1. Given a multivariate Gaussian with mean vector $\mu = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix}$, obtain the square of the Mahalonobis distance of $x = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ from the mean.

   Answer: 1.375

2. Given a multivariate Gaussian distribution over variables X_1, X_2, X_3 with a zero mean vector and covariance matrix $\Sigma = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{bmatrix}$, which pair of variables are most strongly correlated?

   (a) All are equally correlated.
   (b) X_2 and X_3
   (c) X_1 and X_2
   (d) X_1 and X_3

   Answer: (c)

3. A bivariate Gaussian distribution has covariance matrix $\Sigma = \begin{bmatrix} 2 & 0.8 \\ 0.8 & 8 \end{bmatrix}$. What is the correlation coefficient between the two variables?

   (a) 0.05
   (b) 0.2
   (c) 0.04
   (d) 0.8

   Answer: (b)

4. Consider a bivariate Gaussian distribution with mean vector $\mu = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$ and covariance matrix $\Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$. What is the largest variance along any direction in this distribution?

   Answer: 5

5. Which of the following conditions are individually sufficient for the independence of 2 random variables?

   (a) Their joint probability density is the product of their individual probability densities at all sample points.

   (b) Their joint cumulative distribution is the product of their individual cumulative distributions at all sample points.

   (c) Their covariance across the sample space is zero.

   (d) They are uncorrelated with all other variables in the distribution.

   Answer: (a), (b)

6. What does $\Sigma_{i,j}$ of the covariance matrix represent in a multivariate Gaussian distribution?

   (a) $\mathbb{E}[X_i X_j] - \mathbb{E}[X_i]\mathbb{E}[X_j]$

   (b) $\mathbb{E}[(X_i - \mathbb{E}[X_i])(X_j - \mathbb{E}[X_j])]$

   (c) $\mathbb{E}[(X_i - \mathbb{E}[X_i])^2 \mid X_j]$

   (d) None of the above.

   Answer: (a), (b)

7. Suppose a 3 dimensional multivariate Gaussian has a covariance matrix $\Sigma = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Compute the conditional variance of X_1 given $X_2 = 0$.

   Answer: 2.67