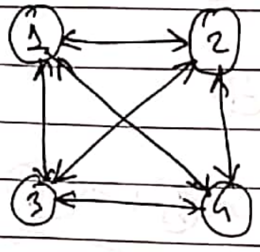
Travelling Salesperson problem:- Dynamic Programming..

salesperson has to visit each city at once & return to the same starting city, with minimum distance.

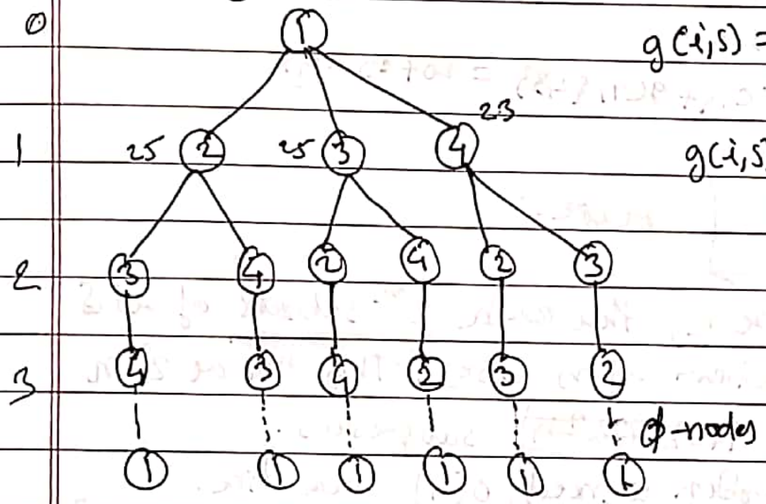


n=4 cities.

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

Cost adjacency matrix.

starting city = 1.



$$g(i,S) = \min_{j \in S} \{ c_{ij} + g(j, S-\{j\}) \}$$

$g(i,S)$ – is the shortest path starting from i & going through all the vertices in S & terminate at vertex 1.

3 $g(2,\phi)$ – Cost of getting from 2 to 1 $= c_{21} + 0 = 5$ $j=1$.

$g(3,\phi) - c_{31} = 6$ $|S|=0$

$g(4,\phi) - c_{41} = 8$

$g(2,\{3\}) = c_{23} + g(3,\phi) = 9+6 = 15$

$g(2,\{4\}) = c_{24} + g(4,\phi) = 10+8 = 18$ $|S|=1$.

2 $g(3,\{2\}) = c_{32} + g(2,\phi) = 13+5 = 18$

$g(3,\{4\}) = c_{34} + g(4,\phi) = 12+8 = 20$

$g(4,\{3\}) = c_{43} + g(3,\phi) = 9+6 = 15$

$g(4,\{2\}) = c_{42} + g(2,\phi) = 8+5 = 13$

j

1 $g(2,\{3,4\}) = \begin{cases} c_{23} + g(3,\{4\}) = 9+20 = 29 & j=3 \\ c_{24} + g(4,\{3\}) & j=4. \\ = 10+15 = 25 \text{ min} & \end{cases}$ $|S|=2$

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$$g(3,\{2,4\}) = \begin{cases} c_{32} + g(2,\{4\}) = 13 + 18 = 31 & \text{min} \quad J=2 \\ c_{34} + g(4,\{2\}) = 12 + 13 = 25 & \text{min} \end{cases}$$

$= 25$

$$g(4,\{2,3\}) = \begin{cases} c_{42} + g(2,\{3\}) = 8 + 15 = 23 & \text{min} \\ c_{43} + g(3,\{2\}) = 9 + 18 = 27 \end{cases}$$

$$g(1,\{2,3,4\}) = \begin{cases} J=2 \\ c_{12} + g(2,\{3,4\}) = 10 + 25 = 35 & \text{min} \\ J=3 \\ c_{13} + g(3,\{2,4\}) = 15 + 25 = 40 & |S| = 3 \\ J=4 \\ c_{14} + g(4,\{2,3\}) = 20 + 23 = 43 \end{cases}$$

$\boxed{1 - 2 - 4 - 3 - 1}$ Route.

For a problem of size n, there can be 2^n subsets of set S & each contains maximum of n cities. Thus there are $2^n \cdot n$ $\therefore$ ~~$T(n) = O(2^n \cdot n)$~~ subproblems.

To solve each subproblem we need, $O(n)$ linear time.

$\therefore T(n) = 2^n \cdot n \cdot n = 2^n \cdot n^2 \Rightarrow O(2^n \cdot n^2)$