Mathematics Assignment - 02
Probability

October 15, 2018

Solve the following:

1. Let X be a random variable with PMF
   $p_x(x) = \frac{x^2}{a} \;\; if \;\; x = -3, -2, -1, 0, 1, 2, 3$
   $p_x(x) = 0 \;\; otherwise$
   Find a, E[X], variance of X

2. A coin is flipped 6 times. Find the probability of getting tail exactly three times.

3. The probability of a telesales representative making a sale on a customer call is 0.15. Find the probability that (a) no sales are made in 10 calls, (b) more than 3 sales are made in 20 calls. Representatives are required to achieve a mean of at least 5 sales each day. (c) Find the number of calls each day a representative should make to achieve this requirement. (d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95.

4. Let X be a random variable that takes values from 0 to 9 with equal probability 0.1. Find the PMF of the random variable Y = X mod(3).

5. A family has 5 natural children and has adopted 2 girls. Each natural child has equal probability of being a girl or a boy, independent of the other children. Find the PMF of the number of girls out of the 7 children.

6. Akira is completing a quiz that consists of 8 multiple choice questions and that has a pass mark of 5. Each question has 6 possible answers, only one of which is correct. Akira by nature is too lazy to study for her quiz and randomly guesses the answers to the questions. With reference to the information given, answer the following parts: a) Find out the probability of achieving full marks by Akira. b) Find out the probability that Akira gets all her answers wrong. c) Find out the probability that Akira passes the quiz.

7. The manager of a industrial plant is planning to buy a machine of either type A or type B. For each day's operation the number of repairs X, that the machine A needs is a Poisson random variable with mean 0.96. The daily cost of operating A is $CA = 160 + 40 \times X^2$ For machine B, let Y be the random variable indicating the number of daily repairs, which has mean 1.12, and the daily cost of operating B is $CB = 128 + 40 \times Y^2$ Assume that the repairs take negligible time and each night the machine are cleaned so that they operate like new machine at the start of each day. Which machine minimizes the expected daily cost?

8. If a Poisson variate X is such that P[X = 1] = 2P[X = 2], find the mean and variance of the distribution.

9. A committee consists of 9 members. What is the probability of having more female members than male members provided the probability of having a male or a female member is equal?

10. On a particular spot on a road, on an average there are 5 accidents/month. Find the probability that (a) there will be exactly three accidents in 2 months of time, (b) there will be more than 2 accidents in a given month (c) there will be no accidents in 3 months (d) at most 2 accidents in 2 months.

11. Isha passes through four traffic lights on her way to work, and each light is equally to be green or red independent of the others. (a) What is the PMF, the mean, and the variance of the number of red lights that Isha encounters? (b) Suppose that each red light delays Isha by exactly two minutes. What is the variance of Isha's commuting time?

12. In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.
    random sample of 10 customers is selected.
    a)find probability that exactly 6 ask for water with their meal, (b) find probability that less than 9 ask for water with their meal.
    c) A second random sample of 50 customers is selected.
    ind the smallest value of n such that $P(X < n) \geqslant 0.9$here random variable represents the number of these customers who ask for water.