Assignment 4 (AE321A) Total Marks: 100

(Practice)

Q1. For a Fixed Wing Aircraft

a). Derive the expression of total lift curve slope of aircraft? [10]

b). Derive the expression of total lift coefficient of aircraft at zero-degree angle of attack? [10]

Q.2. For a Fixed Wing Aircraft

a). Derive the expression of total pitch stability derivative of aircraft? [8]

b). Derive the expression of total pitching moment coefficient of aircraft at zero-degree angle of attack? [8]

c). Prove that the neutral point of aircraft is an equivalent to aerodynamic centre of wing alone [4]

Data for question 3 to 12:

For a Fixed Wing Aircraft, the following data is applicable:

Weight of aircraft = 300 kg, $V_{cruise} = 35m/s$, $\rho_{sea} = 1.2256\ {}^{kg}/_{m^3}$, $C_{L_{\alpha w}} = 4.5279/\text{rad}.$, $C_{L_{\alpha t}} = 4/\text{rad}$, $\alpha_{L=0} = -3°$, $\left(C_{m_{ac}}\right)_w = -0.08$, $\bar{c} = 1\text{m}$, $S_w = 7m^2$, $S_{HT} = 1.5m^2$ $b = 7\text{m}$, $\eta_t = 0.9$, $(X_{ac})_{wing} = 0.25\text{m}$, $(X_{ac})_{Tail} = 3.0\text{m}$ $\tau = 0.4$, $\frac{d\epsilon}{d\alpha} = 0.4118$, $\varepsilon_0 = 1.2375°$, $i_t = -1.0deg$, $i_w = 0.0deg$, static margin (SM)=10%

Useful formula:

$$C_L = C_{L0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e$$

$$C_m = C_{m0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e$$

$$C_{L_{\delta_e}} = \tau.\eta_t.\frac{S_{HT}}{S_w}C_{L_{\alpha t}}$$

$$C_{m_{\delta_e}} = -\tau.\eta_t.\frac{S_{HT}}{S_w}.\frac{\left((X_{ac})_{Tail} - (X_{cg})\right)}{\bar{c}}.C_{L_{\alpha t}}$$

The nomenclature has their usual aerodynamic meaning

Note 1:(neglect the effect of fuselage on stability)

Note 2:(Location of aerodynamic centre of wing and tail has been measured from the wing leading edge)

Q.3. The location of Neutral point in meters of the Fixed Wing Aircraft from the leading edge of the wing? [5]

Q.4.The total lift curve slope of the Aircraft per radian will be? [5]

Q.5.The pitch stability derivative (C_{m_α}) (in per rad.) of the Aircraft? [5]

Q.6. What will be the location of centre of gravity of Aircraft in meters from the wing leading edge? [5]

Q.7.Identyfy the tail Horizontal tail volume ratio of the Fixed Wing Aircraft? [5]

Q.8.What will total pitching moment coefficient at zero degree angle of attack of $((C_{m_0})_{Aircraft})$ of the Fixed Wing Aircraft? [5]

Q.9.What will be the total lift coefficient at zero degree angle of attack of (C_{L0}) in degrees of the Aircraft? [5]

Q.10. The trim lift coefficient at sea level of aircraft will be? [5]

Q.11. With the current design and flight condition of Aircraft, what will be the trim angle of attack in degrees? [10]

Q.12. With above trim angle of attack, elevator deflection required for trim in degrees? [10]

(1)

$$L = L_w + L_t$$

$$\frac{1}{2}\rho V^2 S C_L = \frac{1}{2}\rho V^2 S C_{Lw} + \frac{1}{2}\rho_t V_t^2 S_t C_{Lt}$$

$$C_L = C_{Lw} + \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V^2} \frac{S_t}{S} C_{L\alpha t}\, \alpha_t$$

$$C_L = C_{Lw} + h \frac{S_t}{S} C_{L\alpha t} \left(\alpha_w - i_w - \varepsilon + i_t \right)$$

$$C_L = C_{L0w} + C_{L\alpha w}\alpha_w + h \frac{S_t}{S} C_{L\alpha t} \left(\alpha_w - i_w - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + i_t \right)$$

$$\underline{C_{L0} + C_{L\alpha}\alpha} = C_{L0w} + C_{L\alpha w}\alpha_w + h \frac{S_t}{S} C_{L\alpha t} \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_w$$
$$+ h \frac{S_t}{S} C_{L\alpha t} \left(- i_w - \varepsilon_0 + i_t \right)$$

Comparing ⇒

a) $C_{L0} = C_{L0w} + h \frac{S_t}{S} C_{L\alpha t} \left(i_t - i_w - \varepsilon_0 \right)$

b) $C_{L\alpha} = C_{L\alpha w} + h \frac{S_t}{S} C_{L\alpha t} \left(1 - \frac{d\varepsilon}{d\alpha} \right)$

(2) (a) & (b) → Already explained (Very - Standard)

(c) $$C_{m\alpha} = C_{L\alpha w} \left[\frac{x_{acw}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} \right] - h \frac{S_t}{S_w} C_{L\alpha t} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \times \left(\frac{x_{act}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} \right)$$

at Neutral Point $x_{cg} = x_{NP}$

and $C_{m\alpha} = 0$

Hence,

$$0 = C_{L\alpha w}\left[\frac{x_{acw}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right] - \eta \frac{S_t}{S} C_{L\alpha t}\left(\frac{x_{act}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$$
$$\times \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$\boxed{x_{NP} = \frac{C_{L\alpha w}\, x_{acw} + \eta \frac{S_t}{S} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) x_{act}}{C_{L\alpha w} + \frac{\eta S_{HT}}{S_w} C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right)}}$$

if $S_t = S$

$\eta = 1$, $C_{L\alpha w} = C_{L\alpha t}$ $\Rightarrow$ $\frac{d\epsilon}{d\alpha} = 0$

