WORKED EXAMPLES

1.15 Find the p.d. between the points X and Y in the network shown in Fig. 1.24

Soln.: Let I_1 and I_2 be the loop currents.

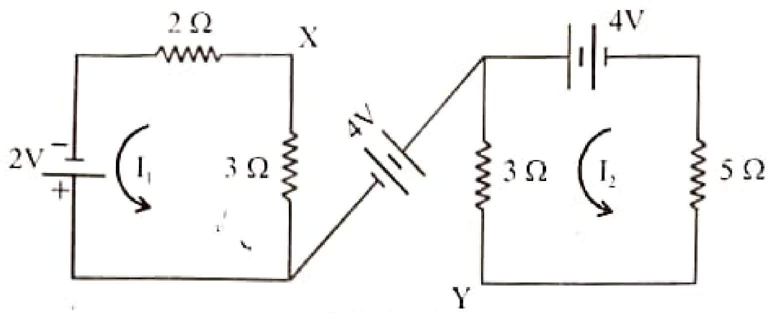


Fig. 1.24

$$I_1 = \frac{2}{2+3} = 0.4 \text{ A} \qquad \text{and} \qquad I_2 = \frac{4}{3+5} = 0.5 \text{ A}$$

$$\therefore \; V_{XY} = +3\,I_2 - 4 - 3\,I_1 = 3 \times 0.5 - 4 - 3 \times 0.4$$

$$= -3.7 \text{ volts} \quad \text{(Y is at a higher potential than X)}$$

1.16 In the network shown in Fig. 1.25 determine the direction and magnitude of current flow in the milli–ammeter A, having a resistance of 10 Ω.

Soln.: Let the current distribution be as shown in Fig. 1.25.

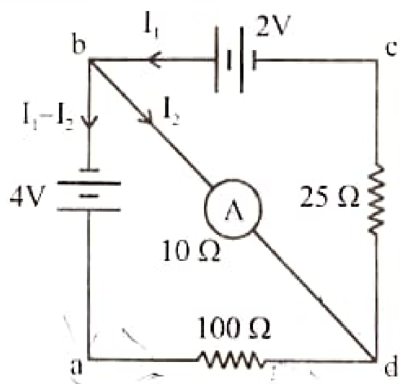


Fig. 1.25

For abda $\qquad -4 - 10\,I_2 + 100\,(I_1 - I_2) = 0$

i.e. $\qquad 100\,I_1 - 110\,I_2 = 4 \qquad$ (i)

For bcdb $\qquad -2 + 25\,I_1 + 10\,I_2 = 0$

i.e. $\qquad 25\,I_1 + 10\,I_2 = 2 \qquad$ (ii)

Solving (i) and (ii), we get

$$I_2 = \frac{\begin{vmatrix} 100 & 4 \\ 25 & 2 \end{vmatrix}}{\begin{vmatrix} 100 & -110 \\ 25 & 10 \end{vmatrix}} = 0.0267 \text{ A} = 26.7 \text{ mA}$$

1.17 In the network shown in Fig. 1.26, find the current flowing in each branch using Kirchhoff's laws.

Soln. : Assume the currents as shown.

For ABDA, $\quad -10\,I_2 - 25\,I_3 + 20\,(I_1 - I_2) = 0$

i.e. $\quad 4\,I_1 - 6\,I_2 - 5\,I_3 = 0 \qquad$ (1.29)

For BCDB, $-15\,(I_2 - I_3) + 5\,(I_1 - I_2 + I_3) + 25\,I_3 = 0$

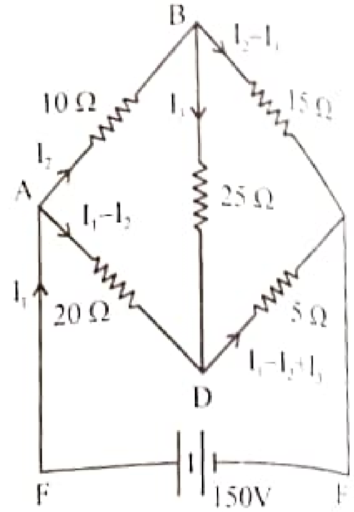


Fig. 1.26

i.e. $I_1 - 4\,I_2 + 9\,I_3 = 0$ (1.30)

