MODULE-3

CHAPTER 5

SINGLE PHASE TRANSFORMERS

5.1 Introduction:

A transformer is a static electrical device, which transfers electrical power from one electrical circuit to the other, which are magnetically coupled together with or without change of voltage and without any change in power and frequency. The basic use of a transformer is to increase or decrease a.c. voltage. If it is used to increase the voltage, it is called a *step-up* transformer. If it is used to decrease the voltage, it is called a *step - down* transformer. If the voltage is not changed, it is called *one to one* transformer.

As the transformer is a static apparatus, there are no moving parts. Hence there are no mechanical losses in a transformer. Hence, the efficiency of a transformer is very high, of the order of 95% to 98%. There are no slots, no teeth and no air gaps. Hence, the maintenance of a transformer is very easy. It is only because of the transformer, a.c. electrical energy is widely used than d.c. electrical energy. The electrical energy is generated, transmitted and distributed and used in the form of a.c. energy.

5.2 Construction:

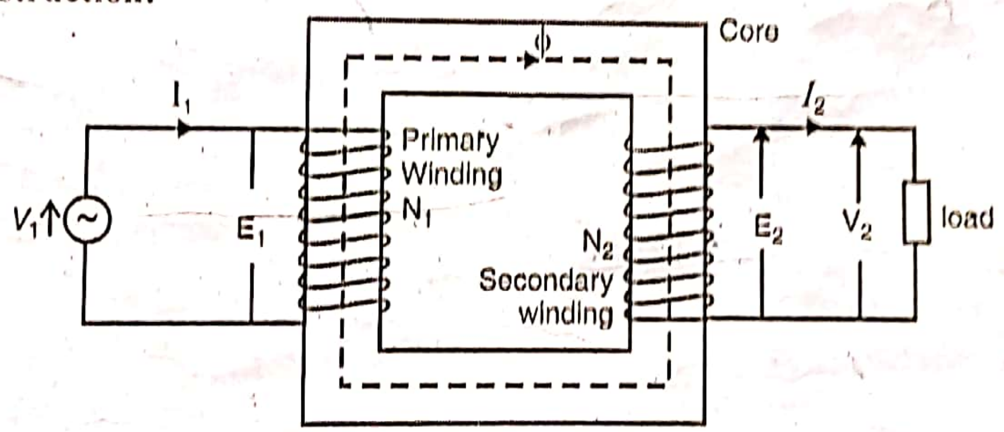

Figure 5.1

A single phase transformer basically consists of two parts (i) windings and (ii) core. There are two windings, which are wound on the two limbs of the core, which are insulated from each other and from the limbs, as shown in Fig. 5.1. The windings are made of copper, so that, they possess a very small resistance. The winding which is connected to the supply is called *primary winding* and the winding which is connected to the load is called *secondary winding*. The primary winding has N_1 number of turns and the secondary winding has N_2 number of turns. The core is made of silicon steel, which has a high relative permeability and low hysteresis co-efficient. The core is laminated to reduce eddy current losses. For small transformers, each lamination is a single piece. For large

E, T, L, I

transformers, each lamination is made of two or more sections like E, T, L or I, so that, when they are joined together, they form a complete lamination. The joints of such laminations are staggered, while forming the core, so that, the reluctance offered by these joints is minimum. The single phase transformer is as shown in Fig.5.1.

5.3 Working Principle:

A single phase transformer works on the principle of mutual induction between two magnetically coupled coils. When the primary winding is connected to an alternating voltage of r.m.s. value V_1 volts, an alternating current flows through the primary winding and sets up an alternating flux ϕ in the material of the core. This alternating flux ϕ, links not only the primary winding but also the secondary winding. Therefore, an e.m.f. e_1 is induced in the primary winding and an e.m.f. e_2 is induced in the secondary winding. e_1 and e_2 are given by,

$$e_1 = -N_1 \frac{d\phi}{dt} \qquad (5.1) \qquad \text{and} \qquad e_2 = -N_2 \frac{d\phi}{dt} \qquad (5.2)$$

$$\therefore \quad \frac{e_2}{e_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1} = K \qquad (5.3)$$

K is known as the transformation ratio of the transformer.

