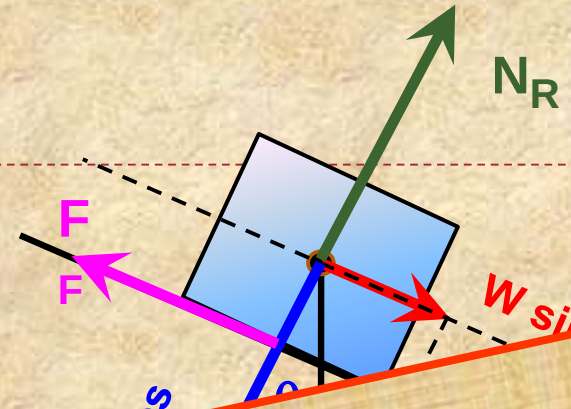
Vishwakarma Institute of Technology

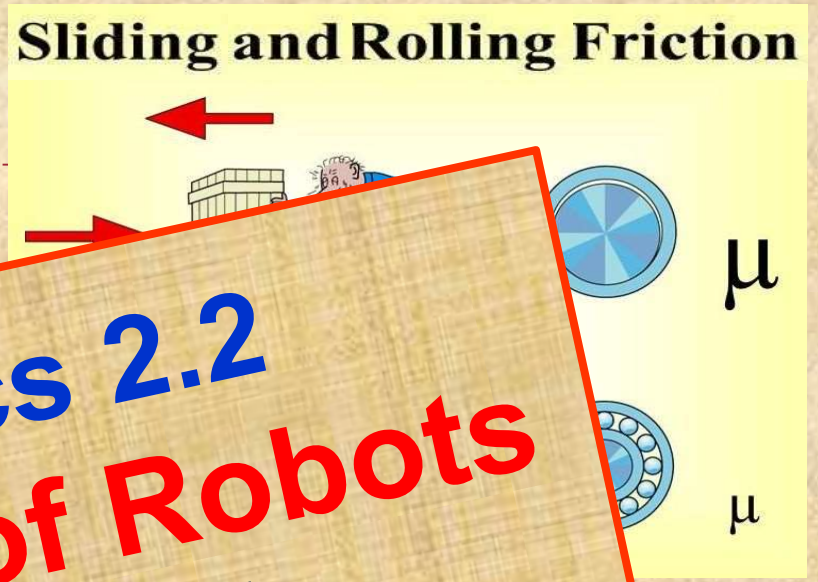
FY - Department of Engineering, Sciences and Humanities

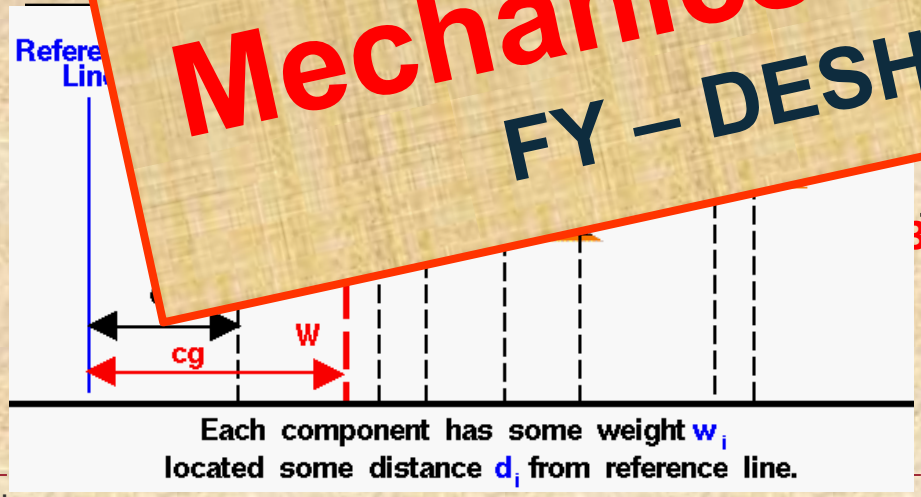
Mechanics 2.2

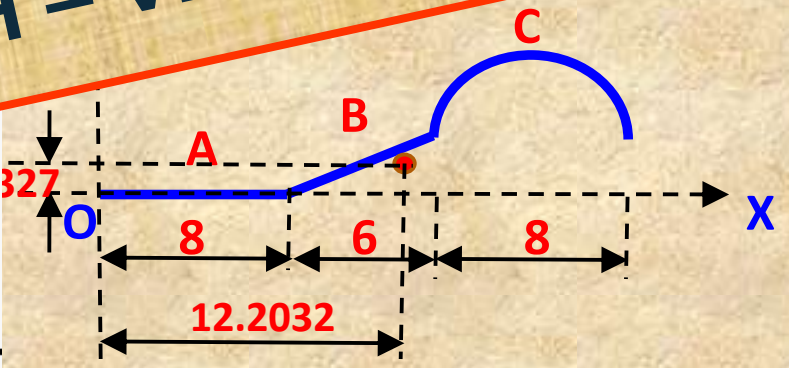
Mechanics of Robots

FY – DESH – VIT









Friction -

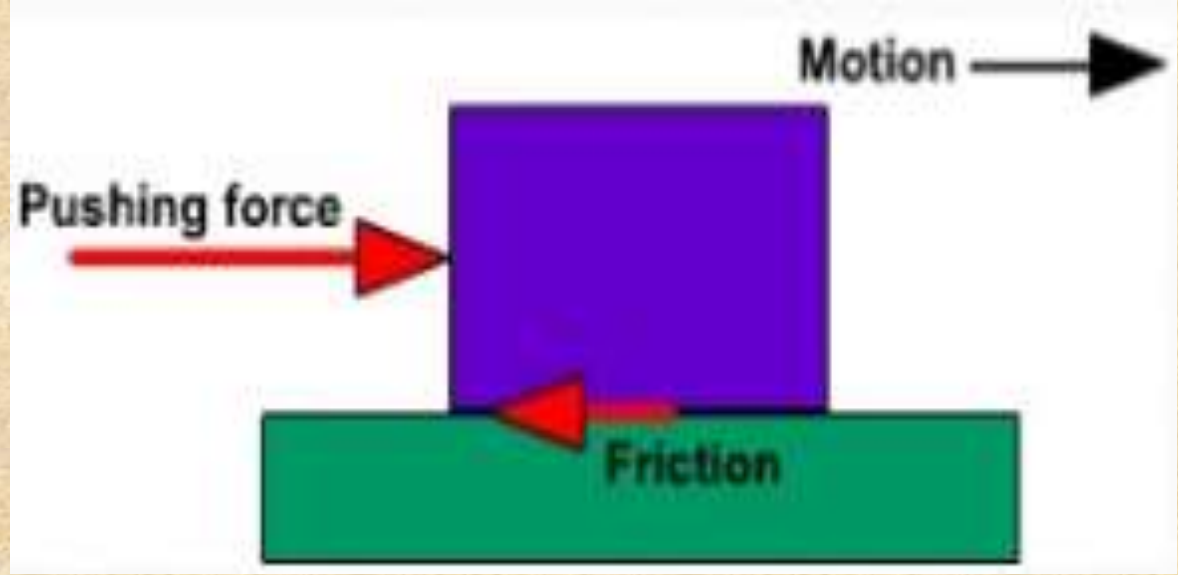
Can you identify the Physical Phenomenon ?

Friction -

Can you identify the Physical Phenomenon ?

Important Concepts about Friction -

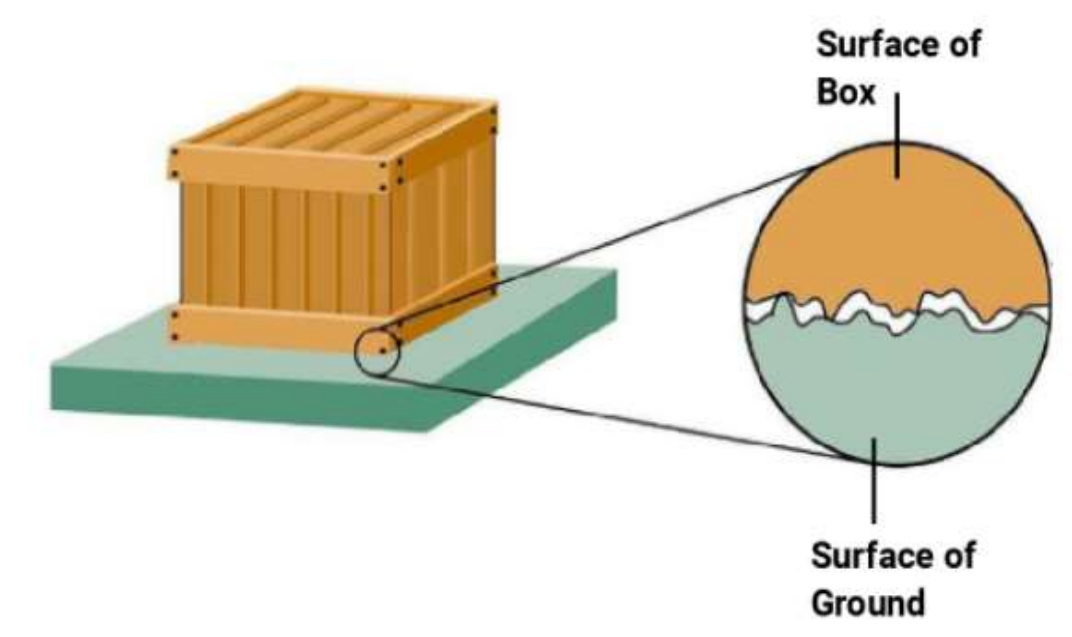
- **Friction always opposes the motion.**
- **This opposition is called as Friction Force.**
- **Friction is a property of the two surfaces in contact.**
- **Friction force is independent of the area of contact of the two surfaces.**



