DIP Assignment - 5

(Theory part)

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Ans-1) : a) To prove : $W = \dfrac{H^*}{|H|^2 + \frac{|N|^2}{|F|^2} + \gamma \frac{|LG|^2}{|F|^2}}$

Proof : Let $Y = \sum\sum |WG - F|^2 + \gamma \sum\sum |LWG|^2$

(Removing k & L from notation for convenience)

Putting $G = FH + N$ in the eq^n

$$Y = \sum\sum |WFH + WN - F|^2 + \gamma \sum\sum |LWG|^2$$

$$= \cancel{\sum\sum |F(}$$

$$= \sum\sum |F(1-WH) - WN|^2 + \gamma \sum\sum |LWG|^2$$

$$=$$

$$\frac{\partial Y}{\partial W} = -2|F|^2(1-WH)^* H + 2|N|^2 W^* + \gamma\, 2|LG|^2 W^*$$

$$= 0$$

$$\Rightarrow -2|F|^2 H + 2|F|^2 W H^* H + 2N^2 W^* + \gamma\, 2|LG|^2 W^* = 0$$

$$\Rightarrow |F|^2 H = |F|^2 W |H|^2 + N^2 W^* + \gamma |LG|^2 W^*$$

Taking ~~[illegible]~~ $*$ both sides

$$|F|^2 H^* = |F|^2 W |H|^2 + |N|^2 W + \gamma |LG|^2 W$$

$$\{\text{as } (A^*)^* = A\}$$

$$\Rightarrow |F|^2 H^* = W\left(|F|^2 |H|^2 + |N|^2 + \gamma |LG|^2\right)$$

$$\Rightarrow W = \frac{\cancel{|F|^2}\, H^*}{\cancel{|F|^2}\left(|H|^2 + \frac{N^2}{|F|^2} + \gamma \frac{|LG|^2}{|F|^2}\right)}$$

$$\therefore \boxed{W = \frac{H^*}{|H|^2 + \frac{|N|^2}{|F|^2} + \gamma \frac{|LG|^2}{|F|^2}}}$$

Hence proved.

Ans-2) :

a) $f(x,y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 10 & 10 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 5 & 15 & 8 \end{bmatrix}$

→ Let G_x, G_y represent the Sobel operator for horizontal and vertical directions, s.t.

$$G_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \text{ \& } G_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

→ Let $f_p(x,y)$ be the zero padded version of $f(x,y)$

s.t.

$$f_p(x,y) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 & 10 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 15 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

→ [illegible]

→ Let resultant ~~img~~ gradient ~~be~~ with horizontal edges be : $\hat{f}_h(x,y) = f_p(x,y) \circledast G_x$

correlation operator

& same for vertical edges be : $\hat{f}_v(x,y) = f_p(x,y) \circledast G_y$

→ Finding $\hat{f}_{h}(x,y)$ for red-highlighted elements = $\hat{f}_{hr}(x,y)$

Let $\hat{f}_{hr}(x,y) = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}$

$$h_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 10 & 10 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = 0+0+20+10$$
$$= 30$$

$$h_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 10 & 10 & 1 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = 0+0+10+20+1$$
$$= 31$$

$$h_{21} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 10 & 10 \\ 0 & 2 & 3 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = -2-1-1+4+3$$
$$= 3$$

$$h_{22} = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 10 & 1 \\ 2 & 3 & 1 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \cancel{-1} \cancel{-2} -1 \cancel{+2} +6 \cancel{+1}$$
$$= 5$$

$$h_{31} = \begin{bmatrix} 0 & 10 & 10 \\ 0 & 2 & 3 \\ 0 & 5 & 15 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = 0 + \cancel{10} + 15 - 20 \cancel{-10}$$
$$= -5$$

$$h_{32} = \begin{bmatrix} 10 & 10 & 1 \\ 2 & 3 & 1 \\ 5 & 15 & 8 \end{bmatrix} \circledast \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \cancel{-10} \; \cancel{-20} \; -1 \; +0 \; +5 \; +8 \; \cancel{+30}$$

$$= 12$$

$$\therefore \boxed{f_{hr}(x,y) = \begin{bmatrix} 30 & 31 \\ 3 & 5 \\ -5 & 12 \end{bmatrix}}$$

→ Finding $\hat{f}_v(x,y)$ for red-highlighted ~~vertices~~ elements = $\hat{f}_{vr}(x,y)$

