Model questions on Maxima and Minima :

Q.1. Find the maximum or minimum value of function
$$xy + \frac{a^3}{x} + \frac{a^3}{y}.$$

Q.2. Find the extreme values of the function :
$$x^3 + 3xy^2 - 3x^2 - 3y^2 + 7.$$

Q.3. Examine the function : $\sin x + \sin y + \sin(x+y)$ for extreme points

Q.4. Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum.

Q.5. A rectangular box open at the top is to have volume of 32 C.C. Find the dimensions of the box requiring least material for its Construction.

Q.6. Find the maximum value of $f = x^2 y^3 z^4$ subject to the Condition $x + y + z = 5$.

Q.7. Find the extreme value of $x^2 + y^2 + z^2$ subject to the Condition $xy + yz + zx = p$.

Q.8. The temperature at any point (x, y, z) in space is $U = K x y z^2$, where K is Constant. Find the highest temp. on the surface of sphere $x^2 + y^2 + z^2 = a^2$.

Q.9. Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

Q.10. Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a Cube.

Answers

Q.1. $3a^2$

Q.2. Max. value = 7, Min. value = 3

Q.3. Max. value = $\frac{3\sqrt{3}}{2}$

Q.4. $x = 40, y = 40, z = 40$.

Q.5. $x = 4, y = 4, z = 2$.

Q.6. Max. value = 83.63

Q.7. Extreme value = b

Q.8. highest temp. = $\frac{k}{8}a^4$

Q.9. Max. distance = 14; Min. distance = 12.

Q.10. $x = y = z$.