CS215 Moodle Quiz 7 **17 October 2024**

1. In class, we defined the covariance matrix as $\mathbb{E}[(\mathbf{X}-\mu)(\mathbf{X}-\mu)^T]$ and assumed that it is invertible. Suppose now that a random vector $\mathbf{X} \in \mathbb{R}^n$ has mean μ and covariance Σ with $\text{rank}(\Sigma) = r < n$ (so it is not invertible). Then which of the following is (are) true?

   (a) There exists a $j \leq n$ such that X_j is a linear combination (of random variables) $\sum_{i=1}^{j-1} a_i X_i$ of the previous components.
   (b) Each constant density set of the density function of $\mathbf{X}$ is an r-dimensional ellipsoid in $\mathbb{R}^n$.
   (c) The support of $\mathbf{X}$ is contained in an r-dimensional subspace of $\mathbb{R}^n$.
   (d) There exists an $r \times n$ matrix A and vector $\pi \in \mathbb{R}^r$ such that $\mathbf{Y} = A\mathbf{X} + \pi$ has covariance identity.

   Answer: (a), (c), (d)

2. Consider a time series $\{x_t\} = x_0, x_1, \cdots$ and let Δ denote the difference operator, defined by $\Delta x_t = x_t - x_{t-1}$. Which of the following statements is (are) true?

   (a) It is possible to have a non stationary time series $\{x_t\}$ which is ARIMA$(0, 1, 1)$.
   (b) The MA(1) model cannot model non stationary time series.
   (c) Every ARIMA$(p, d, 0)$ model can be represented as an AR(p_a) model for some non negative integer p_a.
   (d) $\{x_t\}$ is weakly stationary iff $\{\Delta x_t\}$ is weakly stationary.

   Answer: (a), (b), (c)

3. Consider a time series $\{x_t\} = x_0, x_1, \cdots$ and define the backshift operator B as $Bx_t = x_{t-1}$ and the difference operator Δ as $\Delta = 1 - B$. Suppose that $\{x_t\}$ satisfies the ARIMA$(p, d, 0)$ model, that is, $\Delta^d x_t$ satisfies the AR(p) model. This means that for some constants $\phi_1, \cdots, \phi_p$, $\phi_p(B)(1 - B)^d x_t = w_t$ for every $t \geq p$, where $\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$. Which of the following statements is (are) true?

   (a) If $p = 0$, a linear regression model regressing x_t on $[1, t, t^2, \cdots, t^{d-1}]$ can be used for prediction of future values.
   (b) If $d = 1$, the time series is stationary.

(c) If $d = 0$, an autoregressive model of order p can be used for prediction of future values.

(d) If $p = 0$, an autoregressive model of order d can be used for prediction of future values.

Answer: (a), (c), (d)

4. It is often desirable to remove correlation for statistical analysis, as it simplifies interpretation. Suppose a random vector $\mathbf{X} = (X_1, X_2, X_3)$ has mean 0 and an invertible covariance matrix $\Sigma = \begin{bmatrix} 1 & 0.5 & 0.4 \\ 0.5 & 1 & 0.3 \\ 0.4 & 0.3 & 1 \end{bmatrix}$. We define $Y = A\mathbf{X}$ and wish to make the components of Y uncorrelated. Which of the following transformations A will remove correlation?

(a) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/3 & -1/5 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -2/3 & -4/15 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -2/3 & -2/15 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/3 & -2/15 & 1 \end{bmatrix}$

Answer: (d)

5. Find the correlation coefficient ρ between the two components of random vector $\mathbf{X} = (X_1, X_2)$, where $X_1 = W_1 - W_2$ and $X_2 = W_1 + W_2$. Here W_1 and W_2 are normally distributed with unit variance and expectation 1, 2 respectively.

Answer: 0