CS213/293 Data Structure and Algorithms 2024 IITB India
Quiz 1

Duration : 45 minutes **Total : 30 marks**

1. **(6 marks) Mark the following statements True / False. Also, provide justification.**
   (a) **In open addressed hash table, we can insert a key where there is a Tombstone symbol.**
   (b) **O(1) is the worst case time complexity of dequeue in a queue containing M elements implemented using an array.**
   (c) **A hash function must be difficult to compute.**
   (d) **The number of collisions in a hash table is solely dependent on the table capacity and the hash function.**
   (e) **We can traverse a singly linked list backwards starting from last node to the first node.**
   (f) **The element on the top of the stack has been in the stack for the longest.**

2. **(6 marks) Give an efficient algorithm to reverse a linked list. You must use only 3 extra pointers.**

3. **(8 marks) Suppose we use a hash function h to hash $k_1, ..., k_n$ distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions? More precisely, what is the expected cardinality of $\{(k_i, k_j) : i \neq j \wedge h(k_i) = h(k_j)\}$?**

4. **(10 marks) In the dead of night, a master jewel thief is plotting the heist of a lifetime - stealing the most valuable Fabergé Egg from a towering 100 - storey museum. Each floor of the building has an identical egg, but the higher the floor, the more valuable the egg becomes. However, there's a catch. The thief can steal only one egg and she knows that the most valuable egg at the top may not survive a drop from such a great height. To avoid smashing her prized loot, she must identify the highest floor from which an egg can be dropped without breaking. Armed with two replica eggs from the museum's gift shop - perfectly identical but utterly worthless - the thief devises a plan. These two eggs will be her test subjects, sacrificed in the pursuit of the perfect drop. But time is of the essence, and the thief can't afford to be caught by the museum guards. She needs to figure out the minimum number of test drops required to guarantee finding the highest safe floor. Once an egg is broken, it's gone for good - no replacements, no second chances. She can't use any other method to determine the sturdiness of the eggs.**
   Give an algorithm for the thief to determine, with the least number of drops in the worst case, the highest floor from which an egg can be safely dropped without breaking?
   [Hint: Naive solution that does not work: The thief starts dropping test eggs from floor 1 to 100 and stops when an egg breaks. The worst case number of drops is 100, which is not an optimal solution.]