Mathematics for IT - Assignment 07
Linear Algebra

August 31, 2018

Question 01

State True or False. **Justify your answer**

1. If $AB = 0$, then the column space of B is in the nullspace of A
2. If P is a projection matrix, so is I - P
3. If two matrices have equal reduced row echelon forms, then their column spaces are equal
4. If A is symmetric matrix, then its column space is perpendicular to its nullspace
5. If a subspace S is contained in a subspace V, then $S^{\perp}$ contains $V^{\perp}$
6. If A is an orthogonal matrix, there exists an orthonormal basis of eigenvectors for A
7. If A and B are matrices whose eigenvalues, counted with their algebraic multiplicities, are the same, then A and B are similar
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Question 02

Consider the following system of equations

$$\begin{aligned} x + 2y + 2z &= 2 \\ 2x + 2y + 3z &= 1 \\ 3x + 2y + 4z &= 2 \end{aligned} \tag{1}$$

Find a vector $\mathbf{y}$ for above system such that $A^T\mathbf{y} = 0$ and $\mathbf{y}^T\mathbf{b} = 1$.

Question 03

Let L be the line through the origin in $\Re^3$ which is parallel to the vector $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

1. Find the standard matrix of the orthogonal projection onto L.
2. Find the point on L which is closest to the point (1, 0, 0).

Question 04

Let $\mathbf{x_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{x_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and let P be the plane through the origin spanned by $\mathbf{x_1}$ and $\mathbf{x_2}$. Find an orthonormal basis of P as well as the standard matrix of the orthogonal projection onto P.

Question 05

Inside of $\Re^3$, consider the subset of vectors $\begin{bmatrix} a \\ b \\ a \end{bmatrix}$. satisfying the following requirements. Which of them are subspaces?

1. a and b are both zero.
2. a is zero or b is zero or both are zero.
3. a and b are equal.
4. a, b are both positive, both negative, or both zero.

Question 06

In the vector space of polynomials P_3 , determine if the set S is linearly independent or linearly dependent where

$$S = \{2 + x - 3x^2 - 8x^3.1 + x + x^2 + 5x^3, 3 - 4x^2 - 7x^3\} \tag{2}$$

Question 07

Define the linear transformation $T : \mathcal{C}^3 \to \mathcal{C}^2$, $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 5x_3 \\ -4x_1 + 2x_2 - 10x_3 \end{bmatrix}$

Verify that T is a linear transformation.

Question 08

If $T : \mathcal{C}^2 \to \mathcal{C}^2$, $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Find $T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)$

Question 09

Given $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}$

1. Find an orthonormal basis of $\Re^3$ consisting of eigenvectors for A
2. Find a 3×3 orthogonal matrix S and a 3×3 diagonal matrix D such that $A = SDS^T$

Question 10

Suppose A is a 4×4 identity matrix with its last column removed. A is now 4×3. Project $\mathbf{b} = (1, 2, 3, 4)$ onto the column space of A. Determine the projection matrix P.

Question 11

Find the singular values as well as SVD for $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Question 12

Construct a basis of $\Re^3$ consisting of eigenvectors of the following matrices:

$A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$