A

MA 108 / MA 110 Quiz 3-04-2024 8:15 - 9:05 Maximum Marks: 10

Name :	*RollNo. :*
Divison and Tutorial :	Invig. sign :

General Instructions: 1. This is a question paper cum answer sheet. At the end of the exam, ONLY this sheet will be collected for evaluation. You should write the final answer in the space provided against each question.

2. Books, notes, calculators, mobile phones, electronic devices are NOT permitted.

1. Consider the DE $xy' - xe^{\frac{y}{x}} = y,\ x \neq 0.$ [1+1]

   (i) Let $g(x, y) = 0$ be an implicit solution of the DE. Then

   $\boxed{g(x, y) = e^{-y/x} + \ln|x| - c}$ OR $-\frac{y}{x} = \ln(c - \ln|x|)$

   $c \in \mathbb{R}$

   (ii) If $y = y(x), a < x \leq -\frac{1}{4}$ denotes the solution of the DE with initial condition $y(-1) = 0$, then the minimum possible value of a equals

   $\boxed{-e}$

2. Consider the DE $y + \big(2y + \tan(x + 2y)\big)y' = 0.$ [1+1]

   (i) If $\mu(x, y)$ is an integrating factor for the DE, then

   $\boxed{\mu(x, y) = \cos(x + 2y)}$

   (ii) Let $g(x, y) = 0$ be an implicit solution of the DE. Then

   $\boxed{g(x, y) = y\sin(x + 2y) - c}$, $c \in \mathbb{R}$

3. Let $W(x) = \sin x, x \in (0,\ \pi)$ be the Wronskian of a pair of solutions of the DE $y'' + p(x)y' = 0$, where p is continuous in $(0,\ \pi)$. Then the general solution of the DE is

   $\boxed{y(x) = c_1 \cancel{\sin}\cos x + c_2,\ c_1, c_2 \in \mathbb{R}}$

[1]

4. Let $a, b \in \mathbb{R}$ be such that the DE $y'' + ay' + by = 0$, $x \in \mathbb{R}$ has only bounded solutions. Then all possible values of a, b are [1]

$$\boxed{a = 0, \quad b > 0}$$

5. Consider the DE $x^3y' - x^2y + y^2 \cos\left(\frac{1}{x}\right) = 0$, $x > 0$. Let $y = y(x)$ be a solution of the DE such that $y(\frac{2}{\pi}) = 1$ and $y(\frac{4}{\pi}) = \alpha$. Then [1+1]

(i) for $x \in \mathbb{R}$,

$$y(x) = \frac{\pi x}{2 + \pi - \pi \sin 1/x}, \quad x > 0, \quad \text{undefined for } x \leq 0$$

(ii)

$$\alpha = \frac{4\sqrt{2}}{2\sqrt{2} + (\sqrt{2} - 1)\pi}$$

6. Consider the IVP $y' = x(1 + y), y(0) = 0$. Let $y_n(x)$ denote the nth Picard iterate. Then [1+1]

$$y_2(x) = \frac{x^2}{2} + \frac{x^4}{8}$$

$$y_3(x) = \frac{x^2}{2} + \frac{x^4}{8} + \frac{x^6}{48}$$

B

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1. Consider the DE $xy' - xe^{\frac{2y}{x}} = y,\ x \neq 0.$ [1+1]

   (i) Let $g(x,y) = 0$ be an implicit solution of the DE. Then

   $\boxed{g(x,y) = e^{\frac{-2y}{x}} + 2\ln|x| - c,}$ $c \in \mathbb{R}$ OR $\frac{-2y}{x} = \ln(c - 2\ln|x|)$

   (ii) If $y = y(x), a < x \leq -\frac{1}{4}$ denotes the solution of the DE with initial condition $y(-1) = 0$, then the minimum possible value of a equals

   $\boxed{-\sqrt{e}}$

2. Consider the DE $y + \left(\frac{y}{2} + \tan(x + \frac{y}{2})\right)y' = 0.$ [1+1]

   (i) If $\mu(x,y)$ is an integrating factor for the DE, then

   $\boxed{\mu(x,y) = \cos(x + y/2)}$

   (ii) Let $g(x,y) = 0$ be an implicit solution of the DE. Then

   $\boxed{g(x,y) = y\sin(x + y/2) - c,}$ $c \in \mathbb{R}$

3. Let $W(x) = \cos x, x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ be the Wronskian of a pair of solutions of the DE $y'' + p(x)y' = 0$, where p is continuous in $(-\frac{\pi}{2}, \frac{\pi}{2})$. Then the general solution of the DE is

   $\boxed{y(x) = c_1 \sin x + c_2,\ c_1, c_2 \in \mathbb{R}}$

[1]

4. Let $a, b \in \mathbb{R}$ be such that the DE $y'' + ay' + by = 0$, $x \in \mathbb{R}$ has only bounded solutions. Then all possible values of a, b are [1]

$$a = 0, \quad b > 0$$

5. Consider the DE $x^3 y' - x^2 y + 2y^2 \cos\left(\frac{1}{x}\right) = 0$, $x > 0$. Let $y = y(x)$ be a solution of the DE such that $y(\frac{2}{\pi}) = 1$ and $y(\frac{4}{\pi}) = \alpha$. Then [1+1]

