Solution of Assignment # 1. AE-322.

1. $\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 0$.

(a) $\frac{d}{dt}(y) = \dot{y}$

$\frac{d}{dt}(\dot{y}) = \ddot{y}$

$\frac{d}{dt}(\ddot{y}) = -6y - 11\dot{y} - 6\ddot{y}$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

State-space representation

(b) Characteristic polynomial :-

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix} = 0 \Rightarrow \boxed{\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0}$$

(c) Eigenvalues are $-1, -2, -3$.

(d) Since the roots of the characteristic polynomial have negative real part, the system given in this question is asymptotically stable.

(e) As we conclude in the previous answer, that the given system is asymptotically stable, the response of the system will come to the equilibrium point as when it is perturbed from the equilibrium.

2. (a).

s^3	1	22
s^2	8	20
s^1	19.5	
s^0	20	

There is no sign change in the 1st column of the ~~R~~ Routh table.

⇒ All roots are in open ~~right~~ left half of the s-plane (~~ORHP~~ OLHP).

(OLHP) ⇒ The system is stable.

(b).

s^4	1	3	5
s^3	2	4	
s^2	1	5	
s^1	−6		
s^0	5		

# of sign change = 2 in the 1st column

⇒ There are ~~two~~ 2 roots in the open right half of the s-plane (ORHP)

∴ ⇒ Unstable system.

(c).

s^3	1	2
s^2	1	2
s^1	0.	
s^1	2	
s^0	2	

→ Form aux. polynomial.

$A(s) = s^2 + 2.$

$\frac{dA(s)}{ds} = 2s$

No sign change ⇒ No roots at ORHP.

Order of Aux. polynomial is = 2 ⇒ 2 roots are on jω axis.

∴ System is marginally stable.

(d) There are one sign change in the characteristic polynomial,

⇒ # of real root with +ve sign is 1. (ORHP)

⇒ Unstable system.

2. (e)

s^4	1	3	1
s^3	3	2	
s^2	$7/3$	1	
s^1	$5/7$		
s^0	1		

No sign change in the 1st column.

$\Rightarrow$ All roots are in OLHP.

$\Rightarrow$ Stable system.