Deep Learning IV

CS771: Introduction to Machine Learning

Purushottam Kar

Announcements

Assignment 3 released

Deadline November 23, 2019, 9:59PM IST

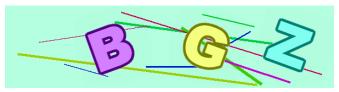
Very close to grade submission deadline – do not rely on "extensions"

Use non-linear methods (kernels, NN) judiciously for this assignment

Do not forget to also explore LwP, linear methods

In ML, good and insightful feature engineering can beat the best of algos ©

Fully exploit the simplicities and structure in given data





















Maybe fruitful to use ML methods for the last step











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Recap of Last Lecture

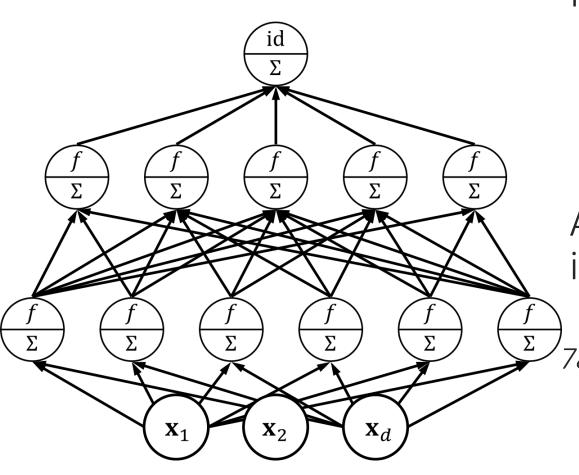
Chain rule for multivariate functions (notion of Jacobian)

Backprogation rule for training DN using GD

Generative models via NN – autoencoders, GANs



Feedforward Networks can be massive



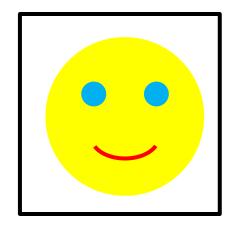
Fully connected layers are powerful

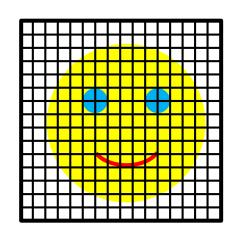
Allow all possible combinations of input dims to create new features which are functions of any subset of $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_d$ New features of the form $f(\mathbf{w}^\mathsf{T}\mathbf{x})$

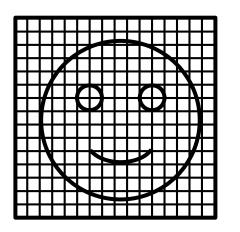
Also very unnecessary for apps where input has lots of structure

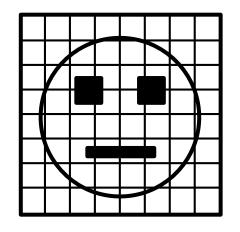
Make networks very bulky — e.g. the AE
784 →1000 →500 →250 →30 →250 →500 →1000 →784
needs 2.8 million edge weights to be learnt
From only 60 thousand data points ⊗
Also require tons of data to train so many
edge weights otherwise NN may overfit

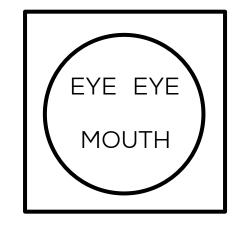
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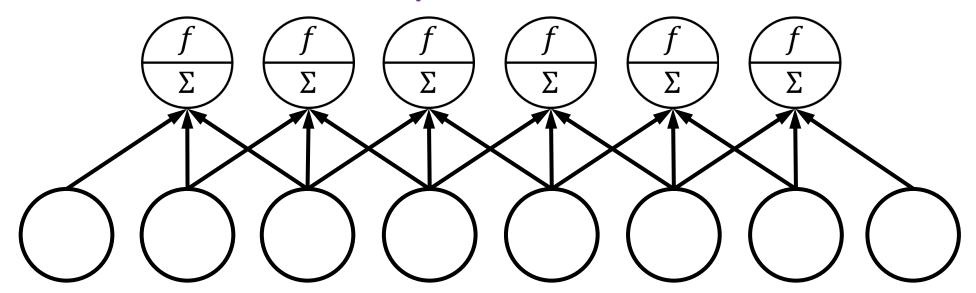
Highly unlikely that top left and bottom right pixels would need to be considered together right at the first hidden layer to detect edges

Only neighboring pixels need to talk to each other to detect edges. Also edge detection happens via "filters" – same filter needs to be applied everywhere

Then, need to aggregate info to detect structures

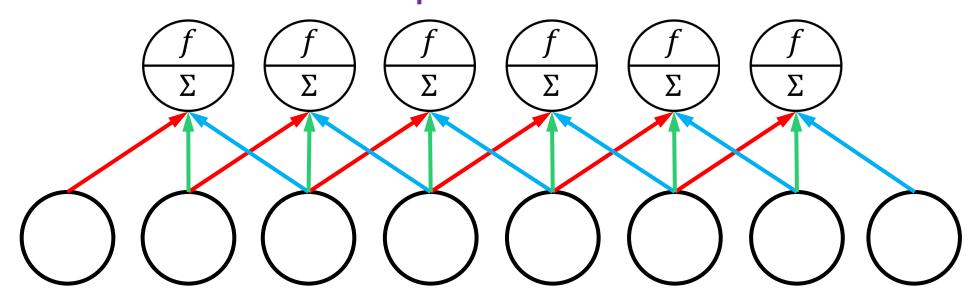
Then, need to detect even higher level features

Distant pixels are jointly considered, but at a much later stage (deeper layer)



Convolutions are at the heart of signal processing and CNNs Convolutions create layers which are sparsely connected Only 18 edges, fully connected layer would have had 48 edges





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Convolutions create layers which are sparsely connected

On top of that they force equality constraints among weights in that layer

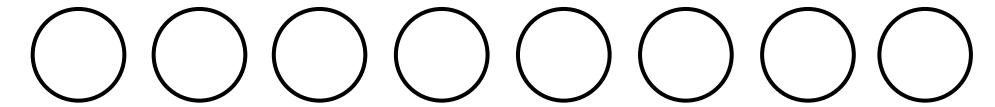
All green edges forced to have the same weight, all red edges forced to have the same weight, all blue edges ...

So effectively only 3 edge weights to be learnt for this layer!

