The Time Value of Money

Economics 43
Stanford University (Spring 2023)
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Outline of Topics

- Time Value of Money: Overview & Intuition
- Future Value & Simple vs. Compound Interest
- Present Value, Net Present Value (NPV), & Opportunity Cost of Capital
- Interest Rate and Yield Definitions
- Series of Cash Flows: The Annuity Formula
- Two Examples of Key Applications: Retirement Savings and Home Mortgages

Time Travel for Money

- Most of finance is about “time travel for money”
- Investing or saving involves giving up money today for (hopefully) more money in the future. It is transporting money from now to the future
- Borrowing is obtaining money today by promising to pay back more money in the future. It is transporting money from the future to the present
- When a bank loans you money, you are borrowing and they are investing. You are sacrificing future money for current money. They are sacrificing current money for future money. Most financial transactions involve both parties moving money in opposite directions in time

Two Examples of Time Travel of Money that Many of You will Face

- Retirement is a period of life without labor market earnings. Most people try to move money from their work years to their retirement years through retirement saving.

- Owning a house is a common goal, but houses are expensive. Most people transfer money from the future to the present in order to buy a house. This involves obtaining a mortgage loan to borrow money to help finance a house purchase.

Time Value of Money: Overview

- **The Time Value of Money** is the increase in an amount of money as a result of interest earned (on bonds, bank accounts or money market funds) or returns (price appreciation and dividends) in the case of equities and mutual funds

- Saving today permits more money in the future (future value)

- Borrowing today = you get the present value today of your future payments

- Saving and spending decisions involve considering trade-offs. Current needs can make borrowing worthwhile. But, watch out for extremely high interest rates such as on credit cards. Saving/investing today can make the future more enjoyable.

The Intuitive Basis for Time Value

There are three reasons why a particular cash flow in the future is worth less than the same cash today.

(1) Individuals tend to prefer present consumption to future consumption.

-People would have to be offered more in future to give up present consumption. Most people are impatient to consume. For instance, they would rather have a European vacation this summer than the same vacation 10 years from now.

-People differ in their degree of time preference for the present

The Intuitive Basis for Time Value (2)

(2) Inflation causes the purchasing power of cash to decrease over time. The higher the inflation rate, the greater the difference in value between a cash flow today and the same cash flow in the future.

One reason why you would rather have $1,000 now than $1,000 in five years, is that prices are likely to be higher 5 years from now and $1,000 will buy less. If prices have doubled in 5 years, then $1,000 will only buy half as much "real stuff."

The Intuitive Basis for Time Value (3)

(3) A promised cash flow might not be delivered for a number of reasons: for instance, the promisor might default on the payment or the promisee might not be around to receive payment.

Any uncertainty (risk) associated with the cash flow in the future reduces the value of the cash flow. For instance, if you lend money to a corporation by buying a bond, that company could declare bankruptcy before you are repaid. You would likely get less than the promised amount, perhaps zero. That is called “credit risk.”

The return on stocks is highly uncertain and therefore risky

Discounting

- The process by which future cash flows are adjusted to reflect these factors is called discounting, and the magnitude of these factors is reflected in the discount rate.

- The discount rate often incorporates all of the above factors. The discount rate can be viewed as a composite of the expected safe real return (reflecting time preferences in the aggregate over the investing population), the expected inflation rate and the uncertainty associated with the future cash flows.

The One-Period Case: Future Value

If you were to invest \$10,000 at 5% interest for one year, your investment would grow to \$10,500

\$10,000 is the principal repayment

\$500 would be interest (\$10,000 • .05)

\$10,500 is the total due (the Future Value). It can be calculated as:

$\$10,500 = \$10,000 \bullet (1.05)$.

The total amount due at the end of the investment is called the *Future Value* (*FV*).

The One-Period Case: Future Value

In the one-period case, the general formula for *FV* can be written as:

$$FV = C_0 \bullet (1 + r)$$

Where C_0 is the initial cash flow and r is the appropriate interest rate for that period.

