BISECTION METHOD

find x_0, x_1 such that $f(x_a) \cdot f(x_b) < 0$

$\Rightarrow$ There will be a root of $f(x) = 0$ in between x_a and x_b

Let
first approximation of the midpoint be

$$= x_1 = \frac{(x_a + x_b)}{2}$$

If $f(x_1) = 0$ Then x_1 is the root

Else root lies b/w x_a & x_1 or x_b & x_1

If $f(x_a) \cdot f(x_1) < 0$
Then root is in b/w x_a & x_1

If $f(x_1) \cdot f(x_b) < 0$
Then root is in b/w x_1 & x_b.

Continue this approximation until you get $|f(x_1)| < eps$

where eps is the error

ALGORITHM

Read x_a, x_b, eps, n

[n is the number of iterations (maximum permissible)]

If $f(x_a)\, f(x_b) > 0$

  print "Interval is Unsuitable"

  exit

For $i = 1$ to n

  $x_1 = \dfrac{x_a + x_b}{2}$

  If $|f(x_1)| <$ eps

    print "{x_1} is the Root"

    exit

  If $f(x_a)\, f(x_1) < 0$

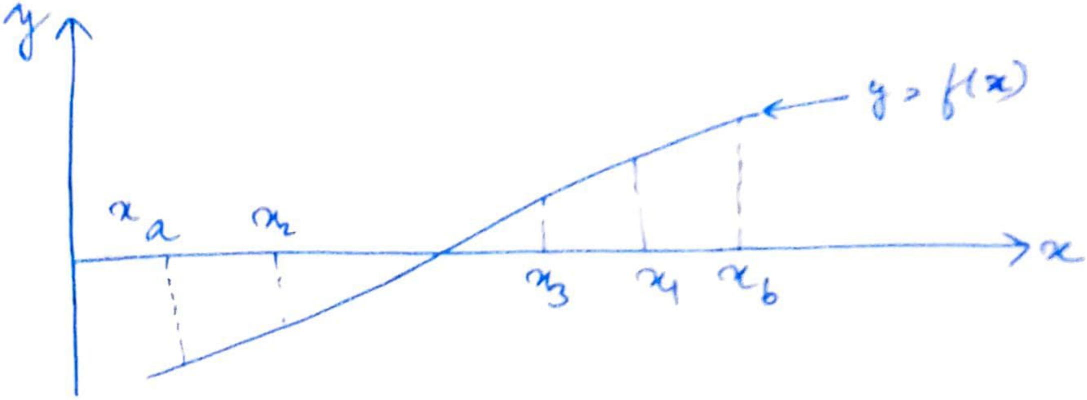
    $x_a = x_1$

  Else

    $x_a = x_1$

Print "No Solution found in n steps".

NOTE :



The interval width is reduced by a factor of $1/2$ at each step

Thus, at the end of n^{th} step, the new interval will be $[a_n, b_n]$ of length $\frac{|b-a|}{2^n}$

Hence, the no. of iterations $\underline{\underline{n}}$ required to achieve an accuracy ϵ is given by,

$$n \geq \frac{\log_e\left(\frac{|b-a|}{\epsilon}\right)}{\log_e 2}$$