AE321A Flight Mechanics

Assignment - 3 **Due date: 5-Nov-2019**

** Note: If you feel inadequate data in any question, please assume the same with proper justification.*
All questions carry equal weightage

Q1. Explain the sign convention for the following parameters of a stable aircraft:

a) C_{m_0} b) C_{m_α} c) $C_{m_{\delta_e}}$ d) C_{L_0} e) C_{L_α} f) $C_{L_{\delta_e}}$

g) $\frac{dC_m}{dC_L}$ h) $\frac{d\delta_e}{dC_L}$ i) C_{h_α} j) $C_{h_{\delta_e}}$ k) $C_{h_{\delta_t}}$ l) $\frac{dC'_m}{dC_L}$

m) C_{y_β} n) $C_{y_{\delta_r}}$ o) C_{l_β} p) $C_{l_{\delta_a}}$ q) $C_{l_{\delta_r}}$ r) $(C_{n_\beta})_{Vt}$

s) $C_{n_{\delta_a}}$ t) $C_{n_{\delta_r}}$ u) $(C_{h_{\delta_r}})_{Vt}$ v) $(C_{h_\beta})_{Vt}$ w) $(C_{n_\beta})_w$

x) Define neutral point and static margin

y) Explain the criteria for longitudinal static stability

z) Explain longitudinal stability contribution by low and high wings

Q2. Find the stick fixed and stick free neutral point of following configuration

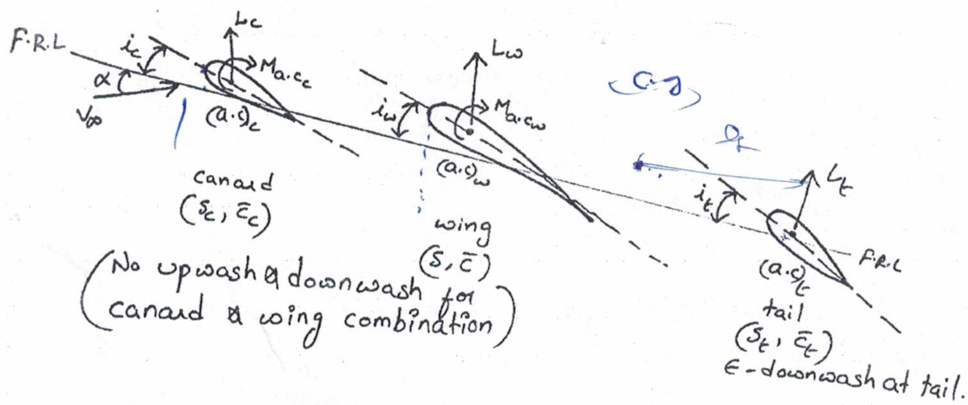


Fig. 1 Canard - Wing - Tail combination

Q3. Explain how addition of canard effects the neutral point of wing and tail combination

Q4. Explain the procedure to determine the stick fixed and stick free neutral points from flight tests with the help of plots (present it in steps)

Q5. Derive the most forward permissible center of gravity of an aircraft during cruise

Q6. An airplane with an all-movable horizontal tail has the following data: $W/S = 289$ kg/m², $S = 30$ m², $C_{L_{max}} = 1.5, \bar{c} = 3\ m, \bar{x}_{ac,w} = 0.25, C_{m_{ac,w}} = -0.05, \alpha_{0L,w} = -2.5$ deg, $i_w = -2$ deg, $C_{L_{\alpha w}} = 5.7/rad, C_{L_{\alpha t}} = 4.58/rad, \varepsilon = 0.4\alpha, l_t = 2.5\bar{c}$ and $\eta_t = 0.8$. Assuming that the most forward and aft permissible center of gravity locations are $0.20\bar{c}$ and $0.35\bar{c}$, determine the tail area and tail setting angle

Q7. What are the functions of a trim tab? How does it affect the floating characteristics of elevator? Explain with a schematic.

Q8. a) Prove that for a stick free stable aircraft $\frac{d\delta_t}{dC_L} > 0$

b) Define stick fixed Static Margin (SM). Prove that $C_{m_\alpha} = -(SM) * C_{L_\alpha}$

Q9. Find the neutral point of the wing alone configuration given in Fig. 2. The cross section of the UAV is NACA 23112 and the plot in Fig. 3 presents the variation of C_L with α. Estimate C_{L_0}, C_{L_α}, C_{m_0} and C_{m_α} of the UAV about CG with a static margin of 10%. Assume $C_{m_{a.c}}$ = 0.01.

Hint: Consider weighted average for parameters such as aspect ratio, mac etc.

