

## What is Probability?

- Probability is a measure of likelihood that a particular event will occur.
- Formula:  $P(E) = \frac{\text{No. of outcomes}}{\text{Total outcomes}}$

Eg: Rolling a dice, what is Prob of 4?

$$P(4) = \frac{1}{6} = 0.1667$$

Outcomes are 6 at a time of throwing

Probability of getting 4 is 1/6

Probability of getting 4

→ Experiment: An activity with uncertain results, like rolling a die or drawing

→ Outcome: A single possible result of

an experiment

→ Sample space: The collection of all possible outcomes

→ Event (E): A specific outcome or a group of outcomes we're interested

→ Probability: A no. from 0 to 1 show how likely an event is.

→ Favourable: The outcomes that satisfy the event we're looking for

3 Eg:-

$$1] P(\text{study hours} < 4) = \frac{3}{20}$$

$$2] P(\text{Pass}) = \frac{11}{20}$$

$$3] P(\text{group-discussion - Yes}) = \frac{12}{20} = \frac{3}{5}$$

4 Empirical Probability:-

What is probability that a student passes in final exam?

$$P(\text{Pass}) = \frac{\text{No. of Pass in final exam}}{20} = \frac{11}{20}$$

Theoretical Probability:-

If each student is equally likely to study any no. of hours from 0-10, what is prob. a random chosen student studies at least 8 hours per week?

$$P(\text{study hours} \geq 8) = \frac{3}{11}$$

5 Random variable

A random variable is a numerical quantity whose value depends on outcome of a random phenomenon or experiment

→ Let  $x = \text{No. of students pass in exam}$

$$P(x \in \{0, 1, 2, 3\})$$

Table:

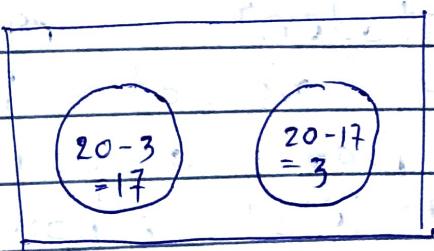
$x$	$P(x)$	$x \times P(x)$
0	0.027	0
1	0.189	0.189
2	0.441	0.882
3	0.343	1.029

$$\text{Mean} = \sum (x_i \times P(x_i)) = 0 + 0.189 + 0.882 + 1.029 = 2.1$$

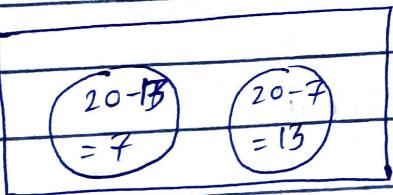
$$\text{Variance} = \sum (x_i^2 \cdot P(x_i)) - \mu^2 = 3 \times 0.7 \times 0.3 = 0.63$$

6 Venn diagram:

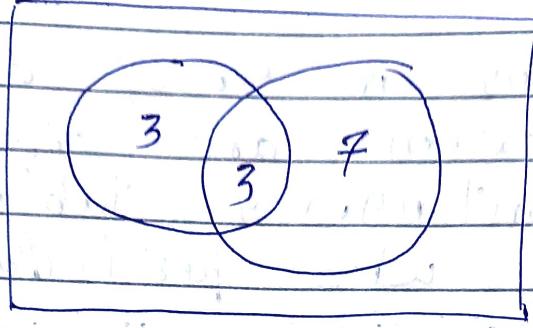
→ Students who study more than 10 hrs



→ Students has 80% attendance



→ Overlap:



7 Contingency table:

	Yes	No	Total
Pass	11	3	14
fail	10	5	15
Total	14	8	20

→ Joint Probability :-  $P(\text{Yes} \cap \text{Pass}) = \frac{11}{20}$

→ Marginal Probability :-  $P(\text{Pass}) = \frac{14}{20} = \frac{7}{10}$

→ Conditional :  $P(\text{Yes} \cap \text{Pass} | \text{Pass}) = \frac{11}{20} \times \frac{10}{7} = \frac{110}{140}$

$$\text{Probability by } = \frac{11}{14}$$

## 8 Understanding relationships

- Participating in group discussion and passing exam are dependent events.
- From contingency table, among students who participated in group discussions, 11 out of 12 are passed, so

$$P(\text{Pass} | \text{Yes}) = \frac{11}{12} = 0.92$$

- Among students who did not pass participated, only 3 out of 8 passed so

$$P(\text{Pass} | \text{No}) = \frac{3}{8} = 0.375$$

## 9 Bayes theorem:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

A

A = Student passed exam

B = Student has high attendance (80%)

$$\rightarrow P(A) = \frac{14}{20} = \frac{7}{10}$$

$$\rightarrow P(B) = \frac{7}{20}$$

$$\Rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{7}{20}}{\frac{7}{10}} = \frac{7 \times 10}{20} = \frac{1}{2}$$

$$\Rightarrow P(B|A) = 1/2$$

$$\Rightarrow P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$$= \frac{\frac{1}{2} \times \frac{7}{10}}{\frac{7}{20}} = \boxed{\frac{1}{2}}$$