MODULE-2

CHAPTER 3

SINGLE PHASE CIRCUITS

3.1 Pure Resistance Circuit:

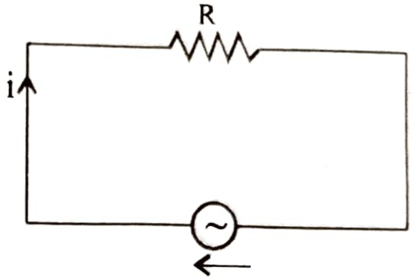


Fig.3.1

In the circuit shown in Fig. 3.1, R is a pure resistance to which an alternating voltage $e = E_m \sin \omega t$ is applied, due to which, an alternating current i flows through it.

Then, by Ohm's law,

$$i = \frac{e}{R} = \frac{E_m \sin \omega t}{R} = I_m \sin \omega t, \quad \text{where,} \quad I_m = \frac{E_m}{R}$$

By observing the equations for voltage, $e = E_m \sin \omega t$ and $i = I_m \sin \omega t$, it can be concluded that, the current is in phase with the voltage. Vectorially, the r.m.s. values of voltage and current are represented as shown in Fig. 3.2.

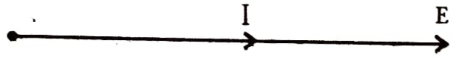


Fig.3.2

The instantaneous power consumed by the resistance is given by,

$$p = e\,i = E_m \sin \omega t\, I_m \sin \omega t = E_m I_m \sin^2 \omega t$$

$$= E_m I_m \left(\frac{1-\cos 2\omega t}{2}\right) = \frac{1}{2} E_m I_m - \frac{1}{2} E_m I_m \cos 2\omega t$$

This equation consists of two parts. The second part $1/2 E_m I_m \cos 2\omega t$ is a periodically varying quantity whose frequency is two times the frequency of the applied voltage and its average value over a period of time is zero. Power is a scalar quantity and hence only its average value has to be taken into account. Hence the power consumed by the resistance is only due to the first part $\frac{1}{2} E_m I_m$.

$$P = \frac{1}{2} E_m I_m = \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = E\,I \qquad (3.1)$$

The waveforms of e, i and p are as shown in the Fig. 3.2

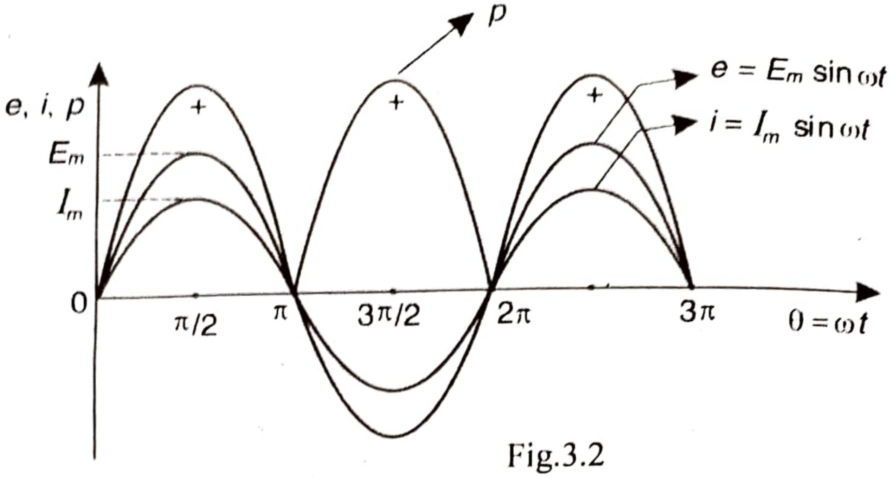


Fig.3.2

3.2 Pure Inductance Circuit:

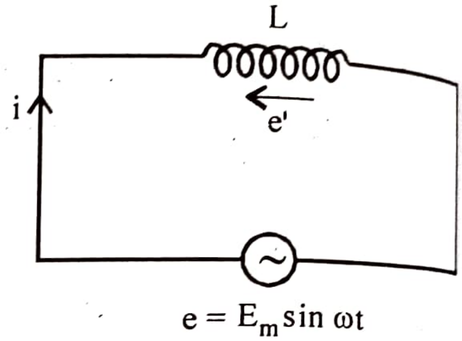


Fig.3.3

Consider a coil of pure inductance L henrys, across which an alternating voltage e = E_m sin ωt is applied, as shown in Fig. 3.3, due to which, an alternating current i flows through it. This alternating current produces an alternating flux, which links the coil and hence, an e.m.f. e^1 is induced in it, which opposes the applied voltage and is given by,