The Two-Period Case: Future Value

Investing \$1 for two periods:

$$FV = [\$1 \bullet (1+r)](1+r)$$
$$= \$1(1+r)^2$$

$$= \underset{\text{(principal)}}{1} + \underset{\text{(simple interest)}}{2r} + \underset{\text{(compound interest)}}{r^2}$$

Compound Interest is the interest you receive on previous interest

The Multiperiod Case: Future Value

The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \bullet (1 + r)^T$$

Where

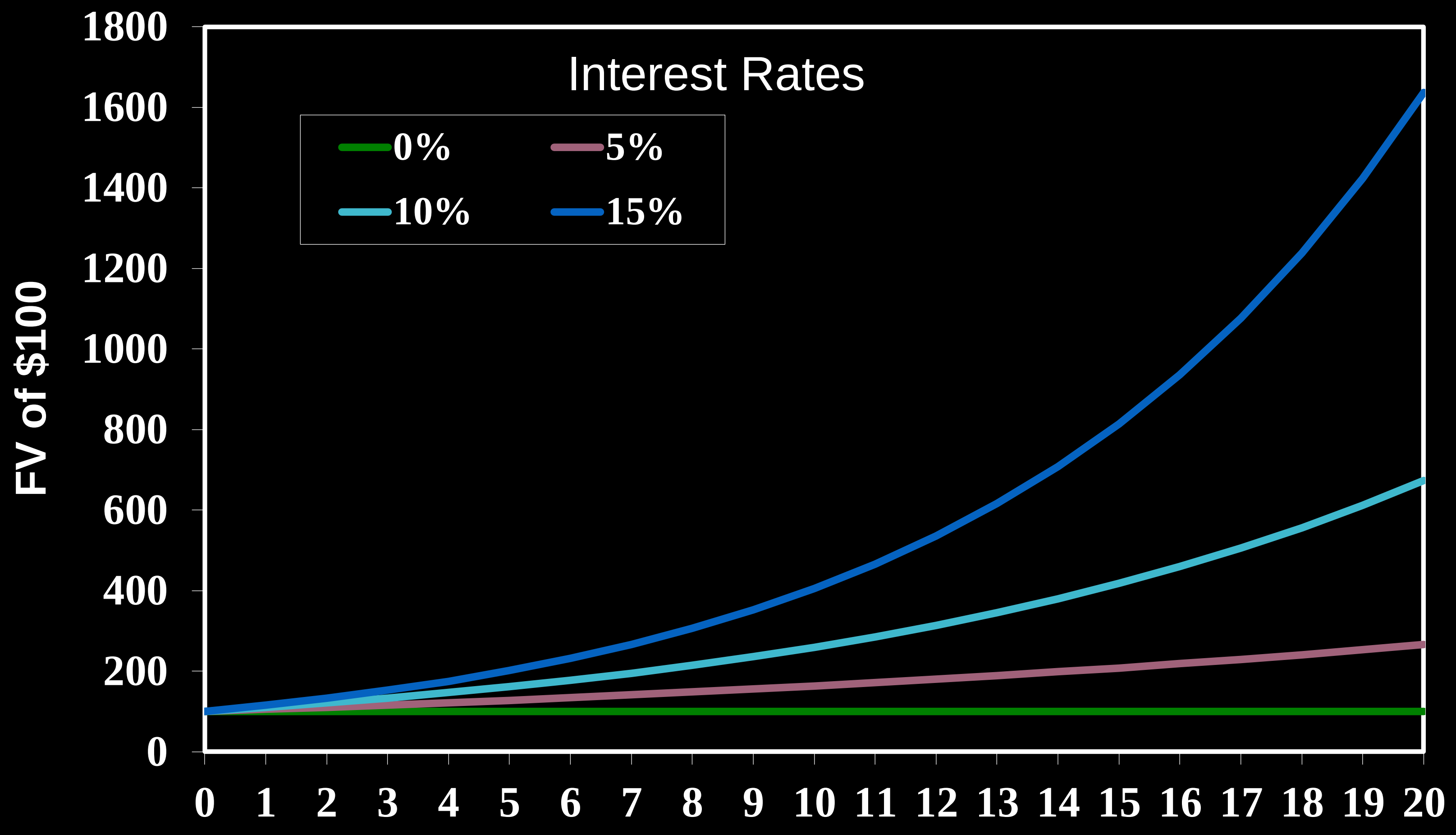
C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Note: "T" can be defined in any unit (years, quarters, months, etc.) as long as "r" is the interest rate for one of those periods. So if we are looking at T in months, then "r" needs to be expressed as a monthly rate.

