Meta Learning III

CS771: Introduction to Machine Learning

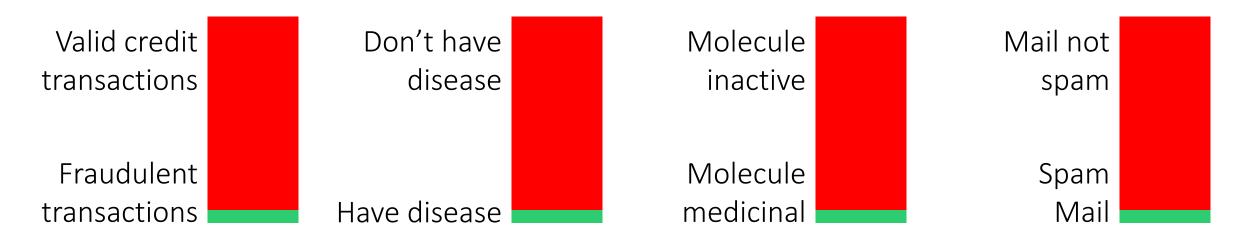
Purushottam Kar

Precap

Learning with Imbalanced Data

Examples of imbalanced data in ML problems
Techniques to handle imbalanced data





In anomaly detection, medical diagnosis, drug discovery, spam filtering and many other domains, data is heavily biased

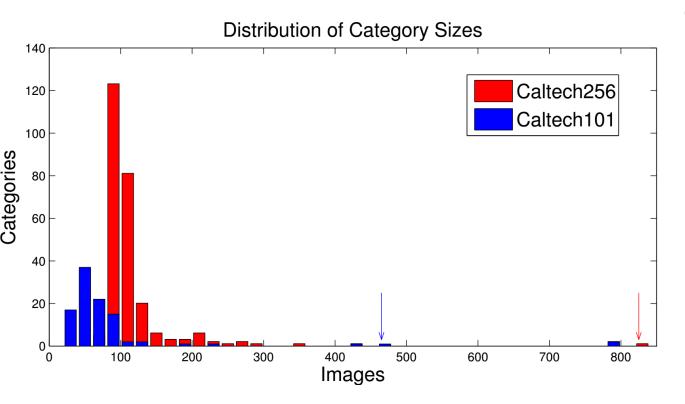
Lots of data may be available in total but very less (0.01% or lesser) may belong to the "rare" class e.g. 100000 red vs 10 green

Trivial for a classifier to get 99.99% accuracy (simply predict everything red)

May be disastrous, e.g. if green class denotes a rare disease or fraudulent transaction Many ML algorithms that are designed to greedily chase classification accuracy may completely ignore green training data points since it is easy to do well on red data points

The whole purpose of learning is lost if the above happens 😊

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Caltech-256 is an object detection dataset with 256 classes

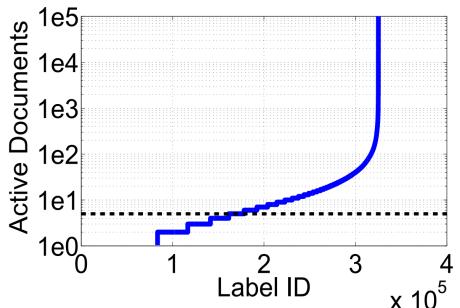
Simple averaging argument shows that 50% of classes must each have less than 0.8% of the data
Lots of data for few classes but most classes impoverished
In fact, rarest class has only 80 images, most popular 800 images

Makes learning challenging

Easier to do well on popular classes with lots of data points e.g. cats

Difficult to handle rare classes





0	0	0	0	0	0	0	0	0	1	0	0
0	1	0	1	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0
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Wikipedia LSHTC is a multilabel dataset with 300000 labels