A fully connected layer would have had 48 edges

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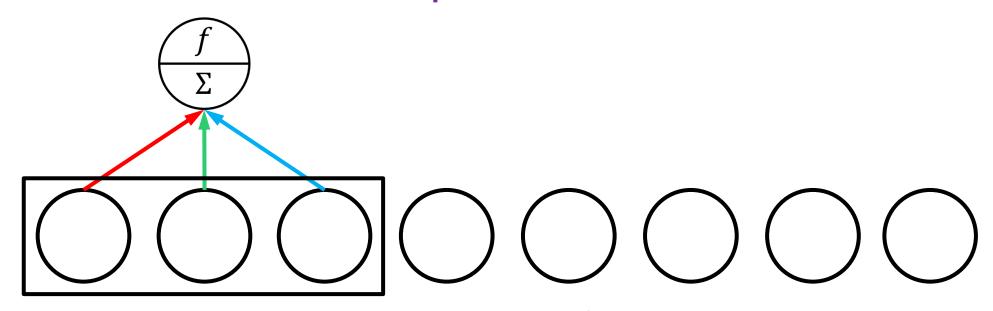
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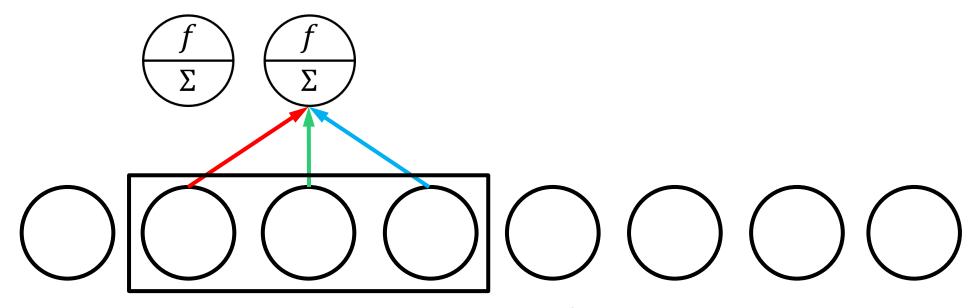
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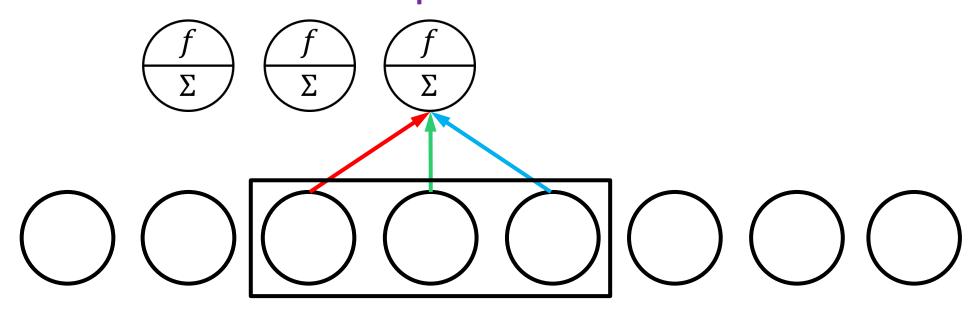
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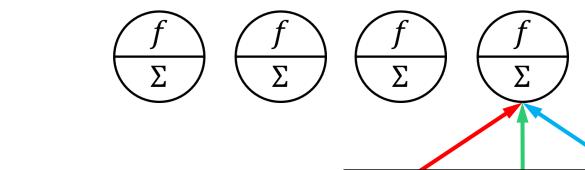
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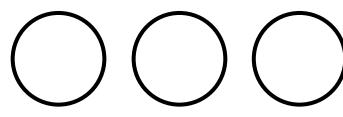
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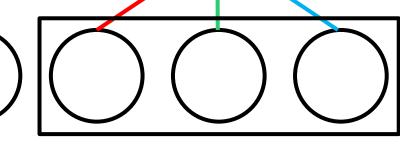
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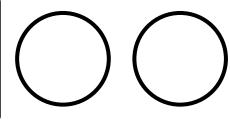
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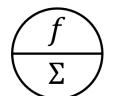
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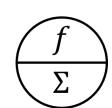
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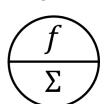
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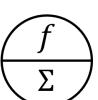
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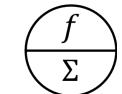


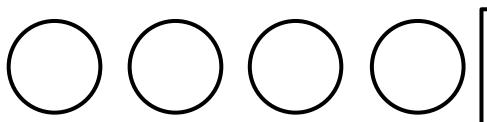


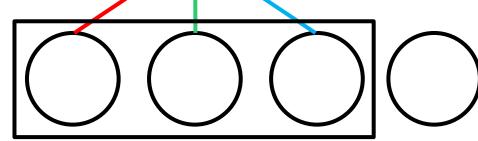












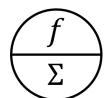
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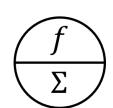
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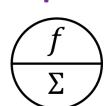
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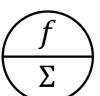
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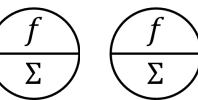


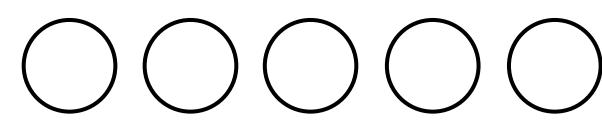


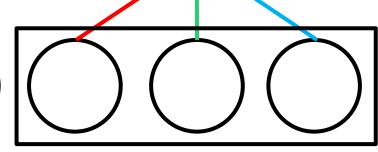












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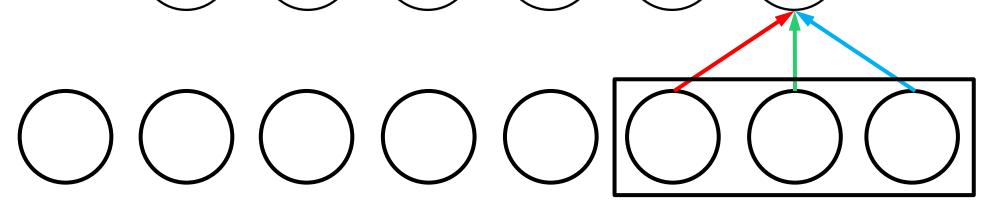
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The formal name for such an operation which sweeps a function (in this case $f(\mathbf{w}^{\mathsf{T}} \cdot)$ across a signal or an array is *convolution*. The vector \mathbf{w} is often called the *kernel* of the convolution – don't confuse this with Mercer kernels though!





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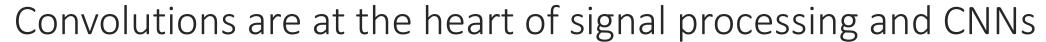
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Convolutions have been around in image processing for decades. Earlier, people used to painstakingly design the kernels by hand (e.g. Canny filters) but CNNs allow us to learn the kernel of the convolution itself.



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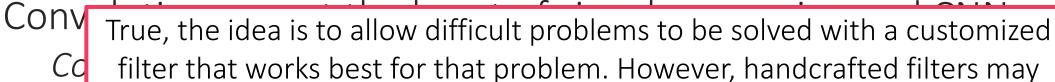
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still offer good results for simple problems (and offer faster training since

learning filters is not an easy task – the backprop becomes complicated)

constrained optimization problem is the key to the success of CNNs

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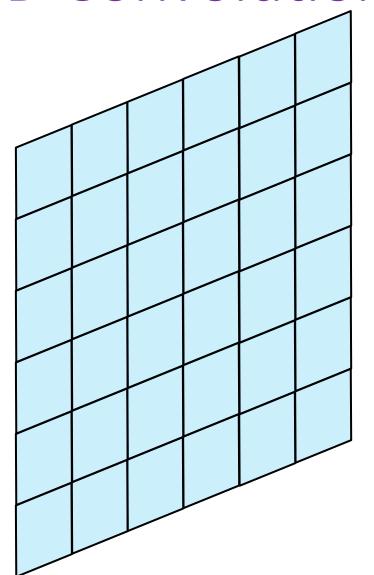
Convolutions make sense for 2D, 3D, nD data as well

A fully conn. layer would've needed 576 weights. A conv. needs only 9 weights

Such local operations are exactly what we need for detecting local patterns

Edges, boundaries, textures
Can apply convolutions to 3D
layers as well





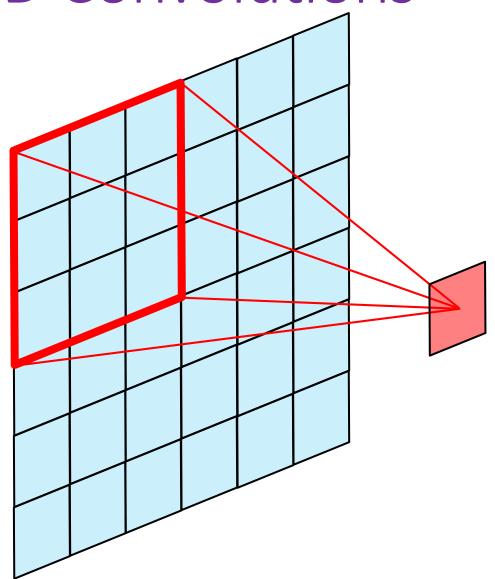
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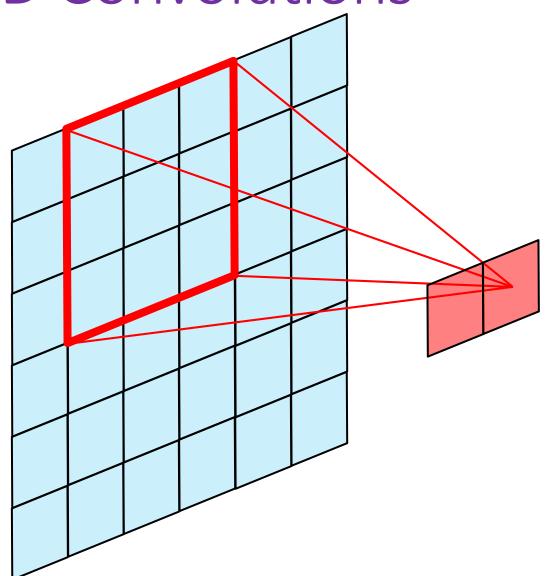
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2D Convolutions

