Measuring the Impact of Shot Sequencing on First Mover Advantage in Penalty Shootouts *

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Abstract

Lorem ipsum.

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1 Introduction

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About 26.5% of football games are tied at the end of the normative 90 minutes. When a winner is required—typically in elimination tournament format – such games will go on to normative 30 additional minutes. If a winner is still not determined, the match will go to a penalty shootout. Here the teams will take alternating penalties, i.e., first team A then team B and so it goes. This sequence is coined ABAB. Team A used to be determined by the outcome of a coin flip, while since 2003 a coin flip determines which team's captain will get to choose the sequence.

Penalty shootouts is however a process that receives some controversy. Criticism include that it only reflects a narrow part of the game—and hence that the criteria does not reflect the rich game, and that it seems to be to a large extent a game of chance—i.e., that it does not really differentiate much even within the narrow slice of the game. On the positive aspects is the spectacular drama—with shooter vs. goalie in person-to-person duels, it is an easy to understand format, and a winner is indeed determined in a reasonable amount of time.

This paper focus on another criticism—namely that team A tends to get an advantage, claimed by some to be an *unfair* aspect of the penalty shootout. This advantage is demonstrated to be 60.5% in 269 matches by AER. Using an extended dataset of 540 matches, MS finds that team A wins in 53.3% of the cases, while with an even larger dataset of 1,001 matches, BOOK finds the team A advantage to be 60.6%.

Wether or not such an advantage is unfair is a question of definition. LITERATURE ON FAIRNESS. The outcome of a football match is determined by factors that roughly can separated into two groups. One is factors internal to the game, such as a kick on the ball, a defender covering space and a shot hitting the goal post. The other is factors external to the game, or *deus ex machina*, such as one team playing against the sun in first half and the second half being played after sunset. In general it is desirable that the outcome of a match is determined by internal factors, and that the impact of external factors is minimized. While a coin flip is probabilistically fair—it is even called a *fair coin*—it is a factor external to the game. Alternative sequences may reduce the impact of the coin flip but potentially at the cost of complexity of less structure in the sequence as ABAB arguable is one of the simplest possible.

IFAB is the governing body of the football rules. To make a major change in the rules requires at least six of the total eight votes, which consist of FIFA's four votes and one vote for each of the four British football associations. IFAB is currently considering alternative sequences for the penalty shootout, and there has been recent experiments with ABBA in XYZ tournaments.

The main objective of this paper is to investigate the impact of alternative sequences of penalty shootouts. A a dataset of X shootouts of shot-by-shot outcomes enables us to do a detailed analysis of the impact of sequence and model alternative sequences.

TO INCLUDE IN THE INTRO:

Example of penalty shootouts in soccer. Evidence in previous work.

This paper:

- Reevaluate first mover advantage in shootouts, expanding the dataset of APH and Kocher Lenz Sutter.
- Analyze shot-by-shot to: (1) understand the underlying mechanisms driving the first mover advantage. (2) use the estimates to evaluate alternative sequences of shots in order to reduce first-mover advantage.

2 Data and Shootout level analysis

This section describes the data collection scheme and provides the empirical estimates of the first-mover advantage, comparing to the results of previous work.

2.1 Data collection

? showed that the data collection process can influence the magnitude and statistical significance of the effect of first-mover advantage. For this reason, the analysis in this work was divided into two data collection schemes. The first dataset is comprised of the same competitions used in **?** and **?**. These data collection starts with the dataset used in **?** (a superset of **?**) and expands using multiple data sources.[1] We refer to this dataset as the APH competitions data. A second data collection scheme incorporates additional adult male competitions for which we could obtain a reasonable number of shootouts (10 shootouts our more during 1970-2017. This dataset is refered as the extended competitions data., and the additional competitions are listed in the Appendix Table 12. The data was collected at two levels of granularity. At a more aggregate level, shootout-level data can be used to test whether the starting team wins more frequently. For this purpose, the sample of shootouts requires (at least) information about the tournament of the match, the team that took the first shot and the outcome of the shootout (which team won). Specific details about the name of the team, the date of the match, the championship round and the final score are not required to conduct the test but can be useful to include as control variables. This level of aggregation has been used to test for a first-mover advantage (**?**, **?**[2]).

More detailed data include the outcome of every shot taken in the shootout. This information can be useful to understand further details of the mechanisms driving the first mover advantage, analyzing how the shooting performance of player is affected by the order of shooting, goal difference between the teams, shooting round, among other factors. This detailed information is available for a subset of the shootouts included in the final sample. **?** also used shot-by-shot information to compare the shooting performance of the starting and second team across rounds. We significantly expanded the number of shootouts to conducts this analysis and also construct new measures that help to explain the underlying mechanism driving the first-mover advantage.

[1] We thank the authors of **?** for providing their dataset.

[2]

<<DESCRIPTION ON HOW THE ONLINE DATA WAS COLLECTED GOES HERE. INDICATE WHAT WAS DONE FOR THE AER COMPETITIONS AS WELL AS THE EXTENDED SET OF COMPETITIONS WE CONSIDER ** ADITYA, PLEASE COMPLETE **>> Most of the data was scraped from worldfootball.net. This was done by going through the schedule page [3] of all competitions where the scores of matches that ended in a shootout are indicated by appending ?pso? to the final score. For the competitions in the data used by **?**, even though the source has the date, score and participating teams for all shootouts, it does not have sequence information for some of them. For each competition, we have sequence data on a higher proportion of shootouts when we look at more recent years <ADD LINK TO TABLE>. The data collected spans a period from September 1970 to June 2017.

The data available through online sources under-represents the shootouts for the Spanish League. This is relevant because **?** shows this tournament has the highest proportion of starting team winners. To collect more data on that tournament, four sources of newspapers were revised. In total, 249 additional matches were collected, for which 186 also had information on the order of the shots. << ADITYA, PLEASE COMPLETE THIS INFO WITH THE ANALYSIS YOU DID>>

Table 1 reports a summary of the shootouts for each of the APH competitions, describing the number of shootouts found where the starting team information was available. The table also reports fraction of matches won by the starting team and the p-value of a one-sided binomial test of the null $H_0 : p = 0.5$ against the one-sided alternative $H_1 : p > 0.5$. The one-sided test is more appropriate in this case, since the test is to identify an advantage of the starting team.

Overall, the data suggests an advantage for the starting team of about 10% (55% vs 45%), that is, 22% more likely to win. The proportion of wins of the starting team is highly statistically significant above 50% at the 0.1% level for the overall sample. This proportion is similar for international and league teams.

The total is sample size is 970, compared to the 540 matches studied in **?**. In most of their analysis, **?** and **?** restrict their sample up to 2003 in order to have a clean natural experiment. Before 2003, the winner of the toin-coss had to start the shootout, but after 2003 the winner could *choose* to go first or second. **?** reports that in most cases the winner of the toss chooses to go first. Although the post-2003 is not a perfect natural experiment, it seems reasonable to use this period to increase the statistical power of the tests, which is considered in part of the analysis reported in **?**, as we do.

<<**ADITYA, NILS: please include here your analysis on the sample selection.

**NOTE MO: Our sample covers until 2016? 2017?. Kocher studies until 2003, they cover 540 matches out of a universe of 709. What is the universe of matches in our case? That is: how many matches with shootouts did we not obtain outcome data because info was not available? How many additional could we get prior to 2003? We should separate the table pre and post 2003. After we do that, we should do a comparison of the pre-2003

[3] `http://www.worldfootball.net/all_matches/wm-2014-in-brasilien`

competition	n	Mean	p-value
League teams			
Champions League	33	60.61%	0.0814
Champions League Qual.	18	66.67%	0.0481
Copa del Rey	256	60.94%	0.0002
Cup Winners Cup	29	58.62%	0.1325
DFB-Pokal	206	50.00%	0.4722
Europa League	90	52.22%	0.2992
Europa League Qual.	42	52.38%	0.3220
FA Community Shield	6	50.00%	0.3438
FA Cup	37	43.24%	0.7443
League Cup	142	58.45%	0.0178
National teams			
Africa Cup	23	52.17%	0.3388
Asian Cup	12	50.00%	0.3872
Copa América	20	65.00%	0.0577
EURO	18	44.44%	0.5927
Gold Cup	12	66.67%	0.0730
World Cup	26	57.69%	0.1635
League Teams	859	55.76%	0.0003
National teams	111	55.86%	0.0918
Total	970	55.77%	0.0001

Table 1: Summary statistics of shootouts for APH competitions, showing the percent of matches where the starting team wins the shootout.

sample with Kocher.>>.

? reevaluates the first-mover advantage using the same tournaments as **?**, expanding the dataset from 156 shootouts to 1001, covering matches up to 2012. << *** NOTE MO: Do a comparison of the samples. On national teams, seems like we have similar composition ***.>>. The data in **?** shows that 60.6% percent of the matches were won by the starting team, similar to the previous result obtained by **?** which uses only pre-2003 matches. The first-mover advantage in **?** seems to be substantially higher that the one obtained with our sample (55.7%), which is in between their estimate and the 53.3% reported in **?**.

