BMS INSTITUTE OF TECHNOLOGY & MANAGEMENT
DEPARTMENT OF MATHEMATICS

MULTIVARIATE CALCULUS AND LINEAR ALGEBRA

(Common to CSE, AI&ML and CSBS Branches)

Tutorial Sheet 1: Module 1: Partial Derivatives

1. If $u = (x3 + y3 + z3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.
2. If $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
3. If $z = xy + yz + zx$, where $x = t\,Cost, y = t\,S\,int, z = t$ $\;find\; \frac{du}{dt}\; at\, t = \frac{\pi}{4}$.
4. If $u = log\left(x^3 + y^3 + z^3 - 3xyz\right)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

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Tutorial Sheet 2: Module 1: Partial Derivatives

1. If $u = f(r,s,t)$ *where* $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$
2. Show that $z(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at (1,1)
3. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
4. If $x = e^v\, Secu, y = e^v\, Tanu$, verify that $\frac{\partial(x,y)}{\partial(u,v)}\frac{\partial(u,v)}{\partial(x,y)} = 1.$

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Tutorial Sheet 3: Module 2: Multiple Integrals

1. Evaluate the following integrals: a) $\int_1^4 \int_0^{\sqrt{4-x}} xy\, dy\, dx.$ b) $\int_0^1 \int_0^1 \frac{dx\, dy}{\sqrt{(1-x)^2(1-y^2)}}.$
2. Evaluate the following integral by changing the order of integration a) $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2+y^2}} dy\, dx.$ b) $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy.$
3. Evaluate the following integrals by transforming to polar coordinates:

   a) $\int\int \left(\frac{1-x^2-y^2}{1+x^2+y^2}\right)^2 dx\, dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1.$

   b) $\int\int \sqrt{a^2 - x^2 - y^2}\; dx\, dy$ over the region bounded by the circle $x^2 + y^2 = ax$ in the first quadrant.

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Tutorial Sheet 4: Module 2: Multiple Integrals

1. Evaluate a) $\int_0^\infty x^3 e^{-4x^2}\, dx$. b) $\int_0^\infty (\log x)^4\, dx$.
2. Prove that a) $\int_0^1 \frac{dx}{\sqrt{(-\log x)}} = \sqrt{\pi}$. b) $\int_0^\infty \frac{e^{-\sqrt{x}}}{x^{7/4}}\, dx = \frac{8}{3}\sqrt{\pi}$.
3. Express the integral $\int_0^1 x^m (1-x^n)^p dx$ in terms of Beta function. Hence evaluate $\int_0^1 x^7 (1-x^4)^3\, dx$.
4. Evaluate a) $\int_0^2 (4-x^2)^{3/2}\, dx$. b) $\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$.

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Tutorial Sheet 5: Module 3: Linear Algebra-I

1. Test the following system for consistency and solve if it is consistent by using Gauss elimination method:
   a) $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$.
   b) $2x_1 - x_2 + 3x_3 = 1$, $-3x_1 + 4x_2 - 5x_3 = 0$, $x_1 + 3x_2 - 6x_3 = 0$.
2. Use Gauss-Jordan method to solve:
   a. $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$.
   b. $2x_1 + x_2 + 4x_3 = 12$, $4x_1 + 11x_2 - x_3 = 33$, $8x_1 - 3x_2 + 2x_3 = 20$.
3. Use L-U decomposition method to solve the following system of equations:
   a) $x + y + z = 1; 3x + y + z = 5; x - 2y - 5z = 25$
   b) $x - y + z = 6; 2x + 4y + z = 3; 3x + 2y - 2z = -2$

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Tutorial Sheet 6: Module 3: Linear Algebra-I

1. Solve the following system of equations by Gauss-Seidel method
   a) $20x + 2y + 6z = 28,\ x + 20y + 9z = -23,\ 2x - 7y - 20z = -57$
   b) $10x + 2y + z = 9,\ 2x + 20y - 2z = -44, -2x + 3y + 10z = 22$
2. Find the eigen values and eigen vectors of the following matrix
$$A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$
3. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and hence find its inverse.

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Tutorial Sheet 7: Module 4: Linear Algebra-II

1. By Power method, find the dominant (largest) eigenvalue and the corresponding eigenvector of the following matrix A = $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by taking the initial approximation to the eigenvector as $[1,0,0]^T$.
2. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation.
3. Verify whether the following matrix is orthogonal or not.

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

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Tutorial Sheet 8: Module 4: Linear Algebra-II

1. Find the inverse transformation of
   $y_1 = x_1,\ y_2 = x_2 \cos\theta + x_3 \sin\theta,\ y_3 = -x_2 \sin\theta + x_3 \cos\theta$
2. Obtain the canonical form of the quadratic form. Also find the rank, index and signature and nature of the quadratic form.
   $8x^2 + 7y^2 - 12xy + 4zx - 8yz + 2z^2$
3. Reduce the following quadratic forms to canonical form by an orthogonal transformation. Hence find the rank, index and signature and nature of the quadratic form.
   a) $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$
   b) $10x_1^2 + 2x_2^2 + 5x_3^2 - 4x_1x_2 - 10x_1x_3 + 6x_2x_3$

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Tutorial Sheet 9: Module 5: Linear Algebra-III

1. Show that W is a subspace of $V(R)$, where $W = \{f(x): f(2) = f(1)\}$.
2. Express the vector $V = (1, -2, 3)$ as linear combination of the vectors $V_1 = (1,1,1), V_2 = (1,2,3), V_3 = (2,-1,1)$.
3. Determine whether the vectors $V_1 = (1,2,3),\ V_2 = (2,5,8),\ V_3 = (2,5,8)$ are linearly independent or not.
4. Determine whether or not each of the following forms a basis in $\mathbb{R}^3$. $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$ and $x_3 = (1, 2, 2)$.

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Tutorial Sheet 10: Module 5: Linear Algebra-III

1. Show that the transformation $T: R^3 \to R^3$ is given by $T(x, y, z) = (y, -x, -z)$ is a linear transformation.
2. Find the linear map $T: R^2 \to R^3$ whose matrix is $A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ relative to the ordered basis $B = \{(1,1), (0,2)\} \in R^2$ and $B' = \{(0,1,1), (1,0,1), (1,1,0)\}$ for R^3.
3. Verify Rank-Nullity theorem for the transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.
4. Consider the following polynomial in $p(t)$ and inner product: $f(t) = t + 2,\ g(t) = 3t - 2,\ h(t) = t^2 - 2t - 3$ and $< f, g > = \int_0^1 f(t)g(t)\, dt$.