$$e = E_m \sin \omega t$$

$$e' = -L \frac{di}{dt} = -e$$

$$\therefore \quad e = L \frac{di}{dt}$$

$$di = \frac{e}{L} dt = \frac{1}{L} E_m \sin \omega t \, . \, dt$$

$$i = \frac{E_m}{L} \int \sin \omega \, t.dt$$

$$= \frac{E_m}{\omega L}(-\cos \omega \, t) \qquad \because -\sin(\pi/2 - \omega t)$$

$$= \frac{E_m}{X_L} \sin(\omega t - \pi/2) \qquad = -\cos \omega t$$

$$= I_m \sin(\omega t - \pi/2) \qquad = \sin(\omega t - \pi/2) \qquad (3.2)$$

Where, $X_L = \omega L = 2\pi f L$ = inductive reactance in ohms

By observing the equations for voltage and current, we find that the current lags the voltage by an angle $\pi/2$. Vectorial representation of the r.m.s. values of voltage and current is as shown in Fig. 3.3.

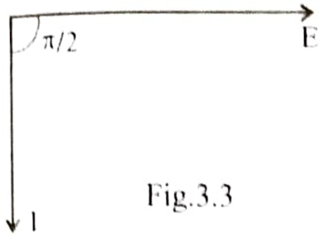


Fig.3.3

The instantaneous power is given by,

$$P = e\,i = E_m \sin \omega t\, I_m \sin(\omega t - \pi/2)$$

$$= E_m I_m \sin \omega t\,(-\cos \omega t) = -\frac{1}{2} E_m I_m \sin 2\omega t \qquad (3.3)$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. Hence, the power consumed by a pure inductance is zero, because, power is a scalar quantity and only its average value has to be considered. The power given to the pure inductance will be stored in the form of an electromagnetic field. The waveforms of e, i and p are as shown in Fig. 3.4.

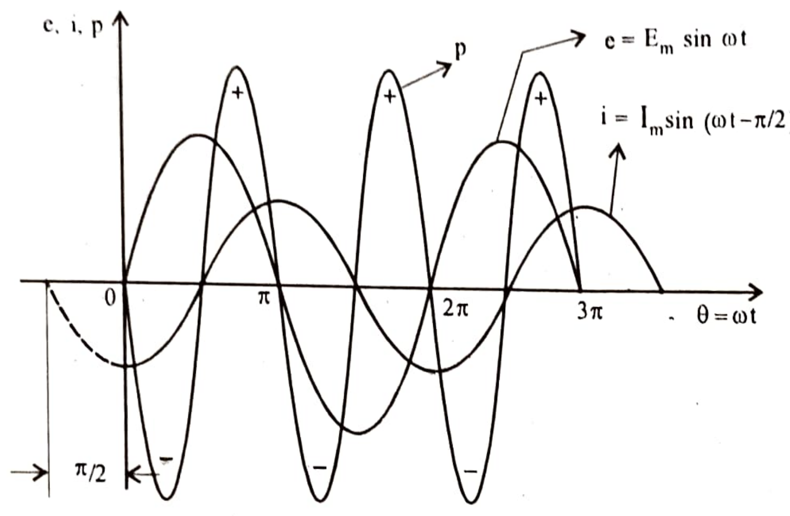
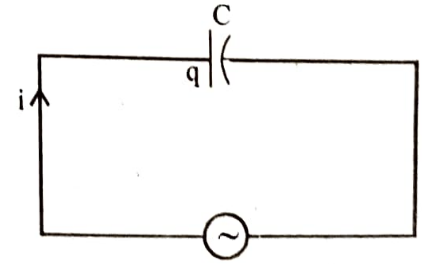


Fig.3.4

The wave from of p consists of alternate lobes of positive and negative power, whose areas are equal and hence cancel out each other. Hence, a pure inductance does not consume any power. The power given is stored in the form of an electromagnetic field.

