RRSDCE, Begusarai

Session (23–27) Course - CSE / DS , Semester - III

Subject - Mathematics III , Topic - 2nd order diff Eq^n.

Module - 5 , Last Date of Submission - 15/03/25

1) Check whether the following functions are linearly independent

(i) $1, \cos 2x, \sin 2x$ (ii) e^{x}, e^{2x}, e^{3x}

(iii) $e^{kx}, 100e^{kx}$ (iv) e^{x}, xe^{x}

(v) x, x^2, x^3 (vi) $\log_e x, (\log_e x)^2, (\log_e x)^3$

2) Find the general solution of the following differential / homogeneous differential Equations

(i) $2\frac{d^2y}{dx^3} - 3\frac{dy}{dx} + y = 0$

(ii) $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

(iii) $y'' + \lambda^2 y = 0$

(iv) $\frac{d^4y}{dx^3} = m^4 y$

(v) $(D^4 + 1)y = 0$

(vi) $\frac{d^5y}{dx^5} - 13\frac{d^3y}{dx^3} + 26\frac{d^2y}{dx^2} + 82\frac{dy}{dx} + 104y = 0$

(vii) $y'' + 4y' + 4y = 0$

(viii) $(D^2 - 1)^2 y = 0$

Solve,

3) $L\dfrac{d^2Q}{dt^2} + R\dfrac{dQ}{dt} + \dfrac{Q}{C} = 0$, given that

$Q = Q_0$ and $\dfrac{dQ}{dt} = 0$ at $t = 0$

& $CR^2 < 4L$.

4) Solve the following differential Equations

(i) $y''' + y = e^x + 2e^{-x}$

(ii) $y'' - 4y' + 3y = \cos x \cos 2x$

(iii) $y'' - 2y' + y = e^x + \cos x$

(iv) $y'' + y = \tan x$

(v) $y^{iv} - y = x$

(vi) $y'' - 2y' + y = x^2 + e^x + \cos x$

(vii) $(D^2 - 1)y = x^2 \cos x$

(vii) $(D^2 + 1)(D-1)^2 y = e^x$

(viii) $(D^3 + 4D^2 + 4D)y = 8e^{-2x}$

5) Solve by method of variation of parameter

(i) $y'' + a^2 y = \sec ax$

(ii) $y'' - 2y' - 3y = xe^x$

(iii) $y'' + y = \tan x$

(iv) $y''' + y = e^{x/2} \sin \dfrac{\sqrt{3}x}{2}$

(v) $(D+1)(D+2)(D+3)(D+4)y = x$

(vi) $y'' - 4y' + 4y = (x+1)e^{2x}$

(vii) $x^2\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 12y = 0$

6) Solve the following differential Equations

(i) $x^3 y''' + 2x^2 y'' + 2y = x + \frac{1}{x}$

(ii) $x^2 y'' - 5xy' + 3y = \ln x$

(iii) $(x+1)^2 y'' + (x+1) y' + y = 4\cos(\ln(1+x))$

(iv) $(2x+5)^2 y'' + 6(2x+5) y' + 8y = x$

(v) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$

7) Solve the following differential Equations

(i) $xy'' - (2x-1) y' + (x-1) y = 0$

(ii) $y'' - \cot x\, y' - (1-\cot x) y = e^x \sin x$

(iii) $y'' - 4xy' + (4x^2-1) y = -3e^{x^2} \sin 2x$

(iv) $(\cos x) y'' + \sin x\, y' - 2(\cos^3 x) y = 2\cos^5 x$

(v) $y'' - 2\tan x\, y' + y = 0$

(vi) $y'' + 2xy' + (x^2+1) y = 0$

(vii) $\cos^2 x\, y'' - 2\sin x \cos x\, y' + \cos^2 x\, y = 0$

8) Find the ordinary regular and singular points of the differential Equations

(i) $x^2 \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

(ii) $(1-x^2) y'' - 2xy' + n(n+1) y = 0$

(iii) $y'' - xy' + 2y = 0$

(iv) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (x-5) y = 0$

(v) $2x^2 \frac{d^2y}{dx^2} + 7x(x+1)\frac{dy}{dx} - 3y = 0$

9) Classify the singular points of the following differential Equations

(i) $x^2 y'' + \sin x \; y'' + \cos x \; y = 0$

(ii) $x^3 (x^2-1) y'' - x(x+1) y' - (x-1) y = 0$

(iii) $x^2 y'' + 2x y' + (x^2 - n^2) y = 0$

10) Find the power series solution of following differential Equation about $x=0$.