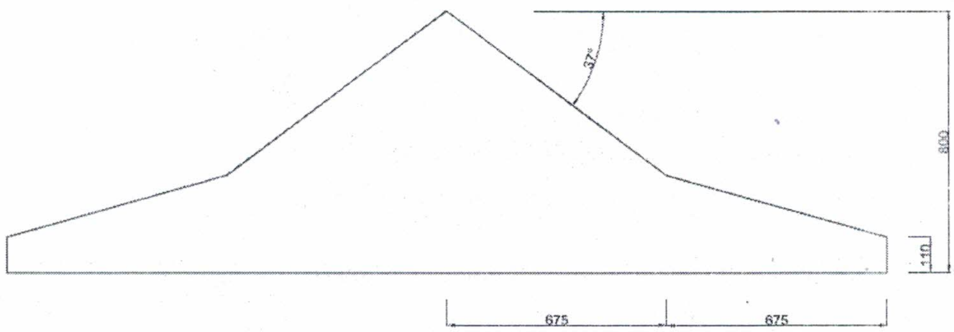


Fig. 2 Wing alone UAV

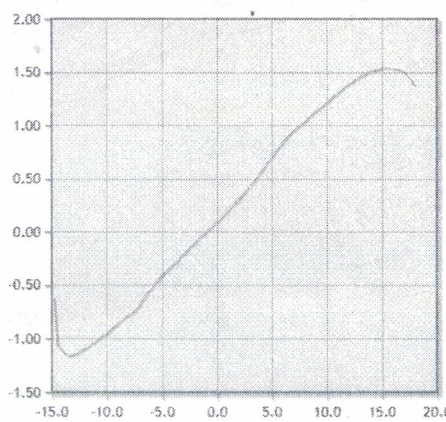


Fig. 3 C_L vs. α of NACA 23112

Q10. Find $C_{L_0}, C_{L_\alpha}, C_{m_0}, C_{m_\alpha}$ and neutral point of the following biplane configuration with cambered wings. Assume that the wings of the biplane are placed at equidistant from the FRL and also negligible drag and downwash at wings.

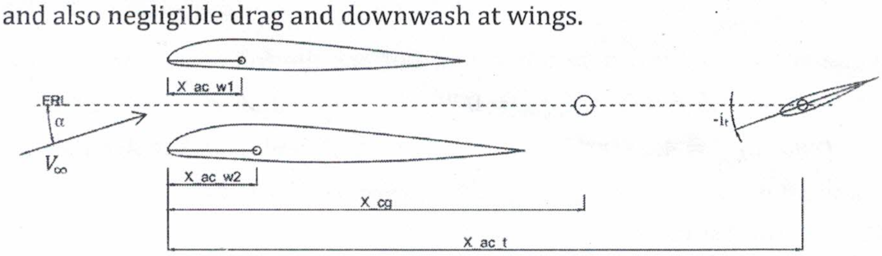


Fig. 4 Biplane UAV

(1)

C_{m_0} (+) C_{m_α} (−) $C_{m_{\delta e}}$ (−) C_{L_0} (+) C_{L_α} (+) $C_{L_{\delta e}}$ (+)

$\frac{dC_m}{dC_L}$ (−) $\frac{d\delta e}{dC_L}$ (−) C_{h_α} (−) $C_{h_{\delta e}}$ (−) $C_{h_{\delta t}}$ (−) $\frac{dC_h}{dC_L}$ (−)

C_{y_β} (−) $C_{y_{\delta r}}$ (+) C_{ℓ_p} (−) $C_{\ell_{\delta a}}$ (+) $C_{\ell_{\delta r}}$ (+) $(C_{n_\beta})_{VT}$ (+)

$C_{n_{\delta a}}$ (−) $C_{n_{\delta r}}$ (−) $(C_{n_{\delta r}})_{VT}$ (−) $(C_{n_\beta})_{VT}$ (+) $(C_{n_\beta})_W$ (−)

New ref point ⇒ A/C center of whole Aircraft
most aft c.g limit.

Longitudinal Stability criteria ⇒ $\boxed{C_{m_\alpha} < 0}$

High wing ⇒ Unstable in pitch
Low wing ⇒ Stable in pitch

(2)

$$C_{m_\alpha} = C_{L_{\alpha w}}\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right] - \eta \frac{S_t \, l_t}{S_w \, \bar{c}} C_{L_{\alpha t}} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$+ \frac{S_c \, l_c}{S_w \, \bar{c}} C_{L_{\alpha c}}$$

where

$$\begin{bmatrix} l_t = x_{ac_t} - x_{cg} \\ l_c = -x_{ac_c} - x_{cg} \end{bmatrix}$$

