Q1. The subgroup generated by any element $a \in G$ is
$\langle a \rangle = \{a^n \mid n \in Z\}$

The number of elements in a subgroup H of G must divide the number of elements in G.

So size of H divides p.

So size of H can be 1 or p.

H must contain the identity element e. So if size of H is 1 $H = \{e\}$.

But the cyclic subgroup generated by $\{e\}$ is $\{e\}$ itself which isnt G.

So, lets take another element $a \in G$, $a \neq e$.

$\langle a \rangle = \{a^n \mid n \in Z\}$.

As $a \neq e$, size of $\langle a \rangle$ is greater than 1.

So, the only possibility is : size of $\langle a \rangle$ is p.

So $|\langle a \rangle| = |G|$. So a is a subgroup of G of same size as G.

So, the cyclic subgroup generated by a is G. Thus G is cyclic.

Q2. Yes, it is possible.

Example: Group G : $(\mathbb{Z}, +)$

Subgroup H : $(\{0\}, +)$ → finite

For each element $a \in G$, the left coset of H corresponding to a is $\{a\}$

So, for each element in G, there is a unique left coset wrt. H. ⇒ There are infinitely many left cosets.