For ABCEFA, $-20\,(I_1 - I_2) - 5\,(I_1 - I_2 + I_3) + 150 = 0$

i.e. $5\,I_1 - 5\,I_2 + I_3 = 30$ (1.31)

Solving equations (1.29), (1.30) and (1.31), we get

$$I_1 = \frac{\begin{vmatrix} 0 & -6 & -5 \\ 0 & -4 & 9 \\ 30 & -5 & 1 \end{vmatrix}}{\begin{vmatrix} 4 & -6 & -5 \\ 1 & -4 & 9 \\ 5 & -5 & 1 \end{vmatrix}} = 12.7\text{ A},\quad I_2 = \frac{\begin{vmatrix} 4 & 0 & -5 \\ 1 & 0 & 9 \\ 5 & 30 & 1 \end{vmatrix}}{\begin{vmatrix} 4 & -6 & -5 \\ 1 & -4 & 9 \\ 5 & -5 & 1 \end{vmatrix}} = 7.03\text{ A}$$

Substituting I_1 and I_2 in equations (1.29), we get $I_3 = 1.72$ A

The various branch currents are :

$I_{AB} = I_2 = 7.03$ A, $I_{BC} = I_2 - I_3 = 7.03 - 1.72 = 5.31$ A,

$I_{DC} = I_1 - I_2 + I_3 = 12.7 - 7.03 + 1.72$, $I_{AD} = I_1 - I_2 = 12.7 - 7.03 = 5.67$ A

$I_{CEFA} = I_1 = 12.7$ A

1.18 In the circuit shown in Fig. 1.27, what is the voltage across cd if (i) switch S is open and (ii) switch S is closed.

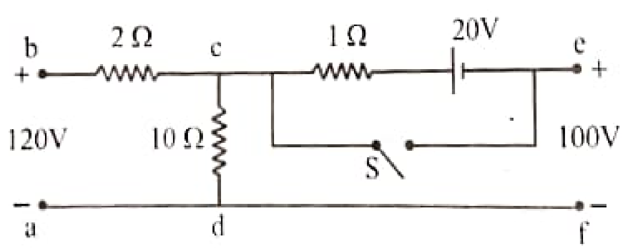


Fig. 1.27

Soln.: i) When S is open, the circuit is written as below.

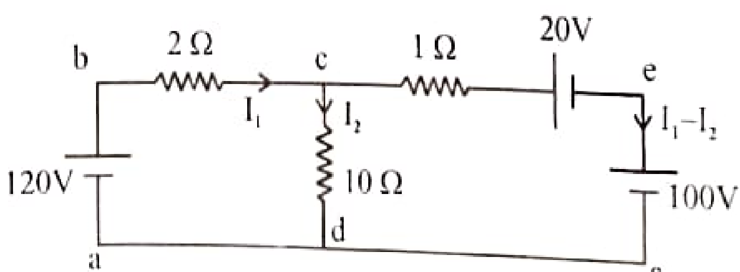


Fig. 1.28

The current distribution is as shown in Fig. 1.28

For abcda, $120 - 2\,I_1 - 10\,I_2 = 0$, i.e. $I_1 + 5\,I_2 = 60$ (1.32)

For dcefd $10\,I_2 - 1\,(I_1 - I_2) - 20 - 100 = 0$

i.e. $-I_1 + 11\,I_2 = 120$

(1.33)

Solving equations (1.32) and (1.33), we get

$I_1 = 3.75$ A and $I_2 = 11.25$ A

Voltage across cd = $V_{cd} = 10 \times I_2 = 10 \times 11.25 = 112.5$ V

ii) When the switch S is closed, no current flows through 1 Ω and 20 V. The resulting circuit is as shown below.

