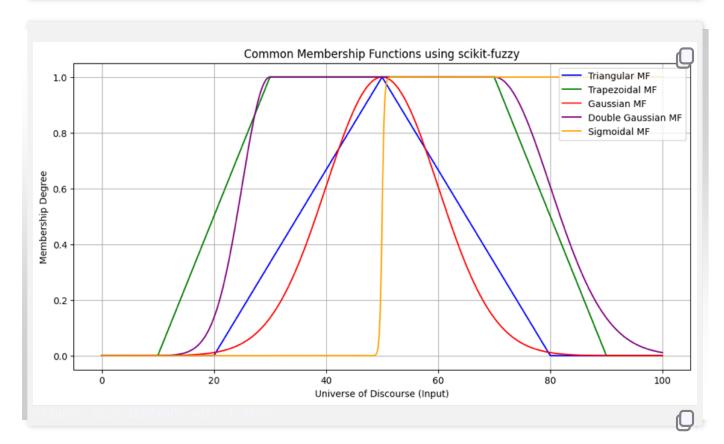
Python Notebook: Fuzzification Defuzzification

Fuzzification and defuzzification Fuzzification and defuzzification are core concepts in fuzzy logic systems. 1. Fuzzification: Fuzzification converts crisp input values (precise values) into fuzzy sets using membership functions. A membership function maps input values to a range between 0 and 1. Example: For a temperature control system. Temperature = 30°C Fuzzy Set: "Cold", "Warm", "Hot" Membership Functions: Cold(30) = 0.2, Warm(30) = 0.7, Hot(30) = 0.1 Common membership functions: Triangular Trapezoidal Gaussian Python Code to Plot Membership Functions using scikit-fuzzy

```
%*capture
!pip install numpy scikit-learn scikit-fuzzy matplotlib
```

```
import numpy as np
import skfuzzy as fuzz
import matplotlib.pyplot as plt
# Define the universe of discourse (input range)
x = np.linspace(0, 100, 500)
# Define common membership functions
# Define Common Membership functions
triangular_mf = fuzz.trimf(x, [20, 50, 80]) # Triangular
trapezoidal_mf = fuzz.trapmf(x, [10, 30, 70, 90]) # Trapezoidal
gaussian_mf = fuzz.gaussmf(x, 50, 10) # Gaussian
gauss2_mf = fuzz.gauss2mf(x, 30, 5, 70, 10) # Double Gaussian
sigmoid_mf = fuzz.sigmf(x, 50, 5) # Sigmoidal
# Plotting the membership functions
plt.figure(figsize=(12, 6))
# Triangular Membership Function
\verb|plt.plot(x, triangular_mf, label='Triangular MF', color='blue')| \\
# Trapezoidal Membership Function
plt.plot(x, trapezoidal_mf, label='Trapezoidal MF', color='green')
# Gaussian Membership Function
plt.plot(x, gaussian_mf, label='Gaussian MF', color='red')
# Double Gaussian Membership Function
plt.plot(x, gauss2_mf, label='Double Gaussian MF', color='purple')
# Sigmoidal Membership Function
plt.plot(x, sigmoid_mf, label='Sigmoidal MF', color='orange')
```

```
# Customize the plot
plt.title('Common Membership Functions using scikit-fuzzy')
plt.xlabel('Universe of Discourse (Input)')
plt.ylabel('Membership Degree')
plt.legend(loc='upper right')
plt.grid(True)
# Show the plot
plt.show()
```



```
Explanation
Triangular Membership Function:
Defined by three points

[a, b, c]

fuzz.trimf(x, [a, b, c])

Trapezoidal Membership Function:
Defined by four points

[a, b, c, d]

fuzz.trapmf(x, [a, b, c, d])

Gaussian Membership Function:
Defined by the mean and standard deviation.

fuzz.gaussmf(x, mean, sigma)

Double Gaussian Membership Function:
Defined by two Gaussian distributions with different means and standard deviations.

fuzz.gauss2mf(x, mean1, sigma1, mean2, sigma2)

Sigmoidal Membership Function:
Defined by the midpoint and slope

fuzz.sigmf(x, midpoint, slope)
```

2. Defuzzification:

Defuzzification converts fuzzy output sets into **crisp values**. Methods include

Centroid (Center of Gravity)