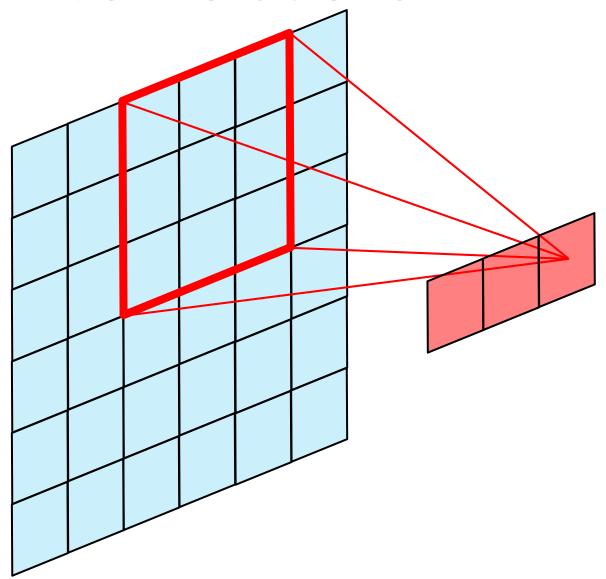


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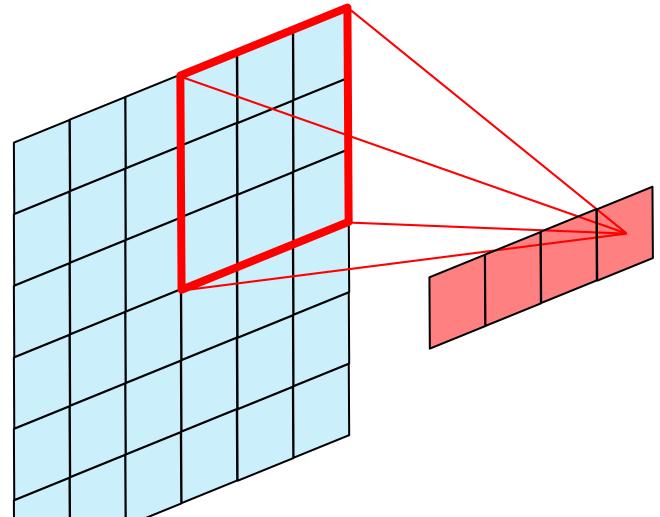
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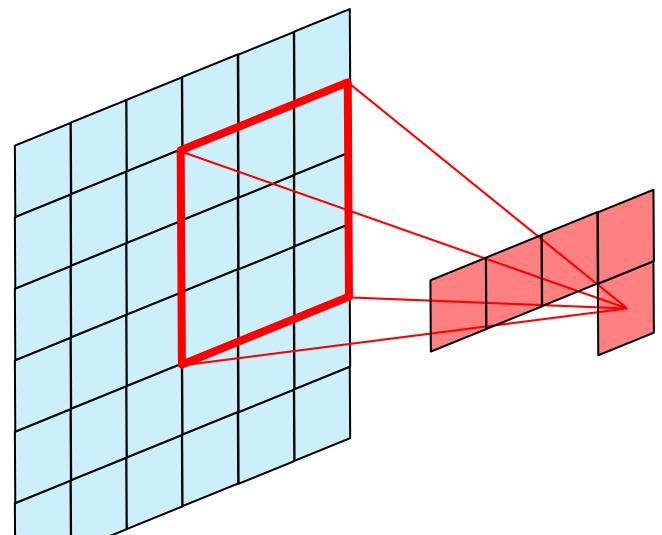
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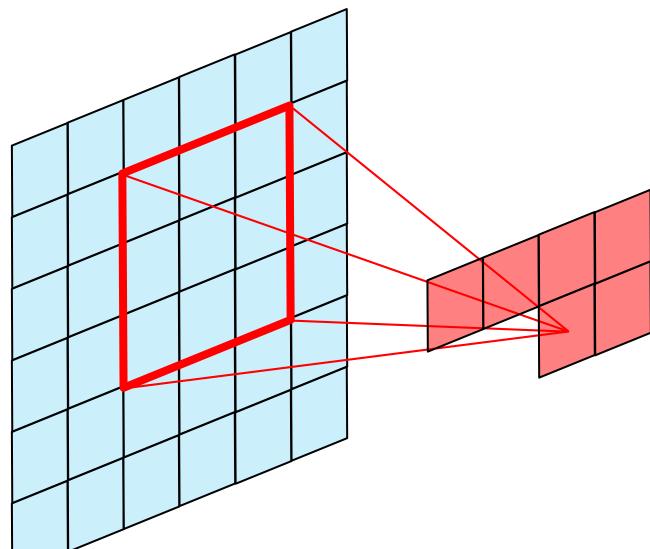
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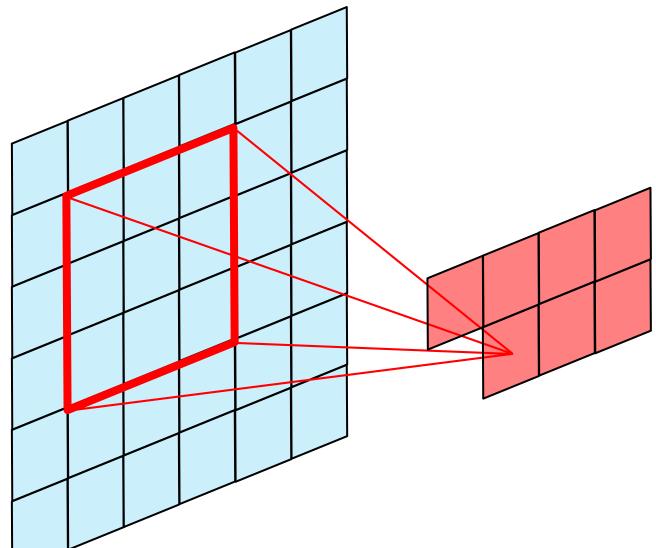


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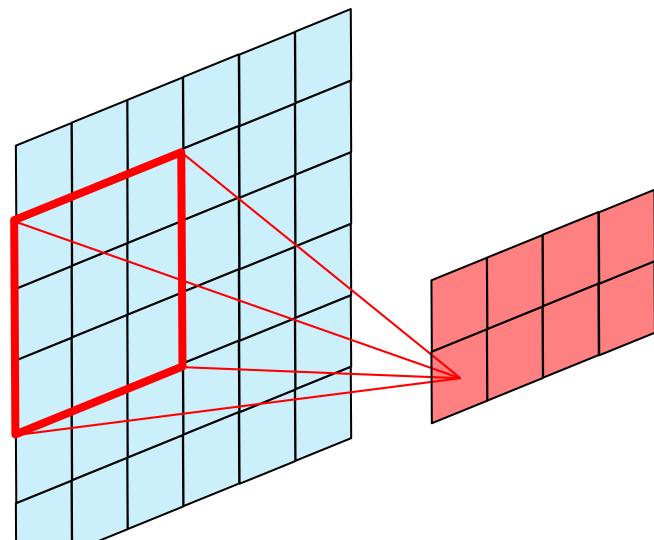


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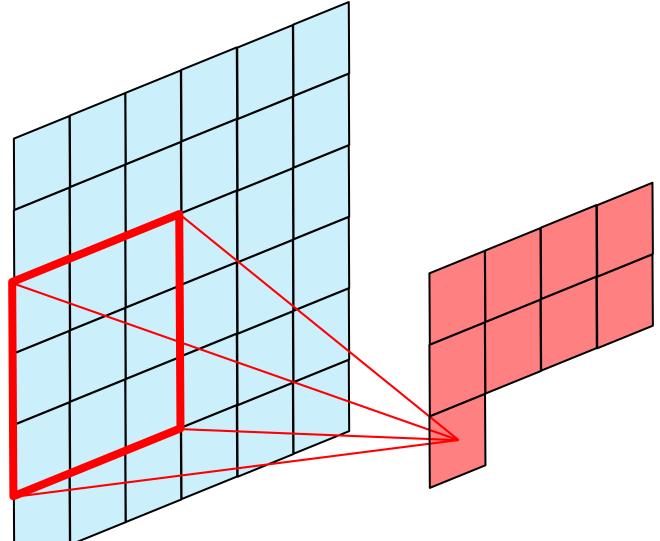