In addition to the simple t-tests reported in Table 1, a linear probability was used to test for the first-mover advantage including dummy variables of year and competition as control variables. The results are summarized in Table 2, corroborating the findings of the summary statistics and suggesting a 10% advantage for the starting team (55% vs. 45%). If we restrict the sample to matches before 2003 (reported in the second column), the effect of starting team is also statistically significant and similar in magnitude.

The estimation was replicated using the extended competition dataset, increasing the number of shootouts to

	(1)	(2)
	WinH	WinHpre2003
FirstShot	0.117***	0.165**
	(0.0330)	(0.0499)
Observations	970	438

Robust standard errors. Includes dummies for competition and year

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: Estimation results for Linear Probability Model, APH competitions

1698. The results are similar, suggesting a 12.5% advantage of the starting team. When restricting the sample to matches before 2003, the effect appears to be larger, above 20%. << ** ADITYA, NILS: in the pre2003 shootouts, the sample increased only from 438 to 534, whereas for the whole period it increased from 970 to 1698. Clearly the new data is mostly for post 2003. It would be useful to have a more detailed description of the competitions that were added pre2003 and perhaps explain why the first mover advantage is so large for these ** >>

Overall, the additional data considered in this study suggests a winning advantage of the first-shooting team of 55% vs. 45% for the second team. This estimate is statistically significant but smaller than the results reported in **?** and **?**; our result is closer to the estimate of **?**.

3 Shot-by-shot empirical analysis

The previous section uses a shootout as the main unit of analysis. The analyses revealed a significant and robust effect of first-mover advantage in the shootouts, which settles part of the controversy reported in previous studies. An additional objective of this work is to conduct a counterfactual analysis to study how a change in the sequence would affect this first mover advantage; this requires a more detailed understanding of the underlying mechanisms driving this advantage. To do so, we use outcomes of individual shots in penalty shootouts to analyze how the performance of shooters is affected by different conditions in which the shots are taken, including the round and partial result of the shootout.

3.1 First-mover advantage by shooting round

For a subset of the shootouts, the outcome of each shot, y_{ijt}, is recorded for match i by team j in round t, where a value equal to one indicates that the shot is scored. Teams are indexed by $j = \{1, 2\}$, where $j = 1$ is the starting team in the shootout. The rounds $t = \{1 \ldots 5\}$ may determine the winner if the accumulated number of goals $Y_{ij5} = \sum_{t=1}^{5} y_{ijt}$ is strictly greater for one of the two teams; otherwise rounds $t \geq 6$ are completed until one of the teams gets a goal advantage at the end of the round.

First, we verify the prevalence of the first mover advantage in this shot-level analysis. A binary variable is

Table 3: Summary statistics of shot performance for first and second shooter, by round. APH competitions data.

Round	first	second	Difference	t-stat	p-value
1	75.7%	74.9%	0.9%	0.43	0.335
	907	907			
2	75.1%	74.3%	0.8%	0.37	0.355
	907	907			
3	74.8%	70.1%	4.6%	2.23	0.013
	907	907			
4	74.3%	70.9%	3.4%	1.59	0.056
	894	817			
5	75.0%	69.3%	5.7%	2.19	0.014
	692	485			
6+	73.4%	70.5%	2.9%	1.10	0.136
	545	545			
Total	74.8%	72.0%	2.8%	3.09	0.001
	4852	4568			

included in the model indicating if the team started first, $ShotFirst$, which is equal to one for team $j = 1$. The overall scoring performance is 73.4%. Table (4) show summary statistics of the fraction of scored shots for the first and second shooter, on each of the first 5 rounds and beyond that (6+). The dataset include 905 shootouts with sequence. Some of these shootouts end before the 5th round; for example, with a score of 3-0 after the second shot in the third round the shootout ends. Therefore, the number of shots decreases for later rounds, and some of them stop with the first shot (so that the second team did not take the shot).

The table reveals that on each round, the shooting performance is higher for the first shot. The difference between first and second shot becomes larger after the second round, which is mostly explained by a reduction in the performance of the second team, dropping from 75% to 70% (approximately). The t-statistic for test of equal proportions is included along with the corresponding p-value of the one-sided t-test. The difference in shooting performance is statistically significant at the 5% level for the third and fifth round and for the overall sample. These results are in line with those found in **?**, who also find a higher difference in shooting performance for the later rounds in the shootout.

A multivariate analysis is conducted to further validate the results. The binary variable y_{ijt} , the outcome of each shot, can be modeled in a generalized linear model of the form:

$$h(\Pr(y_{ijt} = 1)) = \beta X_{ijt} \tag{1}$$

where $h(\cdot)$ is a link function (linear, logistic, probit, etc.). The following analysis uses a linear link function, so that (1) becomes a linear probability model. The following covariates are included in the model:

- Binary variable indicating if the shot is the first on round t.
- Indicator variables for each competition.
- Indicator variables for the calendar year to account for potential trends.
- Indicator variable for each sequence round t

Note that Table 3 reveals an important sample selection problem: shots in later rounds are censored from the sample when the difference in scoring performance between the two teams is large. In this case, second moving teams with lower performance will have fewer observed shots after the third round, and therefore the estimation tends to oversample shots from first moving teams with good performance. This may bias the results towards not finding a first-mover advantage, because the shots in later rounds are more likely to be observed when the difference in scoring performance between the first and second team is smaller: the sample over-represents shootouts where teams were more even.

Two solutions are used to correct for this sample selection problem. The first is to introduce team indicator variables (fixed effects). From the national and club teams in the sample, 54% have repeated shootouts. With team fixed effects, the first mover advantage is identified from these sub-sample of teams.[4] This control is useful to correct for the sample selection problem when team performance is relatively stable. However, our sample covers a long time period and team performance may vary significantly, in which case team fixed effects are not enough to control for time-varying shocks to a team performance. This is corrected by adding a random effect to the model, capturing shocks to team performance in a specific shootout.

With team fixed-effects and team-shootout random effects, the linear probability model is represented through the linear regression:

$$y_{ijt} = \beta X_{ijt} + \phi_j + \xi_{ij} + \varepsilon_{ijt} \tag{2}$$

where ϕ_j is a team fixed-effect and ξ_{ij} is a random effect that captures the performance of team j in match i. Essentially, ξ_{ij} captures a serial correlation in the shot performance for focal team during the shootout. A low ξ_{ij} indicates that the team has lower performance than average and is more likely to miss shots during this shootout.

[4] We found no difference in the shooting performance between the teams with more than one shootout and those with one shootout (72.0% vs. 71.9%, p-value .94). The correlation between the scoring performance and the number of shootouts observed in the sample is very low 0.01.

The shootout is more likely to finish before team j shoots the first five rounds, but including ξ_{ij} in the model accounts for this sample selection. Note that it is not possible to include fixed effect for ξ_{ij}, as it would absorb the first mover advantage, precluding the estimation of the main factor of interest.

Column (1) of Table 4shows the results with random effects but no team fixed effects. Column (2) is a specification with team fixed effects but no random effect and column (3) includes bothteam fixed effects and team-shootout fixed effects (specified in equation (2)). All three specifications show a highly statistically significant effect of the first shooter. Given that the relevance of controlling for selection effects, all the remaining specifications estimated (columns (4)-(6)) include team fixed effects and the random effects.

An additional hypothesis to test is whether the advantage of the first shooter increases as the rounds progress. Column (4) includes an interaction term between shooting first and the round of the shot. The main effect is small and not statistically significant, but the interaction effect is positive and significant. The interpretation is that in the first round the first mover effect is rather small, but it becomes larger as the round progresses. The more flexible specification reported in column (5) interacts the first shooter with five different levels of the round (first round is the base). The main effect of the first-shot indicator is not statistically significant. The interaction effects become larger on the 3rd and subsequent rounds, and is statistically significant in round 5. It appears that as the shootout advances, the difference in shooting performance between the first and second team increases, favoring the first team. Recall that depending on the initial shot performance, the shootout may end early, on the 3rd or 4th round, so it is plausible that the pressure effects starts operating after that round. An additional specification was estimated, including an interaction between the first-shot indicator and the binary indicator $1[j \geq 3]$ (equals one on round 3 onwards). This specification, reported in column (7), shows an insignificant main effect but a significant interaction term. Altogether, these results suggest that the first mover advantage exists and becomes stronger as the rounds of the shootout progress, becoming relevant on the 3rd and subsequent rounds.

Similar models were estimated with the extended sample, reported in the Appendix Table 13. The results are strikingly similar in terms of the magnitude of the coefficients and statistical significance, albeit with higher precision due to the larger sample size. Specifically, the results from the regression that interacts the first-shot indicator with the five round indicators (column (6)) shows that the first mover advantage is not significant in the first two rounds, but increases in magnitude and becomes statistically significant from the third round onwards. This corroborates the presence of a first mover advantage on the later rounds of the shootout (but not in the two initial rounds).

