Branch and Bound algorithms

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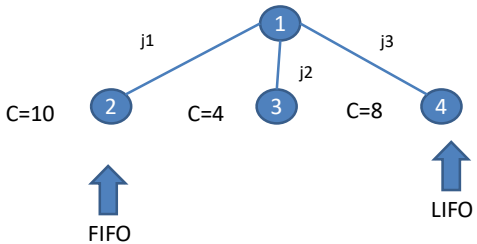
Branch and bound –general strategy

- Used for solving the **optimization problems**
- Mostly for **minimization problem** (but if we negate the function under consideration then maximization problems can also be solved)

$$\text{Find x such that } \min f(x)$$

- It works in a Breadth First manner (as opposed to backtracking)
- For each **branch** it uses **bound** (number) and a given branch is chosen based on the bound, for **further exploration** if multiple choices are there.

State space search tree generation in branch and bound- Least cost approach



Each node has cost C and next node will be selected based on the minimum cost

https://www.youtube.com/watch?v=tKvAniEbeqM

0/1 knapsack problem using branch and bound- Least cost approach

N=4
M=15

P	10	10	12	18
w	2	4	6	9

Find objects to be kept in the sack to maximize the profit. But branch and bound solves minimization problem. Thus, negate all the profits

N=4
M=15

P	-10	-10	-12	-18
w	2	4	6	9

It works by computing lower bound c and upper bound u on each node and takes the decision.
c-lower bound- fractions are allowed(for calculations only)
u- upper bound- fractions are not allowed
-decides a branch to pursue based on bounds

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0/1 knapsack problem using branch and bound- Least cost approach

