D.C. MOTORS

8.1 Introduction:

A direct current motor converts electrical energy into mechanical energy. It is similar in construction to a D.C. generator. Hence, a D.C. motor can be used as a D.C. generator and vice-versa. The external appearance of a D.C. motor may be slightly different from that of the D.C. generator, mainly due to the fact that, the frame of the generator may be partially open, as it is usually located in a clean environment and operated only by skilled workers. A motor is always operated in a dusty environment and usually unskilled workers operate them. Hence, the frames of the motors are usually closed.

8.2 Working Principle:

A D.C. motor works on the principle that "*whenever a current carrying conductor is placed in a magnetic field, it experiences a force*". The magnitude of the force experienced by the conductor is given by,

$$F = BI\ell, \tag{8.1}$$

Where, F = Force experienced in Newtons

B = Flux density of the magnetic field in Wb/m^2

I = Current flowing through the conductor in amperes

ℓ = Length of the conductor in metres

The direction of the force is given by Fleming's left hand rule.

8.3 Fleming's Left Hand Rule:

It states that "*when the thumb, fore finger and the middle finger of the left hand are held mutually perpendicular to each other, the fore finger in the direction of the magnetic field, the middle finger in the direction of the current, then the direction of the thumb indicates the direction of the force experienced by the conductor*".

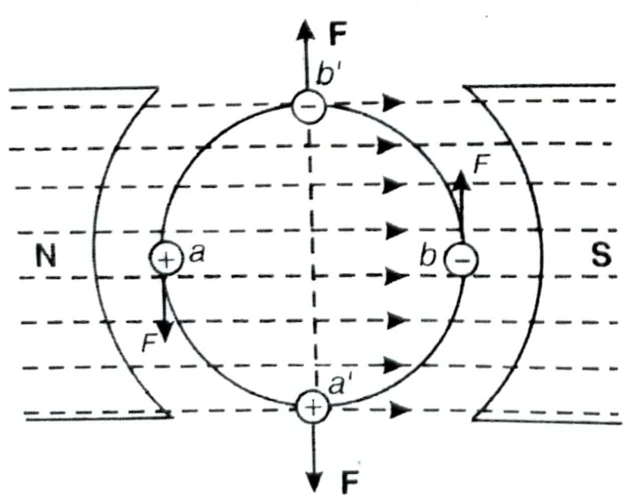


Fig.8.1

Consider a D.C. motor having two poles north and south represented by N and S as shown in Fig. 8.1. There will be conductors placed uniformly in the slots of the armature.

For the sake of explaining the a principle of working of a D.C. motor, only two conductors a and **b**, which are placed under the influence of N-pole and S-pole respectively and which are connected together by an end connection at the back side of the armature and to the commutator segments at front side of the armature are considered. When a D.C. supply is given to the motor terminals, the current flows through the conductors **a** and **b** via the commutator. In conductor **a**, the +ve sign marked indicates that the current is flowing inwards and the –ve sign in conductor **b** indicates that the current is flowing outwards. The direction of the magnetic field is represented by the lines of magnetic force, which emanate from the north pole N and go into the south pole S as shown in the Fig. 8.1.

According to Fleming's left hand rule, the conductor **a** experiences a force F in the downward direction and the conductor **b**, experiences an equal force F in the upward direction. As the two conductors are connected together, the two equal and opposite forces acting on them, constitute a couple, tending to rotate the armature in anti-clockwise direction. Due to the action of this couple, let the armature rotate by 90° in the anti-clockwise direction and the conductors **a** and **b** occupy positions **a'** and **b'** respectively.

In this position, they experience a force F in opposite directions along the same line and hence the torque experienced by them is zero. This position is known as **"dead center"** for the conductors **a** and **b**. If the armature contains only these two conductors, then the armature would stop in the position **a'b'**. But the armature consists of several other conductors which are uniformly distributed in the slots of the armature, which are connected together and experiencing a torque in the anti-clockwise direction. Thus the armature continues to rotate in the anti-clockwise direction.

For the armature to experience a continuous anti-clockwise torque, it is necessary that the directions of currents in conductors **a** and **b** must be reversed, after they cross the positions **a'** and **b'** respectively. Otherwise, the armature experiences a torque in the clockwise direction, thereby producing a pulsating torque in the dead centre position **a' b'**. This reversal of current in conductors **a** and **b** after they cross positions **a'** and **b'** respectively, is effected by the commutator and thus makes the armature to experience a continuous anti-clockwise torque and makes it rotate continuously in the anti-clockwise direction.