The regression models with the BPH sample reported in Table (4) were compared based on statistical tests of fit. An F-test is used to compare the model in column (7) which is nested within the unrestricted model reported in column (6); the test favors the more parsimonious model (column (7), with p-value 0.27 to reject the unrestricted model). The model reported in column (4) is nested within (7), and the F-test favor the latter,

Table 4: Shot-by-shot analysis to validate the first-mover advantage, based on APH competitions.

	(1) BASEre	(2) TeamFE	(3) TeamFEre	(4) InterLin	(5) InterLev	(6) InterBin
FirstShot	0.0310***	0.0402***	0.0427***	0.00464	0.0168	0.0162
	(0.00942)	(0.0116)	(0.0111)	(0.0217)	(0.0217)	(0.0162)
first × Round				0.0121*		
				(0.00597)		
first × Round=1					0	
					(.)	
first × Round=2					-0.00110	
					(0.0290)	
first × Round=3					0.0375	
					(0.0298)	
first × Round=4					0.0383	
					(0.0307)	
first × Round=5					0.0693*	
					(0.0348)	
first × Round=6					0.0333	
					(0.0359)	
first × After2nd						0.0439*
						(0.0197)
Observations	9420	9420	9420	9420	9420	9420

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

rejecting the null model (p-value 0.031). These statistical tests suggest that the preferred model to capture the first mover advantage is column (7), which corroborates that the first mover advantage is significant and stronger on the third and following rounds.

Additional specifications without team effects were estimated, reported in the Appendix. Overall, the magnitude and direction of the effects are similar, but the precision of the estimates are lower and some of them are not statistically significant. Standard errors decrease because: (1) there is no systematic differences in the probability of starting first across teams; (2) team fixed effects explain some of the variation in the shooting performance, so that the variance of the error term decreases.

3.2 Shot importance as a mediating factor of the first-mover advantage

The previous analysis shows that the first-mover advantage arises from a difference in performance between the first and second shooting team in a round, primarily from the third round and beyond. One possible driver of this advantage is the higher pressure faced by the second player: since the scoring probability is higher than 50%, the second shooter is more likely to shoot to catch up with an unfavorable score, which could presumably exert additional psychological pressure affecting his performance.

Exploring this mechanism requires a measure that captures the importance of a shot – that is, a measure that captures the potential consequences of scoring or missing a shot in the final outcome of the shootout. Specifically, we define the importance of a shot as the change in probability of winning from scoring (versus missing the shot), denoted $\Delta Pwin$.

To compute $\Delta Pwin$ for each shot, the possible outcomes in the shootout are captured by a set of states defined by the team taking the shot (indexed by k=1,2 for the first and second team that kicks on each round), the round (t), and the partial score difference s, defined as the number of penalties scored by the first team minus the scored penalties of the second team. Define $q(k, s, t)$ as the probability that team k scores in round t when the score difference before taking the shot is s. In addition, define $w(k, s, t)$ as the probability that team $k = 1$ wins the shootout starting from state (k, s, t).

Figure 1 shows the different states and the corresponding winning probabilities and transition probabilities across states. Note that the network of states includes ending nodes with $w(k, s, t)$ equals one or zero, corresponding to the first or second team winning. Specifically, the ending nodes are

$$w\left(1,3,4\right)=w\left(2,3,4\right)=w\left(1,2,5\right)=w\left(2,2,5\right)=w\left(1,1,6\right)=1,$$

and

$$w\left(1,-3,4\right)=w\left(2,-2,4\right)=w\left(1,-2,5\right)=w\left(2,-1,5\right)=w\left(1,-1,6\right)=0.$$

There is one ending node with winning probability $w\left(1,0,6\right)$ different from zero and one, which represents the start of the decisive rounds (beyond round 5). Its winning probability can be computed by the recursion

$$w\left(1,0,6\right)=q\left(1,0,6\right)\left(1-q\left(2,1,6\right)\right)+\left(1-q\left(1,0,6\right)\left(1-q\left(2,1,6\right)\right)-\left(1-q\left(1,0,6\right)\right)q\left(2,0,6\right)\right)w\left(1,0,6\right).$$

Solving for $w\left(i,s,t\right)$ gives

$$w\left(1,0,6\right)=\frac{q\left(1,0,6\right)\left(1-q\left(2,1,6\right)\right)}{q\left(1,0,6\right)\left(1-q\left(2,1,6\right)\right)+\left(1-q\left(1,0,6\right)\right)q\left(2,0,6\right)}.$$

The recursion equations for the remaining nodes of the network in Figure 1 are given by

$$w\left(k,s,t\right)=q\left(k,s,t\right)w\left(3-k,s+3-2k,t+k-1\right)+\left(q\left(k,s,t\right)\right)w\left(3-k,s,t+k-1\right).$$

The importance of the shot taken in state (k, s, t) is represented in the change in winning probability of team k calculated as:

$$\Delta Pwin(k,s,t)=\begin{cases} w(2,s+1,t)-w(2,s,t) & \text{if team } k=1 \\ -[w(1,s-1,t+1)-w(1,s,t+1)], & \text{if team } k=2 \end{cases}$$

(recall that the winning probability of the second team is $1-w(2,s,t)$, hence the minus sign for the second case).

The main challenge in this calculation is evaluating the probability of scoring for each state, $q(k, s, t)$. A simple approach is to set this probability as constant and independent of the state, equal to the average scoring probability in the overall sample (73.4%). Another approach is to estimate this probability based on the actual transitions rates observed in the data, as described in the Appendix XYXY. << TO DO: calculate DeltaPwin using the scoring probabilities calculated in the previous section and check how much it changes. My hunch is that they are very similar. ADITYA, NILS: If we are including this, you have to calculate DeltaPwin using these transition probabilities and provide them in a similar format.>>

Alternative measures of shot importance were also considered. In some states, missing the shot implies that the team loses or wins with probability one. The covariate $EndIfMiss$ indicates states where scoring lead to a terminal state where the shooting team loses; the covariate $EndIfScore$ indicates shots where the shooting team wins if it scores. It is plausible that this factor externs pressure on the shooter. Note however that this effect applies only to states on the outskirt of the network of Figure 1, leading to an ending node.

Table 5 shows summary statistics of these covariates for the shootouts in the sample. The statistics show that the consequences of missing the shot tend to be bigger for later rounds: $\Delta Pwin$ increases (on average) as the rounds progress. Moreover, this increase in $\Delta Pwin$ is larger for the second team after round 3. A similar pattern is observed for the frequency of shots in which missing implies losing the game: in the fourth round, in 20% of the shots of the second team lead to a loss if missed, compared to only 6% for the first shot in the round. The difference in the fifth round is 57% for the second shot vs. 29% for the first. From rounds 3 onwards, the consequences of the second shot in every round seem to be higher, plausibly exerting more pressure on the shooter.

Several specifications were estimated to measure the effect of $\Delta Pwin$, $EndIfMiss$ and $EndIfScore$ on shooting performance. The specifications are described in Table 6. All the specifications include dummy variables for round number, competition and team fixed effects.

Table 5: Summary statistics of $\Delta Pwin$ and $EndIfMiss$

	$\Delta Pwin$			$EndIfMiss$			$EndIfScore$		
Round	first	second	Diff	first	second	Diff	first	second	Diff
1	27.4	27.4	0.0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	27.6	27.3	-0.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
3	27.4	27.4	0.0	0.0%	3.5%	3.5%	0.0%	0.8%	0.8%
4	28.2	30.7	2.6	6.1%	20.0%	13.9%	8.0%	11.3%	3.3%
5	35.6	50.0	14.4	29.1%	57.1%	28.0%	27.9%	42.9%	15.0%
6	50.0	50.0	0.0	0.0%	73.2%	73.2%	0.0%	26.8%	26.8%

Table 6: Estimation results including proxies of pressure that account for the consequences of the shot (for APH competitions)

	(1)	(2)	(3)
	BASE2	EndMiss2	MissScore2
FirstShot	0.0162	0.0130	0.0137
	(0.0162)	(0.0169)	(0.0171)
first × After2nd	0.0439*	0.0401	0.0311
	(0.0197)	(0.0215)	(0.0204)
EndIfMiss		0.00897	
		(0.0215)	
EndIfScore		0.0242	
		(0.0223)	
$\Delta Pwin$			-0.00187**
			(0.000591)
Observations	9420	8823	8823

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Column (1) is the base specification identical to the last column of table (4), chosen as the preferred model to describe the first shooter advantage. This base model suggests that the first shooter advantage is significant on the 3rd and following rounds, but statistically insignificant in the first two rounds. Specification (2) includes the covariates $EndIfMiss$ and $EndIfScore$ to capture shooter pressure. Both covariates have coefficients that are not statistically different from zero. Note that the effect of first mover advantage (the interaction term first×After2nd) is smaller and with smaller statistical significance (dropped from 0.05 to 0.1, not reported). The specification shown in column (3) includes $\Delta Pwin$ as a measure of pressure – its coefficient is negative and highly statistically significant. A 10% increase in this probability implies a 1.87% decrease in the probability of scoring. Moreover, the first mover advantage is not statistically different from zero at any reasonable significance level. Note that Table 5 shows that at round 5 the average difference in $\Delta Pwin$ between the first and second team is about 14.4%, which implies an overall effect of $14.4\% \times 0.187\% = 2.7\%$. This is about half the difference in shooting performance reported in Table 3 in the 5th round. Hence, the results suggests that the measure of pressure $\Delta Pwin$ mediates a significant portion of the first-mover advantage, accounting for about half of the difference in scoring performance between first and second shooter. Moreover, the remaining first shooter advantage after controlling for pressure is not statistically different from zero after we account for this effect.

