Indian Institute of Technology, Kanpur.

Even semester, 2019 - 2020.

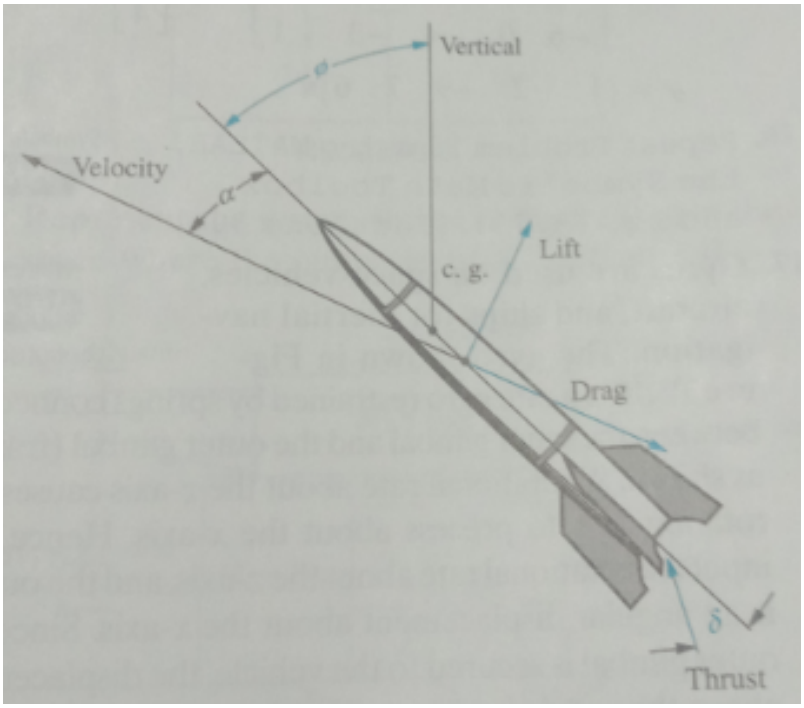
AE 322 Aircraft Control Systems

Assignment 2

Due in class on 14/2/2020 **Answer all questions** **Marks - 20**

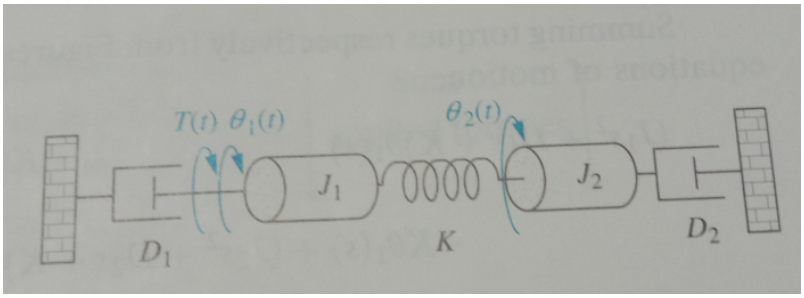
1. A missile in flight, is subject to several forces: thrust, lift, drag and gravity. The missile flies at an angle of attack, α, from its longitudinal axis creating lift. For steering, the body angle from vertical, ϕ, is controlled by rotating the engine at the tail. The transfer function relating ϕ to the angular displacement, δ, of the engine is of the form.

$$\frac{\phi(s)}{\delta(s)} = \frac{K_a + K_a s}{K_0 + K_1 s + K_2 s^2 + K_3 s^3}$$



Obtain the state space representation of the system. **(5marks)**

2. Find the transfer function of the following system in terms of the rotational inertias J_1 and J_2, the bearing constants D_1 and D_2 and the spring constant K. Torque $T(t)$ is the input and $\theta_2(t)$ is the output. **(5 marks)**



3. Write the general form of (a) a first order and (b) a second order system. Identify the time constant, the natural frequency and the damping factor in the equations. Also, vary each of these quantities and solve them in MATLAB using the *RK4* code built in Assignment 1. Analyse all possible system responses giving physical interpretations. **(5 marks)**

4. Derive the rotational kinematics relating p, q and r with the rates of Euler angles $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ as explained in class. Call them equation set I. From here, do the following to appreciate the problem of singularity.

   i) What are the simplified form for p, q and r when $\theta = \pi/2$? Call them set II

   ii) Invert set I and obtain $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ in terms of p, q and r. Call them equation set III.

   iii) Using sets I, II and III show that the rates of Euler angles assume $\frac{0}{0}$ form when $\theta = \pi/2$.

   iv) Use L'Hospital Rule on set II and obtain the simplified Euler angle rates at $\theta = \pi/2$.
   [Hint: $\frac{d}{d\theta}\{.\} = \frac{d}{dt}\{.\}\frac{dt}{d\theta}$. Write tan{.}, sec{.} in terms of sin{.} and cos{.}] **(5 marks)**