8.4 Back E.M.F. (E_b):

Fig. 8.2 symbolically represents a D.C. shunt motor. V is the applied voltage, due to which a current I_a flows through the armature conductors. I_L is the line current and I_{sh} is the current flawing through the shunt field winding

$$I_L = I_a + I_{sh} \tag{8.2}$$

When a current I_a flows through the armature conductors, a torque is produced and the armature rotates. The current I_{sh} flowing through the shunt field winding produces a flux ϕ and hence, an E.M.F. E_b is induced in the armature conductors. The direction of this induced E.M.F. is such as to oppose the applied voltage. Hence, this induced E.M.F. E_b is called as the *back E.M.F.* The directions of the applied voltage V and the back E.M.F. E_b are shown in fig.8.2

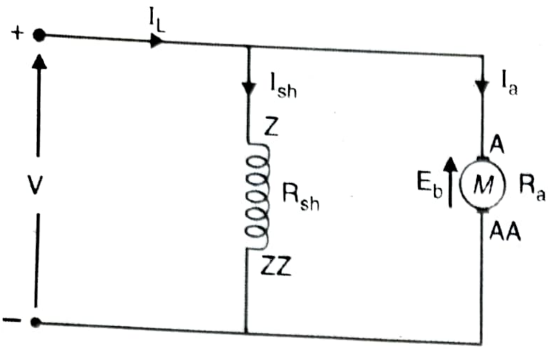


Figure.8.2

The applied voltage V has to drive current through the armature conductors against the opposition of the back E.M.F. and hence work has to be done. This work done is manifested in the form of mechanical power developed by the armature. In the absence of back E.M.F., no mechanical power can be developed by the armature.

The armature current I_a is given by,

$$I_a = \frac{V - E_b}{R_a} \qquad (8.3) \qquad \text{or} \qquad V = E_b + I_a R_a \qquad (8.4)$$

The back E.M.F. is nothing but the induced E.M.F. and hence its equation is,

$$E_b = \frac{\phi ZNP}{60A} \qquad (8.5)$$

Multiplying equation (8.4) by I_a, we get, $\quad V I_a = E_b I_a + I_a^2 R_a \qquad (8.6)$

Where, $V I_a$ = Electrical power input to the armature.

$I_a^2 R_a$ = Copper loss in the armature.

and $E_b I_a$ = Electrical equivalent of the mechanical power developed by the armature, which includes iron losses and mechanical losses.

The efficiency of the D.C. motor is given by

$$\eta = \frac{\text{Mechanical power developed by the armature}}{\text{Electrical power input to the motor}} = \frac{E_b I_a}{V I_a} = \frac{E_b}{V} \qquad (8.7)$$

Higher the value of E_b, higher will be the motor efficiency. The mechanical power developed by the armature is given by,

$$P_m = VI_a - I_a^2 R_a, \qquad \text{i.e.} \qquad \frac{dP_m}{dI_a} = V - 2\, I_a R_a$$

For maximum power to be developed, $\dfrac{dP_m}{dI_a} = 0$

$$\therefore \; V - 2\, I_a R_a = 0 \qquad \text{or} \qquad I_a R_a = \frac{V}{2} \qquad (8.8)$$

But, $E_b + I_a R_a = V$

$$\therefore \; E_b + \frac{V}{2} = V \qquad \text{or} \qquad E_b = \frac{V}{2} \qquad (8.9)$$

But in practice, D.C. motors are not designed to satisfy the conditions given in equations (8.8) and (8.9), because, if $E_b = V/2$, the current flowing through the armature will be far more than the rated current and lot of input power will be wasted in the form of heat, thus increasing the losses and the efficiency of the D.C. motor for the condition $E_b = V/2$ will be less than 50%.

8.5 Torque Equation:

Torque is the turning moment about an axis. It is equal to the product of the force and the radius at which it acts.

