ES1043 MAS Practice/Tutorial Questions

1	Find the rank and nullity of the following. Find the basis for the row space and null space. Also write row space. What is the dimension of column space. Find the basis for column space. $$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$ Answer: $rank = 3$, $nullity = 2$, A basis for the nullspace is $$\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix} \right\}.$$
2	The matrices below are row equivalent. $$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ **1.** Find rank A and dim Nul A. **2.** Find bases for Col A and Row A. **3.** What is the next step to perform to find a basis for Nul A? **4.** How many pivot columns are in a row echelon form of A^T?
3	a) **If the null space of** 5×6 **matrix** A **is 4-dimensional, what is the dimension of column space of** A? b) A **is 6** $\times$ **8, what is the smallest possible dimension of** $Nul(A)$? c) **If a 6** $\times$ 3 $matrix A has rank 3$**, What is the dimension of** $Nul(A)$, $Dim\ Row(A)$, **and** $rank(A^T)$?
4	Find the kernel and range of $T(v) = Av$, where $A = \begin{bmatrix} -1 & 3 & 2 & 1 & 4 \\ 2 & 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix}$. Also state the nullity and rank of T. Also find the bases for kernel and rank of T.
5	Define $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $$T(x_1, x_2, x_3) = (2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + x_3, 2x_1 + 4x_2 + x_3)$$ i) Show that T is a linear map ii) Write the matrix representation of T with respect to the standard basis. iii) If so, find the kernel and range of T. iv) Find the basis and dimension of kernel and range of T.

	v) Is T Invertible? If so, find it's inverse.
6	A) Is there exists a 2×2 singular matrix that maps $(1,2)^T$ into $(2,-3)^T$? If so, find the linear map represented by the matrix.
7	Find a 2×2 that maps $(1,2)^T$ and $(2,-3)^T$ into $(-2,5)^T$ and $(3,2)^T$ respectively.
8	B) Is there exists a 2×2 singular matrix that maps $(1,2)^T$ into $(2,4)^T$? If so, find the linear map represented by the matrix.
9	Let $M_{2\times2}$ be the vector space of all 2×2 matrices. Define $T: M_{2\times2}\rightarrow M_{2\times2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. a) Show that T is a linear transformation. b) Let $B \in M_{2\times2}$ be such that $B^T = B$. Find $A \in M_{2\times2}$ such that $T(A) = B$.
10	Let $B = \{v_1, v_2, v_3\}$ be the basis of $\mathbb{R}^3$, where $v_1 = (-2,1,0)$, $v_2 = (1,2,1)$ and $v_3 = (1,1,1)$. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(v_1) = (2,1,-1)$, $T(v_2) = (-1,1,1)$ and $T(v_3) = (1,0,0)$. Find $T(2,4,-1)$.
11	A) Find the nullity of T a) $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3, rank(T) = 2$ b) $T: \mathbb{R}^5 \rightarrow \mathbb{R}^2, rank(T) = 2$ c) $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4, rank(T) = 0$ d) $T: P_3 \rightarrow P_1, rank(T) = 2$
	B) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation. Use the given information to find the nullity of T a) $rank(T) = 2$ b) $rank(T) = 1$ c) $rank(T) = 0$ d) $rank(T) = 3$
12	Given the transformation $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Find the coordinates of $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ corresponding to $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ in Y. Also describe & find basis, dimension of R(T), Ker(T) Is the above transformations one-one, onto? Justify your answer.

13	Let $T, S: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + y \\ -x + 3y \end{pmatrix}$ and $S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x + y \end{pmatrix}$. i) Find $-3S, 2T + S, T \circ S, S \circ T$ ii) Find rank and nullity of each of the above transformations. iii) Which of the above transformations are one-one, onto? Justify your answer.
14	A) Find a transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that first shear in x_1 direction by a factor of 3 and then reflects about $y = x$. B) Find a transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that first reflects about $y = x$ and then shears by a factor of 3 in x_1 direction. C) Find the standard matrix for T:$\mathbb{R}^3 \to \mathbb{R}^3$, that first reflects about YZ −plane, then rotates the resulting vector in counter clockwise direction through an angle $\frac{\pi}{3}$, about Y - axis and then finally resultant vector is projected on XY −plane.
15	Express the following matrix as a product of elementary matrices. Describe the effect of multiplication by the given matrix in terms of compression, expression, reflection, and shear $A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$.
16	Let T be a linear transformation such that $T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$. Find $T\left(\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}\right)$.

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17	Find the matrix A such that the given set is $Col(A)$ i) $\left\{ \begin{bmatrix} 2s+t \\ r-s+2t \\ 3r+s \\ 2r-s-t \end{bmatrix} : r,s,t \text{ real} \right\}$ ii) $\left\{ \begin{bmatrix} b-c \\ 2b+3d \\ b+3c-3d \\ c+d \end{bmatrix} : b,c,d \text{ real} \right\}$ Answer : i) $\begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \\ 2 & -1 & -1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & 3 & -3 \\ 0 & 1 & 1 \end{bmatrix}$
18	$2x_1 + x_2 - 6x_3 + 2x_4 = 0$ $x_1 + 2x_2 - 3x_3 + 4x_4 = 0$ $x_1 + x_2 - 3x_3 + 2x_4 = 0$ Consider above system find null space and column space of coefficient matrix A
19	State True or False with justification i) A set containing zero vector is linearly independent. ii) A basis is a spanning set that is as large as possible iii) A basis is a linearly independent set that is as large as possible. iv) The additive inverse of a vectors is not unique v) A subspace is also a vector space vi) $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$ is a subspace of $\mathbb{R}^2$ vii) All polynomials of the form $p(t) = a + t^2$,where$a \in R$ is a subspace of P_2. viii) If the columns of an $m \times n$ matrix AspanR^m, then the equation $AX = b$ is consistent for each $b \in R^m$. ix) Suppose a 4×7matrix has four pivot columns. Then $dim(NullA) = 3$. A single vector by itself is linearly dependent.
20	Determine if the following statements true or falls, and justify your answers. $a)$ A linearly independent set in a subspace H is a basis for H. $b)$ The columns of a nonsingular matrix forms a basis for Col(A). $c)$ The null space of an m × n matrix A is a subsapce of $\mathbb{R}^m$. $d)col(A)$is a set of vectors that can be written as $AX$for some$X$. $e)A$ plain in $\mathbb{R}^3$ is two dimensional subspace of $\mathbb{R}^3$. $f)$The dimension of vector space P_3 is 3 $g)$ If the 4×5matrix A has4 pivot columns, then $col(A) = \mathbb{R}^4$. $h)$ If the 6×3matrix A rank 3, then $dimension\ of\ Null(A) = 0$.