3.3 Pure Capacitance Circuit:

Consider a capacitor of pure capacitance C, across which, an alternating voltage $e = E_m \sin \omega t$ is applied as shown in Fig. 3.5, due to which an alternating current i flows, charging the plates of the capacitor with a charge of q coloumbs.



$e = E_m \sin \omega t$

Fig.3.5

$$i = \frac{dq}{dt} = \frac{d(Ce)}{dt} = C\frac{de}{dt} = C\frac{d}{dt}(E_m \sin \omega t)$$

$$= \omega C E_m \cos \omega t = \frac{E_m}{1/\omega C} \sin(\omega t + \pi/2)$$

$$= \frac{E_m}{X_C} \sin(\omega t + \pi/2)$$

$$= I_m \sin(\omega t + \pi/2) \qquad (3.4)$$

Where, $I_m = \dfrac{E_m}{X_C}$

And $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ = capacitive reactance in ohms

By observing the equations for voltage and current, it is observed that the current leads the voltage by an angle $\pi/2$. Vectorially, current and voltage are represented as shown in Fig. 3.6.

Fig.3.6

The instantaneous power is given by,

$$P = e\,i = E_m \sin \omega t \; I_m \sin(\omega t + \pi/2)$$

$$= E_m I_m \sin \omega t \cos \omega t = \frac{1}{2} E_m I_m \sin 2\omega t \qquad (3.5)$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. As power is a scalar quantity, only its average value has to be considered. Hence, the power consumed by a pure capacitance is zero. The waveforms of e, i and p are as shown in Fig. 3.7.

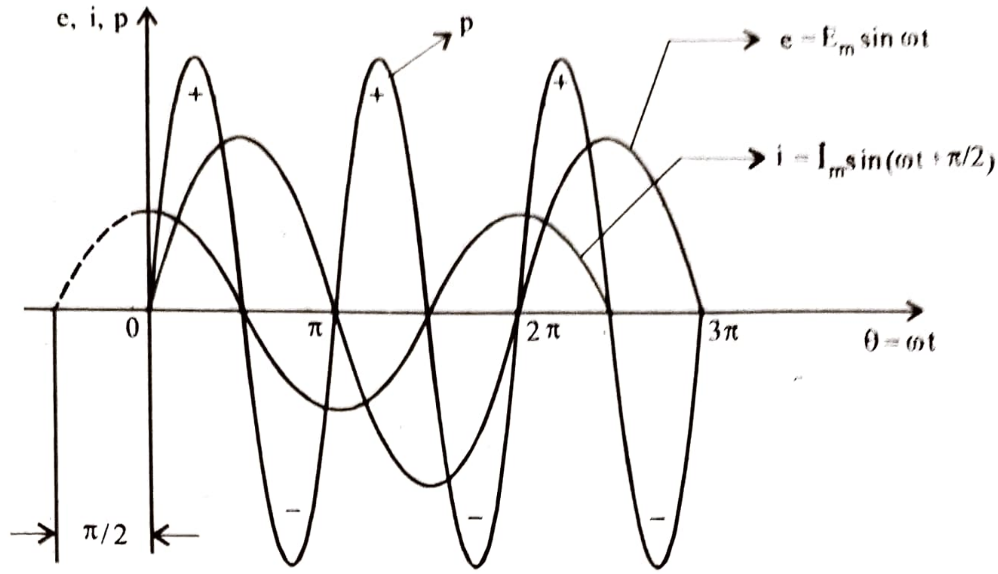
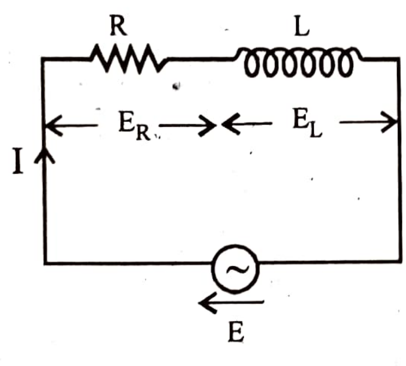


Fig.3.7

The waveform of p consists of alternate lobes of positive and negative power whose areas are equal and whose average value is zero. Hence, a pure capacitance does not consume any power. The power given is stored in the form of an electrostatic field.