When a load is connected to the secondary winding, a current I_2 flows through the load. V_2 is the terminal voltage across the load. As the power transferred from the primary winding to the secondary winding is same,

Power input to the primary winding = Power output from the secondary winding.

$$E_1 I_1 = E_2 I_2$$

(Assuming that the p.f. of the primary is equal to the p.f. of the secondary)

$$\text{i.e.,} \quad \frac{E_2}{E_1} = \frac{I_1}{I_2} = K \qquad (5.4)$$

From the equations (5.3) and (5.4), we can write $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$ (5.5)

The directions of e.m.f.s E_1 and E_2 induced in the primary and secondary windings are such that, they always oppose the primary applied voltage V_1.

5.4 E.M.F. Equation of a Transformer:

When an alternating voltage $v_1 = V_m \sin \omega t$ of r.m.s. value $V_1 = V_m / \sqrt{2}$ is applied to the primary winding of the transformer, the alternating current flowing through the primary winding produces an alternating flux ϕ, which links both the primary winding and the secondary winding. Hence, an e.m.f. e_1 is induced in the primary winding and an e.m.f. e_2 is induced in the secondary winding. The equation for e_1 is

$$e_1 = -N_1 \frac{d\phi}{dt} \tag{5.6}$$

As the primary applied voltage is sinusoidal in nature, the current it drives and the resulting flux produced are also sinusoidal. The equation for the flux is given by,

$$\phi = \phi_m \sin \omega t \tag{5.7}$$

Substituting this value of ϕ in equation (5.6), we get,

$$e_1 = -N_1 \frac{d}{dt}(\phi_m \sin \omega t) = -\omega N_1 \phi_m \cos \omega t = -2\pi f N_1 \phi_m \cos \omega t$$

$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°) \qquad \because \sin(90° - \omega t) = \cos \omega t, \quad \sin(\omega t - 90°) = -\cos \omega t \tag{5.8}$$

From equations (5.7) and (5.8), we find that the induced e.m.f. lags the flux by 90°.

The magnitude of the maximum value of the e.m.f. induced in the primary winding is given by, $E_{m1} = 2\pi f N_1 \phi_m$ (5.9)

The r.m.s. value of the e.m.f. induced in the primary winding is given by,

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = 4.44 f \phi_m N_1 \tag{5.10}$$

Equation (5.10) is the equation for the r.m.s. value of the e.m.f. induced in the primary winding. Similarly the r.m.s. value of the e.m.f. induced in the secondary winding is given by, $E_2 = 4.44 f \phi_m N_2$ (5.11)

Dividing equation (5.11) by (5.10), we get,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K, \text{ the transformation ratio.}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

Also, $\frac{E_1}{N_1} = \frac{E_2}{N_2}$

$\therefore$ the e.m.f induced per turn in both primary winding and the secondary winding is the same.

The total impedance of the transformer as referred to one winding can also be written as,

$$Z_{01} = R_{01} + j\,X_{01} \tag{5.29}$$

= equivalent impedance of the transformer as referred to primary.

$$Z_{02} = R_{02} + j\,X_{02} \tag{5.30}$$

= equivalent impedance of the transformer as referred to secondary.

5.10 Losses in a Transformer

As the transformer is a static apparatus and does not contain any rotating parts, there are no mechanical losses viz friction and windage losses. The losses that occur in a transformer are (i) iron loss and (ii) copper loss.

(i) Iron Loss (W_i): This loss is also called as the *core loss* and this loss occurs in the iron portion i.e. the core of the transformer. Iron loss is of two types (i) eddy current loss and (ii) hysteresis loss.