What causes Friction ? -

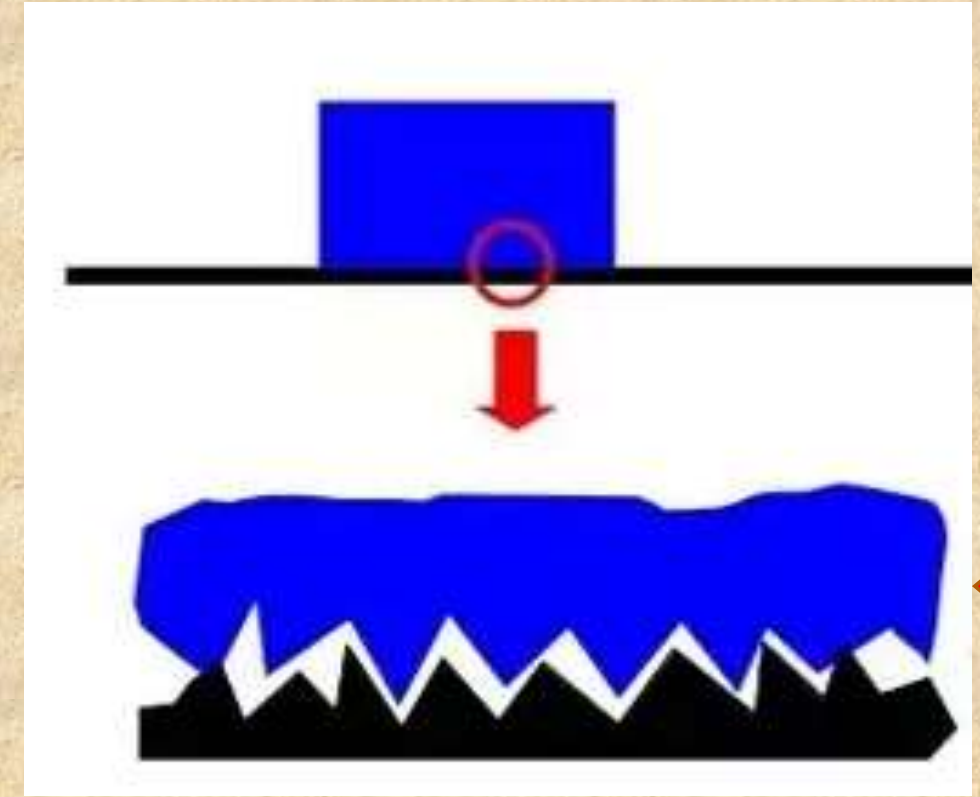
1) Intermolecular force of attraction.
   i.e. Adhesive property between the surfaces.

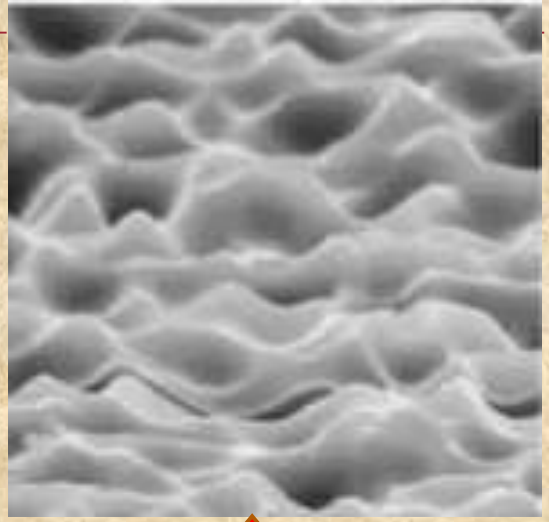
2) Irregularities between the two surfaces.

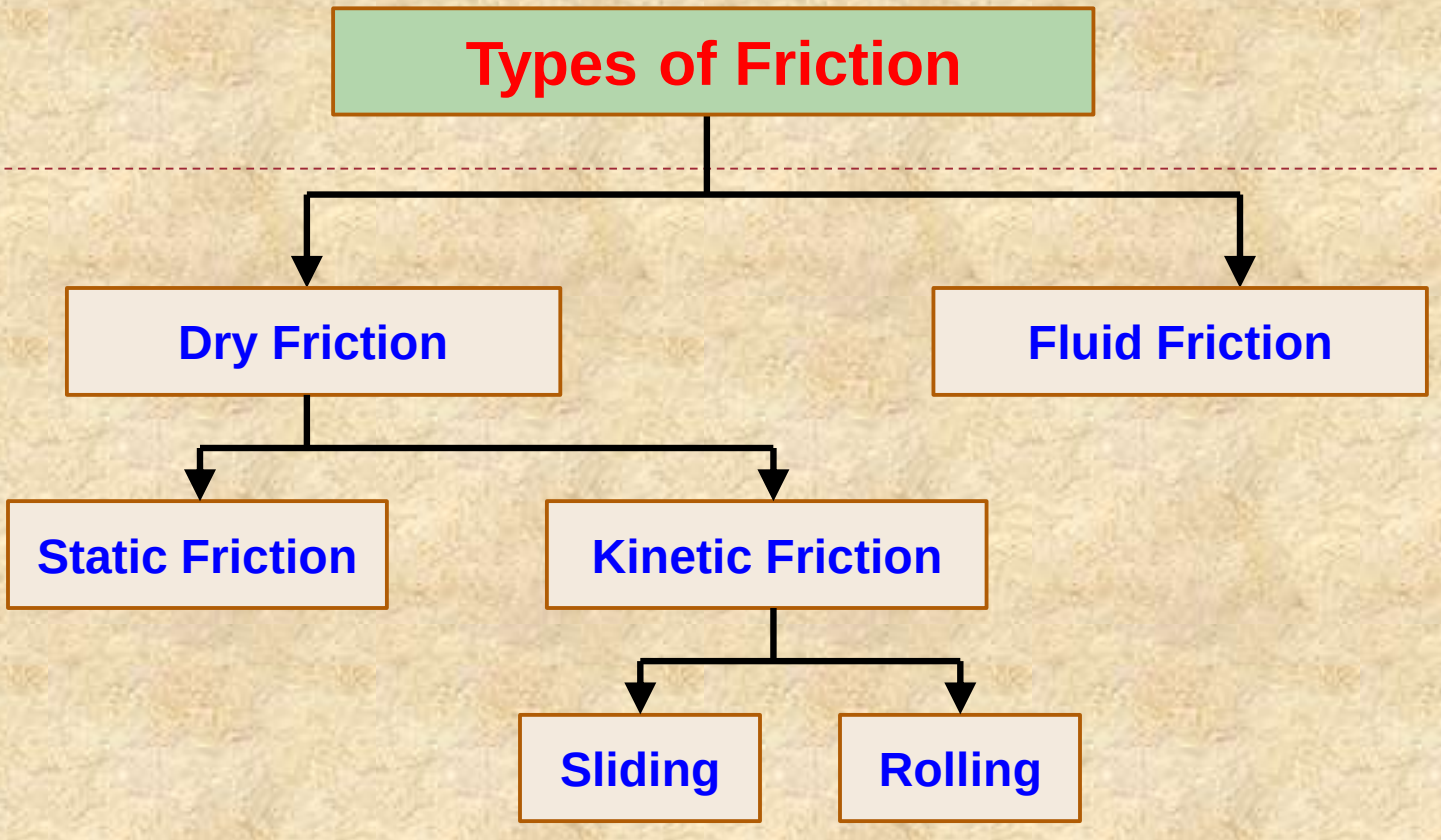


What causes Friction ?

Microscopically magnified surface view






Types of Friction
Dry Friction
Fluid Friction
Static Friction
Kinetic Friction
Sliding
Rolling

Important Terminology about Friction at the inclined plane -

Impending motion

Limiting Equilibrium

Angle of Repose

Limiting Friction (F_L)

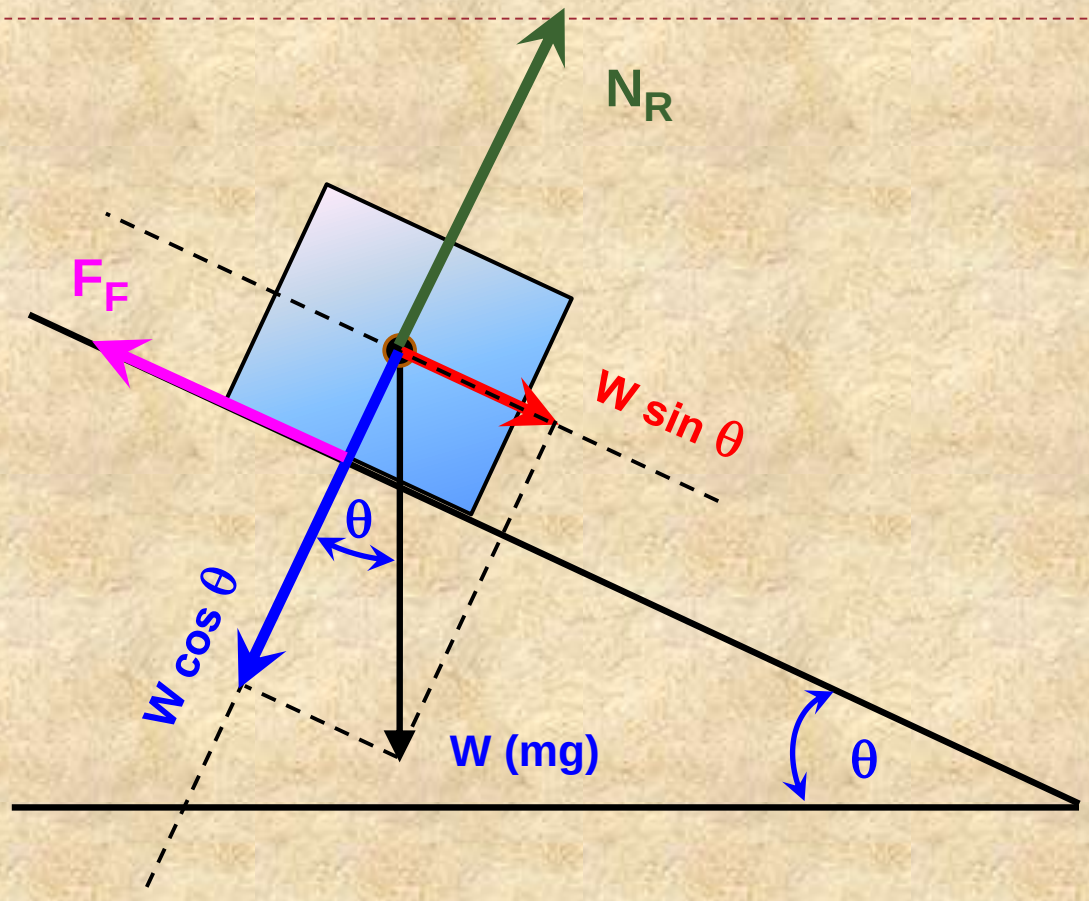
Friction Force

Normal Reaction (N_R)