Future Values with Compounding: Exponential Growth of $100 at Date 0



Note that the value of $100 after compounding at 15% for 20 years is far more than 3X the compound value with 5%

How Long is the Wait to Double Your Money?

If we deposit $5,000 today in an account paying 10%, how long does it take to grow to $10,000 – i.e. how long to double?

$$\$5000 \bullet (1+0.1)^{T} = \$10{,}000$$

$$(1+0.1)^{T} = 2$$

$$\ln(1+0.1)^{T} = \ln 2$$

$$T \bullet \ln(1.1) = \ln 2$$

$$T = 7.27\ years$$

How Long is the Wait? A Simple Rule

The "Rule of 72" can approximate the doubling time of an investment.

Divide 72 by the relevant annual compound interest rate (expressed as a whole number), and you get your approximate answer.

$72/10=7.2$ (last example)

The Rule of 72 is expressed as

$T = 72/r$

If you know r, you can solve for T

If you know T, solve for r

What Rate Is Enough?

Assume the total cost of Stanford for four years of undergraduate education right now is $75,000 per year "all-in," and will grow at 5% per year for 15 years, just in time for when my 3-year old granddaughter would want to apply. That will be about $150,000 per year for 4 years, or approximately $600,000.

So if I wanted to pay for her education, I would need $600,000 in 15 years. (This is just approximate)

Using the rule of 72, if you happened to have $300,000 now, what would the interest rate have to be for your money to double in 15 years?

72/r = 15 => r= *4.8% (or roughly 5% Is this reasonable to achieve?)*

The One-Period Case: Present Value

If you were to be promised \$10,000 one year from now, with interest rates at 5% over the next year, what would your investment be worth today?

Another way to ask this, what would you have to put aside today to meet the \$10,000 obligation?

$\$10,000 = \$9,523.81 \bullet (1.05).$

$$\frac{\$10,000}{1.05} = \$9,523.81$$

The One-Period Case: Present Value

- In the one-period case, the formula for *PV* can be written as:

$$PV = \frac{C_1}{1+r}$$

where *r* is the appropriate interest rate.

This can also be written as <u>PV = FV/(1+r)</u>

The Multi-Period Case: Present Value

- In the multi-period case, the formula for *PV* can be written as:

$$PV = \frac{C_T}{(1+r)^T}$$

where

C_T is cash flow at period T (the future value),

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

<u>Note</u>: "T" can be defined in any unit (years, quarters, months, etc.) as long as "r" is the interest rate for one of those periods. So if we are looking at T in months, then "r" needs to be expressed as a monthly rate.

The Multi-Period Case: Present Value

- In the multi-period case, the formula for *PV* can alternatively be written as:

