Ans-3) :

b) $h(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$f(x,y) = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

Zero padding ~~the~~ $f(x,y)$ by adding 2 rows & columns of 0s to ~~to~~ the top, bottom, left & right respectively.

$$\therefore f(x,y) = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 7 & 0 & 0 \\ 0 & 0 & 8 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6} \end{array}$$

$$\& \; h(x,y) = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Let $w(x,y)$ be the convolution of $f(x,y)$ & $h(x,y)$ s.t.

$$w(x,y) = f(x,y) * h(x,y)$$

$$= \sum_{a=-\infty}^{\infty} \sum_{b=-\infty}^{\infty} f(a,b)\, h(x-a, y-b)$$

→ The shape of w will be $[3+2-1, 3+2-1] = [4,4]$

→ This can be achieved ~~graphically~~ or intuitively by flipping the kernel by 180° s.t.

$$h(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

& multiplying it element-wise with $f(x,y)$ ~~element~~ over a window of size 3X3. Then assigning the corresponding ~~fe~~ ~~vals~~ summations to the output (or convolved) values.

$\rightarrow w(0,0) =$

$$w(0,0) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix} \underbrace{.*}_{\text{element wise multiplication}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 0$$

$\rightarrow w(0,1) =$

$$w(1,0) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 6 & 7 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 6$$

$\rightarrow w(2,0) =$

$\rightarrow w(0,2) = \sum$

$$w(2,0) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6 & 7 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 7$$

$$\rightarrow w(3,0) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 7 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 0$$

$\rightarrow w(0,1) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & 0 & 8 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 6 \cdot 1 + 0 = 6$

$\rightarrow w(1,1) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 6 & 7 \\ 0 & 8 & 9 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 6 \cdot (-4) + 7 \cdot (1) + 8 \cdot (1) + 0$

$= -24 + 7 + 8 = -24 + 15 = -9$

$\rightarrow w(2,1) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 6 & 7 & 0 \\ 8 & 9 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 6 \cdot 1 + 7(-4) + 9(1) + 0$

$= 6 - 28 + 9 = -28 + 15 = -13$

$\rightarrow w(3,1) = \sum \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 0 + 7(1) = 7$

$\rightarrow w\cancel{(2,0)}$ $(0,2)$ $= \Sigma \left\{ \begin{bmatrix} 0 & 0 & 6 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 8 + 0 = 8$

$w(1,2)$

$\rightarrow \cancel{w(2,1)}$ $\cancel{w(0,1)}$ $= \Sigma \left\{ \begin{bmatrix} 0 & 6 & 7 \\ 0 & 8 & 9 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 0 + 6 \cdot 1 + 7 \cdot 0 + 8(-4) + 9$

$= 15 - 32 = -17$

$\rightarrow w(2,2) = \Sigma \left\{ \begin{bmatrix} 6 & 7 & 0 \\ 8 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 6 \cdot 0 + 7 \cdot 1 + 0 + 8 \cdot 1 + 9(-4) + 0$

$= 7 + 8 - 36 = -36 + 15 = -21$

$\rightarrow \cancel{w(2,3)}$ $w(3,2)$ $= \Sigma \left\{ \begin{bmatrix} 7 & 0 & 0 \\ 9 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$

$= 7 \cdot 0 + 9 \cdot 1 = 9$

$$\rightarrow w(0,3) = \Sigma \left\{ \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 0$$

$$\rightarrow w(1,3) = \Sigma \left\{ \begin{bmatrix} 0 & 8 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 0 + 8 \cdot 1 = 8$$

$$\rightarrow w(2,3) = \Sigma \left\{ \begin{bmatrix} 8 & 9 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 8 \cdot 0 + 9 \cdot 1 + 0 = 9$$

$$\rightarrow w(3,3) = \Sigma \left\{ \begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .* \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= 9 \cdot 0 + 0 = 0$$

$$\Rightarrow w(x,y) = \begin{bmatrix} w(0,0) & w(1,0) & w(2,0) & w(3,0) \\ w(0,1) & w(1,1) & w(2,1) & w(3,1) \\ w(0,2) & w(1,2) & w(2,2) & w(3,2) \\ w(0,3) & w(1,3) & w(2,3) & w(3,3) \end{bmatrix}_{4\times 4}$$

Putting in the calculated values :

$$\therefore w(x,y) = \begin{bmatrix} 0 & 6 & 7 & 0 \\ 6 & -9 & -13 & 7 \\ 8 & -17 & -21 & 9 \\ 0 & 8 & 9 & 0 \end{bmatrix}$$

(Ans)

This value perfectly matches with the one calculated by my program in 3. a)