Let $$\hat{f}_{vr}(x,y) = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \\ v_{31} & v_{32} \end{bmatrix}$$

$$v_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 10 & 10 & 10 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = -2+2+10$$

$$\cancel{-2-2-10+2+10}$$

$$= 10$$

$$v_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 10 & 10 & 1 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = -\cancel{2}-10+0\cancel{+2}+1$$

$$= -9$$

$$v_{21} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 10 & 10 \\ 0 & 2 & 3 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = -1+0+1+20+3$$

$$= 23$$

$$v_{22} = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 10 & 1 \\ 2 & 3 & 1 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \cancel{-1}-20\cancel{-2}+0\cancel{+1}\cancel{+2}+1$$

$$= -19$$

$$v_{31} = \begin{bmatrix} 0 & 10 & 10 \\ 0 & 2 & 3 \\ 0 & 5 & 15 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= 0 + 0 + 10 + 6 + 15 = 31$$

$$v_{32} = \begin{bmatrix} 10 & 10 & 1 \\ 2 & 3 & 4 \\ 5 & 15 & 8 \end{bmatrix} \circledast \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= -10 - 4 - 5 + 0 + 1 + 2 + 8$$

$$= -8$$

$$\therefore \boxed{h_{vr}(x, y) = \begin{bmatrix} 10 & -9 \\ 23 & -19 \\ 31 & -8 \end{bmatrix}}$$

→ Finding resultant magnitude of $\hat{f}_r(x,y)$

$$|\hat{f}_r(x,y)| = |\hat{f}_{hr}(x,y)| + |\hat{f}_{vr}(x,y)|$$

$$= \begin{bmatrix} 30+10 & 31+9 \\ 3+23 & 5+19 \\ 5+31 & 12+8 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & 40 \\ 26 & 24 \\ 36 & 20 \end{bmatrix}$$

→ finding resultant direction of $\hat{f}_r(x,y)$

$$\phi[\hat{f}_r(x,y)] = \left.\tan^{-1}\left(\frac{\hat{f}_{vr}(x,y)}{\hat{f}_{hr}(x,y)}\right)\right\} \text{Element wise}$$

$$= \begin{bmatrix} \tan^{-1}\left(\frac{10}{30}\right) & \tan^{-1}\left(\frac{-9}{31}\right) \\ \tan^{-1}\left(\frac{23}{3}\right) & \tan^{-1}\left(-\frac{19}{5}\right) \\ \tan^{-1}\left(\frac{31}{-5}\right) & \tan^{-1}\left(\frac{-8}{12}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 18.43^\circ & -16.19^\circ \\ 82.57^\circ & -75.26^\circ \\ -80.84^\circ & -33.7^\circ \end{bmatrix}$$

Ans: Magnitude of resultant gradient = $\begin{bmatrix} 40 & 40 \\ 26 & 24 \\ 36 & 20 \end{bmatrix}$

Direction of resultant gradient = $\begin{bmatrix} 18.43^\circ & -16.19^\circ \\ 82.57^\circ & -75.26^\circ \\ -80.84^\circ & -33.7^\circ \end{bmatrix}$

Ans-2) : b) Let $M = \begin{bmatrix} 40 & 40 \\ 26 & 24 \\ 36 & 20 \end{bmatrix}$, $\phi = \begin{bmatrix} 18.43° & -16.19° \\ 82.57° & -75.26° \\ -80.84° & -33.7° \end{bmatrix}$

i) For value of 10 at (2,2) in the original matrix $f(x,y)$

$\phi(2,1)$ = Phase at (2,1) in ϕ = 82.57° ∴ Lies in vertical dirn

$M(2,1)$ = Magnitude at (2,1) in M } Corresponds to 10 at (2,2) in $f(x,y)$

= 26

~~M(2,2) = 40~~ $M(1,1) = 40$

$M(3,1) = 36$

As $M(2,1) < \cancel{M(2,2)}\ M(1,1)$

∴ it will be suppressed to 0

∴ $\boxed{g_N(2,1) = 0}$ (Ans)

ii) For value of 10 at (2,3) in $f(x,y)$

$\underbrace{\phi(2,2) = -75.26^\circ}_{\text{Corresponds to 10 at } f(2,3)}$ $\therefore$ Lies in vertical dir^n

$M(2,2) = 24$

$M(1,2) = 40$

$M(3,2) = 20$

As $M(2,2) < M(1,2)$

$\therefore$ it will be suppressed to 0

$\therefore$ $\boxed{g_N(2,2) = 0}$ (Ans)