Boolean Algerbra :

The set $S = \{0, 1\}$ along with the opeeation (•) AND and (+) OR is called Boolean algebra. (Also including NOT opeeation).

Boolean theorems / Identities :

Name	AND	OR
1) Identity law	$1 \cdot A = A$	$0 + A = A$
2) Null law	$0 \cdot A = 0$	$1 + A = 1$
3) Idempotent law	$A \cdot A = A$	$A + A = A$
4) Inverse law	$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$
5) Commutative	$AB = BA$	$A + B = B + A$
6) Associative	$(AB)C = A(BC)$	$(A+B)+C = A+(B+C)$
7) Absorption law	$A(A+B) = A$	$A + AB = A$
8) Distribution law	$A + BC = (A+B)(A+C)$	$A(B+C) = AB + AC$
9) De' Morgan's theorem	$\overline{A \cdot B} = \bar{A} + \bar{B}$	$\overline{A+B} = \bar{A} \cdot \bar{B}$

De' Morgan's theorem:

① The complement of a product is equal to the sum of the complements.

② The complement of a sum is equal to the product of the complements.

Both of these can be illustrated using the truth table as below.

A	B	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Both are equal $\therefore \overline{AB} = \bar{A}+\bar{B}$

A	B	$\overline{A+B}$	$\bar{A}\cdot\bar{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$\rightarrow$ Both are equal

$\therefore \overline{A+B} = \bar{A}\cdot\bar{B}$

Complements of a function

The complement of a function F is $\bar{F}$ and is obtained from an interchange of 0's and 1's in the values of F.

Ex: find $(\overline{A+B+C})$, complement of A+B+C.

Let B+C be x

$$\therefore (\overline{A+x}) = (A+x)'$$
$$= \bar{A}\cdot\bar{x}$$
$$= \bar{A}\cdot(\overline{B+C})$$
$$= \bar{A}\cdot\bar{B}\cdot\bar{C}$$
$$\therefore (\overline{A+B+C}) = \bar{A}\cdot\bar{B}\cdot\bar{C}$$

(ii) Find complement of

① $F_1 = \bar{x} Y \bar{z} + \bar{x} \bar{y} z$

$\bar{F} = \overline{(\bar{x} y \bar{z} + \bar{x} \bar{y} z)}$

$= \overline{\bar{x} y \bar{z}} \cdot \overline{\bar{x} \bar{y} z}$

$\cancel{\therefore \bar{F}_1 =} \cancel{(x \bar{y} z) \cdot (x y \bar{z})}$

$\cancel{= x \bar{y} z}$

$\therefore \bar{F} = (x + \bar{y} + z)(x + y + \bar{z})$

② $F_2 = x(\bar{y}\bar{z} + yz)$

$\bar{F_2} = \overline{x \cdot (\bar{y}\bar{z} + yz)}$

$= \bar{x} + \overline{\bar{y}\bar{z} + yz}$

$= \bar{x} + \overline{\bar{y}\bar{z}} \cdot \overline{yz}$

$\bar{F_2} = \bar{x} + (y+z) \cdot (\bar{y} + \bar{z})$

Basic terminologies in Boolean Algebra

① **Literal** : It is a boolean variable or its complement. If 'x' is a binary variable, then both x & $\bar{x}$ are boolean variables known as literals.

② **Product term** : logical product (AND) of a multiple literals.

Ex : X, XY, XYZ, $\bar{X} Y Z$ etc.

③ **Sum term** : logical sum (OR) of multiple literals.

Ex : X, $X + Y$, $\cancel{Z} X + Y$, $\bar{X} + Y + \bar{Z}$ etc.

④ **SOP** : Sum of Products is the logical OR of multiple product terms.

Ex : $XY + X\bar{Y}Z + X\bar{Y}\bar{Z}$

⑤ **Product of Sums (POS)** : is the logical AND of multiple 'OR' terms.

Ex : $(x+y) \cdot (x+y+\bar{z})(\bar{y}+z)$

7. **Minterm :** A minterm is a special case of product term (AND) that contains all the literals. (input variables)

Ex: $\bar{x}yz$ (for 3 i/ps)

8. **Max term :** A maxterm is a special case sum-term (OR) that contains all of the literals (i/p variables)

Ex: $(x+\bar{y}+z)$ (for 3 i/ps)

9. **Canonical SOP :**

A canonical sum of products is a complete set of minterms, that define when an output variable is logical one.

