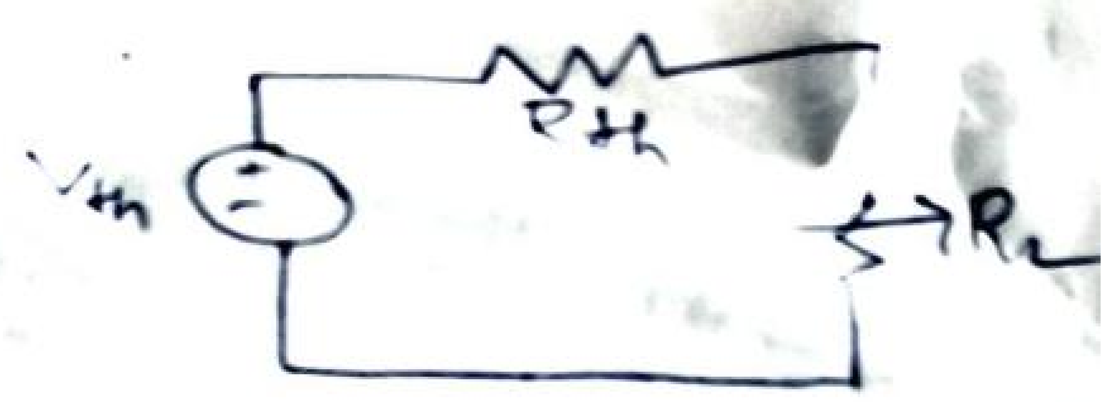
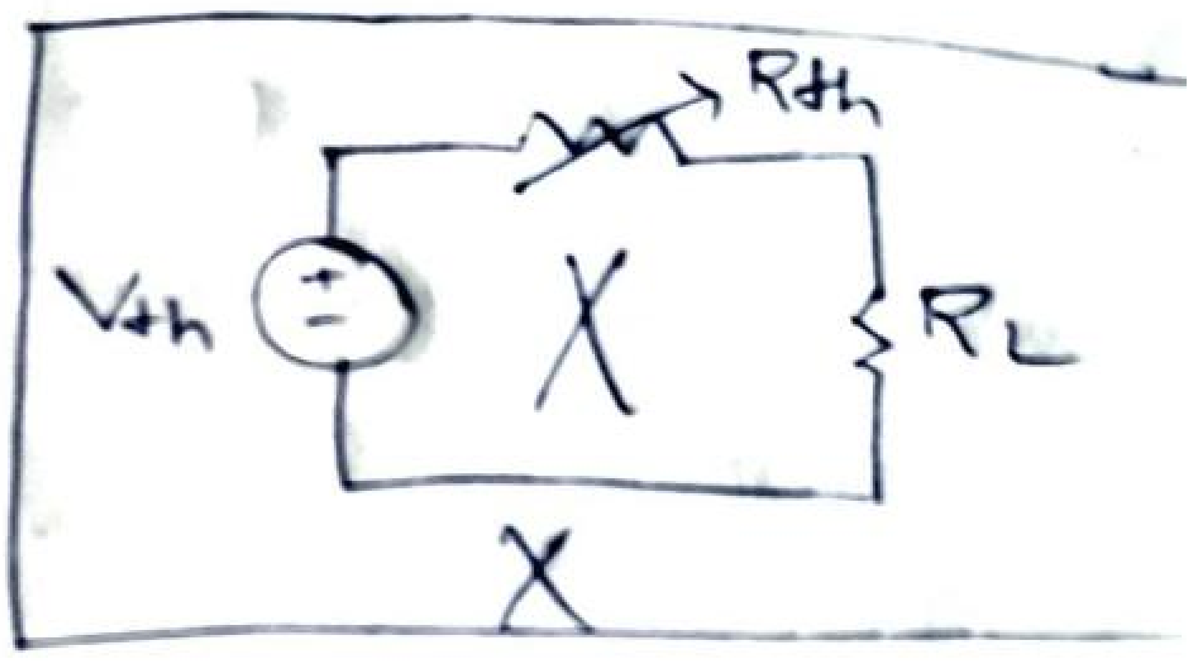
$$P_{max} = \frac{V_{Th}^2}{(R_{Th} + R_{Th})^2} \cdot R_{th}$$



