Master in
Computer Vision
Barcelona

Module: M2

Project: **Chan–Vese Segmentation**

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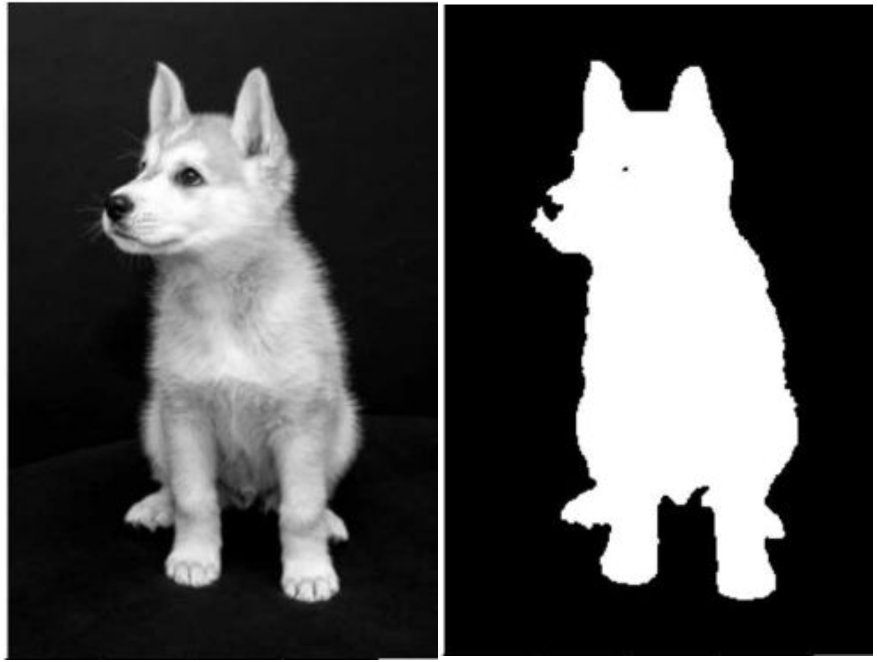
Task

Partition an Image into meaningful segments

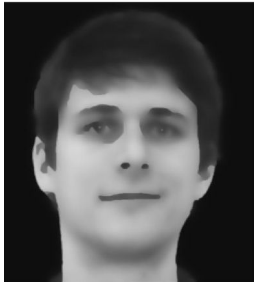


Method: Define Criterias

Mumford-Shah Solution

Criterion	Implication
Segmentation result is an image	Mapping in the same domain $f \rightarrow u$
Segmentation image is similar to the original image	Minimise $\int_{\Omega} (f(x) - u(x))^2 \, dx$
Segmented regions are homogeneous	Minimise $\int_{\Omega \setminus C} \lvert \nabla u(x) \rvert^2 \, dx$
Segmented regions have smooth boundaries	Minimise $\underset{u,C}{\arg\min} \; \mu \, \text{Length}(C)$

f



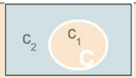
$u(x)$

SOLVE

$$\underset{u,C}{\arg\min} \; \mu \, \text{Length}(C) + \lambda \int_{\Omega} (f(x) - u(x))^2 \, dx + \int_{\Omega \setminus C} \lvert \nabla u(x) \rvert^2 \, dx$$

Method: Define Criterias

Chan-Vese Solution

Criterion	Implication
Segmentation result is an image	Mapping in the same domain $f \rightarrow u$
Segmentation image is similar to the original image	Minimise $\int_{\Omega} \left(f(x) - u(x)\right)^2 dx$
Segmented regions are homogeneous	Minimise $\int_{\Omega \setminus C} \lvert \nabla u(x) \rvert^2 dx$
Segmented regions have smooth boundaries	Minimise $\underset{u,C}{\arg\min} \; \mu \, \mathrm{Length}(C)$
Segmentation image has two regions	Constrain values $u(x) = \{c1, c2\}$ 

$$\underset{c_1, c_2, C}{\arg\min} \; \mu \, \mathrm{Length}(C) + \nu \, \mathrm{Area}(inside(C)) + \lambda_1 \int_{inside(C)} \lvert f(x) - c_1 \rvert^2 dx + \lambda_2 \int_{outside(C)} \lvert f(x) - c_2 \rvert^2 dx$$

