Example-Lipschitz continuity is a sufficient condition

The initial value problem

$$y' = \sqrt{y} + 1,\ y(0) = 0$$

has a unique solution.

- Use separation of variables, one can see that a solution is given by
$$2\sqrt{y} - 2\ln(1 + \sqrt{y}) = x, x \geq 0.$$
- There is atmost one solution.
(Sketch) If y_1, y_2 are two solutions,
consider
$$y(x) = (\sqrt{y_1(x)} - \sqrt{y_2(x)})^2$$
Then $y \geq 0, y' \leq 0$. This implies $y \equiv 0$.
- $f(x, y) = \sqrt{y} + 1$ is doesn't satisfy Lipschitz condition.