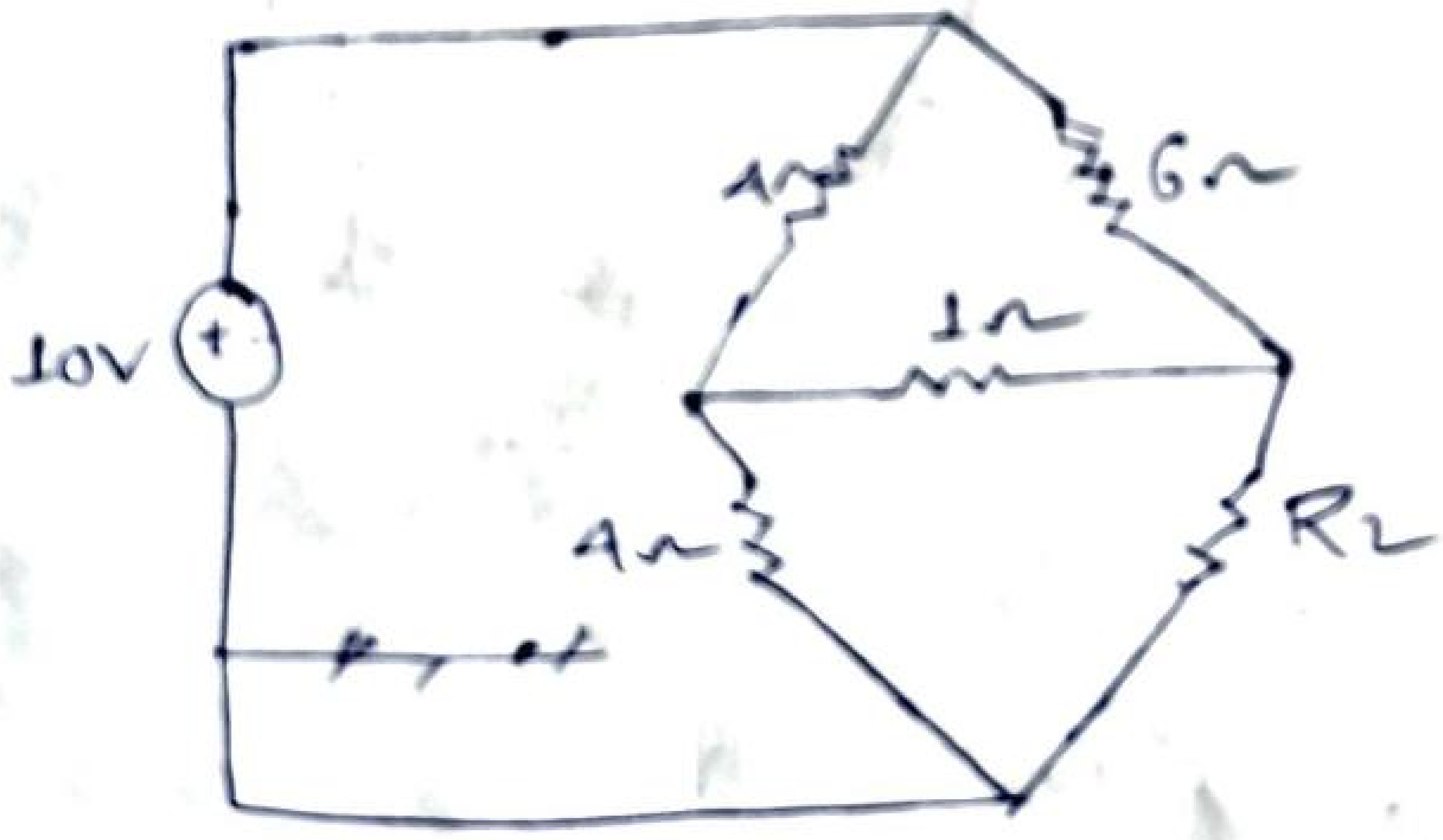
$$= \frac{V_{Th}^2}{4R_{th}^2} \cdot R_{th}$$



$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

∴ Thus for maximum power transfer, load resistance R_L is made equal to internal resistance of the Source R_{th}.

(Q) In the below network, ~~So that~~ Find Value of R_L so that maximum power transfer from it.