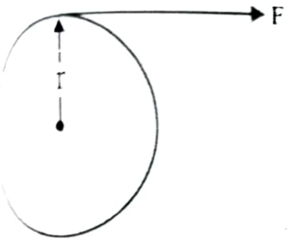


Fig.8.3

Consider the armature of the D.C. motor to have a radius r and let F be the force acting tangential to its surface as shown in Fig. 8.3. The torque exerted by the force F on the armature is given by,

$$T_a = F \times r \;\; \text{Nm} \qquad (8.10)$$

The work done by this force F, in one revolution is given by,

$$W = \text{Force x distance covered in one revolution} = F \times 2\pi r \;\; \text{W–S} \qquad (8.11)$$

The power developed by the armature = Work done in one second

$$= F \times 2\pi r \times \text{number of revolutions per second}$$

$$= F \times 2\pi r \times \frac{N}{60} = \frac{2\pi N}{60}(F \times r) = \frac{2\pi N T_a}{60} \;\; \text{watts} \qquad (8.12)$$

We have already learnt in section 8.4 of this chapter that, the electrical equivalent of the mechanical power developed by the armature of the D.C. motor is equal to $E_b I_a$.

$$\therefore \; \frac{2\pi N T_a}{60} = E_b I_a = \frac{\phi Z N P}{60A} I_a$$

$$\therefore \; T_a = \frac{1}{2\pi}\phi Z I_a\left(\frac{P}{A}\right) \text{Nm} = 0.159\,\phi Z I_a\left(\frac{P}{A}\right) \text{Nm} \qquad (8.13)$$

$$= 0.0163\,\phi Z I_a\left(\frac{P}{A}\right) \text{Kgm} \qquad (8.14)$$

Equation (8.13) gives the *gross torque* developed by the armature, which includes *iron losses* and *mechanical losses* of the motor. The actual torque available at the shaft to do useful work, which is known as *shaft torque* or *useful torque* T_{sh}, is less than T_a by an amount of torque which is equivalent to iron losses and mechanical losses in the D.C. motor.

$$\therefore \quad T_{sh} = T_a - T_L \tag{8.15}$$

Where, T_{sh} = Shaft torque

T_a = Armature torque

T_L = Torque lost due to iron losses and mechanical losses

The useful torque or shaft torque is given by,

Output of the motor in watts $= \dfrac{2\pi N T_{sh}}{60}$ as per equation (8.12)

$$\therefore \quad T_{sh} = \frac{\text{Output of the motor in watts}}{2\pi \frac{N}{60}} \text{ Nm} \tag{8.16}$$

The output of a D.C motor is usually expressed in H.P.

$$\therefore \quad T_{sh} = \frac{\text{Output in H.P.} \times 735.5}{2\pi \frac{N}{60}} \text{ Nm} \tag{8.17}$$

8.6 Types of D.C. motors:

Depending on the way in which the field windings are connected to the armature, D.C. motors are classified into three types.

(1) D.C. shunt motor (2) D.C. series motor and (3) D.C. compound motor

The D.C. compound motor may be further classified into two types:

(a) *Cumulatively* compounded D.C. motor, which may be connected either as *long shunt* or *short shunt*.

(b) *Differentially* compounded D.C. motor, which may be connected either as *long shunt* or *short shunt*.

8.7 D.C. Shunt Motor:

Fig.8.4 represents a D.C. shunt motor. In this type of motor, the shunt field winding is connected across the armature. V is the applied voltage due to which a current I_L flows through the line, a current I_{sh} through the shunt field winding and I_a through the armature conductors. The shunt field current is given by,

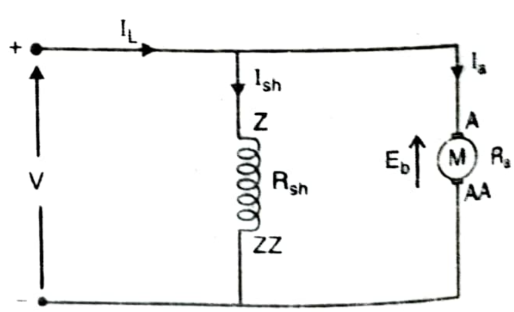


Fig.8.4

$$I_{sh} = \frac{V}{R_{sh}} \tag{8.18}$$

The armature current is given by,

$$I_a = I_L - I_{sh} \tag{8.19}$$

The back E.M.F. induced in the armature is given by,

$$E_b = V - I_a R_a - \text{B.C.D.} - \text{A.R.D.} \tag{8.20}$$

Where, B.C.D. = Brush contact drop and A.R.D. = Armature reaction drop.