3.4 R–L Series Circuit:



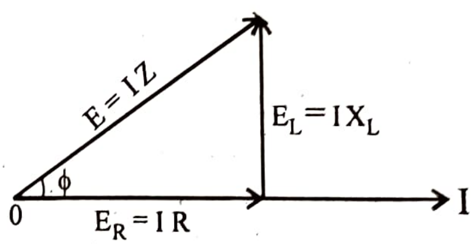


Fig.3.8 Fig.3.9

Consider an R–L series circuit as shown in Fig. 3.8 to which an alternating voltage of r.m.s. value E is applied, due to which an r.m.s. value of current I flows through the circuit. The vector diagram, taking I as the reference vector, is as shown in Fig. 3.9.

The vector diagram consists of three voltages, $E_R = I \cdot R$ which is in phase with the current, $E_L = I X_L$ which leads the current by 90°. The vector sum of these two voltages is the applied voltage $E = I Z$. Here, Z is the *impedance* of the circuit in ohms.

The impedance of an a.c. circuit may be defined as the opposition offered for the flow of alternating current in the circuit. It is the combination of resistance and reactance of the circuit. The impedance triangle for an R–L series circuit is shown in Fig. 3.10.

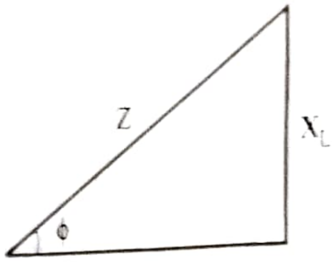


Fig.3.10

$$I = \frac{E}{Z} \quad (3.6)$$

Where. $Z = \sqrt{R^2 + X_L^2}$ (3.7)

ϕ is the power factor angle and is given by

$$\phi = \tan^{-1}\frac{X_L}{R} \quad (3.8)$$

From the vector diagram in the Fig. 3.9, we observe that the current lags the voltage by an angle ϕ. If $e = E_m \sin \omega t$, then, $i = I_m \sin(\omega t - \phi)$ (3.9)

The instantaneous power is given by,

$$p = ei = E_m \sin \omega t \, I_m \sin(\omega t - \phi) = E_m I_m \frac{1}{2}[\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{1}{2} E_m I_m \cos\phi - \frac{1}{2} E_m I_m \cos(2\omega t - \phi) \quad (3.10)$$

The second term in the equation (3.10) is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and its average value is zero. As power is always an average value, only the first term represents the power consumed.

$$\therefore \quad P = \frac{1}{2} E_m I_m \cos\phi = \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = E\,I\cos\phi \quad (3.11)$$

Where, $\cos\phi$ is known as the *power factor* of the circuit.

The phase relation between voltage and current is shown in the vector diagram of the Fig. 3.11.

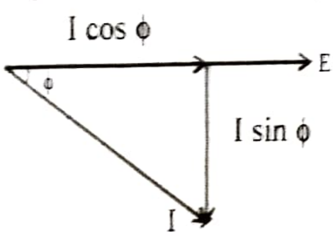


Fig.3.11

The current I can be resolved into two components (i) I cos ϕ, which is in phase with the applied voltage. Hence it is known as the "*in phase*" component. Only this component contributes for the real power consumed by the circuit. Hence it is also known as *real component* or *active component* or *wattful component*. (ii) I sin ϕ, which is in quadrature with the applied voltage is known as "*quadrature component*". This component does not contribute anything for the real power consumed by the circuit. Hence, it is also known as *reactive component* or *wattless component*.

The waveforms of e, i and p are as shown in Fig. 3.12.