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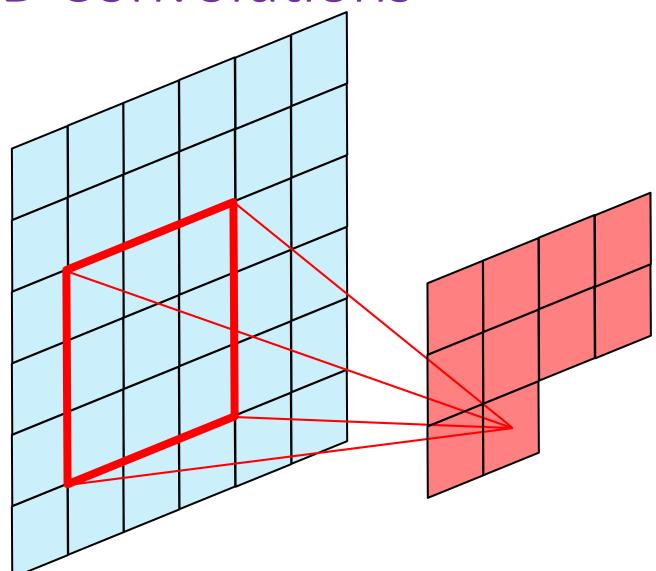


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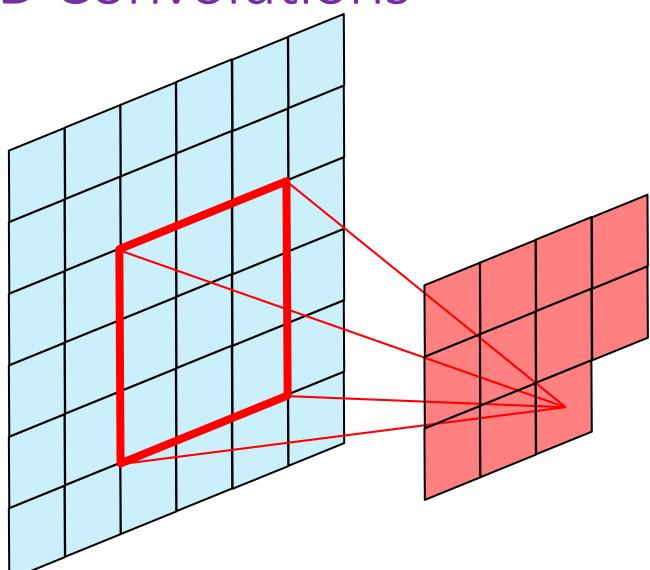
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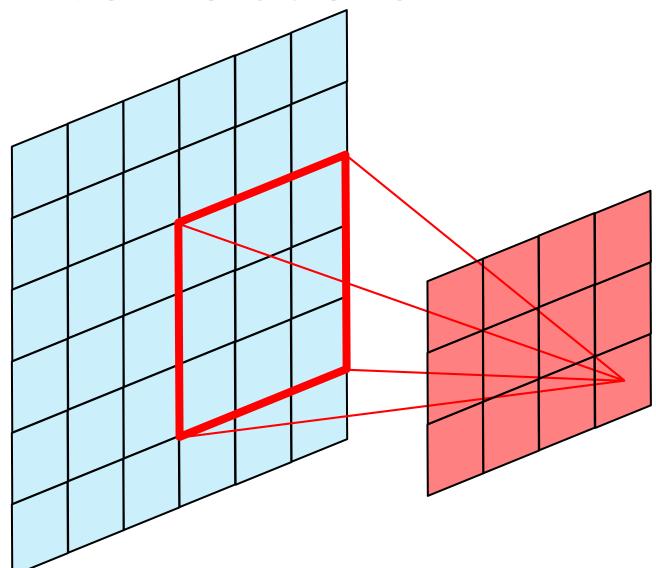


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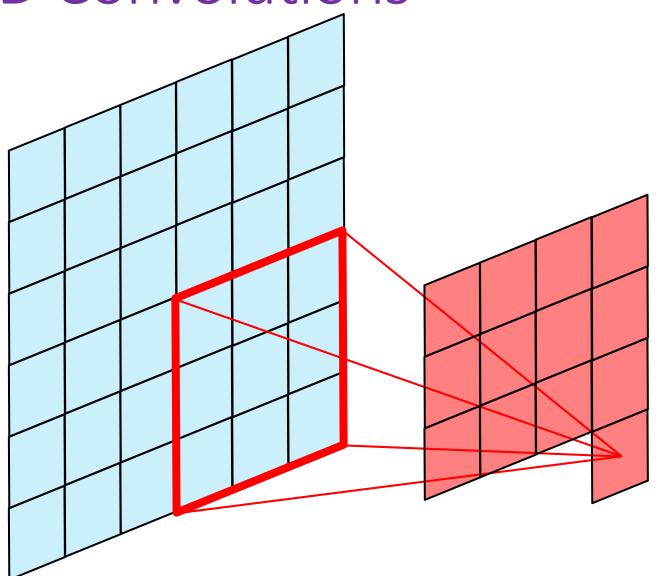


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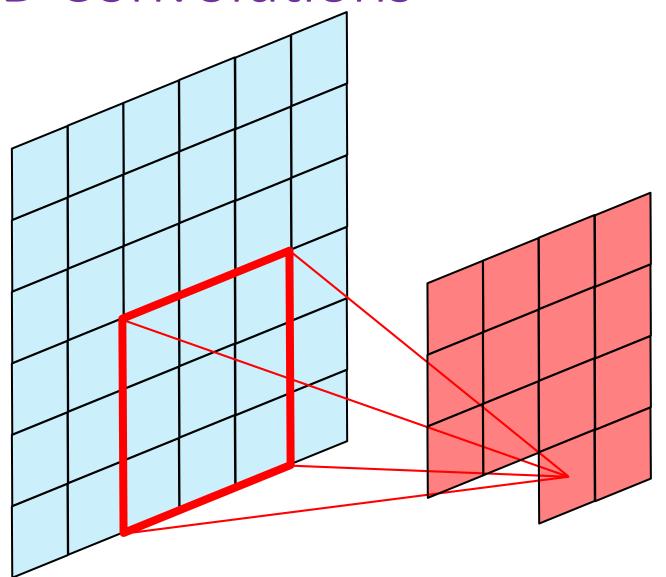


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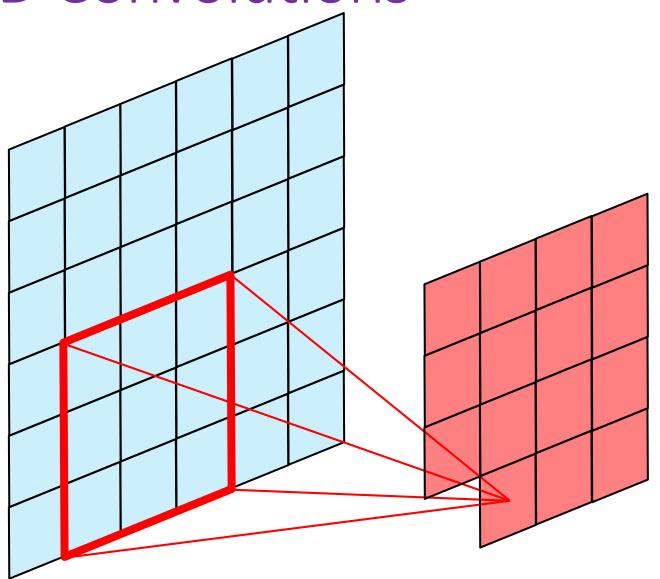


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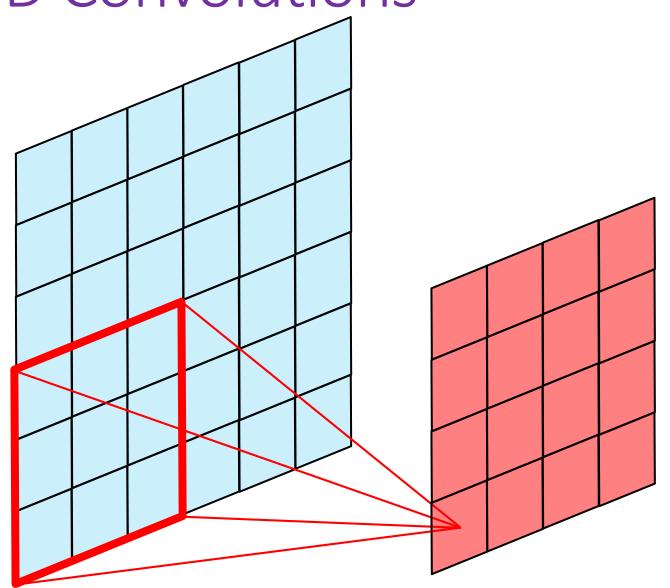


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E.g. Video data is 3D

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Convolutional Neural Network

