CS 228: Logic in CS Autumn 2024

Quiz 3

Total Marks: 30 *30 October 2024*

1. **[10 marks]** Write LTL formulae φ which capture each requirement. Assume $AP = \{a, b\}$.
   (a) Finitely many a's.
   (b) Infinitely many a's and finitely many b's.
   (c) Eventually a and eventually forever $\neg a$.
   (d) There is an a which is never eventually followed by two occurrences of b's.
   (e) There is atleast one a, and b holds continuously since the last occurrence of a.

   For each formula, give a short justification.

Solution

(a) $\neg(\Box\Diamond a)$ or $\Diamond\Box\neg a$

(b) $\Box\Diamond a\ \neg(\Box\Diamond b)$

(c) $\Diamond a \vee \Diamond\Box\neg a$

(d) $\Diamond(a \wedge (\bigcirc(\Diamond b \wedge (\bigcirc\Diamond b))))$

(e) $\Diamond(a \wedge \bigcirc\Box\neg a \wedge \Box b) \vee \Box\Diamond a$

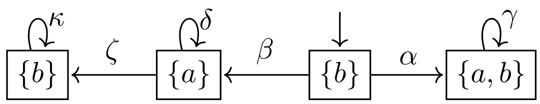
Rubrics

- For (a), (b), and (c) 2 marks has been given in each part for the correct LTL formula. No partial marks have been awarded in (a), (b), and (c).
- For (d), 2 marks has been given for correct LTL formula. 1 mark has been awarded if the LTL formula has 1-2 mistakes.
- For (e), 2 marks for the correct answer. 1 mark has been given if the student has missed the case for infinitely many a's.

Comments.

- There are many possible answers to this question. We have given marks for alternate answers if they are correct.
- A lot of students have misunderstood the alphabet as AP instead of 2^{AP}. You cannot assume that $\neg a \iff b$.
- In (d), the two occurrences of b need not be consecutive.

2. **[10 marks]** Consider the formula $\varphi = \bigcirc\Box(a \wedge \neg b)$ and the transition system TS below over $AP = \{a, b\}$.



Does $TS \models \varphi$? To answer this,

(a) Write down $\neg\varphi$ so that the negation is attached only to symbols from AP.

(b) Draw the NBA $A_{\neg\varphi}$ for $\neg\varphi$. (*You should construct the NBA directly from $\neg\varphi$. DO NOT use the construction from LTL to GNBA*)

(c) Construct the transition system TS' as a product of TS and $A_{\neg\varphi}$. The labels of TS' are the states of $A_{\neg\varphi}$.

(d) Write the persistence property ψ which should be checked on TS'.

(e) Does $TS' \models \psi$? Explain in 2-3 lines your answer for this (whether or not TS' satisfies ψ) and connect it to the given question $TS \models \varphi$ appropriately, and conclude whether or not $TS \models \varphi$.

Solution

(a) $\bigcirc\Diamond(\neg a \vee b)$

3. **[10 marks]** Consider $\varphi = a \mathsf{U} (\bigcirc b \wedge \neg a)$ over $AP = \{a, b\}$.

(a) What is the closure of φ, $Cl(\varphi)$?

(b) Write down all elementary subsets of $Cl(\varphi)$.

(c) Which of these are initial states in a GNBA for φ?

(d) Which of these are accepting states in a GNBA for $\neg\varphi$?

(e) Take a pair of elementary sets B_1, B_2 from the above so that there is no transition from B_1 to B_2. Explain why. Note that you are not asked to draw the GNBA for φ.

Solution

(a) $Cl(\varphi) = \{a, \neg a, b, \neg b, (\bigcirc b \wedge \neg a), \neg(\bigcirc b \wedge \neg a), \bigcirc b, \neg(\bigcirc b), \varphi, \neg\varphi\}$

(b) Taking $\gamma = (\bigcirc b \wedge \neg a)$, the elementary subsets of $Cl(\varphi)$ are

$$
\begin{aligned}
B_1 &= \{a,\, b,\, \bigcirc b, \neg\gamma,\, \varphi\} \\
B_2 &= \{a,\, b, \bigcirc b,\, \neg\gamma,\, \neg\varphi\} \\
B_3 &= \{a,\, b,\, \neg(\bigcirc b),\, \neg\gamma,\, \varphi\} \\
B_4 &= \{a,\, b,\, \neg(\bigcirc b),\, \neg\gamma,\, \neg\varphi\} \\
B_5 &= \{a,\, \neg b,\, \bigcirc b,\, \neg\gamma, \varphi\} \\
B_6 &= \{a,\, \neg b,\, \bigcirc b,\, \neg\gamma, \neg\varphi\} \\
B_7 &= \{a,\, \neg b,\, \neg(\bigcirc b),\, \neg\gamma,\, \varphi\} \\
B_8 &= \{a,\, \neg b,\, \neg(\bigcirc b),\, \neg\gamma,\, \neg\varphi\} \\
B_9 &= \{\neg a,\, b, \bigcirc b,\, \gamma,\, \varphi\} \\
B_{10} &= \{\neg a,\, b,\, \neg(\bigcirc b),\, \neg\gamma,\, \neg\varphi\} \\
B_{11} &= \{\neg a,\, \neg b,\, \bigcirc b,\, \gamma, \varphi\} \\
B_{12} &= \{\neg a,\, \neg b,\, \neg(\bigcirc b),\, \neg\gamma,\, \neg\varphi\}
\end{aligned}
$$

(c) At the initial state, φ must be true. Thus $\textsf{Initial states} = \{B_1, B_3, B_5, B_7, B_9, B_{11}\}$

(d) Notice that $Cl(\varphi) = Cl(\neg\varphi)$. So the elementary sets of φ and $\neg\varphi$ are the same. Hence $\mathcal{F} = F_\varphi = \{B \mid \varphi \notin B \text{ or } (\varphi \in B \text{ and } (\bigcirc b \wedge \neg a) \in B)\} = \{B_2, B_4, B_6, B_8, B_9, B_{10}, B_{11}, B_{12}\}$

Rubrics

- For (a), 2 marks has been given for the correct $Cl(\varphi)$.
- For (b), 0.25 marks has been given for each of the 12 sets and 0.25 marks have been deducted for each set which is not an elementary subset. [For checking, all 8 sets with $\{a, \neg\gamma\}$ as subset and 4 sets with $\{\neg a\}$. In these 4 sets, $\{\bigcirc b, \gamma, \varphi\}$ occur together.]
- For (c), 1.5 marks for the correct answer
- For (d), 1.5 marks for the correct answer
- For (e), 1 mark for the correct answer and 1 mark for justification

Comments.

- If $Cl(\varphi)$ is wrong, everything that follows will be wrong. In such cases, the total marks will be 0. However, in some *exceptional* cases, if you have made elementary sets correct even with wrong $Cl(\varphi)$, we have graded accordingly.
- Part (c) and (d) are graded only if you get full 3 marks in Part (b), as it doesn't make any sense to talk about start states and accepting states, when the state set is itself wrong.

- For (c) and (d), you **must** list down all the relevant states. Just writing "*all states which contains* φ" is not sufficient. Similarly for (d).
- For part (e), we have given 1 mark if the given pair even when justification is wrong. However both the states that you use in (e) should be an elementary set. If a set which is not elementary is used, 0 marks.