Endsem – Model Answers

(1)

(a) The dimension of $[D] = L^2T^{-1}$

(b) ... direction of thermal forces is random

or any direction

(c) Nernst potential changes by $\ln 2$

$\Delta V = \ln\left(\frac{C_1}{C_2}\right)$

$\Delta V' = \ln\left(\frac{2C_1}{C_2}\right) = \ln 2 + \ln\left(\frac{C_1}{C_2}\right)$

(No marks if you have just written equation)

(d) .... proteins that are $>> 5nm$ away will not be relevant

(or $\phi \leq 5nm$ will be relevant) ←

(e) .. always goto a state with minimum free energy

(f) .... in a funnel-like free energy ~~land scape~~ Landscape

"funnel" or anything meaning ✓

(No marks for hills and valleys)

(g) $f_{max} = \ln\left(\frac{c}{k}\right) = -\ln\left(\frac{k}{c}\right)$

$f_{max} \propto \ln c$ or directly proportional

(2) $R = \frac{\rho U a}{\eta}$

a = typical size
U = typical speed

$\rho = 10^3$ SI units

~~$\eta = 10^{-3}$~~ $\eta = 10^{-3}$ SI units

$R \approx 10^6 U a$

$U_{bacterium} \approx 1 mm/s$

$a_{bacterium} \approx 1 mm$

$R_{bacterium} = 10^6 \times 10^{-6} \times 10^{-6}$

$R_{bacterium} \approx 10^{-6}$ ⟸ give mark for anything near this

$U_{human} \approx 0.1 \frac{m}{s}$

$a_{human} \approx 1 m$

$R_{human} = 10^6 \times 0.1 \times 1$

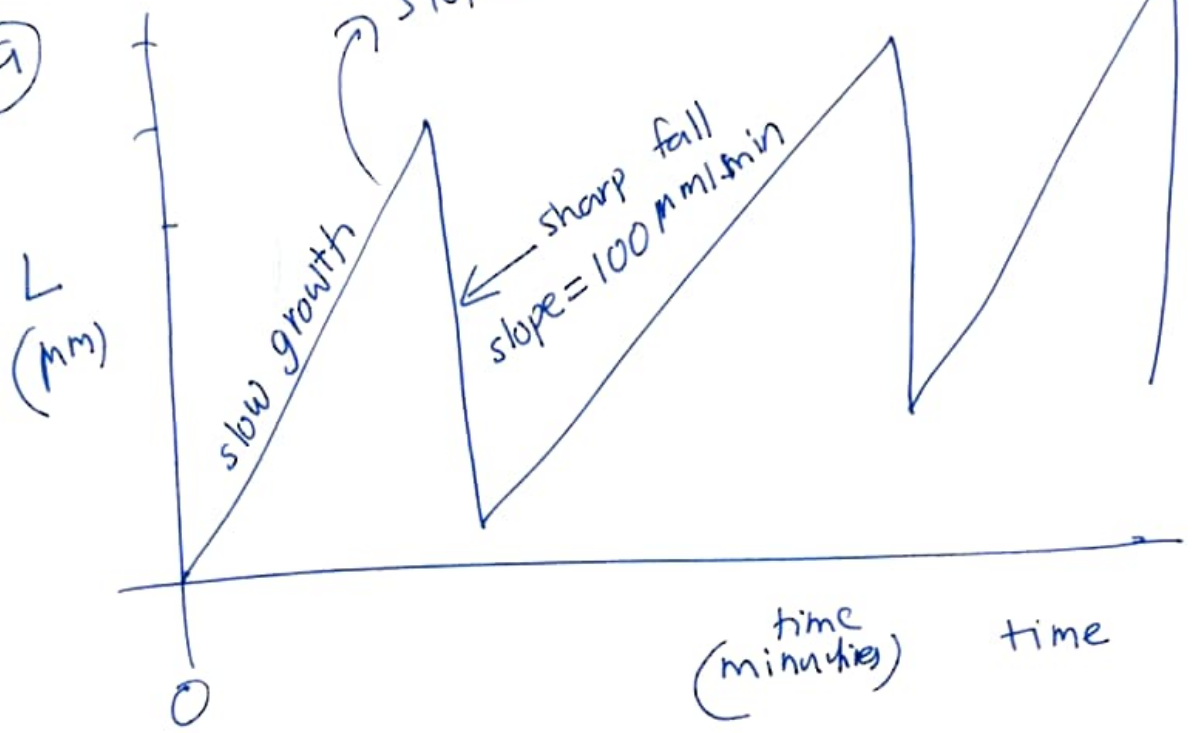
$R_{human} \approx 10^5$

⇑ give mark for anything near this

3

1·5 Mark for plot

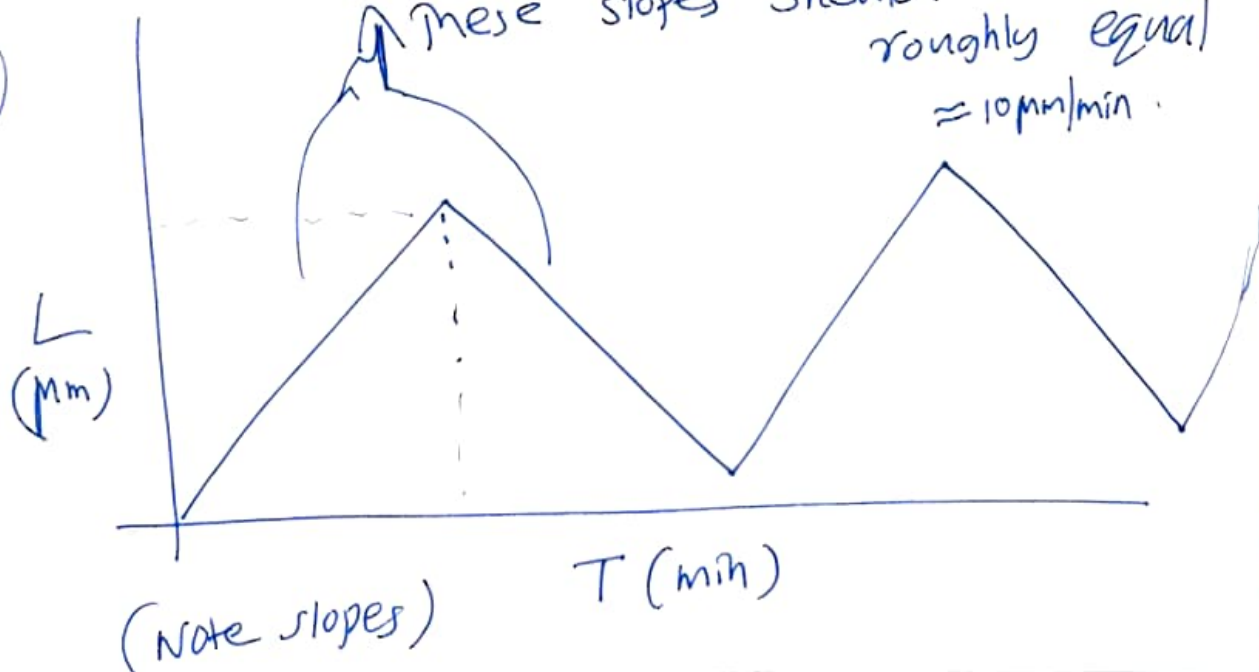
(a)



Check x axis & y axis units & label

→ ½ Mark for label/unit

(b)



(4)

$$n = 100$$

$$p = 0.3$$

at least one $\Rightarrow$ $k \geq 1$

Prob of finding at least one $= \sum_{k=1}^{\infty} G_1(n, k, p)$

$$= 1 - G_1(n, 0, p)$$

$$G_1(n, 0, p) = (0.3)^0 (1-0.3)^n$$

$$= (0.7)^{100}$$

$$\boxed{\text{Ans} = 1 - (0.7)^{100}}$$

5

(A) $C(2) = A e^{-\frac{f2}{K_B T}}$ | A = constant

$C = A e^{-\frac{E}{K_B T}}$ ← ½ mark

~~(A)~~

(B) $J_D = -D \frac{dC}{d2} = -DA e^{-\frac{f2}{K_B T}} \left(\frac{-f}{K_B T}\right)$