Popular where the raw data has strong spatial structure e.g. images have 2D structure, text has linear structure, video has 3D structure

Greatly reduces the number of parameters to be learnt

Layers sparsely connected and aggressive parameter sharing

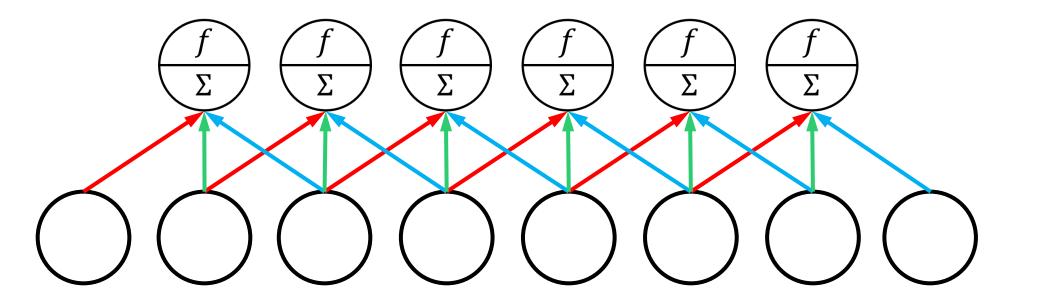
Note: notion of "convolution" used in CNNs is non-standard

Standard notion of convolution of two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is another vector $\mathbf{s} \in \mathbb{R}^n$ denoted as $\mathbf{s} = \mathbf{u} * \mathbf{v}$ such that

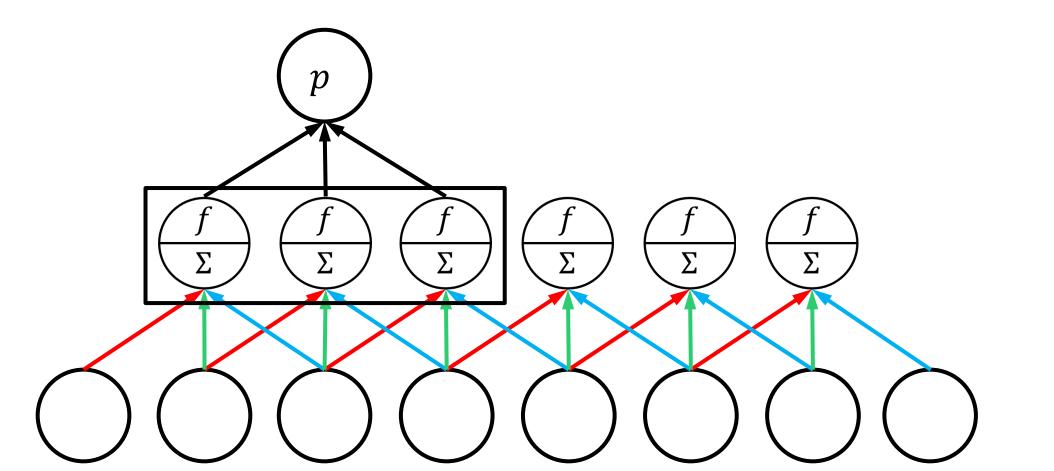
$$\mathbf{s}_i = \sum_{j=1}^{n} \mathbf{u}_j \cdot \mathbf{v}_{i-j}$$

However, CNNs use the definition $(\mathbf{u} * \mathbf{v})_i = \sum_{j=1}^n \mathbf{u}_{i+j} \cdot \mathbf{v}_j$ In signal processing literature this operation is actually called cross-correlation

