Annihilator Method

Exercise: Get candidate solutions by the annihilator method:

1. $y^{(4)} - y^{(3)} - y'' + y' = x^2 + 4 + x \sin x$.
2. $y^{(4)} - 2y'' + y = x^2 e^x + e^{2x}$.

Review Question - recap

Find the candidate solution for

$$y^{(4)} - y^{(3)} - y'' + y' = x^2 + 4 + x \sin x.$$

Note that the auxiliary/ characteristic equation is

$$m^4 - m^3 - m^2 + m = m(m-1)^2(m+1).$$

Thus a basis of Ker L is $\{1, e^x, xe^x, e^{-x}\}$.
$A_1 = D^3$ is the annihilator of $r_1(x) = x^2 + 4$. Since
$A_1 L = D^4(D-1)^2(D+1)$, the candidate solution w.r.t. $r_1(x)$ is

$$x(ax^2 + bx + c),$$

since a constant is a solution of the homogeneous DE $Ly = 0$.
$A_2 = (D^2+1)^2$ is the annihilator of $r_2(x) = x \sin x$. Since
$A_2 L = (D^2+1)^2 D(D-1)^2(D+1)$, the candidate solution w.r.t. $r_2(x)$ is

$$(\alpha x + \beta)\cos x + (\gamma x + \delta)\sin x.$$

So our final candidate is

$$x(ax^2 + bx + c) + (\alpha x + \beta)\cos x + (\gamma x + \delta)\sin x.$$

Review Question - recap

Find the candidate solution for

$$y^{(4)} - 2y'' + y = x^2 e^x + e^{2x}.$$

Note that the AE is

$$m^4 - 2m^2 + 1 = (m-1)^2(m+1)^2.$$

So a basis of Ker L is $\{e^x, xe^x, e^{-x}, xe^{-x}\}$.
$A_1 = D - 2$ is the annihilator of $r_1(x) = e^{2x}$. Since
$A_1 L = (D-2)(D-1)^2(D+1)^2$, the candidate solution w.r.t. $r_1(x)$ is

$$ae^{2x}.$$

$A_2 = (D-1)^3$ is the annihilator of $r_2(x) = x^2 e^x$. Since
$A_2 L = (D-1)^5(D+1)^2$, the candidate solution w.r.t. $r_2(x)$ is

$$x^2(bx^2 + cx + d)e^x.$$

So final candidate would be

$$ae^{2x} + x^2(bx^2 + cx + d)e^x.$$