WORKED EXAMPLES

6.1 The equation for an alternating current is given by $i = 28.28 \sin(314t + 30°)$ A. Find its r.m.s. value, frequency and phase angle.

$$\therefore I = \frac{I_m}{\sqrt{2}} = \frac{28.28}{\sqrt{2}} = 20 \text{ A}, \quad \omega = 2\pi f, \quad \therefore f = \frac{\omega}{2\pi} = \frac{\omega}{2 \times 3.14} = 50 \text{ Hz}$$

Phase angle = 30°

6.2 An alternating current varying sinusoidally with a frequency of 50 Hz has an r.m.s. value of 20 A. (i) write down the equation for the instantaneous value of current. (ii) Find the value at the instant 0.0125 sec, after passing through a +ve maximum value and (iii) At what time measured from the +ve maximum value, will the instantaneous current be 14.14 A.

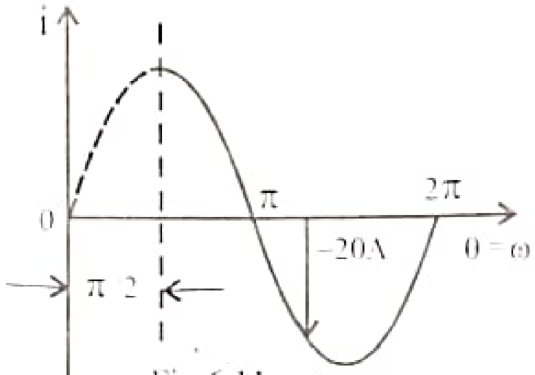

Fig.6.11

i) The time is reckoned from the instant when the current is +ve maximum. Hence, the equation for the instantaneous value of the current is

$i = I_m \sin(\omega t + \pi/2)$

ii) $i = I_m \sin(2\pi f t + \pi/2) = 20\sqrt{2} \sin(360° \times 50 \times 0.0125 + 90°) = -20$ A

iii) $14.14 = 20\sqrt{2} \sin(360° \times 50 \times t + 90°)$,

$\therefore t = \frac{1}{300}$ sec [For $t = \pm \frac{1}{300}$ sec, $i = 14.14$ A]

WORKED EXAMPLES

6.5 A circuit consists of a resistance of 20 Ω, an inductance of 0.05 H connected in series. A supply of 230 V at 50 Hz is applied across the circuit. Find the current, power factor and power consumed by the circuit. Draw the vector diagram.

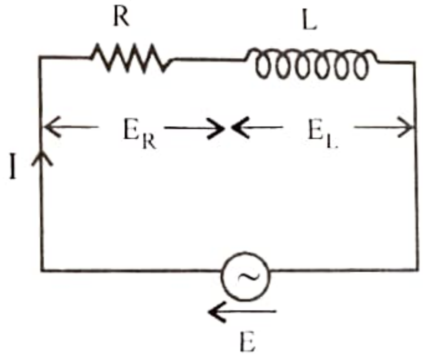

Fig.6.40

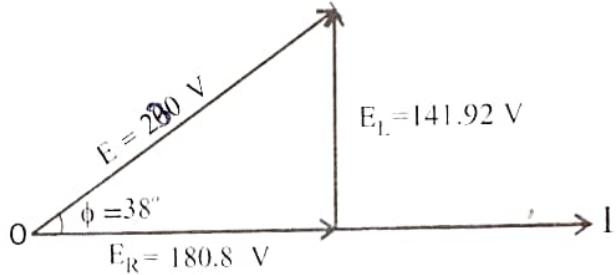

Fig.6.41

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.05 = 15.7\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + (15.7)^2} = 25.42\ \Omega$$

$$I = \frac{E}{Z} = \frac{230}{25.42} = 9.04\ A$$

$$p.f. = \frac{R}{Z} = \frac{20}{25.42} = 0.786 \text{ lagging}$$

$P = E\,I\cos\phi = 230 \times 9.04 \times 0.786 = 1634.43$ watts.

