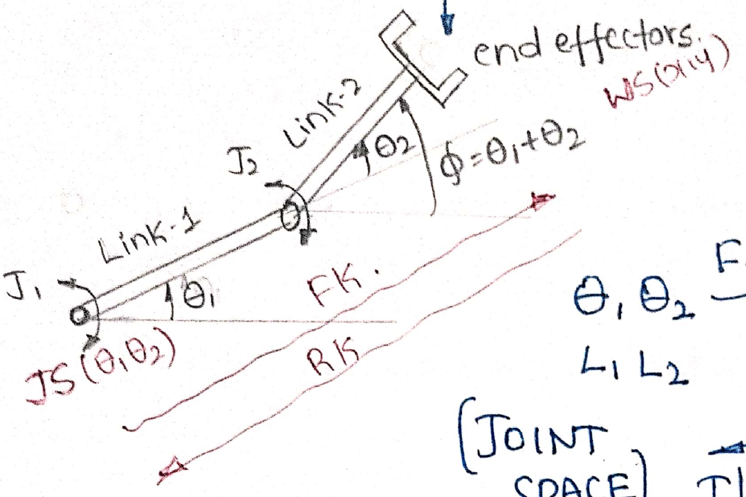
FORWARD AND REVERSE KINEMATICS OF 2D ARM

↳ Motion w/o (with out) external forces.



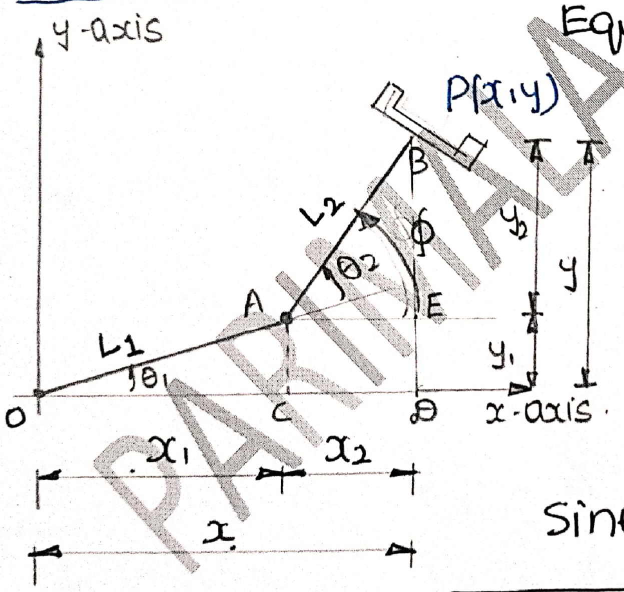
θ = CCW (+ve)

$$\theta_1, \theta_2 \xrightarrow{F.K \,|\, DK} x, y$$ [Absolute c|s] – [Cartesian Coordinate System] – [WORLD SPACE]

$L_1 \; L_2$

[JOINT SPACE] ← I|K (or) RK.

FORWARD KINEMATICS: refers to the use of kinematics Equations of robots to find the position of End Effectors.



$$\phi = \theta_1 + \theta_2$$

$x, y \rightarrow ??$ using $[\theta_1 \; \theta_2 \; L_1 \; L_2]$

In Δ^{le} OAC

$$\sin\theta_1 = \left(\frac{o}{h}\right) = \frac{AC}{OA} = \frac{y_1}{L_1}; \quad \cos\theta_1 = \left[\frac{a}{h}\right] = \frac{OC}{OA}$$

$$\boxed{y_1 = L_1 \sin\theta_1}$$

$$\cos\theta_1 = \frac{x_1}{L_1}$$

$$\boxed{x_1 = L_1 \cos\theta_1}$$

In Δ^{le} AEB: (for link-2)