$$PV = C_T \bullet \frac{1}{(1+r)^T} = C_T \bullet DF$$

Where

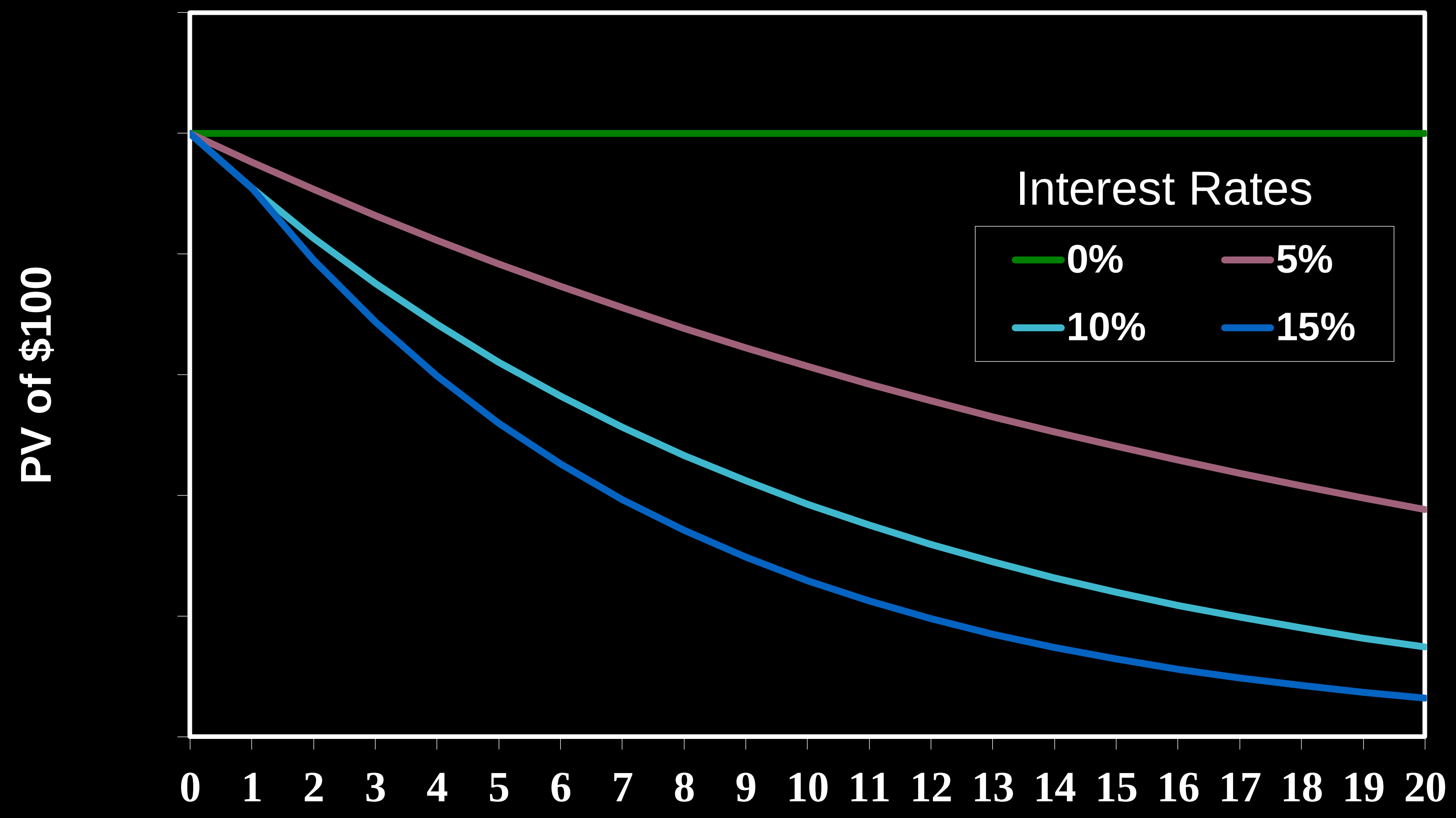
C_T is cash flow at period T,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

$\frac{1}{(1+r)^T}$ = DF or discount factor

Present Values with Compounding: Exponential Decline in value of $100



Discount Rate, Opportunity Cost of Capital

There is a wide range of applications of discounting future cash flows, both for individuals and for corporations

- Comes under heading of Discounted Cash Flow (DCF) analysis

- Involves estimating future positive and negative cash flows, thinking through discount rates, and taking present values

- The discount rate is often referred to as the opportunity cost of capital. It is what the potential investor could earn on an alternative investment with the same riskiness

The Case with Multiple Future Cash Flows at Different Horizons

- Evaluating the Present Value of an investment or project with cash flows at different points in the future (such as 1 year, 2 years, 3 years,…, 25 years) is conceptually simple.   It is just the sum of the present values of each of the future cash flows.

The Net Present Value Rule

- The Net Present Value (*NPV*) of an investment is the present value of the expected future cash flows, less the cost of the investment.

- <u>RULE: Accept positive net present value investments.</u>

- Suppose an investment that promises to pay $10,000 in one year is offered for sale for $9,500. Your opportunity cost of capital (discount rate) is 5%. Should you buy?

- NPV = $10,000/1.05 - $9,500= = $9523.81 - $9,500 =

  $23.81. NPV>0, so yes!

- Having a positive NPV means that the investment has higher value than the alternative represented by the opportunity cost of capital

- This is how most corporate mergers and acquisitions are evaluated. Corporations would/should evaluate all new initiatives with this methodology.