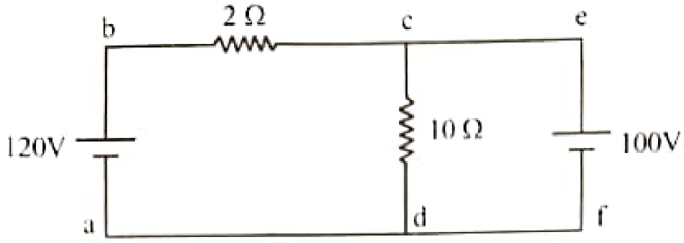


Fig. 1.29

$\therefore V_{cd} = 100$ V

1.19 Find the currents in the various branches of the given network shown in Fig.1.30

Soln.: The currents are distributed in the various branches as shown

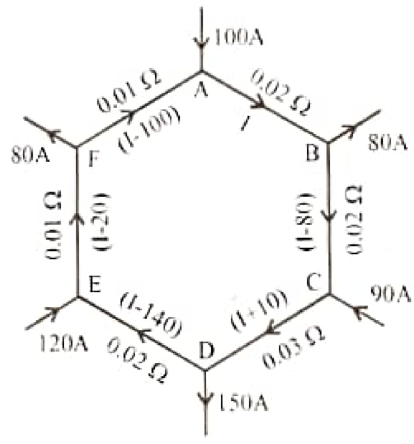


Fig. 1.30

For ABCDEFA

$$-0.02\,I - 0.02\,(I - 80) - 0.03\,(I + 10)$$
$$- 0.02\,(I - 140) - 0.01\,(I - 20) - 0.01(I - 100) = 0$$

On solving, we get, $I = 48.18$ A.

$\therefore I_{AB} = I = 48.18$ A

$I_{BC} = (I - 80) = -31.82$ A or $I_{CB} = 31.82$ A

$I_{CB} = (I + 10) = 58.18$ A

$I_{DE} = (I - 140) = -91.82$ A or $I_{ED} = 91.82$ A

$I_{EF} = (I - 20) = 28.18$ A

$I_{FA} = (I - 100) = -51.82$ A or $I_{AF} = 51.82$ A

1.20 Find the currents I_1, I_2 and I_3 and the voltages V_a, V_b in the network shown in Fig.1.31.

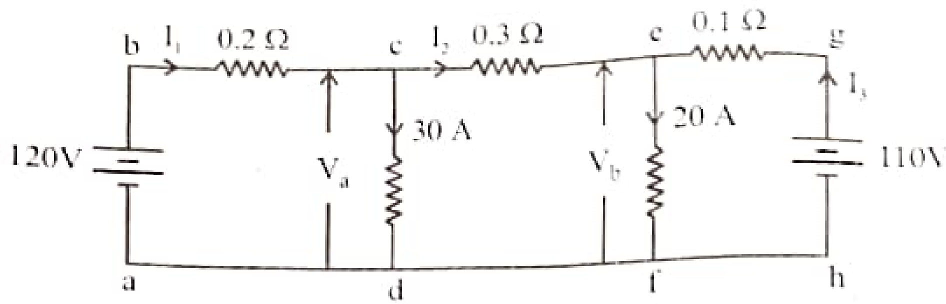


Fig. 1.31

Soln. : From the circuit, we know

$I_2 = I_1 - 30$, $\quad I_3 + I_1 - 30 = 20 \quad$ i.e. $\quad I_3 = 50 - I_1$

For abceghfda , $\quad 120 - 0.2\, I_1 - 0.3\, I_2 + 0.1\, I_3 - 110 = 0$

Substituting for I_2 and I_3, we get

$- 0.6\, I_1 = -24 \quad$ or $\quad I_1 = 40$ A

$\therefore \quad I_2 = 40 - 30 = 10$ A, $\quad I_3 = 50 - 40 = 10$ A

$V_a = 120 - 0.2 \times 40 = 112$ V , $\quad V_b = 110 - 0.1 \times 10 = 109$ V

1.21 For the following network shown in Fig. 1.32, find the currents in all the branches and potential difference across AD and CE.

Soln.: The currents are distributed as shown in the figure.1.32.