To draw the vector diagram, calculate,

$E_R = I\,R = 9.04 \times 20 = 180.8\ V$

$E_L = I\,X_L = 9.04 \times 15.7 = 141.92\ V$

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{15.7}{20} = 38°$$

The vector diagram is shown in Fig. 6.41.

6.6 A circuit consists of a resistance of 25 Ω and a capacitance of 100 μF connected in series. A supply of 200 V at 50 Hz is applied across the circuit. Find the current, power factor and power consumed by the circuit. Draw the vector diagram.

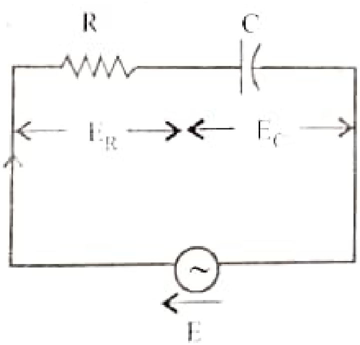

Fig.6.42

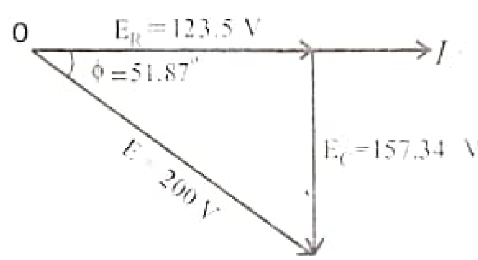

Fig.6.43

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}} = 31.85\ \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{25^2 + 31.85^2} = 40.48\ \Omega$$

$$I = \frac{E}{Z} = \frac{200}{40.48} = 4.94\ A, \qquad p.f. = \frac{R}{Z} = \frac{25}{40.48} = 0.617 \text{ leading}$$

$P = E\,I\cos\phi = 200 \times 4.94 \times 0.617 = 609.59$ watts

To draw the vector diagram, calculate,

$E_R = IR = 4.94 \times 25 = 123.5$ V. $\quad E_C = I\,X_C = 4.94 \times 31.85 = 157.34$ V

$$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{31.85}{25} = 51.87°$$

The vector diagram is shown in Fig. 6.43.

6.7 A circuit consists of a resistance of 10 Ω, an inductance of 16 mH and a capacitance of 150 μF connected in series. A supply of 100 V at 50 Hz is given to the circuit. Find the current, p.f. and power consumed by the circuit. Draw the vector diagram.

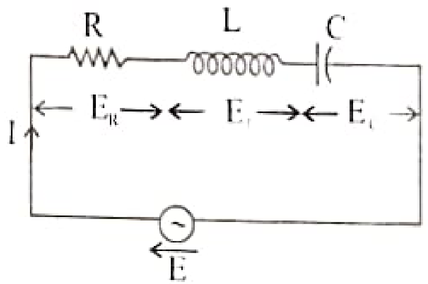

Fig.6.44

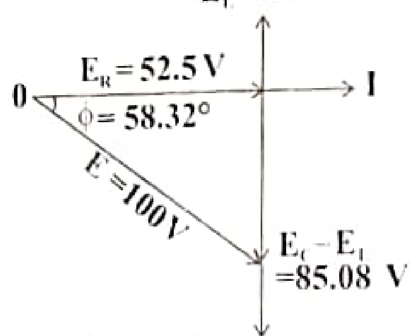

Fig.6.45 $E_C = 111.46V$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 16 \times 10^{-3} = 5.024\ \Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\times 3.14\times 50\times 150\times 10^{-6}} = 21.23\ \Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{10^2 + (21.23 - 5.024)^2} = 19.04\ \Omega$$

$$I = \frac{E}{Z} = \frac{100}{19.04} = 5.25\ A$$

$$p.f. = \frac{R}{Z} = \frac{10}{19.04} = 0.525 \text{ leading}$$

$P = E\,I\cos\phi, = 100 \times 5.25 \times 0.525 = 275.625$ watts.