Ex : $m = \bar{a}bc + a\bar{b}c + \bar{a}\bar{b}c$

Here the three minterms produce logical-1 output.

10. **Canonical POS :**

A canonical POS is a complete set of maxterms that defines when an output is a logical '0' (zero).

Ex : $M = (a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+b+\bar{c})$

SOP Equation to Canonical SOP form:

* Identify the missing literal in each term (AND term / product term).
* logically AND missing term & its complement with the original AND term.
* Expand the term by application of distributive property.

ex: $f(x,y,z) = XY$ is a AND / product term in a system with 3 variables.

Here Z is missing. Therefor multiply XZ with $Z+\bar{Z}$. Since $Z+\bar{Z}=1$, it doesnot alter the value of literal / term.

$$\therefore \quad XY(Z+\bar{Z}) = XYZ + XY\bar{Z}.$$

POS Equation to Canonical form:

* Identify missing literal in each sum term / OR term.
* logically OR the missing terms and its complement with original OR term.
* Then expand to get canonical form

ex: $f(x,y,z) = (x+\bar{y})$

$$f(x,y,z) = x+\bar{y}+z\bar{z}$$

$$= \underline{(x+\bar{y}+z)(x+\bar{y}\bar{z})}.$$

(ii) obtain proper canonical form for the following:

(i) $f(a,b,c) = a\bar{b} + a\bar{c} + bc$

$$a\bar{b}(c+\bar{c}) = a\bar{b}c + a\bar{b}\bar{c}$$
$$a\bar{c}(b+\bar{b}) = ab\bar{c} + a\bar{b}\bar{c}$$
$$bc(a+\bar{a}) = abc + \bar{a}bc$$

$$\therefore f(a,b,c) = a\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} + a\bar{b}\bar{c} + abc + \bar{a}bc.$$

(ii) $f(a,b,c) = (a+\bar{b})(\bar{b}+c)$

$$a+\bar{b}+c\bar{c} = (a+\bar{b}c)(a+\bar{b}+\bar{c})$$
$$\bar{b}+c+a\bar{a} = (\bar{b}+c+a)(\bar{b}+c+a)$$

$$\therefore f(a,b,c) = (a+\bar{b}c)(a+\bar{b}+\bar{c})(\bar{b}+c+a)(\bar{b}+c+a)$$

Generation of switching Equation from truth table

Input	Minterm (m)		Maxterm (M)	
a b c	term	Notation	term	Notation
0 0 0	$\bar{a}\bar{b}\bar{c}$	m0	$a+b+c$	M0
0 0 1	$\bar{a}\bar{b}c$	m1	$a+b+\bar{c}$	M1
0 1 0	$\bar{a}b\bar{c}$	m2	$a+\bar{b}+c$	M2
0 1 1	$\bar{a}bc$	m3	$a+\bar{b}+\bar{c}$	M3
1 0 0	$a\bar{b}\bar{c}$	m4	$\bar{a}+b+c$	M4
1 0 1	$a\bar{b}c$	m5	$\bar{a}+b+\bar{c}$	M5
1 1 0	$ab\bar{c}$	m6	$\bar{a}+\bar{b}+c$	M6
1 1 1	abc	m7	$\bar{a}+\bar{b}+\bar{c}$	M7

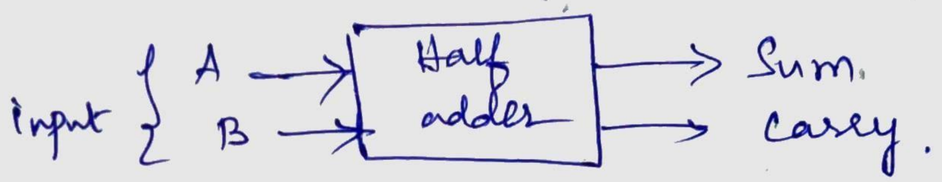
Cannoical SOP form from T-T.

$$f(a,b,c) = \Sigma m(0,1,2,3,4,5,6,7)$$

Canonical POS form from T-T.