8.8 D.C. Series Motor:

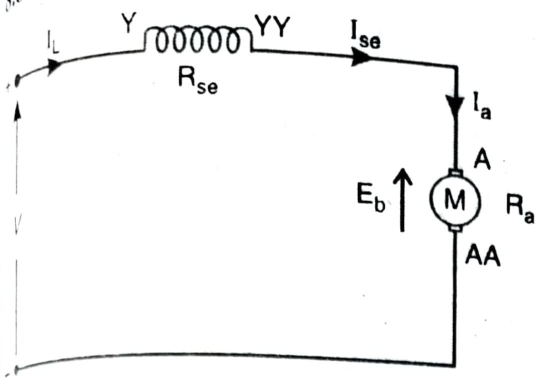


Fig. 8.5

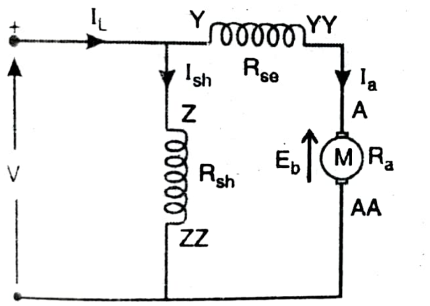
Fig. 8.5 represents a D.C. series motor. In this type of motor, the series field winding is connected in series with the armature. V is the applied voltage due to which a current I_L flows through the line, the series field winding and also through the armature conductors.

$$\therefore \quad I_L = I_{se} = I_a \tag{8.21}$$

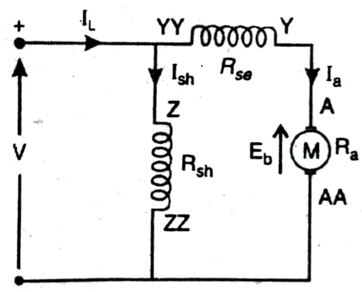
The series field winding carries the armature current and should have very small esistance so that the voltage drop across it is very small. Hence, it is made of a few thick urns of copper. The back E.M.F. induced in the armature is given by,

$$E_b = V - I_a (R_a + R_{se}) - \text{B.C.D.} - \text{A.R.D.} \tag{8.22}$$

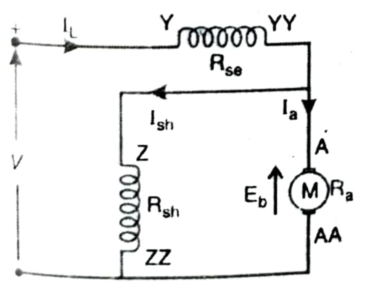
8.9 D.C. Compound Motor:



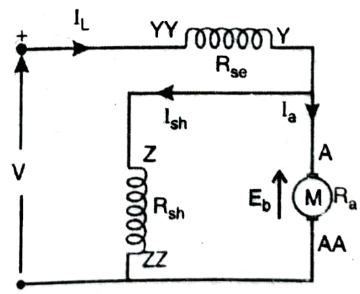
a) Cumulatively compounded long shunt DC motor



b) Differentially compounded long shunt DC motor



c) Cumulatively compounded short shunt DC motor



d) Differentially compounded short shunt DC motor

Fig.8.6

A D.C. compound motor contains both shunt field winding and series field winding. If the fluxes, ϕ_{sh} produced by the shunt field winding and ϕ_{se} produced by the series field winding are in the same direction and are additive, then the motor is said to be *cumulatively compounded*. If the two fluxes oppose each other, then the motor is said to be *differentially compounded.* Depending on the way in which the two field windings are connected, the compound motors can be either *long shunt* or *short shunt*. All the four possible types of D.C. compound motors are shown in Figs. 8.6 (a), (b), (c) and (d).

For cumulatively compounded motors, we observe that the currents enter the positive terminals of the two field windings and hence the fluxes produced by them are in the same direction and they are additive. In the case of differentially compounded motors, the current through the series field winding enters the negative terminal and the current through the shunt field winding enters the positive terminal. Hence, the two fluxes produced are in opposite directions and hence they oppose each other.

For a cumulatively or differentially compounded long shunt, D.C. motor, the following equations hold good.

$$I_{sh} = \frac{V}{R_{sh}} \quad (8.23) \qquad I_a = I_L - I_{sh} \quad (8.24)$$

And $$E_b = V - I_a (R_a + R_{se}) - B.C.D. - A.R.D. \quad (8.25)$$

For a cumulatively or differentially compounded short shunt, D.C. motor, the following equations hold good.

$$I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} \quad (8.26) \qquad I_a = I_L - I_{sh} \quad (8.27)$$

And $$E_b = V - I_L R_{se} - I_a R_a - B.C.D - A.R.D. \quad (8.28)$$

8.10 Characteristics of D.C. Motors:

The characteristics of D.C. motors are studied keeping the applied voltage constant. The following are the three important characteristics of D.C. motors.

i) Electrical characteristic or T_a / I_a characteristic
ii) N / I_a characteristic and
iii) Mechanical Characteristic or N / T_a characteristic.