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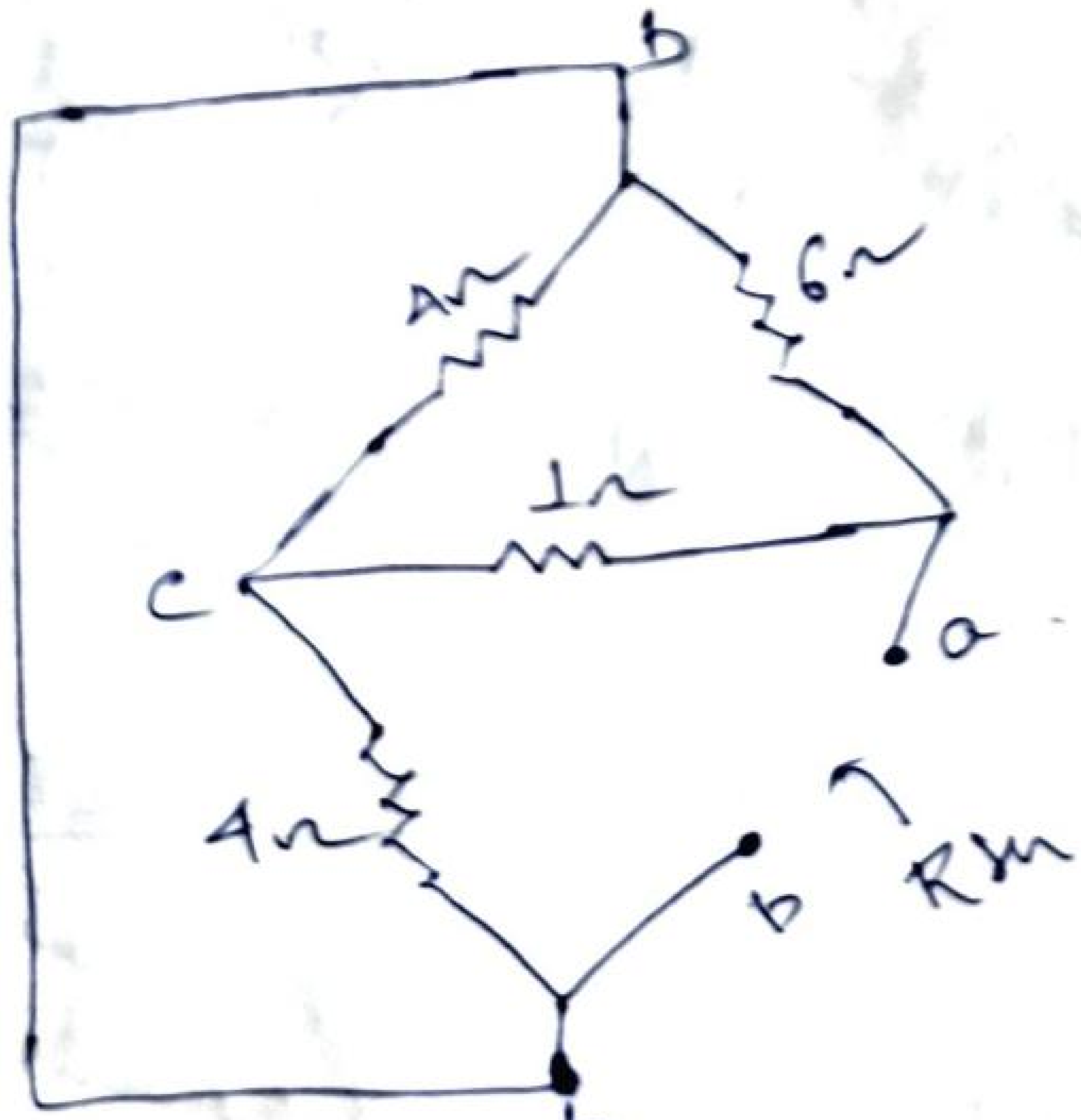
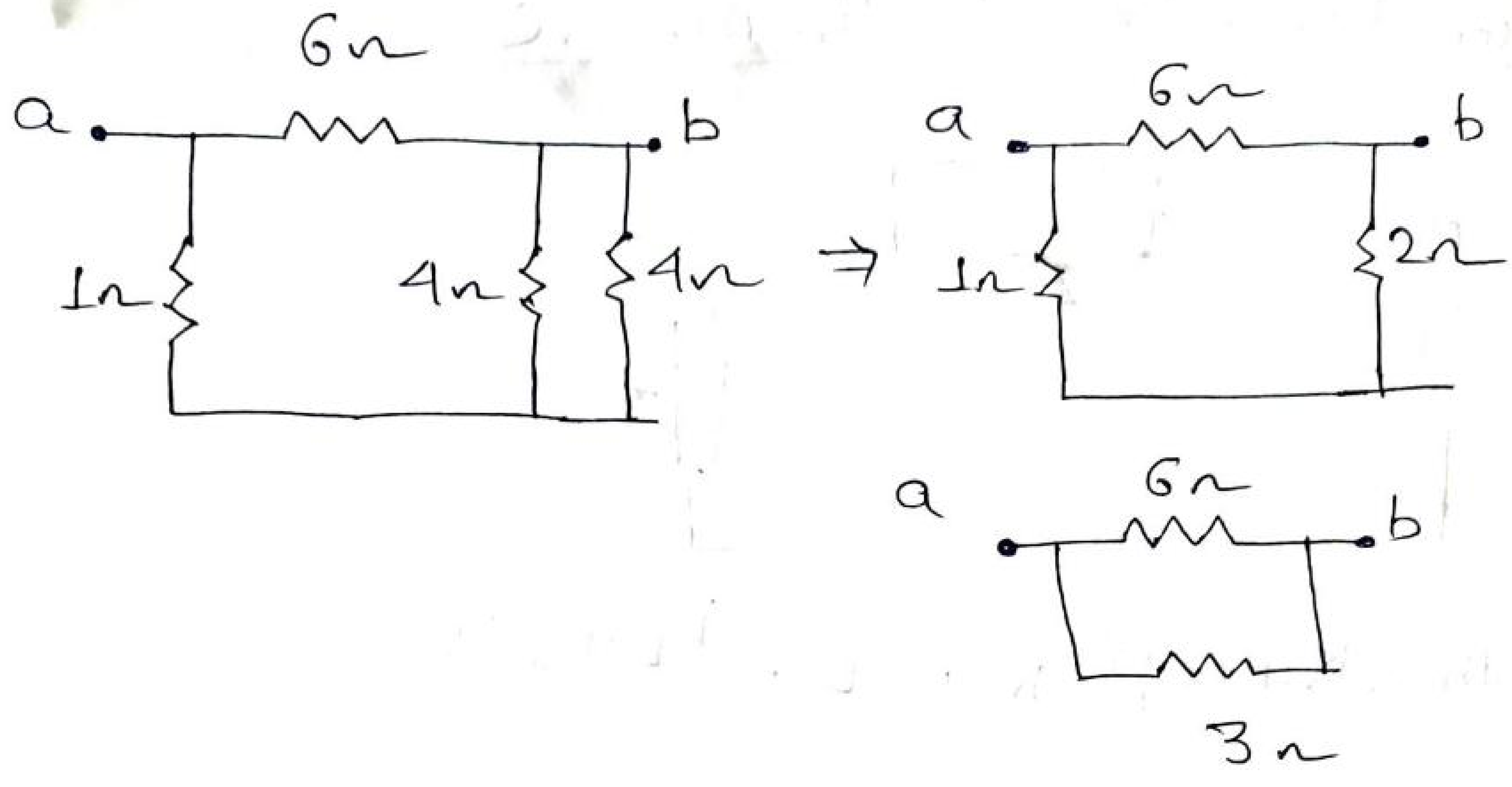