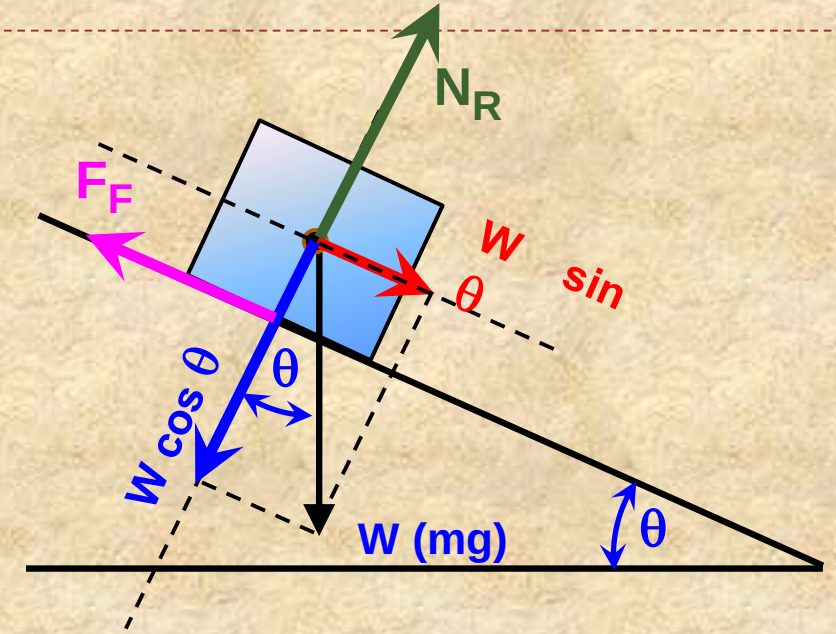
Coefficient of Friction (μ)

Let us study the concepts one by one .....

Various Forces Acting on a System -



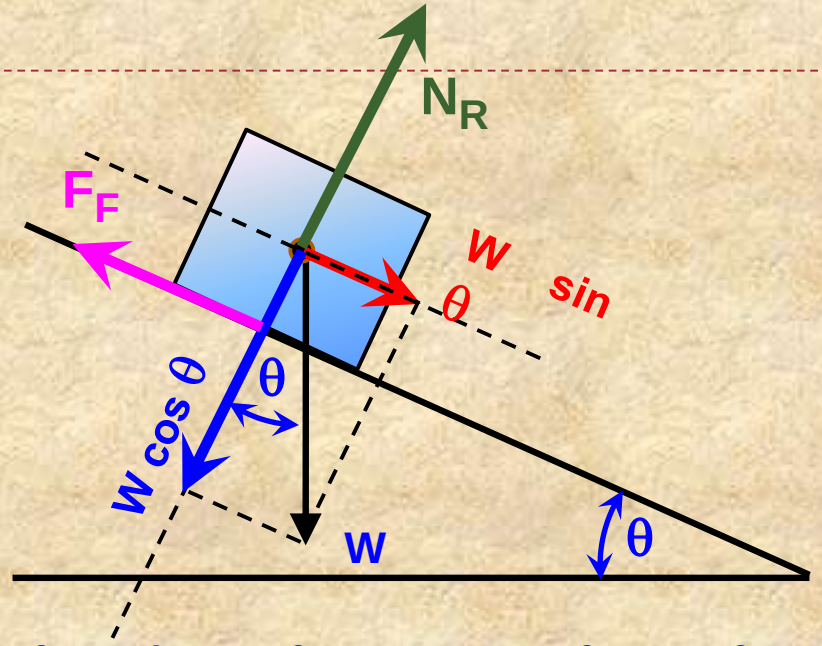
Various Forces Acting on a System – Self Weight W (mg)



The self weight W (mg) of the body always acts downwards, which can be resolved into sin and cos components as shown….

- ✓ The W sin θ tries to pull the body down the slope.
- ✓ The W cos θ tries to balance the normal reaction N_R.

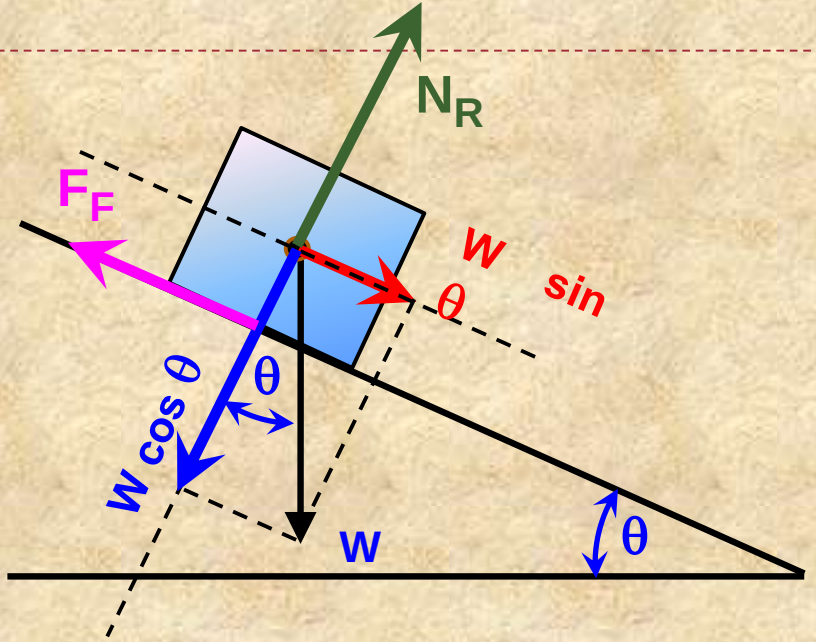
Various Forces Acting on a System – Normal Reaction N_R



The surface is a rigid body and thus, the body can not penetrate into the surface.

- Therefore the surface exerts a force of reaction N_R on the body which is always normal to the surface.
- It is the only force that the surface exerts on the object in the absence of frictional forces.

Various Forces Acting on a System – Friction Force F_F



The friction force F_F is acting opposite to the downward pull of W sin θ.

As long as W sin θ < F_F , the body will remain at rest.

If θ > angle of repose, then W sin θ > F_F

Thus, the body will start coming down because of its own weight.

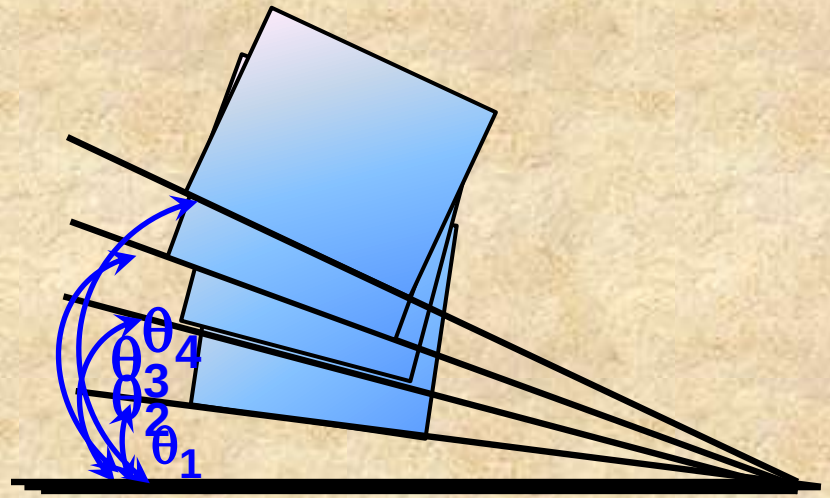
Case 1) $F_F >>> W \sin\theta_1$ $\therefore$ No Motion

Case 2) $F_F >> W \sin\theta_2$ $\therefore$ No Motion

Case 3) $F_F > W \sin\theta_3$ $\therefore$ No Motion

Case 4) $F_F < W \sin\theta_4$ $\therefore$ Motion just starts when

θ_4 = Angle of Repose



Angle of Repose is the minimum angle of inclination of plane at which the block impends its motion **under its own weight**

Important Terminology about Friction at the inclined plane -

1) **Impending motion** = body is on the verge of motion.

2) **Limiting Equilibrium** = When body is in Impending motion.

3) **Angle of Repose** = Angle at which the motion just starts because its own weight.

4) **Limiting Friction (F_L) =** It is the maximum friction force developed at the contacts before the motion just starts.

