

Assignment 3

Group 25

November 17, 2018

Solⁿ1 :

$$P(\text{faulty}) = 1/50$$

Company makes a \$3 profit on the sale of the sale of any making of gadget, but suffers a loss of \$80 for every faulty gadget// $E(X) = 49/50 * 3 + 1/50*(-80)$

$$= 147/50 - 80/50$$

$$= 67/50$$

$$= 1.34$$

Since the expected value is positive, the company can expect to make a profit. On average, they make a profit of \$ 1.54 per gadget produced.

Solⁿ2 :

$$M_x = 1/2*(1 + e^t)$$

$$\text{Variance of X } \text{Var}[x] = M'_x - (M''_x)^2$$

$$M'_x = 1/2*e^t$$

$$M'_x(0) = 1/2$$

$$M''_x = 1/2*e^t$$

$$M''_x(0) = 1/2$$

$$\text{Var}[x] = 1/2 - 1/4$$

$$= 1/4$$

Solⁿ3 :

This time until the first result appears is $p_1 = 1$. The random time until a second (different) result appears is geometrically distributed with parameters of success $p_2 = 1/5$, then $p_3 = 1/4$, $p_4 = 1/3$ and so on..

$$\begin{aligned} E[t] &= \frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} + \frac{1}{p_4} + \frac{1}{p_5} + \frac{1}{p_6} \\ &= \frac{1}{1} + \frac{1}{1/5} + \frac{1}{1/4} + \frac{1}{1/3} + \frac{1}{1/2} + \frac{1}{1/6} \\ &= 14.7 \end{aligned}$$

Solⁿ4 :

$P(Q_1) = 0.8$
 cost = \$100
 $P(Q_2) = 0.5$
 cost = \$200

case 1:

$$\begin{aligned} E[x] &= P(Q_1) * p(Q_2^c) + P(Q_1) * P(Q_2) \\ &= 0.8 * 0.5 * 100 + 0.8 * 0.5 * 300 \\ &= 160 \end{aligned}$$

case 2:

$$\begin{aligned} E[x] &= P(Q_2) * p(Q_1^c) + P(Q_2) * P(Q_1) \\ &= 0.5 * 0.2 * 200 + 0.5 * 0.8 * 300 \\ &= 140 \end{aligned}$$

Solⁿ5 :

The number of tosses is a geometric random variable with parameter

$$p' = p * (1 - q) + (1 - p) * q$$

so the mean is $\frac{1}{p' \text{ Variance is } 1 - p'^2}$

Solⁿ6 :

card 2-9 - 5 points

card 10 and face cards - 10 points

Ace - 15 points

$$\begin{aligned} \text{Total mean} &= E[X_1] + E[X_2] + E[X_3] \\ &= \frac{8}{13} * 5 + \frac{4}{13} * 10 + \frac{1}{13} * 15 \\ &= \frac{95}{13} \end{aligned}$$

Solⁿ7 :

There are $\frac{n}{k}$ groups of size k each. X_i random variable that denotes the number of tests performed in group i.

X_i take value 1 with probability $(1 - p)^k$ and value k+1 with probability $1 - (1 - p)^k$ expected no. of tests is

$$= n \frac{1 * (1 - p)^k + (k + 1) * (1 - (1 - p)^k)}{k * (1 - p)^k + (k + 1) * (1 - (1 - p)^k) = n * (1 - (1 - p)^k) + \frac{1}{k}}$$