$$f(a,b,c) = \pi M(0,1,2,\ldots 7).$$

__Half adder__ : It adds two binary numbers and gives Sum & Carry as output



__Truth table.__

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

from truth table, the minterms for Sum is

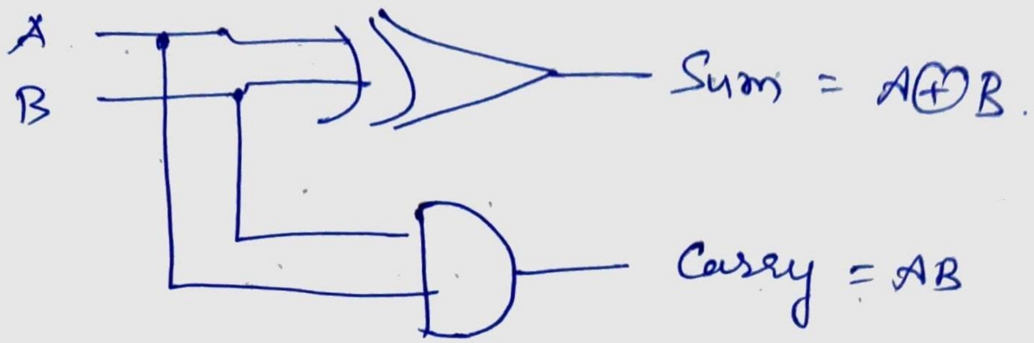
$$Sum = f(A,B) = \bar{A}B + A\bar{B} = A \oplus B$$ Ex-OR

$$Carry = f(A,B) = AB$$ (AND gate)

∴ the circuit has to realize

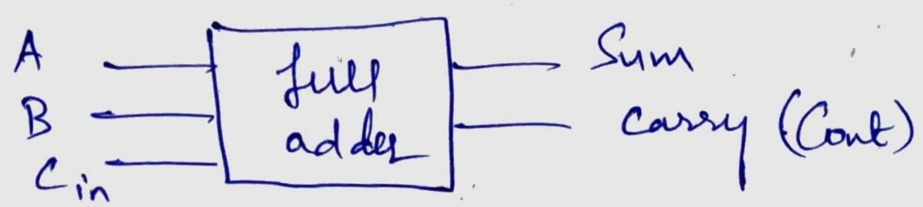
$$Sum = \bar{A}B + A\bar{B} \Rightarrow \text{Ex-OR gate}$$

$$Carry = AB \Rightarrow \text{AND gate}$$



Full adder:

This circuit adds three variables & gives Sum & carry as output.



Input			outputs		
A	B	Cin	Sum	Cout (carry)	
0	0	0	0	0	– m0
0	0	1	1	0	– m_1
0	1	0	1	0	– m_2
0	1	1	0	1	– m_3
1	0	0	1	0	– m_4
1	0	1	0	1	– m_5
1	1	0	0	1	– m6
1	1	1	1	1	– m7.

