Mathematics Assignment - 03
Probability

October 24, 2018

Solve the following:

1. A company makes electronic gadgets. One out of every 50 gadgets is faulty, but the company doesn't know which ones are faulty until a buyer complains. Suppose the company makes a $3 profit on the sale of any working gadget, but suffers a loss of $80 for every faulty gadget because they have to repair the unit. Check whether the company can expect a profit in the long term.

2. Let X be a random variable with moment generating function

$$M_X(t) = \frac{1}{2}(1 + e^t)$$

   What is the variance of X.

3. What is the average number of times it would it take to roll a fair 6 sided die and get all numbers on the die?

4. Consider a quiz game where a person is given two questions and must decide which one to answer first. Question 1 will be answered correctly with probability 0.8, and the person will receive as prize $100, while question 2 will be answered with probability 0.5, and the person will then receive as prize $200. If the first question attempted is answered incorrectly, the quiz terminates i.e. the person is not allowed to attempt the second question. If the first is answered correctly, the person is allowed to attempt the second question. Which question should be answered first to maximize the expected value of the total prize money received?

5. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability p and second with probability q. All tosses are assumed independent.
   (a) Find the PMF, the expected value, and the variance of the number of tosses.

6. In a certain card game, each card has a point value. Numbered cards in the range 2 to 9 are worth five points each. The card numbered 10 and the face cards (jack, queen, king) are worth ten points each. Aces are worth fifteen points each.
   (a) Suppose that you thoroughly shuffle a 52card deck. What is the expected total point value of the three cards on the top of the deck after the shuffle?
   (b) Suppose that you throw out all the red cards and shuffle the remaining 26 card, all black deck. Now what is the expected total point value of the top three cards? (Note that drawing three aces, for example, is now impossible!)

7. The army needs to identify soldiers with a disease called "klep". There is a way to test blood to determine whether it came from someone with klep. The straightforward approach is to test each

soldier individually. This requires n tests, where n is the number of soldiers. A better approach is the following: group the soldiers into groups of k. Blend the blood samples of each group and apply the test once to each blended sample. If the group blend doesn't have klep, we are done with that group after one test. If the group blend fails the test, then someone in the group has klep, and we individually test all the soldiers in the group.

Assume each soldier has klep with probability, p, independently of all the other soldiers.

What is the expected number of tests as a function of n, p, and k? (Assume for simplicity that n is divisible by k.)