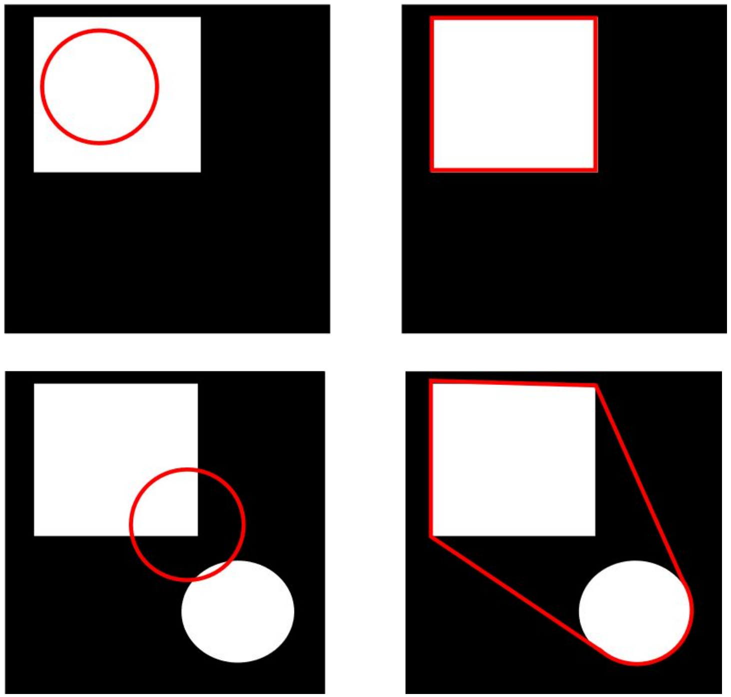
SOLVE

f

$u(x)$

Limitation of Active Contours

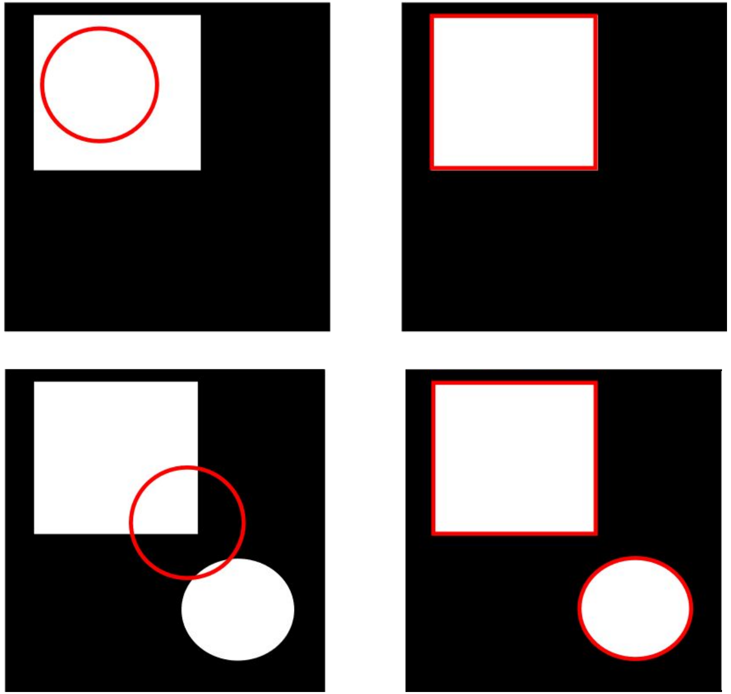
Cannot take into account disconnected components



$$\begin{aligned}\arg\min_{c_1,c_2,C}\ & \mu\,\mathrm{Length}(C) + \nu\,\mathrm{Area}(inside(C)) \\ & + \lambda_1 \int_{inside(C)} |f(x) - c_1|^2\, dx + \lambda_2 \int_{outside(C)} |f(x) - c_2|^2\, dx.\end{aligned}$$

Solution: Active Surfaces

Can take into account any number of disconnected components.



$$\underset{c_1,c_2,C}{\arg\min}\ \mu\,\mathrm{Length}(C)+\nu\,\mathrm{Area}(inside(C))$$
$$+\lambda_1\int_{inside(C)}|f(x)-c_1|^2\,dx+\lambda_2\int_{outside(C)}|f(x)-c_2|^2\,dx.$$