To draw the vector diagram, calculate,

$E_R = IR = 5.25 \times 10 = 52.5\ V$

$E_L = IX_L = 5.25 \times 5.024 = 26.38\ V$

$E_C = I\,X_C = 5.25 \times 21.23 = 111.46\ V$

$$\phi = \tan^{-1}\frac{X_C - X_L}{R} = \tan^{-1}\frac{21.23 - 5.024}{10} = 58.32°$$

The vector diagram is shown in Fig. 6.45.

6.8 An inductive coil takes a current of 33.24 A from 230 V, 50 Hz supply. If the resistance of the coil is 6 Ω, calculate the inductance of the coil and the power taken by the coil.

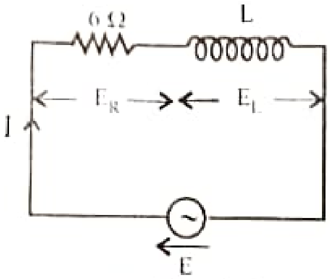

Fig.6.46

$$Z = \frac{E}{I} = \frac{230}{33.24} = 6.92\ \Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{6.92^2 - 6^2} = 3.45\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{3.45}{2 \times 3.14 \times 50} = 0.01098\ \text{H} = 10.98\ \text{mH}$$

$$P = I^2 R = 33.24^2 \times 6 = 6{,}629\ \text{watts}$$

6.9 An e.m.f. given by 100 sin (ωt−π/4) V is applied to a circuit and the current is 20 sin (314t − 1.5708) A. Find the (i) frequency and (ii) circuit elements.

i) $\omega = 314 = 2\pi f, \qquad \therefore \quad f = \frac{314}{2 \times 3.14} = 50\ \text{Hz}$

$$Z = \frac{E}{I} = \frac{100/\sqrt{2}}{20/\sqrt{2}} = 5\ \Omega, \quad \phi_1 = -\frac{\pi}{4}\ \text{rad} = -45°, \quad \phi_2 = -1.5708\ \text{rad} = -90.05°$$

$$\text{p.f.} = \cos\phi = \cos(\phi_1 - \phi_2) = \cos[-45° - (-90.05°)]$$

$$= \cos 45.05° = 0.71\ \text{lagging}$$

$$\cos\phi = \frac{R}{Z} = \frac{R}{5} = 0.71, \qquad \therefore \quad R = 3.55\ \Omega$$

The current lags the voltage. Hence, the other element is inductance.

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 3.55^2} = 3.52\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{3.52}{2 \times 3.14 \times 50} = 0.01\ \text{henry}$$

6.10 Find an expression for the current and calculate the power, when a voltage e = 283 sin 100 πt is applied to a coil having R = 50 Ω and L = 0.159 H.

$$X_L = \omega L = 100\pi \times 0.159 = 49.93\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{50^2 + 49.93^2} = 70.71\ \Omega$$

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{49.93}{50} = 45°\ \text{lagging}$$

$$I_m = \frac{E_m}{Z} = \frac{283}{70.71} = 4\ A$$

The expression for the current is

$$i = I_m \sin(\omega t - \phi) = 4 \sin(100\,\pi\, t - 45°)\ A$$

$$P = E\,I \cos\phi = \frac{283}{\sqrt{2}} \times \frac{4}{\sqrt{2}} \cos\{0-(-45)\} = 400.22 \text{ watts.}$$

6.11 A circuit shown in Fig. 6.47 consists of three branches connected in parallel. One branch consists of a pure resistance of 10 Ω. The second branch consists of a resistance of 20 Ω and an inductance of 0.1 H. The third branch consists of a resistance of 15 Ω and a capacitance of 150 μF. If the circuit is connected across a 230 V, 50 Hz supply, calculate the total current, power and p.f. of the circuit.