As many as 150000 labels occur only in less than 5 documents

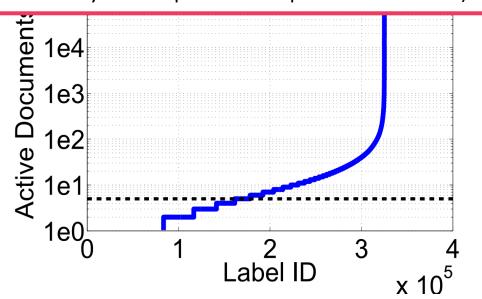
Most popular label ("living persons") occurs in 100000 documents

Similar situation in recommendation — most people like iPhone but most items liked by a very select group of individuals

Lots of train data to find out who likes iPhone, not enough for most other items

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Ironically, whereas we often proudly refer to these as "big-data" problems, they can often be seen as collections of a huge number of "tiny-data" problems (one tiny data problem per rare label) and a few "reasonable-data" problems



I	U	U	V	V	D	D	V	D	D	1	D	U
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	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0
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Get more data for the rare classes so that they don't remain rare ©
Generate artificial data for rare classes to make them more prominent
Change training algorithms that artificially make rare classes prominent

Use different learning strategies for rare classes and popular classes

Includes using loss functions that are aware of class imbalance

A combination of all of the above may be required in practice Extreme classification algos (assignment 2) do indeed practice some of these





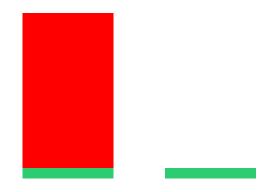






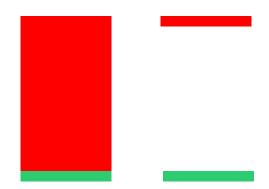




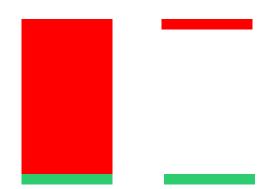












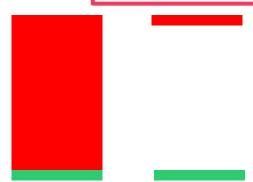
Undersample the popular class so that it does not dominate

Actually throwing out data not a very good idea since it throws away data



A bit pointless to handle label rarity problem by making all labels rare. Would be like a hypothetical administration that aims at equidistribution of wealth but ends up with equidistribution of poverty instead





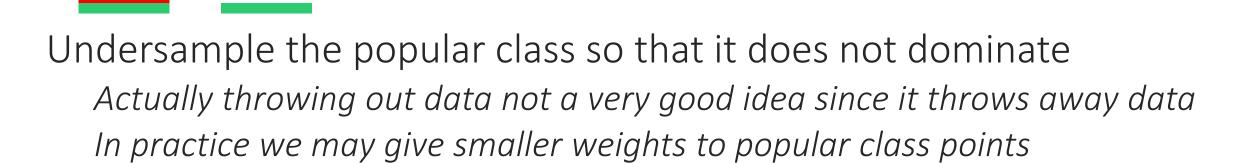
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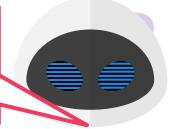
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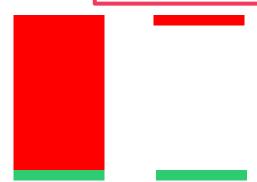






