Aircraft Performance

- The connections between aircraft performance and propulsion system performance.

For a vehicle in steady, level flight,

- the thrust force is equal to the drag force
- the lift is equal to weight.

Any thrust available in excess of that required to overcome the drag can be applied to accelerate the vehicle (increasing kinetic energy) or to cause the vehicle to climb (increasing potential energy).

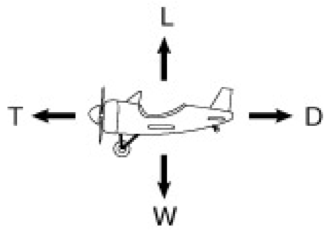

Figure. Force balance for aircraft in steady level flight.

$$T = D,\quad L = W \qquad\qquad W = L = D\frac{L}{D} = T\left(\frac{L}{D}\right)$$

To maximize time aloft (endurance) for a fixed quantity of energy (the fuel), it is necessary to minimize the rate of energy usage (power required = drag*flight velocity). Note that to maximize range, it is necessary to maximize L/D, or for a given weight, to minimize drag.

Vehicle Drag

Recall from fluids that drag takes the form shown below, being composed of a part termed *parasitic drag* that increases with the square of the flight velocity, and a part called *induced drag*, or drag due to lift, that decreases in proportion to the inverse of the flight velocity.

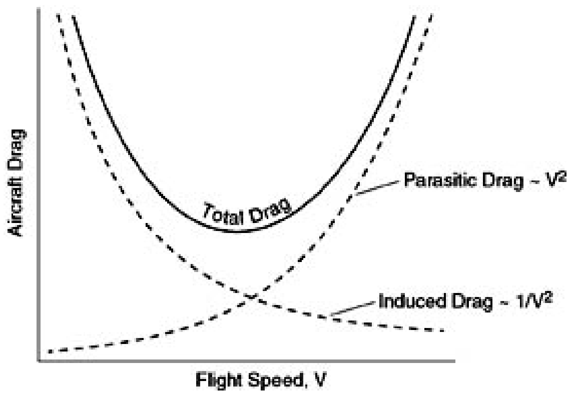

Figure Components of vehicle drag.

Parasitic drag (also called skin friction drag) is drag caused by moving a solid object through a fluid medium (in the case of aerodynamics, more specifically, a gaseous medium). Parasitic drag is made up of many components, the most prominent being form drag. Skin frictional interference drag are also major components of parasitic drag.

In aviation, induced drag tends to be greater at lower speeds because a high angle of attack is required to maintain lift, creating more drag. However, as speed increases the induced drag becomes much less, but parasitic drag increases because the fluid is flowing faster around protruding objects increasing friction or drag. At even higher transonic and supersonic speeds, wave drag enters the picture. Each of these forms of drag changes in proportion to the others based on speed. The combined overall drag curve therefore shows a minimum at some airspeed - an aircraft flying at this speed will be at or close to its optimal efficiency. Pilots will use this speed to maximize the gliding range in case of an engine failure. However, to maximize the gliding endurance, the aircraft's speed would have to be at the point of minimum power, which occurs at lower speeds than minimum drag. At the point of minimum drag, $C_{D,o}$ (drag coefficient of drag created by lift),

e = wing efficiency factor

$$C_D = C_{D_o} + \frac{C_L^{\,2}}{\pi e AR}$$ where $$L = \frac{1}{2}\rho V^2 S C_L$$ and $$D = \frac{1}{2}\rho V^2 S C_D$$

Thus

$$D = \frac{1}{2}\rho V^2 S C_{D_o} + \frac{L^2}{\frac{1}{2}\rho V^2 S}\left(\frac{1}{\pi e AR}\right)$$

or

$$D=\frac{1}{2}\rho V^2 S C_{D_0}+\frac{W^2}{\frac{1}{2}\rho V^2 S}\left(\frac{1}{\pi e AR}\right)$$

The **minimum drag** is a condition of interest. We can see that for a given weight, it occurs at the condition of maximum lift-to-drag ratio

$$D=L\frac{D}{L}=W\left(\frac{D}{L}\right)=W\left(\frac{C_D}{C_L}\right)$$

We can find a relationship for the maximum lift-to-drag ratio by setting

$$\frac{d}{dC_L}\left(\frac{C_{D_0}+\frac{C_L{}^2}{\pi e AR}}{C_L}\right)=0$$

from which we find that

$$C_{L_{\min drag}}=\sqrt{\pi e AR C_{D_0}}$$ and $$C_{D_{\min drag}}=2C_{D_0}$$

$$\left(\frac{C_L}{C_D}\right)_{\max}=\frac{1}{2}\sqrt{\frac{\pi e AR}{C_{D_0}}}$$ and

$$V_{\min drag}=\sqrt{\frac{W}{\frac{1}{2}\rho S C_{L_{\min drag}}}}=\left[4\left(\frac{W}{S}\right)^2\frac{1}{\rho^2}\frac{1}{C_{D_0}}\left(\frac{1}{\pi e AR}\right)\right]^{1/4}$$

