Practice problems # 1 (solution)

2. (i) $V = L_x L_y L_z$

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{L_x L_y L_z}\left[\frac{\partial}{\partial T}\left(L_x L_y L_z\right)\right]_p$$

$$= \frac{L_y L_z}{L_x L_y L_z}\left(\frac{\partial L_x}{\partial T}\right)_p + \frac{L_x L_z}{L_x L_y L_z}\left(\frac{\partial L_y}{\partial T}\right)_p + \frac{L_x L_y}{L_x L_y L_z}\left(\frac{\partial L_z}{\partial T}\right)_p$$

$$= \frac{1}{L_x}\left(\frac{\partial L_x}{\partial T}\right)_p + \frac{1}{L_y}\left(\frac{\partial L_y}{\partial T}\right)_p + \frac{1}{L_z}\left(\frac{\partial L_z}{\partial T}\right)_p$$

$$= 3\delta_T$$

(ii)

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = -v^2\left(\frac{\partial p}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

$$= \frac{B_s}{\rho}$$

$$= \frac{2.82 \times 10^6 \times 10^3}{1100} \ m^2/s^2$$

$$= 2.564 \times 10^6 \ m^2/s^2$$

$$\underline{c = 1601 \ m/s}$$

If the volume change is fast, it is isentropic and if it is slow it is isothermal.

Assuming it is isentropic, $\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = -\beta_s = -\frac{1}{B_s}$

$$\frac{\Delta V}{V} = -\frac{\Delta p}{B_s} = -\frac{1000}{2.82 \times 10^6} = \underline{-3.55 \times 10^{-4}}$$

3. $\dot{m}_1 = \rho_1 A_1 u_1 = 1.2 \times 0.25 \times 25 = 7.5$ kg/s

$\dot{m}_2 = \rho_2 A_2 u_2 = 0.2 \times 0.1 \times 225 = 4.5$ kg/s

$\dot{m}_3 = \rho_3 A_3 u_3 = \dot{m}_1 + \dot{m}_2 = 12$ kg/s

Assume : flow is uniform & steady.

$F_{x,\text{fluid}}$: $\dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(u_2 - u_1)$

$= 0.1(-1) = -0.1$ N

4. The axial force acting on the curved plate is equal & opposite to $F_{x,\text{fluid}}$. Therefore, the external force needed to hold the plate horizontally is -0.1 N ($-$ve x-direction), as shown.

Also, from y-direction momentum balance,

$F_{y,\text{fluid}} = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}(v_2 - v_1)$

$= 0.1 \times 1.73 = 0.173$ N.

5. The lateral force on the plate is in opposite direction ($-$ve y-direction) and thus the external force to hold the plate laterally is $+0.173$ N, as shown.

For the isentropic process, $p_1/p_2 = (T_1/T_2)^{\frac{\gamma}{\gamma-1}}$

(a) therefore, $T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} T_1$

$= 6^{1/3.5} \times 290 = 483.868$ K

The change in the temperature is $\Delta T = 193.868$ K.

(b) 1st law of TD : $du + d(PE) + d(KE) = dq + dw$

Here, vel. changes are neglected $\Rightarrow d(KE) = 0$

Also, assume no change in P.E. $\Rightarrow d(PE) = 0$

$\therefore du = dq + dw$, since the process is isentropic, $dq = 0$

$\therefore du = dw = C_v \Delta T = 717.5 \times 193.868 = 1.39 \times 10^5$ J/kg

(c) work done is negative & value is same as above.

6. Given the flow is adiabatic & frictionless ⇒ isentropic.

Energy eq: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

$C_p = 1004.5$ J/kg-K for air, ⇒ $1004.5 \times 273.15 + \frac{900^2}{2} = 1004.5\, T_2 + \frac{300^2}{2}$

$\Rightarrow T_2 = 631.54$ K.

$\therefore \Delta T = T_2 - T_1 = 358.39$ K.

by isentropic relation, we have ⇒ $\left(p_2/p_1\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$\Rightarrow p_2 = p_1\left(\frac{T_2}{T_1}\right)^{3.5} = 140\left(\frac{631.54}{273.15}\right)^{3.5}$

$= 2.631$ MPa

$\Delta p = p_2 - p_1 = 2.491$ MPa.