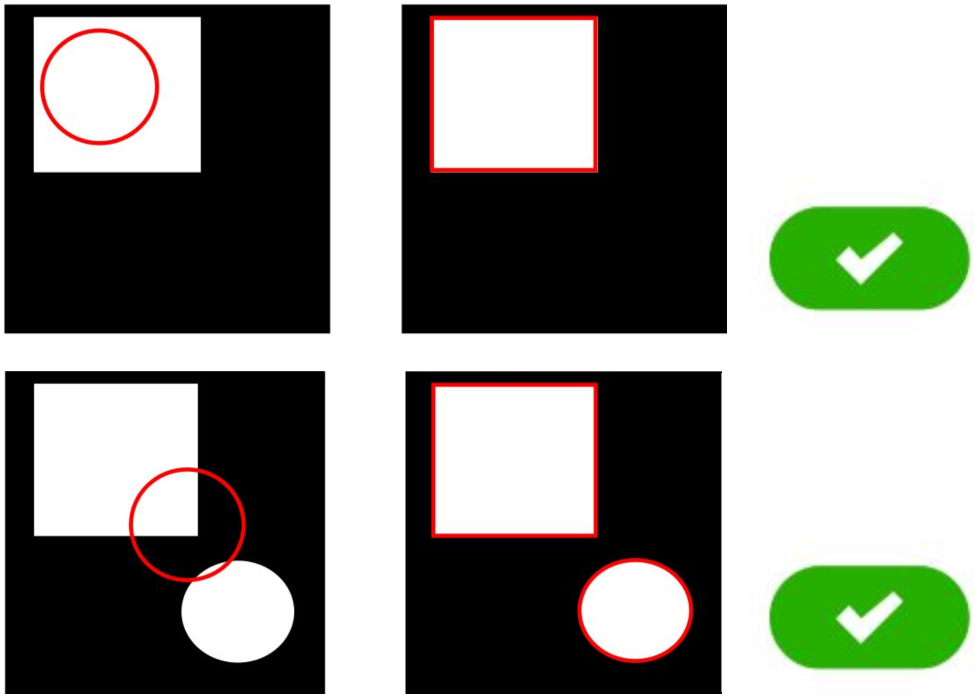
$$\underset{c_1,c_2,\varphi}{\arg\min}\ \mu\int_\Omega \delta(\varphi(x))|\nabla\varphi(x)|\,dx+\nu\int_\Omega H(\varphi(x))\,dx$$
$$+\lambda_1\int_\Omega |f(x)-c_1|^2 H(\varphi(x))\,dx+\lambda_2\int_\Omega |f(x)-c_2|^2(1-H(\varphi(x)))\,dx$$

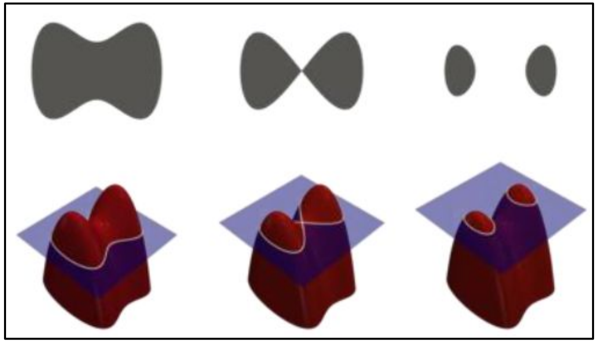
$$\boxed{C=\{x\in\Omega:\varphi(x)=0\}}$$

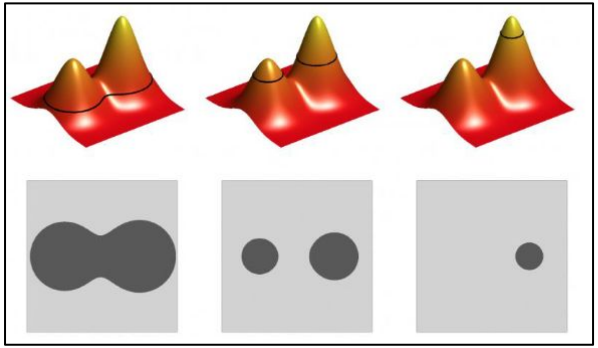
φis a surface that intersects the image at the position of the red contour

Solution: Active Surfaces

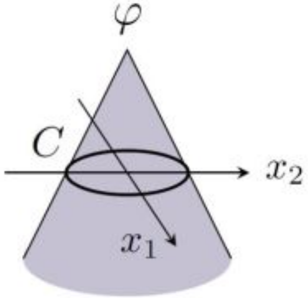
Can take into account any number of disconnected components

