Routh-Hurwitz Criterion: Special Cases

Zero only in the first column

When forming the Routh table, replace the zero with a small number ε and evaluate the fist column for positive or negative values of ε.

Problem: Determine the stability of the closed-loop transfer function:

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3} \qquad (1)$$

s^5	1	3	5
s^4	2	6	3
s^3	$\frac{-\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix}}{2} = \cancel{0} \to \varepsilon$	$\frac{-\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix}}{2} = \frac{7}{2}$	$\frac{-\begin{vmatrix}1 & 0\\ 2 & 0\end{vmatrix}}{2} = 0$
s^2	$\frac{-\begin{vmatrix}2 & 6\\ \varepsilon & \frac{7}{2}\end{vmatrix}}{\varepsilon} = \frac{6\varepsilon-7}{\varepsilon}$	$\frac{-\begin{vmatrix}2 & 3\\ \varepsilon & 0\end{vmatrix}}{\varepsilon} = 3$	$\frac{-\begin{vmatrix}2 & 0\\ \varepsilon & 0\end{vmatrix}}{\varepsilon} = 0$
s^1	$\frac{-\begin{vmatrix}\varepsilon & \frac{7}{2}\\ \frac{6\varepsilon-7}{\varepsilon} & 3\end{vmatrix}}{\frac{6\varepsilon-7}{\varepsilon}} = \frac{42\varepsilon-49-6\varepsilon^2}{12\varepsilon-14}$	$\frac{-\begin{vmatrix}\varepsilon & 0\\ \frac{6\varepsilon-7}{\varepsilon} & 0\end{vmatrix}}{\frac{6\varepsilon-7}{\varepsilon}} = 0$	$\frac{-\begin{vmatrix}\varepsilon & 0\\ \frac{6\varepsilon-7}{\varepsilon} & 0\end{vmatrix}}{\frac{6\varepsilon-7}{\varepsilon}} = 0$
s^0	3	0	0

Label	First Column	$\varepsilon = +$	$\varepsilon = -$
s^5	1	$+$	$+$
s^4	2	$+$	$+$
s^3	ε	$+$	$-$
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	$-$	$+$
s^1	$\frac{42\varepsilon-49-6\varepsilon^2}{12\varepsilon-14}$	$+$	$+$
s^0	3	$+$	$+$

The changes in sign occur in both instances so it doesn't matter which we choose. they both indicate the system is unstable with two poles in the rhp.

Another, less computationally expensive method to use when a zero occurs in the first column is to create the Routh table using the polynomial that has the reciprocal roots of the original polynomial. i.e.

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3} \tag{2}$$

by replacing s with $1/s$.

$$D(s) = 3s^5 + 5s^4 + 6s^3 + 3s^2 + 2s + 1 \tag{3}$$

s^5	3	6	2
s^4	5	3	1
s^3	4.2	1.4	0
s^2	1.33	1	0
s^1	-1.75	0	0
s^0	1	0	0

since there are two sign changes, the system is unstable and has two right half plane (rhp) poles. This is the same as the result obtained by using ε for the original polynomial.

Entire row is zero

Problem: Determine the number of right hand plane poles in the closed-loop transfer function:

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56} \quad (4)$$

s^5	1	6	8
s^4	$\cancel{7} \rightarrow 1$	$\cancel{42} \rightarrow 6$	$\cancel{56} \rightarrow 8$
s^3	$0 \rightarrow$	$0 \rightarrow$	0
s^2			
s^1			
s^0			

Now we are faced with the problem of zeros in the third row.