7. The exit pressure $p_e = p_{16km}$

From standard atmospheric table, $p_{16km} = 10.299$ kPa $= p_e$

$\therefore p_e/p_0 = \frac{10299}{15 \times 101325} = 0.0067762$

By isentropic relation: $p_0/p_e = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$

$\Rightarrow 1 + 0.2 M_e^2 = \left(\frac{1}{0.0067762}\right)^{0.286}$

$\Rightarrow M_e = 3.98$

for $M_e = 3.98$, from isentropic table, we get

$A_e/A^* = 10.53$, $\frac{T_e}{T_0} = 0.23992$

$\therefore T_e = 0.23992 \times (2600 + 273.15) = 689.33$ K.

Exit vel. ⇒ $V_e = M_e a_e = M_e\sqrt{\gamma R T_e} = 3.98\sqrt{1.4 \times 287 \times 689.33}$

$= 2094.6$ m/s

Thrust $= \dot{m}_e V_e = (\rho_e A_e V_e) V_e$

$9000 = \rho_e A_e V_e^2$ ⇒ | $\rho_e = p_e/RT_e = 10299/(287 \times 689.33) = 0.052$ kg/m³

$A_e = \frac{9000}{0.052 \times (2094.6)^2}$

$= 0.0394$ m²

$A^* = A_{th} = \frac{0.0394}{10.53} = 0.00374$ m²

8.

D_1 = Diameter at station 1.
at a distance 1m downstream from station 1, the diameter becomes,

$$D_2 = D_1 + 2\tan\theta\, dx$$

$$\frac{dD}{dx} = 2\tan\theta \quad \Rightarrow \quad A = \frac{\pi}{4}D^2$$

$$\frac{dA}{dx} = \frac{\pi}{4}\, 2D\frac{dD}{dx} = \pi D \tan\theta$$

$$\frac{dA}{dx} = 0.2228 \ m^2/m.$$

(a) for incompressible flow, by cont. we have ⇒ $\rho A V = $ cst.

$$\rho V \frac{dA}{dx} + \rho A \frac{dV}{dx} + AV\frac{d\rho}{dx} = 0$$

$$\rho = \text{cst}, \ \frac{d\rho}{dx} = 0 \quad \Rightarrow \quad \rho V \frac{dA}{dx} + \rho A \frac{dV}{dx} = 0 \quad \text{--- (1)}$$

$$\Rightarrow \ 200 \times 0.222 + 0.2\frac{dV}{dx} = 0$$

$$\Rightarrow \ \frac{dV}{dx} = -222\,(m/s)/m$$

density of flow is ⇒ $\rho = P/RT = \frac{80000}{287 \times 278} = 1.003 \ kg/m^3$

Let's treat the flow is 1D through diffuser,

mom. eq: $\quad V\frac{dV}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx} = -1.003 \times 200(-222)$$

$$= 44.53 \ kPa/m.$$

$$\frac{d\rho}{dx} = 0$$

(b) For compressible, (ρ is variable), but the mom. eq. remains same ⇒ $V\frac{dV}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$

$$\therefore \ \frac{dp}{dx} = -\rho V \frac{dV}{dx}$$

energy eq: $\quad h + \frac{V^2}{2} = h_0 \ \Rightarrow \ c_p T_0 = c_p T + \frac{V^2}{2}$

$$\therefore \ c_p \frac{dT}{dx} + V\frac{dV}{dx} = 0 \ \Rightarrow \ \frac{dT}{dx} = -\frac{V}{c_p}\frac{dV}{dx} \quad (\because T_0 \text{ is cst. for adiabatic process})$$