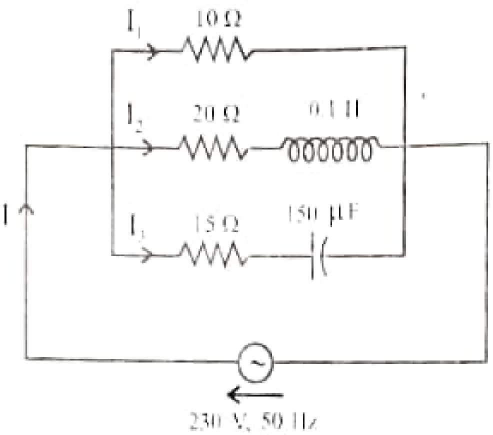

Fig.6.47

This problem may be solved using any one of the three methods discussed earlier.

(i) Method of Vectors:

$$I_1 = \frac{230}{10} = 23\ A$$

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.1 = 31.4\ \Omega$$

$$Z_2 = \sqrt{20^2 + 31.4^2} = 37.23\ \Omega$$

$$I_2 = \frac{230}{37.23} = 6.18\ A$$

$$\phi_2 = \tan^{-1}\frac{X_L}{R_2} = \tan^{-1}\frac{31.4}{20} = 57.5° \text{ lagging}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 150 \times 10^{-6}} = 21.23\ \Omega$$

$Z_3 = \sqrt{15^2 + 21.23^2} = 26\ \Omega$

$I_3 = \dfrac{230}{26} = 8.85$ A, $\quad \phi_3 = \tan^{-1}\dfrac{21.23}{15} = 54.76°$ leading.

I_1, I_2 and I_3 are represented on X–Y plane as shown in Fig. 6.48.

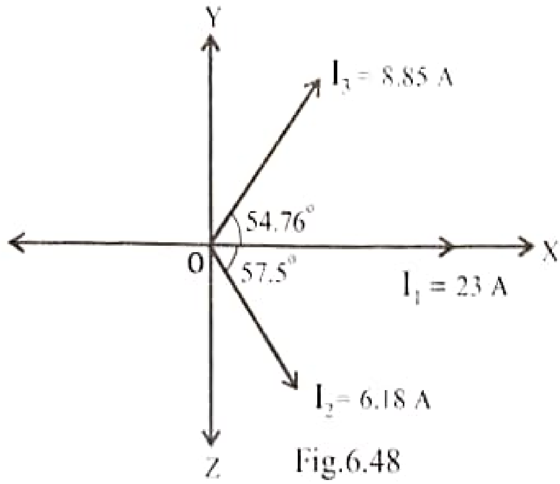

Fig.6.48

$\Sigma X = 23 + 6.18 \cos 57.5° + 8.85 \cos 54.76° = 31.44$

$\Sigma Y = 0 + 8.85 \sin 54.76° - 6.18 \sin 57.5° = 2.02$

$I = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2} = \sqrt{31.44^2 + 2.02^2} = 31.5$ A

$\phi_R = \tan^{-1}\dfrac{\Sigma Y}{\Sigma X} = \tan^{-1}\dfrac{2.02}{31.44} = 3.68°$

p.f. = $\cos\phi = \cos 3.68° = 0.998$ leading

$P = E\,I\cos\phi = 230 \times 31.5 \times 0.998 = 7{,}230$ watts.

(ii) 'j' Method.

$Z_1 = (10 + j0)\ \Omega \qquad I_1 = \dfrac{230}{10 + j0} = 23$ A

$Z_2 = 20 + j31.4 = 37.23\ \angle 57.5°\ \Omega$

$I_2 = \dfrac{230}{37.23\angle 57.5°} = 6.18\ \angle -57.5° = (3.32 - j\,5.21)$ A

