Mathematics Assignment - 06
Eigenvalues, Eigenvectors, SVD and Linear Transformation

August 31, 2018

Solve the following:

1 Find eigenvalues and corresponding eigenvectors for the given matrices and diagonalize the following matrix, if possible.

$$(1.) \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 3 \\ 6 & -6 & -6 \end{bmatrix} \qquad (2.) \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 5 & 3 & 1 & 1 \\ 4 & 7 & 2 & 2 \end{bmatrix} \qquad (3.) \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 24 & -12 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \tag{1}$$

2 Find an SVD of each matrix.

$$(1.) \begin{bmatrix} 7 & 1 \\ 5 & 5 \\ 0 & 0 \end{bmatrix} \qquad (2.) \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} \tag{2}$$

3 Determine whether the following functions are linear transformations.

$$T = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ y \end{bmatrix} \tag{3}$$

4 Determine whether the following transformations T: $E_3 \to E_3$ are linear.

1. T $((x_1, x_2, x_3)) = (x_1, x_1 - x_2, x_2 + x_3)$

2. T $((x_1, x_2, x_3)) = (x_1, x_2, x_2 x_3)$

3. T $((x_1, x_2, x_3)) = (x_1, 0, 0)$

4. T $((x_1, x_2, x_3)) = (1, 0, 0)$

5 Determine which of the matrices are symmetric.

$$(1.) \begin{bmatrix} -6 & 2 & 0 \\ 2 & -6 & 2 \\ 0 & 2 & -6 \end{bmatrix} \qquad (2.) \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 2 & 2 & 1 & 2 \end{bmatrix} \tag{4}$$

6   For the following pairs of matrices, determine whether A is similar to B.

1. $A = \begin{bmatrix} 2 & 6 & 2 \\ 5 & 1 & -1 \\ 4 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7   If A is symmetric positive definite and C is non-singular, Prove that $B = C^T AC$.