Simple Example: Valuing an Office Building

Step 1: Forecast cash flows

Cost of building = C_0 = \$700,000

Sale price in Year 1 = C_1 = \$800,000

Step 2: Estimate opportunity cost of capital

If equally risky investments in the financial market

offer a return of 7%, then opportunity cost of capital = r = 7%

Valuing an Office Building

Step 3: Discount future cash flows

$$PV = \frac{C_1}{(1+r)} = \frac{800{,}000}{(1+0.07)} = \$747{,}664$$

Step 4: Go ahead if PV of payoff exceeds investment

$747,664-$700,000 = $47,664

In the real world, you would take into account the rental value of the office building and its operating costs

Risk and Present Value

- Higher risk projects require a higher rate of return
- Higher required rates of return cause lower PVs

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 12\%$$

$$\text{PV} = \frac{800{,}000}{1+.12} = 714{,}286$$

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 7\%$$

$$\text{PV} = \frac{800{,}000}{1+.07} = 747{,}664$$

Interest Rate Conventions: Compounding Periods

- If you invest \$50 for 3 years at an interest rate of 12% compounded semi-annually, how much will you have in 3 years?

- Compounded semi-annually means that we will compound twice a year, or split the payout into two equal chunks of (12/2)% = 6%.

- \$50 will grow at 6% over 6 semi-annual periods
- So $FV = 50\left(1 + \frac{.12}{2}\right)^{2\times 3} = 70.93$

- Notice that if the investment compounded annually and not semi-annually, it would be worth $FV = 50(1 + .12)^3 = 70.25$

Interest Rate Conventions: Compounding Periods

In general, for an investment compounded m times a period for t periods, the future value is given by

$$FV = C_0\left(1 + \frac{r}{m}\right)^{mt}$$

where C_0 is the initial investment or PV and r is the simple annual interest rate or APR (annual percentage rate).

Credit Card Interest Rates

- The average interest rate on newly issued credit cards last month was 23.65%
- But, this interest is comounded daily. So, each day, the interest rate is 23.65/365 = 0.0648%   (6.48 basis points per day)
- In one year, the total interest charged would be

$$1.000648^{365} - 1 = .2667 \quad \text{that is, } 26.67\%$$

- The 23.65% is called the APR, or Annual Percentage Rate
- The 26.67% is called the EAR, or Effective Annual Rate

Real and Nominal Interest Rates

- Nominal Interest Rates (i)
  - The interest rate expressed in current-dollar terms.

- Real Interest Rates (r)
  - The inflation adjusted interest rate.

- The nominal interest rate you agree on (i) must be based on *expected inflation* (π^e) over the term of the loan plus the real interest rate (r).

$$i = r + \pi^e$$

- This is called the *Fisher Equation* after Irving Fisher, a Yale economist of the first half of the 20th century.
- The higher expected inflation, the higher the nominal interest rate.

Real and Nominal Interest Rates

- Financial markets quote *nominal interest rates*.
- When people use the term interest rate, they are usually referring to the *nominal rate*.
- We cannot directly observe the *real interest rate*; we have to estimate it, because we don’t know for sure what future inflation will be

$$r = i - \pi^e$$

- Today’s nominal interest rate on 10-year U.S. Treasury bonds (roughly 3.3%), translates to a real interest rate of about 0.8% as inflation expectations for the next 10 years are roughly 2.5%

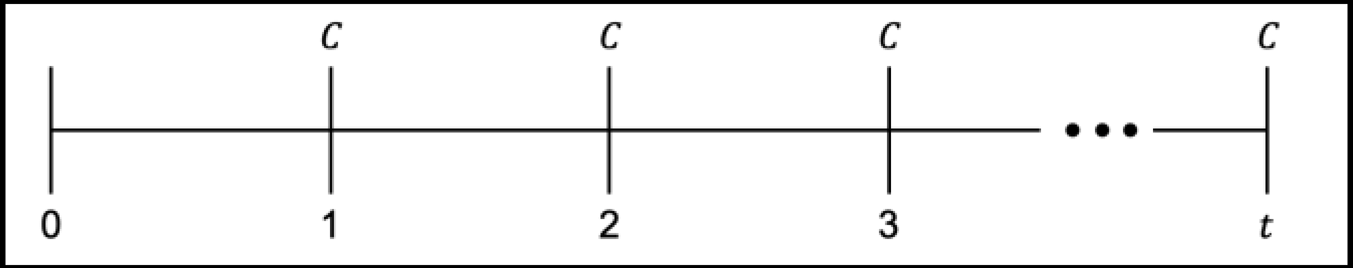
Series of Cash Flows

- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods.

- The annuity model can be used for most purchases on time such as a car loan (often 36, 48 or 60 months) or a home mortgage (often 360 months). It also can be used to model the amount of money you need to save regularly to reach a particular goal.

