

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} D_{-sm}^l(\phi, \theta, 0)$$

$$D_{-sm}^l(\phi, \theta, \gamma) = \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta, \phi) e^{-ism\gamma}$$

$Z_1 \rightarrow Y_1 - Z_2$
convention

$$= \sum_{lm} Y_{lm}(\theta_1, \phi_1) Y_{lm}^*(\theta_2, \phi_2)$$

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{0m}^l(\phi_1, \theta_1) D_{2m}^{l*}(\phi_2, \theta_2, 0)$$

$$\rightarrow D_{MM'}^{J*}(\alpha, \beta, \gamma) = [\hat{D}^{-1}(\alpha, \beta, \gamma)]_{M'M}^J$$

$$= D_{M'M}^J(-\gamma, -\beta, -\alpha)$$

Varshalovich
Pg 72, Eq. 4
Pg 74, Eq. 13

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{0m}^l(\phi_1, \theta_1, 0) D_{m2}^l(0, -\theta_2, -\phi_2)$$

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{02}^l(\alpha, \beta, \gamma) = \sqrt{\frac{2l+1}{4\pi}} D_{20}^{l*}(-\gamma, -\beta, -\alpha)$$

$$\rightarrow D_{MM'}^J(\alpha, \beta, \gamma) = e^{-im\alpha} d_{MM'}^J(\beta) e^{-im'\gamma}$$

Pg 76, eq. 1

$$d_{M'M}^J = (-1)^{M-M'} d_{MM'}^J$$

$$\cancel{D_{02}^l} = \cancel{e^{-im\alpha}}$$

$$= \sum_{lm} \frac{2l+1}{4\pi} e^{-i0\alpha} d_{02}^l(\beta) e^{-i2\gamma}$$

$$= \sum_{lm} \frac{2l+1}{4\pi} e^{+i2\gamma} d_{20}^{l*}(-\beta) e^{+i0\alpha}$$

$$= \sum_{lm} \sqrt{\frac{2l+1}{4\pi}} Y_{l2}(\beta, \alpha) \quad ; \quad q \rightarrow e : \text{Radiating kernel.}$$

$$\sum_{lm} Y_{lm}(\theta_2, \phi_2) Y_{lm}^*(\theta_e, \phi_e) = A1 \quad (\text{Radiate})$$

$EB \rightarrow Q, U$

$$\sum \frac{2l+1}{4\pi} D_{-2m}^l(\phi_2, \theta_2, 0) D_{0m}^{l*}(\phi_e, \theta_e, 0)$$

$$\sum \frac{2l+1}{4\pi} D_{-2m}^l(\phi_2, \theta_2, 0) D_{m0}^l(0, -\theta_e, -\phi_e)$$

$$\sum \frac{2l+1}{4\pi} D_{-20}^l(\alpha_{eq}, \beta_{eq}, \gamma_{eq}) = \sum \frac{2l+1}{4\pi} D_{0-2}^{l*}(-\gamma_{eq}, -\beta_{eq}, -\alpha_{eq})$$

$$= \sum \sqrt{\frac{2l+1}{4\pi}} Y_{l-2}^*(-\beta_{eq}, -\gamma_{eq}) = \sum \sqrt{\frac{2l+1}{4\pi}} Y_{l2}^*(\beta_{eq}, \gamma_{eq})$$

$$= \sum \sqrt{\frac{2l+1}{4\pi}} Y_{l2}(\beta_{eq}, -\gamma_{eq})$$

$Q, U \rightarrow E, B$

$$\sum_{lm} Y_{lm}(\theta_e, \phi_e) Y_{lm}^*(\theta_2, \phi_2) = B1 \quad (\text{Radiate})$$

$$\sum_{lm} \frac{2l+1}{4\pi} D_{0m}^l(\phi_e, \theta_e, 0) D_{2m}^{l*}(\phi_2, \theta_2, 0)$$

$$\sum_{lm} \frac{2l+1}{4\pi} D_{0m}^l(\phi_e, \theta_e, 0) D_{m2}^l(0, -\theta_2, -\phi_2)$$

$$\sum_{lm} \frac{2l+1}{4\pi} D_{02}^l(\alpha_{qe}, \beta_{qe}, \gamma_{qe}) = \sum_{lm} \sqrt{\frac{2l+1}{4\pi}} Y_{l2}(\beta_{qe}, \alpha_{qe})$$

$$B1 = \sum_{lm} Y_{lm}^*(\theta_e, \phi_e) Y_{lm}(\theta_2, \phi_2) = B2 \quad (\text{Convolve})$$

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{0m}^{l*}(\phi_e, \theta_e, 0) D_{-2m}^l(\phi_2, \theta_2, 0)$$

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{-2m}^l(\phi_2, \theta_2, 0) D_{m0}^l(0, -\theta_e, -\phi_e)$$

$$= \sum_{lm} \frac{2l+1}{4\pi} D_{-20}^l(\alpha_{eq}, \beta_{eq}, \gamma_{eq}) = \sum \frac{2l+1}{4\pi} D_{0-2}^{l*}(-\gamma_{eq}, -\beta_{eq}, -\alpha_{eq})$$

$$= \sum \sqrt{\frac{2l+1}{4\pi}} Y_{l-2}^*(-\beta_{eq}, -\gamma_{eq}) = \sqrt{\frac{2l+1}{4\pi}} Y_{l2}(\beta_{eq}, -\gamma_{eq})$$

$$\begin{aligned}
 A) &= \sum_{lm} Y_{lm}(\theta_q, \phi_q) Y_{lm}^*(\theta_e, \phi_e) \quad EB \rightarrow QV \\
 &= \sum_{lm} Y_{lm}^*(\theta_q, \phi_q) Y_{lm}(\theta_e, \phi_e) \\
 &= \sum_{lm} \frac{2l+1}{4\pi} D_{2m}^{l*}(\phi_q, \theta_q, 0) D_{0m}^l(\phi_e, \theta_e, 0) \\
 &= \sum_{lm} \frac{2l+1}{4\pi} D_{0m}^l(\phi_e, \theta_e, 0) D_{m2}^l(0, -\theta_q, -\phi_q) \\
 &= \sum_{lm} \frac{2l+1}{4\pi} D_{02}^l(\alpha_{qe}, \beta_{qe}, \gamma_{qe}) \\
 &= \sum_{lm} \sqrt{\frac{2l+1}{4\pi}} Y_{l2}(\beta_{qe}, \alpha_{qe}) \rightarrow \text{convolution.}
 \end{aligned}$$