N=4
M=15

P	-10	-10	-12	-18
w	2	4	6	9

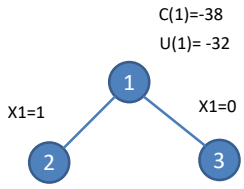
For node 1

-6	3/9
-12	6
-10	4
-10	2

c=-38

-12	6
-10	4
-10	2

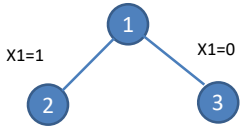
u= -32



3/9*(-18)=- 6

P	-10	-10	-12	-18
w	2	4	6	9

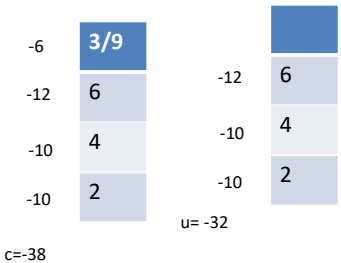
C(1)=-38
U(1)= -32



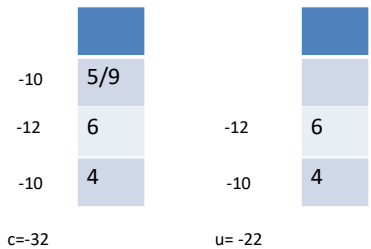
C(2)=-38
U(2)= -32

C(3)=- 32
U(3)= -22

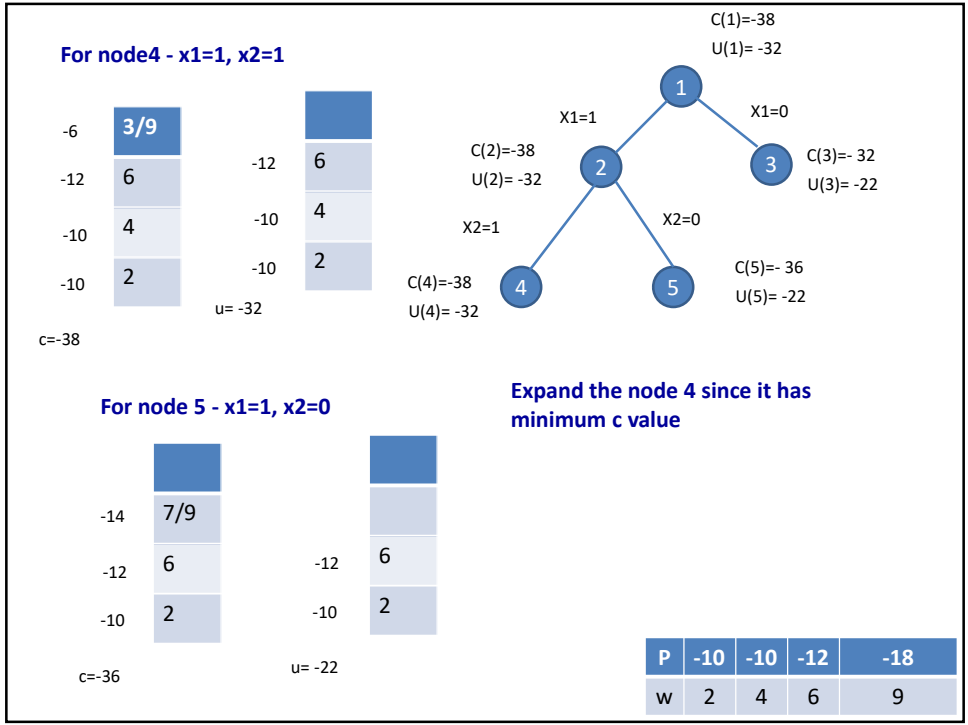
For node2 - x1=1



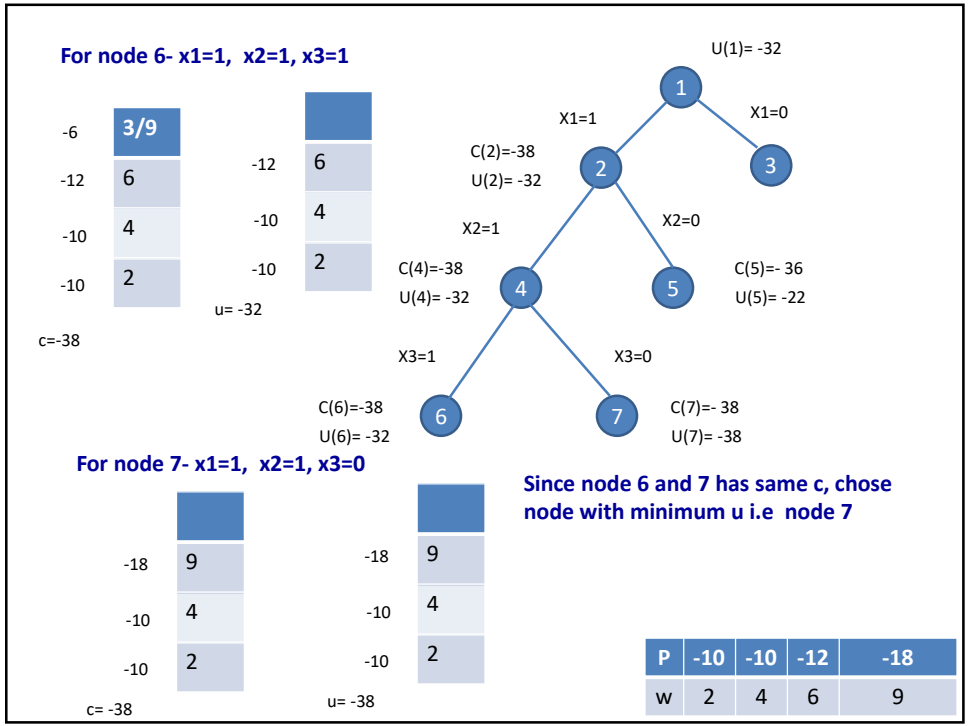
For node 3 - x1=0



Choose the next node for exploration as a node with minimum lower bound c, thus we will pick node 2.



P	-10	-10	-12	-18
w	2	4	6	9


For node 6- x1=1, x2=1, x3=1
-6
3/9
-12
6
-10
4
-10
2
c=-38
-12
6
-10
4
-10
2
u= -32
U(1)= -32
1
X1=1
X1=0
C(2)=-38
U(2)= -32
2
3
X2=1
X2=0
C(4)=-38
U(4)= -32
4
5
C(5)=- 36
U(5)= -22
X3=1
X3=0
C(6)=-38
U(6)= -32
6
7
C(7)=- 38
U(7)= -38
For node 7- x1=1, x2=1, x3=0
-18
9
-10
4
-10
2
c= -38
-18
9
-10
4
-10
2
u= -38
Since node 6 and 7 has same c, chose node with minimum u i.e node 7
P
-10
-10
-12
-18
w
2
4
6
9

Node 8 – x=1, x2=1, x3=0, x4=1

C(8)=- 38

U(8)= -38

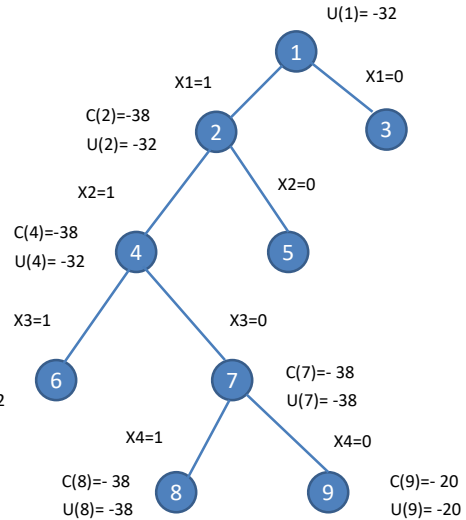
Node 9 – x=1, x2=1, x3=0, x4=0

C(9)=- 20

U(9)= -20

Since node 8 is having minimum c, choose 8

Knapsack instance (x1,x2,x3,x4)= (1,1,0,1)



Maximum profit obtained is = -(-38)=+38

https://www.youtube.com/watch?v=tKvAniEbeqM

Solve following 0/1 knapsack problem using branch and bound- Least cost approach

N=4-objects
M=15- Knapsack capacity

P	10	10	12	9
w	2	4	6	9

- Solve the same problem using **FIFO branch and bound** and **LIFO branch and bound**
  - **(hint updating global upper bound, if a node has lower bound greater than the global upper bound kill that node)**

https://www.youtube.com/watch?v=WXWt5xNrOf4