EOS $\Rightarrow$ $p = \rho R T$

$$\frac{dp}{dx} = R\left[\rho \frac{dT}{dx} + T\frac{d\rho}{dx}\right]$$

$$-\rho V \frac{dV}{dx} = R\left[T\frac{d\rho}{dx} - \frac{\rho V}{c_p}\frac{dV}{dx}\right]$$

$$\frac{d\rho}{dx} = \left[\frac{V}{c_p} - \frac{V}{R}\right]\frac{\rho}{T}\frac{dV}{dx}$$

$$= \left[\frac{200}{1004.5} - \frac{200}{287}\right]\frac{1.003}{278}\frac{dV}{dx}$$

$$= -1.796 \times 10^{-3}\frac{dV}{dx}$$

substituting values for eq-(1), we get,

$$1.003 \times 200 \times 0.222 + 1.003 \times 0.2\frac{dV}{dx} + 0.2 \times 200^2(-1.796 \times 10^{-3})\frac{dV}{dx} = 0$$

$$\Rightarrow \quad 44.5 + \frac{dV}{dx}(0.2006 - 0.07184) = 0$$

$$\Rightarrow \quad \frac{dV}{dx} = -345.6 \text{ (m/s)/m}$$

$$\frac{d\rho}{dx} = -1.796 \times 10^{-3} \times (-345.6)$$

$$= \cancel{0.0620} \quad 0.621 \text{ (kg/m}^3\text{)/m}$$

$$\frac{dp}{dx} = -\rho V\frac{dV}{dx} = -1.003 \times 200 \times (-345.6)$$

$$= 69.3 \text{ kPa/m}$$

9. $M = 2.32$ (at the upstream of the shock),
the area ratio corresponding to this Mach number will provide the area at the shock location.

(a) $M_1 = 2.32$, from isentropic table.

$$A_1/A^* = 2.233$$

$\therefore$ area ~~at~~ the shock location: $A_1 = 2.233 \times 5$

$$= 11.165 \text{ m}^2$$

(b) The Mach number downstream of the shock M_2 given by the normal shock table, for $M_1 = 2.32$ is $M_2 = 0.53$.

For $M_2 = 0.53$, from isentropic table, we have

$$A_2/A^* = 1.286$$

$A_1 = A_2 =$ area at the shock location, we have.

$$A_2^* = \frac{A_2}{1.286} = \frac{11.165}{1.286} = 8.68\ cm^2$$

Therefore, $\dfrac{A_e}{A_2^*} = \dfrac{12.5}{8.68} = 1.44$

from isentropic table, for $\dfrac{A_e}{A_2^*} = 1.44$, the exit Mach number $M_e = 0.45$

(C) For the given nozzle, the area ratio $\dfrac{A_e}{A_{th}}$ is

$$\frac{A_e}{A_{th}} = \frac{A_e}{A^*} = \frac{12.5}{5} = 2.5$$

From isentropic table, for $\dfrac{A_e}{A^*} = 2.5$,

$$M_e = 2.44, \quad p_2/p_{0_2} = 0.0643$$

For complete isentropic flow, $p_{0_2} = p_0 = 700$ kPa,

Then, $p_2 = 0.0643 \times 700 = 45.01$ kPa

The back pressure range for the flow to be completely isentropic is $p_b \leq 45.01$ kPa.

10. Given $p_{0_1} = 5$ atm, $p_{0_2} = 3.6$ atm,

Therefore, $p_{0_2}/p_{0_1} = \dfrac{3.6}{5} = 0.72$

From normal shock table, for $p_{0_2}/p_{0_1} = 0.72$, we have,

$$M_1 = 2.0, \quad p_2/p_1 = 4.5$$

Now, from isentropic table, for $M_1 = 2.0$, we get,

$$p_1/p_{0_1} = 0.1278$$

Hence, the pressure just behind the normal shock at the nozzle exit is

$$p_2 = 4.5\,p_1 = 4.5 \times 0.1278 \times 5$$
$$= 2.876 \text{ atm.}$$

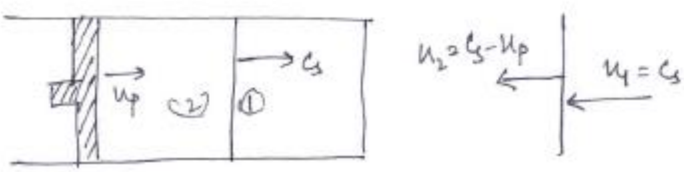
9.

