Mathematics and Statistics ES1043
Vector Spaces : Tutorial Practice Exercise

Q. 1	Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ **a.** Show that the vectors are linearly independent. **b.** Find the unique scalars c_1, c_2, c_3 such that the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ can be written as $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$
Q. 2	Determine whether the given sets of vectors are linearly dependent or independent? $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 10 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \\ 9 \\ 9 \end{bmatrix}.$
Q. 3	Show that $H = \left\{ \begin{bmatrix} a-b+c \\ 2a-b \\ a-b-5c \\ 2a-c \end{bmatrix} : a,b,c \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^4$.
Q. 4	Let U and W are subspaces of a vector space V. Sum of U and W is defined as $U+W=\{u+w \in V:\ u \in U \ and\ w \in W\}$. Show that $U+W$ is a subspace of V.
Q. 5	S={(0,0,1,1),(0,0,0,1),(0,1,1,1),(1,1,1,1)} check whether that S is linearly dependent or independent?
Q. 6	Let vectors $\mathbf{u}=(\lambda, 1, 0)$, $\mathbf{v}=(1, \lambda, 1)$ and $\mathbf{w}=(0, 1, \lambda)$ € $\mathbf{R}^3$ find all real values of λ for which u,v,w are linearly dependent ?
	Determine the values of a such that the matrices $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & -4 \\ a & -2 \end{bmatrix}$ are linearly independent.
Q. 7	For which values of t is each set linearly independent? 1) S ={(t,0,0),(0,1,0), (0,0,1)} 2) S ={(t, t, t) , (t, 1, 0) , (t, 0, 1)}
Q. 8	Determine whether columns of matrix A are linearly independent A= $\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$
Q. 9	Consider the polynomials $P_1=1+x^2$ and $P_2=1-x^2$. Are $\{P_1, P_2\}$ they linearly independent in P_3? Why or why not?
Q.10	Find the matrix A such that the given set is $Col(A)$ i) $\left\{ \begin{bmatrix} 2s+t \\ r-s+2t \\ 3r+s \\ 2r-s-t \end{bmatrix} : r, s, t \text{ real} \right\}$ ii) $\left\{ \begin{bmatrix} b-c \\ 2b+3d \\ b+3c-3d \\ c+d \end{bmatrix} : b, c, d \text{ real} \right\}$

Q. 11	Find a basis for the space spanned by $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix} \right\}$.
Q. 12	a) If the null space of 5×6 matrix A is 4-dimensional, what is the dimension of column space of A? b) A is 6×8, what is the smallest possible dimension of $Nul(\mathrm{A})$? c) If a 6×3 matrix A has rank 3, What is the dimension of $Nul(\mathrm{A})$, dim $Row(A)$, and $rank(A^T)$?
Q. 13	$S = \left\{ \begin{bmatrix} c \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ c \\ -1 \end{bmatrix} \right\}$ where $c \in \mathbb{R}$. (a) Find the value/s of c such that S is a linearly dependent set. (b) Express the vector $\begin{bmatrix} a_1 \\ a_2 \\ 0 \end{bmatrix}$ as a linear combination of the vectors in S where a_1 and a_2 are scalars.
Q. 14	Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_4 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ **a.** Show that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent. **b.** Show that $T = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. **c.** Show that $\mathbf{v}_4$ can be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
Q. 15	Find bases for the four fundamental subspaces of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.
Q.16	Find k such that i) Null space of A is subsapce of R^k and ii) column space of A is

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	subsapce of R^k, where $\mathrm{A}=\begin{bmatrix} 1 & 2 & -1 & 5 \\ 3 & 1 & -1 & 1 \\ 7 & 6 & 5 & 2 \\ 1 & 2 & -3 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$
Q.17	Let $\mathrm{A}=\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$, $u=\begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$, $v=\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$. Determine if u is in $Nul(\mathrm{A})$. Could u be in $col(\mathrm{A})$?Determine if v is in $col(\mathrm{A})$. Could v be in $Nul(\mathrm{A})$?
Q.18	The matrices below are row equivalent. $A=\begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **1.** Find rank A and $\dim \mathrm{Nul}\, A$. **2.** Find bases for $\mathrm{Col}\, A$ and $\mathrm{Row}\, A$. **3.** What is the next step to perform to find a basis for $\mathrm{Nul}\, A$? **4.** How many pivot columns are in a row echelon form of A^T?
Q. 19	The two matrices A and B are row-equivalent. $A=\begin{bmatrix} 2 & -4 & 0 & 1 & 7 & 11 \\ 1 & -2 & -1 & 1 & 9 & 12 \\ -1 & 2 & 1 & 3 & -5 & 16 \\ 4 & -8 & 1 & -1 & 6 & -2 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ i) Find rank of A. ii) Find a basis for a row space of A. iii) Find a basis for a column space of A. iv) Find a basis for a Null space of A. v) Is the last column of A is in the span of the first three columns?
Q. 20	Find a basis and dimension of the solution space of the following. $x_1-3x_2+x_3+x_4=0$; $2x_1+x_2-x_3+2x_4=0$; $x_1+4x_2-2x_3+x_4=0$; $5x_1-8x_2+2x_3+5x_4=0$.
Q. 21	Find a basis and dimension of the solution space of the following. $x_1+4x_2+2x_4-x_5=0, 3x_1+12x_2+x_3+5x_4+5x_5=0, 2x_1+8x_2+x_3+3x_4+2x_5=0$ $5x_1+20x_2+2x_3+8x_4+8x_5=0$
Q. 22	Determine if the following statements true or falls, and justify your answers. a) A linearly independent set in a subspace H is a basis for H. b) The columns of a nonsingular matrix forms a basis for Col(A).

	c) The null space of an $\mathrm{m}\times\mathrm{n}$ matrix A is a subsapce of R^m. $d)$ $col(\mathrm{A})$ is a set of vectors that can be written as AX for some X. $e)$ A plane in R^3 is two dimensional subspace of R^3. $f)$ The dimension of vector space P_3 (set of all polynomials at most of degree 3, is 3. $g)$ If the 4×5 matrix A has 4 pivote columns, then $col(\mathrm{A})=\mathrm{R}^4$. $h)$ If the 6×3 matrix A rank 3, then dimension of $Null(\mathrm{A})=0$.
Q.23	State True or False with justification i) A set containing zero vector is linearly independent. ii) A basis is a spanning set that is as large as possible iii) A basis is a linearly independent set that is as large as possible. iv) The additive inverse of a vectors is not unique v) A subspace is also a vector space vi) $\mathrm{W}=\left\{\begin{bmatrix} x \\ y \end{bmatrix} : \ xy\geq 0\right\}$ is a subspace of R^2 vii) All polynomials of the form $p(t)=a+t^2$, where $a\in\mathrm{R}$ is a subspace of P_2. viii) If the columns of an $m\times n$ matrix A span R^m, then the equation $\mathrm{AX}=b$ is consistent for each $b\in\mathrm{R}^m$. ix) Suppose a 4×7 matrix has four pivot columns. Then $\dim(Null\ \mathrm{A})=3$. x) A single vector by itself is linearly dependent.