The eddy current loss (W_e) occurs due to the flow of eddy currents in the laminations of the core. The eddy .currents are induced in the laminations, because, the alternating flux produced by the primary winding links them. These eddy currents cause power loss in the core and heats up the core of the transformer. The eddy current loss in the core of a transformer is given by an empirical formula due to **Steinmetz** which is given by,

$$W_e = \beta\, B_m^2\, f^2\, t^2\, V \text{ watts} \tag{5.31}$$

Where, W_e = eddy current loss in watts

B_m = maximum value of the flux density in the core in Wb/m^2

f = frequency of the supply in Hz

t = thickness of the laminations in metre

V = volume of the core in m^3

β = a constant, whose value depends on the quality of the magnetic material used for making the core.

To keep the eddy current loss as small as possible, the core is made of thin laminations of high permeability magnetic material, such as silicon steel and they are insulated from one another by coating them with varnish or an oxide layer.

The *hysteresis loss* (W_h) occurs because, the core of the transformer is subjected to cycles of magnetisation. Steinmetz empirical formula for calculating the hysteresis loss in the core of the transformer is given by,

$$W_h = \eta\, B_m^{1.6}\, f\, V \text{ watts} \tag{5.32}$$

Where, W_h = hysteresis loss in watts

B_m = maximum value of the flux density in the core in Wb/m^2

f = frequency of the supply in Hz

V = volume of the core in m^3

η = a constant, whose value depends on the quality of the magnetic material used for making the core.

Hence, Iron losses = Eddy current loss + Hysteresis loss

$$W_i = W_e + W_h$$

$$= \beta\, B_m^2\, f^2 t^2 V + \eta\, B_m^{1.6}\, f V \quad \text{watts} \tag{5.33}$$

From the equation (5.33), we find that the iron loss in the transformer depends on B_m, the maximum value of flux density and f, the frequency of the supply, as other quantities like, the thickness of laminations, volume of the core are constants. As long as the applied voltage remains constant, B_m and f remain constant. Therefore, iron loss in the transformer is considered to be a constant loss at all loads including no load.

(ii) Copper loss (W_{cu}): This loss is due to the resistances R_1 and R_2 of the primary and secondary ~~windings respectively~~.

Total copper loss = copper loss in primary + copper loss in secondary

$$W_{cu} = I_1^2 R_1 + I_2^2 R_2 \text{ watts} \tag{5.34}$$

$$= I_1^2 (R_1 + R_2') \tag{5.35}$$

$$= I_1^2 R_{01} \tag{5.36}$$

$$= I_2^2 R_{02} = I_2^2 (R_2 + R_1') \tag{5.37}$$

From the above equations, we find that the copper losses in the transformer, vary as the square of the currents, I_1 and I_2, which vary with load. Hence, copper loss in the transformer is a variable loss.

The total loss in the transformer is the sum of iron loss and copper loss.

5.11 Efficiency of a Transformer:

The efficiency of a transformer at any load and p.f. is defined as the ratio of the output at the secondary winding to the power input to the primary winding.

$$\text{Efficiency} = \eta = \frac{\text{Power output in watts}}{\text{Power input in watts}}$$

Power input = $V_1 I_1 \cos \phi_1$

Where, V_1 = Primary applied voltage

I_1 = Primary current and

$\cos\phi_1$ = Power factor of the primary

$$\text{Efficiency} = \eta = \frac{\text{Input} - \text{losses}}{\text{Input}} = \frac{\text{Input} - \text{Copper Loss} - \text{Iron Loss}}{\text{Input}}$$

$$= \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1} = 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

The efficiency is maximum, when, $\dfrac{d\eta}{dI_1} = 0$

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 I_1^2 \cos\phi_1} = 0$$

$$\therefore \quad \frac{R_{01}}{\cancel{V_1}\cancel{\cos\phi_1}} = \frac{W_i}{\cancel{V_1} I_1^2 \cancel{\cos\phi_1}}$$

or $\quad W_i = I_1^2 R_{01}$ (5.38)

i.e. $\quad$ Iron loss = Copper loss

Hence, the condition for the maximum efficiency of the transformer is that, the iron loss must be equal to the copper loss.