$Z_3 = 15 - j21.23 = 26\ \angle -54.76°\ \Omega$

$I_3 = \dfrac{230}{26\angle -54.76°} = 8.85\ \angle 54.76° = (5.12 + j\,7.23)$ A

$I = I_1 + I_2 + I_3 = 31.44 + j2.02 = 31.5\ \angle 3.68°$ A

$P = I\cos\phi = 230 \times 31.5 \times \cos 3.68° = 7{,}230$ watts

(iii) Admittance Method

$$G_1 = \frac{1}{R_1} = \frac{1}{10} = 0.1\ \mho, \qquad B_1 = 0$$

$$Z_2^2 = 20^2 + 31.4^2 = 1{,}385.96$$

$$G_2 = \frac{R_2}{Z_2^2} = \frac{20}{1{,}385.96} = 0.0144304\ \mho,\ B_2 = -\frac{X_2}{Z_2^2} = -\frac{31.4}{1{,}385.96} = -0.022557\ \mho$$

$$Z_3^2 = 15^2 + 21.23^2 = 675.7129$$

$$G_3 = \frac{R_3}{Z_3^2} = \frac{15}{675.7129} = 0.0221987\mho,\ B_3 = \frac{X_3}{Z_3^2} = \frac{21.23}{675.7129} = 0.0314186\ \mho$$

$$G = G_1 + G_2 + G_3 = 0.1366291\ \mho,$$

$$B = B_1 + B_2 + B_3 = 0.0087629\ \mho$$

$$Y = G + jB = 0.13666291 + j\,0.0087629 = 0.1369098 \angle 3.67^\circ\ \mho$$

$$I = E\,Y = 230 \times 0.1369098\ \angle 3.67^\circ = 31.49 \angle 3.67^\circ\ \text{A}$$

$$P = E\,I \cos\phi = 230 \times 31.49 \times \cos 3.67^\circ = 7{,}230\ \text{watts}$$

6.12 Find the total current, power and p.f. of the circuit given in Fig. 6.49.

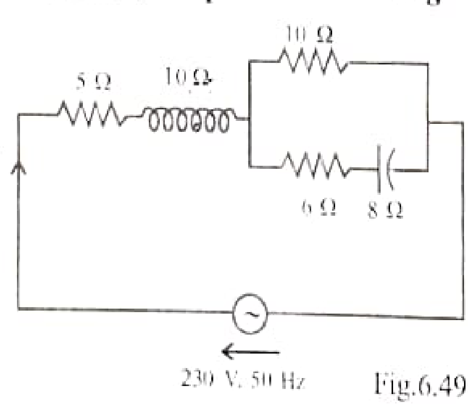

Fig.6.49

$$Z_1 = 5 + j10 = 11.18 \angle 63.43^\circ\ \Omega$$

$$Z_2 = 10\ \Omega$$

$$Z_3 = 6 - j8 = 10 \angle -53.13^\circ\ \Omega$$

$$Z_2 \parallel Z_3 = \frac{10 \times 10\angle -53.13^\circ}{10 + 6 - j8} = (5 - j2.5)\ \Omega$$

$$Z = 5 + j10 + 5 - j\,2.5 = 10 + j7.5 = 12.5 \angle 36.87^\circ\ \Omega$$

$$I = \frac{E}{Z} = \frac{230}{12.5\angle 36.87^\circ} = 18.4 \angle -36.87^\circ\ \text{A}$$

p.f. = cos ϕ = cos 36.87° = 0.8 lagging.

P = E I cos ϕ = 230 x 18.4 x 0.8 = 3,385.6 watts.

6.13 The current has the following steady values in amperes for equal intervals of time changing instantaneously from one value to the next. 0, 10, 20, 30, 20, 10, 0, −10, −20, −30, −20, −10, 0 etc. Calculate the r.m.s. value, the average value and its form factor.

Soln.: $$I = \sqrt{\frac{0^2+10^2+20^2+30^2+20^2+10^2}{6}} = 17.795 \text{ A}$$

$$I_{av} = \frac{0+10+20+30+20+10}{6} = 15 \text{ A}, \quad K_f = \frac{I}{I_{av}} = \frac{17.795}{15} = 1.1863$$

6.14 An alternating voltage of (80+j 60) V is applied across an impedance (3+j 4) Ω. Find the power and power factor of the circuit.