Power Required

Now we can look at the propulsion system requirements to maintain steady level flight since

$$T_{req}=D \quad \text{and} \quad P_{req}=T_{req}V=DV$$

$$P_{req} = \frac{1}{2}\rho V^3 S C_{D_0} + \frac{W^2}{\frac{1}{2}\rho V S}\left(\frac{1}{\pi e AR}\right)$$

Thus the power required (for steady level flight) takes the form

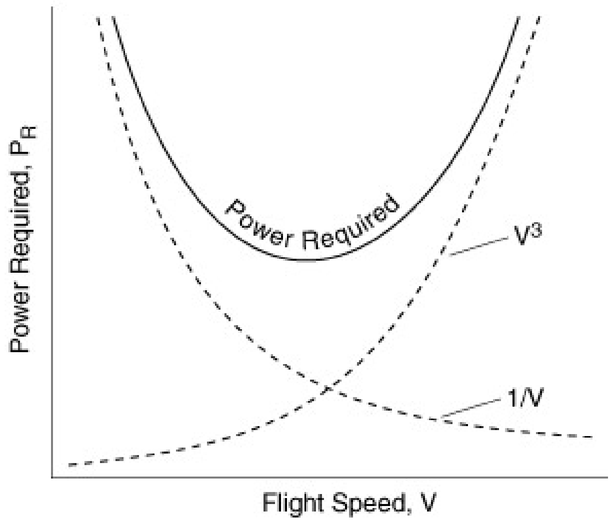

Figure Typical power required curve for an aircraft.

The velocity for minimum power is obtained by taking the derivative of the equation for P_{req} with respect to V and setting it equal to zero.

$$V_{minimum\,power} = \left[\frac{4}{3}\left(\frac{W}{S}\right)^2 \frac{1}{\rho^2}\frac{1}{C_{D_0}}\left(\frac{1}{\pi e AR}\right)\right]^{1/4}$$

As we will see shortly, maximum *endurance* (time aloft) occurs when the minimum power is used to maintain steady level flight. Maximum *range* (distance traveled) is obtained when the aircraft is flown at the most aerodynamically efficient condition (maximum C_L/C_D).

Aircraft Endurance

For a given amount of available fuel energy (Joules), the maximum endurance (time aloft) is obtained at a flight condition corresponding to the minimum rate of energy expenditure (Joules/second), or $Preq_{min}$, as shown in Figure 4.3.

We can determine the aerodynamic configuration which provides the minimum energy expenditure:

$$D = W\frac{D}{L} = W\left(\frac{C_D}{C_L}\right) \qquad \text{so} \qquad P = W\left(\frac{C_D}{C_L}\right)\cdot V$$

where

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}}$$

Then

$$P = \sqrt{\frac{W^3}{\frac{1}{2}\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right)$$

So the minimum power required (**maximum endurance**) occurs when $\frac{C_L^{3/2}}{C_D}$ is a maximum.

With a little algebra we can arrive at an expression for the maximum endurance. Setting

$$\frac{d}{dC_L}\left(\frac{C_{D_0} + \frac{C_L^2}{\pi e AR}}{C_L^{3/2}}\right) = 0$$

we find that

$$C_{L_{minpower}} = \sqrt{3\pi e AR C_{D_0}} \qquad \text{and} \qquad C_{D_{minpower}} = 4C_{D_0}$$

$$\left(\frac{C_L}{C_D}\right)_{minpower} = \sqrt{\frac{3\pi e AR}{16 C_{D_0}}} \qquad \text{and}$$

$$V_{minpower} = \sqrt{\frac{W}{\frac{1}{2}\rho S C_{L_{minpower}}}} = \left[\frac{4}{3}\left(\frac{W}{S}\right)^2 \frac{1}{\rho^2} \frac{1}{C_{D_o}} \left(\frac{1}{\pi e AR}\right)\right]^{1/4}$$

Thus the minimum power (maximum endurance) condition occurs at a speed which is $3^{-1/4}$ = 76% of the minimum drag (maximum range) condition. The corresponding lift-to-drag ratio is 86.6% of the maximum lift-to-drag ratio.