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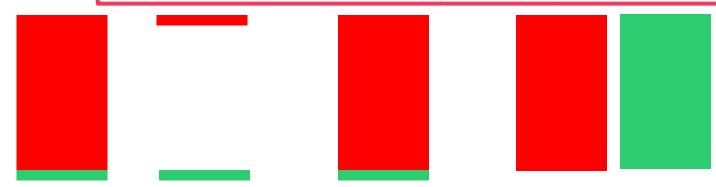
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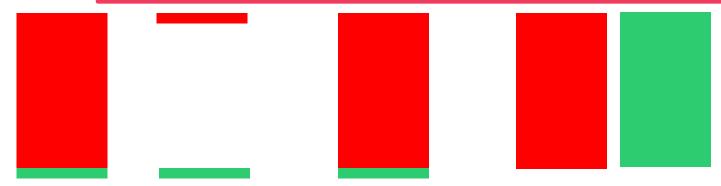
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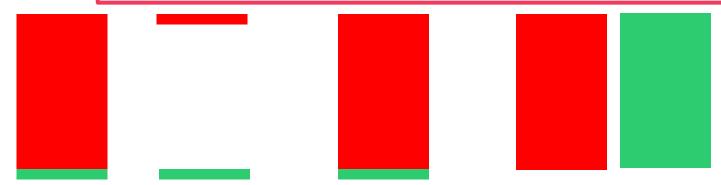
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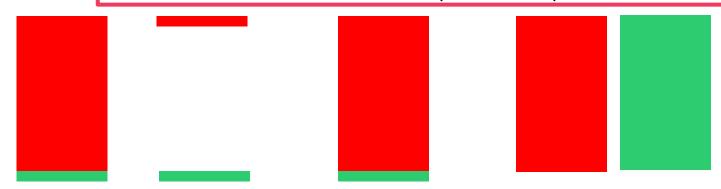
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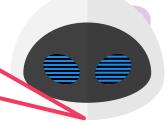
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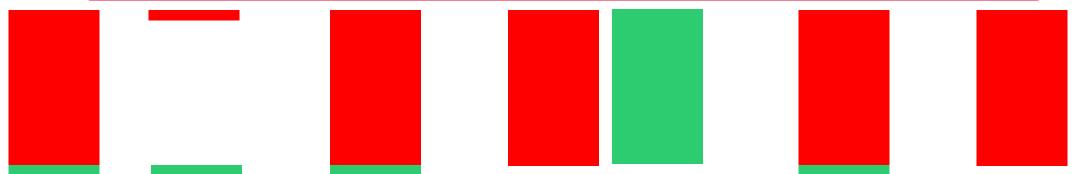
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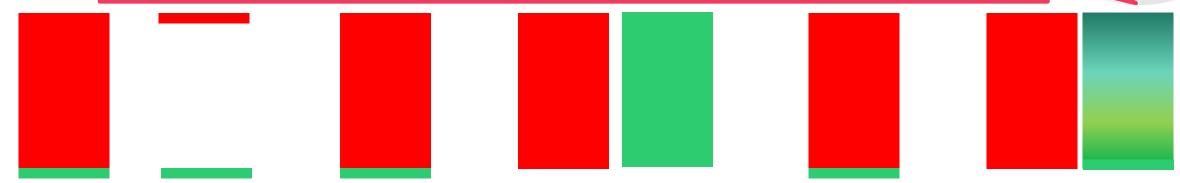
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Imbalance-aware Training Techniques

Simplest are reweighing techniques: add more weight to rare points

Loss functions like hinge loss are very majoritarian — neglect rare classes Introduce a weight per data point (many libraries already support this)

Classical SVM:
$$\min_{\mathbf{w}} \frac{1}{2} \|\mathbf{w}\|_2^2 + C \cdot \sum_{i=1}^n \left[1 - y^i \cdot \langle \mathbf{w}, \mathbf{x}^i \rangle\right]_+$$

Reweighted SVM: give weight C_+ to positives and C_- to negatives

$$\min_{\mathbf{w}} \frac{1}{2} \|\mathbf{w}\|_{2}^{2} + C_{+} \cdot \sum_{i:y^{i}=+1} \left[1 - \langle \mathbf{w}, \mathbf{x}^{i} \rangle \right]_{+} + C_{-} \cdot \sum_{i:y^{i}=-1} \left[1 + \langle \mathbf{w}, \mathbf{x}^{i} \rangle \right]_{+}$$

How to tune C_+ , C_- ?

