

CHAPTER 2

A.C. FUNDAMENTALS

2.1 Introduction:

An alternating current circuit written in short form as a.c. circuit, consists of alternating voltage source or sources due to which, alternating currents flow through the various elements of the circuit viz. the resistance, inductance and capacitance, which may be connected in the circuit independently or in all possible combinations. An alternating voltage or current is a quantity, whose magnitude continuously changes with time but which can have only two directions, either positive or negative. Alternating quantities are usually periodic in nature which can be represented by periodic waveforms. Periodic means recurring and hence during equal periods of time known as time period, the nature of the waveform of an alternating quantity is same. An alternating quantity may have any shape of waveform such as sinusoidal, rectangular, saw tooth etc., but usually, the alternating voltages generated and the alternating currents flowing through the electric circuits are having periodic sinusoidal waveforms.

2.2 Advantages of Sinusoidal Waveform:

also.

$$i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi ft \quad (2.2)$$

2.4 Definitions:

- i) **Instantaneous Value (e):** This is the value of the e.m.f. induced in the conductor at any instant.
- ii) **Amplitude (E_m):** The maximum value of the e.m.f. induced in the conductor is called the amplitude.
- iii) **Cycle of e.m.f.:** A set of positive values together with a set of negative values of the e.m.f. induced in the conductor constitute a cycle of e.m.f. induced.
- iv) **Frequency (f) :** It is defined as the number of cycles of e.m.f. induced in the conductor per second.
- v) **Time Period (T):** It is the time taken to complete one cycle of the e.m.f. induced.

$$T = \frac{1}{f} \quad (2.3)$$

2.5 Effective Value of an Alternating Current (I):

This is also called as the *root mean square value* or *r.m.s. value* in short. It is defined on the basis of the amount of heat produced. The equation for the heat produced in a resistance R , when a current I flows through it for a time t is given by,

$$H = I^2 R t \quad \text{W-S} \quad (2.4)$$

The effective value or r.m.s. value of an alternating current is equal to that steady current, which produces the same amount of heat as produced by the alternating current, when passed through the same resistance for the same time.

2.6 Effective Value of an Alternating Current Represented by any Wave:

2.7 Effective Value of an Alternating Current which is Sinusoidally Varying:

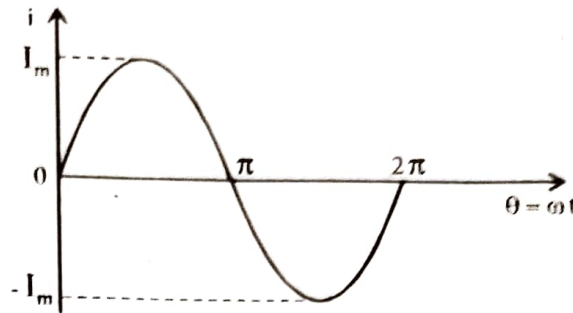


Fig.2.3

The equation for the alternating current representing the sinusoidal waveform shown in Fig. 2.3 is, .

$$i = I_m \sin \theta \quad (2.6)$$

The effective value of this current is given by

$$\begin{aligned} I^2 &= \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2 \theta d\theta = \frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{1 - \cos 2\theta}{2} d\theta \\ &= \frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \frac{I_m^2}{4\pi} [2\pi - 0 - (0 - 0)] = \frac{I_m^2}{2} \end{aligned}$$

$$\therefore I = \frac{I_m}{\sqrt{2}} = 0.707 I_m \quad (2.7)$$

The effective value of the current is 0.707 times its maximum value.

2.8 Average Value of an Alternating Current (I_{av}):

This is defined on the basis of the amount of charge transferred, which is given by,

$$q = I t \quad (2.8)$$

The average value of an alternating current is equal to that steady current, which transfers the same amount of charge, as transferred by the alternating current across the same circuit and in the same time.

2.9 Average Value of an Alternating Current Represented by any Wave:

Consider the waveform shown in fig. 2.2 representing the alternating current. Divide the waveform into n equal parts, so that, the duration of each interval is t / n seconds. Let q be the charge transferred across a circuit in t seconds.

2.10 Average Value of an Alternating Current Represented by a Sine Wave:

The average value of an alternating current represented by the sine wave over one complete cycle is zero.