The same analysis was repeated with the extended sample of competitions, reported in Appendix Table 15. The results are similar in their direction and magnitude, and the statistical significance of several of the coefficients increase due to the higher precision in the estimates. The $EndIfMiss$ coefficient is positive and becomes statistically significant in the specification of column (2). An important change in the estimates is that the interaction of first shot and after 2nd round indicator becomes significant in Column (3) (this is because the estimate is more precise), suggesting that the effect of the first mover advantage is not completely mediated by the the pressure measure $\Delta Pwin$.

Overall, the analysis of individual shots corroborates the presence of a first-mover advantage. The results suggests that: (1) the difference between the first and second shooter on every round becomes more pronounced on the 3rd round onwards; (2) an important portion of this advantage is explained through the higher pressure that the second shooter faces from the consequences of missing a shot, which is much higher for the second shooter from the third round onwards. These results are useful to simulate the impact of changing the shooting sequence, which is analyzed in the next section.

4 Evaluating alternative shooting sequences

Although most of previous work has focused in analyzing data at the shootout level, the analysis on the previous section shows that studying shooter's behavior at the shot level is useful to uncover the mechanisms that drive the first mover advantage. Doing so is critical to analyze counterfactual scenarios of changing the shot sequence, in order to evaluate the impact of these changes in the outcomes of the shootout.

4.1 Modeling round transitions

Although the decision on which team starts shooting is determined by a “fair coin”, the current mechanism if viewed as unfair because it introduces an external factor (the luck of the coin) that affects the outcome of shootout. Hence, the objective is to devise a mechanism that minimizes this impact of these external factor so that when two identical teams are confronted they have similar chances of winning, regardless on how the order of the shots was determined.

The empirical results suggest that the current shooting scheme makes the second team face a higher pressure overall, because the impact on the outcome of, in particular, later shots has a larger effect on the probability of winning the match. This effect can potentially be reduced by alternative sequencing of the teams. On this regard, **?** proposes two alternative schemes:

- ABBA: The team who takes the first shot goes second on even rounds and first on odd rounds. The resulting sequence is ABBAABBA.
- Thue-Morse sequence: this sequence takes an alternating pattern, so that if the two beginning shots where AB, the next would take the opposite order (or mirror image) BA, so the initial four shots are similar to ABBA. However, the next shots take the mirror image as the last four, BAAB, changing the pattern to ABBABAAB. The next eight shots would be the mirror image of these, BAABABBA, and so on. The logic behind the Thue-Morse sequencing scheme is that if a particular order in a subsequence of shots favors one player, then taking the mirror image in the following shots compensates by giving that advantage to the other player.

Analyzing the performance of these alternating sequences requires measuring how the shot performance is affected when the order of shots is altered. Given that the sequencing of shootouts have always operated with the same scheme, observational data is limited to study empirically the effect of order sequencing on shot performance. An alternative option is to run a controlled laboratory experiment to study how shot performance is affected under different shot sequences: **?** report such experiments comparing the current scheme, ABBA and Thue-Morse and report that the latter two reduce the first mover advantage significantly. However, given that the driving mechanism appears to be pressure, it is difficult to replicate the actual pressure faced by players in a high-stakes match into the lab. The best alternative would be to run a field experiment – in fact, FIFA tested the ABBA scheme for the Youth Worldcup in 2017 <<FIND ARTICLE >>, but the small sample sizes precludes taking conclusions on the impact of the effect.

4.2 A network model to analyze outcomes of penalty shootouts

The approach taken here is to use part of the empirical results from the shot-by-shot analysis to simulate the outcome of a shootout under alternative sequencing schemes. In doing so, the following assumptions are made regarding the relationship between shot performance and the shot sequences. First, the scoring probability of a penalty depends only on the round number t and the outstanding score difference (before the shot is taken). Specifically, the score probability is: (i) independent from the path during period $1 \dots t-1$ at which the score difference was reached; (ii) independent on the forthcoming sequence of shots for $t+1$. Second, the order in which a team chooses the order of its shooters is not affected by the sequencing scheme. This latter assumption is required in order to extrapolate the observed performance of shooters under the current scheme into the simulation of alternative sequences. For example, Table 1 shows that the first round has a higher scoring performance that the other rounds, which could be attributed to better players shooting in the first round. The assumption is that the choice of the player quality is independent of the sequencing.

Based on these assumptions, the evolution of a penalty shootout can be modeled as a Markovian system, in a similar way as represented by Figure 1, represented by a binomial network with two possible outcomes coming out of each node. For the purpose of analyzing alternative sequences, the states and the network can be simplified as follows. The state is defined by the round t and the goal difference s calculated as the partial score of the starting team minus the second team (in contrast to the network represented in Figure 1, the team kicking the shot is surpressed). The transitions occur round by round, with three possible transitions out of each node: the goal difference can increase by one (first team scores and the second misses), decrease by one (first team misses and the second scores), or remain constant (both teams miss or score). This means that transitions are in the form of trinomial distributions where the change in score difference from round to round is denoted by $c \in \{-1, 0, 1\}$. Let $p_c(s,t)$ be the probability that the score difference at the start of round $t+1$ is $s+c$ given that the score difference at the start of round t was s. The trinomial network is represented in Figure 2. Then team A's ability to increase the score difference in its favor in round t when already being ahead by s is $p_1(s,t)$, and the counterpart of team B is $p_{-1}(-s,t)$, and so on. It follows that the relevant arcs for comparison are the pairs that form a mirror image around the center of Figure 2.

<<** NILS :

-PLEASE REMOVE THE "A" SUPERSCRIPT FROM THE p's IN THE FIGURE

- WHAT ARE THE q's IN THE FIGURE? WE PREVIOUSLY DEFINED THEM AS SCORING PROBABILITIES, SO SEEMS INCONSISTENT NOTATION HERE. I SUGGEST CHANGING THE NOTATION IN THE FIGURE USING $v(s,t)$ AS THE CONDITIONAL WINNING PROBABILITY **>>

The first teams ability ability to increase the score difference in its favor in round t when already being ahead by s is $p_1(s,t)$, and the counterpart for the second team is $p_{-1}(-s,t)$. It follows that the relevant arcs for comparison are the pairs that form a mirror image around the center of Figure 2. This representation of the network is sufficient to evaluate the impact of alternative sequences, as we show next.

At each node in the network, $v\left(s,t\right)$ represents the probability that the starting team will win the shootout out of state (s,t) at the beginning of round t. All except one ending node of Figure 2, namely $v\left(0,6\right)$, have values 0 or 1, i.e.,

$$v\left(3,4\right)=v\left(2,5\right)=v\left(1,1,6\right)=1,$$

and

$$v\left(-3,4\right)=v\left(-2,5\right)=v\left(-1,6\right)=0.$$

We find $v\left(0,6\right)$ by recursion

$$v\left(0,6\right)=p_{1}\left(0,6\right)+p_{0}\left(0,6\right)v\left(0,6\right).$$

Solving for $v\left(s,t\right)$ gives

$$v\left(0,6\right)=\frac{p_{1}\left(0,6\right)}{1-p_{0}\left(0,6\right)}.$$

The recursion equations for the remaining nodes with two outgoing arcs of the network in Figure 2 are given by:[5] ** NILS: I WILL LET YOU DECIDE ON THE NOTATION, BUT I SUGGEST CHOOSING ONE**

$$v\left(s,t\right)=\left(1-p_{c}\left(s,t\right)\right)v\left(s,t+1\right)+p_{c}\left(s,t\right)v\left(s+c,t+1\right)\text{ for }t\in\{4,5\},|s|=6-t,c=-sign(s)\cdot 1,\quad(3)$$

and for the nodes with three outgoing arcs the recursion equations are

$$v\left(s,t\right)=p_{1}\left(s,t\right)v\left(s+1,t+1\right)+p_{0}\left(s,t\right)v\left(s,t+1\right)+p_{-1}\left(s,t\right)v\left(s-1,t+1\right)\text{ for }s\in\{-2,-1,0,1,2\},t\in\{|s|+1,\ldots\quad(4)$$

4.3 Estimating transition probabilities

The evolution of the shootout shown in Figure **??** requires the transition probabilities $p_c(t,z)$ as primitives. Two approaches are used to estimate these primitives. The first approach is to gather all shots in the sample and calculate the state transitions as sample averages from the observed transitions. This approach is described in further detail in the Appendix.