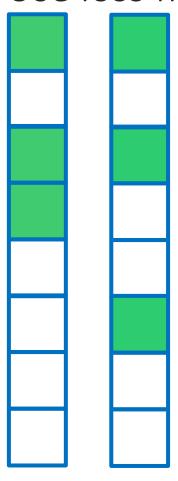
Use inverse popularity. Let $p_+ = \frac{n_+}{n}$, $p_- = \frac{n_-}{n}$ be proportion of pos/neg

Use
$$C_{+} = \frac{1}{p_{+}}$$
, $C_{-} = \frac{1}{p_{-}}$

Other more sophisticated techniques have been discovered recently (see next slides on loss functions for imbalanced training)

Imbalance-aware Training Techniques

Use loss fns that themselves penalize models that neglect rare classes



$$\left| \left\{ i: \begin{array}{ll} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\} \right|$$

$$\left| \left\{ i : \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1 \right. \right\}$$

$$\left| \left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = 1 \end{array} \right\} \right| \quad \left| \left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1 \end{array} \right\} \right| \quad r_{\text{bal}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{r_{\text{TPR}} + r_{\text{TNR}}}{2}$$

$$\left| \left\{ i : \begin{array}{c} \hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1 \end{array} \right. \right\}$$

$$r_{\text{TNR}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{d}{b+d}$$

$$r_{\text{TPR}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$
True Positive Rate (= Recall)

$$r_{\text{TNR}}(\mathbf{y}, \mathbf{y}) = \frac{1}{b+d}$$

$$F(\mathbf{y}, \hat{\mathbf{y}}) = \frac{2 \cdot r_{\text{prec}} \cdot r_{\text{rec}}}{r_{\text{prec}} + r_{\text{rec}}}$$

F-measure

$$r_{
m G}(\mathbf{y}, \widehat{\mathbf{y}}) = \sqrt{r_{
m TPR} \cdot r_{
m TNR}}$$
 G-mean

$$r_{\min}(\mathbf{y}, \hat{\mathbf{y}}) = \min\{r_{\text{TPR}}, r_{\text{TNR}}\}$$

MinPerf

$$\begin{vmatrix} \left\{ i : \hat{\mathbf{y}}_{i} = 1 \\ \mathbf{y}_{i} = 1 \right\} \end{vmatrix} \begin{vmatrix} \left\{ i : \hat{\mathbf{y}}_{i} = 1 \\ \mathbf{y}_{i} = -1 \right\} \end{vmatrix} \begin{aligned} r_{\text{acc}}(\mathbf{y}, \hat{\mathbf{y}}) &= \frac{a+d}{a+b+c+d} \\ &= p_{+} \cdot r_{\text{TPR}} + p_{-} \cdot r_{\text{TNR}} \end{aligned}$$
Classification Accuracy

$$r_{\text{bal}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{r_{\text{TPR}} + r_{\text{TNR}}}{2}$$

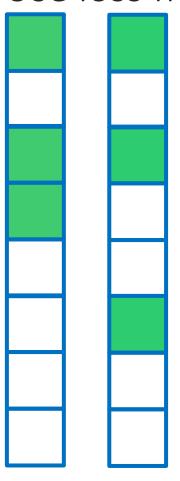
Balanced Accuracy

$$r_{\text{prec}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+b}$$
Precision

$$r_{\text{rec}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$
Recall

This is called the confusion matrix of a classifier and records how many data points that truly belong to class c got classified as class c'. For a multiclass problem with C classes, confusion matrix is $C \times C$

Use loss fris that themselves penanze moders that hegiett rare classes



$$\left| \left\{ i: \begin{array}{l} \hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1 \end{array} \right\} \right|$$

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$$r_{\text{TNR}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{d}{b+d}$$

 $r_{\text{TPR}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$

$$F(\mathbf{y}, \hat{\mathbf{y}}) = \frac{2 \cdot r_{\mathrm{prec}} \cdot r_{\mathrm{rec}}}{r_{\mathrm{prec}} + r_{\mathrm{rec}}}$$
 $r_{\mathrm{G}}(\mathbf{y}, \hat{\mathbf{y}}) = \sqrt{r_{\mathrm{TPI}}}$ G-mean F-measure $r_{\mathrm{Min}}(\mathbf{y}, \hat{\mathbf{y}}) = \min\{r_{\mathrm{Min}}(\mathbf{y}, \hat{\mathbf{y}}) = \min\{r_{\mathrm{Min}}(\mathbf{y},$

$$r_{\mathrm{G}}(\mathbf{y}, \hat{\mathbf{y}}) = \sqrt{r_{\mathrm{TPR}} \cdot r_{\mathrm{TNR}}}$$
 G-mean

$$r_{\text{Min}}(\mathbf{y}, \hat{\mathbf{y}}) = \min\{r_{\text{TPR}}, r_{\text{TNR}}\}$$

