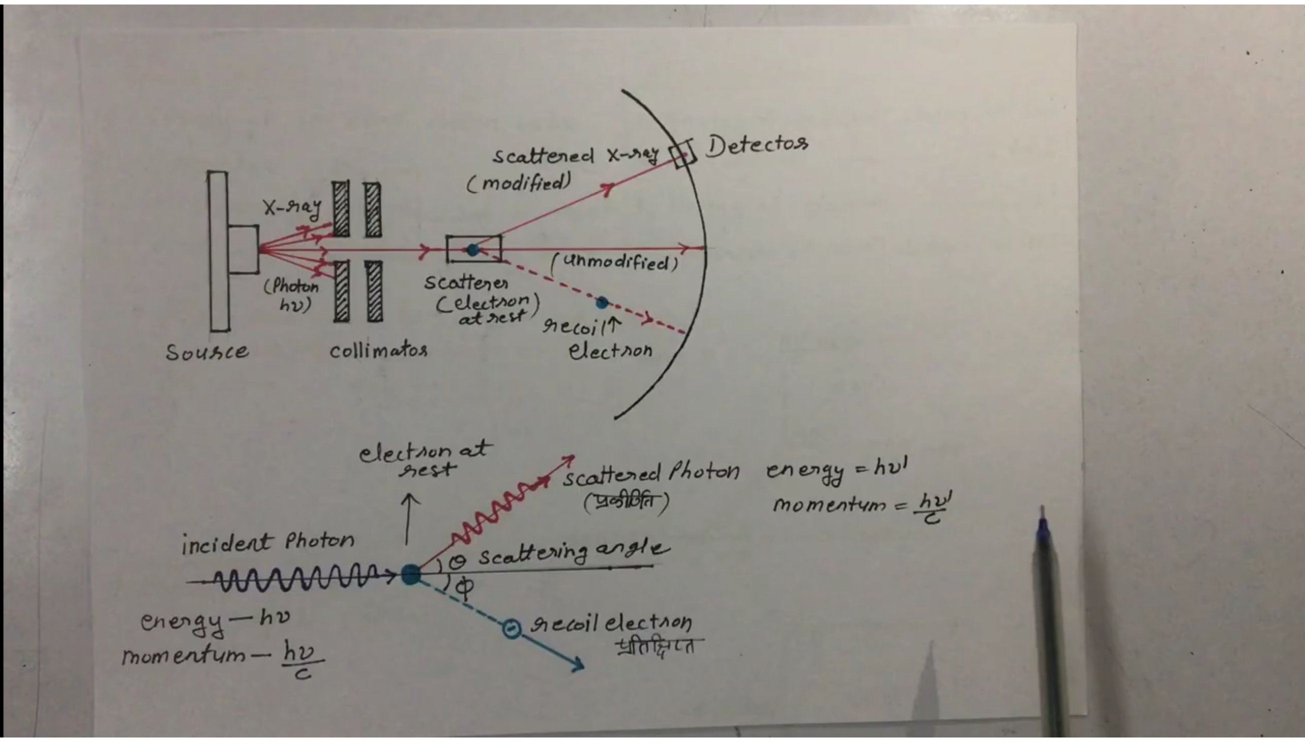
Compton's Effect

Scattering → "When a beam of monochromatic X-rays strikes a target, the X-rays are dispersed in all possible directions. This phenomenon is called as <u>scattering</u>. The angle between the direction of incident & scattered ray is called as <u>scattering angle</u>."

When a monochromatic beam of <u>high frqn</u> radiation is scattered <u>by</u> a substance, the scattered radiation contain two components

- lower frqn / greater wavelength (Modified radiation)
- same frqn / same wavelength (Unmodified radiation)



scattered X-ray
(modified)
Detector
X-ray
(Photon hν)
(unmodified)
Scatterer
(electron)
at rest
recoil
electron
Source
collimator
electron at
rest
scattered Photon
(प्रकीर्णित)
energy = hν'
momentum = hν'/c
incident Photon
θ scattering angle
φ
energy — hν
momentum — hν/c
recoil electron
प्रतिक्षिप्त

Before Collision

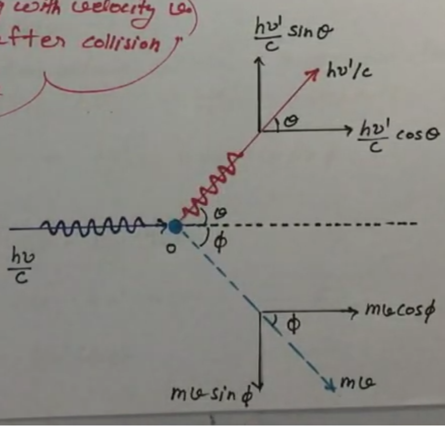
1. Energy of incident Photon $= h\nu$
2. Momentum — " — $= h\nu/c$
3. rest energy of electron $= m_0c^2$
4. momentum of rest electron $= 0$

