1 Introduction

Linear algebra is the math of vectors and matrices. Let n be a positive integer and let R denote the set of real numbers, then R^n is the set of all n-tuples of real numbers. A vector $V \in R^n$ is an n-tuple of real numbers. The notation "$\in$ S" is read "element of S." For example, consider a vector that has three components: $V = (v_1, v_2, v_3) \in (R, R, R)$ i.e R^3. A matrix $A \in R^m \times n$ is a rectangular array of real numbers with m rows and n columns. For example, a 3×2 matrix looks like this: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \in \begin{bmatrix} R & R \\ R & R \\ R & R \end{bmatrix}$

2 Defination

2.1 Vector Operations

The operations we can perform on vectors $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are :

2.1.1 Addition

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

2.1.2 Subtraction

$uv = (u_1 - v_1, u_2 - v_2, u_3 - v_3)$

2.1.3 Scaling

$\alpha u = (\alpha u_1, \alpha u_2, \alpha u_3)$

2.1.4 Norm Length

$$\|u\| = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2}$$

2.1.5 Dot Product

$u\Delta v = u_1v_1 + u_2v_2 + u_2v_2$

2.1.6 Cross Product

$u \times v = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$

2.2 Matrix operations

We denote by A the matrix as a whole and refer to its entries as a_{ij}. The mathematical operations defined for matrices are the following:

2.2.1 Addition (denoted +)

$C = A + B \Leftrightarrow c_{ij} = aij + bij$

2.2.2 Subtraction (the inverse of addition)

$C = A - B \Leftrightarrow c_{ij} = aij - bij$

2.2.3 Matrix Product

The product of matrices $A \in R^{m\times n}$ and $B \in R^{n\times p}$ is another matrix $C \in R^{m\times p}$, given by the formula

$C = AB \Leftrightarrow c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{bmatrix}$$

Note that the matrix product is not a commutative operation: AB $\neq$ BA.

2.2.4 Matrix Transpose (denoted A^T)

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}^T = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

2.2.5 Matrix Trace: Tr[A]

$Tr[A] = \sum_{i=1}^{n} a_{ii}$

2.2.6 Determinant (denoted det(A) or $|A|$)