For the loop ABCDEA,

$-2.5\, I - (I + 10) \times 1 - 2\,(I + 19) - 3\,(I + 17) - 4\,(I + 22) = 0$

i.e. $\quad I = -14.96$ A

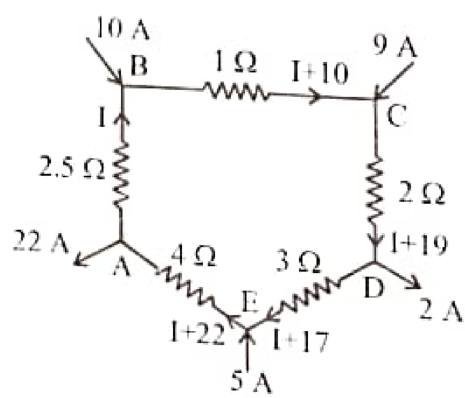


Fig. 1.32

$\therefore \quad I_{AB} = I = -14.96$ A $\quad$ i.e. $\quad I_{BA} = 14.96$ A

$I_{BC} = I + 10 = -14.96 + 10 = -4.96$ A $\quad$ i.e. $\quad I_{CB} = 4.96$ A

$I_{CD} = I + 19 = -14.96 + 19 = 4.04$ A

$I_{DE} = I + 17 = -14.96 + 17 = 2.04$ A

$I_{EA} = I + 22 = -14.96 + 22 = 7.04$ A

$V_{AD} \quad = -3(I + 17) - 4\,(I + 22) = -3 \times 2.04 - 4(7.04) \quad = \quad -34.28$ V

$V_{CE} = 3\,(I + 17) + 2\,(I + 19) = 3 \times 2.04 + 2\,(4.04) \quad = \quad 14.2$ V

1.22 The current in the 6 Ω resistance of the network shown in the Fig. 1.33 is 2A. Determine the currents in all the branches and the applied voltage.

Soln. : Let I be the total current.

Then, $2 = I \times \dfrac{8}{8+6}$

i.e $\quad I = 3.5$ A

$\therefore I_2 \quad = I - 2 \quad = 3.5 - 2 = 1.5$ A

$I_1 = 3.5 \times \dfrac{20}{20+8} = 2.5$ A

$I_3 = I - I_1 = 3.5 - 2.5 = 1$A

$V = 4 \times I + 6 \times 2 + 8 \times I_1 = \quad 4 \times 3.5 + 6 \times 2 + 8 \times 2.5 = 46$ V

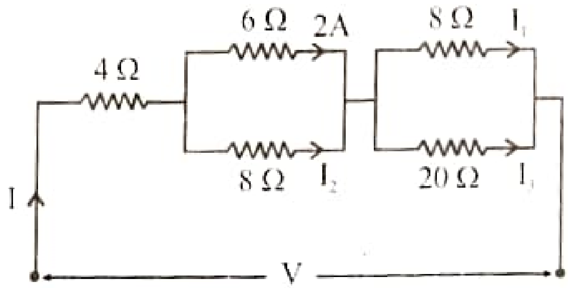


Fig. 1.33

1.23 The circuit in Fig.1.33(a) shows a hollow cube of 12 wires, each having a resistance of r. Find the resistance between any two diagonally opposite corners.

Soln.: Let a current 6I enter at a. Because of the symmetry of the cube, the current distribution in the wires is as shown in the Fig.1.34. A current 6I leaves the diagonally opposite corner g. Let a battery of E volts be connected between a and g. Then, applying Kirchhoff's voltage law for the closed path aefga, we get

$E = r\,(2I + I + 2I) = 5\,I\,r$

If R is the equivalent resistance between a and g, then $E = 6\,I\,R$.