Mean of Maximum (MoM)

```
import numpy as np
import matplotlib.pyplot as plt
# Triangular Membership Function
def triangular_mf(x, a, b, c):
    if x <= a or x >= c:
           return 0
      elif a < x < b:
     return (x - a) / (b - a)
elif b <= x < c:
    return (c - x) / (c - b)
# Trapezoidal Membership Function
def trapezoidal_mf(x, a, b, c, d):
    if x <= a or x >= d:
           return 0
     elif a < x < b:

return (x - a) / (b - a)

elif b <= x <= c:
           return 1
     elif c < x < d:
           \textbf{return} \ (\texttt{d} - \texttt{x}) \ / \ (\texttt{d} - \texttt{c})
# Gaussian Membership Function
def gaussian_mf(x, mean, sigma):
    return np.exp(-((x - mean) ** 2) / (2 * sigma ** 2))
```

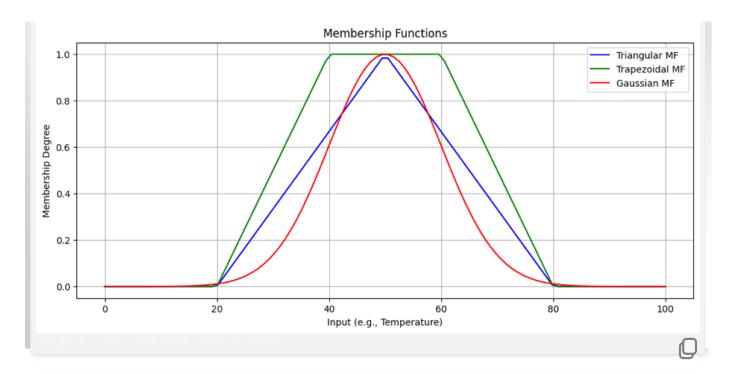
2. Plot Membership Functions

```
# Plotting Membership Functions
x = np.linspace(0, 100, 100)

plt.figure(figsize=(12, 5))

plt.plot(x, [triangular_mf(i, 20, 50, 80) for i in x], label='Triangular MF', color='blue')
plt.plot(x, [trapezoidal_mf(i, 20, 40, 60, 80) for i in x], label='Trapezoidal MF', color='green')
plt.plot(x, [gaussian_mf(i, 50, 10) for i in x], label='Gaussian MF', color='red')

plt.xlabel('Input (e.g., Temperature)')
plt.ylabel('Membership Degree')
plt.legend()
plt.title('Membership Functions')
plt.grid(True)
plt.show()
```



Defuzzification using the Centroid MethodThe centroid method calculates the center of the area under the curve to get a crisp value

```
def centroid_defuzzification(x, mf_values):
    numerator = np.sum(x * mf_values)
    denominator = np.sum(mf_values)
    return numerator / denominator if denominator != 0 else 0

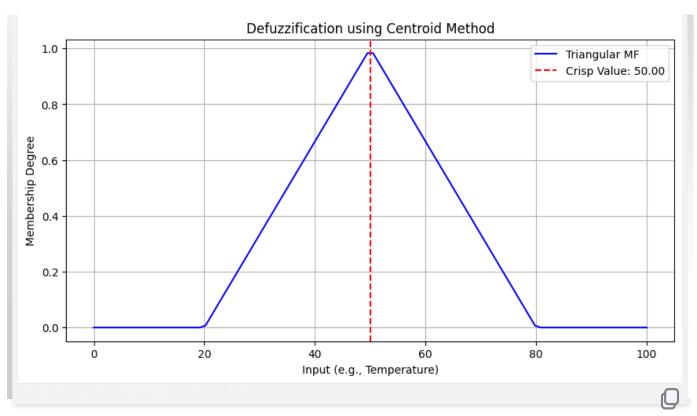
# Example: Defuzzifying a triangular membership function
mf_values = np.array([triangular_mf(i, 20, 50, 80) for i in x])
    crisp_value = centroid_defuzzification(x, mf_values)

print(f"Crisp Output using Centroid Method: {crisp_value}")

# Plot the Defuzzification Result
plt.figure(figsize=(10, 5))
plt.plot(x, mf_values, label='Triangular MF', color='blue')
plt.axvline(crisp_value, color='red', linestyle='--', label=f'Crisp Value: {crisp_value:.2f}')
plt.ylabel('Input (e.g., Temperature)')
plt.ylabel('Membership Degree')
plt.legend()
plt.title('Defuzzification using Centroid Method')
plt.grid(True)
plt.show()
```

