Set - Valued Information System ①

U	C_1	C_2	C_3	D
u_1	{1,2,3,4}	{0,1}	{1,2}	0
u_2	{2,3}	{2,3}	{1}	0
u_3	{1,2,3,4}	{1,2}	{1,2}	1
u_4	{2,3,4}	{0,1,2,3}	{0,1}	0
u_5	{2,4}	{0,1,2}	{0,1}	1

R_{C_1}

Here → objects $U = \{u_1, u_2, u_3, u_4, u_5\}$

$C = \{C_1, C_2, C_3\}$ → conditional attributes

D → decisional attribute

$A = C \cup D$ → attributes

Fuzzy tolerance Relation (degree of Similarity)

$$\mu_{R_C}(u_i, u_j) = \frac{|C(u_i) \cap C(u_j)|}{|\sqrt{|C(u_i)||C(u_j)|}|}$$

Now, degree of similarity of u_1 & u_2 w.r.t attribute $\{C_1\}$.

$$\mu_{R_{C_1}}(u_1, u_2) = \frac{|C_1(u_1) \cap C_1(u_2)|}{|\sqrt{|C_1(u_1)||C_1(u_2)|}|}$$

$$= \frac{|\{1,2,3,4\} \cap \{2,3\}|}{|\sqrt{|\{1,2,3,4\}||\{2,3\}|}|}$$

$$= 1/|\sqrt{2}| = .71$$

$$\mu_{R_{C_1}}(u_2, u_3) = \frac{|C_1(u_2) \cap C_1(u_3)|}{|\sqrt{|C_1(u_2)||C_1(u_3)|}|}$$

$$= \text{———}$$

Similarly we can calculate similarity degree between other objects w.r.t attribute $\{C_1\}$.

And we can find Relational matrix.

R_{C_1} =	U_1	U_2	U_3	U_4	U_5
U_1	1	·71	1	·86	·71
U_2	·71	1	·71	·82	·5
U_3	1	·71	1	·86	·71
U_4	·86	·82	·86	1	·82
U_5	·71	·5	·71	·82	1

Similarly we can find degree of similarity between other objects w.r.t attribute $\{C_2\}$.

$$\mu_{R_{C_2}}(U_1, U_2) = \frac{|C_2(U_1) \cap C_2(U_2)|}{|\sqrt{|C_2(U_1)||C_2(U_2)|}|}$$

$$= \frac{|\{0,1\} \cap \{2,3\}|}{|\sqrt{|\{0,1\}||\{2,3\}|}|}$$

$$= 0$$

Now we can find Relation matrix w.r.t attribute C_2

R_{C_2} =	U_1	U_2	U_3	U_4	U_5
U_1	1	0	·5	·71	·82
U_2	0	1	·5	·71	·40
U_3	·5	·5	1	·71	·82
U_4	·71	·71	·71	1	·86
U_5	·82	·41	·82	·86	1

Now for a set of attributes $B \subseteq C \subseteq A$. (3)

$$\mu_{R_B}(u_i, u_j) = \inf \mu_{R_B}(u_i, u_j)$$

let $B = \{c_1, c_2\}$

$$\mu_{R\{c_1, c_2\}}(u_i, u_j) = \inf \{\mu_{R\{c_1\}}(u_1, u_2), \mu_{R\{c_2\}}(u_1, u_2)\}$$

Relational Matrix w.r.t $B = \{c_1, c_2\}$.

$\mu_{R\{c_1, c_2\}} =$

	u_1	u_2	u_3	u_4	u_5
u_1	1	0	.5	.71	.71
u_2	0	1	.5	.71	0.4
u_3	0.5	0.5	1	0.71	0.71
u_4	0.71	0.71	0.71	1	0.82
u_5	0.71	0.41	.71	.82	1.

Similarly we can find for $\{c_2, c_3\}$, $\{c_1, c_3\}$, $\{c_1, c_2, c_3\}$.

Now we need to find Relational matrix w.r.t decision attribute $\{d\}$. $r_{ij} = 1$ if $u_i = u_j$
$= 0$ if $u_i \neq u_j$

Relation matrix

$$\begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{pmatrix}$$

$$\begin{array}{c|ccccc} & u_1 & u_2 & u_3 & u_4 & u_5 \\ \hline u_1 & 1 & 1 & 0 & 1 & 0 \\ u_2 & 1 & 1 & 0 & 1 & 0 \\ u_3 & 0 & 0 & 1 & 0 & 1 \\ u_5 & 1 & 1 & 0 & 1 & 0 \\ u_5 & 0 & 0 & 1 & 0 & 1 \end{array}$$

Now Step - 3 :

$$|[U_i]_{RC}| = \sum_{j=1}^{n} r_{ij} \quad \text{where } [U_i]_{RC} = \frac{r_{i1}}{U_1} + \frac{r_{i2}}{U_2} + \dots \frac{r_{in}}{U_n}$$

Step

$$|[U_1]_{RC_1}| = 1 + .71 + 1 + .86 + .71$$
$$= 4.28 .$$

$$[U_1]_{RC_1} = \frac{r_{11}}{U_1} + \frac{r_{12}}{U_2} + \dots \frac{r_{15}}{U_5} = \frac{1}{U_1} + \frac{.71}{U_2} + \frac{1}{U_3} + \frac{.86}{U_4} + \frac{.71}{U_5}$$

Step

Step -4. Information entropy of Fuzzy Rough set

$$H(\hat{R}) = \frac{-1}{m} \sum_{i=1}^{m} \log_2 \frac{|[U_i]_{\hat{R}}|}{m}$$

where m is no of objects.