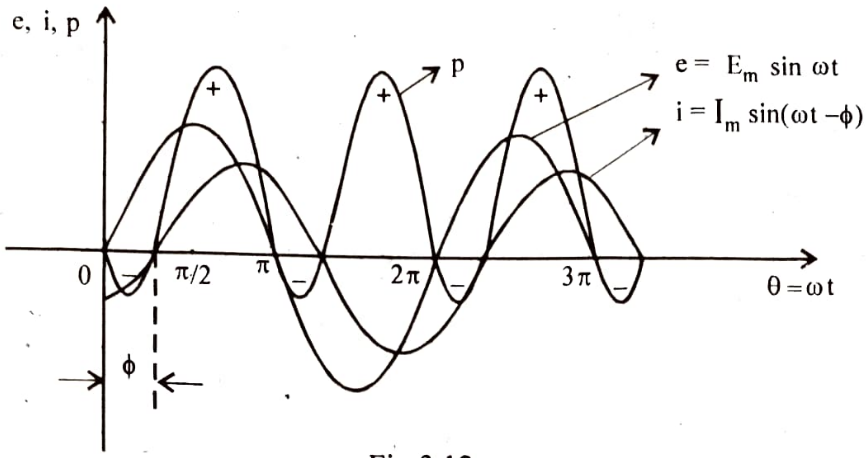


Fig.3.12

The areas of the +ve power lobes is more than the area of the −ve power lobes, indicating that, the power received by the circuit is more than the power returned by the circuit. Hence, the circuit consumes a net power given by,

$P = E\,I\cos\phi$ = Real power in watts (3.12)

$Q = E\,I\sin\phi$ = Reactive power in reactive volt-amperes (3.13)

$S = E\,I$ = Apparent power in volt-amperes (3.14)

3.5 Power Factor of a Circuit:

The p.f. of a circuit can be defined in the following three ways.

i) $\text{p.f} = \cos\phi$ (3.15)

The power factor of a circuit is the cosine of the angle between the voltage and the current.

ii) $\text{p.f} = \dfrac{R}{Z}$ (3.16)

The power factor of a circuit is the ratio of the resistance to the impedance of the circuit.

iii) $\text{p.f} = \dfrac{P}{E\,I}$ (3.17)

The power factor of a circuit is the ratio of the real power to the apparent power. The max. value of p.f. is unity.

Practical importance of Power factor:

The active power consumed by the load in an a.c. circuit is given by $P = E\,I\cos\phi$. If the p.f. of the load is small, the active power generated by an alternator and the active power transmitted and received by the consumer decreases. To generate the same active power from the generator at poor p.f. as at good p.f., the capacity of the generator has to be increased which involves additional investment on generation.

If the p.f. is small, for transmitting a particular power, the current in the transmission line increases and hence, the copper losses (I^2R losses) will increase and the efficiency of transmission decreases.

Due to low p.f, the current carrying capacity of the conductors has to be increased. Hence, large sized conductors have to be used for transmission of electrical power which involves larger investment.

Most of the loads used by the consumers are inductive in nature and normally their p.fs are low. Hence, for the effective use of the supplied energy, the supplying agencies always insist on the consumers to improve the p.fs of their loads to 0.85 or 0.9 by using static condensers of suitable capacities across their loads. The supplying agencies also give some incentive in the tariff to the consumers for improving the p.fs of their loads.

3.6 R–C Series Circuits:

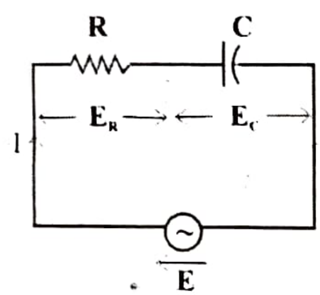


Fig.3.13

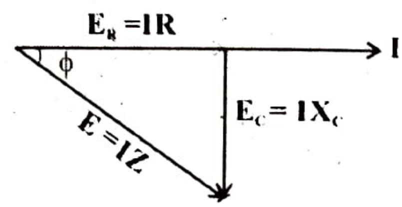


Fig.3.14

Consider an R–C series circuit as shown in Fig. 3.13, to which an alternating voltage of r.m.s. value E is applied, due to which an r.m.s. value of current I flows through the circuit. The vector diagram. is shown in Fig. 3.14.

From the vector diagram, we observe that the current leads the voltage by an angle ϕ.

