Assignment _Eigen Values and Eigen Vectors, Partial Differentiation, Differential Equations

Note: 1) There are total 5 questions.

2) Attempt ONE sub question from Q.1 to Q.5 as per the number shown against your roll number in the attached list.

Q.1	**Attempt the following**
1	Consider, $A=\begin{bmatrix}8&0&3\\2&2&1\\2&0&3\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? (v) If yes, find the spectral and modal matrices.
2	Consider, $A=\begin{bmatrix}-14&1&0\\0&2&0\\1&0&2\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? Why? (v) If yes, find the spectral and modal matrices.
3	Consider, $A=\begin{bmatrix}3&1&0\\-1&4&3\\1&-2&-1\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? Why? (v) If yes, find the spectral and modal matrices.
4	Consider, $A=\begin{bmatrix}4&2&3\\-1&1&-3\\2&4&9\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? Why? (v) If yes, find the spectral and modal matrices.
5	Consider, $A=\begin{bmatrix}1&0&-1\\1&-3&0\\4&-13&1\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? Why? (v) If yes, find the spectral and modal matrices.
6	Consider, $A=\begin{bmatrix}3&2&4\\2&0&2\\4&2&3\end{bmatrix}$. (i) Write the Characteristic Equation of A (ii) Find eigen values and eigen vectors of A. (iii) State Algebraic and Geometric multiplicities of each eigen value (iii) Find eigen values of $A^2+5A\text{-}3I$. (iv) Is A diagonalizable? Why? (v) If yes, find the spectral and modal matrices.
Q.2	**Attempt the following**
1	If $z(x+y)=x^2+y^2$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$
2	If $u(x,t)=Ae^{-gx}\sin(nt-gx)$ satisfies the one dimensional heat equation

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	$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$ where $A,\ g,\ n$ and k are constants, show that $n = 2k^2 g^2$.
3	If $u = x\log(x+r) - r$ where $r^2 = x^2 + y^2$, Is u satisfy Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$?
4	At a given instant the sides of a rectangle are 4 ft. and 3 ft. respectively and they are increasing at the rate of 1.5 ft/sec and 0.5 ft/sec respectively, find the rate at which area is increasing at that instant.
5	Evaluate $f_x,\ f_y$ and f_z at the given point $f(x,\ y,\ z) = x^3yz^2\ \ at\ (1,1,\ 1)$
6	If $u = \tan^{-1}\left(\frac{y}{x}\right)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$, find $\frac{du}{dt}$.
7	If $x = \sqrt{v\,w},\ y = \sqrt{u\,w},\ z = \sqrt{u\,v}$, prove that $x\frac{\partial \Phi}{\partial x} + y\frac{\partial \Phi}{\partial y} + z\frac{\partial \Phi}{\partial z} = u\frac{\partial \Phi}{\partial u} + v\frac{\partial \Phi}{\partial v} + w\frac{\partial \Phi}{\partial w}$, where Φ is a function of x, y, z.
8	Evaluate $f_x,\ f_y$ and f_z at the given point $f(x,\ y,\ z) = \log\left(\frac{xy}{x^2+y^2+z^2}\right)$ at $(1,1,\ 1)$
9	If x increases at the rate of 2 cm/sec at the instant when x = 3 cm and y = 1 cm, at what rate must be y changing in order that the function $2xy^2 - 3x^2y$ shall be neither increasing nor decreasing.
10	If $z = f(x,\ y)$, where $x = e^u\cos v,\ y = e^v\sin v$, prove that (a) $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$ (b) $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2\right]$
Q.3	**Attempt the following**
1	Discuss the stationary values of $u = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.
2	Find the stationary points of the function $f(x,\ y) = y^2 + 4xy + 3x^2 + x^3$ and hence identify these points as maxima/ minima/ saddle points.
3	Find the stationary points of the function $f(x,y) = x^3y^2(1-x-y)$ and hence identify these points as maxima/ minima/ saddle points.
4	Find the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ and hence identify these points as maxima/ minima/ saddle points.
5	Find the stationary points of the function $f(x,y) = x^3 + y^3 - 3xy$ and hence identify these points as maxima/ minima/ saddle points.
Q.4	**Solve by the method of variation of parameter**
1	$(D^2 + D)y = \frac{1}{1+e^x}$
2	$\left(\mathrm{D}^2 - 4\mathrm{D} + 4\right)\mathrm{y} = \mathrm{e}^{2\mathrm{x}}\sec^2\mathrm{x}$
3	$\left(\mathrm{D}^2 - 1\right)\mathrm{y} = \left(1+e^{-x}\right)^{-2}$
4	$y'' - 2y' + 2y = e^x\tan x.$
5	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$

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6	$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$
7	$\frac{d^2y}{dx^2}+a^2y=\sec ax$
8	$\left(D^2-4D+4\right)y=e^{2x}\mathrm{cosec}^2x$
9	$\frac{d^2y}{dx^2}+y=\tan x$
10	$\frac{d^2y}{dx^2}+16y=\cot 4x$
Q.5	**Solve by method of undetermined coefficients**
1	$(D^2-2D)y=e^x+5e^{-2x}.$
2	$(D^2+10D+25)y=100\left(\frac{e^{5x}-e^{-5x}}{2}\right)$
3	$(D^2+D-6)y=6x^3-3x^2+12x$
4	$\frac{d^2y}{dx^2}+7\frac{dy}{dx}+13y=5e^{-3x}+7e^x$
5	$(D^2+4D+5)y=25x^2$
6	$\frac{d^2y}{dx^2}+y=4\sin x$
7	$(D^2-2D+3)y=\cos x.$
8	$\frac{d^2y}{dx^2}+y=2\cos x$
9	$(D^2-2D+3)y=2x^3+5.$
10	$y''-y=\frac{e^x+e^{-x}}{2}$