(a)

$$\frac{p_2}{p_1} = \left(1 + \frac{\gamma-1}{2}\frac{u_p}{a_1}\right)^{\frac{2\gamma}{\gamma-1}}$$

$$a_1 = \sqrt{\gamma R T_1} = 20.04\sqrt{300} = 347 \text{ m/s}$$

11.



$$a_1 = \sqrt{\gamma R T_1} = 347 \text{ m/s}$$

$$\frac{u_1}{u_2} = \frac{c_s}{c_s - u_p} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}, \quad M_1 = \frac{c_s}{a_1}$$

$$\frac{c_s}{c_s - u_p} = \frac{(\gamma+1)(c_s/a_1)^2}{(\gamma-1)\left(\frac{c_s}{a_1}\right)^2 + 2}$$

$$(\gamma-1)\frac{c_s^2}{a_1^2} + 2 = (\gamma+1)\frac{c_s}{a_1^2}(c_s - u_p)$$

$$(\gamma-1)c_s^2 + 2a_1^2 = (\gamma+1)c_s^2 - (\gamma+1)u_p c_s$$

$$2c_s^2 - (\gamma+1)u_p c_s - 2a_1^2 = 0$$

$$c_s^2 - \left(\frac{\gamma+1}{2}\right)u_p c_s - a_1^2 = 0$$

$$M_1^2 - \left(\frac{\gamma+1}{2}\right)\frac{u_p}{a_1} M_1 - 1 = 0$$

$$M_1 = \frac{1}{2}\left[\frac{\gamma+1}{2}\frac{u_p}{a_1} \pm \sqrt{\left(\frac{\gamma+1}{2}\frac{u_p}{a_1}\right)^2 + 4}\right]$$
$$= 1.19$$

+ve sign is considered here, since M_1 cannot be less than 1.

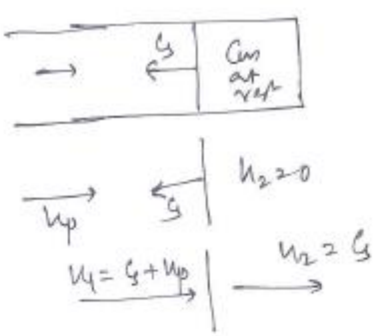
$\therefore$ Hence, $C_s = M_1 a_1 = 413\ m/s$

from normal shock table, for $M_1 = 1.19$, $p_2/p_1 = 1.485$

Thus, the pressure on the face of the piston is

$$p_2 = 1.485 \times 1.0133 \times 10^5 = 1.505 \times 10^5\ N/m^2$$

12.



The vel. of the wave relative to the piston $= C_s$
vel. of air entering the normal shock wave rel. to the shock wave is
$u_1 = C_s + u_p$

$$\Rightarrow M_1 = \frac{C_s + u_p}{a_1}$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{(\gamma+1) M_1^2}{2 + (\gamma+1) M_1^2} = \frac{C_s + u}{C_s}$$

$$C_s = M_1 a_1 - u_p$$

$\therefore$ we get,
$$\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2} = \frac{M_1 a_1}{M_1 a_1 - u_p} = \frac{M_1}{M_1 - u_p/a_1}$$

$$M_1^2 - \left(\frac{\gamma+1}{2}\right)\left(\frac{u_p}{a_1}\right) M_1 - 1 = 0$$

$$a_1 = \sqrt{\gamma R T_1} = 347 \text{ m/s}.$$

solving for M_1, we get, $M_1 = 1.29$ taking only the positive sign, since M_1 is supersonic. Hence,

$$C_s = M_1 a_1 - u_p = 1.29 \times 347 - 150$$

$$= 297.63 \text{ m/s}$$

From shock tables, for $M_1 = 1.29$, we have,

$$p_2/p_1 = 1.775, \quad \frac{T_2}{T_1} = 1.185$$

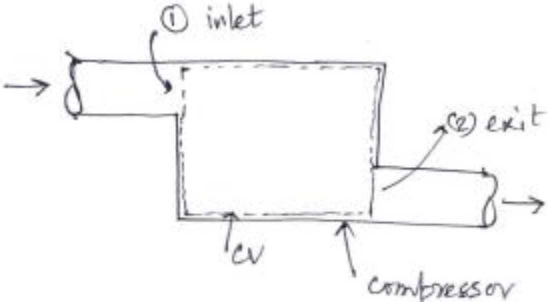
$$\therefore \quad p_2 = 1.775 \times 1.5 \times 10^5 = 2.66 \times 10^5 \text{ N/m}^2$$

$$\therefore \quad T_2 = 1.185 \times 300 = 355.5 \text{ K}.$$

Also, since the gas is at rest, $p_{0_2} = p_2$

$$T_{0_2} = T_2$$

13.



