rk:5

Part A:

1 Each student's performance across subjects can be represented as a vector:

$S_i =$ (Math, Physics, chemistry, biology)

Thus, every student is a 5-d vector R^5

→ Each component corresponds to one subject score

→ The vector captures overall academic performance.

2 Norm 1 & Norm 2 of vector

→ Norm : 1 (Manhattan norm) represents the total academic score across all subjects.

$$\|S_i\|_1 = |x_1| + |x_2| + \dots + |x_5|$$

→ Norm: 2 (Euclidean norm) represents magnitude of a students performance vector:

$$\|S_i\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_5^2}$$

It reflects the overall strength of a students performance.

A norm is a function that measures the magnitude or length of vector

3 Dot Product & angle between 2 student vector

→ The dot product measures similarity between two student's performance:

$S_i . S_j$

→ The angle between vectors indicates how similar or different 2 students are:

Small angle: similar performance pattern

Large angle: different strengths & weakness

→ This is useful for comparing student performance profiles.

→ Angle betn 2 vectors = $\cos\theta = \frac{x \cdot y}{||x|| \, ||y||}$

4 Cross Product:-

→ when considering only 3 subjects:

Maths, Physics, chemistry.

→ The cross product produces a vector orthogonal to both student vectors in 3D space

→ It highlights independent variation between two students performance in those subject.

5 Projection of one vector in another:

→ Vector projector represents how much one student's performance aligns with another's:

→ Projection of student A onto student shows:

The portion of A's performance explained by B's performance pattern

→ This is useful for performance benchmarking.

Part B.

6 Matrix addition 4 multiplication:

→ The dataset can be represented as:

$$X = \begin{bmatrix} \text{Student 1} \\ \text{Student 2} \\ \vdots \\ \text{Student n} \end{bmatrix}$$

Rows: Students
Column: Subject

→ It allows:
Addition: Combining datasets
Multiplication: Transformations 4 similarity analysis

Transpose 4 inverse

→ Transpose - It swaps rows 4 columns, useful for covariance computation

Eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

→ Inverse :- If it exists it helps in solving linear systems.

→ Determinant :- It indicates whether a matrix is invertible 4 whether the data has linear dependency

Eg: $A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$, $|A| = (1\times4) - (2\times3)$

→ $|A| = -2$

Part C:

8 → Exp line, plane 4 hyperplane:

→ Line (1D) :- It defines performance in one subject.

→ Plane (2D) :- It defines performance in 2 subjects.

→ HyperPlane :- full dataset with 5 subject

Date.

9. Show how dimensionality increases from 2D → 3D

→ 2 subjects → 2D plane
3 subjects → 3D plane
5 subjects → 5D hyperplane

→ Higher dimensions allows richer representation but reduce visual interpretability, motivating dimensional reduction.