After Collision

1. Energy of Scattered Photon $= h\nu'$
2. Momentum — " — $= h\nu'/c$
3. Energy of electron $= mc^2$
4. Momentum of recoil electron $= mv$

m – mass of electron moving with velocity v
v – Velocity of electron after collision
& $m = m_0/\sqrt{1-v^2/c^2}$

Component of Momentum before & after collision →



Energy of the system (Photon - electron) before Collision $= h\nu + m_0c^2$ —— (1)

Energy of the system after collision $= h\nu' + mc^2$ —— (2)

Principle of conservation of energy →

Energy before collision = Energy after collision

$$\boxed{h\nu + m_0c^2 = h\nu' + mc^2} \quad \text{—— (3)}$$

Principle of conservation of momentum →

along the direction → Momentum before collision = momentum after collision

$$h\nu/c + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad \text{—— (4)}$$

along ⊥ direction →

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad \text{—— (5)}$$

From eq^n (4) $\quad mv\cos\phi = \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta$

or $\quad mvc\cos\phi = h\nu - h\nu'\cos\theta$ —— (6)

from eq^n (5) $\quad mvc\sin\phi = h\nu'\sin\theta$ —— (7)

Now sq. eq^n (6) & (7) then add we get

$$(m v c \cos\phi)^2 + (m v c \sin\phi)^2 = (h\nu'\sin\theta)^2 + (h\nu - h\nu'\cos\theta)^2$$

$$m^2 v^2 c^2 = h^2\nu'^2 \sin^2\theta + h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta$$

$$m^2 v^2 c^2 = h^2\nu'^2(\sin^2\theta + \cos^2\theta) + h^2\nu^2 - 2h^2\nu\nu'\cos\theta$$

$$m^2 v^2 c^2 = h^2\nu'^2 + h^2\nu^2 - 2h^2\nu\nu'\cos\theta$$

$$\boxed{m^2 v^2 c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta)} \quad \text{--- (8)}$$

Now using eq^n (3)

$$mc^2 = h\nu - h\nu' + m_0 c^2$$

$$mc^2 = h(\nu - \nu') + m_0 c^2 \quad \text{--- (9)}$$

Sq. eq^n (9) we get

$$m^2 c^4 = [h(\nu - \nu') + m_0 c^2]^2$$

$$= h^2(\nu - \nu')^2 + m_0^2 c^4 + 2h(\nu - \nu') m_0 c^2$$

$$= h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2 c^4 + 2h(\nu - \nu') m_0 c^2$$

$$\boxed{m^2 c^4 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu') m_0 c^2 + m_0^2 c^4} \quad \text{--- (10)}$$

eq^n (10) − (8)

$$m^2c^4 - m^2v^2c^2 = h^2(\nu^2+\nu'^2-2\nu\nu'\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - h^2(\nu^2+\nu'^2-2\nu\nu'\cos\theta)$$

$$= \cancel{h^2\nu^2} + \cancel{h^2\nu'^2} - 2h^2\nu\nu' + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - \cancel{h^2\nu^2} - \cancel{h^2\nu'^2} + 2h^2\nu\nu'\cos\theta$$

$$m^2c^2(c^2-v^2) = 2h^2\nu\nu'(\cos\theta-1) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$\frac{m_0^2c^2(c^2-v^2)}{\left(1-\frac{v^2}{c^2}\right)} = \quad \text{—''—}$$

$$\frac{c^2m_0^2c^2\cancel{(c^2-v^2)}}{1\,\cancel{(c^2-v^2)}} = \quad \text{—''—}$$

2. $$\cancel{m_0^2c^4} - \cancel{m_0^2c^4} = +\left[2h^2\nu\nu'(-1+\cos\theta) + 2h(\nu-\nu')m_0c^2\right]$$

$$0 = 2h^2\nu\nu'(\cos\theta-1) + 2h(\nu-\nu')m_0c^2$$

3 $$\cancel{2h}(\nu-\nu')m_0c^2 = \cancel{2h^2}\nu\nu'(1-\cos\theta)$$

(1 $$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{(\nu - \nu')}{\nu\nu'} = \frac{h}{m_0 c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1-\cos\theta) \quad \text{--- (11)}$$