For steady flow,

$$\dot{m}_1 = \dot{m}_2$$

or $\rho_1 Q_1 = \rho_2 A_2 V_2$

$$\Rightarrow \rho_2 \frac{\pi}{4} d_2^2 V_2 = \rho_1 Q_1$$

$$\Rightarrow d_2 = \sqrt{\frac{\rho_1}{\rho_2} \frac{Q_1}{\frac{\pi}{4} V_2}}$$

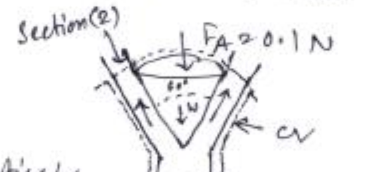
However, $\rho_1/\rho_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$

$$\Rightarrow d_2 = \sqrt{\left(\frac{p_1}{p_2}\right)^{1/n} \frac{Q_1}{\frac{\pi}{4} V_2}} = \sqrt{\left(\frac{1}{10}\right)^{1/1.4} \frac{30}{\frac{\pi}{4}(30) \times 60}}$$

$$= 0.064 \text{ m}.$$

14.

To determine the mass of the conical deflector we use the stationary, non-deforming CV as shown in Fig. Application of the vertical direction component of the linear momentum eq. to the contents of CV yields



$$\dot{m}(-V_1 + V_2 \cos 30^\circ) = -F_A - W_{cone}$$

$$\Rightarrow W_{cone} = m_{cone} \cdot g = \dot{m}(V_1 - V_2 \cos 30^\circ) - F_A = \rho_1 A_1 V_1 (V_1 - V_2 \cos 30^\circ) - F_A \quad \cdots ①$$

However, $V_1 = V_2$, $A_1 = \frac{\pi}{4} D_1^2$

$$\therefore \text{eq-(1)} \Rightarrow m_{cone} = \rho \frac{\pi}{4} \frac{D_1^2}{g} V_1 (V_1 - V_1 \cos 30^\circ) - F_A/g$$

$$= (1.23) \frac{\pi}{4} \frac{(0.1)^2}{9.81} \times 30 (30 - 30 \cos 30^\circ) - \frac{0.1}{9.81}$$

$$m_{cone} = 0.108 \text{ kg}$$

15.

(a) $dp = -\rho V dV$, $d\rho = \rho \tau dp$

$\Rightarrow dp = \frac{d\rho}{\rho\tau}$

Combining, $\frac{d\rho}{\rho\tau} = -\rho V dV$

$\Rightarrow \frac{d\rho}{\rho} = -\tau \rho V^2 \frac{dV}{V}$.

(b) $\tau_s = \frac{1}{\gamma p} = \frac{1}{1.4(1.01\times10^5)} = 7.07\times10^{-6}\ m^2/N$.

$\frac{d\rho}{\rho} = -\tau_s \rho V^2 \frac{dV}{V} = -(7.07\times10^{-6})\times1.23\times10\times0.01$

$\simeq -8.7\times10^{-6}$

(c) $\frac{d\rho}{\rho} = -8.7\times10^{-2}$

By increasing the velocity of a factor of 100, the fractional change in density is increased by factor of 10^4. This is just another indication of why high-speed flows must be treated as compressible.

16. (A).

(a) $\frac{p_0}{p} = \frac{1.22\times10^5}{1.01\times10^5} = 1.21$, from Table $\Rightarrow M_\alpha = 0.53$

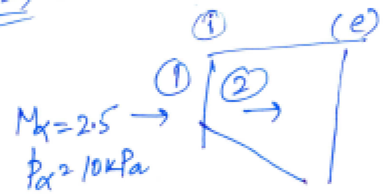
(b) $\frac{p_0}{p} = \frac{7222}{2116} = 3.413$, from Table $\Rightarrow M_\alpha = 1.45$

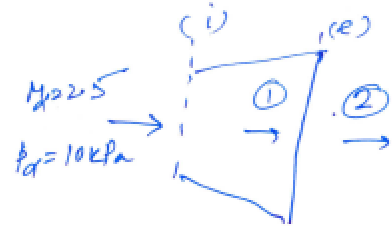
However, since this is supersonic, a normal shock sits in front of the Pitot tube. Hence, p_0 is now the total pressure behind the normal shock. Thus, $\boxed{M_\alpha = 1.5}$

for $p_{02}/p_{01} = 3.412 \Rightarrow M_\alpha = 1.5$

(c) $\frac{p_{02}}{p_1} = \frac{13107}{1020} = 12.85$, $M_\alpha = 3.1$

(B).