5) **Friction Force** will be maximum when body is on the verge of Impending Motion.

6) The force required to maintain motion with uniform velocity **is less than** the force required to set a body into motion (Limiting Static Friction Force = F_L)

Important Terminology about Friction at the inclined plane -

7) Normal Reaction (N_R) – It is the force that the surface exerts on the object in the absence of frictional forces. The Limiting Frictional Force (F_L) is proportional to the Normal Reaction (N_R)

$$F_L \propto N_R$$

8) Coefficient of Friction (μ) - It is the ratio of the Limiting Frictional force (F_L) resisting the motion of two surfaces in contact to the Normal reaction (N_R) pressing the two surfaces.

$$\text{Coeff. of Friction } \mu = \frac{\text{Limiting Frictional Force}}{\text{Normal Reaction}} = \frac{F_L}{N_R}$$

Statics and Kinetic Friction -

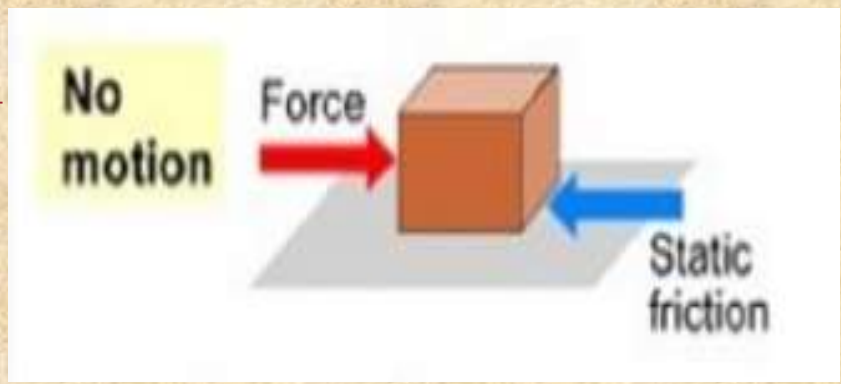
Static Friction – It is the friction experienced by the surface of the body when the body is at rest or in Equilibrium.

The force of Static Friction keeps a stationary object at Rest or in Equilibrium.

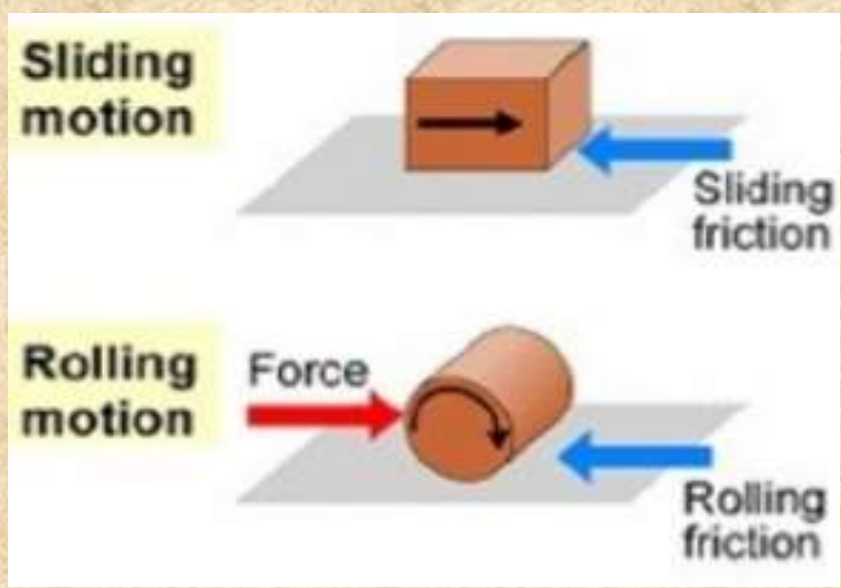
Kinetic Friction – It is the friction experienced by the surface of the body when the body is in Motion.

- Sliding over the surface is Sliding Friction.
- Rolling over the surface is Rolling Friction.

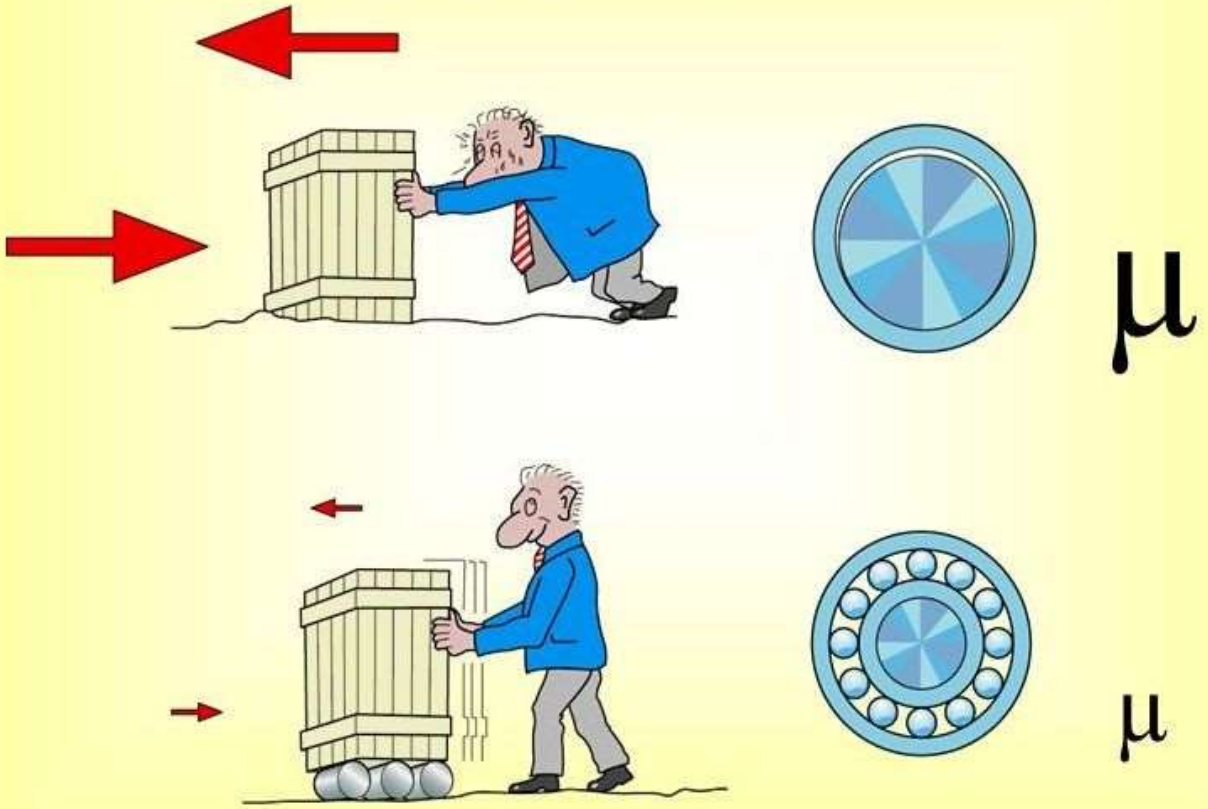
Static Friction



Kinetic Friction



Sliding and Rolling Friction



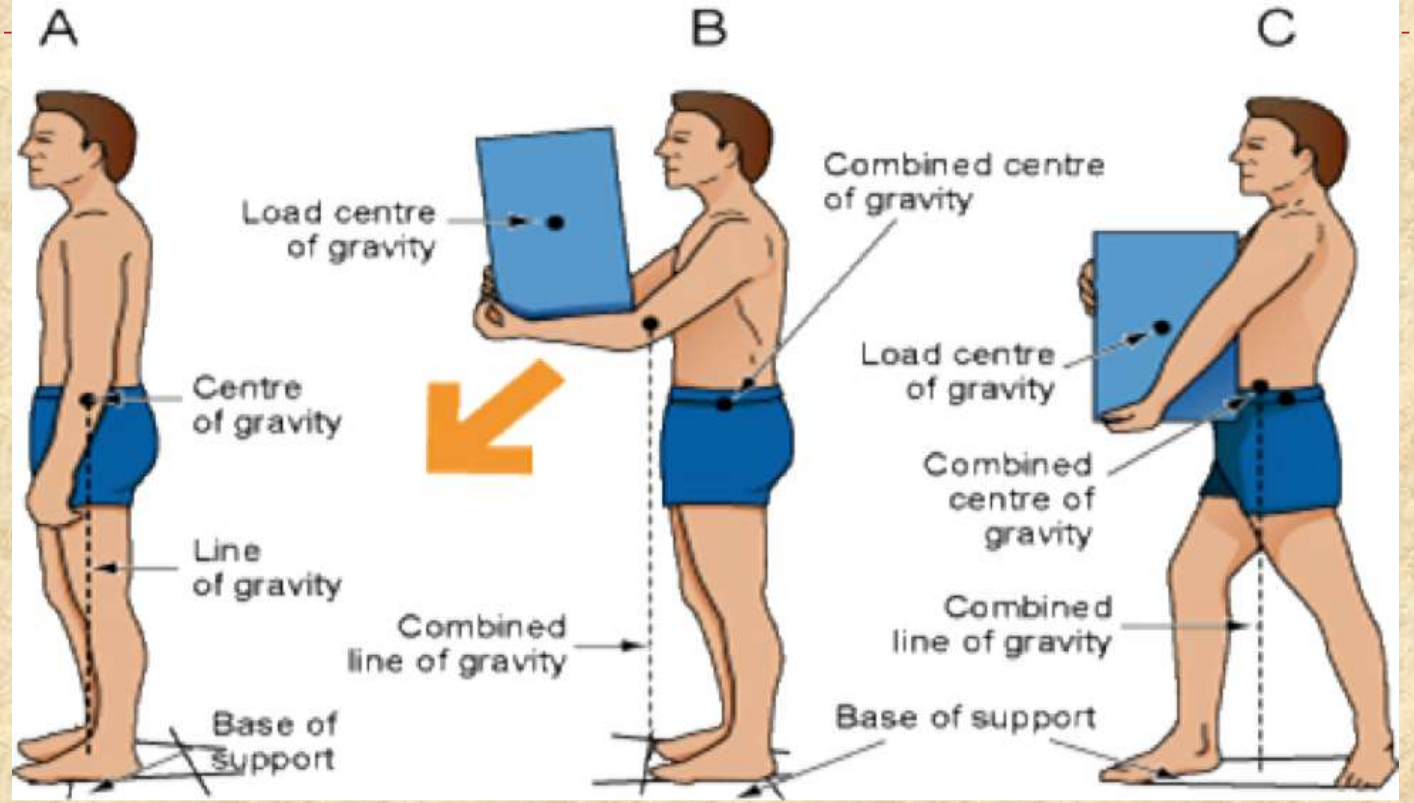
Centre of Gravity (CG) -

- The centre of gravity (C.G.) of a body is that point at which the whole weight of the body can be assumed to be concentrated

- The C.G. is a function of the Gravity.