The copper losses of the transformer may also be written as equal to $I_2^2 R_{02}$

For maximum efficiency

$$W_i = I_2^2 R_{02}$$

or $\quad I_2 = \sqrt{\dfrac{W_i}{R_{02}}}$ → Efficiency of transformer is maximum. (5.39)

Equation (5.39) gives the load current for which the efficiency of the transformer is maximum. The kVA of the transformer at which the maximum efficiency occurs is derived as follows.

Let W_i = iron loss of the transformer

W_{cu} = full load copper loss

The efficiency is maximum when $W_{cu} = W_i$. As W_i is constant, the kVA output at maximum efficiency is that kVA at which $W_{cu} = W_i$.

Let x = kVA output at which efficiency is maximum, Then, $W_{cu} \alpha$ (full load kVA)^2 (5.40)

As W_i is equal to the copper loss at x kVA

$W_i \alpha x^2$ (5.41)

$x \rightarrow$ kVA output at which η is max

From equations (5.40) and (5.41), we get

$$\left(\frac{x}{\text{Full load kVA}}\right)^2 = \frac{W_i}{W_{cu}}$$

$$\therefore \quad x = \text{Full load kVA}\sqrt{\frac{W_i}{W_{cu}}} = \text{Full load kVA}\sqrt{\frac{\text{Iron Loss}}{\text{Full load copper loss}}} \quad (5.42)$$

The efficiency at any load and p.f. is given by

$$\eta_x = \frac{x \times kVA \times 1000 \times p.f.}{x \times kVA \times 1000 \times p.f. + W_i + x^2 W_{cu}}$$

Where x = load expressed as a fraction of full load

$x = 1$ for full load and $x = \frac{1}{2}$ for half full load

5.12 Regulation of a transformer:

The regulation of a transformer is defined as the rise in the secondary terminal voltage, when the full load is thrown off, keeping the primary voltage constant.

It is also defined as the change in the secondary terminal voltage from no load to full load, keeping the primary voltage constant.

$\therefore$ Regulation $= {}_0V_2 - V_2$

Where, ${}_0V_2$ = No load secondary terminal voltage

V_2 = full load secondary terminal voltage.

$$\% \text{ Regulation} = \frac{{}_0V_2 - V_2}{V_2} \times 100 \quad (5.43)$$

The vector diagram of the transformer on load as referred to the secondary winding is shown in fig.5.9.

$$= \left(\frac{I_2 R_{02}}{V_2} \times 100\right)\cos\varphi_2 \pm \left(\frac{I_2 X_{02}}{V_2} \times 100\right)\sin\varphi_2$$

$$= V_r \cos\phi_2 \pm V_x \sin\phi_2 \qquad (5.47)$$

Where, $V_r = \frac{I_2 R_{02}}{V_2} \times 100 =$ Percentage resistance drop.

And $V_x = \frac{I_2 X_{02}}{V_2} \times 100 =$ Percentage reactance drop.

5.13 Types of Transformers:

Depending on the way in which the primary and secondary windings are wound on the core, the transformers are classified into two types. (i) Core type and (ii) Shell type.

(i) Core type transformer: In this type of transformer, the windings surround a considerable part of the core as shown in Fig. 5.10. The primary winding and the secondary winding are shown as connected on separate limbs but in actual construction the two windings are inter-leaved to reduce the leakage flux. i.e. half the primary winding and half the secondary winding are placed concentrically on one limb, the low voltage winding nearer to the core. The coils used are former would and are of cylindrical type. The coils may be circular or rectangular oval. For small sized core-transformers, the core is rectangular in shape but for large sized core-transformers cruciform core is used. The cylindrical coils are wound in helical layers, which are insulated from each other. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. The core is always laminated to reduce the eddy current losses. Core type transformers are used to handle low and medium voltages.