at Neutral point $C_{m\alpha} = 0$

$$x_{cg} = x_{NP}$$

$$0 = C_{L\alpha w}\left[\frac{x_{NP}}{\bar{c}} - \frac{x_{acw}}{\bar{c}}\right] - \eta \frac{S_t\,(x_{act} - x_{NP})}{S_w \bar{c}} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$+ \frac{(-x_{acc} - x_{NP})\,S_c}{S_w \bar{c}} C_{L\alpha c}$$

$$\Rightarrow \left[\frac{C_{L\alpha w}}{\bar{c}} + \eta \frac{S_t}{S_w \bar{c}} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{S_c}{S_w \bar{c}} C_{L\alpha c}\right] x_{NP}$$

$$= C_{L\alpha w} \frac{x_{acw}}{\bar{c}} + \eta \frac{S_t}{S_w \bar{c}} x_{act} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$+ \frac{x_{acc}\, S_c\, C_{L\alpha c}}{S_w \bar{c}}$$

$$\frac{x_{NP}}{\bar{c}} = \frac{\left[C_{L\alpha w} \frac{x_{acw}}{\bar{c}} + \eta \frac{S_t}{S_w} \frac{x_{act}}{\bar{c}} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{S_c}{S_w} C_{L\alpha c}\left(\frac{x_{acc}}{\bar{c}}\right)\right]}{\left[\frac{C_{L\alpha w}}{\bar{c}} + \frac{\eta S_t}{S_w \bar{c}} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{S_c}{S_w \bar{c}} C_{L\alpha c}\right]}$$

Stick free case $\Rightarrow$

$$\cancelto{0}{C_h} = \cancelto{0}{C_{h_0}} + C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta e}} \delta e$$

$$(\delta e)_{free} = -\frac{C_{h_{\alpha_t}}}{C_{h_{\delta e}}} \alpha_t$$

$$\Rightarrow C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta e}} \delta e_{free}$$

$$\Rightarrow C_{L_t} = C_{L_{\alpha_t}} \alpha_t \left(1 - \frac{C_{L_{\delta e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta e}}}\right)$$

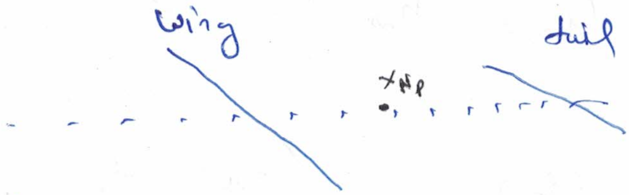
$$\left[C_{L_{\alpha_t}}' = C_{L_{\alpha_t}} \left(1 - \frac{C_{L_{\delta e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta e}}}\right) \right.$$

$$\left. \Rightarrow f = \left(1 - \frac{C_{L_{\delta e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta e}}}\right) \right]$$

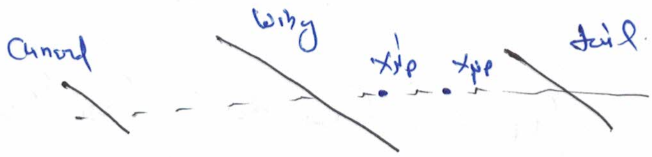
So $\Rightarrow$

$$\left[\frac{x_{NP}}{\bar{c}} = \frac{C_{L_{\alpha_w}} \frac{x_{ac_w}}{\bar{c}} + \frac{\eta S_t}{S_w} \frac{x_{ac_t}}{\bar{c}} C_{L_{\alpha_t}}' \left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{S_c}{S_w} C_{L_{\alpha_c}} \left(\frac{x_{ac_c}}{\bar{c}}\right)}{\frac{C_{L_{\alpha_w}}}{\bar{c}} + \frac{\eta S_t}{S_w \bar{c}} C_{L_{\alpha_t}}' \left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{S_c}{S_w \bar{c}} C_{L_{\alpha_c}}} \right]$$