**** STOPPED HERE ****

The second approach is parametric and considers that transitions are generated as the outcome of two shots. The transition probabilities $p_c(s,t)$ characterize the distribution of the change in the goal difference, $c=\{-1,0,1\}$. This underlying r.v. is the difference of the score of the two shots in the round, $c=Y_1-Y_2$, where Y_1 equals one when the first shot is scored, zero otherwise (similar for the second shot Y_2). Hence, the distribution of c is

[5] Can also use $c=-s/|s|$ here.

uniquely determined by the distribution of Y_1 and Y_2. Following the notation used in section 3.2, the distribution of these two r.v.s are modeled as:

- $q(1, s, t)$, the scoring probability of the first shot in state (s, t).
- $q_2(2, s + Y_1, t)$, the scoring probability of the second shot in state (s, t) when the first shot outcome is Y_1. In some terminal nodes the game ends before the second shot of the round is taken. The probability q_2 is undefined for those states.

The transition probabilities for the network are given by:

$$\begin{aligned} p_1(s,t) &= q_1(s,t) \times (1 - q_2(s+1,t)) \\ p_{-1}(s,t) &= q_2(s,t) \times (1 - q_1(s,t)) \\ p_0(s,t) &= 1 - p_1(s,t) - p_{-1}(s,t) \end{aligned}$$

The shot scoring probabilities q_1 and q_2 are estimate with a logistic regression that includes dummy variables for round interacted with dummies for goal difference and dummies for first/second shooter. An additional dummy variable is included in the second shot of each round indicating if the first shot was score or miss (Y_1). This is a saturated model that provides sufficient flexibility to accomodate different patterns of the transition probabilities. As additional controls, the model includes dummy variables for the type of competition.

4.4 Statistical tests to compare trinomial transitions

<<** NOTE MO: I DON'T UNDERSTAND WHAT THIS ANALYSIS IS ADDING TO THE MAIN OBJETIVE OF THE PAPER. ADITYA AND NILS , PLEASE WRITE THE MOTIVATION AND THE CONCLUSIONS FROM THIS SECTION**>>

Let $\bar{p}$ denote the expected change. Specifically, for $s \geq 0$,

$$\bar{p}_c(s,t) = p_1(s,t) - p_{-1}(s,t),$$

and

$$\bar{p}_{-c}(-s,t) = p_{-1}(-s,t) - p_1(-s,t).$$

We here to statistical (one sided) tests using bootstrapping with 100,000 samples of the dataset.

AER competitions:

<<** SOME OF THE TESTS GIVE DIFFERENT RESULTS DEPENDING ON HOW $p_c(s,t)$ WAS CALCULATED. WHICH IS THE CORRECT ESTIMATOR TO USE?? YOU CAN USE SIMULATION TO EVALUATE THE PERFORMANCE OF THE TWO ESTIMATORS AND CHOOSE ONE**>>

test	t	$\lvert s\rvert$	$\bar{p}_c\,(s,t)$	$\bar{p}_{-c}\,(-s,t)$	p-value	$\bar{\rho}_c$	$-\bar{\rho}_{-c}$	p-value	
one trinomial	1	0	0.0074		0.3645	0.0074		0.3605	
	2	0	0.0001		0.5116	-0.0077		0.6298	
	3	0	0.0272		0.1859	0.0508		0.0325*	
	4	0	0.0373		0.1480	0.0145		0.3218	
	5	0	0.1100		0.0022**	0.1092		0.0009***	
	6	0	0.0270		0.1706	0.0270		0.1666	
		0	0.0348		0.0024**	0.0335		0.0018**	
two trinomials	2	1	0.1002	0.0861	0.4190	0.0959	0.0530	0.2547	
	3	1	0.0632	-0.0538	0.0270*	0.0167	-0.0269	0.2161	
	4	1	0.0221	-0.0140	0.2842	0.0535	-0.0181	0.1038	
	3	2	-0.0230	-0.0766	0.3460	0.0054	-0.1664	0.0769.	
		1	0.0618	0.0061	0.0657.	0.0554	0.0027	0.0650.	
two binomials	4	2	-0.0564	-0.1694	0.0204*	-0.0725	-0.1881	0.0030**	
	5	1	-0.1600	-0.1748	0.3456	-0.1609	-0.1768	0.3007	

Some comments about these tests: It seems like the comparison of two trinomials is not as sharp as it could be as it does not utilize the ordinal nature of the transitions. <NR: ref. earlier discussions and notes. Could one possibility be to do some kind of likelihood test?>

For the comparison of two binomials, the expected value of transition is misleading in this table. <NR: should we do this in separate table?> The expected values are the value of the arc going towards the centre of the network.

<< ADITYA, NILS: SOME CONCLUSIONS NEED TO BE ADDED HERE >>

4.5 Calculation of winning probabilities for alternative shot sequences

The structure of the trinomial network (Figure 2) and the estimated parameters can be used to analyze the outcome of alternative sequence schemes, specifically ABBA and Thue-Morse (TM). For notation, Team A refers to the team that takes the first shot in the first round, and Team B the second shot.

A sequence scheme is defined by the vector $\gamma = (\gamma_1, \gamma_2, \gamma_3, \ldots)$ that define the team that take the first shot in every round t. Define $\gamma_t = 1$ when team A goes first in round t (by definition, $\gamma_1 = 1$) and $\gamma_t = -1$ if team B goes first in round t. The evolution of the penalty shootout under sequence scheme γ is defined by adjusting the transition probabilites of the trinomial network so that $\tilde{p}_c\,(s,t) = p_{\gamma_t c}\,(\gamma_t s, t)$.

To calculate the conditional winning probabilities $v(s,t)$ for the TM sequence, we compute $v^{TM}\,(0,6)$ by doing the recursion until convergence << *** THE LAST STATEMENT IS NOT CLEAR **>>, while for ABBA we get the probability that team A will win the shootout, conditioned of it being extended past the initial 5 rounds, by the recursion:

<<** SHOULD ALL THE p's BELOW HAVE TILDE?**>>

$$v^{ABBA}(0,6) = p_{-1}(0,6) + p_0(0,6)\left(p_1(0,6) + p_0(0,6)\, v^{ABBA}(0,6)\right).$$

Solving for $v^{ABBA}(0,6)$ gives

$$\begin{aligned} v^{ABBA}(0,6) &= \frac{p_{-1}(0,6) + p_0(0,6)\, p_1(0,6)}{1 - p_0^2(0,6)} \\ &= \frac{(1 - p_0(0,6) - p_1(0,6)) + p_0(0,6)\, p_1(0,6)}{(1 - p_0(0,6))(1 + p_0(0,6))} \\ &= \frac{1 - p_1(0,6)}{1 + p_0(0,6)}. \end{aligned}$$

All the remaining transitions can be calculated using equations (3) and (4) replacing $p_c(s,t)$ by the adjusted transition $\tilde{p}_c(s,t)$.

4.6 Hypothesis testing

The main objective of this analysis is to evaluate whether alternative sequencing schemes can help in reducing the gap in the winning probability between the team that take the first shot and other team. Based on Figure 2, denote by $W^{ABAB} = v_1^{ABAB}(0)$, $W^{ABBA} = v_1^{ABBA}(0)$ and $W^{TM} = v_1^{TM}(0)$ the winning probabilities of the starting team under the current, ABBA and Thue-Morse sequencing schemes, respectively. We seek to compare these winning probabilities and test whether the alternative schemes ABBA and Thue-Morse help to reduce the first mover advantage. Specifically, the following pairwise hypothesis tests are evaluated:

- Test 1: $H_0:\ W^{ABAB} = W^{ABBA}$ versus the alternative $H_A: W^{curr} > W^{ABBA}$
- Test 2: $H_0:\ W^{ABAB} = W^{TM}$ versus the alternative $H_A: W^{curr} > W^{TM}$
- Test 3: $H_0:\ W^{TM} = W^{ABBA}$ versus the alternative $H_A: W^{TM} < W^{ABBA}$

Rejecting the Null implies that one of the schemes reduces the first mover advantage. In addition, it is of interest to test if a first mover advantage prevails under each of the alternative sequence schemes:

- Test 4: $H_0:\ W^{ABAB} = 0.5$ versus the alternative $H_A: W^{ABAB} > 0.5$
- Test 5: $H_0:\ W^{ABBA} = 0.5$ versus the alternative $H_A: W^{ABBA} > 0.5$
- Test 6: $H_0:\ W^{TM} = 0.5$ versus the alternative $H_A: W^{TM} > 0.5$

Failing to reject the Null implies that the sequence scheme was effective in eliminating the first mover advantage.

These hypothesis test can be conducted with a point estimate of W and its corresponding confidence interval. A valid point estimate fo can be obtained by first estimating the transition probabilities $p_x(t, z)$ using one of the methods described in Section 4.3. Using these estimates, compute the estimate of W via recursion, using the corresponding sequencing schemes (using the recursion for each sequence scheme described in the previous section). The challenge is to construct a confidence interval for this point estimate. Although the confidence intervals of the transition probabilities $p_x(t, z)$ can be computed through standard methods (i.e. asymptotic confidence intervals), the estimators $\hat{W}$ are a convoluted function of the transition probabilities and therefore its standard error and asymptotic distribution is difficult to compute.