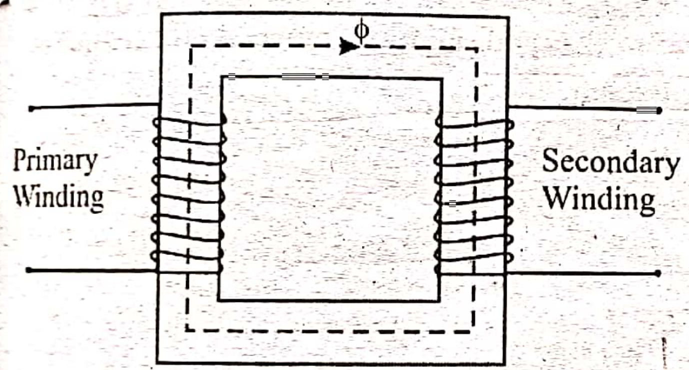

Fig.5.10

(ii) Shell type transformer: In this type of transformer, the core surrounds a considerable portion of the windings as shown in Fig 5.11. The coils are former wound which are multilayer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper. The core is rectangular in shape and is laminated to reduce eddy current losses. The primary winding and the secondary

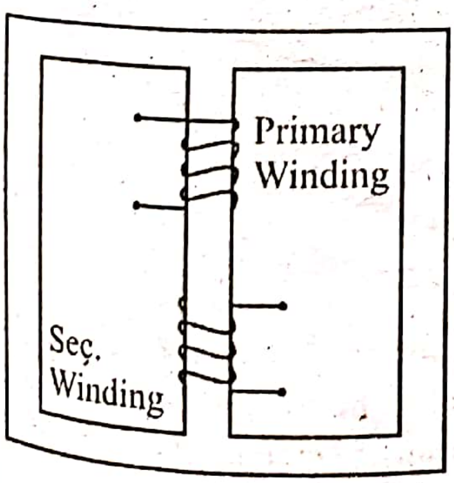

Fig.5.11

winding are wound on the central limb as shown in Fig. 5.11. The choice of core or shell type transformer mainly depends on voltage rating, kVA rating, weight, insulation stress, heat distribution etc. Shell type of transformers are used for handling very high voltages.

WORKED EXAMPLES

5.11 **A 50 kVA, 2500/250 V, single phase transformer has a primary winding resistance of 3 Ω and a reactance of 5 Ω. The secondary winding resistance and reactance are 0.02 Ω and 0.03 Ω respectively. Find (i) equivalent resistance, reactance and impedance as referred to primary winding (ii) equivalent resistance, reactance and impedance as referred to secondary winding (iii) total copper loss in the transformer.**

i) $R_{01} = R_1 + \frac{R_2}{K^2}$, $K = \frac{E_2}{E_1} = \frac{250}{2500} = \frac{1}{10} = 0.1$

$$= 3 + \frac{0.02}{(0.1)^2} = 5\,\Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 5 + \frac{0.03}{(0.1)^2} = 8\,\Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{5^2 + 8^2} = 9.43\,\Omega$$

ii) $R_{02} = R_2 + K^2 R_1 = 0.02 + (0.1)^2 3 = 0.05\,\Omega$

$$X_{02} = X_2 + K^2 X_1 = 0.03 + (0.1)^2 5 = 0.08\,\Omega$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{(0.05)^2 + (0.08)^2} = 0.094\,\Omega$$

iii) $I_1 = \frac{50 \times 1000}{2500} = 20\text{ A}$

Total copper loss $= I_1^2 R_{01} = 20^2 \times 5 = 2000$ W

5.12 **A 250/500 V transformer has the primary winding resistance of 0.25 Ω and reactance of 0.5 Ω and the corresponding values for the secondary winding are 1 Ω and 2 Ω respectively. Find the secondary terminal voltage, when supplying (a) 12 A, at a p.f. of 0.8 lagging and (b) 12 A, at a p.f. of 0.8 leading.**