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÷ Maths Assignment ÷

Q1 What is Matrix ? Define at least 5 Matrix ?

Ans Matrix is a collection of rows & Columns.

a) Row Matrix ⇒ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}_{1\times 3}$

b) Column Matrix → $\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}_{3\times 1}$

c) Scalar Matrix ⇒ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$. Diagonals Elements are Same & other Elements are Zero

d) Square Matrix ⇒ no. of rows = no. of Columns.

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$

e) Rectangular Matrix → No. of rows ≠ no. of Columns.

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$

Q2 Find the Multiplication

$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 5 \end{bmatrix}$

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$$AB = \begin{bmatrix} -2+6 & 1-4+9 & 3-8+15 \\ 3-2 & 2+6-3 & 6+12-5 \\ 1+4 & -3+2+6 & -9+4+10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 6 & 10 \\ 1 & 5 & 13 \\ 5 & 5 & 5 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1+0 & -2 & 2-4 \\ -1 & 0 & -2 \\ 2 & -1 & 4-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ -1 & 0 & -2 \\ 2 & -1 & 2 \end{bmatrix}$$

Q 3 find the determinant

a) $A = \begin{bmatrix} 4 & 7 \\ -5 & 9 \end{bmatrix} \Rightarrow |A| = 36+35$

$= 71$ Ans.

b) $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix} \Rightarrow 3(12-3) - 2(4-5) + 2(3-15)$

$\Rightarrow 3(9) - 2(-1) + 2(-12)$

$\Rightarrow 27 + 2 - 24 \Rightarrow 5$ Ans.

c) $A = \begin{bmatrix} -5 & 8 & -5 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix} \quad |A| = -5(12-12) - 8(6-6) + 5(12-12)$

$= 0$ Ans.

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Q1 Find the cofactors:?

a) $A \Rightarrow \begin{bmatrix} 11 & 9 \\ 21 & -16 \end{bmatrix}$

- Minors → $M_{11} = -16$, $M_{12} = 21$, $M_{21} = 9$, $M_{22} = 11$

Cofactors → $A_{11} = -16$, $A_{12} = -21$, $A_{21} = -9$, $A_{22} = 11$

b) $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

- Minors → $M_{11} = -3$, $M_{12} = 1$, $M_{13} = 5$, $M_{21} = -1$, $M_{22} = -1$, $M_{23} = -1$, $M_{31} = 7$, $M_{32} = -5$, $M_{33} = -13$

- Cofactors → $A_{11} \Rightarrow -3$, $A_{12} \Rightarrow -1$, $A_{13} \Rightarrow 5$, $A_{21} \Rightarrow +1$, $A_{22} = -1$, $A_{23} = 1$, $A_{31} = 7$, $A_{32} = 5$, $A_{33} = -13$.

Q2 Find the adjoint & inverse

a) $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 2 \end{bmatrix}$, $|A| = 1(18-12) - 1(2-3) + 2(4-9)$

$= 6 + 1 - 10$

$|A| \Rightarrow -3$

- Minors $A_{11} \Rightarrow 6$, $A_{12} \Rightarrow -1$, $A_{13} \Rightarrow -5$, $A_{21} = -6$, $A_{22} = 0$, $A_{23} = 3$, $A_{31} = -15$, $A_{32} = 1$, $A_{33} = 8$

- Cofactor → $M_{11} = 6$, $M_{12} = 1$, $M_{13} = -5$, $M_{21} = 6$, $M_{22} = 0$, $M_{23} = -3$, $M_{31} = -15$, $M_{32} = -1$, $M_{33} = 8$

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$$\text{adj } A = \begin{bmatrix} 6 & 6 & -15 \\ 1 & 0 & -1 \\ -5 & -3 & 8 \end{bmatrix}$$

$$A^{-1} \Rightarrow \frac{1}{|A|}(\text{adj } A) = \frac{1}{-3}\begin{bmatrix} 6 & 6 & -15 \\ -1 & 0 & -1 \\ -5 & -3 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -2 & 5 \\ -1/3 & 0 & 1 \\ 5 & 1/3 & 8/3 \end{bmatrix}$$

b) $$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}, \quad |A| = 1(-28+30) - 1(-18)$$
$$= 2 + 18 \Rightarrow 20$$

- Minor $\Rightarrow$ $A_{11} \Rightarrow 2$, $A_{21} \Rightarrow -6$, $A_{31} = 4$
  $A_{12} \Rightarrow -21$, $A_{22} \Rightarrow -7$, $A_{32} = 8$
  $A_{13} \Rightarrow -18$, $A_{23} \Rightarrow -6$, $A_{33} = 4$
- Cofactor $\Rightarrow$ $M_{11} \Rightarrow 2$, $A_{21} \Rightarrow 6$, $A_{31} = 4$
  $M_{12} \Rightarrow 21$, $A_{22} = -7$, $A_{32} = -8$
  $M_{13} \Rightarrow -18$, $A_{23} = 6$, $A_{33} = 4$

- $$\text{adj } A = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

- $$A^{-1} \Rightarrow \frac{1}{20}\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

Ques Solve the following Eq^n by Cramer's rule:

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Q1. $2x+3y-4z=-8$

$3x+2y+4z=3$

$3x-4y+5z=18$

$$A=\begin{bmatrix}2 & 3 & -4\\ 3 & 2 & 4\\ 3 & -4 & 5\end{bmatrix} \quad B=\begin{bmatrix}-8\\ 3\\ 18\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$|A| = 2(10+16) - 3(15-12) - 4(-12-6)$

$\Rightarrow 52 - 9 + 72$

$= 115$

$$\Delta x = \begin{bmatrix}-8 & 3 & -4\\ 3 & 2 & 4\\ 18 & -4 & 5\end{bmatrix} \Rightarrow -8(10+16) - 3(15-72) - 4(-12+36)$$

$\Rightarrow -208 + 171 + 192$

$\Delta x \Rightarrow 155$

$x \Rightarrow 155/115$

$$\Delta y = \begin{bmatrix}2 & -8 & -4\\ 3 & 3 & 4\\ 3 & 18 & 5\end{bmatrix} \Rightarrow 2(15-72) + 8(15-12) - 4(54-9)$$

$\Rightarrow -114 + 24 - 180$

$\Rightarrow -294 + 24 \Rightarrow -270$

$\Delta y = -270/115 \Rightarrow$

$$\Delta z = \begin{bmatrix}2 & 3 & -8\\ 3 & 2 & 3\\ 3 & -4 & 18\end{bmatrix} \Rightarrow 2(36+12) - 3(54-9) - 8(12-6)$$

$\Rightarrow 96 - 135 + 144$

$\Rightarrow 105$

$z = 105/115$

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Q1. $x+y+z=11$

$2x-6y-z=0$

$3x+4y+2z=0$

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & -6 & -1\\ 3 & 4 & 2\end{bmatrix} \quad B=\begin{bmatrix}11\\0\\0\end{bmatrix} \quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$|A| = 1(-12+4) - 1(4+3) + 1(8+18)$

$\Rightarrow -8-7+26 \Rightarrow 11.$

$$\Delta x=\begin{bmatrix}11 & 1 & 1\\ 0 & -6 & -1\\ 0 & 4 & 2\end{bmatrix} \Rightarrow 11(-12+4) \Rightarrow -88$$

$x \rightarrow -88/11 \Rightarrow -8$

$$\Delta y=\begin{bmatrix}1 & 11 & 1\\ 2 & 0 & -1\\ 3 & 0 & 2\end{bmatrix} \Rightarrow 11(4+3) \Rightarrow 77$$

$y = 77/11 \Rightarrow 7.$

$$\Delta z=\begin{bmatrix}1 & 1 & 11\\ 2 & -6 & 0\\ 3 & 4 & 0\end{bmatrix} = 11(8+18) \Rightarrow 286$$

$z = \frac{286}{11} \Rightarrow 26.$

Q2. Determine the Rank of Matrix.

11) $$A=\begin{bmatrix}2 & 1 & -1\\ 0 & 3 & -2\\ 2 & 4 & -3\end{bmatrix}$$

~~$R_2 \leftrightarrow R_3$~~

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix}2 & 1 & -1\\ 0 & 3 & -2\\ 0 & 4 & -2\end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

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$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$ Rank $\Rightarrow 2$

b.

$$\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

$R_3 \rightarrow R_3 + 2R_1$

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Rank $\Rightarrow 1$.

c.

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ Rank $= 2$.

d.

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 2 & 3 & -3 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 0 & 5 & -9 & -16 \\ 0 & 4 & -6 & -20 \\ 0 & 9 & -18 & -43 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 4R_1$

$R_4 \rightarrow R_4 - 9R_1$

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$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 0 & 5 & -9 & -16 \\ 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 11 \end{bmatrix} \quad \text{Rank} = 3.$$

Q.1 Solve the following are Consistant & Solve them.

a) $x_1 - x_2 + x_3 = 2$

$3x_1 - x_2 - 2x_3 = -6$

$3x_1 + x_2 + x_3 = -18.$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & -2 \\ 3 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ -6 \\ -18 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$M = \left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 3 & -1 & -2 & -6 \\ 3 & 1 & 1 & -18 \end{array}\right] \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 2 & -5 & -12 \\ 0 & 4 & -2 & -24 \end{array}\right] \quad R_3 \to R_3 - 2R_2$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 2 \\ 0 & 2 & 5 & -12 \\ 0 & 0 & 8 & 0 \end{array}\right] \quad \begin{matrix} \rho(A) = 3 \\ \rho(M) = 2 \end{matrix}$$

$$\boxed{x_3 = 0}$$

$$2x_2 - 5x_3 = -12$$

$$2x_2 = -18$$

$$x_2 = -6$$

$$x_1 - x_2 + x_3 = 2$$

$$x_1 + 6 = 2$$

$$x_1 = 4$$

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Q1. $x_1 + x_2 + x_3 = 6$

$x_1 + 2x_2 + 3x_3 = 14$

$x_1 + 4x_2 + 9x_3 = 36$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 14 \\ 36 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$M_2 = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 9 & 36 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$M_2 = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 8 & 30 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$M = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$\rho(A) = 3$

$\rho(A) = 3$

No. Variable $\to 3$

$2x_3 = 6$

$\boxed{x_3 = 3}$

$x_2 + 2x_3 = 8$

$x_2 + 6 = 8$

$\boxed{x_2 = 2}$