③



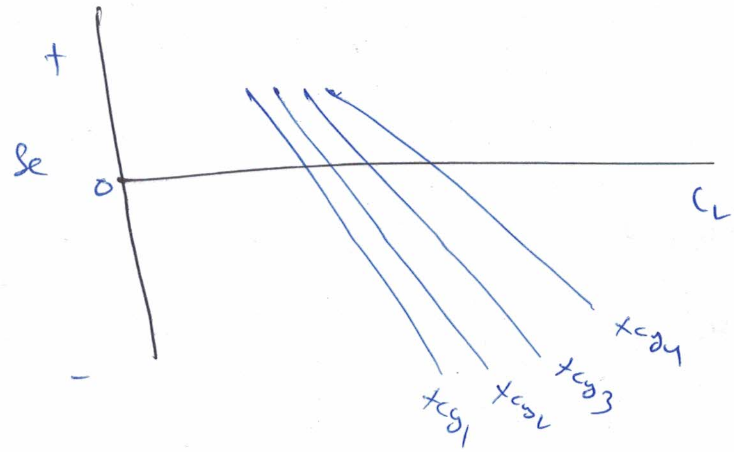
Addition of Canard



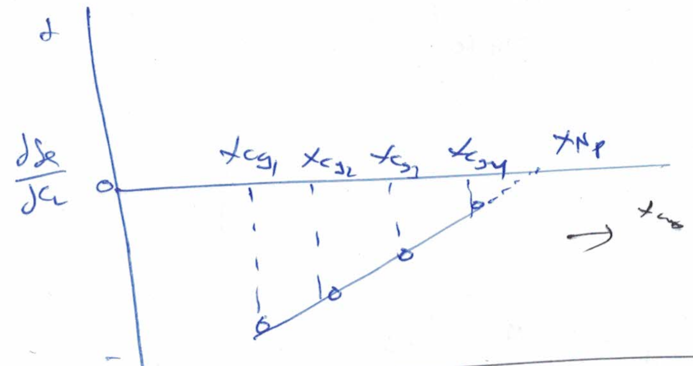
i) Static margine will ~~decrease~~ reduce (New neutral point shift towards the Nose,

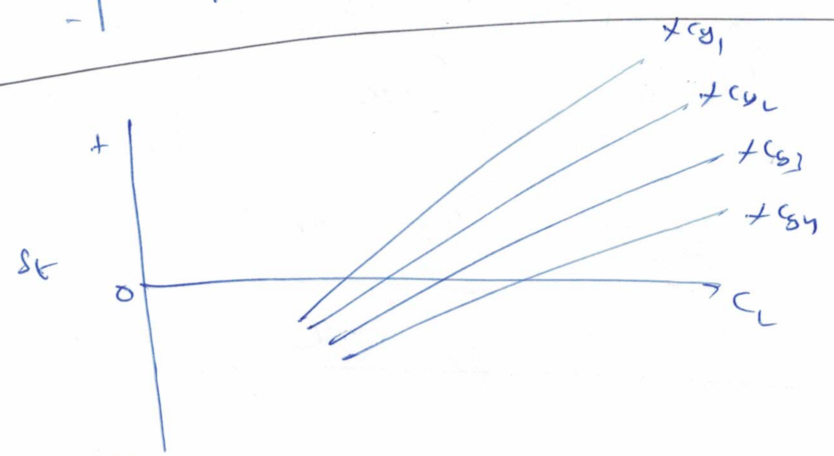
mathematically, it can be concluded from the Assignment solution of Question No. 2 of their

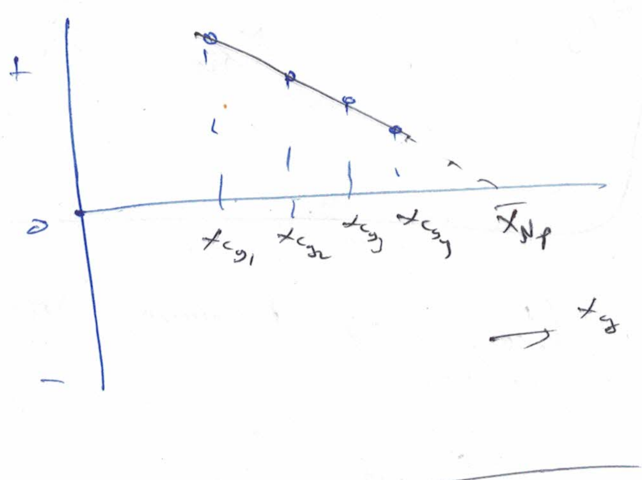
(4)



Stick fixed







Stick-free

(5)

$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{m\delta e}\delta e$$

at trim $C_m = 0$

$$\delta e = -\frac{C_{m0} - C_{m\alpha}\alpha}{C_{m\delta e}}$$

$$= -\frac{C_{m0}}{C_{m\delta e}} + \left(-\frac{C_{m\alpha}}{C_{m\delta e}}\right)\alpha$$