Annuities

- The idea of an annuity is that there are a fixed number of CONSTANT payments set at certain time intervals for a set amount of time. This can be what you save each month, or pay in home mortgage or car payments every month, or what you receive semi-annually in interest payments on a bond you own. The examples are endless.
- So we can see that the present value of an annuity is given by



$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t} = \sum_{i=1}^{t} \frac{C}{(1+r)^i}$$

<u>Note</u>:

The first “C” is one period from today.

Annuities

- The most useful version of this formula can be expressed as follows:

$$PV = \frac{C}{r}\left(1 - \frac{1}{(1+r)^t}\right)$$

If you were saving C per period, starting one period from now, how much would that accumulate to after t periods?

$$FV = (C/r)((1+r)^t - 1)$$

This is because FV always equals PV times $(1+r)^t$

Example #1: Saving for Retirement

Suppose you turn 25 years old today and are considering your retirement needs.
You expect to retire at age 65 and actuarial tables suggest that you will live to be 100.

You want to move to Portugal when you retire. You estimate that it will cost you $500,000 to make the move (on your sixty-fifth birthday) and cover your first year living expenses. After that, your expenses will be $100,000 a year for the subsequent 35 years. You expect to earn 5% per year on your money

(1) How much will you need to have saved by your retirement date to be able to afford this course of action?

(2) How much would you have to set aside each year, starting one year
from now and ending on your 65th birthday, to be able to afford this retirement plan?

Example #1: Saving for Retirement

We know that it will cost $500,000 on your 65th birthday and then for the next 35 years it will cost $100,000 per year. This can be modeled as an annuity of retirement costs, with present value measured on the day of retirement – i.e. your 65th birthday. Therefore, we have

$$\$500{,}000 + \frac{100{,}000}{0.05}\left(1 - \frac{1}{(1+0.05)^{35}}\right) = \$2{,}137{,}419.43$$

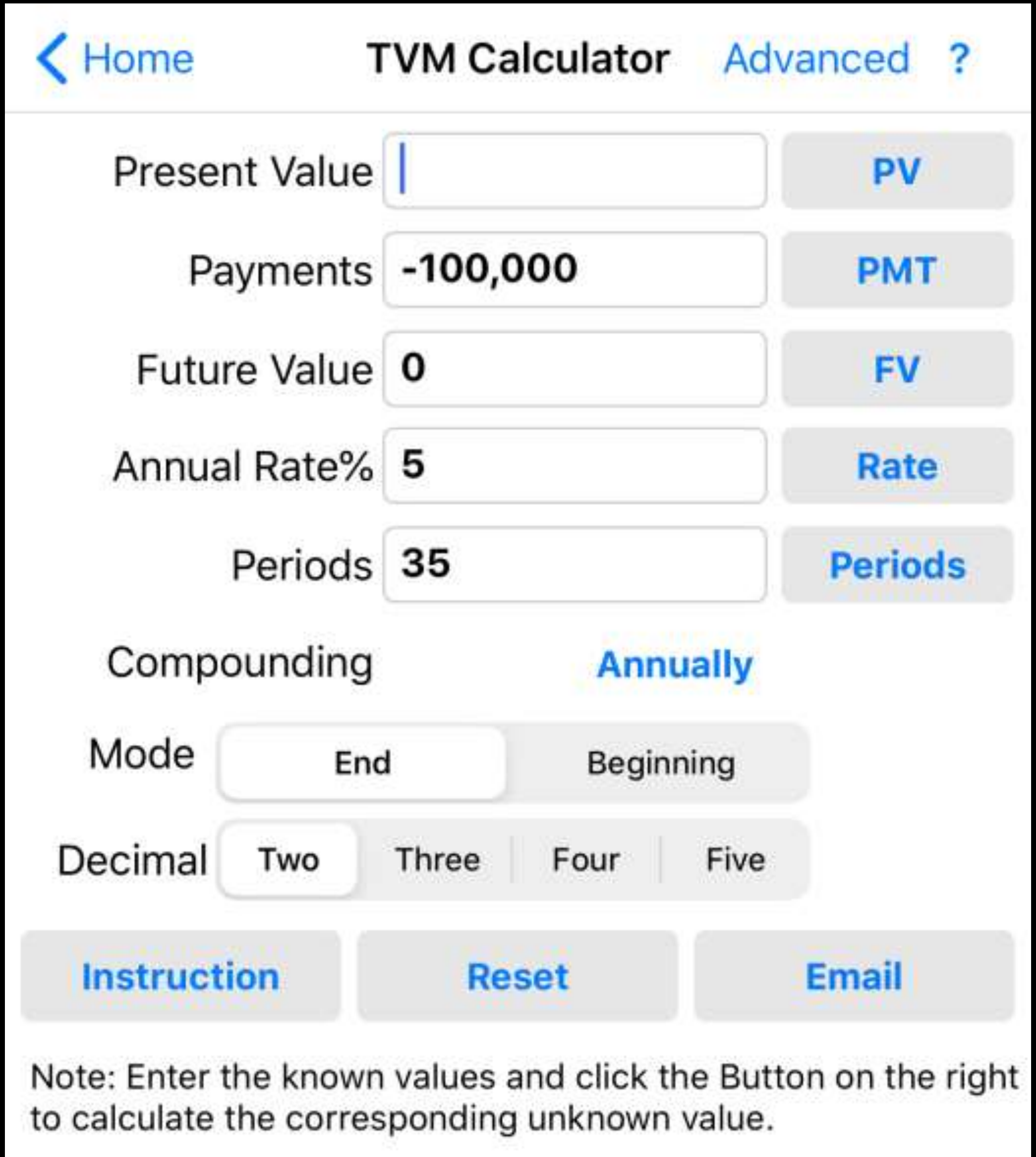
In this case, C=$100,000, r=0.05, and t=35 years. We solve for present value.