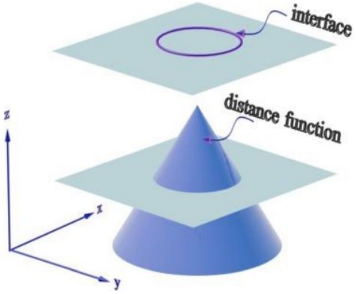






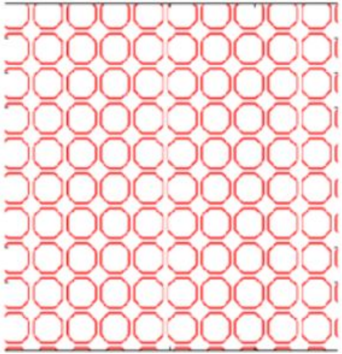
Code: Initial phi





$$\varphi(x) = r - \sqrt{x_1^2 + x_2^2}$$

```
%%Initial phi
phi_0=(-sqrt( ( X-round(ni/2)).^2 + (Y-round(nj/2)).^2)+50);
```



$$\varphi(x) = \sin(\tfrac{\pi}{5}x_1)\sin(\tfrac{\pi}{5}y)$$

```
%%% Checkerboard
phi_0=sin((pi/5)*X).*sin((pi/5)*Y);
```

Code: Phi Boundary Conditions

The boundary condition is enforced by duplicating pixels near the borders

$$\varphi_{-1,j} = \varphi_{0,j}, \quad \varphi_{M,j} = \varphi_{M-1,j}, \quad \varphi_{i,-1} = \varphi_{i,0}, \quad \varphi_{i,M} = \varphi_{i,M-1}$$

```
%Boundary conditions
phi(1,:)   = phi_old(2,:);
phi(end,:) = phi_old(end-1,:);

phi(:,1)   = phi_old(:,2);
phi(:,end) = phi_old(:,end-1);
```

Code: Heaviside step function

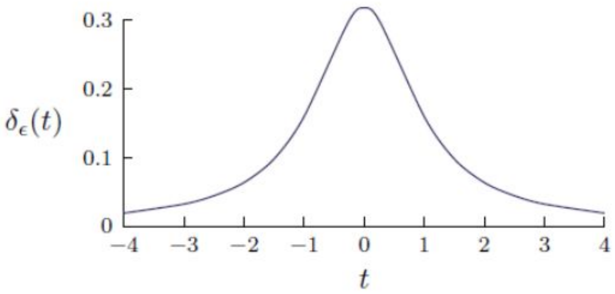
$$H(t) = \begin{cases} 1 & t \geq 0, \\ 0 & t < 0, \end{cases} \quad \delta(t) = \frac{d}{dt} H(t)$$

Regularized heaviside

$$H_\epsilon(t) = \frac{1}{2}\left(1 + \frac{2}{\pi} \arctan\left(\frac{t}{\epsilon}\right)\right)$$

```
%Regularized Heaviside step function
H = 1/2.*(1+(2/pi).*atan(phi_old./epHeaviside));
```

Dirac



$$\delta_\epsilon(t) := \frac{d}{dt} H_\epsilon(t) = \frac{\epsilon}{\pi(\epsilon^2 + t^2)}$$

```
%Dirac function of x
y = epsilon ./ (pi*(epsilon^2 + x.^2));
```

Code: c1 and c2

Average of the pixels intensity values inside each region

$$c_1 = \frac{\int_\Omega f(x) H\big(\varphi(x)\big)\, dx}{\int_\Omega H\big(\varphi(x)\big)\, dx}, \qquad c_2 = \frac{\int_\Omega f(x)\big(1 - H\big(\varphi(x)\big)\big)\, dx}{\int_\Omega \big(1 - H\big(\varphi(x)\big)\big)\, dx}.$$

```
%Fixed phi, Minimization w.r.t c1 and c2 (constant estimation)
c1 = sum(sum(I.*H)) ./ sum(sum(H));
c2 = sum(sum(I.*(1-H))) ./ sum(sum(1-H));
```