Soln.: $$I = \frac{E}{Z} = \frac{80+j60}{3+j4} = \frac{100\angle 36.87^\circ}{5\angle 53.13^\circ} = 20 \angle -16.26^\circ \text{ A}$$

$$\text{Power} = I^2 R = 20^2 \times 3 = 1{,}200 \text{ W}, \quad \text{p.f.} = \frac{R}{Z} = \frac{3}{5} = 0.6 \text{ lagging}$$

6.15 A current i = 10 sin (314t − 10°) A produces a potential drop v = 220 sin (314t+20°) V in a circuit. Find the values of the circuit parameters assuming a series combination.

Soln.: $$Z = \frac{V}{I} = \frac{\frac{220}{\sqrt{2}}\angle 20^\circ}{\frac{10}{\sqrt{2}}\angle -10^\circ} = 22 \angle 30^\circ \ \Omega = (19.05 + j\,11)\ \Omega$$

$$\therefore \ R = 19.05\ \Omega \quad \text{and} \quad X_L = 11\ \Omega, \quad f = \frac{314}{2\pi} = 50 \text{ Hz}$$

$$\therefore \ L = \frac{X_L}{2\pi f} = \frac{11}{2\pi \times 50} = 0.035 \text{ H} \quad \text{or} \quad 35 \text{ mH}$$

6.16 A voltage of 177.2 V is applied to a series circuit consisting of a resistor, an inductor and a capacitor. The respective voltages across these components are 170 V, 150 V and 100 V and the current is 4 A. Find the power factor of the circuit.

Soln.:

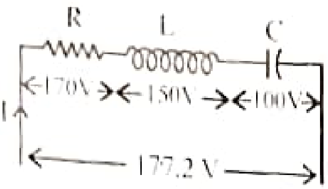

Fig.6.50

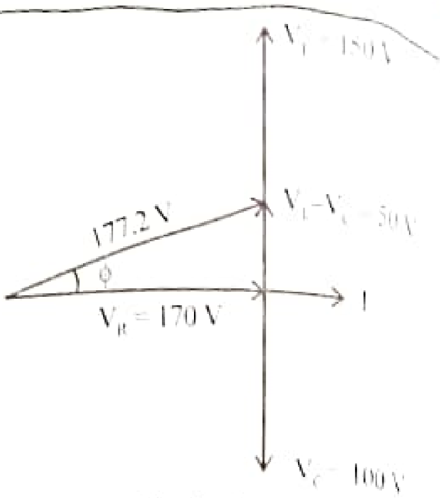

Fig.6.51

From the vector diagram in Fig.6.54

$$\text{Power factor} = \cos\phi = \frac{170}{177.2} = 0.9595 \text{ lagging}$$

6.17 In a parallel RLC circuit, when a line voltage of 250 V at 50 Hz is applied, the currents in the resistance, inductance and capacitance are 1.5 A, 2.5 A and 1.7 A respectively. Calculate the line current, power factor of the circuit and power supplied.

Soln.: $V = 250\ \angle 0°$ V (reference)

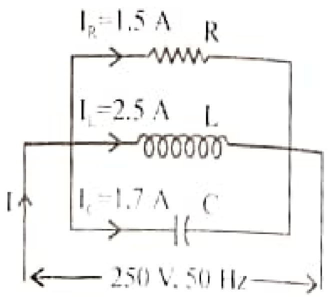

Fig.6.52

$\therefore I_R = 1.5$ A

$$I_L = 2.5\ \angle -90°\ \text{A} = -j\,2.5\ \text{A}$$

$$I_C = 1.7\ \angle +90°\ \text{A} = j\,1.7\ \text{A}$$

$$I = I_R + I_L + I_C = 1.5 - j\,2.5 + j\,1.7$$

$$= (1.5 - j\,0.8)\ \text{A} = 1.7\ \angle -28.07°\ \text{A}$$

$$\text{p.f.} = \cos\phi = \cos(\phi_1 - \phi_2) = \cos[0 - (-28.07)] = \cos 28.07° = 0.8824 \text{ lagging}$$

$$P = E\,I\cos\phi = 250 \times 1.7 \times 0.8824 = 375.02\ \text{W}$$

6.18 Two impedances (150 − j 157) Ω and (100 + j 110) Ω are connected in parallel across 200 V, 50 Hz supply. Find branch currents, total current and total power consumed in the circuit. Draw the phasor diagram.

