Introduction to Electronics Engg.

Module - 4

Boolean Algebra
&
Logic Circuits.

Dr. Suma Umesh
EEE Dept.

I. Binary Numbers

In a digital electronics system, the active devices are used to operate as switches such as diodes, transistors. and they will have only two states; 'ON' and 'OFF'.
These states are represented by binary numbers '1' and '0'

In general, a number in base-r system has coefficients multiplied by the power of r expressed as:

$$\boxed{a_n r^n + a_{n-1} r^{n-1} + \cdots a_1 r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2}\, a_{-3} \cdot r^{-3} \cdots + a_{-m} \cdot r^{-m}}$$

The coefficients 'a' range from (0 to r−1)

* The binary number system has base-2 or radix-2.

* In binary number, each digit (0 and 1) is called bit.

* The left most bit of a binary number is called Most Significant Bit [MSB] and the right most bit of a binary number is called Least Significant Bit [LSB].

Ex:

101101

MSB (Highest Weight) — LSB (least weight)

* The weight one each digit, in the increa order is $2^0\ 2^1\ 2^2\ 2^3 \ldots$ so on...

Ex

$(10110)_2$ has the weight on each digit as:

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$
1 0 1 1 0
$\longleftarrow$
Increasing order of weight

Different Number Systems:

(i) **Binary number** has a radix r of 2.
As r=2, the digits are only 0 and 1

(ii) **Decimal Number System** has a radix r = 10.
As r = 10, the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

(***) Digital systems work only on binary numbers
Since binary are often very long, two short-hand notations namely octal and hexadecimal.

(iii) **Octal number System:** has a radix r=8
As r=8, the digits are 0, 1, 2, 3, 4, 5, 6, 7.

(iv) **Hexa deimal number System** has a radix r = 16 and the decimal digits 0 to 9 are used as the first ten digits followed by 6 letters A, B, C, D, E and F, which represents Values 10, 11, 12, 13, 14 and 15 respectively.

(10) Decimal	(2) Binary	(8) Octal	(16) Hexa decimal
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	~~08~~ 10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number Base Conversion:

Computer uses binary number system. In reality, we use decimal numbers for transactions. Octal and hexa deimal are two intermediatary number systems.

Hence there is a need to convert numbers from one base to the other base.

They are:

(1) Binary to Octal

(2) Binary to Decimal

(3) Binary to Hexa deimal.

II [illegible] Conversion

(4) Octal to Binary
(5) octal to ~~Hexa~~ Decimal
(6) octal to Hexa decimal
(7) Decimal to Binary
(8) Decimal to octal
(9) Decimal to Hexadecimal
(10) Hexadecimal to Binary
(11) Hexadecimal to octal
(12) Hexa decimal to Decimal.

I. Binary to Octal Conversion

Step 1 : Make group of 3 bits starting from LSB for Integer part and from MSB for fractional part. Add 0's if required

Step 2 : write equivalent octal number for each group of 3 bits.

ex: (i) $(10101101.0111)_2 = (\quad)_8$

add 0   0 10 | 101 | 101 . 011 | 1 00   add 0

$\leftarrow . \rightarrow$

weight on each group is $2^2 2^1 2^0 \Rightarrow 4 2 1$

2 5 5 . 3 4

$\therefore$ Ans = $(255.34)_8$

(ii) $(1101011110111101 \cdot 101110)_2 = (\quad)_8$

weight in 421

$\underleftarrow{110}\ \underleftarrow{101}\ \underleftarrow{111}\ \underleftarrow{011}\ \underleftarrow{101} \cdot \underrightarrow{101}\ \underrightarrow{110}$

6 5 7 3 5 . 5 6

$\therefore$ Ans = $(65735.56)_8$

(iii) $(11011011110)_2 = (\quad)_8$

$\underline{011}\ \underline{011}\ \underline{011}\ \underline{110}$

3 3 3 6

$\therefore$ Ans = $(3336)_8$.