8.11 Characteristics of D.C. Shunt Motors:

(i) T_a / I_a Characteristic:

Fig. 8.7 represents a D.C. shunt motor on load. In a D.C. shunt motor, the field current I_{sh} remains constant irrespective of the load connected to the motor, because, the applied voltage remains constant for all loads. Hence the flux produced also remains constant.

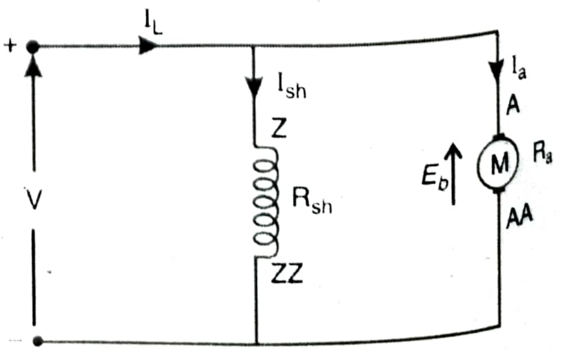


Fig.8.7

$$T_a = 0.159\,\phi\, Z\, I_a \left(\frac{P}{A}\right)$$

In the above equation, Z, P, A and ϕ are constants. $\therefore\ T_a \,\alpha\, I_a$ (8.29)

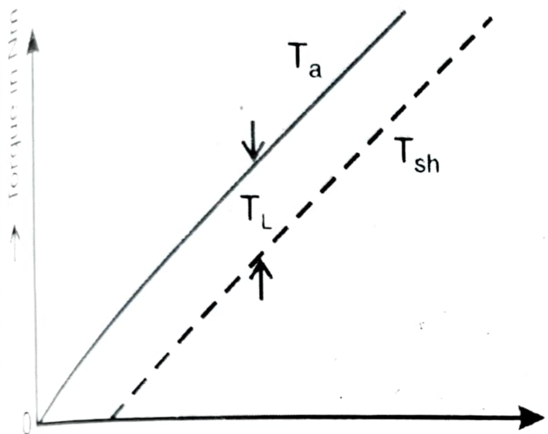


Fig.4.8

Hence T_a / I_a characteristic is a straight line passing through the origin as shown fig. 8.8.

The shaft torque T_{sh} is always less than the armature torque T_a due to iron losses and mechanical losses in the D.C. motor. Hence it is shown always to be less than T_a by a torque T_L, which is the torque lost due to the above losses. From the characteristic, we learn that, a D.C. shunt motor has a medium starting torque and hence does not suit where, very large loads are required to be started.

(ii) N / I_a Characteristic:

The equation for the back E.M.F. E_b is given by,

$$E_b = \frac{\phi Z N P}{60A}, \quad \therefore \quad N \,\alpha\, \frac{E_b}{\phi} \qquad (8.30)$$

as the other quantities are constant.

For D.C. shunt motor, ϕ is constant

$$\therefore\ N \,\alpha\, E_b \ \ \alpha \ \ V - I_a R_a \qquad (8.31)$$

As I_a increases, $I_a R_a$ increases and hence the speed decreases. But the drop I_aR_a is very small compared to V and hence, the decrease of speed as the armature current increases is also small. The variation of speed with respect to armature current is as shown in Fig.8.9.

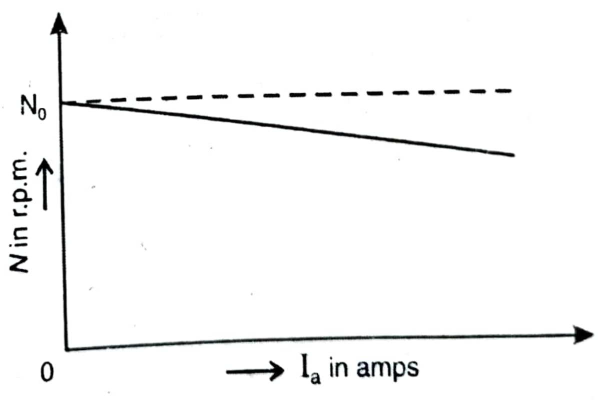


Fig.8.9

N_0 is the no load speed of the motor. From this characteristic, we understand that the change in the speed of a D.C. shunt motor is small, when the armature current or load on the motor is increased. Hence for all practical purposes, a D.C. shunt motor may be considered as almost a constant speed motor.