$\therefore 5\,I\,r = 6\,I\,R \qquad$ or $\quad R = \dfrac{5}{6}\,r.$

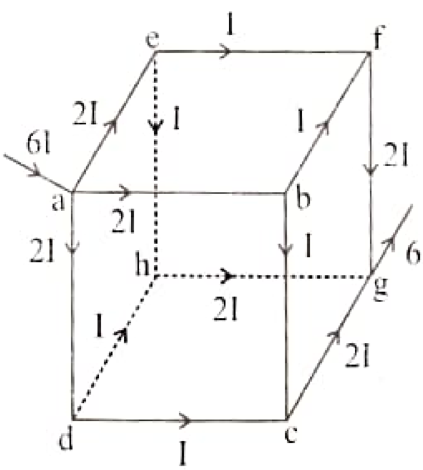


Fig.1.34

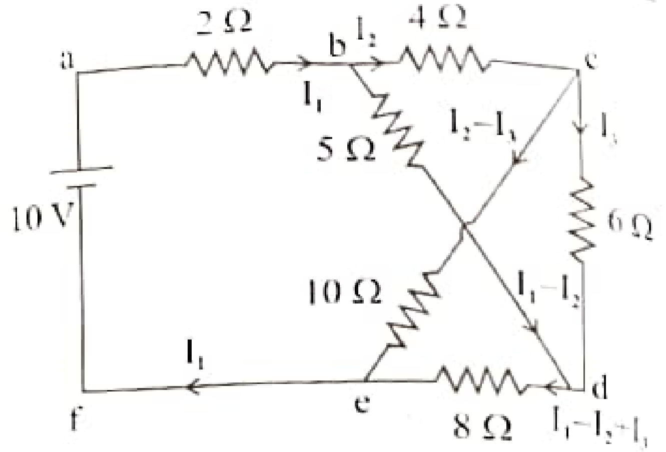
1.24 Find the current in the various branches of the circuit shown in Fig.1.35.

Soln.: The current distribution in the various branches of the circuit is as shown in the Fig.1.35.

For the loop abdefa:

$-2I_1 - 5\,(I_1 - I_2) - 8\,(I_1 - I_2 + I_3) + 10 = 0$

i.e $15I_1 - 13I_2 + 8I_3 = 10$ --- (1)



For the loop bcdb:

$-4I_2 - 6I_3 + 5(I_1 - I_2) = 0$

i.e. $5I_1 - 9I_2 - 6I_3 = 0$ --- (2)

For the loop cdec:

$-6I_3 - 8(I_1 - I_2 + I_3) + 10(I_2 - I_3) = 0$

i.e. $4I_1 - 9I_2 - 12I_3 = 0$ --- (3)

Solving equations (1),(2) and (3), we get

$I_1 = 2$ A, $I_2 = 1.33$ A and $I_3 = -1.59$ A.

The currents in the various branches are:

$I_{efab} = I_1 = 2A$, $I_{bc} = I_2 = 1.33$ A, $I_{cd} = I_3 = -1.59$ A i.e. $I_{dc} = 1.59$ A,

$I_{bd} = I_1 - I_2 = 2 - 1.33 = 0.67$ A

$I_{ce} = I_2 - I_3 = 1.33 - (-1.59) = 2.92$ A

$I_{de} = I_1 - I_2 + I_3 = 2 - 1.33 - 1.59 = -0.92$ A i.e. $I_{ed} = 0.92$ A

WORKED EXAMPLES

1.1 Two resistances 20 Ω and 40 Ω are connected in parallel. A resistance of 10 Ω is connected in series with the combination. A voltage of 200 V is applied across the circuit. Find the current in each resistance and the voltage across 10 Ω. Find also the power consumed in all the resistances.

Soln. : The total resistance of the circuit is given by

$$R = 10 + \frac{20 \times 40}{20 + 40} = 23.33\ \Omega$$

$$I = \frac{200}{23.33} = 8.57\ \text{A}$$

$$I_1 = 8.57 \times \frac{40}{40 + 20} = 5.71\ \text{A}$$

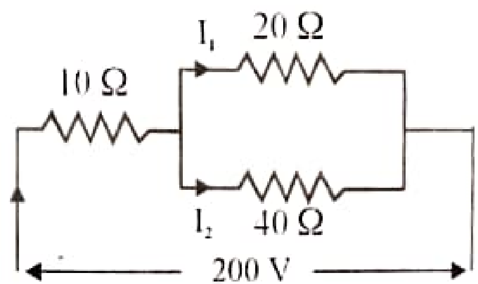


Fig.1.8

$I_2 = 8.57 - 5.71 = 2.86$ A, $\quad V_{10\Omega} = 8.57 \times 10 = 85.7$ volts

$P_{10\Omega} = 8.57^2 \times 10 = 734.45$ watts , $\quad P_{20\Omega} = 5.71^2 \times 20 = 652.08$ watts

$P_{40\Omega} = 2.86^2 \times 40 = 327.18$ watts

1.2 A D.C. arc has a voltage current relation given by $V = 20 + \frac{40}{I}$. It is connected in series with a resistor. The total voltage applied is 120 V. If the voltage across the arc is half the voltage across the resistor, find the value of the resistor.