(iv) $(0 \cdot 11110101101)_2 = (\quad)_8$

$0 \cdot \underline{111}\ \underline{101}\ \underline{011}\ \underline{010}$

0 · 7532

$\therefore$ Ans = $(0 \cdot 7532)_8$.

(v) $(1101101110 \cdot 1001101)_2 = (\quad)_8$

$\underline{001}\ \underline{101}\ \underline{101}\ \underline{110} \cdot \underline{100}\ \underline{110}\ \underline{100}$

1 5 5 6 . 4 6 4

$\therefore$ Ans = $(1556.464)_8$

Solve. ① $(1101011110)_2$ ② $(11101101110 \cdot 11101)_2$
③ $(101010101111100)_2$ ④ $(1101111010 \cdot 011101)_2$

II. Binary to Decimal conversion

Step1 : For the integer part multiply the binary digit with its weight, which is in increasing order starting from LSB. For the fractional part, multiply the binary digit with its weight, which is in increasing order (−ve weight) from MSB.

Step2 : Add all digits seperately for integer & fraction part.

Ex. $\ldots 2^4\, 2^3\, 2^2\, 2^1\, 2^0 \,.\, 2^{-1}\, 2^{-2}\, 2^{-3}\, 2^{-4} \ldots$

(i) $(11010)_2 = (\ )_{10}$

$2^4\, 2^3\, 2^2\, 2^1\, 2^0 \rightarrow$ weight

$1\,1\,0\,1\,0 \rightarrow$ digits

$2^4 \times 1 + 2^3 \times 1 + 0 + 2^1 \times 1 + 0$

$16 + 8 + 2 + 0 = 26$

$\therefore (11010)_2 = (26)_{10}$

(ii) $(11011.1101)_2 = (\ )_{10}$

$2^4\, 2^3\, 2^2\, 2^1\, 2^0 \quad 2^{-1}\, 2^{-2}\, 2^{-3}\, 2^{-4}$

$1\,1\,0\,1\,1 \,.\, 1\,1\,0\,1$

$2^4 \times 1 + 2^3 \times 1 + 0 + 2^1 \times 1 + 1 \times 1 \,.\, 1 \times \frac{1}{2} + 1 \times \frac{1}{2^2} + 0 + 1 \times \frac{1}{2^4}$

$16 + 8 + 0 + 2 + 1 \,.\, 0.5 + 0.25 + 0 + 0.0625$

$(27.8125)_{10}$

Ans

(iii) $(111110101)_2 = (\ )_{10}$

$2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0$
1 1 1 1 1 0 1 0 1

$= \cancel{2^8}\ 2^8 \times 1 + 2^7 \times 1 + 2^6 \times 1 + 2^5 \times 1 + 2^4 \times 1 + 0 + 2^3 \times 1$
$+ 2^2 \times 1 + 0 + 2^0 \times 1$

$= 256 + 128 + 64 + 32 + 16 + 0 + 8 + 4 + 0 + 1$

$= (501)_{10}$ Ans

(iv) $(11101.01)_2 = (\ )_{10}$

$2^4 2^3 2^2 2^1 2^0 \quad 2^{-1} 2^{-2}$
1 1 1 0 1 . 0 1

$1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 + 2^0 \times 1 \cdot 0 + 1 \times 2^{-2}$

$16 + 8 + 4 + 0 + 1 \cdot 0 + 1 \times \frac{1}{2^2}$

$(29.25)_{10}$

(v) $(10001.101)_2 = (\ )_{10}$

$2^4 \times 1 + 0 + 0 + 0 + 1 \times 2^0 \cdot 1 \times 2^{-1} + 0 + 1 \times 2^{-3}$

$16 + 1 \cdot 0.5 + 0.125$

$(17.625)_{10}$ Ans

Solve ① $(1101)_2$ ② $(10101)_2$ ③ $(101010.111)_2$
④ $(1011.11)_2$

III. Binary to Hexadecimal Conversion

The Base for hexadecimal number is 4th power of base for binary i.e $2^4 = 16$.