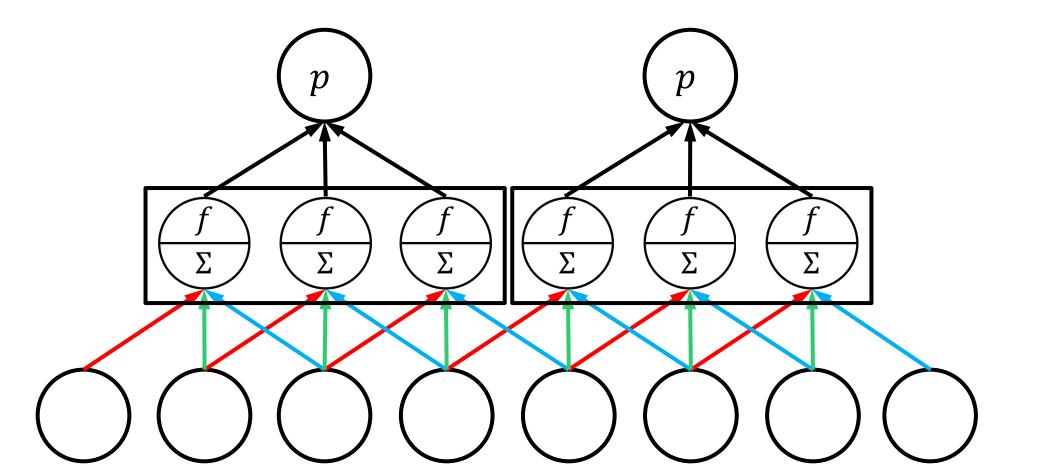




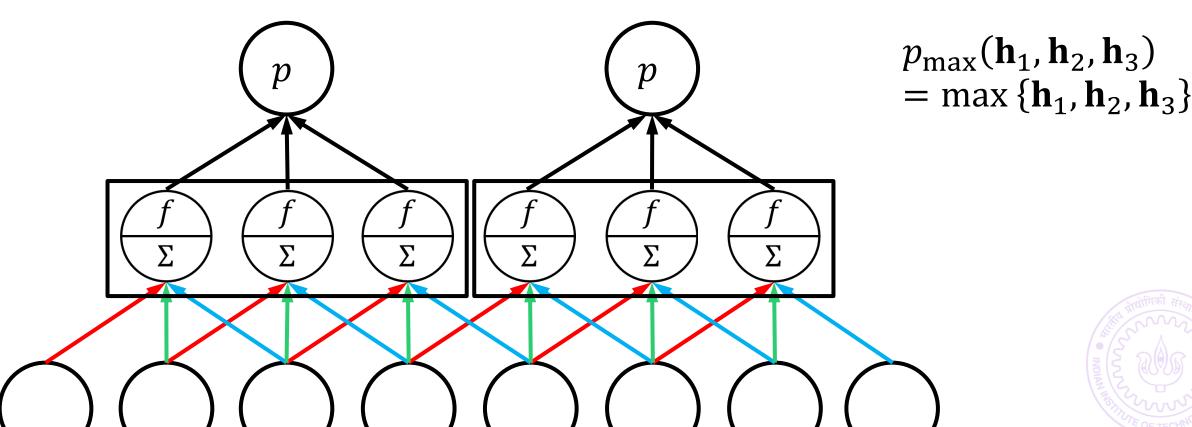




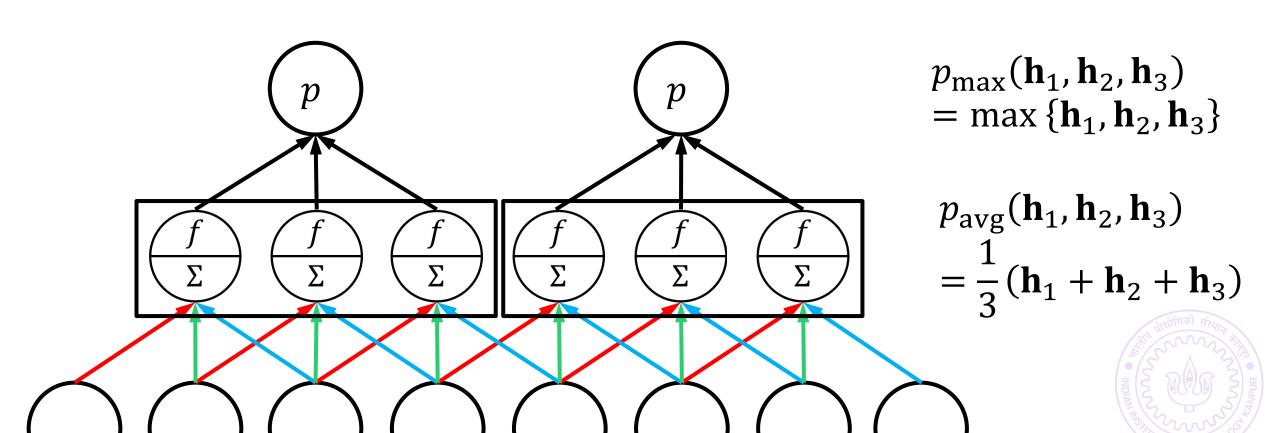




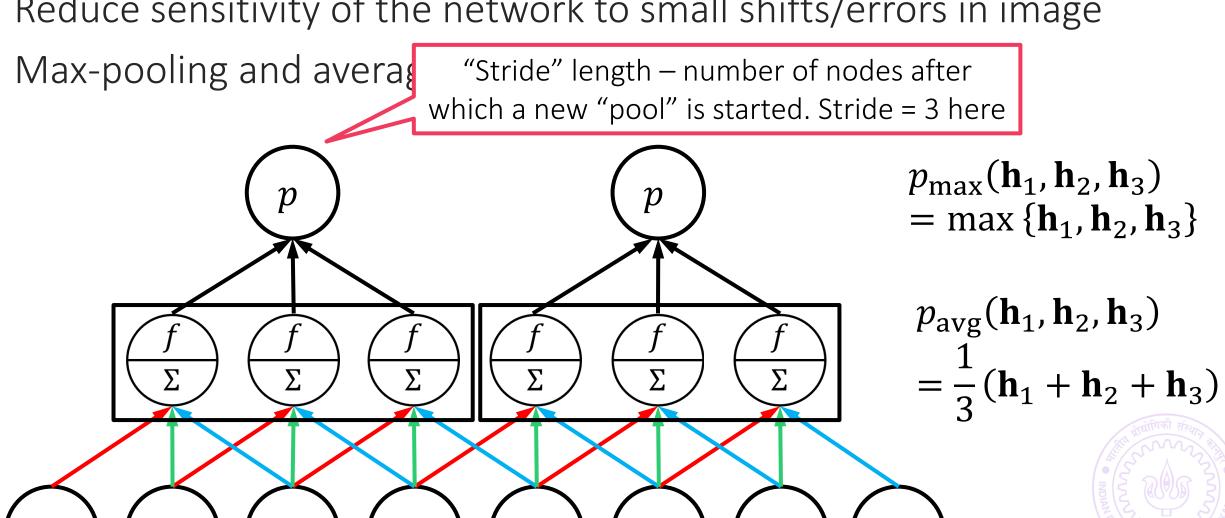








Reduce sensitivity of the network to small shifts/errors in image









44

Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



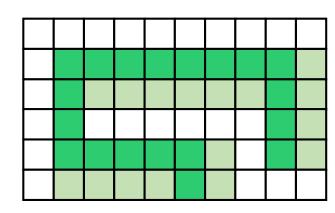
44

Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



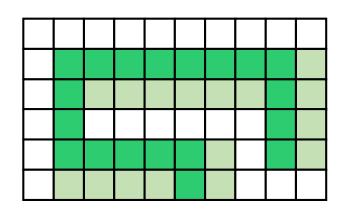


Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



$$= +1$$

$$= +0.5$$

$$\Box = 0$$

$$= 0$$
 (padded)

$$= -0.5$$

$$= -1$$



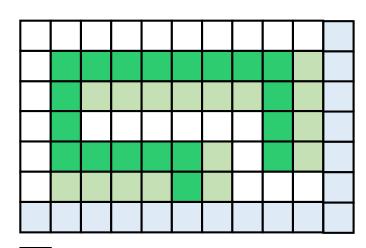
44

Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



$$= +1$$

$$= +0.5$$

$$\Box = 0$$

$$= 0$$
 (padded)

$$= -0.5$$

$$= -1$$



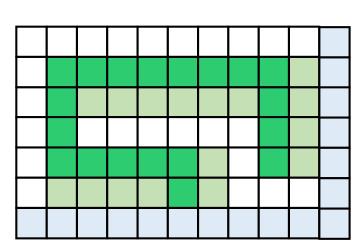
44

Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



= +1

= +0.5

 $\square = 0$

= 0 (padded)

= -0.5

= -1

-1
Detects
horizontal
edges!



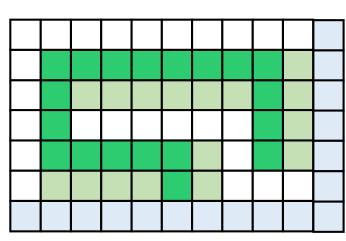
44

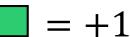
Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)





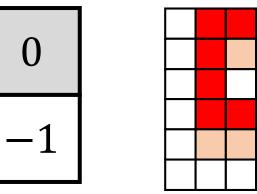
$$= +0.5$$

= 0

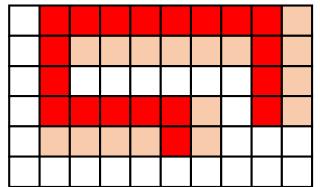
= 0 (padded)

= -0.5

= -1



Detects horizontal edges!





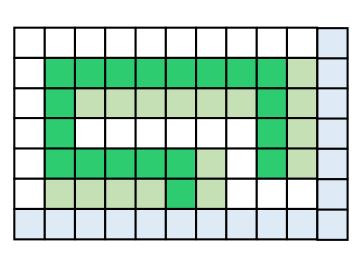
44

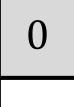
Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)



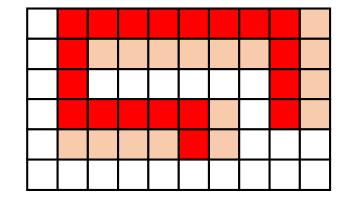


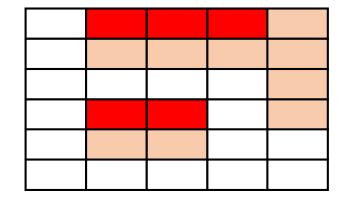


Detects

horizontal

edges!





= +1

= +0.5

 $\square = 0$

= 0 (padded)

= -0.5

= -1



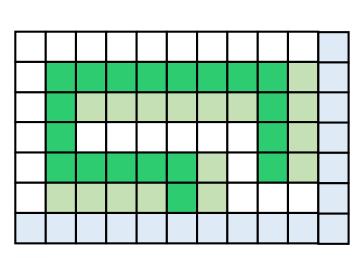
44

Raw Image



Convolved Image

Max Pooling (stride 1x2)



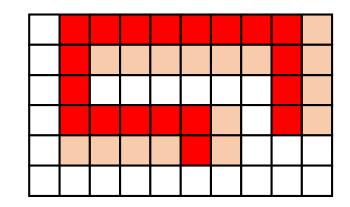


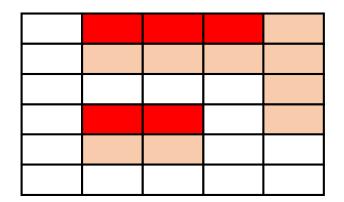


Detects

horizontal

edges!





= +1

= +0.5

 $\square = 0$

= 0 (padded)

= -0.5

= -1

0 -1

Detects vertical edges!

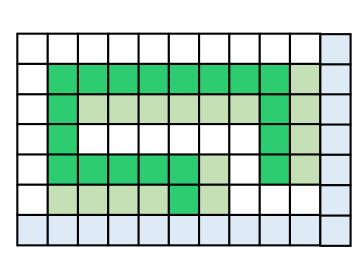


Raw Image

Kernels

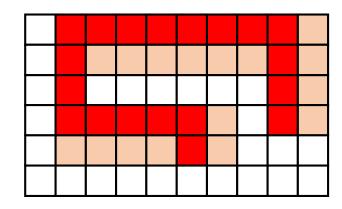
Convolved Image

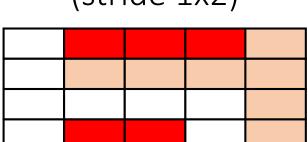
Max Pooling (stride 1x2)







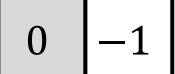




= +0.5

= 0 (padded)

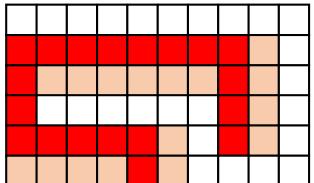
Detects horizontal edges!

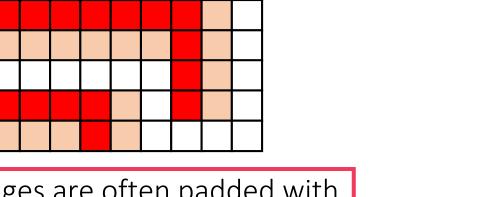


Detects

vertical

edges!