Sum = $\Sigma m(1, 2, 4, 7)$

Carry = $\Sigma m(3, 5, 6, 7)$

Sum = $\bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$

$$Sum = C_{in}(AB + \bar{A}\bar{B}) + \bar{C}_{in}(\bar{A}B + A\bar{B})$$

$$C_{in}(\overline{A \oplus B}) + \bar{C}_{in}(A \oplus B)$$

$$\boxed{Sum = A \oplus B \oplus C_{in}}$$

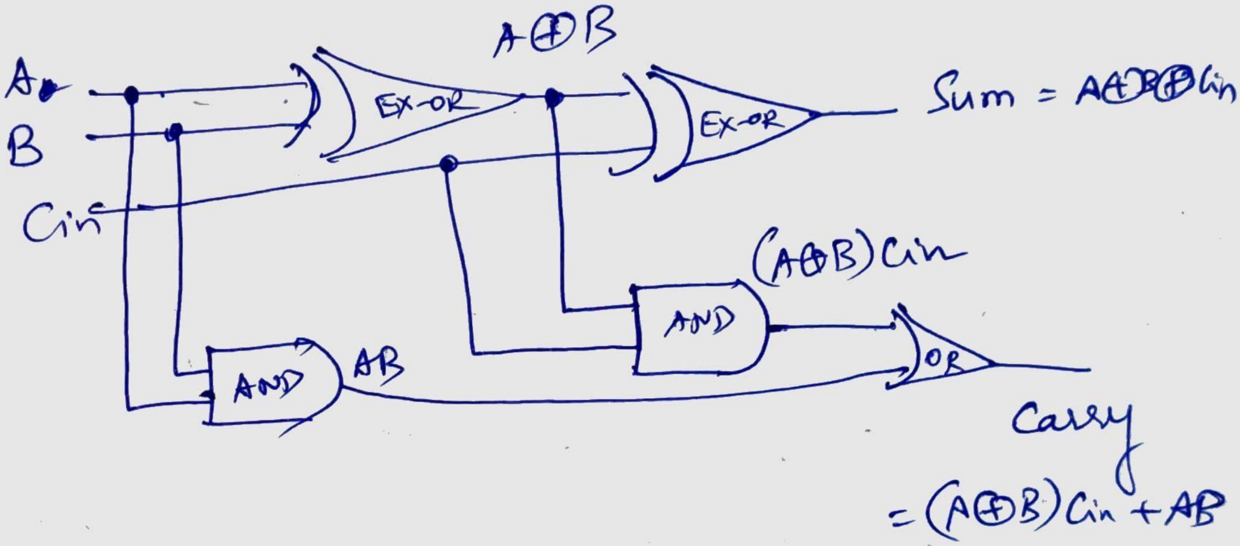
$$C_{out} = \Sigma m(3, 5, 6, 7)$$

$$= \bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC_{in}$$

$$= C_{in}(\bar{A}B + A\bar{B}) + AB(C_{in} + \bar{C}_{in})$$

$$= C_{in}(A \oplus B) + AB(1)$$

$$C_{out} = \boxed{(A \oplus B)C_{in} + AB}$$



(Full adder circuit)

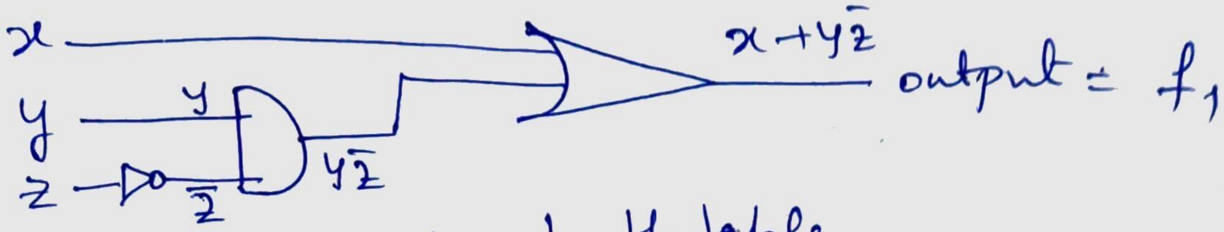
Realization of Boolean Equations using logic gates and truth tables

① $F_1 = x + y\bar{z}$.

There are three variables / literals

$y\bar{z}$ is realized through AND gate

then $x + y\bar{z}$ thro' an OR gate



Output through truth table

x	y	z	$\bar{z}$	$y\bar{z}$	$x+y\bar{z}$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	0	0	1

output.

(2) $F_2 = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}$

Before implementing it is a good idea to go for Simplification of Boolean Equations, so that no. of gates required to implement will be reduced.

We use Boolean theorems & laws for Simplification.

The above equation has 3 terms and 8 literals ($x, \bar{x}, y, \bar{y}, z, \bar{z}$)