$\because C_{m\alpha} = -(S.M)\,C_{L\alpha}$

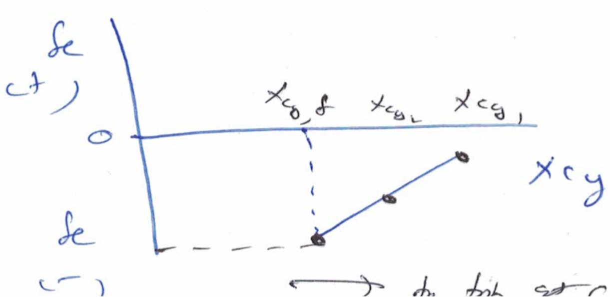
$$= -\frac{C_{m0}}{C_{m\delta e}} + \frac{(S.M)\,C_{L\alpha}\,\alpha}{C_{m\delta e}}$$

$$= \delta e_0 - \frac{(\bar{x}_{cg} - \bar{x}_{NP})}{C_{m\delta e}}\,C_L$$

$$\boxed{\bar{x}_{cg,f} = \cancel{x_{NP}}\;\bar{x}_{NP} - \left(\delta e_{max} - \delta e_0\right)\frac{C_{m\delta e}}{C_{L_{max}}}}$$

$(\delta e)_{max} \rightarrow$ maximum upward elevator deflection

$C_L \rightarrow C_{L_{max}}$



(6) $W/S = 289 \ kg/m^2$, $C_{L_{max}} = 1.5$, $\bar{c} = 3m$, $\bar{x}_{ac} = 0.25$

$S = 30 m^2$, $C_{m_{ac}} = -0.05$, $\alpha_{0.L.w} = -2.5°$, $i_w = -2°$

$C_{L\alpha w} = 5.7/rad$, $C_{L\alpha t} = 4.58/rad$, $\varepsilon = 0.4\alpha$, $l_t = 2.5\bar{c}$

$h_t = 0.8$

$\Rightarrow \quad x_{NP} = 0.35\bar{c}$

$\bar{x}_{cg.f} = 0.20\bar{c}$

$$C_{m_0} = C_{m_{ac}} + \cancel{C_{m_{0f}}}^{0} - a_t V_H h_t (\alpha_{w0L} - i_w + i_t)$$

$$C_{m_0} = -0.05 - 4.58 \times \frac{S_t \times 2.5 \times 3}{30 \cdot 3} \times 0.8 (-2.5 + 2 + i_t) \pi/180$$

$$C_{m_0} = -0.05 - 0.38166 S_t (-0.5 + i_t) \times \pi/180$$

$$C_{m_0} = -0.05 - 0.00666133 S_t (-0.5 + i_t) \quad — (1)$$

$$C_{m_\alpha} = C_{L\alpha}\left[\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right] + (C_{m_\alpha})_f - h V_H C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$0 = 5.7\left[\frac{x_{NP}}{c} - \frac{x_{ac}}{c}\right] + 0 - 0.8 \cdot \frac{S_t \cdot 2.5 \times 3}{30 \cdot 3} \times 4.58 (1 - 0.4)$$

$$0 = 5.7 [0.35 - 0.25] - 0.1832 S_t$$

$$\boxed{S_t = 3.111 \ m^2}$$

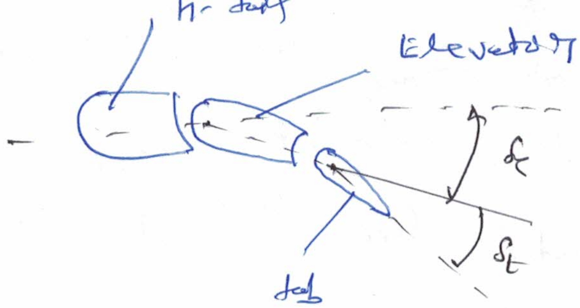
from eqn (1)

$$-0.5 + i_t = -3.01557$$

$$\boxed{i_t = -2.51557°}$$

(7)

trim tab ⇒



function → to reduce the hinge moment
or oppose the hinge moment

→ Improve the handling quality

→ alter the pressure distribution over the entire Horizontal tail surface.

→ tab is used for achieving $C_h = 0$

$$C_h = C_{h_0} + C_{h_\alpha} \alpha_t + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t$$

$$\boxed{\delta_t = - \frac{C_{h_\alpha} \alpha_t + C_{h_{\delta_e}} \delta_e}{C_{h_{\delta_t}}}}$$

$C_h = 0$
C_{h_0} is small

⇒ it only affects the hinge moment

$$\boxed{\frac{\partial \delta_t}{\partial C_L} > 0}$$

(8) (9)

$$C_h = C_{h_0} + C_{h_\alpha}\alpha_s + C_{h_{\delta e}}\delta e + C_{h_{\delta t}}\delta_t$$

$\Rightarrow$ at $C_h = 0$ (floating elevator)

$$\delta e_f = -\frac{C_{h_0} + C_{h_\alpha}\alpha_s + C_{h_{\delta t}}\delta_t}{C_{h_{\delta e}}}$$