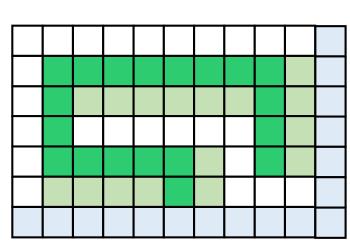
44

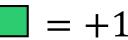
Raw Image

Kernels

Convolved Image

Max Pooling (stride 1x2)





$$= +0.5$$

$$\square = 0$$

$$= 0$$
 (padded)

$$= -0.5$$

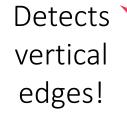
$$= -1$$

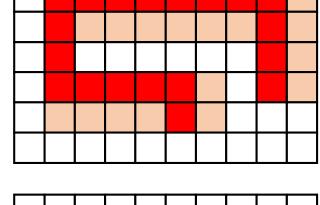


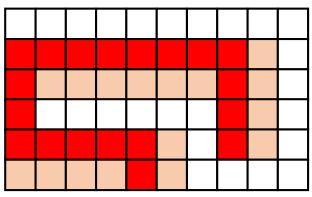


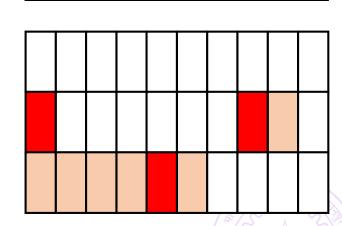
Detects horizontal edges!











Images are often padded with zero pixels so that convolved image is of same size

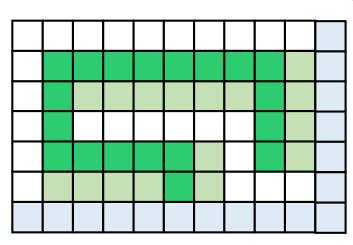


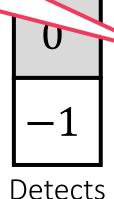
CS771: Intro to ML

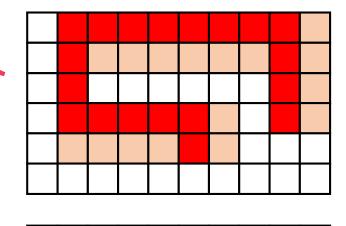
We may verify that 2x2 stride leads to too much info loss

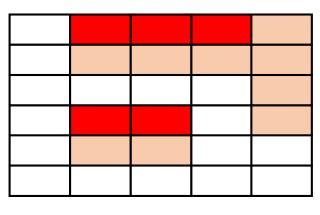
Convolved Image

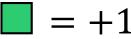
Max Pooling (stride 1x2)

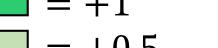


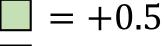












$$\square = 0$$

$$= 0$$
 (padded)

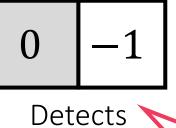
$$= -0.5$$

$$= -1$$



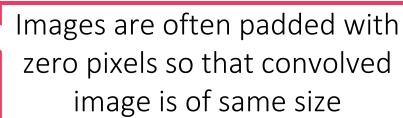
horizontal

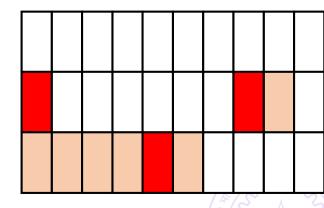
edges!



vertical

edges!