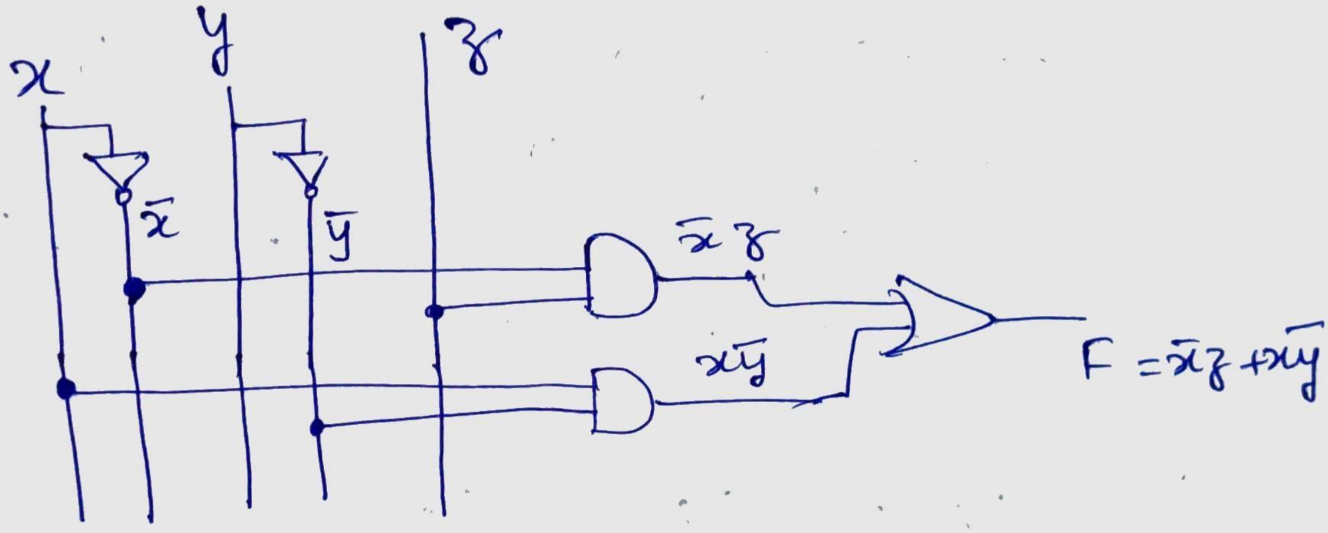
Simplification of Boolean Equations

(i) $F = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}$

$= \bar{x}z(y+\bar{y}) + x\bar{y}$

$= \bar{x}z(1) + x\bar{y}$

$= \bar{x}z + x\bar{y}$ Simplified Equations
2 terms
4 literals

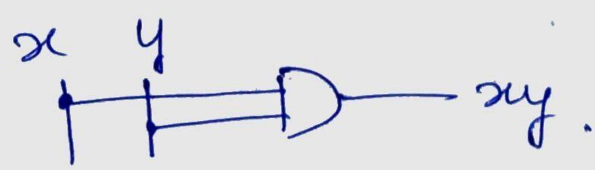


(ii) Simplify the following boolean equation to a minimum literals & implement the ckt.

(a) $x(\bar{x}+y)$

$x\bar{x} + xy$

$0 + xy = \underline{xy}$

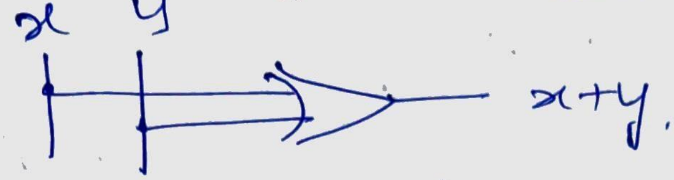


(b) $x + \bar{x}y.$

using distributive property

$(x+\bar{x})(x+y)$

$1\cdot(x+y) = \underline{\underline{x+y}}.$



(c) $(x+y)(x+\bar{y})$

$= xx + x\bar{y} + xy + y\bar{y}$ $\qquad \because y\bar{y} = 0.$

$= x + x\bar{y} + xy + 0$

$= x(1+\bar{y}+y)$ $\qquad \because 1+y+\bar{y} = 1$

$= x(1) = \underline{\underline{x.}}$

(d) $(\overline{A+B})(\bar{A}+\bar{C})(\bar{B}+C)$

Apply Demorgan's theorem to 1st term

$(\bar{A}\cdot\bar{B})(\bar{A}+\bar{C})(\bar{B}+C)$

$\cancel{\bar{A}\bar{A}\bar{B}}$

$(\bar{A}\bar{B}\bar{A} + \bar{A}\bar{B}\bar{C})(\bar{B}+C)$

$(\bar{A}\bar{B} + \bar{A}\bar{B}\bar{C})(\bar{B}+C)$

$\cancel{\bar{A}\bar{B}(1+\bar{C})}$

$[\bar{A}\bar{B}(1+\bar{C})](\bar{B}+C)$

$(\bar{A}\bar{B})(\bar{B}+C)$

$\bar{A}\bar{B}\bar{B} + \bar{A}\bar{B}C.$

$= \bar{A}\bar{B}$ // $\quad \bar{A}\bar{B}(1+C) \quad \bar{A}\bar{B} + \bar{A}\bar{B}C.$