$\therefore$ by grouping 4 digits of binary number and converting each group digits to its hexadecimal equivalent, we can convert binary to its equivalent hexadecimal number.

Step 1 : group the digits into 4 digits each starting from LSB in integer part and starting from MSB in fractional part. add 0's if necessary.

Step 2 : Write the equivalent hexa number for each group.

weight is $\boxed{2^4 2^3 2^1 2^0}$

ex : (i) $(1101101110 \cdot 1001101)_2 = (\;)_{16}$

$\leftarrow \cdot \rightarrow$

$\underline{0011}\ \underline{0110}\ \underline{1110} \cdot \underline{1001}\ \underline{1010}$

3 6 E . 9 A

(refer table in pg-4)

$\therefore$ Ans $= (36E \cdot 9A)_{16}$

(ii) $(110111101 \cdot 01)_2$

421

8421 8421 8421 . 8421 weight

$\underline{0001}\ \underline{1011}\ \underline{1101} \cdot \underline{0100}$

1 B D . 4

Ans $= (1BD \cdot 4)_{16}$

(iii) $(1101111010111101)_2 = (\ )_{16}$?

$\underline{0110}\ \underline{1111}\ \underline{0101}\ \underline{1101}$

6 F 5 D

Ans $= (6F5D)_{16}$

(iv) $(0.110,0101,11011)_2 = (\ )_{16}$

$0.\ \underline{1101}\ \underline{0101}\ \underline{1101}\ \underline{1000}$

$0.\ D5D8$

$\therefore$ Ans $= (0.D5D8)_{16}$

(v) $(0001\,0010\,0100\,1000\,1001,1010,1101,1111)_2$

$\underline{0001}\ \underline{0010}\ \underline{0100}\ \underline{1000}\ \underline{1001}\ \underline{1010}.\ \underline{1101}\ \underline{1111}$

1 2 4 8 . 9 A D F

Ans $= (12489ADF)_{16}$

Solve.

① $(110101101)_2$ ② $(1111111111111)_2$

③ $(1011\,0111\,110.\,110001.010011)_2$

IV Octal to Binary.

To convert a given octal to a binary, just replace each octal digit by its 3-bit binary equivalent. $2^2 2^1 2^0 \Rightarrow 421$ pattern.

Ex: (i) convert $(27633)_8$ into binary.

$$\begin{array}{ccccccc} 2 & 7 & 6 & 3 & 3 & = & (\ )_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 010 & 111 & 110 & 011 & 011 & & \end{array}$$

$\therefore (27633)_8 = 010111110011011$ binary.

(ii) $(725.25)_8 = (\ )_2$

$(111\,010\,101.\ 010\,101)_2$ Ans.

(iii) $(673.124)_8 = (\ )_2$

$= (110\,111\,011.\ 001\,010\,100)_2$

(iv) $(4532)_8 = (\ )_2$

$= (100\,101\,011\,010)_2$

Solve

① $(673.124)_8$ ② $(7463.245)_8$

③ $(674.35)_8$

V Octal to Decimal conversion

An octal can be converted to decimal equivalent by multiplying each octal digit by its ~~po~~ positional weightage. which are $8^n \cdots 8^3\, 8^2\, 8^1\, 8^0$

Ex: (i) $(6327.4051)_8 = (\quad)_{10}$

$$\begin{array}{cccccccccc} 8^3 & 8^2 & 8^1 & 8^0 & & 8^{-1} & 8^{-2} & 8^{-3} & 8^{-4} \\ 6 & 3 & 2 & 7 & . & 4 & 0 & 5 & 1 \end{array}$$

$= 6 \times 512 + 3 \times 64 + 8 \times 2 + 7 \times 1 + 4 \times 8^{-1} + 0 + 5 \times \frac{1}{8^3} + 1 \times \frac{1}{8^4}$