Here m = 5.

How to calculate entropy. $P \subseteq C$ conditional attribut

$P = \{C_1\}$.

St

$$H(P) = \frac{-1}{m} \sum_{i=1}^{m} \log_2 \frac{|[U_i]_P|}{m}$$
$$= -\frac{1}{5} \left\{ \log_2 \frac{|[U_1]_{RC_1}|}{5} + \log_2 \frac{|[U_2]_{RC_1}|}{5} + \dots + \log_2 \frac{|[U_5]_{RC_1}|}{5} \right\}.$$

$$= \_\_\_\_\_\_$$

$$\_\_\_\_\_\_$$

Similarly we can calculate for $\{C_2\}$, $\{C_3\}$, $\{C_1, C_2\}$, $\{C_1, C_3\}$, $\{C_1, C_2, C_3\}$. or D

$P \subseteq C$, D - decisional attribute.

The conditional entropy of D condition to P — (5).

St

$$H(D|P) = -\frac{1}{m}\sum_{i=1}^{m} \log_2 \frac{|[u_i]_{\hat{P}} \cap [u_i]_D|}{|[u_i]_P|}$$

St

$|[u_i]_P \cap [u_i]_D| \Rightarrow$ take $P = \{C_1\}$.

$$[U_1]_{C_1} = \frac{1}{U_1} + \frac{.71}{U_2} + \frac{1}{U_3} + \frac{.86}{U_4} + \frac{.71}{U_5}.$$

$$[U_1]_D = \frac{1}{U_1} + \frac{1}{U_2} + \frac{0}{U_3} + \frac{1}{U_4} + \frac{0}{U_5}$$

$$[U_1]_{C_1} \cap [U_1]_D = \frac{1}{U_1} + \frac{.71}{U_2} + \frac{0}{U_3} + \frac{.86}{U_4} + \frac{0}{U_5}$$

$$|[U_1]_P \cap [U_1]_D| = 1 + .71 + 0 + .86 + 0$$

$= \_\_\_\_$

$$H(D|P) = -\frac{1}{5}\left\{ \log_2 \frac{|[U_1]_P \cap [U_1]_D}{|[U_1]_P|} + \log_2 \frac{|[U_2]_P \cap [U_2]_D|}{|[U_2]_P|} + \dots + \log_2 \frac{|[U_5]_P \cap [U_5]_D|}{|[U_5]_P|} \right\}.$$

$= \_\_\_\_$

$$\hat{H}(D|P) = \frac{-1}{m}\sum_{i=1}^{m}\log\frac{|[u_i]_{\hat{P}} \cap [u_i]_{\hat{D}}|}{|[u_i]_{\hat{P}}|}$$

Step-6 Mutual information of P and D has the following definition.

$$\hat{I}(P;D) = \hat{H}(D) - \hat{H}(D|P)$$

Step-7. $(U, C \cup D)$ be a fuzzy decision system. $\forall\, c \in C-P$. The gain of attribute c, $\hat{G}(c,P,D)$ has the following definition:

$$\hat{G}(c,P,D) = \hat{I}(P \cup \{c\};D) - \hat{I}(P;D)$$
$$= \hat{H}(D|P) - \hat{H}(D|P \cup \{c\})$$

If $P = \phi$, $\hat{G}(c,P,D) = \hat{H}(D) - \hat{H}(D|\{c\}) = I(\{c\};D)$

Attribute c is more significance for decision attribute D, and then $G(c,P,D)$ is higher.

Step-8. The Mutual information gain ratio has the following definition:

$$\hat{G}R(c,P,D) = \frac{\hat{G}(c,P,D)}{\hat{H}(\{c\})}$$
$$= \frac{\hat{H}(D|P) - \hat{H}(D|P \cup \{c\})}{H(\{c\})}.$$

If $P = \phi$

$$\hat{G}R(c,P,D) = \frac{\hat{H}(D) - \hat{H}(D|\{c\})}{H(\{c\})}$$

tember 12,

Set - Valued data-

Algorithm: $\hat{GR}$ algorithm for reduct in a set valued information System-

Input : A Incomplet set valued decision system $= (U, C \cup D)$

Output: One attribute set P

```
Initialize : P ← φ, start ← 1;
while start do
    for each attribute c ∈ C - P do
        Calculate the gain radion of the attribute a,
        GR^ = (C, P, D);
    end
    Select the attribute a which its GR^ = (C, P, D) has
    max value;
    if GR^ = (C, P, D) > 0 then
        P ← P ∪ {c};
    else
        start ← 0;
    end
end
obtain one reduct P.
```