Code: Solving Chan-Vese minimization

$$\varphi_{i,j}^{n+1} \leftarrow \Big[\varphi_{i,j}^{n} + dt\,\delta_\epsilon(\varphi_{i,j}^{n})\big(A_{i,j}\varphi_{i+1,j}^{n} + A_{i-1,j}\varphi_{i-1,j}^{n+1} + B_{i,j}\varphi_{i,j+1}^{n} + B_{i,j-1}\varphi_{i,j-1}^{n+1}$$
$$- \nu - \lambda_1(f_{i,j} - c_1)^2 + \lambda_2(f_{i,j} - c_2)^2\big)\Big]$$
$$\Big/ \; \big[1 + dt\,\delta_\epsilon(\varphi_{i,j})(A_{i,j} + A_{i-1,j} + B_{i,j} + B_{i,j-1})\big].$$

```
%%Equation 22, for inner points
phi(2:end-1, 2:end-1) = ...
      (phi_old(2:end-1, 2:end-1) + dt .* delta_phi(2:end-1, 2:end-1) ...
   .* (A(2:end-1, 2:end-1) .* phi_old(3:end, 2:end-1) ...
   +   A(1:end-2, 2:end-1) .* phi(1:end-2, 2:end-1) ...
   +   B(2:end-1, 2:end-1) .* phi_old(2:end-1, 3:end) ...
   +   B(2:end-1, 1:end-2) .* phi(2:end-1, 1:end-2) ...
   -   nu ...
   -   lambda1 .* ((I(2:end-1, 2:end-1)-c1).^2) ...
   +   lambda2 .* ((I(2:end-1, 2:end-1)-c2).^2))) ...
   ./ (1 + dt .* delta_phi(2:end-1, 2:end-1) ...
   .*  (A(2:end-1, 2:end-1) ...
   +    A(1:end-2, 2:end-1) ...
   +    B(2:end-1, 2:end-1) ...
   +    B(2:end-1, 1:end-2)));
```

Code: Coefficients A and B (I)

$$A_{i,j} = \frac{\mu}{\sqrt{\eta^2 + (\nabla_x^+ \varphi_{i,j})^2 + (\nabla_y^0 \varphi_{i,j})^2}}, \quad B_{i,j} = \frac{\mu}{\sqrt{\eta^2 + (\nabla_x^0 \varphi_{i,j})^2 + (\nabla_y^+ \varphi_{i,j})^2}},$$

```
%A and B estimation (A y B from the Pascal Getreuer's IPOL paper "Chan
%Vese segmentation
A = mu ./ sqrt(eta^2 + phi_iFwd.^2 + phi_jcent.^2);
B = mu ./ sqrt(eta^2 + phi_icent.^2 + phi_jFwd.^2);
```

Code: Coefficients A and B (II)

Phi derivatives

$$A_{i,j} = \frac{\mu}{\sqrt{\eta^2 + (\boxed{\nabla_x^+ \varphi_{i,j}})^2 + (\boxed{\nabla_y^0 \varphi_{i,j}})^2}}, \quad B_{i,j} = \frac{\mu}{\sqrt{\eta^2 + (\boxed{\nabla_x^0 \varphi_{i,j}})^2 + (\boxed{\nabla_y^+ \varphi_{i,j}})^2}},$$

$\nabla_x^+ \varphi_{i,j}$

```
%derivatives estimation
%i direction, forward finite differences
phi_iFwd  = DiFwd(phi_old, hi);
phi_iBwd  = DiBwd(phi_old, hi);
```

$\nabla_y^+ \varphi_{i,j}$

```
%j direction, forward finitie differences
phi_jFwd  = DjFwd(phi_old, hj);
phi_jBwd  = DjBwd(phi_old, hj);
```

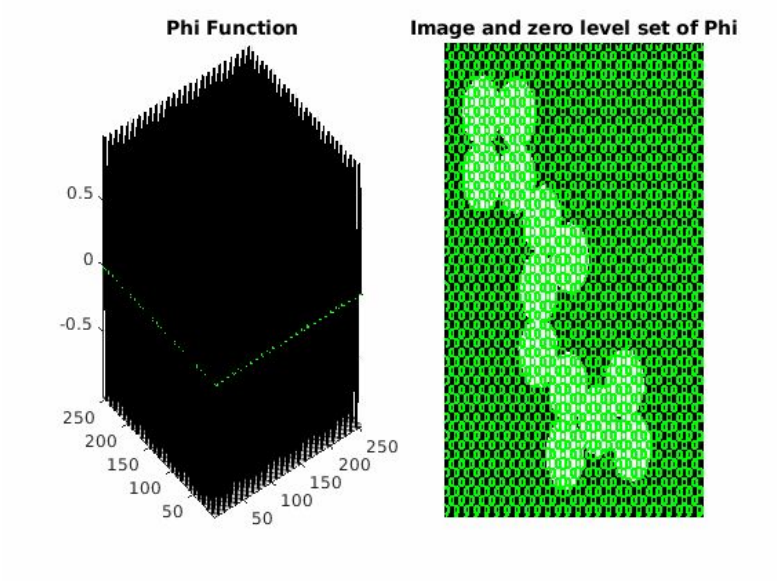
$\nabla_x^0 \varphi_{i,j}$

$\nabla_y^0 \varphi_{i,j}$

```
%centered finite diferences
phi_icent   = (phi_iFwd+phi_iBwd) / 2;
phi_jcent   = (phi_jFwd+phi_jBwd) / 2;
```