$R_{ab} \Rightarrow 6 \parallel 3$

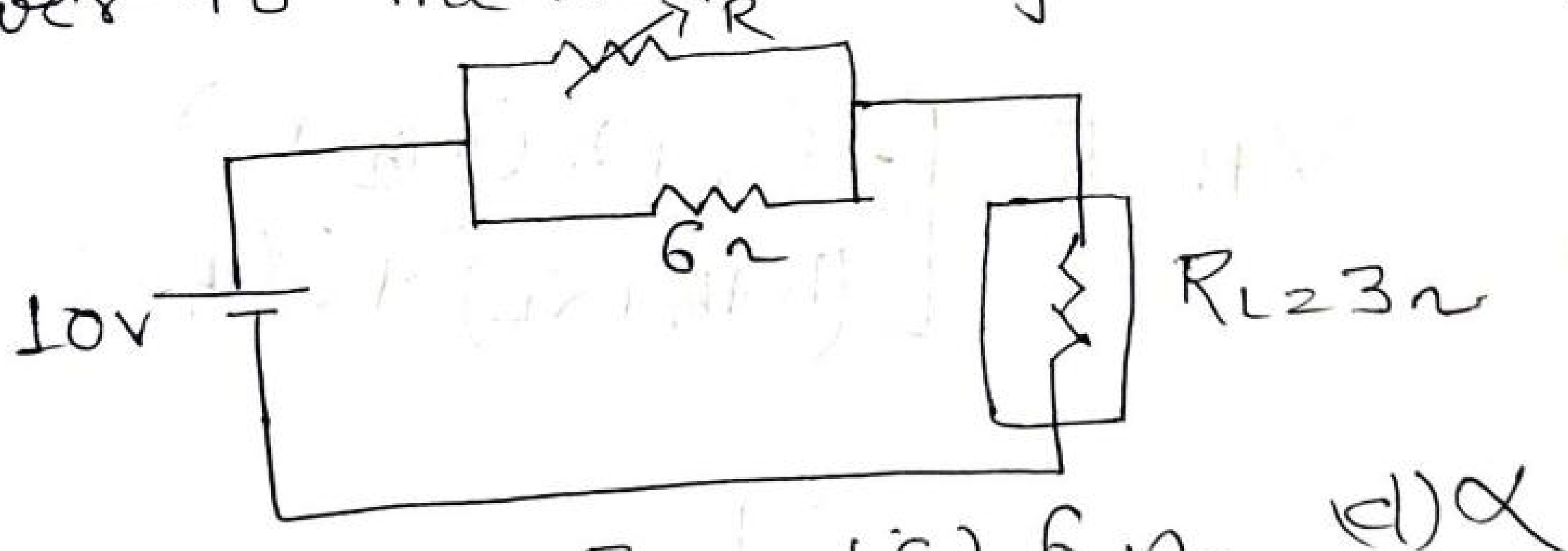
$\Rightarrow 2$

$R_{th} = R_{ab} = 2\Omega$

For maximum power, $R_L = R_{th}$.

2Ω.

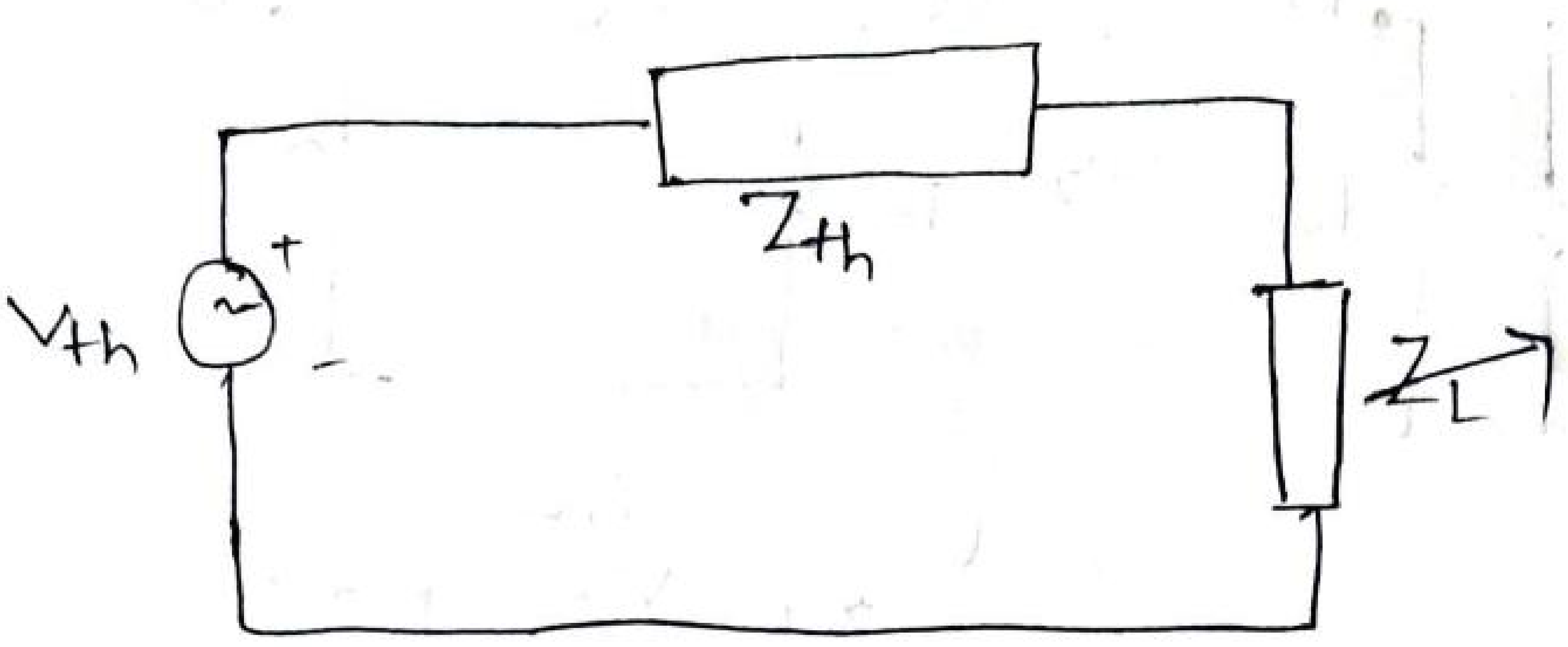
(Q) Find the Value of R required for the transfer of maximum power to the load having resistance of 3Ω.



