Complements in Binary Number Systems

Before proceeding, please Note that

In Binary addition:

$0+0 = 0$
$0+1 = 1$
$1+0 = 1$
$1+1 = 10 \Rightarrow$ sum $= 0$
carry $= 1$.

The complements are used for subtraction in binary number system.

There are two complents in binary ($r=2$) system. 2's complements & 1's complement.

1's complement Representation:

It is the number ~~system~~ that results when we change all 1's to zeros and the 0's to 1's

2's complement Representation:

The 2's complement is the binary number that results when we add 1 to the 1's complement of the number.

2's complement = 1's complent +1

Ex: Find 1's complement of the following:

(i) $(111001)_2$

1's complement $\Rightarrow$ 000110

(ii) $(1\,1\,0\,1\,0\,\;1\,0\,1\,0\,1)_2$

1's complement = 0 0 1 0 1 0 1 0 1 0

(iii) $(15)_{10}$ = 1's complement ?

Convert to binary first

(iv) $(15)_{10} = (\overset{8\,4\,2\,1}{1\,1\,1\,1})_2$

1's complement of 1111 is 0000

2	15
2	7 – 1
2	3 – 1
	1 – 1

(v) $(28)_{10}$ ⇒ Step 1 – convert to binary.

$(28)_{10} = (\overset{16\,8\,4\,2\,1}{1\,1\,1\,0\,0})_2$

1's complement of 11100 is 00011

2	28
2	14 – 0
2	7 – 0
2	3 – 1
	1 – 1

Find 2's complement of the following:

(i) $(1\,1\,0\,0\,0\,1\,0\,0)_2$

1 1 0 0 0 1 0 0

1's complement ⇒ 0 0 1 1 1 0 1 1

Add 1 1

0 0 1 1 1 1 0 0 ⇒ 2's complement

11000100 2's complements is 00111100

(ii) $(1\,0\,1\,0\,1\,1)_2$

1's complement of 1 0 1 0 1 1

0 1 0 1 0 0

Add 1 1

0 1 0 1 0 1 ⇒ 2's complement.

Try ① $(0111000)_2$ ② $(111001)_2$

Binary Subtraction using 1's complement method.

To perform A−B, the following steps are followed:

1. Take 1's complement of B (Subtrahend)
2. Perform A (minuend) + 1's complement of B
3. If carry is generated, then result is positive; Add carry to the result to get final result.
4. If carry is not generated, then result is negative and in the 1's complement form.

Ex:

(i) Perform $(5)_{10} - (3)_{10}$ $[5-3=2]$

5 in binary 0101

3 in binary 0011

1's complement of $(3)_{10} \Rightarrow (0011)_2 \Rightarrow 1100$.

$(5)_{10} - (3)_{10}$ = 5 in binary + 1's compl. of 3 in binary

5	0 1 0 1
1's compl. of 3	1 1 0 0
+	
	1 0 0 0 1

cy generated

∴ add it to get final answer

0 0 0 1
+ 1
0 0 1 0 $\Rightarrow (2)_{10}$

(ii) Perform $(28)_{10} - (19)_{10}$ using 1's complement

16 8 4 2 1

$28 = 11100$

$19 = 10011$

2 | 28
2 | 14 – 0
2 | 7 – 0
2 | 3 – 1
1 – 1

1's complement of 19 is 01100

∴ add 28 + 1's compl. of 19 in binary

$28 \rightarrow$ 11100

1's compl of 19 + 01100

1 01000

01000 + 1 = 01001 $= (09)_{10}$

∴

(iii) Perform $(15)_{10} - (28)_{10}$ using 1's complement

$(15)_{10} = (01111)_2$

$(28)_{10} = (11100)_2$

15 − 28 = −13

1's complement of 28

11100
00011

add with 15

01111
+ 00011
10010

↳ no carry generated.

∴ a negative number and take

1's complement of the result-

Result — 10010

1's complement — 01101 = $-(13)_{10}$

(iv) Given $M = 1010100$ and $N = 1000100$,

Perform $M - N$ using 1's complement-

$N \Rightarrow 1000100$

1's compl of $N \Rightarrow 0111011$

$M \Rightarrow 1010100$

$+$ ———————

$\boxed{1}\ 0001111$

CY is generated ∴ add CY to result-

0001111

1

———————

00010000 → Result final.

Binary Subtraction using 2's Complement:

To perform subtraction, of 2 binary numbers in 2's complement method for A - B, follow the steps as below:

1. Take 2's complement of B
2. Perform A + 2's complement of B ⇒ result.
3. If carry is generated, then result is positive. IGNORE the CARRY.
4. If carry is NOT generated, then the result is negative and in 2's complement form.

Ex: (i) Perform $(5)_{10} - (3)_{10}$ using 2's complement method of subtraction.

5 → 0101

3 → 0011

2's complement of 0011

1's compt 1100

$$\begin{array}{r} 1100 \\ +\quad\ 1 \\ \hline 1101 \end{array} \Rightarrow \text{2's compl.}$$

∴ 5 + 2's compl. of 3.

$$\begin{array}{r} 0101 \\ 1101 \\ \hline [1]\,0010 \end{array}$$

↳ Discared the carry.