Bootstrapping provides a feasible approach to conduct these hypothesis tests. The main idea of bootstrapping is to compute the distribution of the estimators $\hat{p}$ and $\hat{W}$ by simulating the sampling process behind the estimation. The sampling process is a follows: (1) sample n *shootouts* from a population of matches; (2) using the sample of shootouts and its corresponding shots round-by-round, estimate the transition probabilities $\hat{p}_x(t, z)$ through some method (parametric or non-parametric). (3) Compute $\hat{W}$ using the estimates $\hat{p}_c(t, z)$ through recursion (using the corresponding sequence scheme).

The following algorith is used to construct the confidence interval of the estimator $\hat{W}$:

(a) Take a re-sample of size n (with replacement) from the original sample of matches (some matches could get repeated). Denote the shootouts in this resample S_r (the subscript indexes the resample)

(b) Using the resample S_r, estimate $\hat{p}^{(r)}$ (for all states (s, t) and $c \in \{-1, 0, 1\}$).

(c) Compute the estimator $\hat{W}^{(r)}$ using the estimated $p^{\hat{(r)}}$, for each sequencing schemes.

(d) Store the estimates of the current run, $\hat{W}^{(r)} = (\hat{W}^{curr,(r)}, \hat{W}^{ABBA,(r)}, \hat{W}^{TM,(r)})$. Repeat to take a new resample $s + 1$.

(e) Iterate for many (e.g. $R = 1000$) resamples.

The point estimates $\{\hat{p}^{(s)}\}_s$ and $\{\hat{W}^{(s)}\}_s$ provide an empirical distribution of the estimators $\hat{p}_x(t, z)$ and $(\hat{W}^{curr}, \hat{W}^{ABBA}, \hat{W}$
Two estimators are constructed, the first using the non-parametric estimates of the trinomial transitions (estimates reported in Table 7)and the second using the parametric estimates (reported in Table 8). In general, both estimators provide similar results. As expected, the current sequencing ABAB gives the largest point estimate, slightly above 54%, similar to the first mover advantage estimated in section 2. The 99% confidence interval is [50.7, 58.32]), which does not contain the Null value 50%, therefore Test 4 is rejected at the 1% significance level. These results are consistent with those obtained with the shootout level analysis described in section 2, both in terms of magnitude and statistical significance of the first mover advantange in the ABAB sequence scheme.

sequence	average	0.01	0.05	0.50	0.95	0.99
ABAB	0.5456	0.5070	0.5188	0.5456	0.5725	0.5832
ABBA	0.5250	0.4908	0.5010	0.5250	0.5491	0.5591
T-M	0.4805	0.4465	0.4564	0.4805	0.5045	0.5142

Table 7: Bootstrap distribution of the estimator of winning probabilities of the starting team, under three different possible shot sequences, using **non-parametric** estimator of the trinomial transition probabilities $p_c(s,t)$.

** NILS, ADITYA: Please add the additional percentiles to this table so that it looks similar to the previous one **

Sequence	mean	p1	p5
Current	0.5472	0.5172	0.5260
ABBA	0.5050	0.4998	0.5013
Thue-Morse	0.4972	0.4919	0.4939

Table 8: Bootstrap distribution of the estimator of winning probabilities of the starting team, under three different possible shot sequences, using **parametric** estimator of the trinomial transition probabilities $p_c(s,t)$.

For the ABBA scheme, both estimators give smaller point estimates of the winning probability, close to 50%. The 99% confidence interval contains 50%, therefore Test 5 cannot be rejected. However, it is rejected at the 5% significance level, providing some evidence that ABBA does not fully eliminate the first mover advantage. For Thue-Morse, the point estimate is even smaller, and both the 99% and 95% confidence interval also contains 50%, failing to reject Test 6. Overall, the estimates predict that for ABBA and Thue-Morse the first mover advantage is smaller, and in Thue-Morse both teams have equal probabilities of winning .

To conduct the pairwise Test 1, the statistic $t^{(r)} = \hat{W}^{curr,(r)} - \hat{W}^{ABBA,(r)}$ is computed for each bootstrapped sample r. The Null hypothesis $W^{ABAB} = W^{ABBA}$ is rejected at the α significance level if the α bottom percentile of the sample $t^{(r)}$ is greater than zero (one-sided test). A similar test can be conducted for Test 2 and 3. Table 9 report the results for these tests. The difference in winning probability of ABAB and ABBA is positive and statistically significant at the 1% level. Similar results is obtained for the difference between ABAB and Thue-Morse sequences. Overall, Test 1 and 2 suggest that the probability of winning of the starting team is significantly (statistically) greater under the current sequence scheme relative to ABBA or Thue-Morse. The difference is in the order of 4-5% reduction for the alternative schemes. In contrast, the results suggest that ABBA and Thue-Morse have about the same winning probability for the starting team, with a difference in probability less than 0.8% (ABBA is slightly greater). The 99% confidence interval of the difference contains the Null value zero, indicating that the Null of equal winning probabilities cannot be rejected at that significance level; it is though rejected at the 5% significance level. In any case, the difference appears to be much smaller relative to the difference with respect to the current scheme.

** NILS, ADITYA: Please add similar calculations of the Table below using the non-parametric estimates **

Overall, the counterfactual simulations analyzed in this section suggest that the first mover advantage can be significantly reduced under the alternative sequencing of shoots studied, both ABBA and Thue-Morse. The

		mean	p1	p5	p95	p99
Test 1	ABAB- ABBA	0.0422	0.0154	0.0234	0.0612	0.0682
Test 2	ABAB - TM	0.0499	0.0168	0.0276	0.0718	0.0826
Test 3	ABBA - TM	0.0077	-0.0017	0.0011	0.0149	0.0183

Table 9: Boostrap distribution of the differences in winning probabilities between alternative sequence mechanisms, based on parametric estimators of $p_c(s,t)$

advantage of Thue-Morse over ABBA appears to be very small. Given the simplicity of ABBA, which is also used in other sports such as tie-break in Tennis, it seems to be a preferable sequence scheme to reduce the effect of external factors on the outcome of the shootout.

5 Conclusions

The main conclusions so far:

- We find a first mover advantage in penalty shootouts, based on an expanded dataset including the same competitions as previous work and also expanding to other competitions.
- We conduct a detailed analysis shot-by-shot to uncover the underlying mechanism that drives the first mover advantage. We construct a measure of pyschological pressure that explains most of the first mover advantage.
- We conduct simulations, calibrated with the observed data, to estimate the effect of alternative sequence schemes on the winning probability of the starting team. These simulations show that using a simple scheme such as ABBA reduces significantly the winning probability of the first team.
- More complicated sequence schemes such as Thue-Morse, have marginal impact in reducing the first mover advantage relative to ABBA, but in the simulations it appears as fully eliminating the first mover advantage.

A Estimation of scoring probabilities using state transitions

Let $n_x = \sum_{i=1}^{n} \mathbf{1}_{x_j}$ be a counter function of the number of shootouts that for shootout i satisfy the logical test x_i. Let $(k, s_1, s_2, t)_i$ be true if and only if shootout i travels the path involving that team k's shot makes the score difference change from s_1 to s_2 in round t. Then $n_{k,s,s,t}$ denote the number of shots taken by team k in round t that is missed (i.e., it goes from score difference s to the same score difference s), and let $n_{k,s,s+3-2k,t}$ denote the shots taken by team i in round t that is scored (i.e., it goes from score difference s and to score difference $s+1$ if team $k=1$ and to score difference $s-1$ if team $k=2$).

** NOTATION BELOW NEEDS TO BE CHANGED. Index j -> i , index i -> k **

Let $q(i,s,t)$ be the probability that team i score in round t from a score difference of s. From the data, this is estimated by the relevant proportion

$$\hat{q}(i,s,t) = \frac{n_{i,s,s+3-2i,t}}{n_{i,s,s,t} + n_{i,s,s+3-2i,t}}, t = 1, \ldots, 5.$$

For $t = 6, \ldots$ we assume that the scoring probabilities are independent of the round and us 6 as the round number, which gives

$$\begin{aligned} \hat{q}(1,0,6) &= \frac{n_{1,0,1,t}}{n_{1,0,0,t} + n_{1,0,1,t}}, \\ \hat{q}(2,s,6) &= \frac{n_{2,s,s-1,t}}{n_{2,s,s,t} + n_{2,s,s-1,t}}, t = 6, \ldots. \end{aligned}$$

When dropping an argument of a function, it represents the overall value across all values of this argument. For example, $q(i,s)$ would mean the scoring probability of team i when the score difference is s taken across all rounds t.