$J_D = -DA \left(\frac{-f}{K_B T}\right) e^{-\frac{f2}{K_B T}}$

$J_D = \frac{+Df}{K_B T} C$

5 C

$$V = \frac{f}{6\pi\eta R}$$

$$V = \frac{f}{6\pi\eta R}$$

5D

$$J_D = -J_f \quad \text{or} \quad |J_D| = |J_f|$$

$$\frac{D f C}{K_B T} = \frac{f C}{6\pi\eta R}$$

$$D = \frac{K_B T}{6\pi\eta R}$$

6 a

$$\eta_0(x) = Ax^4 - Bx^2$$

$$\frac{d\eta_0}{dx} = 4Ax^3 - 2Bx$$

minima/maxima $\Rightarrow$ $\frac{d\eta_0}{dx} = 0$

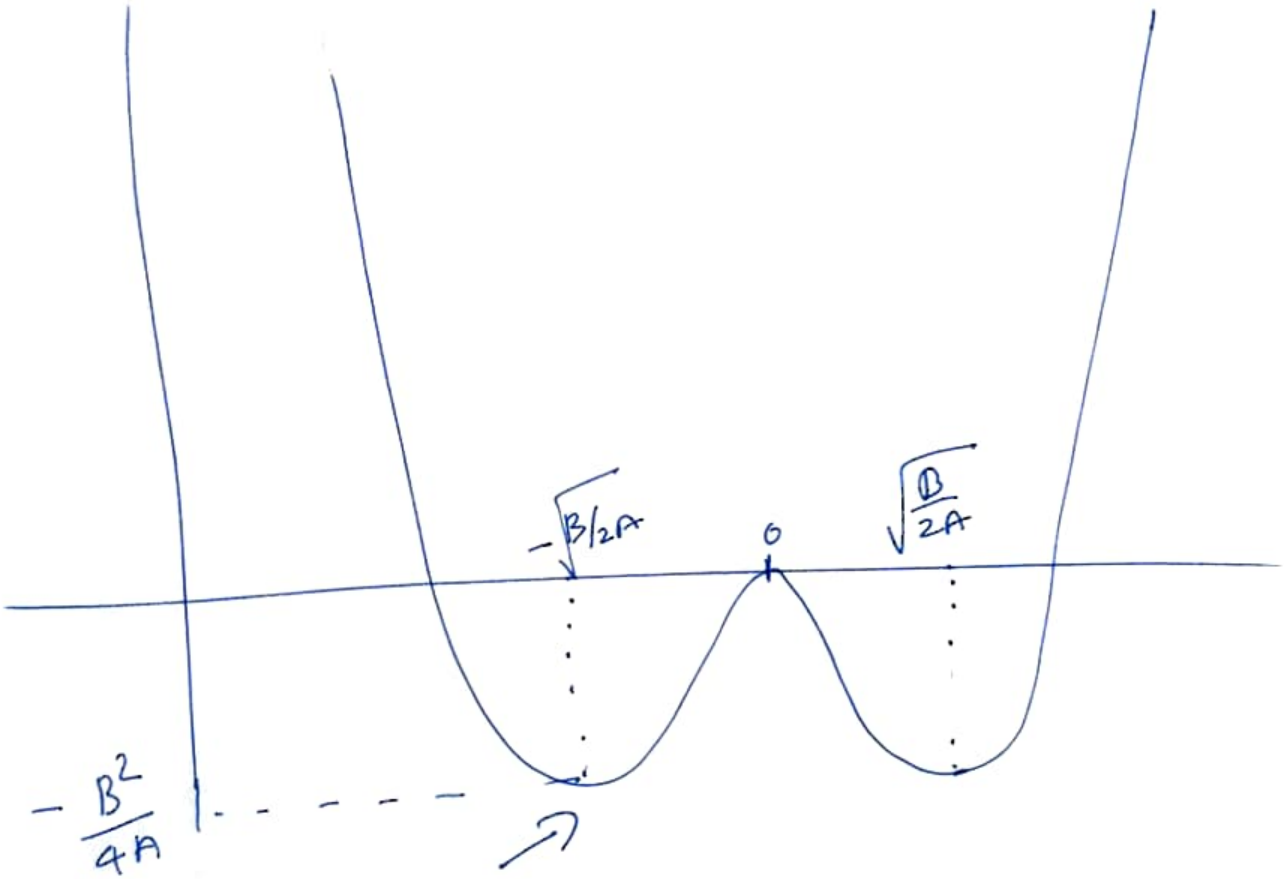
$$4Ax^3 - 2Bx = 0$$

$$x^2 = \frac{2B}{4A}$$

$$x = \pm\sqrt{\frac{B}{2A}} \leftarrow \text{minima}$$

or $x = 0 \leftarrow$ maxima

Pg 9



$G_0(\text{minima}) =$

$f(\text{minima}) = A\left(\frac{B}{2A}\right)^2 - B\frac{B}{2A} = \frac{B^2}{4A} - \frac{B^2}{2A}$

$G_0(\text{at minima}) = -\frac{B^2}{4A}$

Pg 10

6b

$$R = R_0 e^{-\frac{G_{hierisht}}{k_B T}} = R_0 e^{-\frac{(G_2 - G_{min})}{k_B T}}$$

$$R = R_0 e^{-\frac{B^2}{4 A k_B T}}$$

1 Mark

6c very ~~slow~~ slow when

$$\frac{B^2}{4A} > k_B T$$

$$B^2 > 4 A k_B T$$

Pg 11

Prob A = p = Prob T

Prob G = p/2 = Prob C

A = T, G = C

Total Prob = 1

$p + p + \frac{p}{2} + \frac{p}{2} = 1$

$2p + p = 1 \Rightarrow p = \frac{1}{3}$

$P_{of\ G} = \frac{1}{6}$

$S = k \sum_i -p_i \ln p_i$

$= k - \left(p_A \ln p_A + p_T \ln p_T + p_C \ln p_C + p_G \ln p_G \right)$

$= -k \left[\frac{1}{3} \ln \frac{1}{3} + \frac{1}{3} \ln \frac{1}{3} + \frac{1}{6} \ln \frac{1}{6} + \frac{1}{6} \ln \frac{1}{6} \right)$

$$S = -\left(\frac{2}{3} \ln \frac{1}{3} + \frac{2}{6} \ln \frac{1}{6} \right)$$

1 mark for finding $p_{of\ A} = p_T = \frac{1}{3}$

$p_G = p_C = 1/6$

1 mark for S= answer

Pg 12

AA, AT, AG, AC, TT, TA, TG, TC, GG, GA, GT, GC

CC, CT, CG, CA

= 16 possibilities ← 1 mark

all are equally probable

$S = k \ln 16$ ← 1 mark