Engineering Mathematics ES1032
Linear Transformation

Practice Exercise	
Q.1	Find a 2×2 that maps $(1,\ 2)^T$ and $(2,\ -3)^T$ into $(-2,\ 5)^T$ and $(3,\ 2)^T$ respectively.
Q. 2	A) Is there exists a 2×2 singular matrix that maps $(1,\ 2)^T$ into $(2,\ -3)^T$? If so, find the linear map represented by the matrix.
	B) Is there exists a 2×2 singular matrix that maps $(1,\ 2)^T$ into $(2,\ 4)^T$? If so, find the linear map represented by the matrix.
Q. 3	Let $M_{2\times 2}$ be the vector space of all 2×2 matrices. Define $T: M_{2\times 2}\to M_{2\times 2}$ by $T(A)=A+A^T$, where $A=\begin{bmatrix} a & b \\ c & d\end{bmatrix}$. a) Show that T is a linear transformation. b) Let $B\in M_{2\times 2}$ be such that $B^T=B$. Find $A\in M_{2\times 2}$ such that $T(A)=B$.
Q.4	Let $B=\{v_1,\ v_2,\ v_3\}$ be the basis of R^3, where $v_1=(-2,\ 1,\ 0)$, $v_2=(1,\ 2,\ 1)$ and $v_3=(1,\ 1,\ 1)$. Let $\mathrm{T}:\mathrm{R}^3\to\mathrm{R}^3$ such that $T(v_1)=(2,1,-1)$, $T(\mathrm{v}_2)=(-1,\ 1,\ 1)$ and $\mathrm{T}(\mathrm{v}_3)=(1,\ 0,\ 0)$. Find $T(2,\ 4,\ -1)$.
Q.5	Check whether the following matrices are orthogonal or not. $a)\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ \frac{-2}{\sqrt{45}} & \frac{-4}{\sqrt{45}} & \frac{5}{\sqrt{45}} \end{bmatrix}$ $\quad b)\ \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$
Q.6	A) Find the nullity of T a) $T:\mathrm{R}^4\to\mathrm{R}^3,\ rank(T)=2$ b) $T:\mathrm{R}^5\to\mathrm{R}^2,\ rank(T)=2$ c) $T:\mathrm{R}^4\to\mathrm{R}^4,\ rank(T)=0$ d) $T:P_3\to P_1,\ rank(T)=2$
	B) Let $T:\mathrm{R}^3\to\mathrm{R}^3$ be a linear transformation. Use the given information to find the nullity of T a) $rank(T)=2$ b) $rank(T)=1$ c) $rank(T)=0$ d) $rank(T)=3$
Q.7	Given the transformation $\begin{bmatrix} y_1 \\ y_2 \\ y_3\end{bmatrix}=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & -2\end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3\end{bmatrix}$. Find the coordinates $\begin{bmatrix} x_1 \\ x_2 \\ x_3\end{bmatrix}$ of in X corresponding to $\begin{pmatrix} 1 \\ 2 \\ -1\end{pmatrix}$ in Y.
Q.8	Let $T,S:\mathrm{R}^2\to\mathrm{R}^2$ defined as $T\begin{pmatrix} x \\ y\end{pmatrix}=\begin{pmatrix} 2x+y \\ -x+3y\end{pmatrix}$ and $S\begin{pmatrix} x \\ y\end{pmatrix}=\begin{pmatrix} x \\ x+y\end{pmatrix}$. i) Find $-3S,\ 2T+S, T\circ S, S\circ T$ ii) Find rank and nullity of each of the above transformations. iii) Which of the above transformations are one-one, onto? Justify your answer.
Q.9	A) Find a transformation from R^2 to R^2 that first shears in x_1 direction by a factor of 3 and then reflects about $y=x$.

	B) Find a transformation from R^2 to R^2 that first reflects about $y = x$ and then shears by a factor of 3 in x_1 direction.
	C) Find the standard matrix for $\mathrm{T}:\mathrm{R}^3 \to \mathrm{R}^3$, that first reflects about YZ - plane , then rotates the resulting vector in counterclockwise direction through an angle $\frac{\pi}{3}$, about Y - axis and then finally resultant vector is projected on XY - plane.
Q.10	Express the following matrix as a product of elementary matrices. Describe the effect of multiplication by the given matrix in terms of compression, expression, reflection and shear. $$A=\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}.$$