<< TO BE COMPLETED BY NILS AND ADITYA>>

B Estimation of transition probabilities for the Trinomial Network

When estimating p, there are two alternatives. The one that may come first to mind is using only data from shootouts that have score difference s at the start of round t, which gives

$$
\begin{aligned}
\hat{p}_1(s,t) &= \frac{n_{1,s,s+1,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{1,s,s+1,t\wedge 2,s+1,s+1,t}}{n_{2,s+1,s+1,t}+n_{2,s+1,s,t}}, \\
\hat{p}_0(s,t) &= \frac{n_{1,s,s+1,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{1,s,s+1,t\wedge 2,s+1,s,t}}{n_{2,s+1,s+1,t}+n_{2,s+1,s,t}} + \frac{n_{1,s,s,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{1,s,s,t\wedge 2,s,s,t}}{n_{2,s,s,t}+n_{2,s,s-1,t}}, \\
\hat{p}_{-1}(s,t) &= \frac{n_{1,s,s,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{1,s,s,t\wedge 2,s,s-1,t}}{n_{2,s,s,t}+n_{2,s,s-1,t}}.
\end{aligned}
$$

<<** NOTE MO: In the text, given that the results are exactly the same using both methods, I decided to remove the results of ρ to simplify the reading. Below is the discussion and proposition supporting this.>>
But by second thought, based on the markovian argument, notice that for $t = 2, 3, 4, 5$ there is a second branch leading into the state $2, s+1, t$, namely $n_{1,s+1,s+1,t}$. And, similarly, there is a second branch leading into state $2, s, t$, namely $n_{1,s-1,s,t}$. This gives the following estimates of the trinomial probabilities (we use ρ here instead of p for these estimates):

$$
\begin{aligned}
\hat{\rho}_1(s,t) &= \frac{n_{1,s,s+1,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{2,s+1,s+1,t}}{n_{2,s+1,s+1,t}+n_{2,s+1,s,t}}, \\
\hat{\rho}_0(s,t) &= \frac{n_{1,s,s+1,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{2,s+1,s,t}}{n_{2,s+1,s+1,t}+n_{2,s+1,s,t}} + \frac{n_{1,s,s,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{2,s,s,t}}{n_{2,s,s,t}+n_{2,s,s-1,t}}, \\
\hat{\rho}_{-1}(s,t) &= \frac{n_{1,s,s,t}}{n_{1,s,s,t}+n_{1,s,s+1,t}} \cdot \frac{n_{2,s,s-1,t}}{n_{2,s,s,t}+n_{2,s,s-1,t}}.
\end{aligned}
$$

Out initial motivation of also considering ρ as an alternative way to estimate the trinomial transition probabilities was to check if the results of the recursions (and alternative sequences) were robust in terms of the two alternatives. But as the following proposition establish, for the recursions used to calculate the conditional winning probabilities $v(s,t)$, the two alternative estimations actually gives the same results.

Proposition 1. *The two estimators of $v(s,t)$, the conditional winning probability of the first team in round t with goal difference s, that result from using the estimators $\hat{p}$ and $\hat{\rho}$ of the transition probabilities $p_c(s,t)$ yield identical estimates.*

Proof: We will prove this result using forward recursion. Let $m_p(s,t)$ and $m_\rho(s,t)$ be the number of shootouts that visits state (s,t) based on the p and ρ transition probabilities, respectively. The starting node is given the total number of shootouts $m_p(0,1) = m_\rho(0,1) = J$. Since the expressions are the same, we state the recursions only based on the p transition probabilities. For nodes with only one incoming arc,

$$m_p(s,t) = p_c(s-c,t-1)\, m_p(s-c,t-1).$$

For nodes with two incoming arcs:

$$m_p(s,t) = p_c(s-c,t-1)\, m_p(s-c,t-1) + p_0(s,t-1)\, m_p(s,t-1).$$

For nodes with three incoming arcs:

$$m_p(s,t) = p_1(s-1,t-1)\, m_p(s-1,t-1) + p_0(s,t-1)\, m_p(s,t-1) + p_{-1}(s+1,t-1)\, m_p(s+1,t-1).$$

For proving the proposition, it is sufficient to establish that the number of shootouts flowing through each of the states are the same for both sets of trinomial transitions, i.e., $m_p(s,t) = m_\rho(s,t)$ for all states (s,t).
First note that for the transition probabilities in round 1 and in rounds 6+ are equal, i.e., $p_c(0,1) = \rho_c(0,1)$ and $p_c(0,6) = \rho_c(0,6)$.
We then consider the transition from round t to $t+1$. For notational convenience, we drop the round indicator t for the n-notation and also define $n^0_{2,s,\cdot} = n_{1,s,s\wedge 2,s,\cdot}$ and $n^+_{2,s,\cdot} = n_{1,s-1,s\wedge 2,s,\cdot}$. Also let $n_{1,s}$ be the number of shootouts that started the round (dropping t) at score difference s.
For trinomial nodes with only one incoming arc this is obvious. We will first show that the results hold for a node with two incoming arcs. We have

$$m_p(s,t+1) = m_p(s,t)\left[\frac{n_{1,s,s+1}}{n_{1,s}} \cdot \frac{n_{2,s+1,s}}{n_{1,s,s+1}} + \frac{n_{1,s,s}}{n_{1,s}} \cdot \frac{n^0_{2,s,s}}{n_{1,s,s}}\right] + m_p(s-1,t)\,\frac{n_{1,s-1,s}}{n_{1,s-1}} \cdot \frac{n^+_{2,s,s}}{n_{1,s-1,s}},$$

and

$$\begin{aligned} m_\rho(s,t+1) &= m_\rho(s,t)\left[\frac{n_{1,s,s+1}}{n_{1,s}} \cdot \frac{n_{2,s+1,s}}{n_{1,s,s+1}} + \frac{n_{1,s,s}}{n_{1,s}} \cdot \frac{n^0_{2,s,s} + n^+_{2,s,s}}{n_{1,s,s} + n_{1,s-1,s}}\right] \\ &\quad + m_\rho(s-1,t)\,\frac{n_{1,s-1,s}}{n_{1,s-1}} \cdot \frac{n^0_{2,s,s} + n^+_{2,s,s}}{n_{1,s,s} + n_{1,s-1,s}}. \end{aligned}$$

Assume that $m_p(s,t) = m_\rho(s,t) = n_{1,s}$ and $m_p(s-1,t) = m_\rho(s-1,t) = n_{1,s-1}$. Then it would follow that $m_p(s,t+1) = m_\rho(s,t+1) = n_{2,s+1,s} + n^0_{2,s,s} + n^+_{2,s,s}$. For three incoming arcs:

$$\begin{aligned} m_p(s,t+1) &= m_p(s+1,t)\,\frac{n_{1,s+1,s+1}}{n_{1,s+1}} \cdot \frac{n^0_{2,s+1,s}}{n_{1,s+1,s+1}} + m_p(s,t)\left[\frac{n_{1,s,s+1}}{n_{1,s}} \cdot \frac{n^+_{2,s+1,s}}{n_{1,s,s+1}} + \frac{n_{1,s,s}}{n_{1,s}} \cdot \frac{n^0_{2,s,s}}{n_{1,s,s}}\right] \\ &\quad + m_p(s-1,t)\,\frac{n_{1,s-1,s}}{n_{1,s-1}} \cdot \frac{n^+_{2,s,s}}{n_{1,s-1,s}}, \end{aligned}$$

Table 10: Additional specifications for the shot-by-shot analysis, without including team Fixed Effects

Table 11: Estimation results of goal difference without Team Fixed Effects

and

$$\begin{aligned} m_{\rho}(s,t+1) &= m_{\rho}(s+1,t)\frac{n_{1,s+1,s+1}}{n_{1,s+1}}\cdot\frac{n_{2,s+1,s}^{0}+n_{2,s+1,s}^{+}}{n_{1,s+1,s+1}} \\ &+m_{\rho}(s,t)\left[\frac{n_{1,s,s+1}}{n_{1,s}}\cdot\frac{n_{2,s+1,s}^{0}}{n_{1,s,s+1}+n_{2,s+1,s}^{+}}n_{1,s+1,s+1}+n_{1,s,s+1}+\frac{n_{1,s,s}}{n_{1,s}}\cdot\frac{n_{2,s,s}^{0}+n_{2,s,s}^{+}}{n_{1,s,s}+n_{1,s-1,s}}\right] \\ &+m_{\rho}(s-1,t)\frac{n_{1,s-1,s}}{n_{1,s-1}}\cdot\frac{n_{2,s,s}^{0}+n_{2,s,s}^{+}}{n_{1,s,s}+n_{1,s-1,s}}. \end{aligned}$$

Assume that $m_p(s+1,t) = m_{\rho}(s+1,t) = n_{1,s+1}$, $m_p(s,t) = m_{\rho}(s,t) = n_{1,s}$ and $m_p(s-1,t) = m_{\rho}(s-1,t) = n_{1,s-1}$. Then it would follow that $m_p(s,t+1) = m_{\rho}(s,t+1) = n_{2,s+1,s}^{0} + n_{2,s+1,s}^{+} + n_{2,s,s}^{0} + n_{2,s,s}^{+}$.