(e)

$$f = ABC + \bar{A} + A\bar{B}C$$

$$= AC[B+\bar{B}] + \bar{A}$$

$$= AC\cdot 1 + \bar{A} = AC + \bar{A}$$

$$= (\bar{A}+A)(\bar{A}+C)$$

$$= 1\cdot(\bar{A}+C)$$

$$= \underline{\underline{\bar{A}+C}}$$

(f)

$$Y = (A+B)(A+C)$$

$$= A\cdot A + AC + AB + BC$$

$$= A + AC + AB + BC$$

$$= A[1+C+B] + BC$$

$$= A\cdot 1 + BC = \underline{\underline{A+BC}}$$

(g)

$$Y = (A+B)(A+\bar{B})(\bar{A}+C)$$

$$= (AA + A\bar{B} + AB + B\bar{B})(\bar{A}+C)$$

$$= (A + A\bar{B} + AB + 0)(\bar{A}+C)$$

$$= A[1+B+\bar{B}](\bar{A}+C)$$

$$= A(\bar{A}+C)$$

$$= A\bar{A} + AC = 0 + AC = \underline{\underline{AC}}$$

(h)

$$y = A\bar{B} + \overline{(\bar{A}+\bar{B}+C\cdot\bar{C})}$$

$$= A\bar{B} + \overline{(\bar{A}+\bar{B})} \qquad (\because C\bar{C} = 0)$$

$$= A\bar{B} + \bar{\bar{A}}\cdot\bar{\bar{B}} \qquad (\because \overline{A+B} = \bar{A}\cdot\bar{B})$$

$$= A\bar{B} + AB \qquad \bar{\bar{A}} = A$$

$$= A(\bar{B}+B) = \underline{\underline{A}}$$

(i) $y = \overline{(C + D)} + \bar{A} C \bar{D} + A\bar{B}C + AC\bar{D}$

$= \bar{C} \cdot \bar{D} + \bar{A} C \bar{D} + A\bar{B}C + AC\bar{D}$

$= \bar{C}\bar{D} + C\bar{D}(\bar{A} + A) + A\bar{B}C$

$= \bar{C}\bar{D} + C\bar{D} + A\bar{B}C$

$= \bar{D}(C + \bar{C}) + A\bar{B}C$

$= \underline{\underline{\bar{D} + A\bar{B}C}}$

(j) $y = \overline{(A\bar{B} + \bar{A}B)}\,(A+B)$

$= \overline{A\bar{B}} \cdot \overline{\bar{A}B}\,(A+B)$ ∵ Demorgan's theorem.

$= (\bar{A} + B)(A + \bar{B})(A+B)$

$= (A\bar{A} + \bar{A}\bar{B} + AB + B\bar{B})(A+B)$

$(0 + AB + \bar{A}\bar{B} + 0)(A+B)$

$= (AB + \bar{A}\bar{B})(A+B)$

$= AAB + ABB + \bar{A}A\bar{B} + \bar{A}\bar{B}B$

$= AB + AB + 0 + 0$ ∵ $A\bar{A} = 0$, $AA = A$

$= \underline{\underline{AB}}$

(k) $y = AB + \overline{AC} + A\bar{B}C(AB + C)$

$= AB + \bar{A} + \bar{C} + A\bar{B}CAB + A\bar{B}CC$

$= AB + \bar{A} + \bar{C} + 0 + A\bar{B}C$

$= A(B + \bar{B}C) + \bar{A} + \bar{C}$

$= A(B + C) + \bar{A} + \bar{C}$

$= AB + AC + \bar{A} + \bar{C}$

$= \bar{A} + AB + \bar{C} + AC.$

$= (\bar{A} + A)(\bar{A} + B) + (\bar{C} + A)(\bar{C} + C)$

$= \bar{A} + B + \bar{C} + A = 1 + B + \bar{C}$

$= 1$