Assignment 2

Group 25

November 17, 2018

$Sol^n 1:$

Let X be a random variable
P(X) = x^2/a if x = -3, -2, -1, 0, 1, 2, 3
= 0, otherwise

P(-3) = 9/a, P(-2) = 4/a, P(-1) = 1/a, p(0) = 0
P(1) = 1/a, P(2) = 4/a, P(3) = 9/a

$\sum$ P_x(x) = 7

$\frac{9}{a} + \frac{4}{a} + \frac{1}{a} + 0 + \frac{1}{a} + \frac{4}{a} + \frac{9}{a} = 1$

$\frac{28}{a} = 1$

a = 28

E[X] = $\sum$ x.P_x(x)

= -3(9/28) + -2(4/28) + ... + 3.(9/28) = 0

Var[X] = E[X^2] - $[E[X]]^2$

= 196/28 = 7

$Sol^n 2:$

n = 6, Tail = 3

P(getting tail exactly thrice) = ($^6C_3$ $(1/2)^3$ (1/2)) / $(1/2)^6$

= $^6C_3$ / 8

$Sol^n 3:$

P(sale) = 0.15, P(q) = 0.85

a) P(No sale in 10 calls) = $^10C_0$ p^0 q^{1}0

= (0.85)^{1}0

b) x is greater than 3

P(more than 3 sales) = 1 - [P(0) + P(1) + P(2) +P(3)]

= 1 - [(0.85)^{2}0 + 20*0.15*(0.85)^{1}9 + 190*(0.15)^2*(0.85)^{1}8 + 1140(0.15)^3(0.85)^{1}7]

c) np $\geq$ 5

n(0.15) $\geq$ 5

n $\geq$ 33.33

n = 34

d) P(X $\geq$ 1) $\geq$ 0.95

1 - P(X=0) $\geq$ 0.95
0.05 $\geq$ P(X=0)
0.05 $\geq$ (O.85)^n
n = 19

$Sol^n 4:$

Y = Xmod(3)
X = 0 Y = 0
X = 1 Y = 1
X = 2 Y = 2
X = 3 Y = 0
X = 4 Y = 1
X = 5 Y = 2
X = 6 Y = 0
X = 7 Y = 1
X = 8 Y = 2
X = 9 Y = 0
P(Y=0) = P(X=0) + P(X=3) + P(X=6) + P(X=9)
= $\frac{4}{10}$
= $\frac{2}{5}$

P(Y=1) = P(X=1) + P(X=4) + P(X=7)
= $\frac{3}{10}$
P(Y=2) = P(X=2) + P(X=5) + P(X=8)
= $\frac{3}{10}$

$Sol^n 5:$

5 natural children + 2 adopted girls

girls out of 7 = 2,3,4,5,6,7

P(X=2)=$^7C_2$ $(\frac{1}{2})^2$ $(\frac{1}{2})^5$

P(X=3)=$^7C_3$ $(\frac{1}{2})^3$ $(\frac{1}{2})^4$

P(X=4)=$^7C_4$ $(\frac{1}{2})^4$ $(\frac{1}{2})^3$

P(X=5)=$^7C_5$ $(\frac{1}{2})^5$ $(\frac{1}{2})^2$

P(X=6)=$^7C_6$ $(\frac{1}{2})^6$ $(\frac{1}{2})^1$

P(X=7)=$^7C_7$ $(\frac{1}{2})^7$ $(\frac{1}{2})^0$

$Sol^n 6:$

No. of ques. = 8
passing = 5
p(correct) = $\frac{1}{2}$ = p
q = 1-p = $\frac{5}{6}$

P(full marks) = $^8C_8$ $(\frac{1}{6})^8$ $(\frac{5}{6})^0$

P(all wrong)=$(\frac{5}{6})^8$

P(pass) = $^8C_5$ $(\frac{1}{6})^8$ $(\frac{5}{6})^3$ + $^8C_6$ $(\frac{1}{6})^6$ $(\frac{5}{6})^2$ + $^8C_7$ $(\frac{1}{6})^7$ $(\frac{5}{6})^1$ + $^8C_8$ $(\frac{1}{6})^8$ $(\frac{5}{6})^0$

$Sol^n 8:$

$P\,(X=1) = 2P\,(X=2)$

$\frac{\lambda^{x_1} e^{-\lambda}}{x_1!} = \frac{2\lambda^{x_2} e^{-\lambda}}{x_2!}$

$\frac{\lambda^1 e^{-\lambda}}{1!} = \frac{2\lambda^2 e^{-\lambda}}{2!}$

$\lambda = 1$

mean = variance = 1

$Sol^n 9:$

Total members = 9

$P\left(woman\right) = p = \frac{1}{2}$

$P\left(man\right) = 1 - p = \frac{1}{2}$

$P\left(women > men\right) = \binom{9}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^4 + \binom{9}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^3 + \binom{9}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^2 + \binom{9}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^1 + \binom{9}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^0$

$Sol^n 11:$

X = number of red light
$p = \frac{1}{2}, q = \frac{1}{2}$

$X = 0 \Rightarrow P_x\left(X\right) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$

$X = 1 \Rightarrow P_x\left(X\right) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3$

$X = 2 \Rightarrow P_x\left(X\right) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

$X = 3 \Rightarrow P_x\left(X\right) = \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$

$X = 4 \Rightarrow P_x\left(X\right) = \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$

$E[X] = \sum xp\left(x\right)$

$var\left(x\right) = E[X^2] - E[X]^2$

delay = 2X

Var(delay) = var(2X)

= 4var(X)