$= 3072 + 192 + 16 + 7 + \frac{4}{8} + 0 + \frac{5}{512} + \frac{1}{4096}$

$= 3072 + 192 + 16 + 7 + 0.5 + 0.00976 + 2.44 \times 10^{-4}$

$= (3287.51000414)_{10}$

(ii) $(514.24)_8 = (\quad)_{10}$

$$\begin{array}{cccccc} 8^2 & 8^1 & 8^0 & & 8^{-1} & 8^{-2} \\ 5 & 1 & 4 & . & 2 & 4 \end{array}$$

$5 \times 64 + 1 \times 8 + 4 \times 1 + 2 \times \frac{1}{8} + 4 \times \frac{1}{64}$

$= 320 + 8 + 4 + 0.25 + 0.0625$

$= (332.3125)_{10}$

VI Octal to Hexadecimal Conversion

To convert an octal number to hexadecimal, first convert it to Binary and then convert from binary to Hexa.

Octal
↓ (group of 3 bits)
Binary
↓ (group into 4 bits)
Hexa.

Ex: (i) convert $(3576)_8$ to hexadecimal

3 5 7 6 → octal

011 101 111 110 → binary

to convert to hexa, group into 4 bits

$\underbrace{0111}_{7}\ \underbrace{0111}_{7}\ \underbrace{1110}_{E}$

$\therefore (3576)_8 = (77E)_{16}$

(ii) $(273)_8 = (\quad)_{16}$

VII Decimal to Binary Conversions

To convert decimal to binary, divide the number by base - 2 until you get reminder less than - 2 and arrange the digits as in the example below.

Ex: (i) $(734)_{10} = (\ )_2$

2	734	
2	367	— 0
2	183	— 1
2	91	— 1
2	45	— 1
2	22	— 1
2	11	— 0
2	5	— 1
2	2	— 1
	1	— 0

arrange bits

$\therefore \quad (734)_{10} = (1011011110)_2$

For the fractional portion, multiply the fractional portion by 2 and take ~~the integer part~~ and continue to multiply until fractional part becomes zero or stop after few iterations and arrange the integer part of product as shown

VIII. ... to Octal Conversion

(ii) Convert $(0.705)_{10}$ into binary.

Soln:

$$0.705 \times 2 = 1.41$$
$$0.41 \times 2 = 0.82$$
$$0.82 \times 2 = 1.64$$
$$0.64 \times 2 = 1.28$$
$$0.28 \times 2 = 0.56$$
$$0.56 \times 2 = 1.12$$
$$0.12 \times 2 = 0.24$$
$$0.24 \times 2 = 0.48$$
$$0.48 \times 2 = 0.96$$
$$0.96 \times 2 = 1.92$$

$\therefore (0.705)_{10} = (1011010001)_2$

(iii) Convert $(1593.875)_{10}$ to binary.

Take integer & fractional part seperately and do the conversion.

(a)

2	1593	
2	796	– 1
2	396	– 0
2	199	– 0
2	99	– 1
2	49	– 1
2	24	– 1
2	12	– 0
2	6	– 0
2	3	– 0
	1	– 1

(b) 0.875

$$0.875 \times 2 = 1.75$$
$$0.75 \times 2 = 1.5$$
$$0.5 \times 2 = 1.00$$

$\therefore$ Ans $= (11000111001 \cdot 111)_2$

(iv) $(2533.634)_{10} = (\quad)_2$

ⓐ

2	2533	
2	1266	— 1
2	633	— 0
2	316	— 1
2	158	— 0
2	79	— 0
2	39	— 1
2	19	— 1
2	9	— 1
2	4	— 1
2	2	— 0
	1	— 0

$0.634 \times 2 = \underline{1}.2$

$0.268 \times 2 = \underline{0}.53$

$0.536 \times 2 = \underline{1}.07$

$0.072 \times 2 = \underline{0}.14$

$0.144 \times 2 = \underline{0}.28$

$0.288 \times 2 = \underline{0}.5$

$(0.634)_{10} = (101000)_2$

$(2533)_{10} = (100111100101)_2$

$\therefore (2533.634)_{10} = (100111100101.101000)_2$

Solve : ① $(4187.37)_{10}$ ② $(657.129)_{10}$

$(7132.87)_{10}$.

