CS215 SAFE Quiz 3 **29 August 2024**

1. Let X be a negative binomial random variable with parameters r and p. Given that
$$E[X] = \frac{50}{3} \text{ and } Var(X) = \frac{350}{9}$$
find r and p. Recall that for a negative binomial distribution
$$E[X] = \frac{r}{p} \text{ and } Var(X) = \frac{r(1-p)}{p^2}$$
.

   (a) 6, 0.3
   (b) 5, 0.3
   (c) 7, 0.7
   (d) 6, 0.5

   Answer: (b)

2. Let $p_i = P\{X = i\}$ and suppose that $p_1 + p_2 + p_3 = 1$. If $E[x] = 2$, what values of p_1, p_2 and p_3 maximises $Var(X)$?

   (a) $\frac{1}{2}, 0, \frac{1}{2}$
   (b) $\frac{1}{2}, \frac{1}{2}, 0$
   (c) $\frac{1}{2}, \frac{1}{2}, \frac{1}{4}$
   (d) $0, 1, 0$

   Answer: (a)

3. We say that m_p is the $100p$ percentile of the distribution function F if $F(m_p) = p$. Find m_p for $p = \frac{1}{2}$ for the distribution having density function $f(x) = 2e^{-2x}, x \geq 0$. If m_p is expressed using the expression $a * ln(b)$ then find $10a + 20b$.

   Answer: 5

4. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding,

what is the probability that the message will be incorrectly decoded? (Assume independence in transmission error among binary digits. By majority decoding we mean that the message is decoded as "0" if there are at least three zeros in the message received and as "1" otherwise.) Provide answer correct up to 3 decimal places.

Answer: 0.057

5. If X and Y are binomial random variables with respective parameters $(n, p), (n, 1-p)$ and $0 < p < 1$, select the incorrect statements.

   (a) $P\{X \leq k\} = P\{Y > n - k\}$

   (b) $P\{X = k\} = P\{Y = n - k\}$

   (c) $P\{X = k = 1\} = \frac{p}{1-p}\frac{n-k}{k+1}P\{X = k\}, k = 0, ..., n$

   (d) as k goes from 0 to n, $P\{X = k\}$ reaches its largest value when k is any integer less than or equal to $(n + 1)p$

   (e) None of the above

   Answer: (d)