िrisp Output using Centroid Method: 50.0





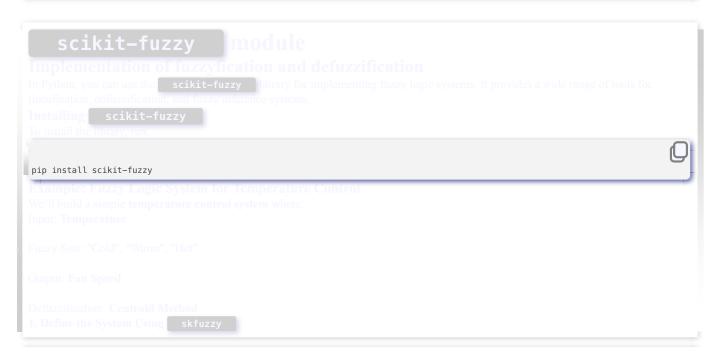
Explanation of Code
Fuzzification:
We define three membership functions: Triangular, Trapezoidal, and Gaussian.

Each function maps a crisp input to a membership degree between 0 and 1.

Plotting:
We visualize the membership functions to understand how they behave.

Defuzzification (Centroid Method):
For the given membership values, we compute the centroid as the crisp value.

This helps convert fuzzy outputs (like "somewhat warm") into a crisp number (e.g., 47.5°C).



%capture
!pip install scikit-fuzzy

```
import numpy as np
import skfuzzy as fuzz
import matplotlib.pyplot as plt

# Define the universe of discourse (input and output ranges)
x_temp = np.arange(0, 101, 1)  # Temperature (0 to 100°C)
x_speed = np.arange(0, 101, 1)  # Fan Speed (0 to 100%)

# Define fuzzy membership functions for temperature
temp_cold = fuzz.trimf(x_temp, [0, 0, 50])
temp_warm = fuzz.trimf(x_temp, [30, 50, 70])
temp_hot = fuzz.trimf(x_temp, [50, 100, 100])

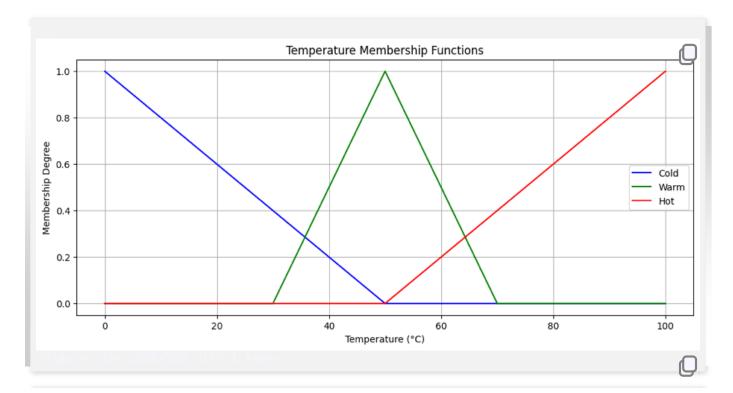
# Define fuzzy membership functions for fan speed
speed_low = fuzz.trimf(x_speed, [0, 0, 50])
speed_medium = fuzz.trimf(x_speed, [30, 50, 70])
speed_high = fuzz.trimf(x_speed, [50, 100, 100])
```

2. Visualize the Membership Functions

```
# Plot the membership functions for temperature
plt.figure(figsize=(12, 5))

plt.plot(x_temp, temp_cold, label='Cold', color='blue')
plt.plot(x_temp, temp_warm, label='Warm', color='green')
plt.plot(x_temp, temp_hot, label='Hot', color='red')

plt.title('Temperature Membership Functions')
plt.xlabel('Temperature (°C)')
plt.ylabel('Membership Degree')
plt.legend()
plt.grid(True)
plt.show()
```



3. Perform Fuzzy Inference (Using Rules)

Rule 1: If temperature is cold, then fan speed is low.

Rule 2: If temperature is warm, then fan speed is medium

Rule 3: If temperature is hot, then fan speed is high

```
# Example input temperature
temp_input = 45

# Fuzzification: Get membership degrees
cold_level = fuzz.interp_membership(x_temp, temp_cold, temp_input)
warm_level = fuzz.interp_membership(x_temp, temp_warm, temp_input)
hot_level = fuzz.interp_membership(x_temp, temp_hot, temp_input)

print(f"Cold: {cold_level}, Warm: {warm_level}, Hot: {hot_level}")

# Apply rules (taking the minimum of input membership degrees)
activation_low = np.fmin(cold_level, speed_low)
activation_medium = np.fmin(warm_level, speed_medium)
activation_high = np.fmin(hot_level, speed_high)

# Combine the outputs using the maximum (aggregation)
aggregated = np.fmax(activation_low, np.fmax(activation_medium, activation_high))
```