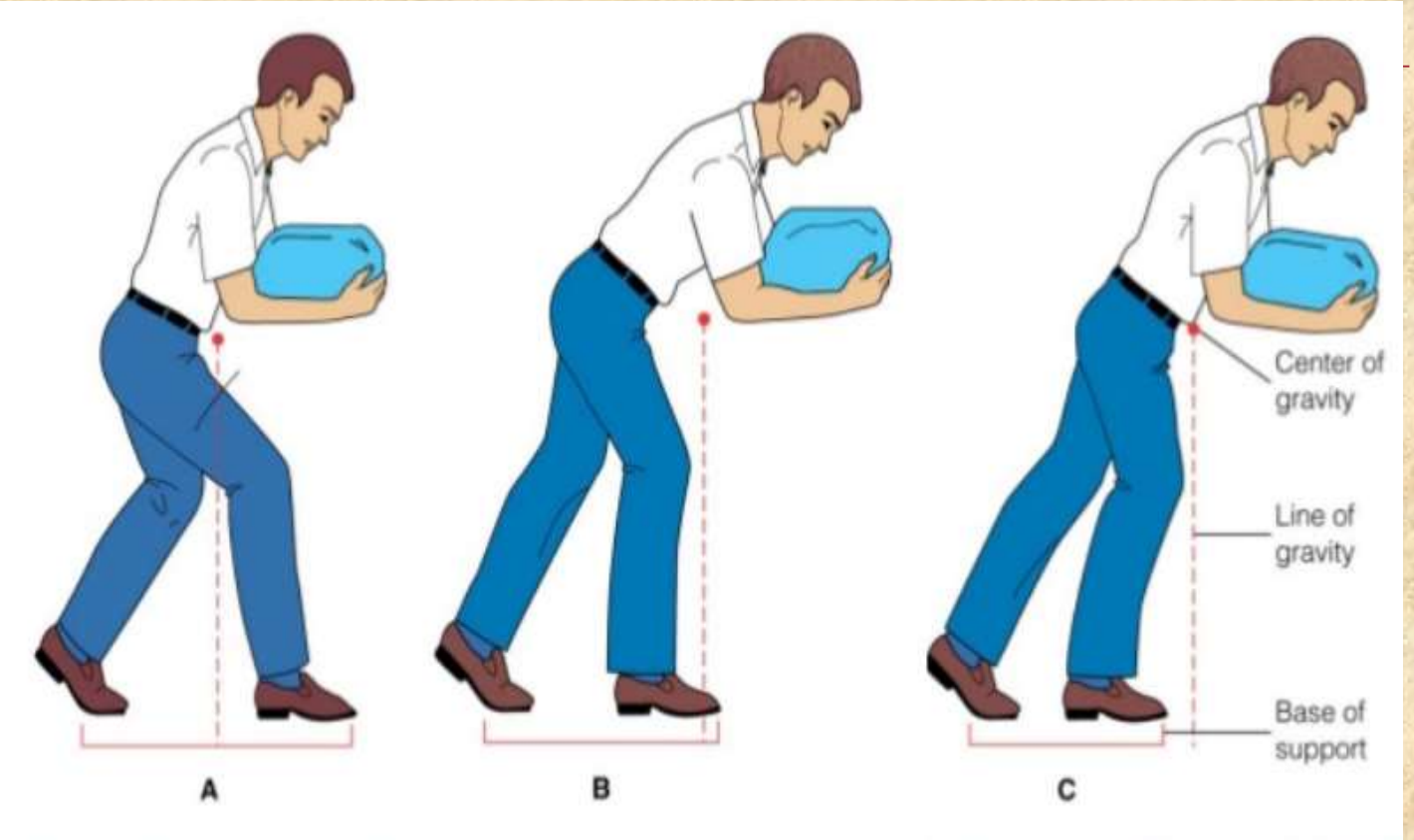
- The point of C.G. may not necessarily lie on the body itself.

Concept of CG -



- **The point of C.G. changes as humans can occupy variable positions with a load lifted.**

Concept of CG -



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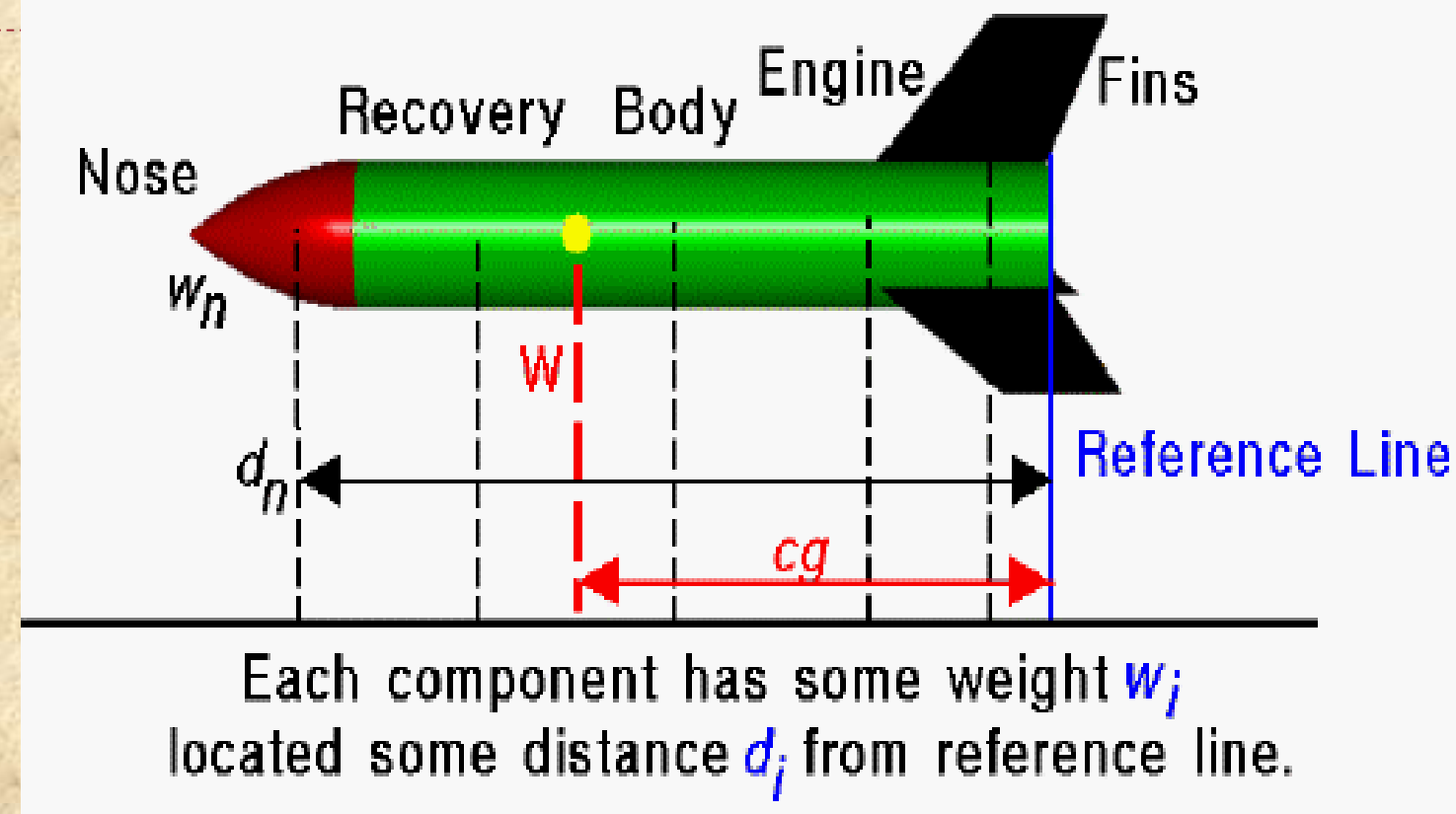
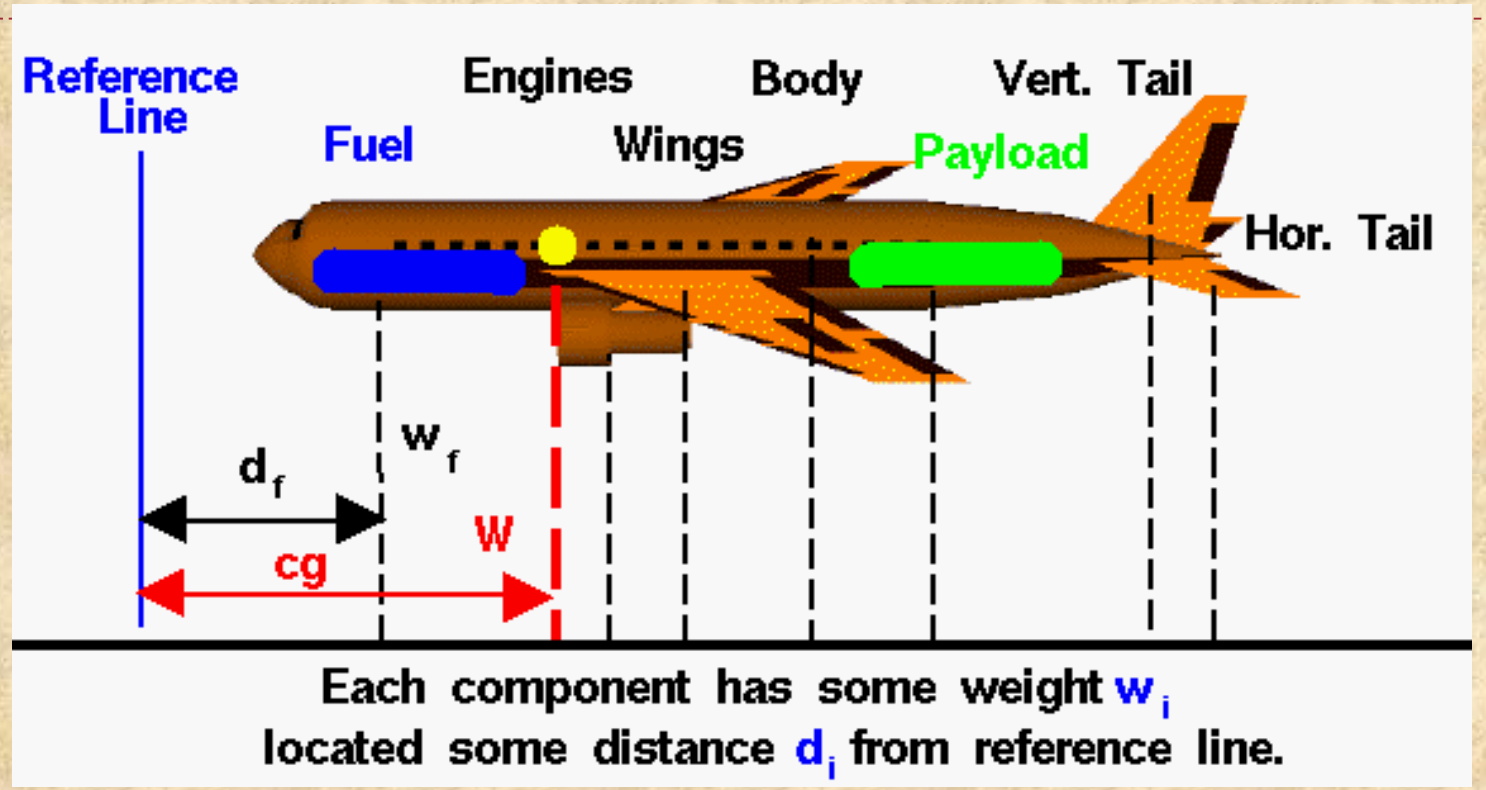
Concept of CG -