C_{h_0} is small

$$\Rightarrow \quad C_{m_t} = -C_{L_t} V_H \eta_t$$

$$= -a_t(\alpha_s + \tau\,\delta e_f)V_H \eta_t$$

$$\frac{dC_{m_t}}{dC_L} = -\frac{a_t}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \tau\frac{C_{h_\alpha}}{C_{h_{\delta e}}}\right)V_H \eta_t$$

δ_t for $C_h = 0$

$$\delta_t = -\frac{C_{h_\alpha}\alpha_s + C_{h_{\delta e}}\delta e}{C_{h_{\delta t}}}$$

Substituting δe expression

$$\Rightarrow \quad \delta_t = -\frac{1}{C_{h_{\delta t}}}\left(C_{h_\alpha}\alpha_s + C_{h_{\delta e}}\left[\delta e_0 - \left(\frac{dC_m}{dC_L}\right)_{fix}\frac{C_L}{C_{m_{\delta e}}}\right]\right)$$

$$\frac{d\delta_t}{dC_L} = -\left(\frac{C_{h_{\delta e}}}{C_{h_{\delta t}}}\right)\left(\frac{1}{C_{m_{\delta e}}}\right)\left(\bar{x}_{cg} - \bar{x}_{N_0}\right)$$

$\because \quad \bar{x}_{cg} - \bar{x}_{N_0} > 0, \quad C_{h_{\delta e}},\ C_{h_{\delta t}},\ C_{m_{\delta e}} < 0$

$\therefore \quad \boxed{\frac{d\delta_t}{dC_L} > 0}$

(b)

Static margin ⟹

$$S.M = x_{NP} - x_{cg}$$

$$\because \quad C_{m_\alpha} = C_m\, C_{L_\alpha}\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right] - \eta V_H \frac{S_t}{S} C_{L_{\alpha t}} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$x_{cg} = x_{NP}$$

$$C_{m_\alpha} = 0$$

or

$$C_m = C_L\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right] + C_{m_f} - C_{L_{\alpha t}} V_H \eta_t$$

$$\frac{dC_m}{dC_L} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + \left(\frac{dC_m}{dC_L}\right)_f - \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) V_H \eta_t$$

or

$$\frac{dC_m}{dC_L} = \frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}$$

when $x_{cg} = x_{NP}$

$$\frac{dC_m}{dC_L} = 0$$

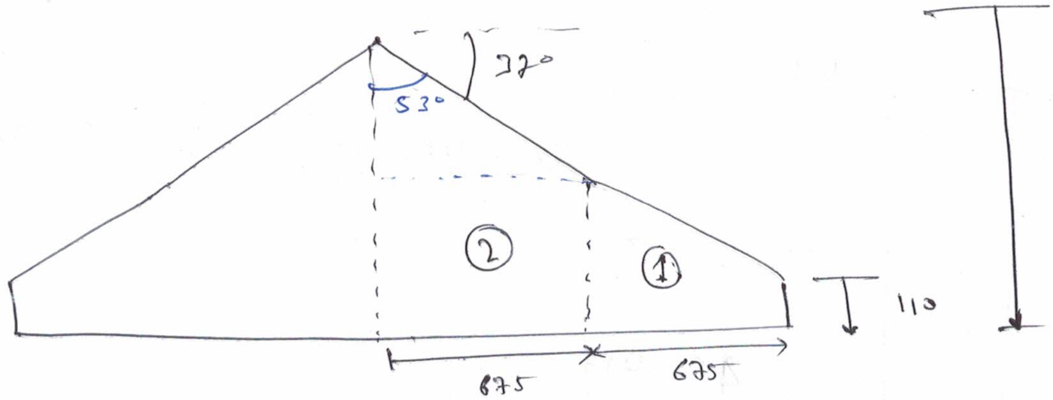
$$\frac{x_{NP}}{c} - \frac{x_{cg}}{c} = -\frac{dC_m}{dC_L}$$

$$\bar{x}_{NP} - \bar{x}_{cg} = -\frac{\frac{dC_m}{d\alpha}}{\frac{dC_L}{d\alpha}}$$

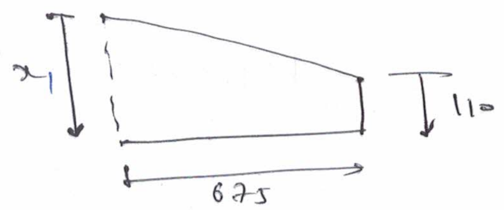
$$(S.M) = -\frac{C_{m_\alpha}}{C_{L_\alpha}}$$

$$\boxed{C_{m_\alpha} = -(S.M)\, C_{L_\alpha}}$$

(9)



portion (1)



