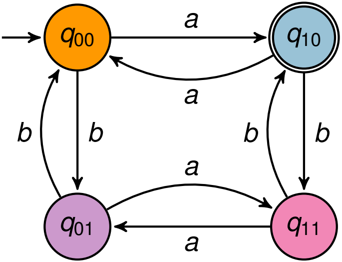
CS 228 : Logic in Computer Science

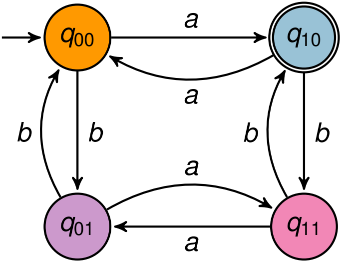
Krishna. S

Language Acceptance : Proof



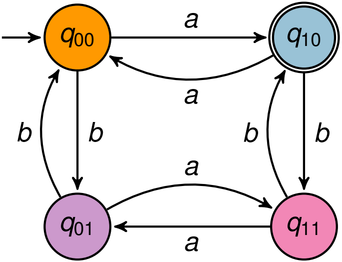
- Prove by induction on $|w|$

Language Acceptance : Proof



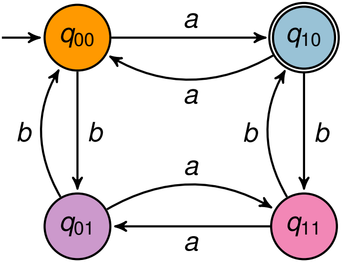
- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\delta}(q_{00}, \epsilon) = q_{00}$

Language Acceptance : Proof



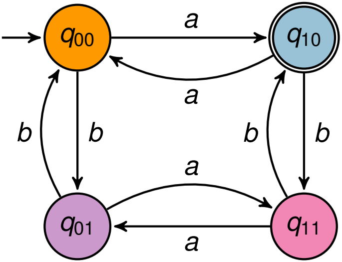
- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\delta}(q_{00}, \epsilon) = q_{00}$
- Assume the claim for $x \in \Sigma^*$, and show it for $xc, c \in \{a, b\}$.

Language Acceptance : Proof



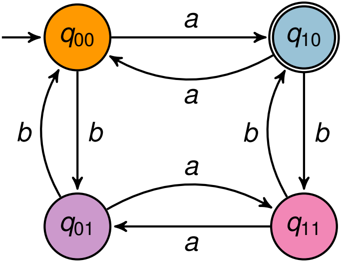
- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$

Language Acceptance : Proof



- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$
- By induction hypothesis, $\hat{\delta}(q_{00}, x) = q_{ij}$ iff
  - parity of i and $|x|_a$ are the same
  - parity of j and $|x|_b$ are the same

Language Acceptance : Proof



- Case Analysis : If $|x|_a$ odd and $|x|_b$ even, then $i = 1, j = 0$
  - $\delta(q_{10}, a) = q_{00}, \delta(q_{10}, b) = q_{11}$
  - $|xa|_a$ is even and $|xa|_b$ is even
  - $|xb|_a$ is odd and $|xb|_b$ is odd
- Other Cases : Similar
- $\hat{\delta}(q_{00}, x) = q_{10}$ iff $|x|_a$ odd and $|x|_b$ even

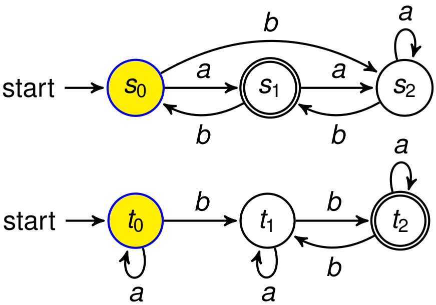
Closure Properties : DFA

Closure under Complementation

- If L is regular, so is $\overline{L}$
  - Let $A = (Q, q_0, \Sigma, \delta, F)$ be the DFA such that $L = L(A)$
  - For every $w \in L$, $\hat{\delta}(q_0, w) = f$ for some $f \in F$
  - For every $w \notin L$, $\hat{\delta}(q_0, w) = q$ for some $q \notin F$
  - Construct $\overline{A} = (Q, q_0, \Sigma, \delta, Q - F)$
    - $w \in L(\overline{A})$ iff $\hat{\delta}(q_0, w) \in Q - F$ iff $w \notin L(A)$
    - $L(\overline{A}) = \overline{L(A)}$

Closure under Intersection

- *aaab*



Closure under Intersection

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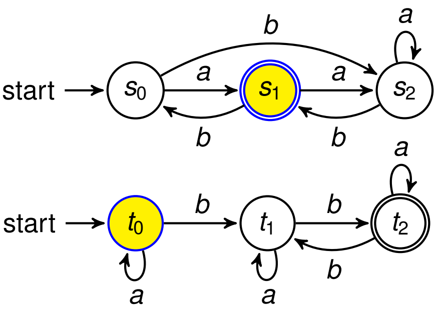