VIII. Decimal to Octal Conversion

Here also, the integral & fractional part are treated seperately and then results are combined to obtain the desired octal number.

* Integer part is obtained by repeated-division method. The decimal number is divided repeatedly by octal base (8) till Quotient = 0.

* Fractional part is obtained by repeated-multiplication with 8. The subsequent decimal fraction is multiplied by 8 till the product is 0 or until required accuracy.

Ex: (i) $(2003)_{10} = (\ )_8$

$$\begin{array}{r|l}
8 & 2003 \\
8 & 250 - 3 \\
8 & 31 - 2 \\
 & 3 - 7
\end{array}$$

$\therefore (2003)_{10} = [3723]_8$

(ii) $(0.705)_{10} = (\ )_8$

$0.705 \times 8 = 5.64$

$0.64 \times 8 = 5.12$

$0.12 \times 8 = 0.96$

$0.96 \times 8 = 7.68$

$0.68 \times 8 = 5.44$

$\therefore (0.705)_{10} = (0.55075)_8$

(iii) Convert $(11582 \cdot 875)_{10}$ to octal

(a)

$$
\begin{array}{r|l}
8 & 11582 \\
\hline
8 & 1447 - 6 \\
\hline
8 & 180 - 7 \\
\hline
8 & 22 - 4 \\
\hline
 & 2 - 6
\end{array}
$$

$(11582)_{10} = (26476)_8$

(b) $0 \cdot 875 \times 8 = \underline{7} \cdot 00$ $\quad (0 \cdot 875)_{10} = (0 \cdot 7)_8$

$\therefore (11582 \cdot 875)_{10} = (26476 \cdot 7)_8$

(iv) $(378 \cdot 93)_{10} = (\quad)_8$

$$
\begin{array}{r|l}
8 & 378 \\
\hline
8 & 47 - 2 \\
\hline
 & 5 - 7
\end{array}
\Rightarrow (572)_8
$$

$$
\begin{aligned}
0 \cdot 93 \times 8 &= 7 \cdot 44 \\
0 \cdot 44 \times 8 &= 3 \cdot 52 \\
0 \cdot 52 \times 8 &= 4 \cdot 16 \\
0 \cdot 16 \times 8 &= 1 \cdot 28
\end{aligned}
\Rightarrow (7341)_8.
$$

$\therefore (378 \cdot 93)_{10} = (572 \cdot 7341)_8$

Solve $(658 \cdot 825)_{10}$ $\quad (9532 \cdot 237)_{10}$

IX. Decimal to HexaDecimal conversion:

The procedure is similar to previous method.

* the integer part is successively divided by 16 until quotient is zero

* fractional part is successively multiplied by 16 until product is 0 or upto the required accuracy.

Ex: (i) $(48530)_{10} = (\quad)_{16}$

$$
\begin{array}{r|l}
16 & 48530 \\
\hline
16 & 3033 - 2 \\
\hline
16 & 189 - 9 \\
\hline
 & 11 - 13 \\
 & \text{B} \quad\;\; \text{D}
\end{array}
$$

$\Rightarrow (BD92)_{16}$.

(ii) $(675.625)_{10} = (\quad)_{16}$

Integer Part

$$
\begin{array}{r|l}
16 & 675 \\
\hline
16 & 42 - 3 \\
\hline
 & 2 - 10 \rightarrow A
\end{array}
$$

$\Rightarrow 2A3$

fractional part

$0.625 \times 16 = 10.0 \Rightarrow 0.A$.