Consider the sinusoidal waveform as shown in Fig. 2.3, representing the alternating current. Its average value is given by,

$$I_{av} = \frac{1}{\pi} \int_0^{\pi} i \, d\theta = \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta = \frac{I_m}{\pi} [-\cos \theta]_0^{\pi} = \frac{2}{\pi} I_m = 0.637 I_m \quad (2.10)$$

\therefore The average value of an alternating current is 0.637 times its maximum value.

2.11 Form Factor (K_f):

The form factor of an alternating quantity represented by the sinusoidal waveform is defined as the ratio of its r.m.s. value to its average value.

$$K_f = \text{Form Factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I}{I_{av}} = \frac{0.707 I_m}{0.637 I_m} = 1.11, \text{ for sine wave}$$

2.12 Peak Factor (K_p):

The peak factor of an alternating quantity, represented by the sinusoidal waveform is defined as the ratio of its maximum value to its r.m.s. value.

$$K_p = \text{Peak factor} = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I} = \frac{I_m}{0.707 I_m} = 1.414, \text{ for a sine wave}$$

2.13 Vector Representation of an Alternating Quantity:

Consider a vector OA whose magnitude is equal to I_m , the maximum value of the alternating current, as shown in Fig. 2.4 (a). Let this vector rotate in the anticlockwise direction with an angular velocity ω , same as that of the alternating current. The projection of the vector OA on Y-axis is zero, when it is in position 1 i.e. when $\theta = 0$.

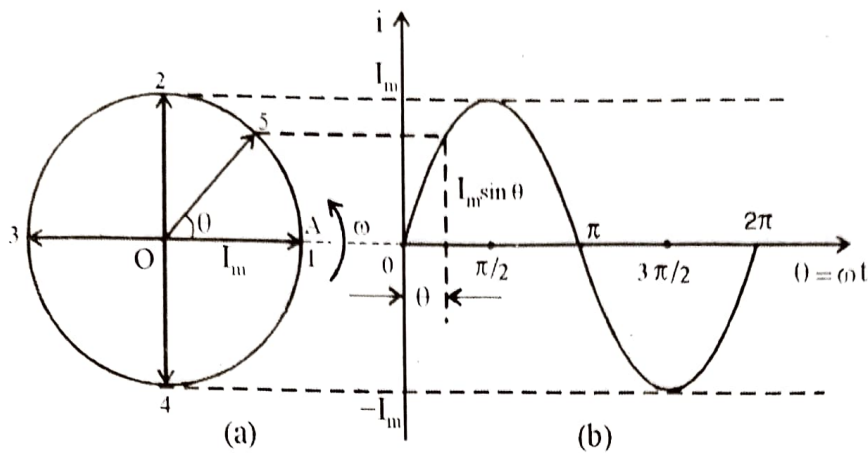


Fig.2.4

Let the vector be rotated to position 2, where $\theta = \pi/2$. The projection of the vector on Y axis in this position is I_m . Let the vector be rotated to position 3, where $\theta = \pi$. The projection of the vector on Y axis in this position is zero. In position 4, where $\theta = 3\pi/2$, the projection of this vector on Y axis is $-I_m$. Again in position 1, where $\theta = 2\pi$, the projection of this vector on Y axis is zero. At any position 5, at an angle θ from the reference axis, the projection of this vector on Y axis is $I_m \sin \theta$. When all the projected values of this vector on Y axis, for various values of θ , are plotted and the curve is traced, it gives a sine wave as shown in Fig. 2.4 (b), whose equation is $i = I_m \sin \theta$, which is also the equation for a sinusoidally varying current.

From the above discussion, we can conclude that, an alternating quantity can be vectorially represented by means of a rotating vector (i) whose magnitude is equal to the maximum value of the alternating quantity (ii) which rotates with the same angular velocity as that of the alternating quantity and (iii) whose projection on Y axis at any instant, represents the instantaneous value of the alternating quantity.

However, if the alternating quantities are represented by rotating vectors, the analysis of a.c. circuits becomes very difficult. Therefore, they are normally represented as any other ordinary vectors, having r.m.s. value as magnitude and phase angle as direction.

2.14 Phase of an Alternating Quantity:

The phase of an alternating quantity at any instant, is the angle through which the rotating vector representing the alternating quantity has rotated through, from the reference axis. When the rotating vector OA, which represents the alternating quantity, is along X axis as shown in Fig. 2.5 (a) at position 1, its phase at that instant is zero. At any other instant, say at position 5, the phase of the alternating quantity is θ . The phase of the alternating quantity θ , varies from 0 to 2π .