iii) N / T_a Characteristic:

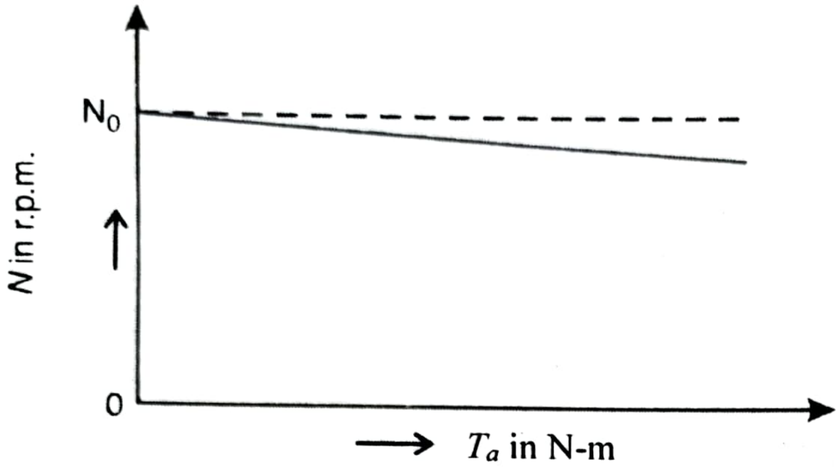


Fig.8.10

From the equation (8.29), we know that $T_a \propto I_a$ and hence, N/T_a characteristic is similar to N/I_a characteristic as shown in fig.8.10. N_0 is the no load speed of the motor.

8.12 Characteristics of D.C. Series Motors:

(a) T_a / I_a Characteristic:

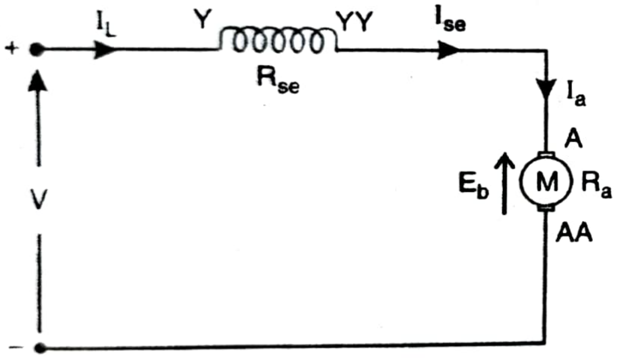


Fig.8.11

Fig. 8.11 represents a D.C. series motor on load. As the load on the motor increases, the current through the series field winding also increases and hence the flux produced also increases. The torque equation of a D.C. motor is given by,

$$T_a = 0.159\, \phi\, Z\, I_a \left(\frac{P}{A}\right)$$

Or $T_a \propto \phi I_a$, but $\phi \propto I_a$, $\therefore$ $T_a \propto I_a^2$

But after saturation, the flux remains constant and $T_a \propto I_a$.

The variation of T_a with respect to I_a is as shown in Fig. 8.12.

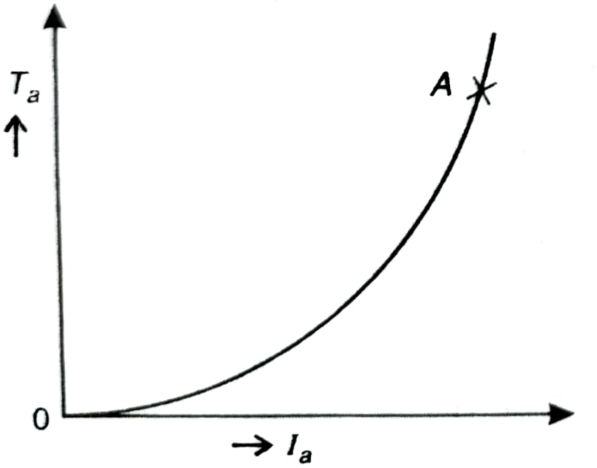


Fig.8.12

Upto saturation i.e., upto point A, $T_a \propto I_a^2$ and hence the curve is a parabola. After saturation i.e., beyond point A, $T_a \propto I_a$ and hence the curve is a straight line. From the characteristic, we find that the starting torque of a D.C. series motor is very high.

