

Homework7

2024-04-18

Question 1

Step 1:

Model for data: $X \sim \text{Normal}(\mu = 16.00, \sigma)$

Step 2:

Null Hypothesis:

H_0 = Coca Cola cans have 16.00 ounces of Cola.

Alternative Hypothesis:

H_a = Coca Cola cans have < 16.00 ounces of Cola.

Step 3:

We will pick the t statistic as our test statistic.

Step 4:

```
sample <- c(15.997, 16.005, 15.981, 15.954, 15.986, 16.021, 15.985, 16.001, 16.018, 16.056)
```

```
s_mean <- mean(sample)
```

```
s_sd <- sd(sample)
```

```
s_mean
```

```
## [1] 16.0004
```

```
s_sd
```

```
## [1] 0.02755278
```

Sample mean, $\bar{x} = 16.0004$, sample standard deviation $s = 0.02755278$, Hypothesized mean $\mu = 16.00$, Sample Size = 10.

```
Observed_tvalue <- (16.0004 - 16.00)/(0.02755278/sqrt(10))
```

```
Observed_tvalue
```

```
## [1] 0.04590865
```

```
t = 0.04590865
```

Step 5:

Computing p-value under H_0 ,

Degrees of freedom = 9

```
df <- 9
p_val <- pt(Observed_tvalue, df)
p_val
```

```
## [1] 0.5178072
```

p-value under $H_0 = 0.5178072$

Step 6:

Significance level $\alpha = 0.05$.

Step 7:

$p_value = 0.5178072 > \alpha$

Therefore, we cannot reject the hypothesis that the mean fill of Coca Cola cans is 16.00 oz. The sample data does not provide enough evidence to prove that it is filled with less than 16.00 oz of Cola.

Question 2

Step 1:

Model for data: $X \sim \text{Binomial}(n, p = 0.7)$

Step 2:

Null Hypothesis:

$H_0 = 70\%$ of college students in the US are stressed.

Alternative Hypothesis:

$H_a =$ The percentage of college students in the US that are stressed is less than 70%.

Step 3:

We will pick the z statistic as our test statistic.

Step 4:

Sample proportion $\hat{p} = 130/200 = 0.65$.

Hypothesized proportion $p_0 = 0.7$.

Sample size $n = 200$

Observed z value = $(\hat{p} - p_0) / \sqrt{p_0(1 - p_0)} / n$

```
z <- (0.65 - 0.7) / sqrt((0.7 * 0.3) / 200)
z
```

```
## [1] -1.543033
```

Step 5:

Computing p-value under H_0 ,

```
p <- 1 - pnorm(-1.543033, mean = 0, sd = 1)
p
```

```
## [1] 0.9385886
```

p-value under $H_0 = 0.9385886$

Step 6:

Significance level $\alpha = 0.05$.

Step 7:

$p_value = 0.9385886 > \alpha$

Therefore, we cannot reject the hypothesis that 70% of college students in the US are stressed. The sample data does not provide enough evidence to prove that the number of stressed college students is different than 70%.

Question 3

Step 1:

Model for data: $X \sim \text{Normal}(\mu = 2, \sigma = 0.812)$ for ages 18 - 50.

$Y \sim \text{Normal}(\mu = 1.85, \sigma = 0.837)$ for ages 50 and above.

Step 2:

Null Hypothesis:

H_0 = The mean cell phone data usage among people above 50 years of age is greater than or equal to that in people of ages 18 - 50.

Alternative Hypothesis:

H_a = The mean cell phone data usage among people above 50 years of age is less than that in people of ages 18 - 50.

Step 3:

We will pick the t statistic as our test statistic.

Step 4:

Sample size of customers from 18-50 (n_1) = 350

Sample size of customers > 50 (n_2) = 150

Sample mean of n_1 (\hat{X}) = 2

Sample mean of n_2 (\hat{Y}) = 1.85

Let $\hat{w} = (n_1(\hat{X}) + n_2(\hat{Y})) / (n_1 + n_2)$

```
w <- ((350*2) + (150*1.85))/700
w
```

```
## [1] 1.396429
```

t-value = $(\hat{X} - \hat{Y}) / \sqrt{\hat{w}(1 - \frac{n_1}{n_1+n_2})(\sigma_1^2/n_1 + \sigma_2^2/n_2)}$

```
t <- (2 - 1.85) / sqrt(w*(1 - (350/(350+150))) * (0.812/350) + (0.837/150))
t
```

```
## [1] 1.853135
```

Step 5:
Computing p-value under H_0 ,

```
p_val <- pt(t, df = 350 + 150 - 2)
p_val
```

```
## [1] 0.9677729
```

p-value under $H_0 = 0.9677729$

Step 6:
Significance level $\alpha = 0.05$.