Soln.: For the arc, $40 = 20 + \frac{40}{I}$,

$\therefore I = 2A$

for the resistor $I\,R = 80$ i.e. $2\,R = 80$

$\therefore R = 40\ \Omega$

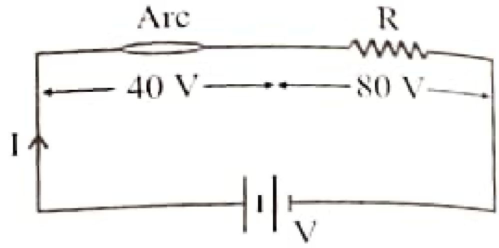


Fig. 1.9

1.3 Three resistors A, B and C are connected in parallel taking a total current of 12A from the supply. If $I_B = 2\ I_A$, $I_C = 3.5\ I_B$ and total power drawn is 3 kW, calculate (a) current drawn by each resistor (b) supply voltage and (c) power consumed by each resistance.

Soln.: (a) $I_A + I_B + I_C = I = 12$ A

$I_A + 2\ I_A + 3.5\ I_B = 12$

$3\ I_A + 3.5\ (2\ I_A) = 12$

$\therefore$ $I_A = 1.2$ A, $I_B = 2.4$ A and $I_C = 8.4$ A

(b) $P = 3{,}000$ W $= V\,I$

$\therefore V = \frac{3000}{12} = 250$ V

(c) $P_A = 250 \times 1.2 = 300$ watts

$P_B = 250 \times 2.4 = 600$ watts, $P_C = 250 \times 8.4 = 2{,}100$ watts

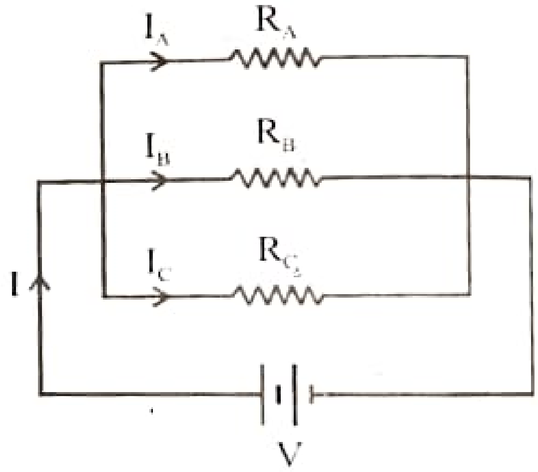


Fig. 1.10

1.4 In the series parallel circuit shown in Fig. 1.11, find (i) the voltage drop across 4 Ω resistor and (ii) the supply voltage.

Soln. : Let current I, I_1 and I_2 flow in the branches as shown in the Fig. 1.11