1. Form a new polynomial using the entries in the row above zeros. The polynomial will start with power of s in that row, and continue by skipping every other power of s, i.e.

$$P(s) = s^4 + 6s^2 + 8 \tag{5}$$

2. Next we differentiate the polynomial with respect to s and obtain

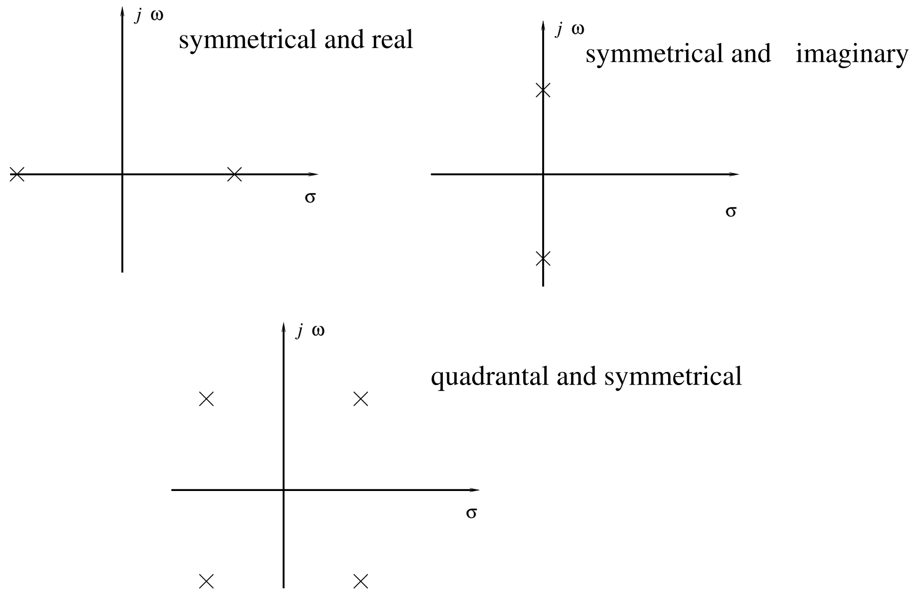
$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0 \tag{6}$$

3. Finally the row with all zeros in the Routh table is replaced with the coefficients in Eq.(6), and continue the table.

s^5	1	6	8
s^4	$\cancel{7} \to 1$	$\cancel{42} \to 6$	$\cancel{56} \to 8$
s^3	$\cancel{0} \to \cancel{4} \to 1$	$\cancel{0} \to \cancel{12} \to 3$	$\cancel{0} \to 0$
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
s^0	8	0	0

We see no sign changes hence no rhp poles.

• Why does an entire row of zeros occur? When a purely odd or even polynomial is a factor of the original polynomial. (s^4+6s^2+8 is an even polynomial as it only has even power of s.)



• Some polynomial only have roots symmetrical about the origin.

• Routh table from the even polynomial ($s^4 \rightarrow s^0$) is a test of the even polynomial.

• The rows of zeros indicates the possilbility of $j\omega$ roots.

Look at a larger example:

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20} \tag{7}$$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$-\cancel{1}0 \rightarrow -1$	$-\cancel{2}0 \rightarrow -2$	$\cancel{1}0 \rightarrow 1$	$\cancel{2}0 \rightarrow 2$	0
s^5	$\cancel{2}0 \rightarrow 1$	$\cancel{6}0 \rightarrow 3$	$\cancel{4}0 \rightarrow 2$	0	0
s^4	1	3	2	0	0
s^3	$\cancel{0} \rightarrow \cancel{4} \rightarrow 2$	$\cancel{0} \rightarrow \cancel{6} \rightarrow 3$	$\cancel{0} \rightarrow 0$	0	0
s^2	$\frac{3}{2}$	2	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

1. Rows of zeros of s^3, Form a new polynomial using the entries in the row above zeros, i.e. s^4.

$$P(s) = s^4 + 3s^2 + 2 \tag{8}$$

2. Next we differentiate the polynomial with respect to s and obtain

$$\frac{dP(s)}{ds} = 4s^3 + 6s + 0 \tag{9}$$

3. Finally the row with all zeros in the Routh table is replaced with the coefficients in Eq.(9), and continue the table.

• As the entries from s^4 to s^0 are looking at the even polynomials and there are no sign changes. So all four poles must exits on the $j\omega$ axis.

• There are two sign changes from s^8 to s^4, so there are 2 right hand plane poles and 2 left hand plane poles.

• Final results: 2rhp, 2lhp, 4 $j\omega$ poles.