CS215 Moodle Quiz 6 **10 October 2024**

1. Consider the use of matrix form of least squares solution for linear regression. Mark all of the following that are correct.

   (a) The solution to the matrix form is $B = (X^T X)^{-1} X^T Y$

   (b) Matrix inversion has high time complexity.

   (c) Matrix form is not sensitive to outliers.

   (d) Parameters cannot be obtained using matrix form if any two features are perfectly correlated.

   Answer: (a), (b), (d)

2. For a linear regression model

$$y = \beta_0 + \beta_1 x^2 + \epsilon$$

   where B_0 and B_1 are estimators of β_0 and β_1 respectively. Choose the correct options.

   (a)

$$B_1 = \frac{n \sum_i x_i^2 Y_i - n\bar{Y} x_i^2 \sum_i}{n \sum_i x_i^4 - (\sum_i x_i^2)^2}$$

   (b)

$$B_0 = \bar{Y} - B_1 \bar{x}$$

   (c)

$$B_0 = \bar{Y} - \frac{B_1 \sum_i x_i^2}{n}$$

   (d)

$$B_1 = \frac{\sum_i x_i Y_i - n\bar{x}\bar{Y}}{\sum_i x_i^2 - n\bar{x}^2}$$

   Answer: (a), (c)

3. Given a kernel K and a positive number h, called the bandwidth, the kernel density estimator is defined to be

$$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{h} K(\frac{x - X_i}{h})$$

.

Consider the following statements:

Statement 1: As $h \to 0$, $\hat{f}_n$ consists of a set of spikes and height of spikes tends to infinity.

Statement 2: As $h \to \infty$, $\hat{f}_n$ tends to a uniform density.

Mark all the statements that apply.

(a) Statement 1 is true
(b) Statement 1 is false
(c) Statement 2 is true
(d) Statement 2 is false

Answer: (a), (c)

4. Suppose in the simple linear regression model

$$Y = \alpha + \beta x + \epsilon, 0 < \beta < 1$$

if $x < \frac{\alpha}{1-\beta}$ then $E[Y]$ lies in the range

(a) (α, β)
(b) $(\alpha, 1 - \beta)$
(c) $(x, \frac{\alpha}{1-\beta})$
(d) $(\frac{\alpha}{1-\beta}, x)$

Answer: (c)

5. Recall that estimated coefficients in linear regression, A and B corresponding to α and β, follow a normal distribution. Let their distribution be

$$A \sim \mathcal{N}(\alpha, 25) \text{ and } B \sim \mathcal{N}(\beta, 49)$$

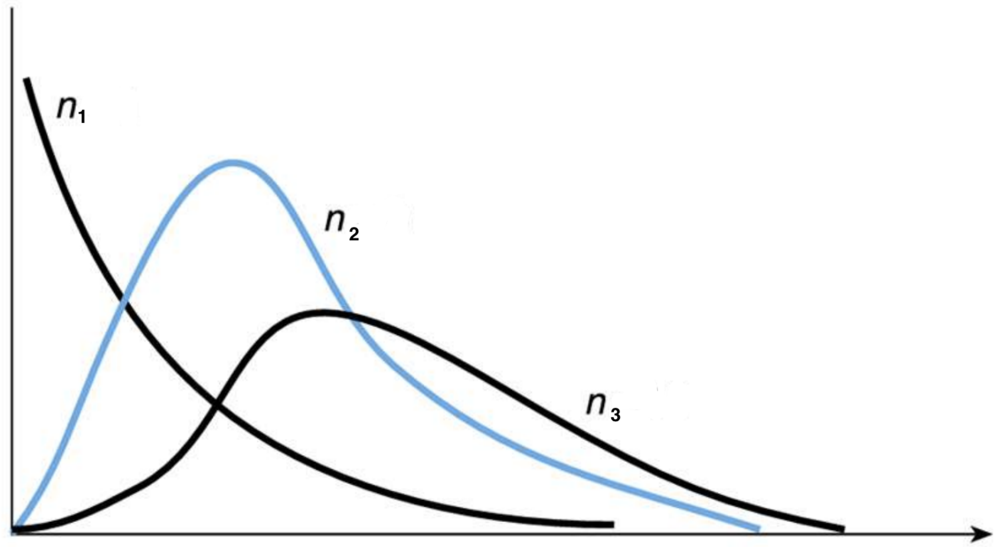
and given that

$$\text{risk} = \text{bias}^2 + \text{variance}$$

If $risk(A) + risk(B) = \alpha^x + \beta^y + 72$, find $x + y$.

Answer: 0

6. Let X be a chi-square random variable with n degrees of freedom. Below is a plot for different values of n. What is the relation between n_1, n_2 and n_3? Figure shows chi-square density function with different degrees of freedom.



(a) $n_1 > n_2 > n_3$

(b) $n_1 < n_2 > n_3$

(c) $n_1 < n_2 < n_3$

(d) Cannot say

Answer: (c)

7. Suppose you are fitting a regression function for one-dimensional data, and the true distribution is

$$P(Y|x) \sim \mathcal{N}(\mu_x = 2x + 1, \sigma^2)$$

If you fit a kernel regression with a Gaussian Kernel on the dataset

$$D = \{(1, 3.1), (2, 3), (3, 6.8)\} \text{ with } h = 1$$

then what is the maximum predicted value by the fitted kernel regression model on this dataset?

Answer: 6.8