NLD Project
Logistic Modelling of Economic Data
The Growth of IBM

Manan Pareek (202101018),* Suyash Bhagat (202101085),† and Aditya Singhania (202101086)‡
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India

In this project, we numerically and analytically analyze the growth of IBM using the logistic equation for modeling and real data of IBM's annual revenue, human resource count and net annual profit from 1914 to 2006. We tried to find a correlation between IBM's annual revenue growth to the expansion of its human resource base from 1914 to 2006. We also statistically analysed the accuracy of the model by finding mean and standard deviation.

I. EQUATIONS

The logistic equation as follows,

$$\dot{x} \equiv \frac{dx}{dt} = f(x) = ax - bx^2 \tag{1}$$

represents the basic model of a nonlinear function $f(x) = ax - bx^2$, where a and b are fixed parameters. Taking initial condition as $x(0) = x_0$ and $k = \frac{a}{b}$, and solving the equation, we have,

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

Using $x = k/2$, we get the non-linear time scale as follows:

$$t_{nl} = \frac{1}{a}ln(\frac{k}{x_o} - 1) \tag{3}$$

With the logistic equation the revenue is $R \equiv R(t)$, with R measured in US dollars and t in years, the logistic model for the revenue growth is,

$$\dot{R} \equiv \frac{dR}{dt} = \mathcal{R}(R) = \rho_1 R - \rho_2 R^2 \tag{4}$$

Here, a and b translate to ρ_1 and ρ_2 respectively, with $K_R = \rho_1/\rho_2$.

Similarly, the Human Resource equation is as follows,

$$\dot{H} \equiv \frac{dH}{dt} = \mathcal{H}(H) = \eta_1 H - \eta_2 H^2 \tag{5}$$

Also, $H \equiv H(t)$ and a and b translates to η_1 and η_2 respectively, with $K_H = \eta_1/\eta_2$.

To analyze the correlated growth of R and H, we have set down a coupled autonomous dynamical system as $\dot{R} = \mathcal{R}(H, R)$ and $\dot{H} = \mathcal{H}(H, R)$. Then, defining $\mathcal{U} = H^{-1} - k_H^{-1}$, $V = R^{-1} - k_R^{-1}$ and $\beta = \rho/\eta_1$, the H-R phase solutions are transformed to a compact power-law form as

$$V \sim \mathcal{U}^{\beta} \tag{6}$$

*Electronic address: 202101018@daiict.ac.in
†Electronic address: 202101085@daiict.ac.in
‡Electronic address: 202101086@daiict.ac.in

II. GRAPHS

1. Annual revenue of IBM vs time (in years) plot

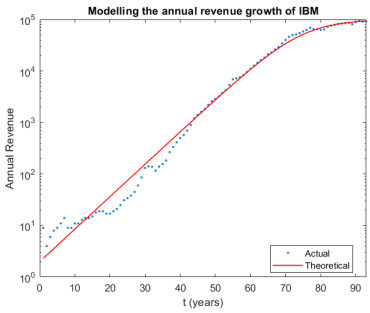

Fig. 1

The dotted curve models the annual revenue growth of IBM, using the company data from 1914 to 2006. The smooth curve is the logistic function, as given by Equation(1). The parameter values to fit the revenue growth are $\rho 1 = 0.145$ year^{-1} and $k_R = 100billion$ (the predicted maximum revenue that IBM can earn). Saturation of the revenue growth starts on the time scale of 75-80 years.

2. Human resources strength of IBM vs time (in years) plot

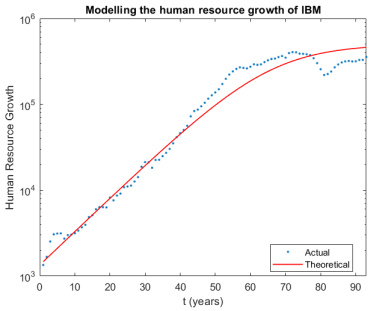

Fig. 2

The dotted curve models the human resource growth of IBM, using the company data from 1914 to 2006. The smooth curve is the logistic function, as given by Equation[1]. The parameter values to fit the revenue growth are $\eta_1 = 0.09$ year^{-1} and $k_H = 500000$ (the

predicted maximum employees of IBM). The human resource graph declines around 75-80 years.

3. Net annual earnings of IBM(in millions of dollars)

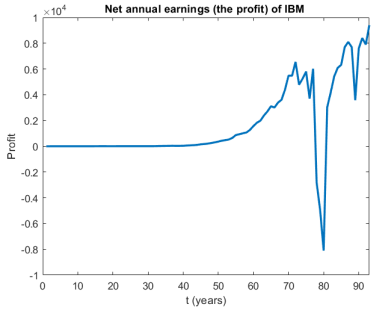

Fig. 3

The net annual earnings (the profit P) of IBM grow steadily till about 75-80 years (the early years of the 1990s). Around this time IBM suffered major losses in its net earnings ($ 8 billion in 1993), and this time scale corresponds closely to the time scale for the onset of nonlinear saturation in revenue growth, which is also 75-80 years.

4. Correlation log-log plot of annual revenue and human resource strength of IBM

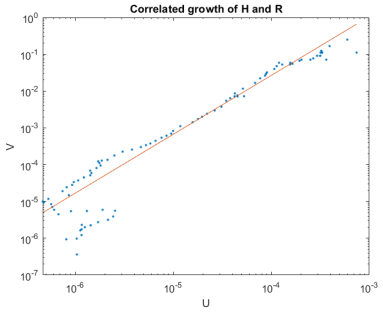

Fig. 4

This is the plot for u-v curve which shows the correlated growth of H and R. For dotted graph, $U = H^{-1} - k_H^{-1}$, $V = R^{-1} - k_R^{-1}$. For the line graph, $V \approx U^{\beta}$ and $\beta = \rho_1/\eta_1 \approx 1.6$

III. STATISTICS

Annual Revenue

- Annual Revenue Mean: 0.025
- Annual Revenue Standard Deviation: 0.4870

Human Resource Count

- Human Resource Count Mean: 0.0901
- Human Resource Count Standard Deviation: 0.2980

IV. CONCLUSIONS

- The modelled logistic equation $\dot{x} \equiv \frac{dx}{dt} = f(x) = ax - bx^2$ fits very nicely (low amount of deviations) to the given data of annual revenue and human resource count of IBM with respective arbitrary constants. They seem to reach their saturation level as observed in the graphs at high values of t. For Annual revenue, this value is found to be $\$10^{11}$.
- Through figure 3, it can be noticed that there is a major loss in profits around t=80 years, i.e., around the year 1990. These scenario can also be seen through the fall in the human resource curve in its actual (original) data.
- We can also conclude that Annual revenue is proportional to human resource count and is of increasing nature due to a straight line graph ($V \approx U^{\beta}$ and $\beta(slope) \approx 1.6$).

[1] Arnab K. Ray, *Logistic modelling of economic dynamics* (November 18, 2024).