This eq^n shows scattered frq^n < incident frq^n

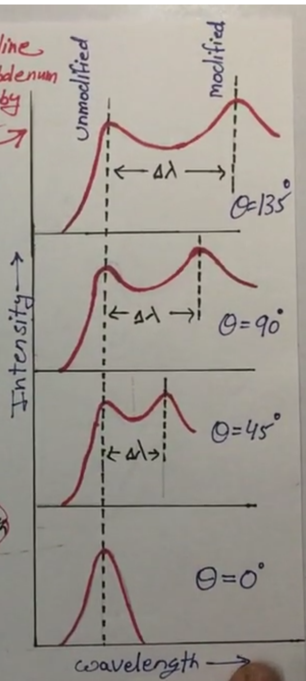
$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0 c^2}(1-\cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)} \quad \text{--- (12)} \star$$

1. When $\theta = 0$, $\Delta\lambda = \lambda' - \lambda = 0$ (no scattering along the direction)
2. When $\theta = \frac{\pi}{2}$, $\Delta\lambda = \frac{h}{m_0 c} = 0.02426\,Å$ (This diff. in wavelength is compton wavelength)
3. When $\theta = \pi$, $\Delta\lambda = \frac{2h}{m_0 c} = 0.4852\,Å$

As θ varies 0 — $180°$, wavelength of scattered photon varies

For K_α line of a molybdenum scattered by graphite

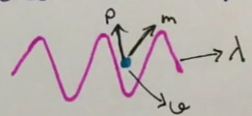


de-Broglie Hypothesis

In 1924, Louis de-Broglie suggested → " A wave to be associated with each moving material particle (गतिशील द्रव्य कण) which is called the matter wave."

The wavelength of this wave is determined by the momentum of the particle. If p is the momentum of the particle, the wavelength of the wave associated with it is given as

$$\boxed{\lambda = \frac{h}{p}}$$



Note : These waves can travel through vacuum like E.M. waves, but these waves are different from E.M. waves because, these are associated with all types of charged & neutral moving material particles.

Expression for de-Broglie wave length →

Consider ν is the freq^n of Photon

& Energy of Photon $E = h\nu$ — ①

If mass of particle is converted into energy, the energy is given by Einstein's mass-energy relation (theory of relativity)

energy of Photon is $E = mc^2$ — ②

from eq^n ① & ②

$$h\nu = mc^2$$

$$m = \frac{h\nu}{c^2} \quad \text{— ③}$$

Now momentum of Photon $p = mc$

$$p = \frac{h\nu}{c^2} \cdot c$$

$$p = \frac{h\nu}{c}$$

$$p = \frac{h}{\lambda} \qquad \left[\because \lambda = \frac{c}{\nu}\right]$$

$$\boxed{\lambda = \frac{h}{p}}$$

Special cases: $\lambda = \frac{h}{p}$

1) If a particle of mass 'm' is moving with non relativistic velocity "v" then momentum of particle $p = mv$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{mv}$$

$\lambda = \frac{h}{mv}$

$[v \to c, \; m = m_0/\sqrt{1 - v^2/c^2}]$

2) If K.E of the particle of mass "m" is "K"

$$K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}}$$

so $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2K}{m}}} = \frac{h}{\frac{m\sqrt{2K}}{\sqrt{m}}} = \frac{h}{\sqrt{m}\sqrt{2K}} = \frac{h}{\sqrt{2mK}}$ $\quad \lambda = \frac{h}{\sqrt{2mK}}$

3) According to the theory of gas the avg K.E. of the material particle is given by

$$KE = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$\therefore \frac{1}{\cancel{2}}mv^2 = \frac{3}{\cancel{2}}kT$$

$$v^2 = \frac{3}{m}kT$$

$$v = \sqrt{\frac{3}{m}kT}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{3}{m}kT}} = \frac{h}{\sqrt{3mkT}}$$

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

4) If a charged Particle (charge q) is accelerated through a Pot^n diff. of V volt, $K = qV$ Then

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

5) If the moving material Particle is electron & this electron is accelerated through a potential difference of V volts so K.E of material Particle is

$$eV = \frac{1}{2}mv^2$$

$$mv^2 = 2eV$$

$$v^2 = \frac{2eV}{m}$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

For electron → $h = 6.6 \times 10^{-34}$ Js

$m = 9.1 \times 10^{-31}$ Kg

$e = 1.6 \times 10^{-19}$ C

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} = \sqrt{\frac{150}{V}} \text{ A}^\circ$$

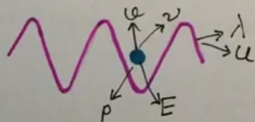
$$\lambda = \frac{12.27}{\sqrt{V}} \text{ A}^\circ$$

WAVE PACKET

Matter Wave
→ Particle nature
→ Wave nature

According to de-Broglie hypothesis "A wave is associated with each moving material particle" whose wavelength is given as $\lambda = \frac{h}{p}$