Shock at inlet:

$M_\alpha = M_1 = 2.5$, $\dfrac{p_{o_2}}{p_{o_1}} = 0.4990 = \dfrac{A_1^*}{A_2^*}$

$\dfrac{p_1}{p_{o_1}} = 0.05853$

$$\frac{A_e}{A_e^*} = \frac{A_e}{A_2^*} = \left(\frac{A_e}{A_1}\right)\left(\frac{A_1}{A_1^*}\right)\left(\frac{A_1^*}{A_2^*}\right) = (3)(2.6367)(0.4990)$$
$$= 3.9471$$

$M_\alpha = 0.1486$

$\dfrac{p_e}{p_{o_2}} = 0.9847$, $\dfrac{p_{o_2}}{p_{o_1}} = 0.4990$.

$p_{o_1} = \dfrac{10}{0.05853} = 170.8526\ kPa$, $p_{o_2} = 0.4990 \times 170.8526$
$= 85.2554\ kPa$

$p_e = \dfrac{p_e}{p_{o_2}} \times p_{o_2} = 0.9847 \times (85.2554) = 83.9510\ kPa$

$p_{oi} - p_{oe} = p_{o_1}\left(1 - \dfrac{p_{o_2}}{p_{o_1}}\right) = 170.8526(1 - 0.4990)$
$= 85.5972\ kPa$.

Shock at exit:

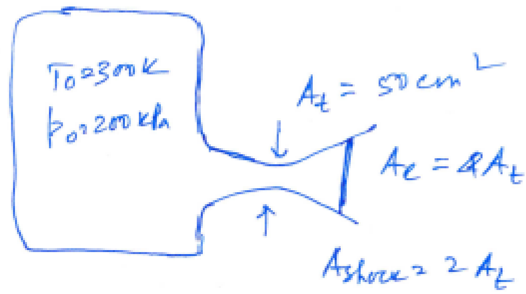
$\dfrac{A_1}{A_i} = 3.0$, $\dfrac{A_i}{A_1^*} = 2.6367$, $\dfrac{A_1}{A_1^*} = 3 \times 2.6367$
$= 7.9101$

$M_1 = 3.6649$, $M_2 = 0.4451$

$p_2 = \dfrac{p_2}{p_1} \times \dfrac{p_1}{p_{o_1}} \times \dfrac{p_{o_1}}{p_i} \times p_i = (15.5038)(0.0104)\left(\dfrac{1}{0.05853}\right) \times 10$
$= 27.5482\ kPa$

$p_{oi} - p_{oe} = p_{oi}\left(1 - \dfrac{p_{o_2}}{p_{o_1}}\right) = 170.8526(1 - 0.1847)$
$= 139.2961\ kPa$.

17.



(a) $50\ cm^2$

(b) For shock, $M_1 = 2.20$, $\frac{p_{o2}}{p_{o1}} = \frac{A_1^*}{A_2^*} = 0.6281$

$$A^*_{shock\ to\ exit} = A_2^* = \frac{50}{0.6281} = 79.6052\ cm^2$$

(c) $\frac{A_e}{A_2^*} = \frac{200}{79.6052} = 2.5124$, $M_e = 0.2383$

(d) $p_{oe} = p_{o1}\left(\frac{p_{o2}}{p_{o1}}\right) = 200(0.6281) = 125.6200\ kPa$

(e) $p_e = p_{oe}\left(\frac{p_e}{p_{oe}}\right) = 125.62(0.9613) = 120.3548\ kPa$

(f) $T_e = T_0\left(\frac{T_e}{T_0}\right) = 300\,(0.9888) = 296.64\ K$

$$V_e = 0.2383\sqrt{1.4 \times 287 \times 296.64}$$

$$= 82.2704\ m/s.$$