∴ $(5)-(3) = (0010)_2 = (02)_{10}$

(ii) Perform $(3)_{10} - (5)_{10}$

3 ⇒ 0011

5 ⇒ 0101

2's compl of 5 ⇒ 0101

$$\begin{array}{r} 1010 \\ +\quad\ 1 \\ \hline 1011 \end{array}$$

∴

$$\begin{array}{r} \overset{1\,1}{} \\ 0011 \\ +\ 1011 \\ \hline 1110 \end{array}$$

↳ NO carry ∴ the number is negative.

Take 2's compl. of result 1110 → 0001

$$\begin{array}{r} 0001 \\ +\quad\ 1 \\ \hline 0010 \end{array} = (2)_{10}$$

(iii) $M = 1000100$, $N = 1010100$, perform $M-N$ using 2's complement method.

$N = 1010100$

1's compl $\Rightarrow 0101011$

$+\quad\quad 1$

2's compl. 0101100

1000100

11110000

No carry.

$\therefore$ Perform 2's complement again.

11110000

0001111

$+\quad\quad 1$

$-\quad 0010000$

Answer is -10000

(-ve sign because no carry)

Try ① $[4]_{10} - [9]_{10}$ ② ~~(1)~~ $1101 - 1010$

③ $M = 1010100$, $N = 1000100$

Perform $M-N$ using 2's complement.

(ii) Find 10's complement of $(0.3267)_{10}$

0.3267

10's complement is given by

$r^n - N$

$n = 0$; no integer part

$\therefore \quad r = 10$

$10^0 - 0.3267$

$= 1 - 0.3267$

$= \underline{0.6733}$ Ans.

(iii) $(25.639)_{10}$

$n = 2 \Rightarrow$ two digits in integer part

$r^n - N \Rightarrow$ 10's compl.

$r = 10, \quad N = 25.639.$

$r^n = 10^2 = 100$

$\therefore$ 10's complement is

$\Rightarrow 100 - 25.639$

$\Rightarrow \underline{74.361}$ Ans

(iv) $(13579)_{10}$

$n = 5$

10's comp $\Rightarrow r^n - N$

$10^5 - 13579 = \underline{86421}$ Ans

(iii) $M = 1000100$, $N = 1010100$, perform $M-N$ using 2's complement method.

$N = 1010100$

1's compl $\Rightarrow 0101011$

$+\quad\quad\quad 1$

2's compl. 0101100

1000100

11110000

No carry.

$\therefore$ Perform 2's complement again.

11110000

0001111

$+\quad\quad\quad 1$

$-\quad 0010000$

Answer is -10000

(-ve sign because no carry)

Try ① $[4]_{10} - [9]_{10}$ ② 1101 − 1010

③ $M = 1010100$, $N = 1000100$

Perform $M-N$ using 2's complement.

(ii) Find 10's complement of $(0.3267)_{10}$

0.3267

10's complement is given by

$r^n - N$

$n = 0$; no integer part

$r = 10$

$10^0 - 0.3267$

$= 1 - 0.3267$

$= \underline{0.6733}$ Ans.

(iii) $(25.639)_{10}$

$n = 2 \Rightarrow$ two digits in integer part

Hint $r^n - N \Rightarrow$ 10's compl.

$r = 10, \quad N = 25.639.$

$r^n = 10^2 = 100$

$\therefore$ 10's complement is

$\Rightarrow 100 - 25.639$

$\Rightarrow \underline{74.361}$ Ans

(iv) $(13579)_{10}$

$n = 5$

10's comp $\Rightarrow r^n - N$

$10^5 - 13579 = \underline{86421}$ Ans

Decimal Number System

Complements

→ 9's complement
→ 10's complement

The general representation for 10's complement is

$$\begin{cases} r^n - N & \text{for } N \neq 0 \\ 0 & \text{for } N = 0 \end{cases} \Rightarrow \text{10's complement}$$

where

$r \Rightarrow$ base

$n \Rightarrow$ no. of digits in integer part

$N \Rightarrow$ Given Number.

To find 9's complement general formula is

$$r^n - r^{-m} - N \Rightarrow \text{9's complement}$$

where r^n → r - base
→ n → no. of digits in integer part.

r^{-m} → r - base
→ m is no. of digits in fractional part

N → given number

Ex:

(i) Find 10's complement of $(52520)_{10}$

Wkt $r^n - N$. is for 10's compl.

n = no. of digits = 5

10's complement = $r^N - N$

$= 10^5 - 52520$

$= 100000 - 52520$

$= 47480$ // Ans

(v) 8374.59

10's compl is $r^n - N$

$n = 4$

$\Rightarrow 10^4 - 8374.59$

$\Rightarrow 10000 - 8374.59$

$\Rightarrow$ ~~8374.2376~~ ~~Ans~~

$\Rightarrow$ 1625.41 Ans

Try ① 17850.5684 ② 0000 ③ 0.1352
④ 900900.

9's complement of a number

Formula is $r^n - r^{-m} - N$.

n = no. integer portion digit

m = no. of fractional portion digit.