The point of C.G. in a missile.

Concept of CG -



The point of C.G. in an Aircraft.

Concept of CG -

The location of CG has to be continuously monitored in a Robotic system.

Centre of Mass (CM) -

- The centre of mass (C.M.) is a point at which the entire mass of the body can be assumed to be concentrated.
- The C.M. is independent of gravity.
- C.M. = C.G. if density of mass is uniform.

Centre of Volume (CV) -

- The centre of volume is defined as the point at which the whole volume of the body can be assumed to be concentrated.
- C.V. = C.G. if density of mass is uniform.

Centroid of Line and Area -

- The earth's attraction has no effect on the lines, curves or geometrical figures having area because they do not possess mass or volume.

- Therefore, the centre of gravity, centre of mass and centre of volume does not apply to lines, curves or such other geometrical figures.

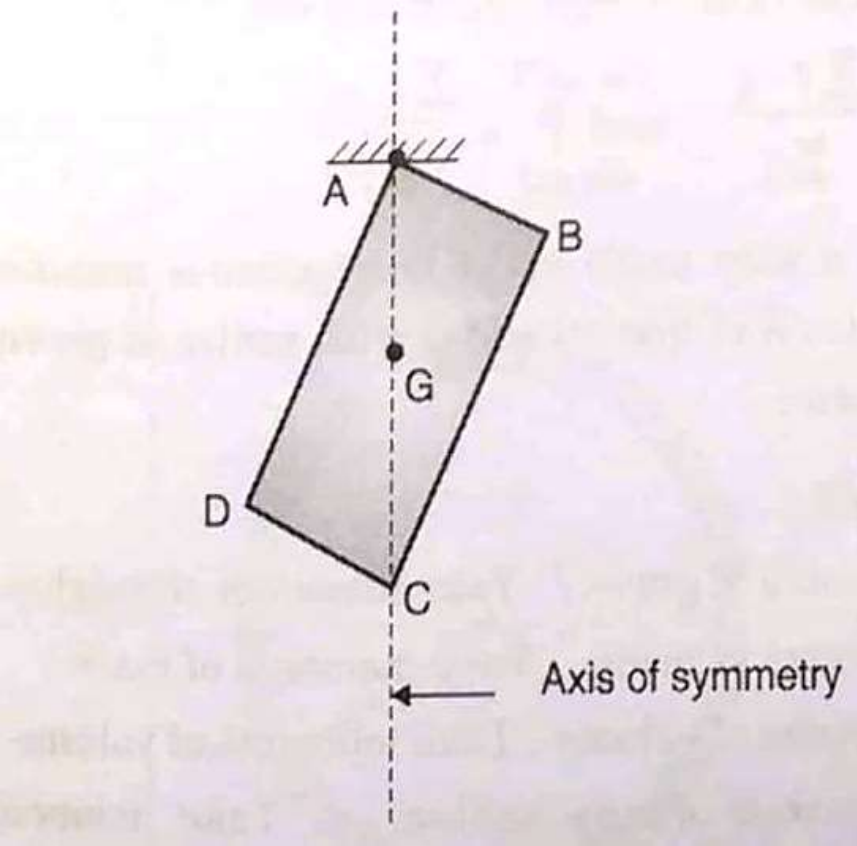
- Centroid applies to two dimensional plane laminas or figures and one dimensional lines, wires, rods etc.

Centroid of Line and Area -

- **Centroid of line** is defined as the point at which whole length of the line may be assumed to be concentrated

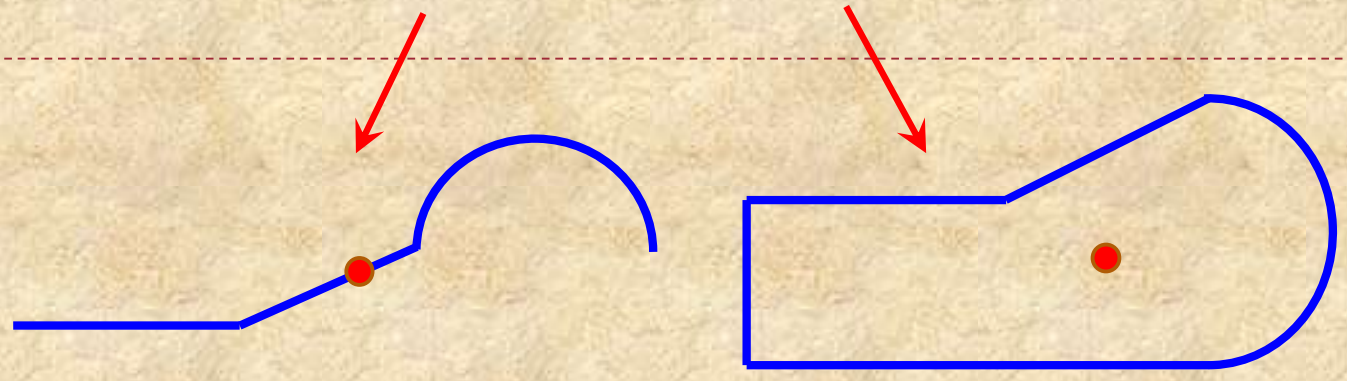
- **Centroid of area (plane lamina or figure)** is defined as the point, at which the whole area of the lamina or figures may be assumed to be concentrated

Axis of Symmetry –



- **Axis of Symmetry** is the line which divides a body into 2 parts, so that the moments of these parts about the axis of symmetry are equal and opposite.

Centroid of Curves and Areas



Centroid is that imaginary point in the Curve or Area where whole of the Curve or Area can be assumed to be concentrated.

It is not necessary that the Centroid should lie on the Curve or inside the Area.