If, $e = E_m \sin \omega t$ (3.18)

then, $i = I_m \sin (\omega t + \phi)$ (3.19)

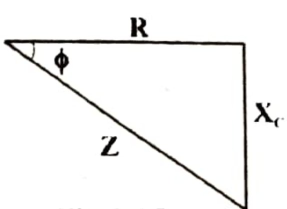


Fig.3.15

The current in the circuit is given by, $I = \dfrac{E}{Z}$ (3.20)

The impedance triangle is as shown in the Fig. 3.15.

$$Z = \sqrt{R^2 + X_C^2} \tag{3.21}$$

$$\phi = \tan^{-1} \frac{X_C}{R} \tag{3.22}$$

The instantaneous power is given by,

$$p = e\,i = E_m \sin \omega t \; I_m \sin (\omega t + \phi) = E_m I_m \frac{1}{2}[\cos(-\phi) - \cos(2\omega t + \phi)]$$

$$= \frac{1}{2} E_m I_m \cos\phi - \frac{1}{2} E_m I_m \cos(2\omega t + \phi) \qquad (3.23)$$

The second term in the equation for p is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and whose average value is zero. Hence, it does not contribute to the average value of power consumed by the circuit. The average power consumed by the circuit is only due to the first term.

$$\therefore \quad P = \frac{1}{2} E_m I_m \cos\phi = \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = E\,I\cos\phi \qquad (3.24)$$

The waveforms of e, i and p are as shown in Fig. 3.16.

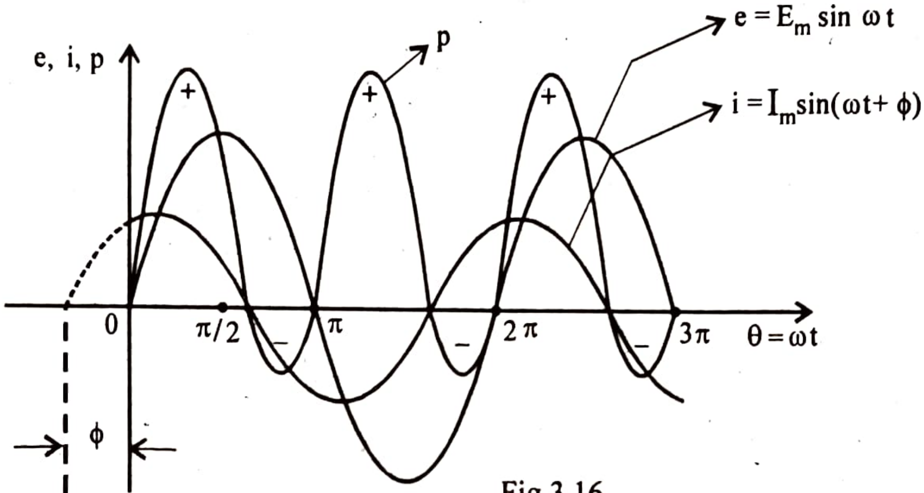


Fig.3.16

3.7 R–L–C Series Circuit:

Consider an R – L – C series circuit as shown in Fig.3.17, to which an alternating voltage of r.m.s. value E is applied due to which, an r.m.s. current I flows through the circuit. Three cases of the circuit can be discussed.

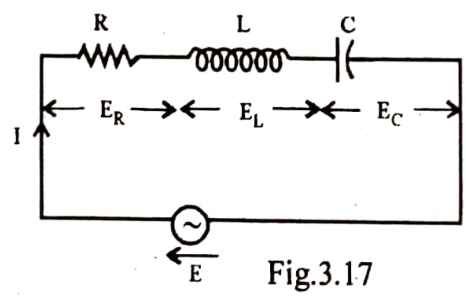
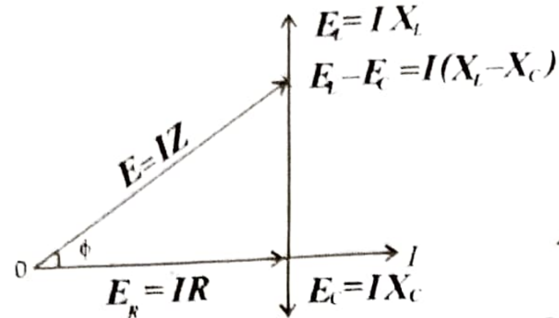


Fig.3.17

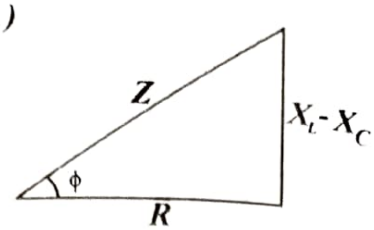
Case 1; When $X_L > X_C$

When the inductive reactance is more than the capacitive reactance, the vector diagram of the circuit as shown in Fig. 3.18.