Ex: (i) Find 9's complement of $(52590)_{10}$

$(52590)_{10}$

$n = 5$

$m = 0$

$\therefore r^5 - r^{-0} - 52590$

$10^5 - 1 - 52590$

$100000 - 1 - 52590$

$= 47409$ Ans.

(ii) $(0.3267)_{10}$

$n = 0$

$m = 4$

9's complement $\quad r^n - r^{-m} - N$

$10^0 - 10^{-4} - 0.3267$

$= 1 - 0.0001 - 0.3267$

$= \underline{0.6732}$ Ans.

(iii) $(8374.59)_{10}$

$n = 4$

$m = 2$

9's compl $\Rightarrow \quad r^n - r^{-m} - N$

$10^4 - 10^{-2} - 8374.59$

$10000 - 0.01 - 8374.59$

$= \underline{1625.409}$ Ans.

(iv) $(17850.6584)_{10}$

$n = 5$; $m = 4$

9's complement $\quad r^n - r^{-m} - N$

$10^5 - 10^{-4} - 17850.6584$

$= 100000 - 0.0001 - 17850.6584$

$= \underline{82149.3415}$ Ans

Try ① $(13759)_{10}$ ② 0.1035

Subtraction using 10's complements

To Perform subtraction using 10's complement, the following steps have to be followed:

1. Find 10's complement of Subtrahend.
2. Add with Minuend
3. If carry is generated, discard it, because the number is positive.
4. If there is No carry, the no. is negative and once again 10's complement to get final result.
5. (Note: the no. of digits in minuend & Subtrahend has to match)

Ex:

(i) Perform the following using 10's complement

$(487)_{10} - (354)_{10}$

Step 1. Find 10's compl. of Subtrahend

354

$r = 3$

$10^3 - 354 = \underline{646}$

Step 2. Add with minuend

```
  11
  487   minuend
+ 646   10's compl of 354
-----
 1133
```

↳ Discard carry.

∴ Ans $= \underline{(133)_{10}}$

(ii) $(8437)_{10} - (39)_{10}$

1. find 10's complement of $(39)_{10}$

M = 8437 minuend
S = 0039. subtrahend

find 10's compl. of 0039

$r^n - N$.

$n = 4$

$10000 - 0039 = \underline{\underline{9961}}$

2. Add minuend & subtrahend (10's compl. of)

$$\begin{array}{r} {\scriptstyle 1}\\ 8437 \\ +\ 9961 \\ \hline \boxed{1}\,8398 \end{array}$$

↳ CY = 1 & discard it

∴ Ans = $\underline{\underline{8398}}$

(iii) $(309)_{10} - (1447)_{10}$

M = 309

S = 1447

1. 10's complement of 1447

$10^4 - 1447 = 10000 - 1447$

$= (\underline{8553})_{10}$

2.

$$\begin{array}{r} {\scriptstyle 1}\\ 0309 \\ +\ 8553 \\ \hline 8862\ (\underline{8862}) \end{array}$$

↳ No Carry; so take 10's compl. again.

$10^4 - 8862$

$10000 - 8862 = \underline{1138}$

Ans = $-(\underline{1138})_{10}$

(iv) $(72532)_{10} - (3250)_{10}$

1. Match digits of subtrahend with minuend.

$$03250$$

$$10^5 - 03250 = 96750$$

$$\therefore \quad \begin{array}{r} 72532 \\ +\ 96750 \\ \hline 1\,69282 \end{array}$$

↳ discard. cy.

$\therefore$ Ans = 69282

Try: ① $(3250)_{10} - (72532)_{10}$ ② $(5)_{10} - (3)_{10}$.

Subtraction using 9's complement-

Perform the following steps for subtraction using 9's complement.

1. Take 9's compl. of subtrahend
2. Add with minuend. & get result
3. If carry is generated, add it to the result obtained.
4. If no carry, result is negative & hence take complement once again to get final result.

① Perform 9's complement and Subtract- $(487)_{10} - (354)_{10}$ using 9's complement.

$$\begin{array}{r} 487 \\ -\ 354. \end{array}$$

Find 9's complement of $(354)_{10}$

$$= r^m - r^n - N$$

$$m = 3$$
$$n = 0$$

$$= 10^3 - 10^0 - 354$$

$$= 1000 - 1 - 354 = 645$$

$$\begin{array}{r} 487 \\ +\ 645 \\ \hline 1132 \\ \hookrightarrow +1 \\ \hline (133)_{10} \end{array}$$

$\therefore$ $487 - 354 = \underline{133}$

② $(309)_{10} - (1447)_{10}$

$$\begin{array}{r} 0309 \\ -\ 1447 \\ \hline \end{array}$$

9's complement of 1447

$$= 10^4 - 10^0 - 1447 = \underline{8552}$$

$$\begin{array}{r} 0309 \\ +\ 8552 \\ \hline 8861 \end{array}$$

↳ No carry, negative result.

$\therefore$ Perform 9's complement again on result

$8861 \Rightarrow 10^4 - 1 - 8861 = -\underline{1138}_{10}$