(a) 0 (b) 3Ω (c) 6Ω (d) α

$I = \frac{V_{th}}{R_{th} + R_L}$ $P = I^2 R_L$

R	$6 \parallel R$	$I = \frac{10}{6 \parallel R + 3}$	$P_{3\Omega} = I^2 \times 3$
0	0	$\frac{10}{3}$	33.3W.
3	2	$\frac{10}{5} = 2$	12W
6	3	$\frac{10}{6}$.	8.33.



$$Z_{th} = R_{th} + jX_{Th} \quad , Z_L = R_L + jX_L$$

$$I = \frac{V_{th}}{Z_{th} + Z_L}$$

$$|I| = \frac{V_{th}}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}}$$

$$P = |I|^2 R_L$$

$$P = \frac{V_{th}^2 \cdot R_L}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}$$

$$\frac{dP}{dX_L} = V_{th}^2 R_L \left[\frac{-[0 + 2(X_{Th}+X_L)]}{\left((R_{th}+R_L)^2 + (X_{th}+X_L)\right)^2} \right] = 0$$

$$\boxed{Z_L = Z_{th}^*}$$

∴ It will deliver maximum power to Variable Comple load. When load impedance is equal to Complex of Source impedance.

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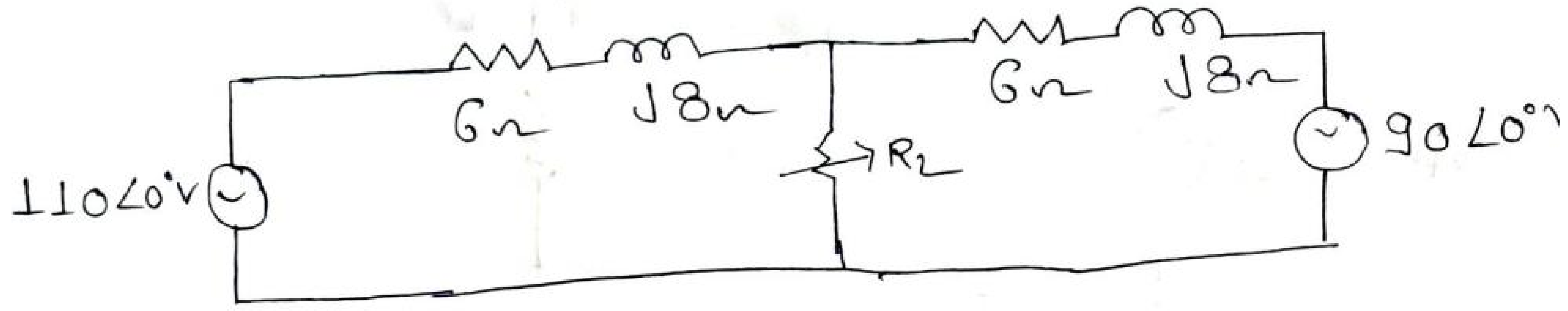
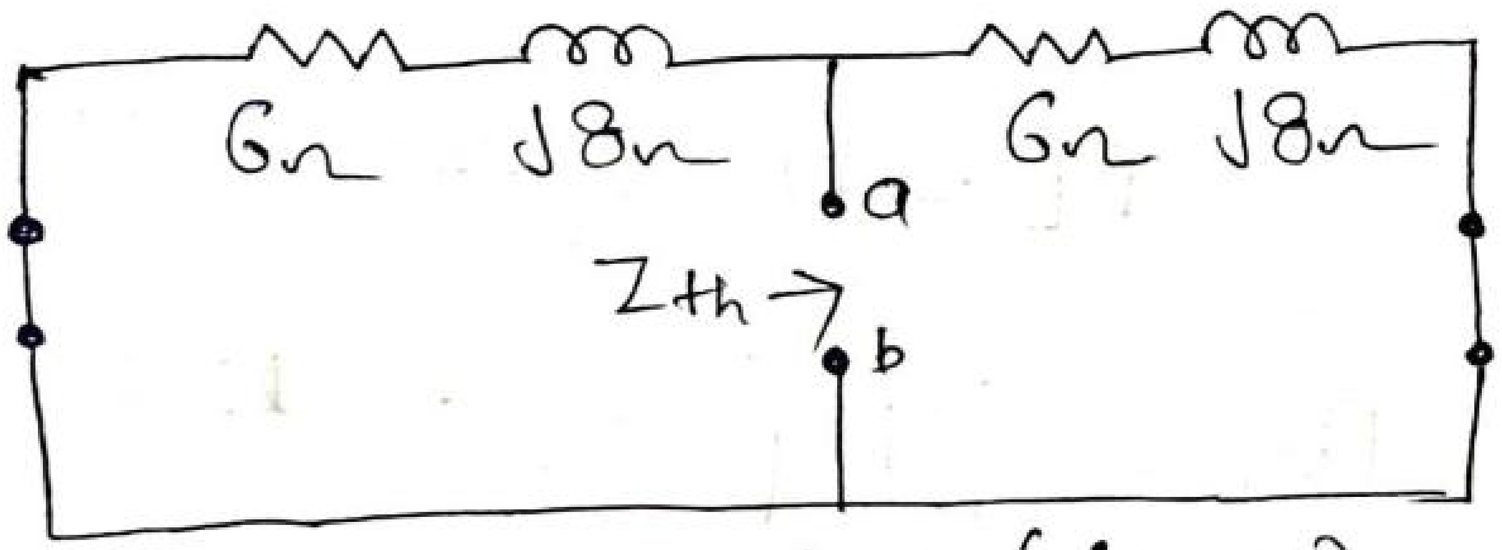
(Q) Find the maximum power, dissipation in load resistance