Triangle of voltages

Fig.3.18



Impedance triangle

Fig.3.19

From the vector diagram, we observe that the current lags the voltage by an angle ϕ. The impedance triangle is shown in Fig. 3.19

$$I = \frac{E}{Z} \tag{3.25}$$

Where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (3.26)

The circuit is similar to an R–L series circuit.

If, $e = E_m \sin \omega t$ (3.27)

then, $i = I_m \sin(\omega t - \phi)$ (3.28)

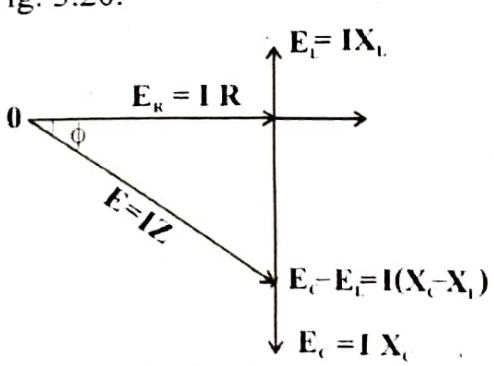
Hence, it can be proved that the power consumed is given by,

$$P = E\,I \cos\phi \tag{3.29}$$

Equation 3.29 is proved in the case of an R–L Series Circuit of section 3.4.

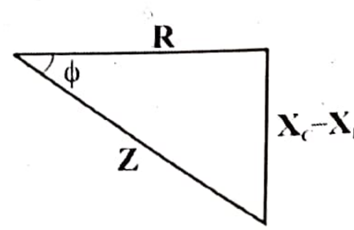
Case 2; When $X_L < X_C$

When the inductive reactance is less than the capacitive reactance, the vector diagram is as shown in Fig. 3.20.



Triangle of voltages

Fig.3.20



Impedance triangle

Fig.3.21

From the vector diagram shown in Fig. 3.20, it is observed that, the current leads the voltage by an angle ϕ. The impedance triangle is shown in Fig. 3.21.

$$I = \frac{E}{Z} \qquad (3.30)$$

Where, $Z = \sqrt{R^2 + (X_C - X_L)^2}$ (3.31)

The circuit is similar to an R–C series circuit.

If, $e = E_m \sin \omega t$ (3.32)

then, $i = I_m \sin(\omega t + \phi)$ (3.33)

Hence, it can be proved that the power consumed is given by

$P = E\, I \cos \phi$ (3.34)

Equation 3.34 is proved in the case of an R–C series circuit of section 3.6.

Case 3; When $X_L = X_C$

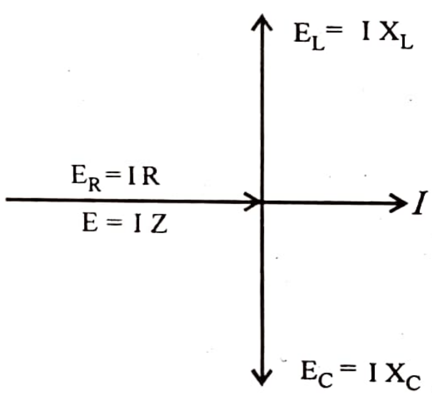


Fig.3.22

When the inductive reactance is equal to the capacitive reactance, the vector diagram is as shown in Fig. 3.22. E_L and E_C cancel each other. The current is in phase with the voltage and the circuit behaves as a pure resistance circuit. Hence, $Z = R$.

If, $e = E_m \sin \omega t$

then, $i = I_m \sin \omega t$ (3.35)

Hence, the power consumed is given by $P = E\, I$, which is proved in the case of a pure resistance circuit, of section 3.1.