Steps to determine Centroid of Lines and Curves -

1) Select and finalise reference axes if not given.
2) Check whether given figure is symmetrical about either X or Y axis.
3) If symmetrical about X axis, then $\bar{y} = 0$ and vice versa.
4) Divide the given curve into different line segments or curves of simple and standard shapes.
5) Obtain and tabulate results of ….
   a) Lengths of line segments (L).
   b) Position Centroid of line segments w.r.t. the reference points X or Y.
   c) Take moments of line segments about X and Y axes. i.e. L*x and L*y
6) Coordinates of the Centroid are given by……

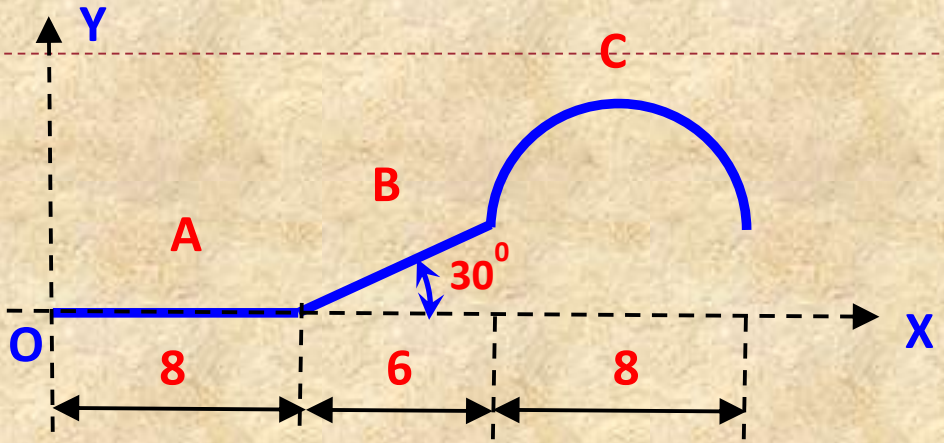
$$\bar{x} = \frac{\sum L^*x}{\sum L} \qquad \bar{y} = \frac{\sum L^*y}{\sum L}$$

Steps to determine Centroid of Lines and Curves -

Sr. No.	Shape	Length (L) mm	Distance of Centroid from Y axis (x) mm	Distance of Centroid from X axis (y) mm	L* x (mm^2)	L* y (mm^2)
1						
2						
3						
		Σ L			Σ (L * x)	Σ (L * y)

1) L * x = Moment of Line segment about X axis.
2) L * y = Moment of Line segment about Y axis.

Steps to determine Centroid of Lines and Curves -



1) Divide the given curve into 3 sections A, B and C as shown.

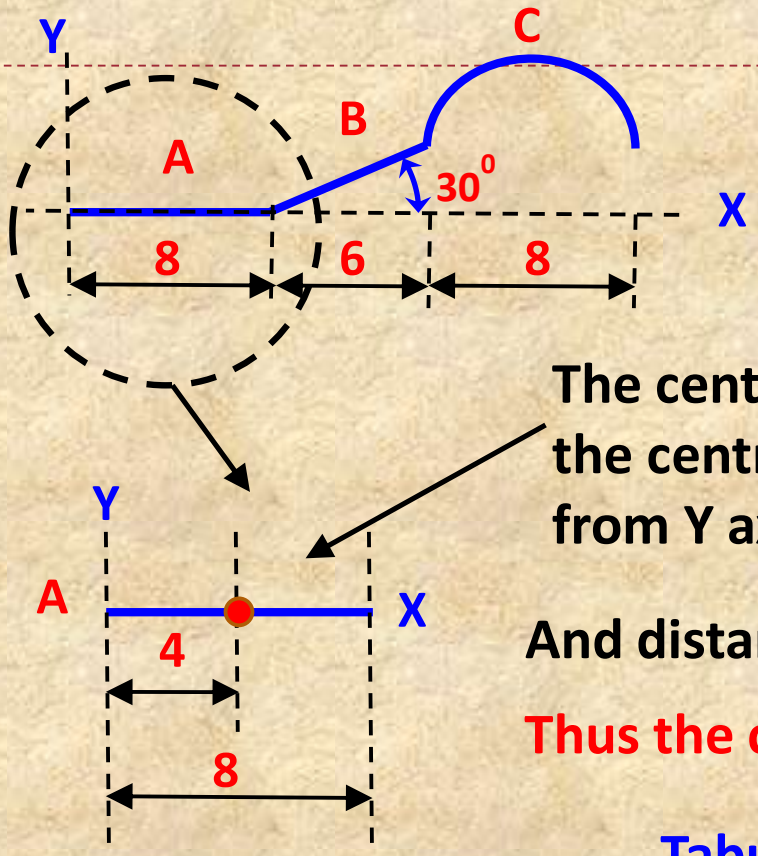
2) Find centroids of the 3 sections separately.

3) Find the distance of each centroid from the X and Y axes.

4) Tabulate the results.

5) Find the Centroid of the given curve using the formulae.

Steps to determine Centroid of a Curve – Section "A"



The centroid of section A will be at the centre of the length. i.e. 4 cm from Y axis. i.e. x = 4 cm.

And distance from X axis i.e. y = 0 cm.

Thus the co-ordinates are (4, 0)

Tabulate these values.....

Summary Table -

Sr. No.	Section	Length (L) mm	Distance of Centroid from Y axis (x) mm	Distance of Centroid from X axis (y) mm	L* x (mm^2)	L* y (mm^2)
1	A	8	4	0	32	0
2						
3						
		Σ L			Σ (L * x)	Σ (L * y)

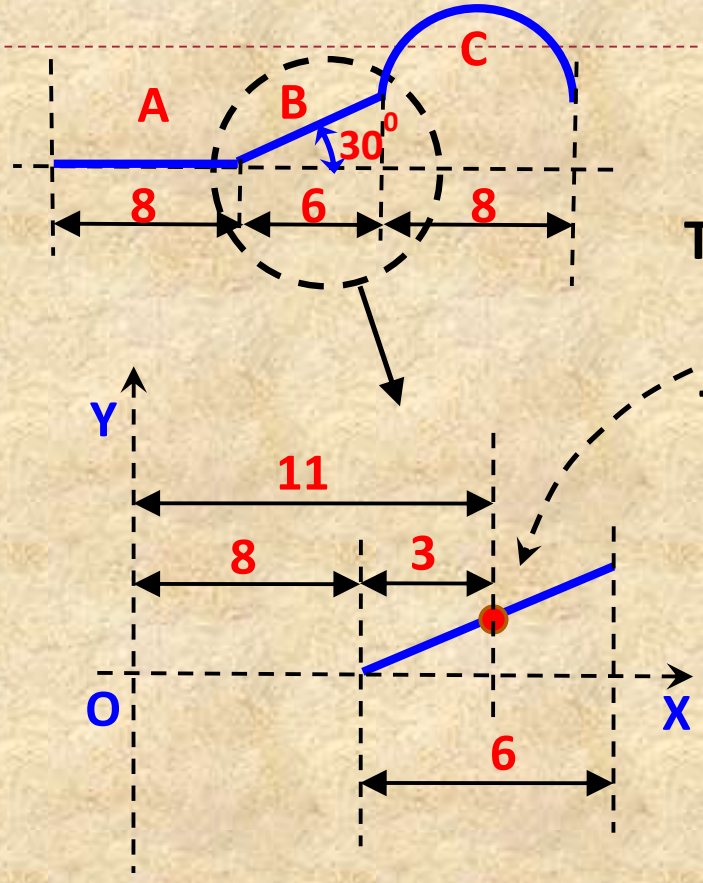
8 x 4 = 32

8 x 0 = 0

Section A …. done …….

Repeat same procedure for other sections ….

Steps to determine Centroid of a Curve – Section "B"



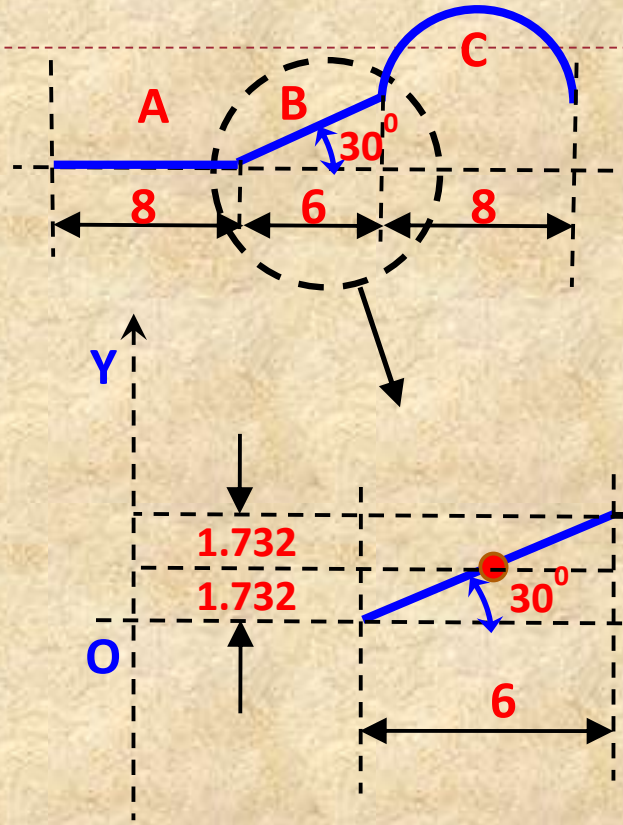
The centroid of section B will be at the centre of the section B.

Thus, total distance of Centroid from Y axis will be

$x = 8 + 3 = 11$ cm

Similarly, let us find the vertical distance y from X axis.

Steps to determine Centroid of a Curve – Section "B"



Thus, total distance of Centroid from Y axis will be

x = 8 + 3 = 11 cm

To find the Y coordinate, find the vertical distance.

Thus, y = 1.732 cm

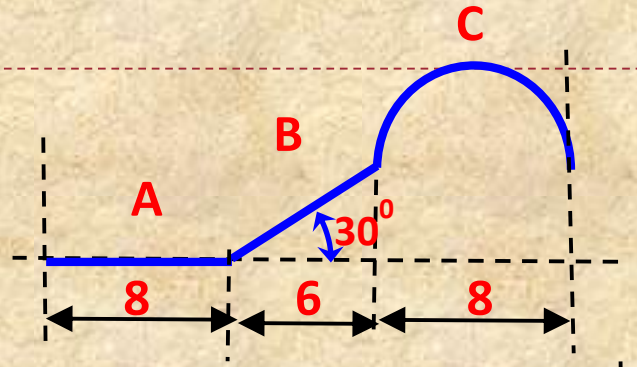
Thus, the co-ordinates are = (11, 1.732)

Tabulate these values.....