omլցը Cold: 0.1, Warm: 0.75, Hot: 0.0

4. Defuzzification Using Centroid Method

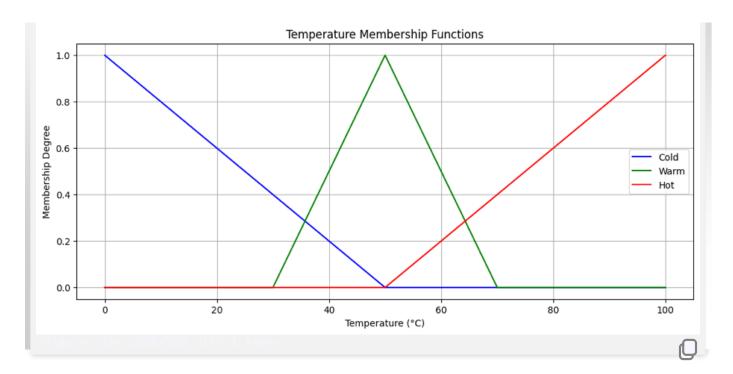
2. Visualize the Membership Functions

```
# Plot the membership functions for temperature
plt.figure(figsize=(12, 5))

plt.plot(x_temp, temp_cold, label='Cold', color='blue')
plt.plot(x_temp, temp_warm, label='Warm', color='green')
plt.plot(x_temp, temp_hot, label='Hot', color='red')

plt.title('Temperature Membership Functions')
plt.xlabel('Temperature (°C)')
plt.ylabel('Membership Degree')
plt.legend()
plt.grid(True)
plt.show()
```

0



3. Perform Fuzzy Inference (Using Rules)
Rule 1: If temperature is cold, then fan speed is low

Rule 2: If temperature is warm, then fan speed is medium,

Rule 3: If temperature is hot, then fan speed is high

```
# Example input temperature
temp_input = 45

# Fuzzification: Get membership degrees
cold_level = fuzz.interp_membership(x_temp, temp_cold, temp_input)
warm_level = fuzz.interp_membership(x_temp, temp_warm, temp_input)
hot_level = fuzz.interp_membership(x_temp, temp_warm, temp_input)

print(f"Cold: {cold_level}, Warm: {warm_level}, Hot: {hot_level}")

# Apply rules (taking the minimum of input membership degrees)
activation_low = np.fmin(cold_level, speed_low)
activation_medium = np.fmin(warm_level, speed_medium)
activation_high = np.fmin(hot_level, speed_high)

# Combine the outputs using the maximum (aggregation)
aggregated = np.fmax(activation_low, np.fmax(activation_medium, activation_high))
```

4. Defuzzification Using Centroid Method

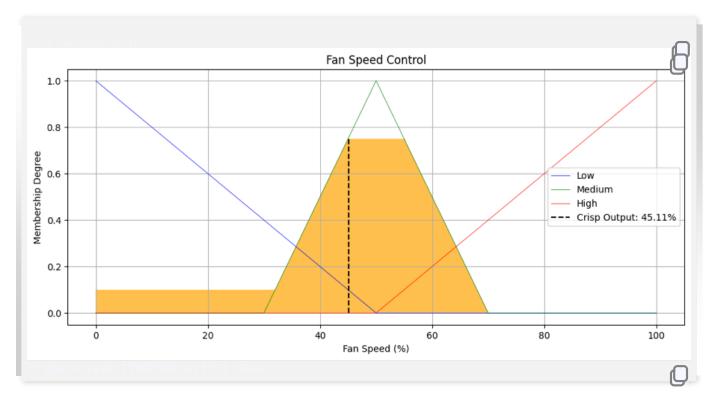
```
# Defuzzify the aggregated result to get a crisp fan speed
fan_speed = fuzz.defuzz(x_speed, aggregated, 'centroid')
fan_speed_activation = fuzz.interp_membership(x_speed, aggregated, fan_speed)
```

```
print(f"Crisp Fan Speed: {fan_speed:.2f}%")

# Plot the aggregated output and the defuzzified result
plt.figure(figsize=(12, 5))

plt.plot(x_speed, speed_low, 'b', linewidth=0.5, label='Low')
plt.plot(x_speed, speed_medium, 'g', linewidth=0.5, label='Medium')
plt.plot(x_speed, speed_high, 'r', linewidth=0.5, label='High')
plt.fill_between(x_speed, 0, aggregated, facecolor='orange', alpha=0.7)
plt.plot([fan_speed, fan_speed], [0, fan_speed_activation], 'k', linestyle='--', label=f'Crisp Output: {fan_speed:.2f}%')

plt.title('Fan Speed Control')
plt.xlabel('Fan Speed (%)')
plt.ylabel('Membership Degree')
plt.legend()
plt.grid(True)
plt.show()
```