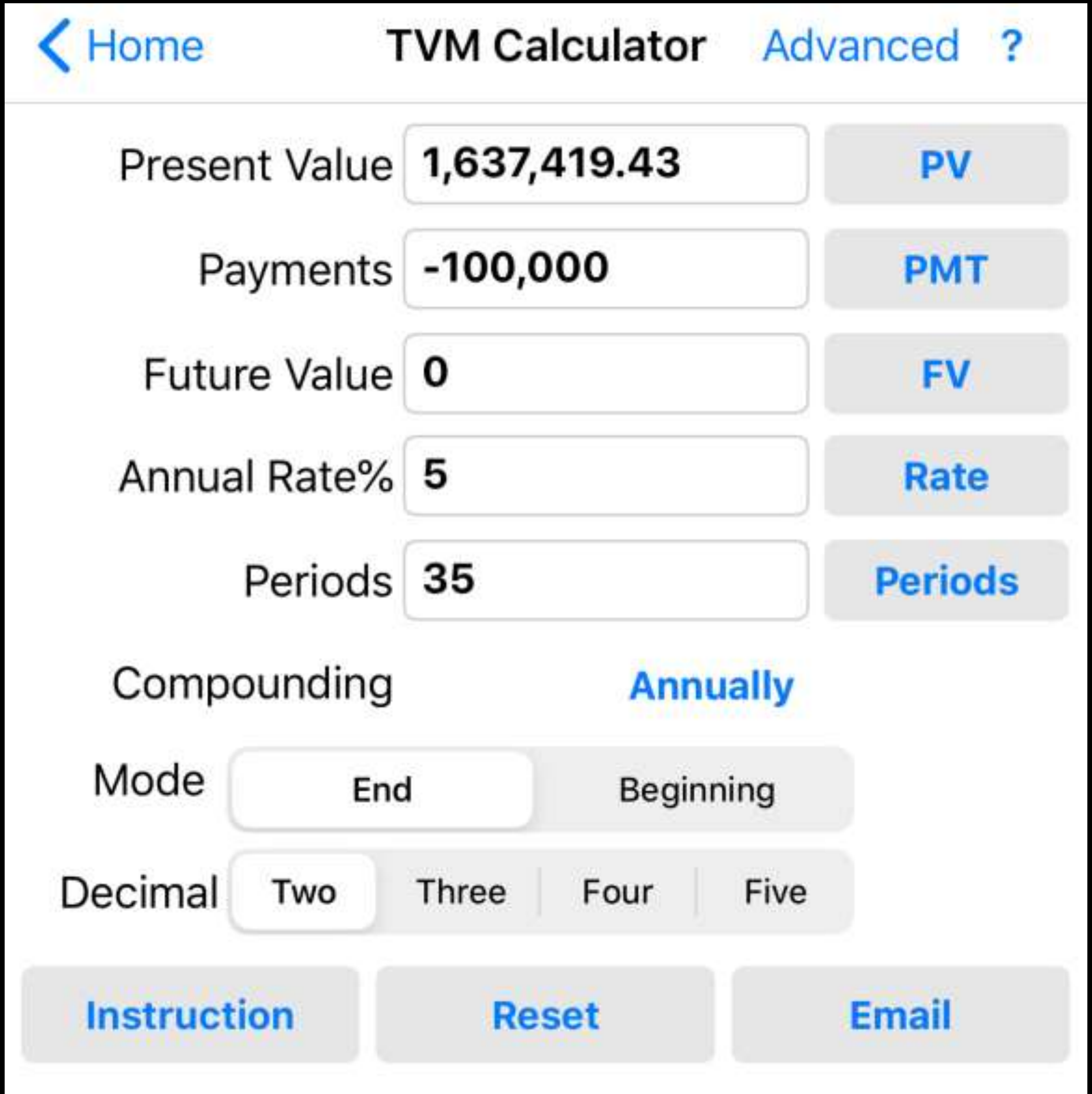
(1)So the amount of money that we need to have saved
by your 65th birthday is $2,137,419.43.