$x_{act} = x_{acw}$

$$x_{NP} = \frac{(x_{acw} + 1 \times 1 \times x_{acw})\, C_{L\alpha w}}{C_{L\alpha w}(1 + 1)}$$

$$\boxed{x_{NP} = x_{acw}}$$

3 to 12

Given, $W = 300\,kg$, $V_c = 35\,m/s$, $\rho_{sea} = 1.2256\,kg/m^3$

$C_{L\alpha w} = 4.5279/rad$, $C_{L\alpha t} = 4/rad$, $\alpha_{L=0} = -3^\circ$

$(C_{mac})_w = -0.08$, $\bar{c} = 1\,m$, $S_w = 7\,m^2$

$S_{HT} = 1.5\,m^2$, $b = 7\,m$, $\eta_t = 0.9$, $(x_{ac})_{wing} = 0.25\,m$

$(x_{ac})_t = 3\,m$, $z = 0.4$, $\frac{d\varepsilon}{d\alpha} = 0.4118$

$\varepsilon_0 = 1.2375^\circ$, $i_t = -1.0^\circ$, $i_w = 0.0^\circ$

$S.M = 10\%$

(3)

$$X_{NP} = \frac{C_{L\alpha w}\, x_{ac_w} + \eta \frac{S_{HT}}{S_w} C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) x_{ac_t}}{C_{L\alpha w} + \eta \frac{S_{HT}}{S_w} C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)} \quad \text{--- (1)}$$

Substituting all the values present in eq (1),

we have,

$$\boxed{X_{NP} = 0.5005\,m} \rightarrow \text{from Wing L.E}$$

(4) Total lift $\Rightarrow$

$$(C_{L\alpha})_{Aircraft} = C_{L\alpha w} + \eta \frac{S_{HT}}{S_w} C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$= 4.5279 + \frac{0.9 \times 1.5}{7}\, 4\,(1 - 0.4118)$$

$$= \underline{4.9817\,/rad}$$

(5)

$$C_{m_\alpha} = -(S.M)\,(C_{L_\alpha})_{Aircraft}$$

$$= -(0.1) \times 4.9817$$

$$= -0.49817$$

(6)

$$S.M = \bar{x}_{Np} - \bar{x}_{cg}$$

$$\frac{x_{cg}}{\bar{c}} = \bar{x}_{Np} - S.M$$

$$= 0.5005 - 0.1$$

$$x_{cg} = 0.4005\,\bar{c}$$

$$= 0.4005 \times 1$$

$$\left[x_{cg} = 0.4005\right]$$

(7)

$$V_H = \frac{S_{HT}\,(x_{ac_t} - x_{cg})}{S_w\,\bar{c}} = \frac{1.5\,(3 - 0.4005)}{7 \times 1}$$

$$\left[V_H = 0.5570\right]$$

(8)

$$(C_{m_0})_{Aircraft} = C_{m_{ac_w}} + C_{L_{0w}}\,(x_{cg} - x_{ac_w})\cdot\frac{1}{\bar{c}}$$

$$+ \eta\,\frac{S_{HT}}{S_w}\,C_{Lt}\,\frac{(x_{ac_t} - x_{cg})}{\bar{c}}\,(\varepsilon_0 - i_t) \quad -(2)$$

Substituting the values in eqn (2)

$$\left[(C_{m_0})_{Aircraft} = 0.0340\right]$$

(9)

$$(C_{L_0})_{Aircraft} = (C_{L_0})_W + \eta \frac{S_{HT}}{S_W} C_{L_{\alpha t}} (i_t - \epsilon_0)$$

$$= 0.2371 + 0.9 \times \frac{1.5}{7} \times 4 (-1 - 1.2375^\circ) \times \pi/180$$

$$= 0.2070$$

(10)

$$L = W = \frac{1}{2} \rho V^2 S C_L$$

$$C_L = \frac{2W}{\rho V^2 S} = \frac{2 \times 300 \times 9.81}{1.2256 \times 35^2 \times 7}$$

$$\left[C_L = 0.5603 \right]$$

for (11) & (12)

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta e}} \delta e \quad -(3)$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta e}} \delta e \quad -(4)$$

at trim $C_m = 0$

Eqn (3) & (4) can be written as —

$$\begin{bmatrix} \alpha \\ \delta e \end{bmatrix} = \begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{bmatrix}^{-1} \begin{bmatrix} C_L - C_{L_0} \\ -C_{m_0} \end{bmatrix} \quad -(5)$$

2)

$$C_{L\delta e} = +2 h_t \frac{S_{HT}}{S_w} C_{L\alpha t}$$

$$= +0.4 \times 0.9 \times \frac{1.5}{7} \times 4$$

$$= \underline{0.3086}$$

$$C_{m\delta e} = -2 h_t \frac{S_{HT}}{S_w} \left(\frac{x_{ac t} - x_{cg}}{\bar{c}} \right) C_{L\alpha t}$$

$$= -0.4 \times 0.9 \times 0.5570 \times 4$$

$$= \underline{-0.8021}$$

from eq^n (5)

$$\begin{bmatrix} \alpha \\ \delta e \end{bmatrix} = \begin{bmatrix} 4.9817 & 0.3086 \\ -0.4982 & -0.8021 \end{bmatrix}^{-1} \begin{bmatrix} 0.5603 - 0.207 \\ -0.0340 \end{bmatrix} \times \frac{180}{\pi}$$

$$\boxed{\begin{aligned} \alpha &= 4.071^\circ \\ \delta e &= -0.1002^\circ \end{aligned}}$$