Explanation Fuzzification: We calculate the membership degrees of the input temperature (45°C) for "Cold", "Warm", and "Hot". Inference: Using the rules, we compute the activation levels for each fan speed (Low, Medium, High). Aggregation: The outputs from all rules are aggregated using the maximum operation. Defuzzification: We use the centroid method to calculate the erisp fan speed Output Example Cold: 0.1, Warm: 0.8, Hot: 0.0 Crisp Fan Speed: 47.62% Conclusion The scikit-fuzzy library simplifies working with fuzzy systems. You can experiment by changing the input temperature, membership functions, and rules to explore how different settings affect the output. This code also visualizes the fuzzy inference process.

References

scikit-fuzzy docs (https://scikit-fuzzy.readthedocs.io/en/latest/)

scikit-fuzzy overview (https://pythonhosted.org/scikit-fuzzy/overview.html)

Example from Book

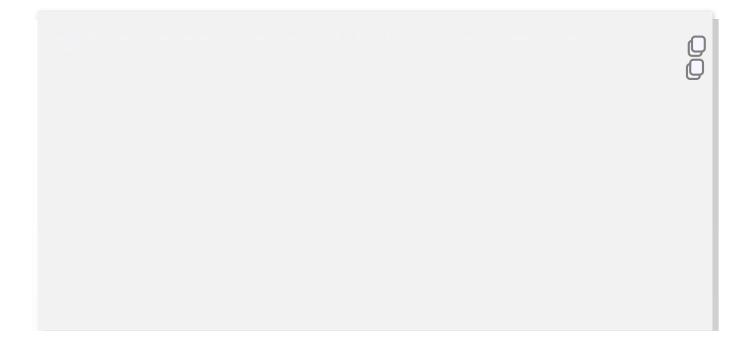
```
Enter the number of fuzzy sets: 1
Enter the points in the range (e.g., 0 1 4 5):
Enter the corresponding membership values (e.g., 0 0.3 0.3 0):
0 0.3 0.3 0
Defuzzification Results:
i) Center of Area (Centroid): 2.416666666666665
ii) Center of Sums (COS): 2.5
iii) Mean of Maximum (MOM): 2.5
```

