1. Let S be a sample space. Which of the following is/are correct about S?

   (a) An element of S is called an event.
   (b) A subset of S is called an event.
   (c) An element of S is called an outcome.
   (d) An outcome is drawn in each trail of an experiment.

   Answer: (b), (c), (d)

2. Let n be an odd integer. The geometric median of a set of points $x_1, ..., x_n$ (assumed to be sorted in increasing order) is any point y minimizing the equation $\sum_{i=1}^{n} |x_i - y|$. The statistical median is the value $x_{(n+1)/2}$. Suppose that we are given a set of sorted points $x_1, ..., x_n$ where each x_i is distinct for which g is the geometric median and s is the statistical median. Then:

   (a) $s > g$ for all x_i.
   (b) $s < g$ for all x_i.
   (c) $s = g$ for all x_i.
   (d) s may be equal to g for some x_i.

   Answer: (c)

3. Let X be an experiment with sample space S, and associated probability distribution p. We run the experiment T times independently. Define $n_S(A)$ to be the size of event $E \subseteq A$, and $n_T(A)$ to be the number of times event E occurred over the T trials. Then for large enough T, which of the following relations are true?

   (a) $p = \frac{n_S(A)}{|S|}$
   (b) $p = \frac{n_T(A)}{|S|}$
   (c) $p = \frac{n_S(A)}{T}$
   (d) $p = \frac{n_T(A)}{T}$

   Answer: (d)

4. Given a set of data points $x_1, ..., x_n \in \mathbb{R}$ we define a k minimizer to be any point $y \in \mathbb{R}$ that minimizes $\sum_{i=1}^{n} d_k(x_i, y)$, where for each $k \geq 0$, $d_k(x_i, y) := |x_i - y|^k$. It can be shown that the mean is the k_1) minimizer, the median is the k_2 minimizer, and the mode is the k_3 minimizer for some $k_1, k_2, k_3 \in \mathbb{Z}$. Evaluate $10^4 k_1 + 10^2 k_2 + k_3$.

   Answer: 20100

5. Consider a room with a line of n bulbs. Initially, half of them are on, half of them are off. A magician enters the room with a rod. The rod primarily sucks power when tapped on a bulb. It can sometimes leak power, lighting up a bulb that was off upon tapping. In particular, the probability that on tapping, a bulb is turned on from off is 0.3, and that a bulb is turned off from on is 0.8. Now, fix integer $i > 0$. The following is repeated i times in succession: the magician taps each bulb with the rod, one after the other along the line. After the i rounds of tapping, she picks an integer r uniformly at random from $1, 2, ..., n$. She wins if the rth bulb is now off. Which of the following are true?

   (a) If $i = 1$, she wins with probability $\frac{3}{4}$.
   (b) If $i = 2$, she wins with probability $\frac{31}{40}$.
   (c) As i grows larger, she wins with probability approaching $\frac{64}{73}$.
   (d) As i grows larger, she wins with probability approaching $\frac{8}{11}$.

   Answer: (a), (d)