(i) $\frac{dy}{dx} + y = 0$

(ii) $\frac{d^2y}{dx^2} + y = 0$

(iii) $y'' + x y' + x^2 y = 0$

(iv) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$

(V) $(1-x^2) y'' - 2x y' + 2y = 0$

11) Find a series solution of $y'' - xy = 0$ about $x=0$ statisfying the initial condition $y(0) = 1, \quad y'(0) = 2$.

12) Find the power series solution about $x=2$ of the differential Equation

$$y'' + (x-1) y' + y = 0$$

13) Find the power series sol^n about $x = 1$ of differential Eq^n

$$x y' - y = 0.$$

14) Using Frobenius method, solve following differential Equation

(i) $(2x+x^3)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0$

(ii) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$

(iii) $x(x-2)\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$, About $x=2$

(iv) $2(1-x)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$, about $x=1$.

(v) $x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + (x^2+6)y = 0$

15) Express $P(x) = 4P_3(x) + P_2(x) + 2P_1(x) + 5P_0(x)$ as polynomial in 'x' where $P_m(x)$ is Legendre's polynomials.

16) Express $f(x) = x^4 + 2x^3 - 6x^2 + 5x - 3$ in the terms of Legendre's polynomials.

17) Establish orthogonal property for Legendre's polynomials or prove that

(i) $\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = 0$ if $m \neq n$

(ii) $\int_{-1}^{1} [P_n(x)]^2\,dx = \frac{2}{2n+1}$, if $m=n$.

18) State and prove Rodrigues Formula.

19) Find generating function for Legendre polynomials.

20) Show that

(i) $P_n(1) = 1$ (ii) $P_n(-x) = (-1)^n P_n(x)$

(iii) $P_n'(1) = \frac{n(n+1)}{2}$

(iv) $(2n+1) x P_n = (n+1) P_{n+1} + n P_{n-1}$

(v) $n P_n = (2n-1) x P_{n-1} - (n-1) P_{n-2}$

(vi) $n P_n = x P_n' - P_{n-1}'$

(vii) $(2n+1) P_n = P_{n+1}' - P_{n-1}'$

(viii) $(n+1) P_n = P_{n+1}' - x P_n'$

(ix) $(1-x^2) P_n' = n(P_{n-1} - x P_n)$

(x) $(1-x^2) P_n' = (n+1)(x P_n - P_{n+1})$

21) Show that $J_n(x)$ is an even function for 'n' is even and $J_n(x)$ is an odd function for 'n' is odd.

22) Show that

(i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

(iii) $J_3(x) = \left(\frac{8}{x^2} - 1\right) J_1(x) - \frac{4}{3} J_0(x)$

(iv) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{1}{x^2}(3 - x^2)\sin x - \frac{3}{x}\cos x \right]$

(v) $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{1}{x^2}(3 - x^2)\cos x + \frac{3}{x}\sin x \right]$

(vi) $\frac{d}{dx}\left[x J_n(x) J_{n+1}(x) \right] = x\left[J_n^2(x) - J_{n+1}^2(x) \right]$

(vii) $\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x) \right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$

(23) Prove that

$$J_0^2 + 2(J_1^2 + J_2^2 + \cdots) = 1$$

24) Prove that

$$J_0 J_1 + 3 J_1 J_2 + 5 J_2 J_3 + \cdots = \frac{x}{2}$$

25) Prove that

$$\int_0^\infty \frac{J_n(x)}{x}\, dx = \frac{1}{n}$$

——— x ——— x ———