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Branch – Data Science & CSE– Sem III

Subject: Mathematics III   Course Code - 105102   Assignment (Module-2 & 6)

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Module-2

Limit, Continuity & Differentiability for function of several variables

1) Evaluate: $\lim_{(x,y)\to(0,0)} \left[\frac{xy}{\sqrt{x^2+y^2}}\right]$.
2) Let $f(x,y) = \frac{xy}{x^2+y^2}$, show that the repeated limits exist at $(0,0)$ and are equal, but the double limit does not exist.
3) Find $\lim_{(x,y)\to(0,0)} \left(y\sin\frac{1}{x} + x\sin\frac{1}{y}\right)$.
4) Discuss the continuity of the
$$f(x,y) = \begin{cases} \frac{x-y}{x+y}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
5) Discuss the continuity of the
$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
6) Discuss the continuity of the following function at $(-1,c)$
$$f(x,y) = \begin{cases} \frac{x^2y}{|1+x|}, & x \neq -1 \\ y, & (x,y) = (-1,c) \end{cases}$$
7) Discuss the continuity of the following function at (0,0)
$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^3+y^3}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
8) Discuss the continuity of the following function $f(x,y)$ at point (0,0):
$$f(x,y) = \begin{cases} \frac{\sin\sqrt{|xy|}-\sqrt{|xy|}}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
9) Show that the function $f(x,y) = \sqrt{x^2+y^2}$ is not differentiable at (0,0).
10) For the function
$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
Show that $f_{xy}(0,0)$ and $f_{yx}(0,0)$ are equal or not.

11) For the function
$$f(x,y) = \begin{cases} \frac{xy(2x^2+3y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
check whether $f_{xy}(0,0)$ and $f_{yx}(0,0)$ are equal or not.

Partial Derivatives, Euler’s Formula, Total Derivatives & Change of variable

12) Let $w = x^3 - 3x^2y + y^2$. Compute the partial derivatives of w with respect to both variables x, y at the point $P_0(-2, 3)$.
13) Find the partial derivative of $w = \tan^{-1}\frac{y}{x}$, with respect to x and y.
14) Given that $w = x^2 + y^2 + z^2$, $x = e^{2t}\cos t$, $y = e^{2t}\sin t$ and $z = e^{2t}$, find $\frac{dw}{dt}$, using chain rule.
15) Given that $w = f(u,v)$, $u = x + y$ and $v = x - y$, show that $\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \left(\frac{\partial f}{\partial u}\right)^2 - \left(\frac{\partial f}{\partial v}\right)^2$.
16) If $z(x + y) = x^2 + y^2$ show that
$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$
17) If $u = f\left(\frac{y}{x}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.
18) State and prove Euler’s theorem.
19) If $u = \log\frac{x^2+y^2}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.
20) If $u = \sqrt{x^2 + y^2 + z^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2}{u}$.
21) If $\frac{1}{u} = \sqrt{x^2 + y^2 + z^2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
22) If $w = \sin^{-1}u$, $u = \frac{x^2+y^2+z^2}{x+y+z}$, then find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$.
23) If $u = xy\, f\left(\frac{y}{x}\right) + yz\, g(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u$.
24) If $u = f(x,y)$ is a homogeneous function of degree n, then prove that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$
25) If $u = \frac{x^2y^2}{x^2+y^2}$, then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$.
26) If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
27) Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar co-ordinates.

Maxima, Minima of two variables

28) Find the maximum and minima of the function $x^3 + y^3 - 3x - 12y + 10$.
29) Investigate whether $f(x,y) = x^2 + 2y^4 - 3xy^2$ has an extreme point at (0,0).
30) Find all the stationary points of the function
$$u = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$
Also examine whether they are maxima or minima at $(4,0)$ and $(6,0)$.
31) Find the maximum and minimum values of $x^3 + y^3 - 3axy$.
32) Examine the extreme values of the function $\sin x + \sin y + \sin(x + y)$.

Method of Lagrange Multipliers

33) Find the extreme value of $f(x,y,z) = xyz$, when $x + y + z = a,\ a > 0$.
34) Find the minimum value of $f(x,y,z) = x^2 + y^2 + z^2$ subject to condition $xyz = a^3$.
35) Use Lagrange method to determine the minimum distance from the origin to the plane $3x + 2y + z = 12$.
36) Find the maximum value of $\sin A\ \sin B \sin C$ for a ΔABC.

37) Find the extreme values of
$$f(x,y,z) = 2x + 3y + z$$
such that $x^2 + y^2 = 5$ and $x + z = 1$.
38) Find the maximum value of $f(x,y,z) = xyz$ under the constraints $x^2 + z^2 = 1$ and $y - x = 0$.

Taylor's and Maclaurin's Theorem with remainders of several variables

39) State Taylor's theorem of two variables with remainder.
40) Expand e^{xy} at $(0,0)$.
41) Expand e^{xy} at $(1,1)$.
42) Obtain the Taylor's series approximation to the function
$$f(x,y) = xy^2 + y\ \sin(x+y)$$
about the point $(1,1)$.
43) Show that $e^y \log(1+x) = x + xy - \frac{x^2}{2}$.

Module 6

Q.1 Form the partial differential equation by eliminating the arbitrary constants from following

i) $z = ax + by + a^2 + b^2$
ii) $z = (x-a)^2 + (y-b)^2$
iii) $z = (x^2 + a)(y^2 + b)$
iv) $z = ax + a^2y^2 + b$

Q.2 Form the partial differential equation by eliminating arbitrary functional form from following

i) $z = f(x^2 - y^2)$
ii) $z = (x+y)f(x^2 - y^2)$
iii) $z = f(x + at) + g(x - at)$
iv) $f(x + y + z, x^2 + y^2 + z^2) = 0$

Q3. Find complete integral of the following:

i) $z = e^p$
ii) $p^2 - q^2 = 4$
iii) $pq = p + q$
iv) $px + qy + pq = 0$
v) $x^2p^2 + y^2q^2 = z$

Q4. Find complete integral and singular solution of the following partial differential equation

i) $z = px + qy + p^2 + q^2$

ii) $z = px + qy + \frac{p}{q}$

iii) $z = px + qy + p^2 - q^2$

iv) $z = px + qy + p^2q^2$

Q5. Find the complete integral of the following partial differential equation

i) $p^2 = zq$

ii) $p + q = \frac{z}{a}$

iii) $z = p^2 + q^2$

iv) $zpq = p + q$

v) $z = pq$

Q6. Find the complete integral of the following PDE using Lagrange's method

i) $\sin(x+y)p + \cos(x+y)q = z$

ii) $p \tan x + q \tan y = \tan z$

iii) $x(y-z)p + (z-x)q = x - y$

iv) $(y+z)p + (z+x)q = x + y$

v) $(y-z)p + (z-x)q = x - y$

vi) $x^2(y-z)p + (z-x)q = x - y$

vii) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

viii) $p + q = x + y + z$

ix) $(mz - ny)p + (nx - lz)q = ly - mx$

x) $yzp + zxq = xy$

Q7. Find the complete integral of the following using Charpit's method

i) $2xz + q^2 = x(px + qy)$

ii) $(p^2 + q^2)y = zq$

iii) $pxy + pq + qy = yz$

iv) $z = px + qy + p^2 + q^2$

v) $(p+q)(xp + yq) = 1$

vi) $p^2 + q^2 - 2px - 2qy + 1 = 0$

vii) $(p+q)(px + qy) = 1$

viii) $x^2p + y^2q = (x+y)z$