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Classification Accuracy

$$r_{\text{bal}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{r_{\text{TPR}} + r_{\text{TNR}}}{2}$$

Balanced Accuracy

$$r_{\text{prec}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+b}$$
Precision

$$r_{\text{rec}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$

Hinge, logistic loss promote higher classification accuracy

 $r_{\rm acc}(\mathbf{y}, \hat{\mathbf{y}}) = p_+ \cdot r_{\rm TPR} + p_- \cdot r_{\rm TNR}$

However, if $p_+ \to 0$ then $r_{acc}(\mathbf{y}, \hat{\mathbf{y}}) \approx p_- \cdot r_{TNR}$ which is why these loss functions have no incentive to do well on rare classes

Even a useless model that predicts everything -ve will get $r_{\rm acc}(\mathbf{y}, \hat{\mathbf{y}}) \approx p_- \to 1$

Other reward functions heavily penalize such neglectful behavior

Easy to verify that the above useless model will get heavily penalized since it would have a=0 i.e. $r_{TPR}=0$, $r_{prec}=0$, $r_{rec}=0$ which means that it would get $r_{Min}(\mathbf{y},\hat{\mathbf{y}})=\min\{r_{TPR},r_{TNR}\}=0$ and F-measure = 0

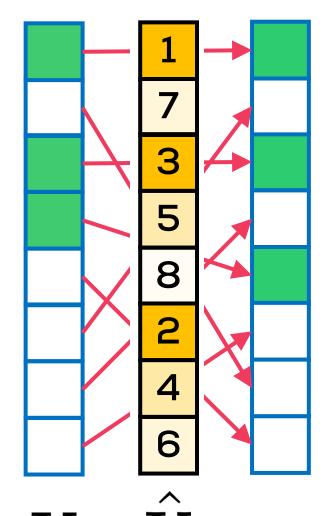
Using these reward functions in training requires advanced opt. techniques

Narasimhan et al. On the Statistical Consistency of Plug-in Classifiers for Nondecomposable Performance Measures, NIPS 2014

Narasimhan et al. Optimizing Non-decomposable Performance Measures: A Tale of Two Classes. ICML 2015

Imbalance-aware Training Techniques





Another technique is to treat classification problems as ranking problems instead

For binary problems, this is done by asking the model to assign scores to data points and try to give high scores to +ve points and low scores to -ve points

Next, data points are ranked according to their scores

Note: here we do not threshold scores at 0 to arrive labels

This ranking is used to assign loss/reward to the model

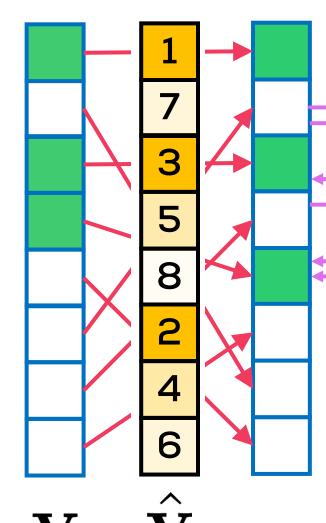
Model is rewarded if lots of positives are in top positions

Model is penalized if negatives pollute the ranking

Such loss fns are key in multiclass/label learning too For multiclass/label, instead of ranking data points, we

ask model to rank classes from most likely to least likely for each data point e.g. rank items for each user

Imbalance-aware Training Techniques



Bipartite Ranking Loss (AUC loss)