$I_1 = \frac{50}{10} = 5A$, $I_2 = \frac{50}{8} = 6.25$ A

$I = I_1 + I_2 = 5 + 6.25 = 11.25$ A

i) $V_{4\Omega} = 11.25 \times 4 = 45$ V

ii) $24\ \Omega \parallel 12\ \Omega = \frac{24 \times 12}{24 + 12} = 8\ \Omega$

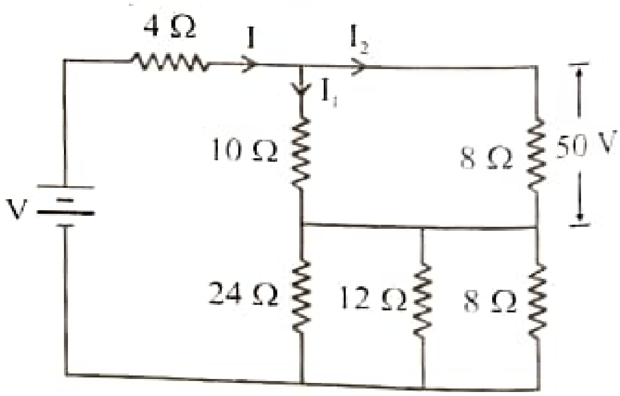


Fig. 1.11

$$8\,\Omega \parallel 8\,\Omega = \frac{8\times 8}{8+8} = 4\,\Omega\,, \qquad 10\,\Omega \parallel 8\,\Omega = \frac{10\times 8}{10+8} = 4.44\,\Omega$$

Total resistance = R = 4 + 4.44 + 4 = 12.44 Ω

Supply voltage = V = 12.44 x I = 12.44 x 11.25 = 140 volts

1.5 A resistance R is connected in series with a parallel circuit comprising 20 Ω and 48 Ω. The total power dissipated in the circuit is 1,000 W and the applied voltage is 250 V. Calculate R.

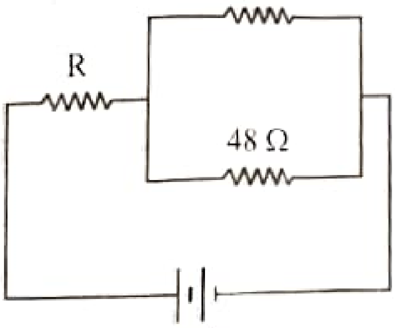


Fig. 1.12

Soln.: $P = \dfrac{V^2}{R_T} = \dfrac{250^2}{R_T} = 1{,}000$

$\therefore R_T$ = total resistance $= \dfrac{250^2}{1{,}000} = 62.5\,\Omega$

$R_T = R + \dfrac{20\times 48}{20+48}$ i.e. $62.5 = R + 14.12$

$\therefore R = 48.38\,\Omega$

1.6 In the given circuit shown in Fig. 1.13, calculate (a) the total current (b) current in 5 Ω and (c) the power dissipated in 6 Ω and 7 Ω.

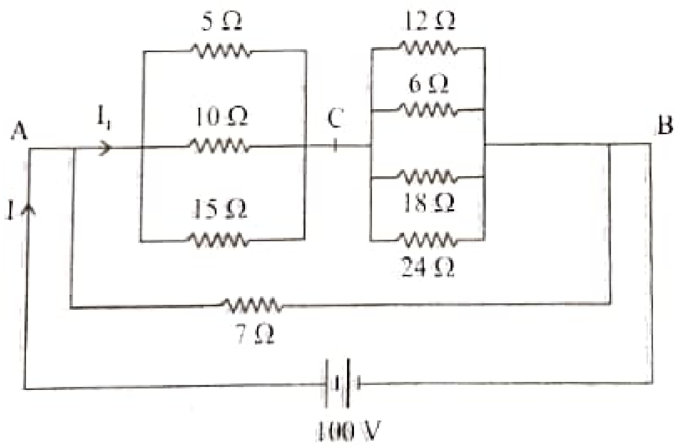


Fig. 1.13

Soln.

a) $\dfrac{1}{R_{AC}} = \dfrac{1}{5} + \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{11}{30}\,\Omega\,,$ $\qquad \therefore R_{AC} = 2.73\,\Omega$

$\dfrac{1}{R_{CB}} = \dfrac{1}{12} + \dfrac{1}{6} + \dfrac{1}{18} + \dfrac{1}{24} = \dfrac{25}{72}\,\Omega\,,$ $\qquad \therefore R_{CB} = 2.88\,\Omega$

$R_{ACB} = R_{AC} + R_{CB} = 2.73 + 2.88 = 5.61\,\Omega$

The total resistance $= R_T = \dfrac{5.61 \times 7}{5.61+7} = 3.11\ \Omega$, $I = \dfrac{100}{3.11} = 32.15$ A

b) $I_1 = 32.15 \times \dfrac{7}{7+5.61} = 17.85$ A , $E_{AC} = 17.85 \times R_{AC} = 17.85 \times 2.73 = 48.73$ V