Objective. $P_{max} = |I_L|^2 R_L$

$$I_L = \frac{V_{th}}{Z_{th}}$$

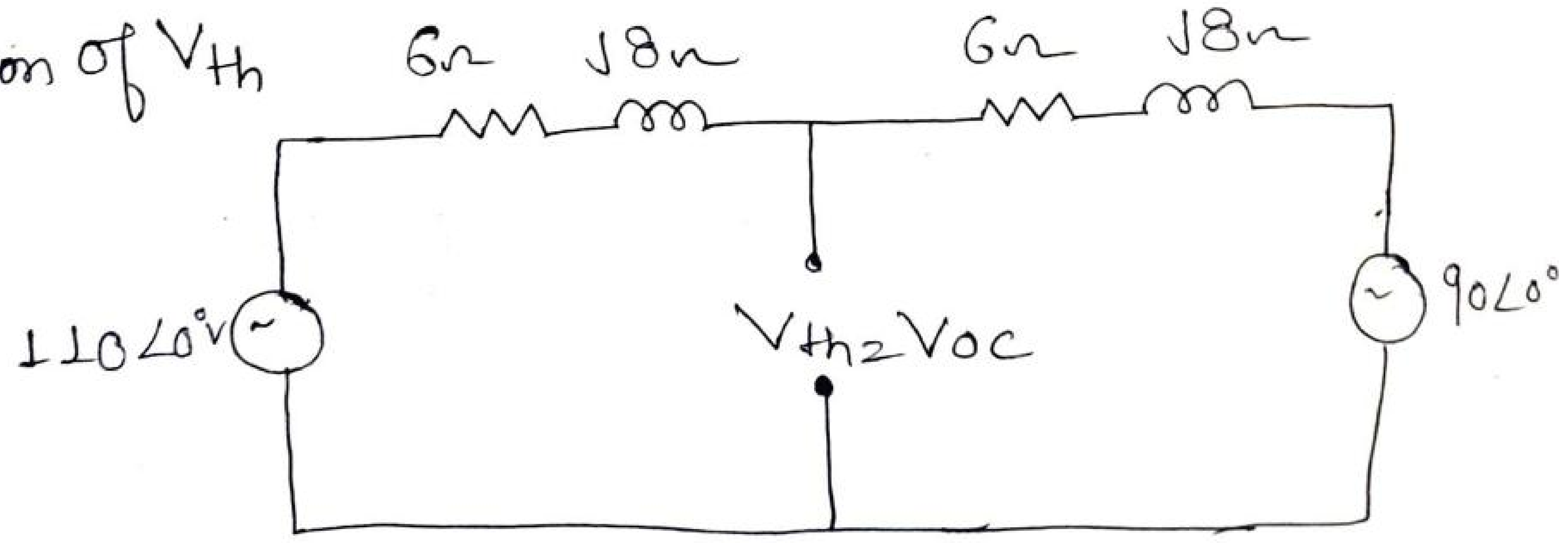
(i) Calcu. of Z_{th}



$$Z_{th} = (6+j8) \parallel (6+j8)$$

$$Z_{th} = (3+j4)\,\Omega$$

(ii) Calculation of V_{th}



$$\frac{V_{th} - 110\angle 0^\circ}{6+j8} + \frac{V_{th} - 90\angle 0}{6+j8} = 0$$