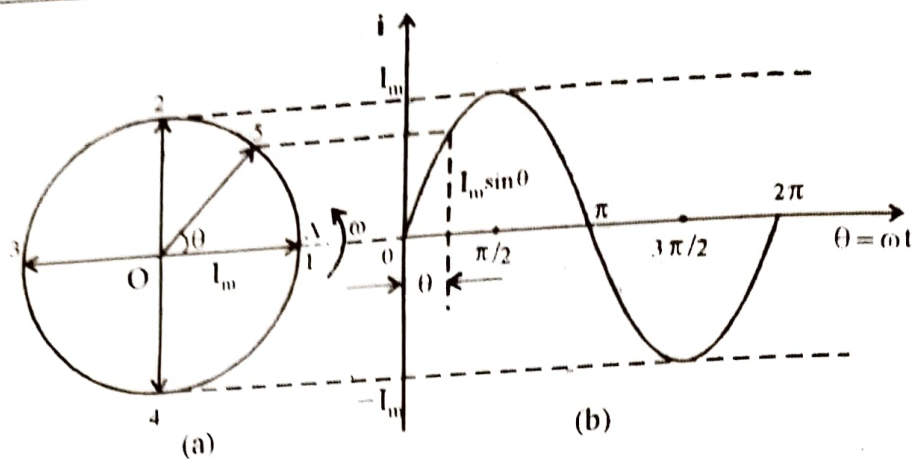


Fig.2.5

2.15 Phase Difference:

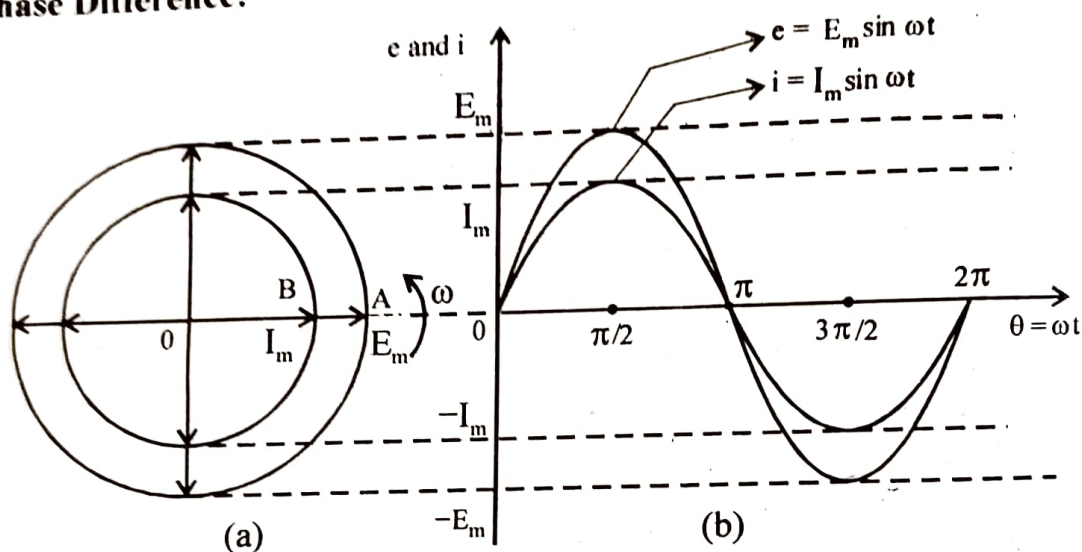


Fig.2.6

The phase difference between two alternating quantities is the angle difference between the two rotating vectors, representing the two alternating quantities.

In Fig. 2.6 (a), the rotating vector OA represents the alternating voltage and the rotating vector OB represents the alternating current. Both of them rotate together with an angular velocity ω and hence, the phase difference between them is zero. The waveforms of the alternating voltage and alternating current are as shown in Fig. 2.6 (b). In such a case, the two alternating quantities are said to be in phase with each other.

