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Group Member Names

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Introduction

System of Linear Equations [Lay]

A linear equation in the variables $x_1, ..., x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b \tag{1}$$

where b and the coefficients $a1, ...a_n$ are real or complex numbers, usually known in advance.

The subscript n may be any positive integer.

Solution of system of linear equations

The graphs of the above equations are lines, which we denote by L_1 and L_2 . A pair of numbers. (x_1, x_2) satisfies both equations in the system if and only if the point .x 1 ; x 2 / lies on both L_1 and L_1.

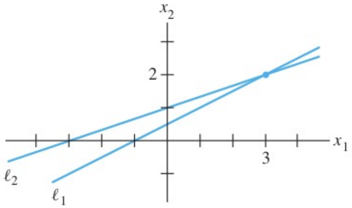

Figure: Exactly one solution

Matrix Notation

The essential information of a linear system can be recorded compactly in a rectangular array called a matrix. Given the system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

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with the coefficients of each variable aligned in columns, the matrix

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}$$

Solved Examples

A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Let the sample space S be $S = \{(b, b), (b, g), (g, b), (g, g)\}$, and all outcomes are equally likely. ((b, g) means, for instance, that the older child is a boy and the younger child a girl.)

Solution

Letting B denote the event that both children are boys, and A the event that at least one of them is a boy, then the desired probability is given by

$$P(A|B) = \frac{P(AB)}{P(B)} \tag{2}$$

$$= \frac{P(\{(b,b)\})}{P(\{(b,b),(b,g),(g,b)\})} \tag{3}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \tag{4}$$

Poisson Random Variable [Ross2014]

Definition

A random variable X, taking on one of the values $0, 1, 2, ...$, is said to be a Poisson random variable with parameter λ, if for some $\lambda > 0$,

$$p(i) = PX = i = e^{\lambda}\frac{\lambda^i}{i!} \qquad i = 0, 1, ... \tag{5}$$

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An important property of the Poisson random variable is that it may be used to approximate a binomial random variable when the binomial parameter n is large and p is small.

Basic Probability

Definition [Ross2014]

Consider an experiment whose sample space is S. For each event E of the sample space S, we assume that a number $P(E)$ is defined and satisfies the following three

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1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. For any sequence of events $E_1, E_2, ...E_n$ that are mutually exclusive, that is, events for which $E_n E_m = \phi$ when $n \leq m$, then

$$P\left(\cup E_n\right) = \sum_{n=1}^{\infty} P(E_n) \tag{6}$$

Problem

If the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$, what is the probability that no accidents occur today ?

Solution

$$P\{X = 0\} = \exp^3 \approx 0.05$$

Lay, D. C.
Linear algebra and its applications, 1997.

Ross, S. M. (2014).
Introduction to probability models.
Academic press.