For $t = 2$ the result follows directly. By induction, it follows for the rest of the network. □

Given this result, the estimator $\hat{p}_c(s,t)$ is used as the non-parametric estimator of the transition probabilities.

C Additional specifications

C.1 Expanded set of competitions

1	Friendlies	21	Svenska Cupen	41	Campionato	61	Pokal Slovenije
2	Coupe de France	22	A-League	42	Steypakappingin	62	AFC Champions League
3	Kubok	23	Primera División	43	Kup Crne Gore	63	Copa MX
4	Coppa Italia	24	Puchar Polski	44	Playoff	64	Gold Cup Quali.
5	ÖFB-Cup	25	Coupe	45	Football League Trophy	65	Major League Soccer
6	Taça	26	Cupa Romaniei	46	Supercup	66	FAI Cup
7	Copa Argentina	27	Welsh Cup	47	Irish Cup	67	Club World Cup
8	Cup	28	Taça da Liga	48	Challenge Cup	68	Lengjubikarinn
9	Pohár FAČR	29	CAF Champions League	49	FFA Cup	69	Superkubok
10	Coupe de la Ligue	30	Nogometni Kup	50	Primera A		
11	Copa Libertadores	31	Bikar	51	UI-Cup		
12	Kypello Elladas	32	Suomen Cup	52	Cupa Moldova		
13	KNVB beker	33	Copa Conmebol	53	AFC Cup		
14	Cupen	34	Kup BiH	54	Liigacup		
15	Türkiye Kupasi	35	Kupa	55	Super Cup		
16	Slovensky Pohar	36	Eesti Karikas	56	Copa do Brasil		
17	Kup Srbije	37	Magyar Kupa	57	Emperor's Cup		
18	Beker van België	38	Kup na Makedonija	58	Relegation		
19	Copa Sudamericana	39	State Cup	59	David Kipiani Cup		
20	DBU Pokalen	40	Africa Cup Qual.	60	Latvijas kauss		

Table 12: Additional competitions in the extended dataset

	(1) WinH	(2) WinHpre2003
FirstShot	0.125***	0.211***
	(0.0253)	(0.0447)
Observations	1698	534

Robust standard errors. Includes dummies for competition and year

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: Estimation results for Linear Probability Model using extended set of competitions

Table 14: Shot-by-shot analysis to validate the first-mover advantage using extended set of competitions.

	(1)	(2)	(3)	(4)	(5)	(6)
	BASEre	TeamFE	TeamFEre	InterLin	InterLev	InterBin
FirstShot	0.0274***	0.0309***	0.0328***	-0.00964	-0.00428	0.00463
	(0.00694)	(0.00881)	(0.00833)	(0.0160)	(0.0161)	(0.0120)
first × Round				0.0135**		
				(0.00441)		
first × Round=1					0	
					(.)	
first × Round=2					0.0177	
					(0.0215)	
first × Round=3					0.0489*	
					(0.0223)	
first × Round=4					0.0549*	
					(0.0226)	
first × Round=5					0.0686**	
					(0.0259)	
first × Round=6					0.0540*	
					(0.0263)	
first × After2nd						0.0465**
						(0.0145)
Observations	17202	17202	17202	17202	17202	17202

Standard errors in parentheses

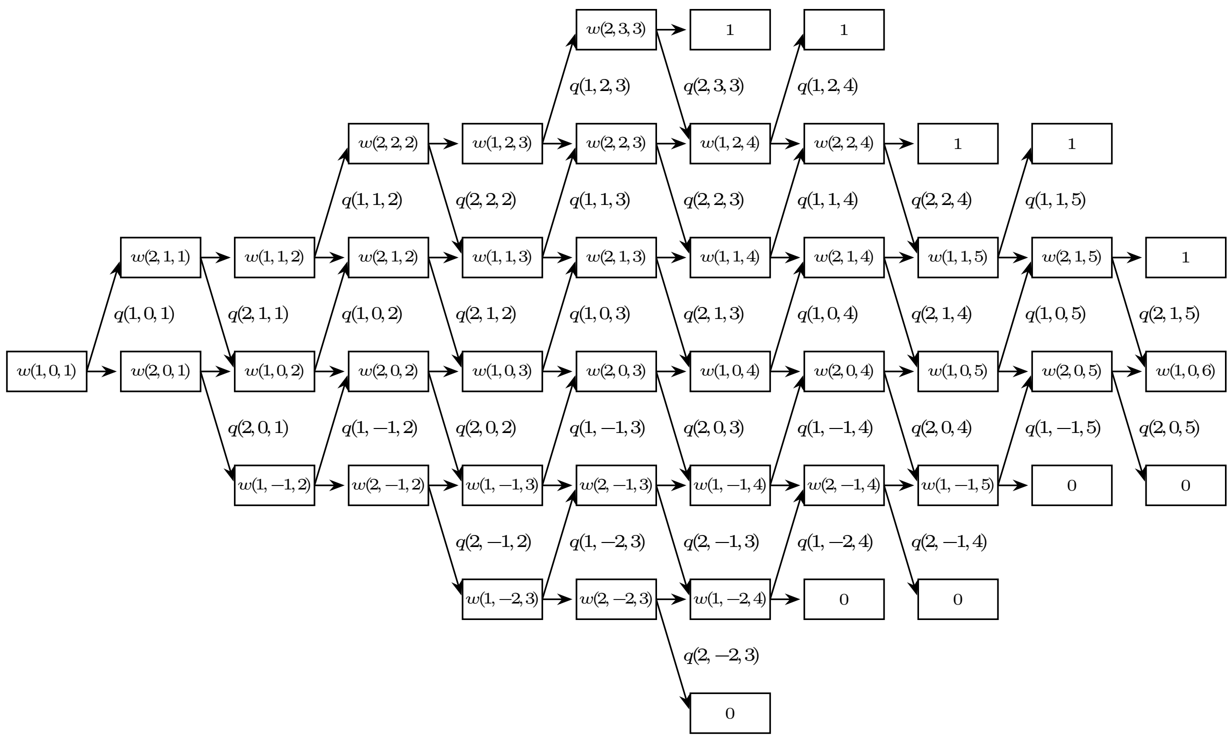
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 15: Estimation results including proxies of pressure that account for the consequences of the shot using the extended set of competitions

	(1) BASE2	(2) EndMiss2	(3) MissScore2
FirstShot	0.00463	0.00123	0.00147
	(0.0120)	(0.0123)	(0.0124)
first $\times$ After2nd	0.0465**	0.0513***	0.0359*
	(0.0145)	(0.0155)	(0.0148)
EndIfMiss		0.0365*	
		(0.0156)	
EndIfScore		0.00611	
		(0.0167)	
$\Delta Pwin$			-0.00244***
			(0.000435)
Observations	17202	16605	16605

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$



1

Figure 1: Binomial network state and transition probabilities for first 5 rounds of penalty shootout.

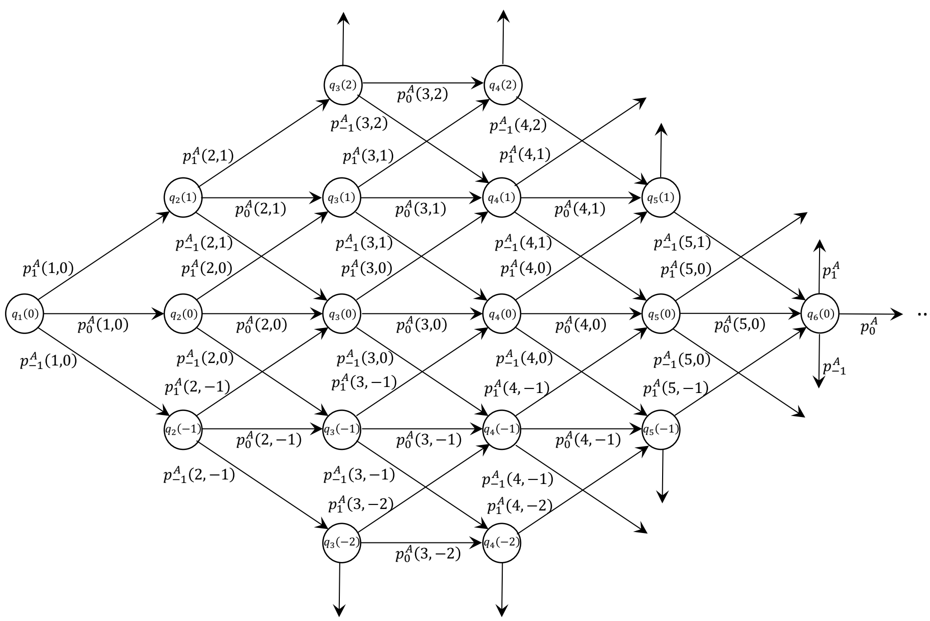


Figure 2: Trinomial network state and transition probabilities for first 5 rounds of penalty shootout.