(i) for $x \in \mathbb{R}$,

$$y(x) = \frac{\pi x}{2(1+\pi - \pi \sin 1/x)}, \quad x > 0; \text{ not defined otherwise}$$

(ii)

$$\alpha = \frac{2\sqrt{2}}{\sqrt{2} + (\sqrt{2}-1)\pi}$$

6. Consider the IVP $y' = 2x(1+y), y(0) = 0$. Let $y_n(x)$ denote the nth Picard iterate. Then [1+1]

$y_2(x)$	$=$	$x^2 + \frac{x^4}{2}$
$y_3(x)$	$=$	$x^2 + \frac{x^4}{2} + \frac{x^6}{6}$.

(C)

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   (i) Let $g(x, y) = 0$ be an implicit solution of the DE. Then

   $$g(x, y) = e^{\frac{y}{x}} + \ln|x| - C, \; C \in \mathbb{R} \quad \text{OR} \quad \frac{y}{x} = \ln(C - \ln|x|)$$

   (ii) If $y = y(x), a < x \leq -\frac{1}{4}$ denotes the solution of the DE with initial condition $y(-1) = 0$, then the minimum possible value of a equals

   $$-e$$

2. Consider the DE $y + (3y + \tan(x + 3y))y' = 0.$ [1+1]

   (i) If $\mu(x, y)$ is an integrating factor for the DE, then

   $$\mu(x, y) = \cos(x + 3y)$$

   (ii) Let $g(x, y) = 0$ be an implicit solution of the DE. Then

   $$g(x, y) = y\sin(x + 3y) + C, \; C \in \mathbb{R}$$

3. Let $W(x) = \sin x, x \in (0, \pi)$ be the Wronskian of a pair of solutions of the DE $y'' + p(x)y' = 0$, where p is continuous in $(0, \pi)$. Then the general solution of the DE is

   $$y(x) = C_1 \cos x + C_2, \; C_1, C_2 \in \mathbb{R}$$

   [1]

4. Let $a, b \in \mathbb{R}$ be such that the DE $y'' + 2ay' + 2by = 0$, $x \in \mathbb{R}$ has only bounded solutions. Then all possible values of a, b are [1]

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5. Consider the DE $x^3 y' - x^2 y + \sqrt{2}\, y^2 \cos\left(\frac{1}{x}\right) = 0$, $x > 0$. Let $y = y(x)$ be a solution of the DE such that $y(\frac{2}{\pi}) = 1$ and $y(\frac{4}{\pi}) = \alpha$. Then [1+1]

(i) for $x \in \mathbb{R}$,

$$y(x) = \frac{\pi x}{\sqrt{2}\left(\pi + \sqrt{2} - \pi \sin 1/x\right)}, \ x > 0; \ \text{not defined otherwise}$$

(ii)

$$\alpha = \frac{4}{2 + (\sqrt{2} - 1)\pi}$$

6. Consider the IVP $y' = \frac{1}{2}x(1 + y), y(0) = 0$. Let $y_n(x)$ denote the nth Picard iterate. Then [1+1]

$y_2(x)$	=	$\frac{x^2}{4} + \frac{x^4}{32}$
$y_3(x)$	=	$\frac{x^2}{4} + \frac{x^4}{32} + \frac{x^6}{384}$

(D)

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   (i) If $\mu(x, y)$ is an integrating factor for the DE, then

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   $\boxed{y(x) = c_1 \sin x + c_2 ,\ c_1, c_2 \in \mathbb{R}}$

[1]

4. Let $a, b \in \mathbb{R}$ be such that the DE $y'' - 2ay' - 2by = 0$, $x \in \mathbb{R}$, $y'(0) = 1$ has only bounded solutions. Then all possible values of a, b are [1]

$$\boxed{a = 0, \quad b < 0}$$

5. Consider the DE $x^3y' - x^2y + \frac{1}{2}y^2\cos\left(\frac{1}{x}\right) = 0$, $x > 0$. Let $y = y(x)$ be a solution of the DE such that $y(\frac{2}{\pi}) = 1$ and $y(\frac{4}{\pi}) = \alpha$. Then [1+1]

(i) for $x \in \mathbb{R}$,

$$y(x) = \frac{2\pi x}{4 + \pi - \pi \sin 1/x}, x > 0; \text{ not defined otherwise}$$

(ii)

$$\alpha = \frac{8\sqrt{2}}{4\sqrt{2} + \pi(\sqrt{2} - 1)}$$

6. Consider the IVP $y' = 3x(1+y), y(0) = 0$. Let $y_n(x)$ denote the nth Picard iterate. Then [1+1]

$y_2(x)$	=	$\frac{3x^2}{2} + \frac{9x^4}{8}$
$y_3(x)$	=	$\frac{3x^2}{2} + \frac{9x^4}{8} + \frac{27x^6}{48}$