$$\tan 53 = \frac{675}{800 - x_1}$$

$$800 - x_1 = \frac{675}{\tan 53}$$

$$\boxed{x_1 = 291.35}$$

$$\bar{c}_1 = \frac{2}{3} x_1 \left[\frac{1 + d_1 + d_1^2}{1 + d_1} \right]$$

$$d_1 = \frac{110}{291.35} = 0.3775$$

$$\left[\bar{c}_1 = 214.3327 \right], \quad S_1 = \frac{1}{2}\, 675 \,(110 + 291.35)$$

$$S_1 = 135455.625$$

portion (2)

$$\bar{c}_2 = \frac{2}{3} x_2 \left(\frac{1 + d_2 + d_2^2}{1 + d_2} \right)$$

$$d_2 = \frac{291.35}{800}$$

$$d_2 = 0.3641875$$

$$= \frac{2}{3} \times 800 \left(\frac{1 + 0.364 + 0.364^2}{1 + 0.364} \right)$$

$$\left[A_f = \frac{b^2}{5} = 14.47 \right]$$

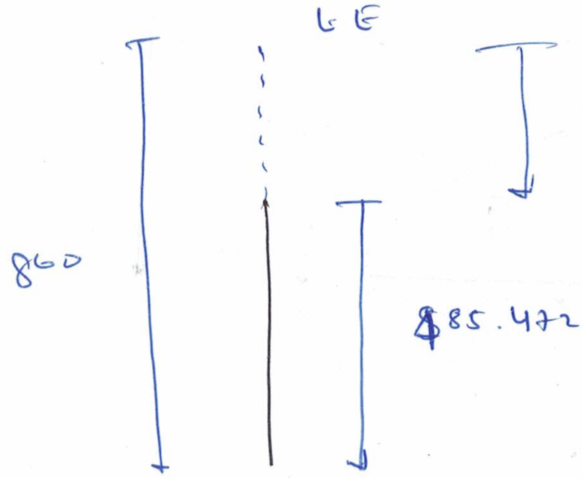
$$\left[\bar{c}_2 = 585.1864 \right]$$

$$S_2 = \frac{1}{2} \times 675 \,(291.35 + 800)$$

$$= 368330.625$$

$$\bar{c} = \frac{S_1 \bar{c}_1 + S_2 \bar{c}_2}{S_1 + S_2} = \frac{135455.625 \times 214.33 + 368330.625 \times 585.1864}{135455.625 + 368330.625}$$

$$\boxed{\bar{c} = 485.472}$$



$800 - 485.472 = 314.5274$

$x_{ac} = \frac{\bar{c}}{4} = \frac{485.472}{4} = 121.368$

from wing LE

$= 314.5274 + 121.368$

$= 435.8954$ mm

from fig. 3

$\alpha_{L=0} = -1^\circ$

$(C_{L\alpha})_{2D} = 6.1/\text{rad}$

$(C_{L\alpha})_{3D} = \frac{(C_{L\alpha})_{2D}}{\frac{(C_{L\alpha})_{2D}}{\pi e AR} + 1}$

$\boxed{C_{L\alpha} = 5.3783/\text{rad}}$

$C_{L0} = C_{L\alpha}(\cancel{\alpha}^{0} - \alpha_{L=0}) = C_{L\alpha}(-\alpha_{L=0})$

$= 5.3783 \times (-1) \times \pi/180 = \boxed{0.09386}$

$C_{m0} = C_{m_{ac}} + C_{L0}\left[\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right]$

$= 0.01 + 0.09386(0.1)$

$\boxed{C_{m0} = 0.00613}$

$C_{m\alpha} = C_{L\alpha}\left[\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right]$

Given, $S.M = 0.1$

$C_{m\alpha} = -S.M \cdot C_{L\alpha}$

$S.M = -\frac{\frac{dC_m}{d\alpha}}{\frac{dC_L}{d\alpha}}$

$x_{NP} - x_{cg} = 0.1$

$\boxed{C_{m\alpha} = -0.1 \times 5.3783 = -0.53783}$

(10)

$$L = L_{w_1} + L_{w_2} + L_t$$

$$\frac{1}{2}\rho V^2 S_w C_L = \frac{1}{2}\rho V^2 S_{w_1} C_{Lw_1} + \frac{1}{2}\rho V^2 S_{w_2} C_{Lw_2} + \frac{1}{2}\rho V_t^2 S_t C_{Lt}$$

$$C_L = C_{Lw_1} \cdot \frac{S_{w_1}}{S_w} + C_{Lw_2} \cdot \frac{S_{w_2}}{S_w} + \eta \frac{S_t}{S_w} C_{L\alpha t} (\alpha_t)$$