(b) N / I_a **Characteristic:**

From equation (8.30), we know that, $N \; \alpha \; \dfrac{E_b}{\phi} \; \alpha \; \dfrac{V - I_a(R_a + R_{se})}{\phi}$ (8.31)

From the equation (8.31), we find that as the load on the motor increases, there are two factors which influence the speed of the motor.

(i) $I_a(R_a + R_{se})$ increases and hence the speed decreases.

(ii) The flux ϕ also increases due to which the speed decreases.

But it has been observed that the decrease of speed due to the first factor is negligibly small as compared to the decrease in speed due to the second factor. Hence for all practical purpose, we can say

$$N \; \alpha \; \frac{1}{\phi} \quad \text{but} \quad \phi \; \alpha \; I_a, \quad \therefore \; N \; \alpha \; \frac{1}{I_a} \tag{8.32}$$

The variation of speed with respect to I_a is as shown in fig. 8.13.

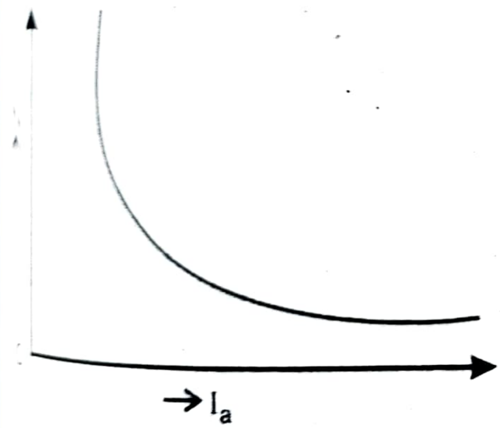


Fig.8.13

From the characteristic, we observe that, as the load increases, the speed decreases over a wide range. Hence, a D.C. series motor is considered as a variable speed motor.

At no load, I_a is very small and hence the speed will be dangerously high as per equation (8.32). Hence, if a D.C. series motor is started without any load on it, the speed is very high and it may run out of the foundation due to the centrifugal forces set up. Hence, a D.C. series motor should never be started without load.

(c) N / T_a **Characteristic:**

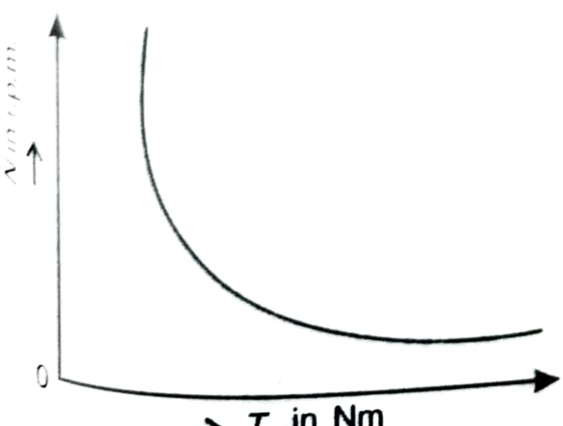


Fig.8.14

From the N / I_a characteristic of a D.C. series motor shown in fig.8.13, we find that, when I_a is small, N is very high.

But, $T_a \; \alpha \; I_a^2$ or $I_a \; \alpha \; \sqrt{T_a}$

$$\therefore \; N \; \alpha \; \frac{1}{\sqrt{T_a}} \tag{8.33}$$

For smaller values of T_a, N is very large and for higher value of T_a the speed decreases. The variation of speed with respect to torque T_a is as shown in Fig.8.14.

8.14 Applications of D.C. Motors:

(a) D.C. Shunt Motors:

From the study of the characteristics of a D.C. shunt motor, we realize that it has a medium starting torque and its speed remains almost constant from no load to full load. Hence, it is used where constant speed is required and the starting torque required is not very high, such as for lathes, centrifugal pumps, fans, reciprocating pumps, drilling machines, boring machines, spinning and weaving machines etc.,

(b) D.C. Series Motors:

From the study of the characteristics of D.C. series motor, we realize that, it has a very high starting torque and its speed varies widely from no load to full load. Hence, it is used where very high starting torque and variable speed is required, such as for electric traction work, electric locomotives, trolleys, cranes, hoists, conveyors, air compressors, vacuum cleaners, hair driers, sewing machines etc.,