$$V_{th} = 100\angle 0^\circ$$

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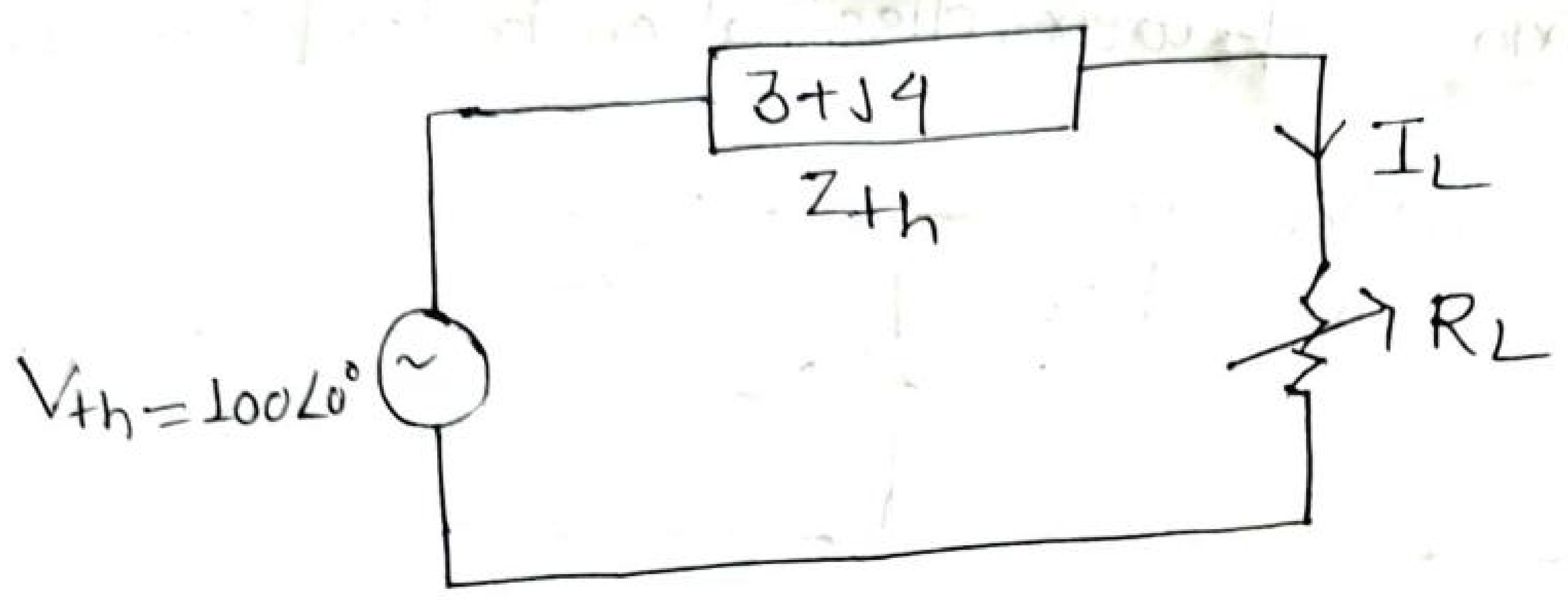

Maxm. power transfered to Load

$R_L = |Z_{th}| = \sqrt{3^2+4^2} = 5\Omega$

$$I_L = \frac{V_{th}}{3+j4+5} = \frac{100}{8+j4}$$

$$I_L = 11.18 \angle -26.56^\circ$$

$$P_{RL} = |I_L|^2 R_L = 11.18^2 \times 5$$
$$= 625W$$

Z_S	Z_L	Condition for MPT	P_L (maxm. power transfer to load)
$R_S + j0$	$R_L + j0$	$R_L = R_S$	$\lvert I^2 \rvert R_L = \frac{V_S^2}{4R}$
$R_S + jX_S$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$	$\lvert I^2 \rvert R_L$
$R_S + jX_S$	$R_L + jX_L$	$X_L + X_S = 0$	$\lvert I^2 \rvert R_L = \frac{V_S^2 R_L}{(R_S + R_L}$
$R_S + jX_S$	$R_L + jX_L$	$Z_L = Z_S^*$	$\lvert I^2 \rvert R_L = \frac{V_S^2}{4R_S}$
$R_S + jX_S$	$R_L + j0$	$R_L = \sqrt{R_S^2 + X_S^2}$	$\lvert I^2 \rvert R_L$
$R_S + j0$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + X_L^2}$	$\lvert I^2 \rvert R_L$