Step 7:
 $p_value = 0.9677729 > \alpha$

Therefore, we cannot reject the hypothesis that the mean cell phone usage of those above the age of 50 is greater than or equal to those between ages 18 and 50. The statistical data does not provide enough evidence to prove that the phone usage of those above 50 is less than those from 18 - 50.

Question 4

Step 1:
Model for data: $X \sim \text{Binomial}(n = 150, p = 14/150)$ for Apple iPhone.
 $Y \sim \text{Binomial}(n = 125, p = 15/150)$ for Samsung Galaxy.

Step 2:
Null Hypothesis:
The proportion of returned iPhones is \geq the proportion of returned Samsung Galaxy phones.

Alternative Hypothesis:
 H_a = The proportion of returned iPhones is $<$ the proportion of returned Samsung Galaxy phones.

Step 3:
We will pick the z statistic as our test statistic.

Step 4:
Sample size of Apple iPhones (n_1) = 150
Sample size of Samsung Galaxy (n_2) = 125
Sample Proportion of returned iPhones (p_1) = $14/150 = 0.093$
Sample Proportion of returned Samsung Galaxy (p_2) = $15/125 = 0.12$

Let $\hat{w} = (p_1 + p_2) / (n_1 + n_2)$

```
w <- (0.093 + 0.12)/275
w
```

```
## [1] 0.0007745455
```

Observed z value = $(p_1 - p_2) / \sqrt{\hat{w}(1 - \hat{w}) / (1/n_1 + 1/n_2)}$

```
z <- -0.027/sqrt(0.000773945579268 * 0.0146666666667)
z
```

```
## [1] -8.01388
```

Step 5:

Computing p-value under H_0 ,

```
p <- pnorm(-8.01388, mean = 0, sd = 1)
p
```

```
## [1] 5.557258e-16
```

p-value under $H_0 = 5.557258e-16$

Step 6:

Significance level $\alpha = 0.05$.

Step 7:

$p_value = 5.557258e-16 < \alpha$

Thus, we can reject the hypothesis that the proportion of returned iPhones is greater than or equal to the proportion of returned Samsung Galaxy phones. The sample data provides enough statistical evidence to prove that Apple iPhones have a smaller chance of being returned compared to Samsung Galaxy phones.

Question 5

```
library(UsingR)
```

```
## Loading required package: MASS
```

```
## Loading required package: HistData
```

```
## Loading required package: Hmisc
```

```
##
```

```
## Attaching package: 'Hmisc'
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##      format.pval, units
```

```
data(babies)
```

```
m_age <- babies$age
```

```
d_age <- babies$dage
```

Step 1:

Model for data: $X \sim \text{Normal}(\mu, \sigma)$ for Mother's age.

$Y \sim \text{Normal}(\mu, \sigma)$ for Father's age.

Step 2:

Null Hypothesis:

$H_0 =$ The mean ages of mothers and fathers are equal.

Alternative Hypothesis:

H_a = The mean ages of mothers and fathers are different.

Step 3:

We will pick the t statistic as our test statistic.

Step 4:

```
X = mean(m_age)
Y = mean(d_age)
s1_square = var(m_age)
s2_square = var(d_age)
n1 = length(m_age)
n2 = length(d_age)

t <- (X - Y) / sqrt((s1_square / n1) + (s2_square / n2))
t
```

```
## [1] -11.0671
```

```
t = -11.0671
```

Step 5:

Computing p-value under H_0 ,

```
p <- pnorm(-11.0671, mean = 0, sd = 1)
p
```

```
## [1] 9.05844e-29
```

```
p-value under  $H_0$  = 9.05844e-29
```

Step 6:

Significance level $\alpha = 0.05$.

Step 7:

```
p_value = 9.05844e-29 <  $\alpha$ 
```

Thus, we can reject the hypothesis that the mean ages of mothers and fathers are equal. The sample data provides enough statistical evidence to prove that the mean ages of mothers and fathers are different.