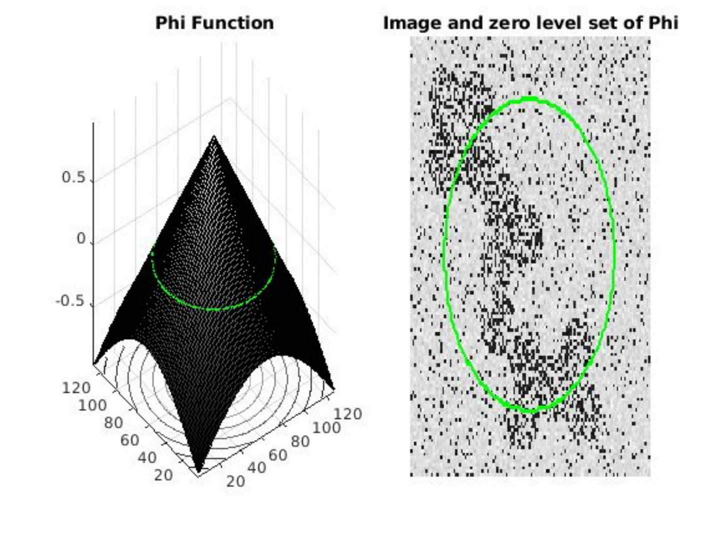
Results - Circles

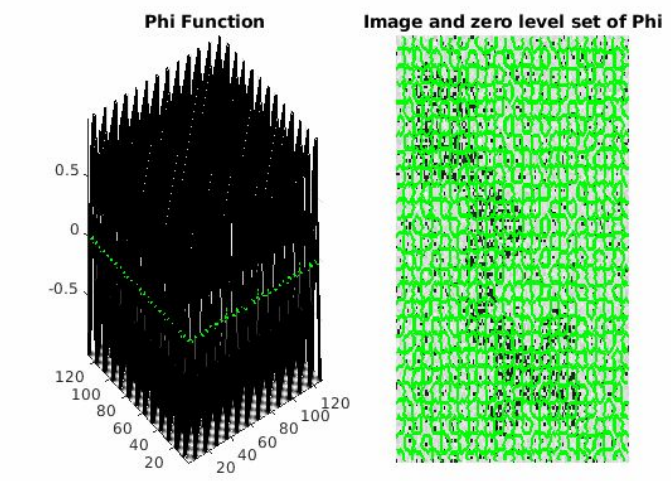
- The Chan-Vese approach yields good results when applied to images that have well defined homogeneous regions as shown below.



Results - Circles with Noise

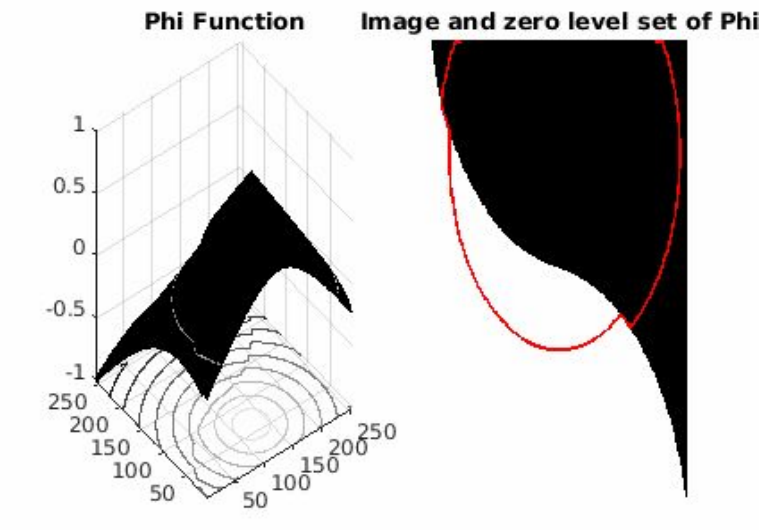
- The introduction of noise in the image produces worse results since it affects the distribution of pixels in the image, hence making the regions less homogeneous and the transition between c1 and c2 less clear.