$\therefore (675.625)_{10} = (2A3.A)_{16}$

(iii) $(532.65)_{10} = (\quad)_{16}$

$$\begin{array}{r|l} 16 & 532 \\ \hline 16 & 33 \; - 4 \\ \hline & 2 \; - 1 \end{array} \Rightarrow (214)_{16}$$

$$\begin{array}{lcll} 0.65 \times 16 & = & 10.4 & \rightarrow A \\ 0.4 \times 16 & = & 6.4 & \rightarrow 6 \\ 0.4 \times 16 & = & 6.4 & \rightarrow 6 \end{array} \Rightarrow (0.A66)_{16}$$

$\therefore (532.65)_{10} = (214.A66)_{16}$

(iv) $(894867)_{10} = (\quad)_{16}$

$$\begin{array}{r|l} 16 & 894867 \\ \hline 16 & 55929 \; - 3 \\ \hline 16 & 3495 \; - 9 \\ \hline 16 & 218 \; - 7 \\ \hline & 13 \; - A \\ \hline & \rightarrow D \end{array}$$

$\Rightarrow \underline{\underline{(DA793)_{16}}}$.

Solve

① $(8899)_{10}$ ② $(7084.95)_{10}$

③ $(0.368)_{10}$ ④ $(4477.85)_{10}$

X. Hexadecimal to Binary Conversion

Replace ~~the~~ each hexadecimal digit by its 4-bit binary equivalent.

Ex : (i) $(8BE6.7A)_{16} = (\quad)_2$

8	B	E	6	.	7	A
1000	1011	1110	0110	.	0111	1010

(ii) $(306.D)_{16}$

0111 0000 0110 • 1101

(iii) $(ABCD.F)_{16}$

1010 1011 1100 1101 • 1111

XI Hexadecimal to Octal conversion

* First convert the hexanumber to binary and then binary to octal by grouping into 3-digits.

Ex: Convert $(3576)_{16}$ to octal.

Hexa → Binary → octal.

3 5 7 6

00 0011 0101 0111 0110

3 2 5 6 6

octal
↓
421

$\therefore (3576)_{16} = (32566)_8$

(ii) $(ABCD)_{16} = (\quad)_8$

$$\begin{array}{cccccc} & A & B & C & & D \\ 00 & 1010 & 1011 & 1100 & & 1101 \\ 1 & 2 & 5 & 7 & 1 & 5 \end{array}$$

$\therefore (ABCD)_{16} = (125715)_8.$

Solve: ① $(A283 \cdot 3C)_{16}$ ② $(B387 \cdot A8)_{16}$

XII. Hexadecimal to Decimal Conversion:

A hexadecimal number is converted to decimal equivalent by multiplying each digit by its positional weight. which is $\ldots 16^3\, 16^2\, 16^1\, 16^0$ for integer part and $16^{-1}\, 16^{-2}\, 16^{-3} \ldots$ for fractional part.

Ex: (i) $(5C7)_{16} = (\quad)_{10}$

$$\begin{aligned} 5C7 &= 5\times16^2 + C\times16^1 + 7\times16^0 \\ &= 5\times256 + 12\times16 + 7\times1 \\ &= 1280 + 192 + 7 \\ &= \underline{\underline{(1479)}}_{10}. \end{aligned}$$

(ii) $(AC8 \cdot 32)_{16} = (\quad)_{10}$

$A\times16^2 + C\times16^1 + 8\times16^0 + 3\times16^{-1} + 2\times16^{-2}$

$10\times256 + 12\times16 + 8 + \frac{3}{16} + \frac{2}{256}$

$2560 + 192 + 8 + 0.1875 + 7.81\times10^{-3}$

$= (2760.1953)_{10}$

Solve

① $(B2B.39)_{16}$ ② $(F13.2C)_{10}$.

Complements

Complements are used in digital system/computer for simplifying subtraction operation and also for logical manipulations.

There are two types of complements for each base-r number system:

(i) r's complement
(ii) (r-1)'s complement.

Hence for

✓ Binary → 2's complement, 1's complement

✓ Decimal → 10's complement, 9's complement.

Octal → 8's complement, 7's complement

Hexadecimal → 16's complement, 15's complement.