Soln.: $Z_1 = (150 - j\,157)\ \Omega = 217.14\ \angle -46.31°\ \Omega$

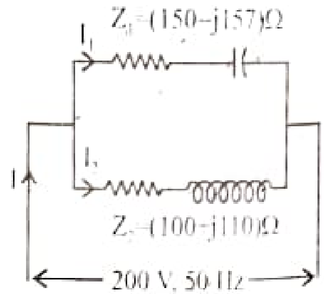

Fig.6.53

$$Z_2 = (100 + j\,110)\ \Omega = 148.66\ \angle 47.73°\ \Omega$$

$$I_1 = \frac{E}{Z_1} = \frac{200}{217.14\angle -46.31°} = 0.921\angle 46.31°\ \text{A}$$

$$= (0.636 + j0.666)\ \text{A}$$

$$I_2 = \frac{E}{Z_2} = \frac{200}{148.66\angle 47.73°} = 1.345\angle -47.73°\ \text{A}$$

$$= (0.905 - j0.995)\ \text{A}$$

$I = I_1 + I_2 = (0.636 + j0.666 + 0.905 - j0.995)\ A = 1.541 - j0.329$

$= 1.576 \angle -12.05^\circ\ A$

$P = E\ I \cos\phi = 200 \times 1.576 \cos[0^\circ - (-12.05^\circ)]$

$= 308.26\ W$

The phasor diagram is as shown in Figure.6.57.

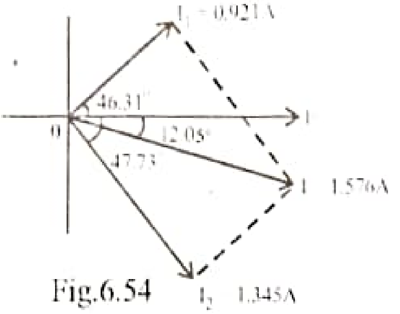

Fig.6.54

6.19 A current drawn by a capacitor of 20 μF is 1.382 A from 220 V A.C. supply. What is the supply frequency?

Soln.: $X_C = \dfrac{V}{I} = \dfrac{220}{1.382} = 159.19\ \Omega$

$X_C = \dfrac{1}{2\pi f C}$ i.e. $159.19 = \dfrac{1}{2\pi \times f \times 20 \times 10^{-6}}$, $\therefore\ f = 50\ Hz$

6.20 A coil of power factor 0.6 is in series with a 100 μF capacitor. When connected to a 50 Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.

Soln.: $I\,Z = I\,X_C$

$\therefore\ Z = X_C = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$= 31.83\ \Omega$ = impedance of the coil

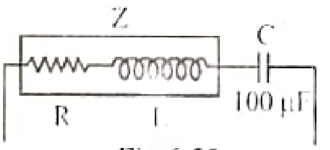

Fig.6.55

$\cos\phi = 0.6$, $\therefore\ \phi = 53.13^\circ$, $\sin\phi = 0.8$

$R = Z\cos\phi = 31.83 \times 0.6 = 19.098\ \Omega$

$X_L = Z\sin\phi = 31.83 \times 0.8 = 25.46\ \Omega$, $L = \dfrac{X_L}{2\pi f} = \dfrac{25.46}{2\pi \times 50} = 0.081\ H$

