Model questions on Gradient, Curl and Divergence

Q.1. If $\phi = x^2+y-z-1$, find grad ϕ at $(1,0,0)$. Also find its magnitude.

Q.2. If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ and $r = |\vec{r}|$ prove that

(i) $\nabla f(r) = f'(r)\nabla r$ (ii) $\nabla r = \left(\frac{1}{r}\right)\vec{r} = \hat{r}$

(iii) $\nabla r^n = n r^{n-2}\vec{r}$ (iv) $\nabla \vec{r} = 3$

(v) $\nabla(\vec{a}\cdot\vec{r}) = \vec{a}$, where $\vec{a}$ is Constant vector.

Q.3. Find the directional derivative of $\phi = 3x^2+2y-3z$ at $(1,1,1)$ in the direction $2\hat{i}+2\hat{j}-\hat{k}$

Q.4. What is the greatest rate of increase of $\phi = xyz^2$ at the point $(1,0,3)$.

Q.5. Find the unit vector normal to the surface of sphere $x^2+y^2+z^2=1$.

Q.6. In what direction from $(3,1,-2)$ is the directional derivative of $\phi = x^2y^2z^4$ maximum and what is its magnitude?

Q.7. Find the rate of change of $f(x,y,z) = xyz$ in the direction normal to the surface $x^2y+y^2x+yz^2=3$ at the point $(1,1,1)$

Q.8. The temperature of points in space is given by $T(x,y,z) = x^2+y^2-z$. A mosquito located at $(1,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?

Q.9. Find div $\vec{f}$, if $\vec{f} = \text{grad}(x^3+y^3+z^3-3xyz)$

Q.10. Find div $(3x^2\hat{i}+5xy^2\hat{j}+xyz^3\hat{k})$ at the point $(1,1,1)$

Q.11. Find divergence and curl of following vector at $(2,-1,1)$
$\vec{F} = (xyz)\hat{i}+(3x^2y)\hat{j}+(xz^2-y^2z)\hat{k}$.

Q.12. Find 'a' such that $(3x-2y+z)\hat{i}+(4x+ay-z)\hat{j}+(x-y+2z)\hat{k}$ is solenoidal.

Q.13. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$. Show that $\vec{F} \cdot \text{Curl}\,\vec{F} = 0$

Q.14. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both (a) solenoidal (b) irrotational.

Q.15. Prove the following :- (i) $\text{div}(\text{grad} f) = \nabla^2 f$

(ii) $\text{Curl}(\text{grad}\,\phi) = \vec{0}$ (iii) $\text{div}(\text{Curl}\,\vec{V}) = 0$

(iv) $\text{Curl}(\text{Curl}\,\vec{f}) = \text{grad}(\text{div}\,\vec{f}) - \nabla^2 \vec{f}$

(v) $\nabla^2 u = 0$ if $u = x^2 - y^2$

Q.16. Prove that $\nabla^2(r^n) = n(n+1) r^{n-2}$

Answers

Q.1. $\nabla\phi = 2\hat{i} + \hat{j} - \hat{k}$; $|\nabla\phi| = \sqrt{6}$

Q.3. $\frac{19}{3}$

Q.4. 9

Q.5. $\vec{r}$

Q.6. $96(\hat{i} + 3\hat{j} - 3\hat{k})$ and $96\sqrt{19}$

Q.7. $\frac{9}{\sqrt{29}}$

Q.8. $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$

Q.9. $6(x+y+z)$

Q.10. 19

Q.11. $\text{div}\,\vec{F} = 14$ and $\text{curl}\,\vec{F} = 2\hat{i} - 3\hat{j} - 14\hat{k}$

Q.12. $a = -5$