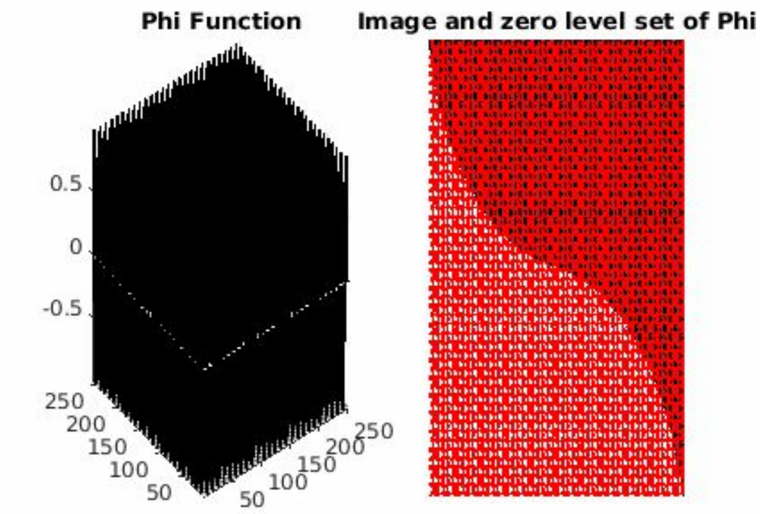




Results - Phantom 17

As stated in previous slides, images with well defined homogeneous binary regions yield good segmentation results. In this case we can observe how a checkerboard initialization results in a faster convergence.