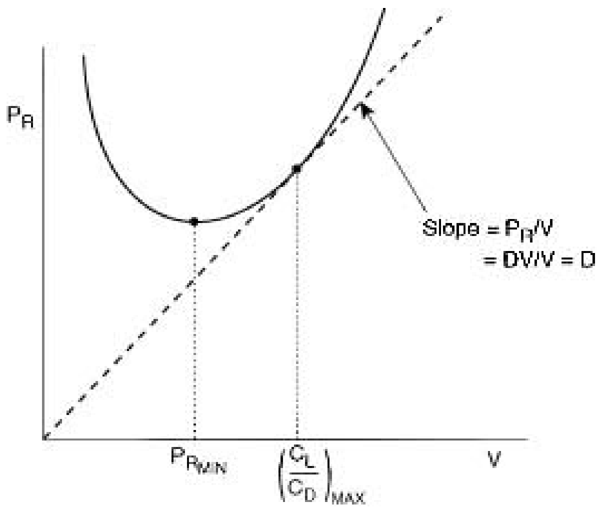

Figure: Relationship between condition for maximum endurance and maximum range.

$$\left(\frac{L}{D}\right)_{\dot{m}_n power} = \sqrt{\frac{3\pi eAR}{16CD_o}}$$

Continuing

$$D_{minimum\,power} = W\left[\frac{16}{3}\frac{C_{D_o}}{\pi e AR}\right]^{1/2}$$

which can be substituted into

Specific Impulse (I or I_{sp}):

$$I = \frac{\text{thrust}}{\text{fuel weight flow rate}} \qquad \text{(units of seconds)}$$

$$\frac{dW}{dt} = \dot{m}_f \cdot g = \frac{-T\dot{m}_f \cdot g}{T} = \frac{-T}{Isp} = \frac{-D}{Isp}$$

Such that, for maximum endurance

$$\frac{dW}{dt} = \frac{-W}{Isp}\left[\frac{16}{3}\frac{C_{D_o}}{\pi e AR}\right]^{1/4}$$

which can be integrated (assuming constant Isp) to yield

$$\boxed{t_{max} = Isp\left[\frac{16}{3}\frac{C_{D_o}}{\pi e AR}\right]^{-1/4} \ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

Climbing Flight

Any excess in power beyond that required to overcome drag will cause the vehicle increase kinetic or potential energy. We consider this case by resolving forces about the direction of flight and equating these with accelerations.

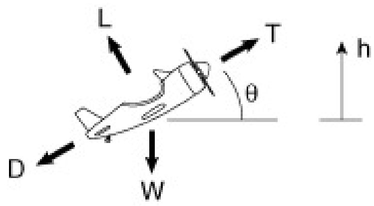

Figure Force balance for an aircraft in climbing flight.

$$L - W\cos\theta = \frac{W}{g}V\frac{d\theta}{dt}$$ where $V\frac{d\theta}{dt}$ is the accel. normal to the flight path

$$T - D - W\sin\theta = \frac{W}{g}\frac{dV}{dt}$$ where $\frac{dV}{dt}$ is the accel. tangent to the flight path

So the change in height of the vehicle (the **rate of climb**, **R/C**) is:

$$R/C = \frac{dh}{dt} = V\sin\theta = V\left(\frac{T-D}{W}\right) - \frac{V}{g}\frac{dV}{dt}$$

which is instructive to rewrite in the form

$$TV - DV = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

or

$$P_{available} - P_{required} = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

in words:

excess power = change in potential energy + change in kinetic energy

<u>**For steady climbing flight**</u>,

$$R/C = V\left(\frac{T-D}{W}\right) = \frac{P_{avail} - P_{req}}{W}$$

and the time-to-climb is

$$t = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

where

$P_{available} = \eta_{prop} P_{shaft}$ for example, and $P_{required} = DV$

The power available is a function of the propulsion system, the flight velocity, altitude, etc. Typically it takes a form such as that shown in Figure 4.6. The shortest time-to-climb occurs at the flight velocity where P_{avail} P_{req} is a maximum.

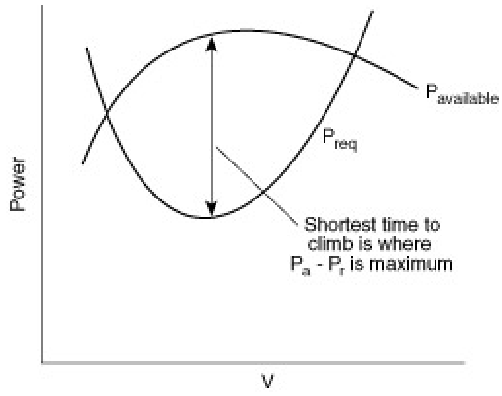

Figure: Typical behavior of power available as a function of flight velocity.

Figure Lockheed Martin F-16 performing a vertical accelerated climb.