Steps to determine Centroid of Line -

Sr. No.	Shape	Length (L) mm	Distance of Centroid from Y axis (x) mm	Distance of Centroid from X axis (y) mm	L* x (mm^2)	L* y (mm^2)
1	A	8	4	0	32	0
2	B	6	11	1.732	66	10.392
3						
		Σ L			Σ (L * x)	Σ (L * y)

Steps to determine Centroid of a Curve – shape "C"

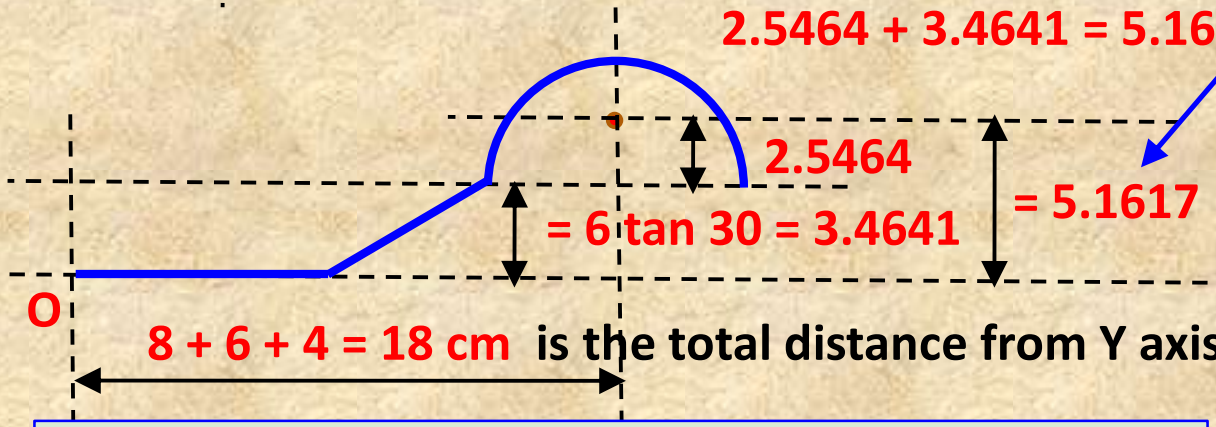


Centroid of semicircle is at (2R /π) from the X axis.

2R/π = 2*4/π = 2.5464 cm

Total distance from X axis =

2.5464 + 3.4641 = 5.1617 cm



is the total distance from Y axis

L = Perimeter of semicircle πD/2 = π*8/2 = 12.5663

Tabulate these values…..

Steps to determine Centroid of a Curve – shape "C"

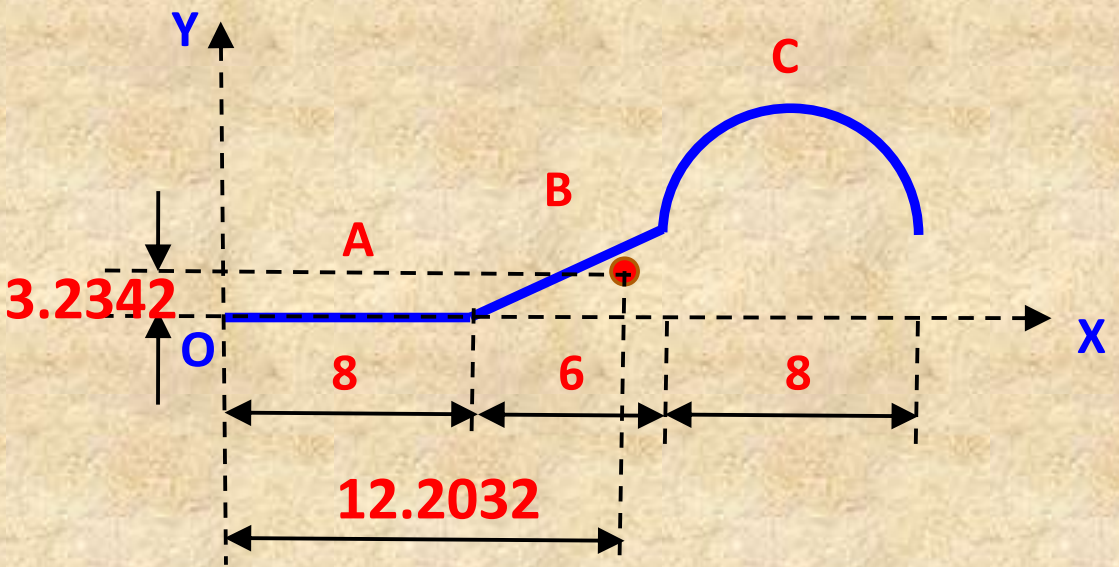
Sr. No.	Shape	Length (L) mm	Distance of Centroid from Y axis (x) mm	Distance of Centroid from X axis (y) mm	L* x (mm^2)	L* y (mm^2)
1	A	8	4	0	32	0
2	B	6	11	1.732	66	10.392
3	C	12.5663	18	6.0105	226.1946	75.5297
		26.5663			324.1946	85.9217
		ΣL			$\Sigma (L*x)$	$\Sigma (L*y)$

Coordinates of the Centroid are..

$$\bar{x} = \frac{\Sigma L*x}{\Sigma L} = \frac{324.1946}{26.5663} = 12.2032 \text{ cm}$$

$$\bar{y} = \frac{\Sigma L*y}{\Sigma L} = \frac{85.9217}{26.5663} = 3.2342 \text{ cm}$$

Final Location of Centroid of a Curve



IMP Note - It is not necessary that the Centroid should lie on the Curve or inside the Area.

Steps to determine Centroid of Area -

1) Select and fix reference axes if not given.
2) Check whether given figure is symmetrical about either X or Y axis. If symmetrical about X axis, then $\bar{y} = 0$ and vice versa.
3) Divide the figure into different areas of simple shapes.
4) Take moments of areas about X and Y axes.
5) Coordinates of the Centroid are given by……

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

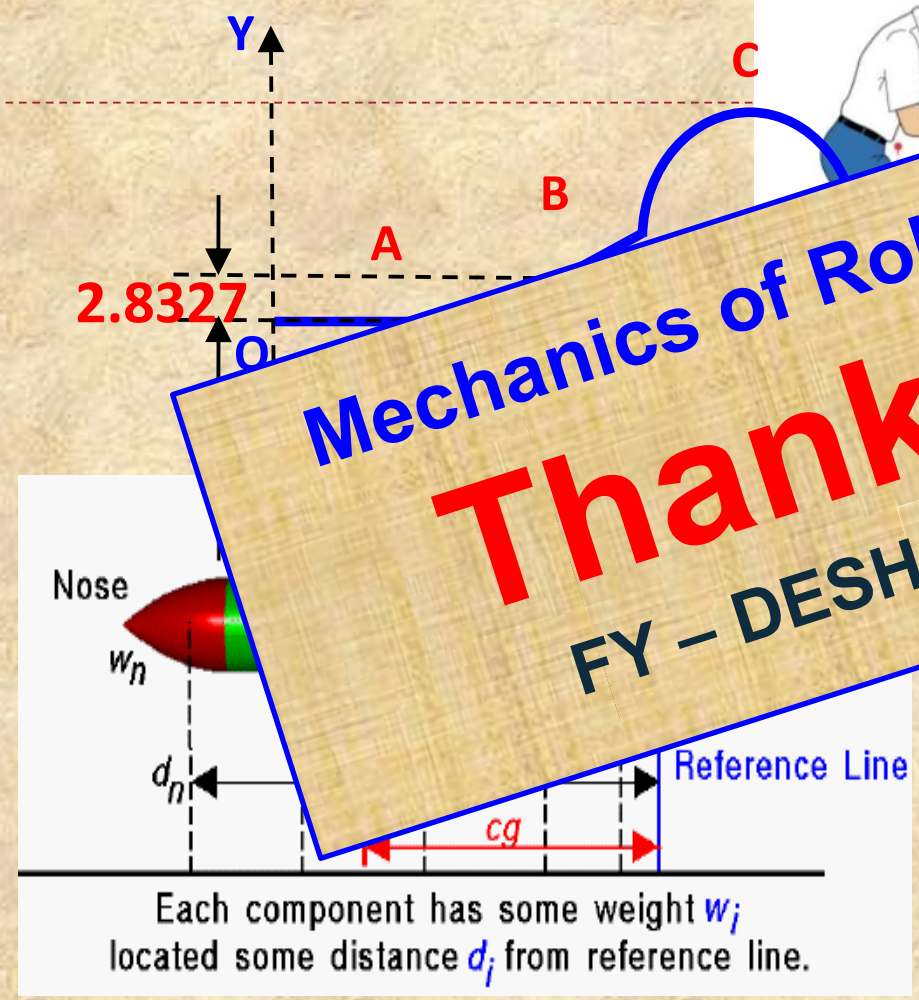
Steps to determine Centroid of Area -

Sr. No.	Shape	Area (A) mm^2	Distance of Centroid from Y axis (x) mm	Distance of Centroid from X axis (y) mm	A* x (mm^3)	A* y (mm^3)
1						
2						
3						
		ΣA			$\Sigma (A * x)$	$\Sigma (A * y)$

1) A * x = Moment of Area about X axis.
2) A * y = Moment of Area about Y axis.

Coordinates of Centroid are .....

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$



Mechanics of Robots - 2

Thanks !

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