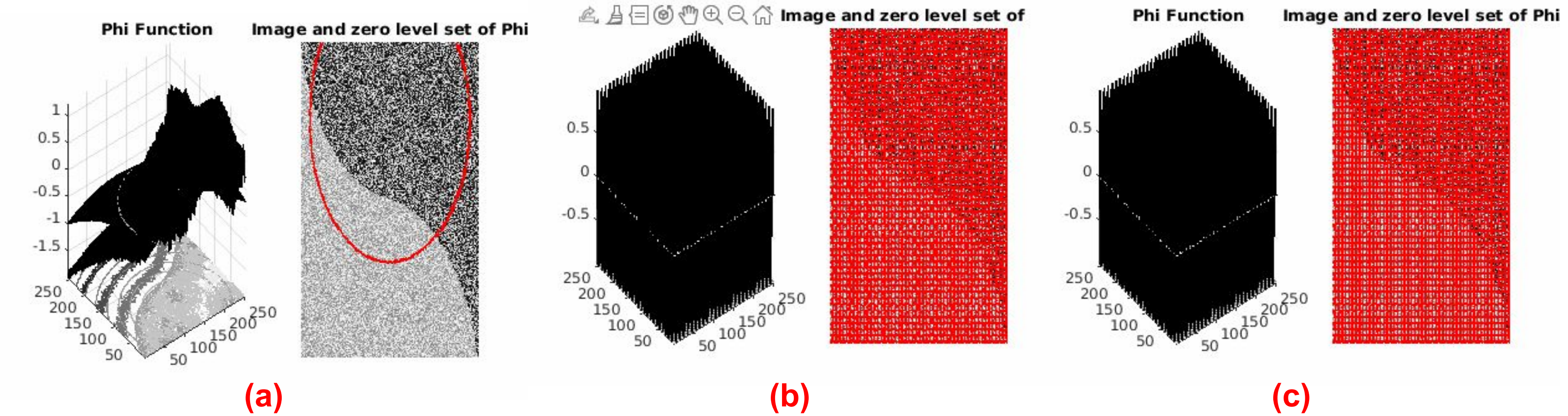




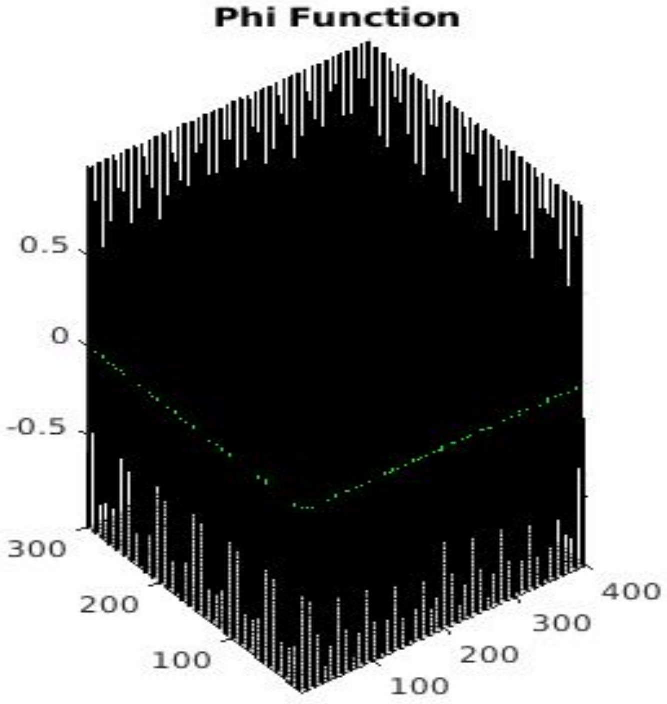
Results - Phantom 18

Different values of *mu* for a noisy images will produce different outcomes:

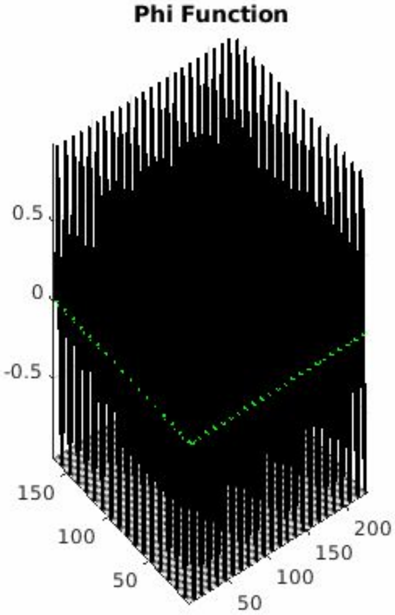
- In figure **b)** a phi with a checkerboard initialization and a small *mu* (which gives less importance to the length of the region contours) results in lots of undesired noise segmentation.
- In figure **c)** the same initialization with a bigger *mu* value will give more importance to the length of the contours which lowers the effect of noise in the final segmentation.
- Default initialization of phi (figure **a)**) with the same *mu* has shown to be less sensitive to noise.

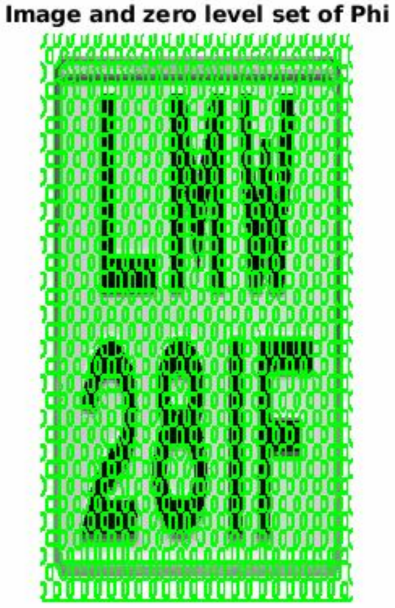


Results - Hola Carola



Results - Custom image





Conclusions: Advantages

1. The model can be successfully applied to images with two regions which have a distinct mean of pixel intensity.
2. The model utilizes local image information efficiently
3. It has a simplified functional which is easier to minimize
4. The segmentation can be performed when there are no edges separating the regions since the algorithm works on pixel value proportions.

Conclusions: Limitations

- The algorithm can only segment the images into two regions, since it works under the assumption that the edges (denoted by C) in the image can be represented by φ ($C = \{x \in \Omega: \varphi(x) = 0\}$).
- The Chan-Vesel approach is dependent on the initialisation of the level set function (φ) as the minimisation problem is non-convex.
- To ensure fast convergence, the algorithm requires that the initial curve is close to the boundaries, otherwise the convergence is too slow.
- For noisy images, too precise manual adaptation is required to reach convergence, which is not very useful in some applications.
- In the Chan-Vese model, pixel intensities are assumed to be statistically homogenous in each region. As it assumes implicitly a Gaussian intensity distribution for each region Ωi, if the intensities with inside C or outside C are not homogenous, the constants c1 and c2 will not be accurate. In other words, the model generally fails to segment images with intensity inhomogeneity, which is often the case in medical imaging (especially in MR images where an intensity bias can be observed).