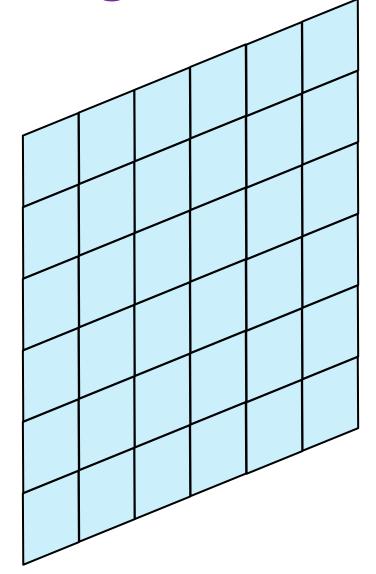


CS771: Intro to ML



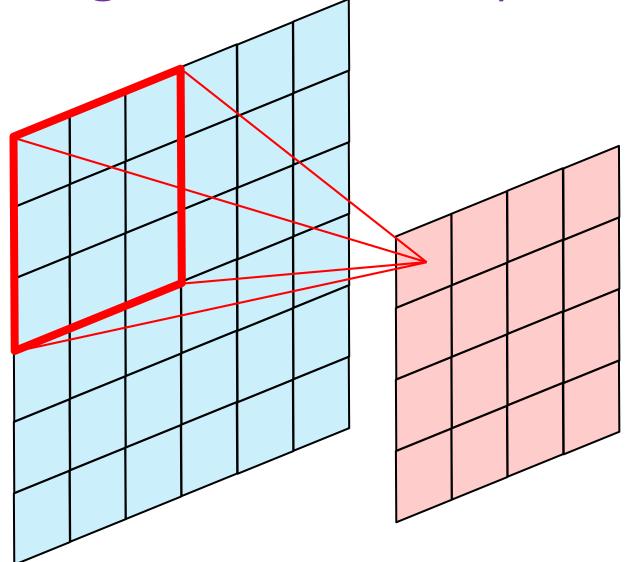






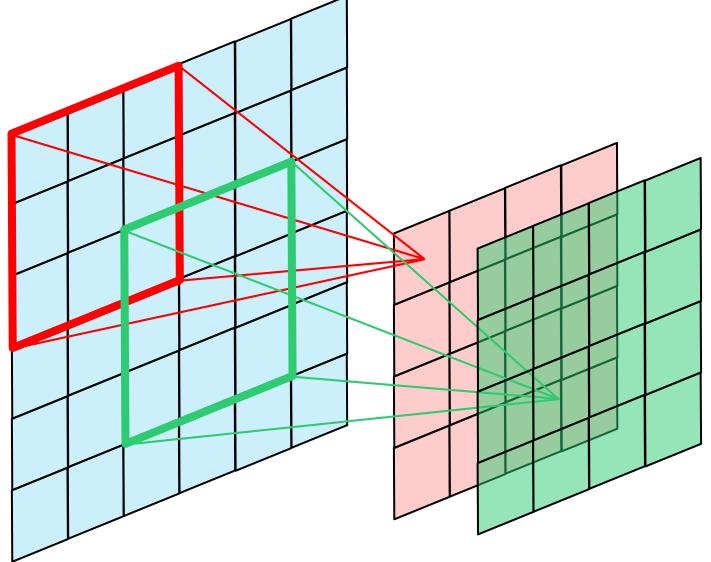






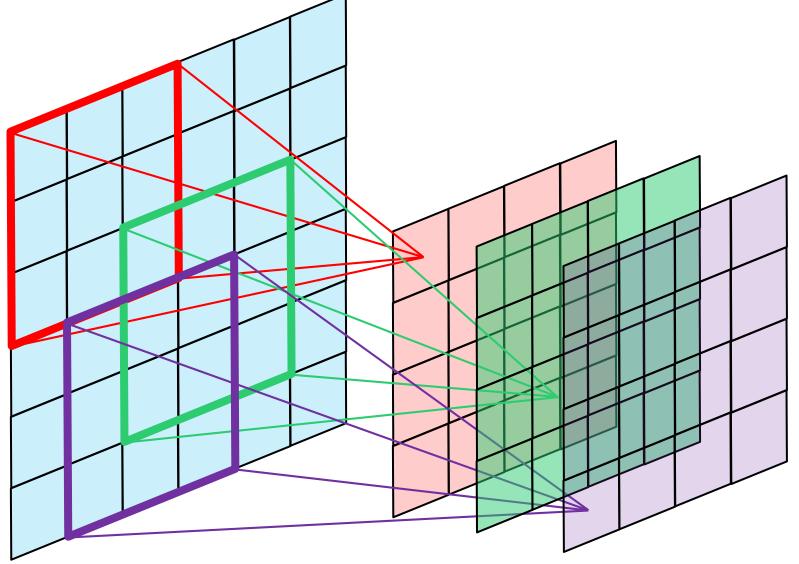




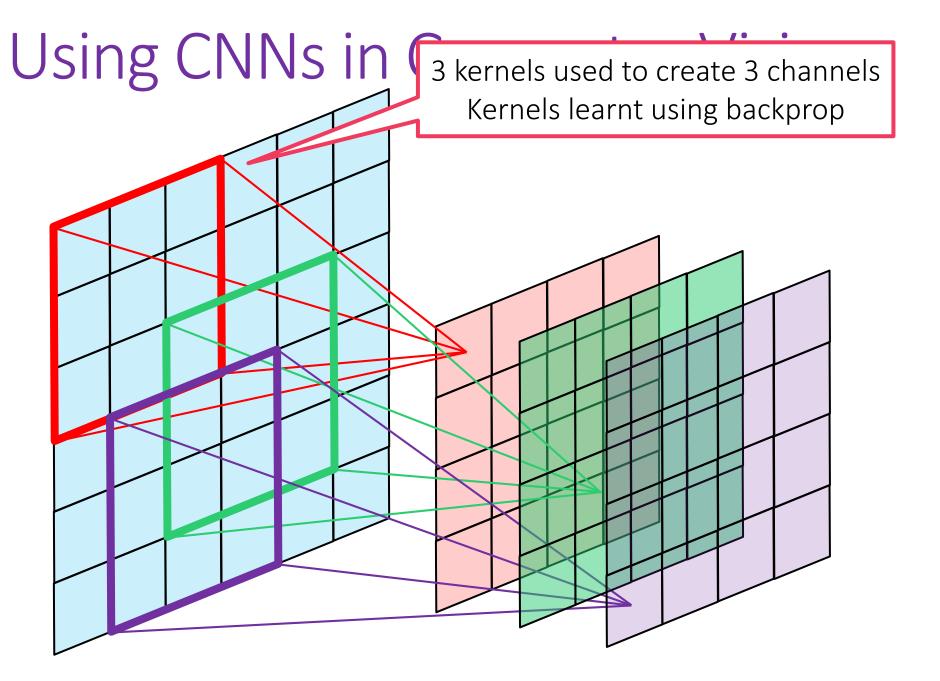






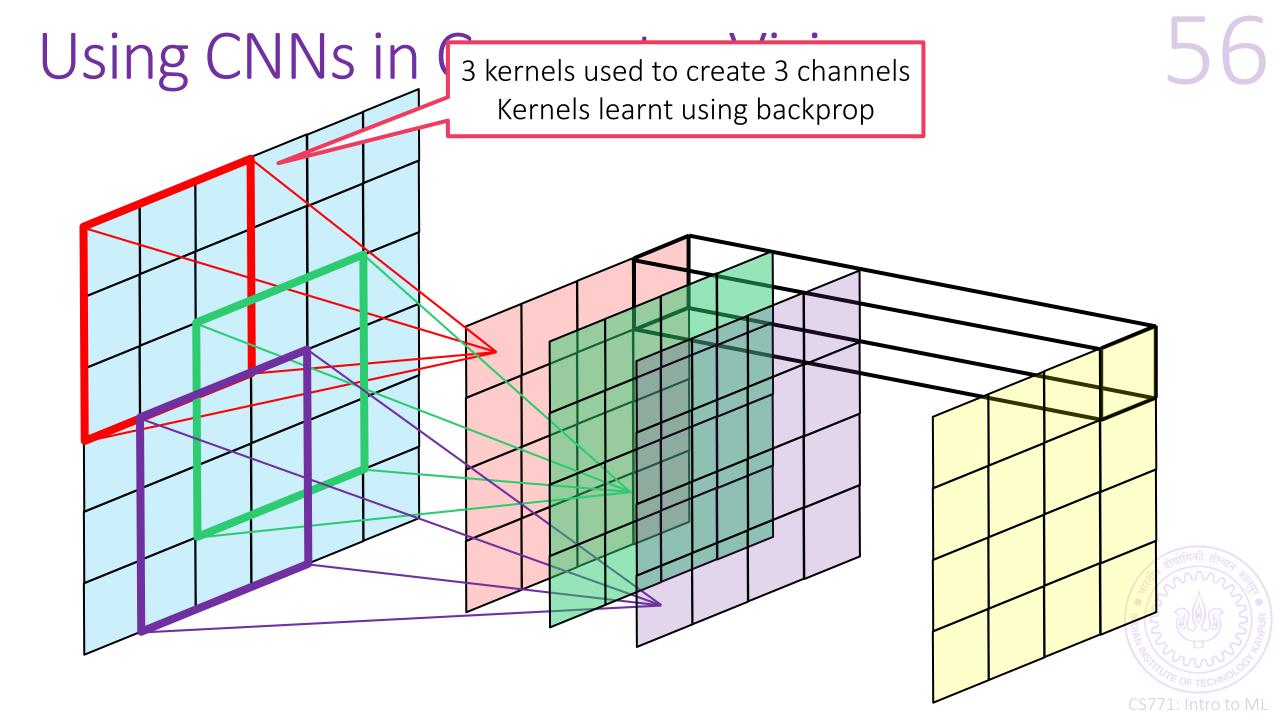


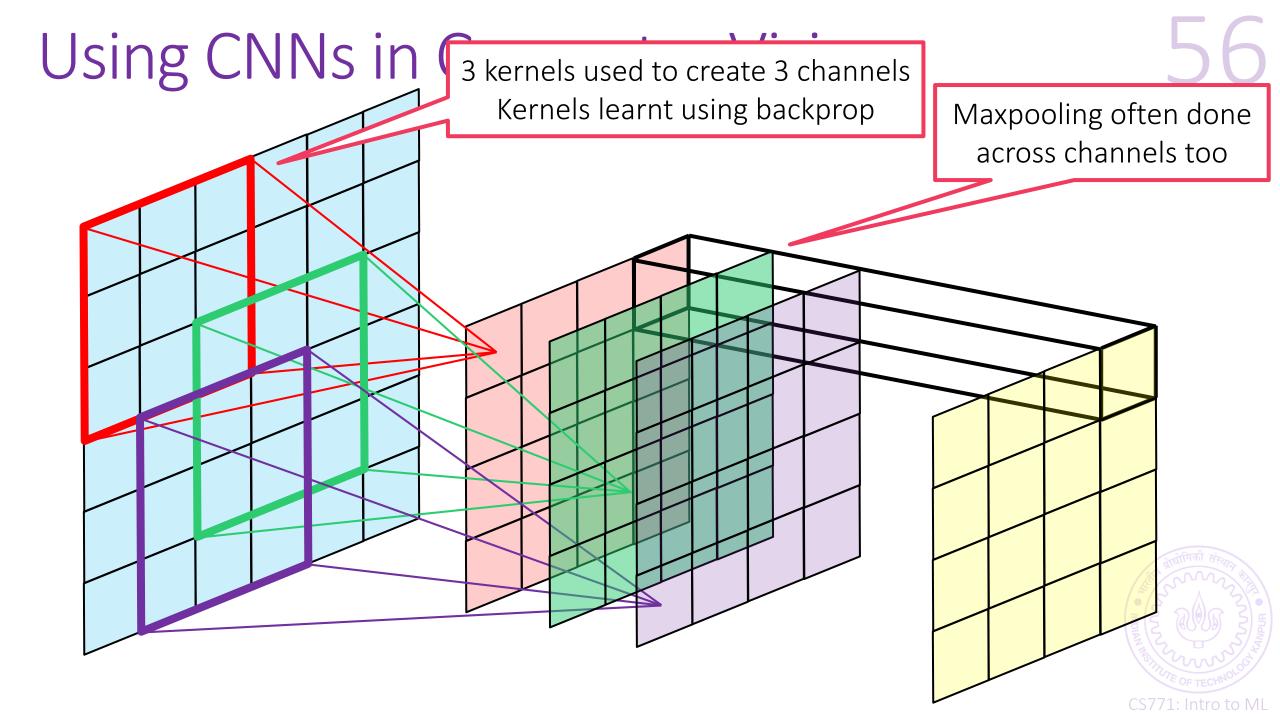












Using CNNs in 3 kernels used to create 3 channels
Kernels learnt using backprop

Maxpooling often done across channels too

Think of each channel (kernel) acting as a feature e.g. one kernel may tell if there is a vertical edge or not, another might tell if there is a diagonal edge or not. Maxpooling across channels is like asking if at least one of the features is active or is it the case that all features are inactive (no edges)

CS771: Intro to N

Using CNNs in 3 kernels used to create 3 channels

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True. However, note that even CNNs usually have a top layer that is dense i.e. weights that connect the last hidden layer to output layer are usually dense since it is assumed that by then all features that had to be learnt, have been learnt and so now all we need is a linear function over those.