$I_{5\Omega} = \dfrac{48.73}{5} = 9.75$ A

c) $E_{CB} = 100 - E_{AC} = 100 - 48.73 = 51.27$ V, $P_{6\Omega} = \dfrac{E_{CB}^2}{6} = \dfrac{51.27^2}{6} = 438.1$ W

$P_{7\Omega} = \dfrac{E_{AB}^2}{7} = \dfrac{100^2}{7} = 1{,}428.57$ W

1.7 A current of 30 A flows through two ammeters A_1 and A_2 connected in series. The p.d. across the two ammeters are 0.3 V and 0.6 V respectively. Find how the same current will divide when they are connected in parallel.

Soln. : Let R_1 and R_2 be the resistances of the ammeters A_1 and A_2 respectively.

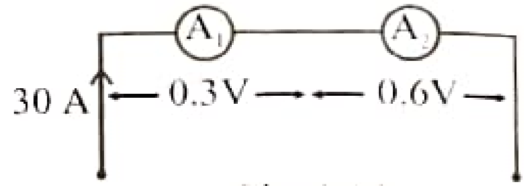


Fig. 1.14

$30\ R_1 = 0.3$, $\therefore R_1 = 0.01\ \Omega$

$30\ R_2 = 0.6$, $\therefore R_2 = 0.02\ \Omega$

The two ammeters are now connected in parallel as shown in Fig. 1.15.

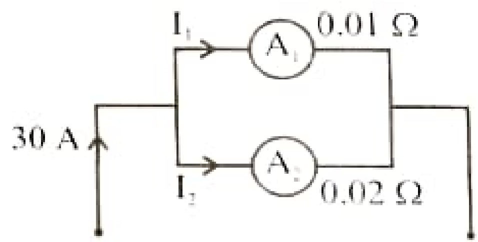


Fig. 1.15

$I_1 = 30 \times \dfrac{0.02}{0.02+0.01} = 20$ A

$I_2 = 30 - 20 = 10$ A

1.8 A voltage of 200 V is applied to a tapped resistor of 500 Ω. Find the resistance between the tapping points connected to a load, needing 0.1 A at 25 V. Also calculate the total power consumed.

Soln.: Let the resistor be tapped between points B and C and x be the resistance between these points.

$\therefore \quad R_{BC} = x\ \Omega$

Then, $(I - 0.1)\,x = 25$ i.e $Ix - 0.1\,x = 25$ (1.14)

Also, $I\,(500 - x) = 200 - 25 = 175$ i.e. $500\,I - I\,x = 175$ (1.15)

Adding equations (1.14) and (1.15), we get

$$500\,I - 0.1\,x = 200, \quad \text{or} \quad I = \frac{200 + 0.1x}{500} \tag{1.16}$$

Substituting I given by equation (1.16) in equation (1.14), we get

$$\frac{200 + 0.1x}{500}\,x - 0.1\,x = 25$$

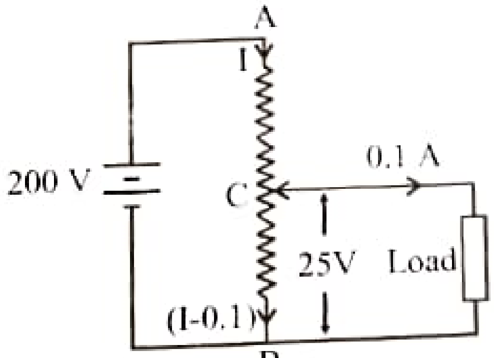


Fig. 1.16

on simplification, we get

$$0.1\,x^2 + 150\,x - 12{,}500 = 0$$

$$\therefore\ x = \frac{-150 \pm \sqrt{150^2 + 4 \times 0.1 \times 12{,}500}}{2 \times 0.1} = 79.15\ \Omega$$

(considering only the positive value)

Substituting this value in equation (1.14), we get, $\quad I = 0.42$ A

Total power consumed $= 200 \times I = 200 \times 0.42 = 84$ watts