**MINIMUM SPANNING TREE** : Minimum cost tree which connects all the vertices in a graph

## KRUSKAL'S ALGORITHM

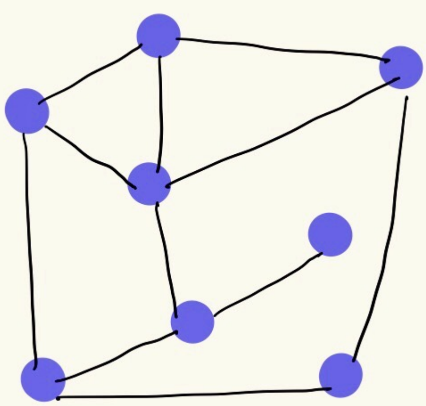
KEY CONCEPT : Repeatedly add the next lightest edge if this doesn't produce a cycle.

## PRIM'S ALGORITHM

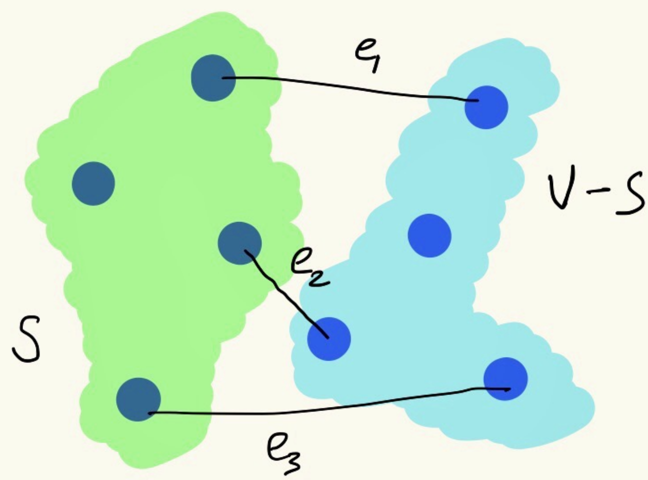
KEY CONCEPT : Repeatedly attach a new vertex to the current tree by a lightest edge

## CUT PROPERTY

Let $S$ be any subset of vertices and let $e$ be the min cost edge with exactly one endpoint in $S$. Then Minimum spanning tree of the graph contains $e$.



$G \rightarrow$ Graph.




$e = \min(e_1, e_2, e_3)$

$\Rightarrow e \in MST(G)$

# KRUSKAL'S ALGORITHM

$\forall u \in V$ :

MakeSet$(u)$

$X \leftarrow$ empty set

Sort all edges $E$ by weight

$\forall \{u, v\} \in E$ in non-decreasing order

if $Find(u) \neq Find(v)$

add $\{u, v\}$ to $X$

Union $(u, v)$

return $(X)$

checks whether a cycle is forming

Joins connected components

## IMPLEMENTATION

### SIMPLE ARRAY

① QuickFind
② Weighted Quick Union

| | Find | Union |
|---|---|---|
| QuickFind | $O(1)$ | $O(N)$ |
| Weighted Quick Union | $O(\log N)$ | $O(\log N)$ |

## RUNNING TIME ANALYSIS

① Sorting edges $= O(|E| \log |V|)$

② Processing edges $=$

$$2|E| \, T(\text{find}) + |V| . T(\text{Union})$$

$$= O(|E| \log |V|)$$

Running Time $= O(|E| \log |V|)$

# PRIM'S ALGORITHM

$\forall\, u \in V$:

  $cost[u] \leftarrow \infty$

  $parent[u] \leftarrow$ None

Pick initial vertex $u_0$

$cost[u] \leftarrow 0$

$PrioQ \leftarrow$ MakeQueue$(V)$

while $PrioQ$ is not empty :-

  $v \leftarrow$ ExtractMin$(PrioQ)$

  $\forall\, \{v, z\} \in E$:

    if $z \in PrioQ$ and $cost[z] > w(v, z)$:

      $cost[z] \leftarrow (v, z)$

      $parent[z] \leftarrow v$

      ChangePriority$(PrioQ, z, cost[z])$

# IMPLEMENTATION

PrioQ → Priority Queue

1. SIMPLE ARRAY
2. BINARY HEAP BASED

| | EXTRACT MIN | CHANGE PRIORITY |
|---|---|---|
| SIMPLE ARRAY | $O(V)$ | $O(1)$ |
| BINARY HEAP | $\log\lvert V\rvert$ | $\log\lvert V\rvert$ |

## RUNNING TIME ANALYSIS

$$|V| \cdot T(\text{EXTRACT MIN}) + |E| \cdot T(\text{Change Priority})$$

ARRAY: $O(|V|^2)$  BINARY HEAP: $O(|E| \log |V|)$