6.21 A sinusoidally varying current of average value 18.019 A is flowing in a circuit to which a voltage of peak value 141.42 V is applied. Determine (i) Z = R ± j X (ii) Power. V lags I by π/6 radians. (VTU – July/ Aug 2005)

Soln.: $\dfrac{I_{av}}{I_m} = 0.637$ i.e. $\dfrac{18.019}{I_m} = 0.637$, $\therefore\ I_m = 28.287\ A$

$|Z| = \dfrac{V_m}{I_m} = \dfrac{141.42}{28.287} = 5\ \Omega$

As voltage lags current, $Z = R - j\,X_C = Z\angle -\phi$

$\therefore\ Z = 5\angle -30^\circ = (4.33 - j\,2.5)\ \Omega$

$$P = \frac{E_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi = \frac{141.42}{\sqrt{2}} \times \frac{28.287}{\sqrt{2}} \cos(-30^\circ) = 1{,}732.2 \text{ W}$$

6.22 Three voltages are connected as shown in Figure.6.59. If $V_a = (17.32 + j\,10)$ V, $V_b = 30\angle 80^\circ$ V, and $V_c = 15\angle -100^\circ$ V. Find (i) V_{12} (ii) V_{23} and (iii) V_{34}.

Soln.: i) $V_{12} = V_b - V_c = 30\angle 80^\circ - 15\angle -100^\circ$

$= 45\angle 80^\circ$ V

ii) $V_{23} = V_c - V_b + V_a$

$= 15\angle -100^\circ - 30\angle 80^\circ + 17.32 + j\,10 = 35.6\angle -74.51^\circ$ V

iii) $V_{34} = -V_a + V_b = -(17.32 + j\,10) + 30\angle 80^\circ = 23\angle 121.78^\circ$ V

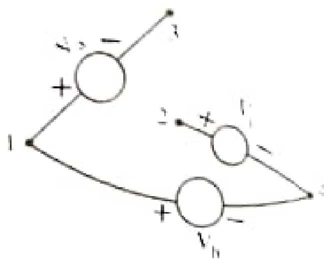

Fig.6.56

6.23 In a circuit supplied from 50 Hz, the voltage and current have maximum values of 500 V and 10 A respectively. At t = 0, their respective values are 400 V and 4 A, both increasing positively. (i) Write expressions for instantaneous values (ii) Find the angle between V and I and (iii) I at t = 0.015 sec.

Soln.: i) $e = E_m \sin(\omega t + \phi_1)$ i.e. $400 = 500 \sin(\omega t + \phi_1)$

When $t = 0$, $400 = 500 \sin\phi_1$, $\therefore \phi_1 = 53.13^\circ$

The equation for voltage is $e = 500 \sin(\omega t + 53.13^\circ)$ volts

$i = I_m \sin(\omega t + \phi_2)$ i.e. $4 = 10 \sin(\omega t + \phi_2)$

When $t = 0$, $4 = 10 \sin\phi_2$, $\therefore \phi_2 = 23.57^\circ$

The equation for current is : $i = 10 \sin(\omega t + 23.57^\circ)$ amps

ii) $\phi = \phi_1 - \phi_2 = 53.13^\circ - 23.57^\circ = 29.55^\circ$

iii) $i = 10 \sin(\omega t + 23.57^\circ) = 10 \sin(360 \times 50 \times 0.015 + 23.57)$

$= 10 \sin 293.57^\circ = -9.1$ A

6.24 The equation of an alternating current is given by i = 42.42 sin 628 t. Calculate (i) maximum value (ii) frequency (iii) r.m.s. value (iv) average value and (v) form factor.

Soln.: i) $I_m = 42.42$ A ii) $2\pi f = 628$, $\therefore f = 100$ Hz

iii) $I = 0.707\, I_m = 0.707 \times 42.42 = 30$ A

iv) $I_{av} = 0.637\, I_m = 0.637 \times 42.42 = 27$ A

v) Form factor $= \frac{I}{I_{av}} = \frac{30}{27} = 1.11$