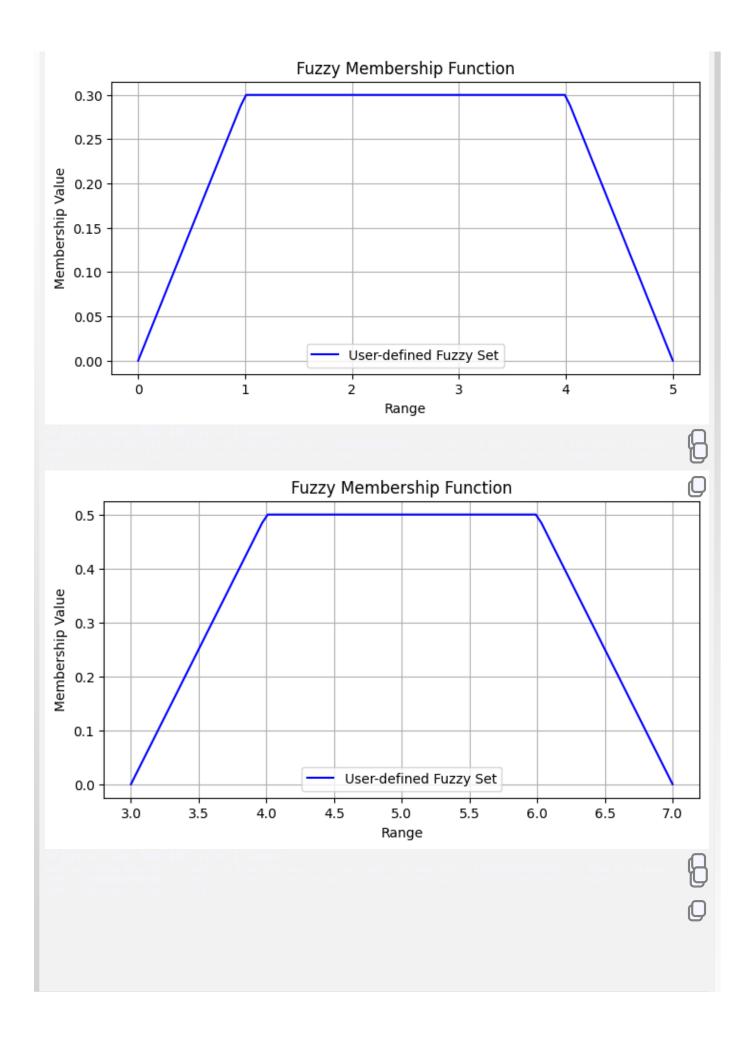
```
import numpy as np
import skfuzzy as fuzz
import matplotlib.pyplot as plt

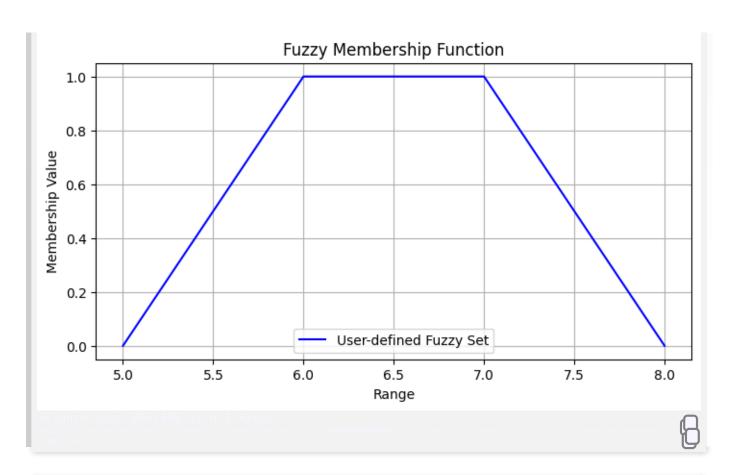
# Function to ask user for membership function points and values
def get_membership_input():
    print("Enter the points in the range (e.g., 0 1 4 5): ")
    points = list(map(float, input().strip().split()))

    print("Enter the corresponding membership values (e.g., 0 0.3 0.3 0): ")
    values = list(map(float, input().strip().split()))
```

```
if len(points) != len(values):
         raise ValueError("The number of points must match the number of membership values.")
    return np.arrav(points). np.arrav(values)
# Plot the fuzzy membership function
def plot_membership(points, values):
    x = np.linspace(points[0], points[-1], 100)  # Generate smooth x-axis
    y = np.interp(x, points, values) # Interpolate the membership values
    plt.figure(figsize=(8, 4))
    ptt.rigure(rigsize=(0, 4/)
plt.plot(x, y, label="User-defined Fuzzy Set", color="b")
plt.xlabel("Range")
plt.ylabel("Membership Value")
plt.title("Fuzzy Membership Function")
plt.legend(loc="best")
    plt.grid(True)
    plt.show()
    return x, y
# Calculate defuzzification methods
def defuzzification(x, y):
    # i) Center of Area (Centroid)
    centroid = fuzz.defuzz(x, y, 'centroid')
    # ii) Center of Sums (COS) - Approximated as weighted average
    numerator = np.sum(y * x)
    denominator = np.sum(y)
    cos = numerator / denominator if denominator != 0 else np.nan
     # iii) Mean of Maximum (MOM)
    mom = fuzz.defuzz(x, y, 'mom')
    print(f"\nDefuzzification Results:")
    print(f" i) Center of Area (Centroid): {centroid}")
print(f" ii) Center of Sums (COS): {cos}")
    print(f" iii) Mean of Maximum (MOM): {mom}")
# Main function
def main():
    num_sets = int(input("Enter the number of fuzzy sets: "))
    for i in range(num_sets):
         print(f"\nCreating Fuzzy Set {i + 1}")
         points, values = get_membership_input()
         x, y = plot_membership(points, values)
         defuzzification(x, y)
if __name__ == "__main__":
    main()
```







```
Combined fuzzy sets

Combined fuzzy sets

Combined fuzzy sets

Combined fuzzy sets

Control of page further fuzzy sets

Control of page fuzzy sets further fuzzy sets by specifying points and corresponding membership values for each set them of fuzzy sets and the combined fuzzy set among the maximum of all sets

Overlapping Regions