Two alternating quantities are said to be *in phase* with each other, when their corresponding values occur at the same time. The equations for the voltage and current can be written as

$$e = E_m \sin \omega t \quad (2.11)$$

$$\text{and } i = I_m \sin \omega t \quad (2.12)$$

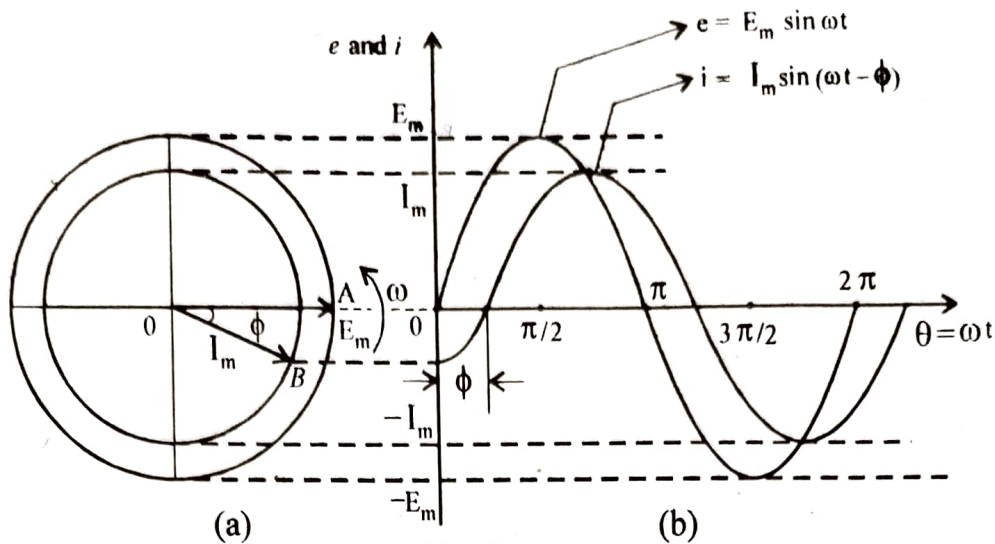


Fig.2.7

Fig. 2.7 (a) represents the two alternating quantities, the voltage and the current, by their rotating vectors OA and OB and Fig. 2.7 (b) represents their waveforms. The two vectors always rotate with an angle difference of ϕ , the current vector always lagging the voltage vector by an angle ϕ .

The current is said to *lag* the voltage by an angle ϕ , when the corresponding values of the current occur later by an angle ϕ , than the corresponding values of voltage. The equations for current and voltage in such a case can be written as,

$$e = E_m \sin \omega t \quad (2.13)$$

$$\text{and } i = I_m \sin (\omega t - \phi) \quad (2.14)$$

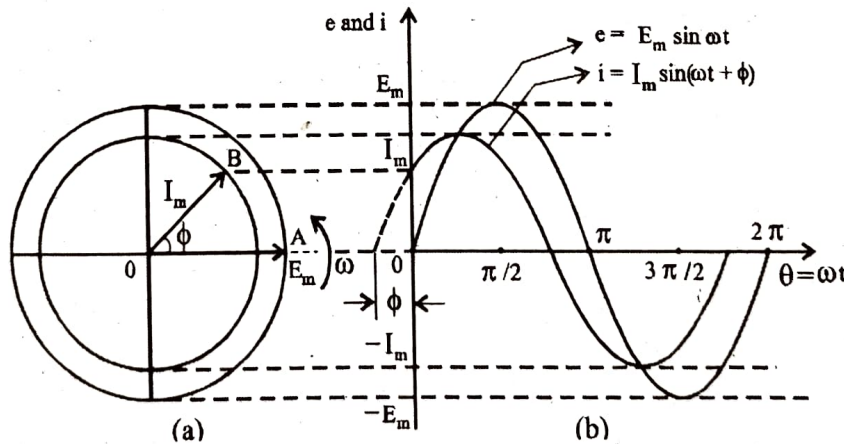


Fig.2.8

Fig. 2.8 (a) represents the two alternating quantities, the voltage and the current by their rotating vectors OA , and OB respectively and Fig. 2.8 (b) represents their waveforms. The two vectors always rotate with an angle difference of ϕ , the current vector always leading the voltage vector by an angle ϕ .

The current is said to *lead* the voltage by an angle ϕ , when the corresponding values of current occur earlier by an angle ϕ than the corresponding values of voltage.

The equations for current and voltage can be written as

$$e = E_m \sin \omega t \quad (2.15)$$

and $i = I_m \sin (\omega t + \phi) \quad (2.16)$

The phase difference between the voltage and current is usually expressed in terms of whether the current lags or leads the voltage.