# MA110 - Linear Algebra & Differential Equations: Quiz 1 Answer Key

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**Code:** **X** **Max. marks: 20**

**Tutorial Batch: D 0 T 0** **February 2nd, 2024**

INSTRUCTIONS

1. FAILURE TO FOLLOW INSTRUCTIONS WILL RESULT IN A DEDUCTION OF 2 MARKS.
2. ONLY ANSWERS WRITTEN IN THE BOXES PROVIDED WILL BE GRADED. Be careful when entering your answers, you will not be given a spare question paper in case you make mistakes.
3. Please write your roll no., division and tutorial batch on the answer sheet.
   Exams without roll numbers will be awarded ZERO marks.
4. No work is required to be shown. You may work out the details in the rough sheets, which will not be collected.
5. In Questions (6) - (9), one or more options are correct. The correct options should be entered in the given boxes as follows: $\boxed{\text{(a)}}$, or $\boxed{\text{(a), (c)}}$, etc. The marking scheme followed is:
   (i) All correct options, and no incorrect options chosen: 2 marks
   (ii) Some correct options, and no incorrect options chosen: 1 mark
   (iii) If even one chosen option is incorrect: 0 marks

$\boxed{\text{EXAMS BEGINS}}$

(1) A solution to $x + 2y - z = 6$ in $\mathbb{R}^3$ is $\boxed{(\ 0\ 3\ 0\ )^T}$.
The geometric object given by the set of all solutions is a $\boxed{\text{plane}}$ in $\mathbb{R}^3$, and a subspace of $\mathbb{R}^3$ parallel to it is given by $\boxed{x + 2y - z = 0}$. [2]

(2) Let $D$ be a $5 \times 6$ matrix with pivot columns 1, 4, 5, and $C = P_{12}P_{23}D$.
Then the number of pivots of $C$ is $\boxed{3}$ and
the pivot columns of $C$ will be $\boxed{\text{1, 4, 5,}}$. [1]

NOTE: *Permuting rows does not change the columns in which the pivots occur.*

(3) Let be $C = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 6 & n \end{pmatrix}$ where $n$ is the last digit of your roll number. [2]

Then $\boxed{C = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 6 & \boxed{\text{n}} \end{pmatrix}}$. The echelon form of $C$ is $\begin{pmatrix} 1 & \boxed{\text{-1}} & \boxed{0} \\ 0 & \boxed{3} & \boxed{\text{-1}} \\ 0 & 0 & \boxed{\text{n+2}} \end{pmatrix}$.

NOTE: *Replace $n$ by the last digit of your roll number.*

(4) Let $A = \begin{pmatrix} 1 & 2 & 2 & 5 \\ 2 & 4 & -1 & 0 \\ 0 & 0 & 5 & 10 \end{pmatrix}$. The rank of $A$ is $\boxed{2}$, the number of special solution(s) of $Ax = 0$ is $\boxed{2}$, and the special solution(s) is/are $\boxed{\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -2 \\ 1 \end{pmatrix}}$. [2]

NOTE: *Calculate !*

(5) Fill in the blanks with the correct answer from the given options:
Let $\mathcal{P}$ be the set of all polynomials in $X$ with real coefficients. [2]
Let $S = \{f \in \mathcal{P} : \deg(f) = 2 \text{ or } f = 0\}$. Then:
S $\boxed{\text{is}}$ closed under scalar multiplication. (OPTIONS: is/is not).
S $\boxed{\text{is not}}$ a subspace of $\mathcal{P}$. (OPTIONS: is/is not).

NOTE: *Not closed under vector addition, $x^2 + (-x^2 + x) = x \notin \mathcal{P}$.*

(6) Let $S = \{(x, y) \in \mathbb{R}^2 \mid xy > 0\}$. Which of the following imply that $S$ is not a subspace of $\mathbb{R}^2$?
a) $S$ is not closed under addition.
b) $S$ is not closed under scalar multiplication.
c) $(0, 0)$ is not an element of $S$.
d) $S$ is not a subset of $\mathbb{R}^2$.
The correct options are $\boxed{\text{(a), (b), (c)}}$. [2]

NOTE: *$S$ is not closed under addition $(1, 1) + (-2, -1) = (-1, 0) \notin S$ which implies $S$ is not a subspace, $S$ is not closed under scalar multiplication, $0.(1, 1) = (0, 0) \notin S$ and $S$ does not contain $(0, 0)$. In this example, $S$ is a subset of $\mathbb{R}^2$ and hence (d) will not imply that $S$ is not a subspace.*

(7) Which of the following is the rank of a matrix $F$?
a) The number of free variables in $Fx = 0$.
b) The number of columns of $F$ containing a pivot.
c) The number of non-zero rows of the row reduced form of $F$.
d) The number of rows of $F$ containing a pivot.
e) The number of non-zero columns in the row reduced form of $F$.
The correct options are $\boxed{\text{(b), (c), (d)}}$. [2]

NOTE: *Any row or column can contain at most one pivot.*

(8) Let $\mathcal{M}$ be the vector space of $2 \times 2$ matrices with real entries, under the standard operations. Which of the following is a subspace of $\mathcal{M}$?
a) The set of all skew-symmetric $2 \times 2$ matrices.
b) The set of all scalar multiples of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.
c) The set of all non-invertible $2 \times 2$ matrices.
d) The set $\left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$.
The correct options are $\boxed{\text{(a), (b)}}$. [2]

NOTE: *Verify that the set of all scalar multiples are of the form* $\begin{pmatrix} \lambda & \lambda \\ \lambda & \lambda \end{pmatrix}$, *for all* $\lambda \in \mathbb{R}$ *and will be closed under linear combinations.*
*Skew symmetric matrices are closed under linear combinations.*
*The matrices* $e_{11}$ *and* $e_{22}$ *are not invertible, but their sum is identity which is invertible, hence the set in c) is not a subspace.*
*The set in part (d) is not closed under scalar multiples.*

(9) Which of the following are valid row reduced forms for 3 by 4 matrices?

a) $\begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

The correct options are $\boxed{\text{(b), (c))}}$. [2]

NOTE: *The pivot columns are of the form* $e_i$ *in the reduced form.*

(10) Let $F = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 3 & 0 \\ 7 & -9 & -4 \end{pmatrix}$ [3]

(i) A special solution to $Fx = 0$ is
(ii) A vector $b$ such that $Fx = b$ is consistent
(iii) The particular solution to $Fx = \begin{pmatrix} 3 & -3 & 3 \end{pmatrix}^T$ is

(a) $\begin{pmatrix} -2 & 0 & 1 \end{pmatrix}^T$
(b) $\begin{pmatrix} -3 & 2 & 0 \end{pmatrix}^T$
(c) $\begin{pmatrix} 3 & 2 & 0 \end{pmatrix}^T$
(d) $\begin{pmatrix} -2 & -2 & 1 \end{pmatrix}^T$

Match each option on the left with the correct option on the right:
(i) - $\boxed{\text{(d)}}$ (ii) - $\boxed{\text{(b)}}$ (iii) - $\boxed{\text{(c)}}$

NOTE: *Calculate !*