The code computes the combined fuzzy set mang the maximum of all sets

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Overlapping Regions

The code computes the combined fuzzy sets and the combined fuzzy sets and the combined set maximum of all sets

Overlapping Regions

The code combined fuzzy sets and corresponding membership values for maximum of all sets

Overlapping Regions

The code combined fuzzy sets and the combined fuzzy sets and corresponding member
```

```
Enter the points (e.g., 0 1 4 5):
2 4 6
Enter the corresponding membership values (e.g., 0 0.3 0.3 0):
0 0.8 0

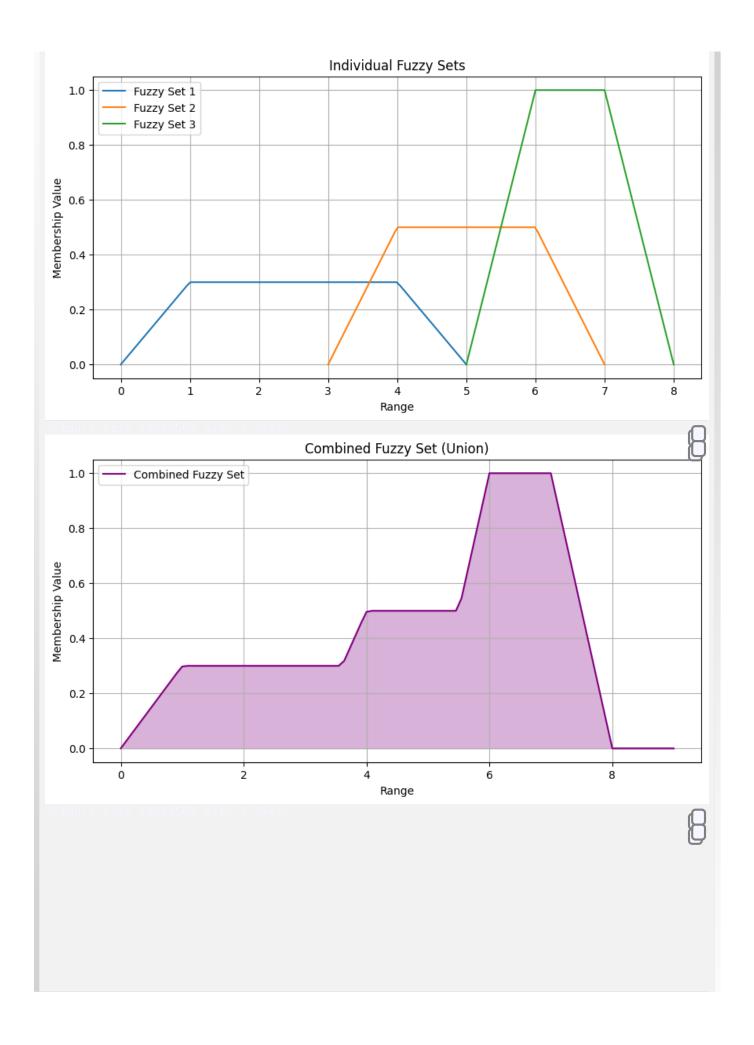
Defuzzification Results:
Fuzzy Set 1:
Centroid: 2.5, COS: 2.5, MOM: 2.0
Fuzzy Set 2:
Centroid: 4.0, COS: 4.0, MOM: 4.0
Combined Fuzzy Set:
Centroid: 3.5, COS: 3.5, MOM: 3.0

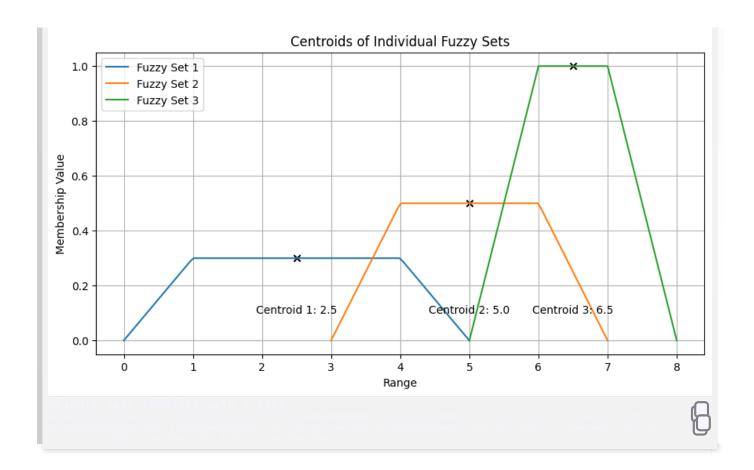
Code: Multi-Set Fuzzy Membership with Combined Analysis
```

```
import numpy as np
import nampy as np
import skfuzzy as fuzz
import matplotlib.pyplot as plt
# Function to ask user for membership function points and values
def get_membership_input():
    print("\nEnter the points (e.g., 0 1 4 5): ")
points = list(map(float, input().strip().split()))
     print(f"Your Input: {points}")
     print("Enter the corresponding membership values (e.g., 0 0.3 0.3 0): ")
     values = list(map(float, input().strip().split()))
     print(f"Your Input: {values}")
     if len(points) != len(values):
          raise ValueError("The number of points must match the number of membership values.")
     return np.array(points), np.array(values)
# Plot individual fuzzy sets
def plot_individual_sets(fuzzy_sets):
    plot_individual_sets(tuzzy_sets):
plt.figure(figsize=(10, 5))
for i, (x, y) in enumerate(fuzzy_sets):
    plt.plot(x, y, label=f"Fuzzy Set {i + 1}")
plt.xlabel("Range")
plt.ylabel("Membership Value")
     plt.title("Individual Fuzzy Sets")
     plt.legend(loc="best")
     plt.grid(True)
     plt.show()
# Plot the combined fuzzy set (union)
def plot_combined_set(x, combined_membership):
    plt.figure(figsize=(10, 5))
     plt.plot(x, combined_membership, label="Combined Fuzzy Set", color="purple")
     plt.fill_between(x, combined_membership, alpha=0.3, color="purple")
plt.xlabel("Range")
plt.ylabel("Membership Value")
     plt.title("Combined Fuzzy Set (Union)")
     plt.legend(loc="best")
     plt.grid(True)
     plt.show()
# Plot centroids of individual triangles/squares
def plot_centroids(fuzzy_sets):