$$C_{L0} + C_{L\alpha}\alpha = C_{L0w_1} \frac{S_{w_1}}{S_w} + C_{L0w_2} \frac{S_{w_1}}{S_w} + C_{L\alpha w_1} \frac{S_{w_1}}{S_w} + C_{L\alpha w_2} \frac{S_{w_2}}{S_w}$$

$$+ \eta \frac{S_t}{S_w} C_{L\alpha t} \left(\alpha_w - i_w - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + i_t \right)$$

comparing $\Rightarrow$

$$C_{L0} = C_{L0w_1} \cdot \frac{S_{w_1}}{S_w} + C_{L0w_2} \frac{S_{w_2}}{S_w} + \eta \frac{S_t}{S_w} C_{L\alpha t} (i_t - \varepsilon_0 - i_w)$$

$$C_{L\alpha} = C_{L\alpha w_1} \frac{S_{w_1}}{S_w} + C_{L\alpha w_2} \frac{S_{w_2}}{S_w} + \eta \frac{S_t}{S_w} C_{L\alpha t} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_m = C_{m w_1} + C_{m w_2} + C_{m t}$$

$$C_{m\alpha} = C_{L\alpha w_1} \frac{S_{w_1}}{S_w} \left[\frac{x_{cg}}{\bar{c}_1} - \frac{x_{ac w_1}}{\bar{c}_1} \right] + C_{L\alpha w_2} \frac{S_{w_2}}{S_w} \left[\frac{x_{cg}}{\bar{c}_2} - \frac{x_{ac w_2}}{\bar{c}_2} \right]$$

$$- \eta V_H C_{L\alpha t} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_{m0} = C_{L0w_1} \frac{S_{w_1}}{S_w} \left[\frac{x_{cg}}{\bar{c}_1} - \frac{x_{ac}}{\bar{c}_1} \right] + C_{L0w_2} \frac{S_{w_2}}{S_w} \left[\frac{x_{cg}}{\bar{c}_2} - \frac{x_{ac}}{\bar{c}_2} \right.$$

$$+ \eta V_H C_{L\alpha t} (i_w + \varepsilon_0 - i_t)$$

$$\boxed{V_H = V_{H_1} + V_{H_2}}$$

at Neutral point $C_{m\alpha} = 0$

$$0 = C_{L\alpha w_1} \frac{S_{w_1}}{S_w}\left[\frac{x_{NP}}{\bar{c}_1} - \frac{x_{acw_1}}{\bar{c}_1}\right] + C_{L\alpha w_2} \frac{S_{w_2}}{S_w}\left(\frac{x_{NP}}{\bar{c}_2} - \frac{x_{acw_2}}{\bar{c}_2}\right)$$

$$S_t l_t / S\bar{c}$$

$$- h V_{H_1} C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) - h V_{H_2} C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$0 = \left[C_{L\alpha w_1} \frac{S_{w_1}}{S_w}\cdot\frac{1}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\cdot\frac{1}{\bar{c}_2}\right] x_{NP}$$

$$- \left(C_{L\alpha w_1}\frac{S_{w_1}}{S_w}\cdot\frac{x_{acw_1}}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\frac{x_{acw_2}}{\bar{c}_2}\right)$$

$$- h\left(\frac{x_{act} - x_{NP}}{S_{w_1}\bar{c}_1}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$- h\left(\frac{x_{act} - x_{NP}}{S_{w_2}\bar{c}_2}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$0 = \left[C_{L\alpha w_1}\frac{S_{w_1}}{S_w}\cdot\frac{1}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\cdot\frac{1}{\bar{c}_2} + \frac{h}{S_w\bar{c}_1}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{h\left(1 - \frac{d\varepsilon}{d\alpha}\right)}{S_w\bar{c}_2}\right] x_{NP}$$

$$- \left(C_{L\alpha w_1}\frac{S_{w_1}}{S_w}\cdot\frac{x_{acw_1}}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\frac{x_{acw_2}}{\bar{c}_2} + h\frac{x_{act}}{S_{w_1}\bar{c}_1}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{h\, x_{act}}{S_{w_2}\bar{c}_2}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right)$$

$$x_{NP} = \frac{C_{L\alpha w_1}\frac{S_{w_1}}{S_w}\cdot\frac{1}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\cdot\frac{1}{\bar{c}_2} + h\left(1 - \frac{d\varepsilon}{d\alpha}\right)\cdot\left[\frac{1}{S_{w_1}\bar{c}_1} + \frac{1}{S_{w_2}\bar{c}_2}\right]}{C_{L\alpha w_1}\frac{S_{w_1}}{S_w}\frac{x_{acw_1}}{\bar{c}_1} + C_{L\alpha w_2}\frac{S_{w_2}}{S_w}\frac{x_{acw_2}}{\bar{c}_2} + \ldots}$$