So $\lambda = \frac{h}{p} = \frac{h}{mv}$ —(1)



from quantum cond^n energy of particle $E = h\nu$

$\therefore \nu = \frac{E}{h} = \frac{mc^2}{h}$ —(2) ($E = mc^2$ → Einstein's mass-energy)

so velocity of debroglie wave $u = \nu\lambda$

$$u = \frac{\cancel{m}c^2}{\cancel{h}} \times \frac{\cancel{h}}{\cancel{m}v} = \frac{c^2}{v}$$

$$u = \frac{c^2}{v}$$

but Einstein's theory of relativity "no material particle can have speed/velocity greater than velocity of light" i.e. $v < c$

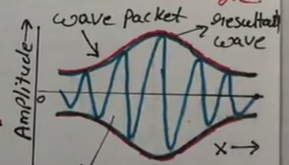
Conclusion : $u = \frac{c^2}{v}$

1) u will be greater than speed of light c [ie $u > c$] which is impossible.

2) if $u > v$, the particle left behind. means only one wave is not associated with material Particle.

Hence "Schrodinger assumed that a moving material particle is equivalent to a wave packet, instead of a single wave." so

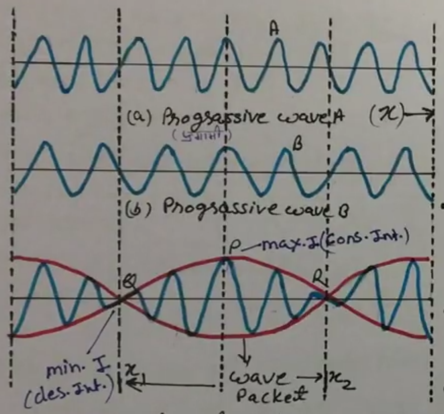
- Wave packet is a group of several waves of slightly different velocities & different wavelength.
- amplitude & Phase of component waves are such that they interfere constructively in limited region where Particle found, & the interfere destructively where Particle not found (outside region) & resultant amplitude abruptly falls to zero.



Several progressive waves of slightly diff. λ / diff. v

Superposed & travel in from of group of waves

Formation of Wave Packet



(c) Resultant wave

Progressive waves of equal amplitude, slightly different "λ"

$\lambda_{(B)} > \lambda_{(A)}$

at P Point → A & B waves are in same phase

at Q & R Point → —"— opposite phase

- The spread of amplitude of resultant wave with distance determines the size of wave packet.
- If the velocity of all superposing component wave is same → velocity of wave packet will be same
- If the velocity of all component wave is differ → velocity of wave packet will be differ

"The velocity of component waves of a wave packet is called the wave velocity or Phase velocity "v_p" while the velocity of the wave packet is called the group velocity "v_g""

Equation of a Wave Packet

Consider two waves y_1 & y_2

amplitude $\longrightarrow$ same i.e. "a"

frequencies $\longrightarrow \nu_1$ & ν_2

wavelength $\longrightarrow \lambda_1$ & λ_2

angular frqn $\rightarrow \omega_1 = 2\pi\nu_1$ & $\omega_2 = 2\pi\nu_2$

Propagation cons. $\rightarrow$ $k_1 = \frac{2\pi}{\lambda_1}$ & $k_2 = \frac{2\pi}{\lambda_2}$
(संचरण नियतांक)

here $y_1 = a\sin(\omega_1 t - k_1 x)$

& $y_2 = a\sin(\omega_2 t - k_2 x)$

Using Principle of superposition - The resultant wave motion at any Point "x" & at any time "t" is given by

$$Y = y_1 + y_2$$

$$= a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$Y = 2a \sin\left\{\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right\} \cos\left\{\frac{\omega_1 t - k_1 x - \omega_2 t + k_2 x}{2}\right\}$$

$$Y = 2a \sin\left\{\frac{(\omega_1+\omega_2)t - (k_1+k_2)x}{2}\right\} \cos\left\{\frac{(\omega_1-\omega_2)t - (k_1-k_2)x}{2}\right\}$$

$$Y = 2a \sin\left\{\frac{(\omega_1+\omega_2)}{2}t - \frac{(k_1+k_2)}{2}x\right\} \cos\left\{\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right\}$$

[$\because \frac{\omega_1+\omega_2}{2} = \omega$, $\frac{k_1+k_2}{2} = k$, $\omega_1-\omega_2 = \Delta\omega$, $k_1-k_2 = \Delta k$]

so $Y = 2a \sin(\omega t - kx) \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$

or $$\boxed{Y = 2a \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \sin(\omega t - kx)}$$

I II

here amplitude of wave packet — $A = 2a\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$

& its Phase — $(\omega t - kx)$

ω & k are mean angular freq^n & propagation cons. of the wave.

—x—