     plt.figure(figsize=(10, 5))
     for i, (x, y) in enumerate(fuzzy_sets):
    centroid = fuzz.defuzz(x, y, 'centroid')
    plt.plot(x, y, label=f"Fuzzy Set {i + 1}")
          plt.scatter(centroid, fuzz.interp_membership(x, y, centroid), color="black", marker="x")
          plt.text(centroid, 0.1, f"Centroid {i + 1}: {round(centroid, 2)}", ha='center')
    plt.xlabel("Range")
plt.ylabel("Membership Value")
plt.title("Centroids of Individual Fuzzy Sets")
     plt.legend(loc="best")
     plt.grid(True)
     plt.show()
# Calculate defuzzification methods
\textbf{def defuzzification}(\textbf{x, y}) \colon
     {\tt centroid = fuzz.defuzz(x, y, 'centroid')}
```

```
cos = np.sum(y * x) / np.sum(y) if np.sum(y) != 0 else np.nan
    mom = fuzz.defuzz(x, y, 'mom')
    \textbf{return} \text{ centroid, } \cos, \text{ mom}
# Main function
def main():
    num_sets = int(input("Enter the number of fuzzy sets: "))
    fuzzy_sets = []
    # Collect all fuzzy sets from the user
    for i in range(num_sets):
        print(f"\nCreating Fuzzy Set {i + 1}")
         points, values = get_membership_input()
         x = np.linspace(points[0], points[-1], 100)
         y = np.interp(x, points, values)
fuzzy_sets.append((x, y))
    # Plot individual fuzzy sets
    plot_individual_sets(fuzzy_sets)
    # Compute the union of all fuzzy sets
    x_{union} = np.linspace(0, 9, 100)
    \verb|combined_membership| = \verb|np.maximum.reduce([np.interp(x_union, x, y) | \textit{for} x, y | \textit{in} fuzzy\_sets])|
    # Plot the combined fuzzy set (union)
    plot_combined_set(x_union, combined_membership)
    # Plot centroids of individual triangles/squares
    plot_centroids(fuzzy_sets)
    # Defuzzification for individual and combined fuzzy sets
print("\nDefuzzification Results:")
    for i, (x, y) in enumerate(fuzzy_sets):
         centroid, cos, mom = defuzzification(x, y)
print(f" Fuzzy Set {i + 1}:")
print(f" Centroid: {centroid}, COS: {cos}, MOM: {mom}")
    # Defuzzification for the combined fuzzy set
    combined_centroid, combined_cos, combined_mom = defuzzification(x_union, combined_membership)
    print(f"\nCombined Fuzzy Set:")
    print(f" Centroid: {combined_centroid}, COS: {combined_cos}, MOM: {combined_mom}")
            _ == "<u>__</u>main<u>__</u>":
if __name_
    main()
```

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DOWNLOAD SOURCE (https://upessocs.github.io/lectures/softcomputing/fuzzification_defuzzification.ipynb,