Closure under Intersection

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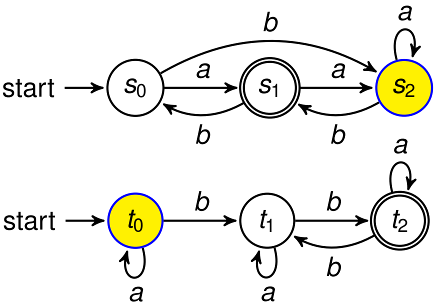

Closure under Intersection

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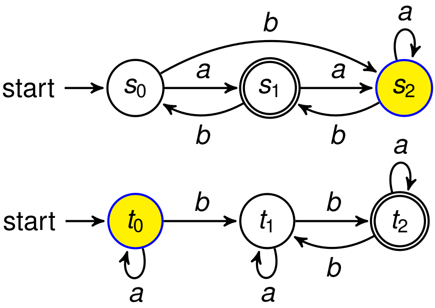
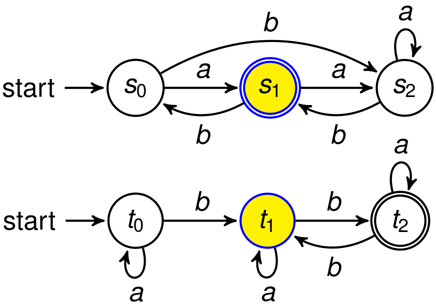


Closure under Intersection

- *aaa**b*



Closure under Intersection

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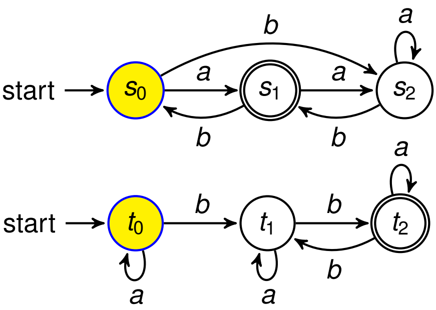

Closure under Intersection

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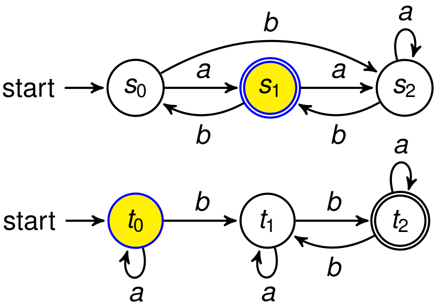
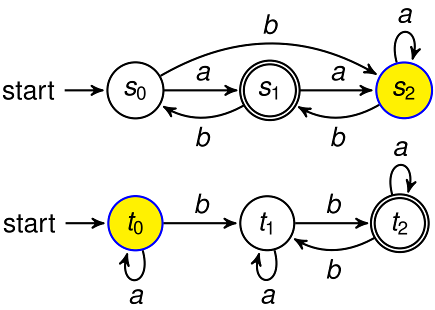


Closure under Intersection

- *a**a**bba*



Closure under Intersection

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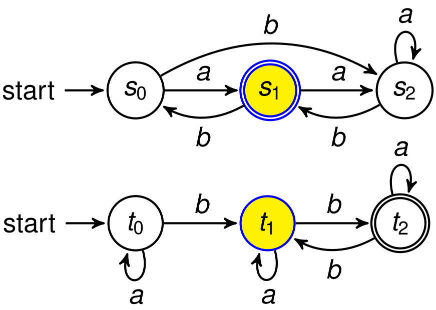

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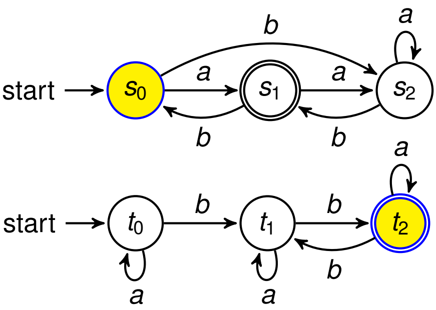

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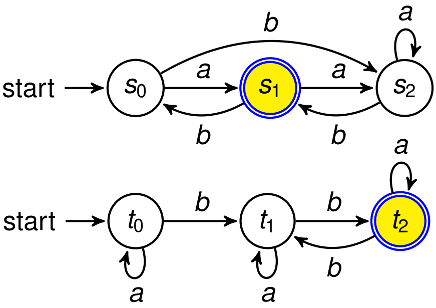

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta}_1(p, x), \hat{\delta}_2(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$ iff $(\hat{\delta_1}(q_0, x), \hat{\delta_2}(s_0, x)) \in F_1 \times F_2$

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$ iff $(\hat{\delta_1}(q_0, x), \hat{\delta_2}(s_0, x)) \in F_1 \times F_2$ iff $\hat{\delta_1}(q_0, x) \in F_1$ and $\hat{\delta_2}(s_0, x) \in F_2$

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$ iff $(\hat{\delta_1}(q_0, x), \hat{\delta_2}(s_0, x)) \in F_1 \times F_2$ iff $\hat{\delta_1}(q_0, x) \in F_1$ and $\hat{\delta_2}(s_0, x) \in F_2$ iff $x \in L(A_1)$ and $x \in L(A_2)$

Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$

Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta}_1(p, x), \hat{\delta}_2(q, x))$

$x \in L(A)$ iff $x \in L(A_1)$ or $x \in L(A_2)$