$$\phi = \theta_1 + \theta_2$$

$$\sin(\theta_1+\theta_2) = \frac{EB}{AB} = \frac{y_2}{L_2}$$

$$\therefore \boxed{y_2 = L_2 \sin(\theta_1+\theta_2)}$$

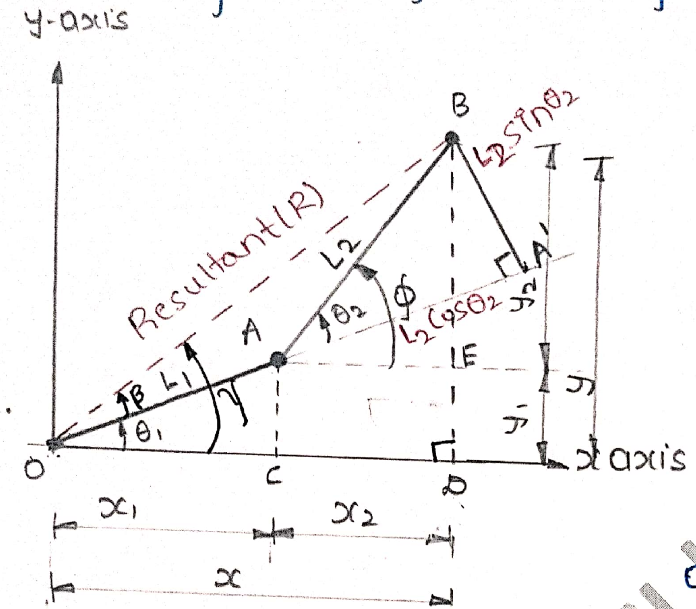
$|||^{ly}$: $$\cos(\theta_1+\theta_2) = \frac{AE}{AB} = \frac{x_2}{L_2}$$

$$\boxed{x_2 = L_2 \cos(\theta_1+\theta_2)}$$

$$\boxed{\begin{aligned} x &= x_1 + x_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) \\ y &= y_1 + y_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) \end{aligned}}$$

REVERSE KINEMATICS: (Inverse Kinematics)

It refers to the use of kinematics equations to determine the joint space from world space which provides desired position for each of end effectors.



θ_1, θ_2 ?? from (x, y)

Join OB (Resultant-R)

In Δ^{le} OBD

$$OB^2 = OD^2 + BD^2$$

$$\boxed{R^2 = x^2 + y^2} \longrightarrow (i)$$

Extend Link-1 (OA) to AA' which meet a $\perp^r$ line drawn from B.

In Δ^{le} OA'B

$$OB^2 = (OA')^2 + (A'B)^2$$

$$OB^2 = (OA + AA')^2 + (A'B)^2$$

$$OB^2 = [L_1 + L_2\cos\theta_2]^2 + [L_2 \sin\theta_2]^2$$

$$OB^2 = L_1^2 + L_2^2\cos^2\theta_2 + 2L_1L_2\cos\theta_2 + L_2^2\sin^2\theta_2$$

$$OB^2 = L_1^2 + L_2^2[\cos^2\theta_2 + \sin^2\theta_2] + 2L_1L_2\cos\theta_2$$

In Δ^{le} AA'B

$$\sin\theta_2 = \frac{A'B}{AB} = \frac{A'B}{L_2}$$

$$\boxed{A'B = L_2 \sin\theta_2}$$

$$\cos\theta_2 = \frac{AA'}{AB} = \frac{AA'}{L_2}$$

$$\boxed{AA' = L_2\cos\theta_2}$$

$$OB^2 = L_1^2 + L_2^2 + 2L_1L_2\cos\theta_2$$

$$x^2 + y^2 = L_1^2 + L_2^2 + 2L_1L_2\cos\theta_2$$

From Eq(i)

$OB = R = x + y$

$$\cos\theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\boxed{\therefore \theta_2 = \cos^{-1}\left[\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right]} \quad \cdots \text{(ANS)}$$

To find θ_1:

Defining: β:

In Δ^{le} $OA'B$:

$$\tan\beta = \left[\frac{o}{a}\right] = \frac{A'B}{OA'} = \frac{L_2\sin\theta_2}{OA + AA'} = \frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}$$

$$\therefore \boxed{\beta = \tan^{-1}\left[\frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}\right]}$$

In Δ^{le} OBD:

$$\tan\gamma = \left[\frac{o}{a}\right] = \frac{BD}{OD} = \frac{y}{x}$$

$$\gamma = \tan^{-1}\left[\frac{y}{x}\right].$$

From Figure: $\gamma = \beta + \theta_1$

$$\therefore \theta_1 = \gamma - \beta$$

$$\boxed{\theta_1 = \tan^{-1}\left[\frac{y}{x}\right] - \tan^{-1}\left[\frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}\right]}$$

$$\therefore \boxed{\theta_2 = \cos^{-1}\left[\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right]}$$

Find step.