Counts # times a -ve point was ranked above a +ve point divided by product of # +ves and # -ves

In this cartoon this mistake was committed 3 times and #+ves=3, #-ves=5 so AUC loss is 3/(3*5)=0.2

Precision@k

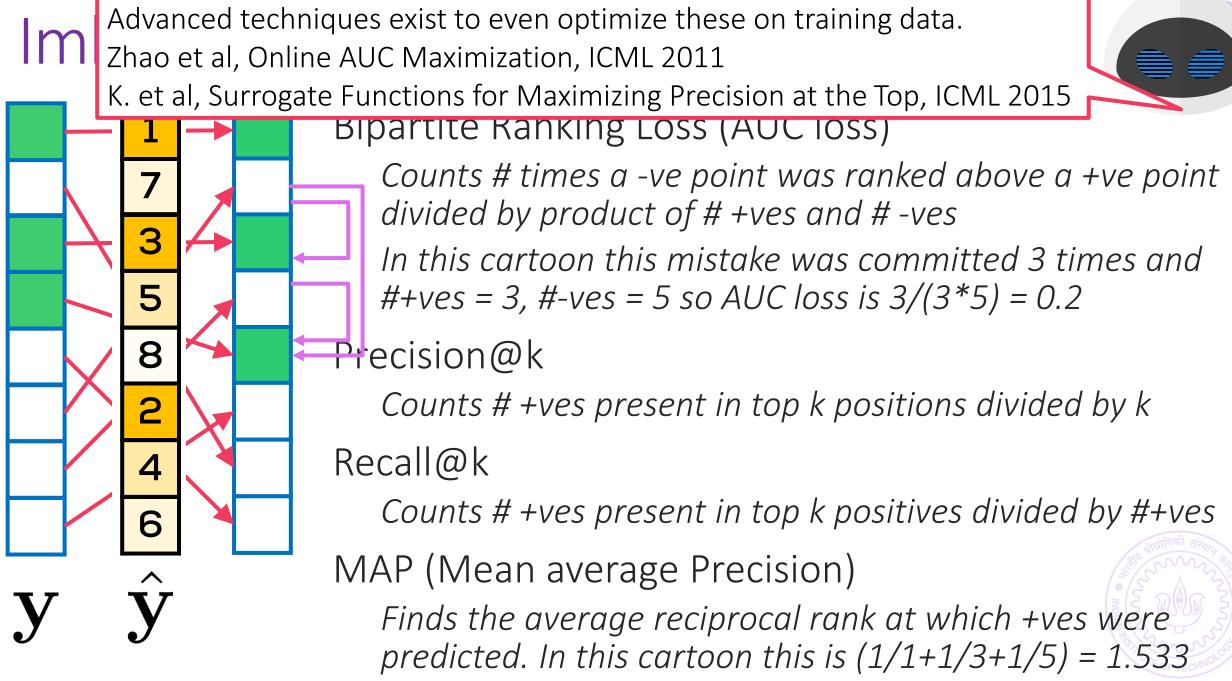
Counts # +ves present in top k positions divided by k

Recall@k

Counts # +ves present in top k positives divided by #+ves

MAP (Mean average Precision)

Finds the average reciprocal rank at which +ves were predicted. In this cartoon this is (1/1+1/3+1/5) = 1.533



Hybrid Learning Strategies

It is folklore in ML that discriminative models do better when lots of data is available but generative models do better with less data

Ng and Jordan, On Discriminative vs. Generative classifiers, NIPS 2001

For popular classes with lots of data, safe to use discriminative techniques like kernels, trees, deep nets along with one-vs-all etc

For rare classes with less data, may try generative techniques like naive Bayes, GMM or even LwP

Hybrid strategies are often used in recommendation systems

Rocchio classifiers, reranking algorithms etc

Use a standard discriminative algorithm e.g. trees etc to rank all labels but then use LwP-style techniques to boost the ranks of rare labels so that they do not get lost

Jain et al. Extreme Multi-label Loss Functions for Recommendation, KDD 2016