Using EZ Financial Calculator

- App is free on iOS and Android app stores
- Today we will use the Time Value of Money (TVM) calculator
- Simply fill in the known amounts and hit the button for the unknown
- The convention is that money you pay is entered as negative numbers and money you receive as positive numbers
- The next slide contains an example for the current retirement saving problem

Using Our Calculator App for this problem



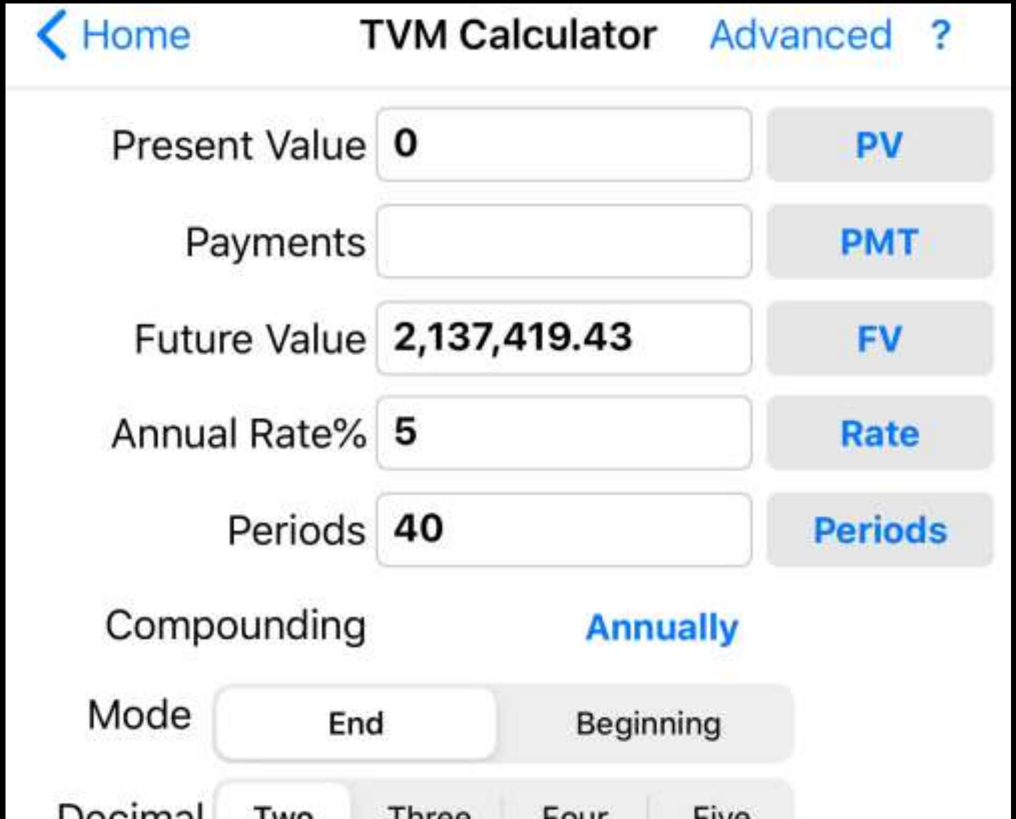


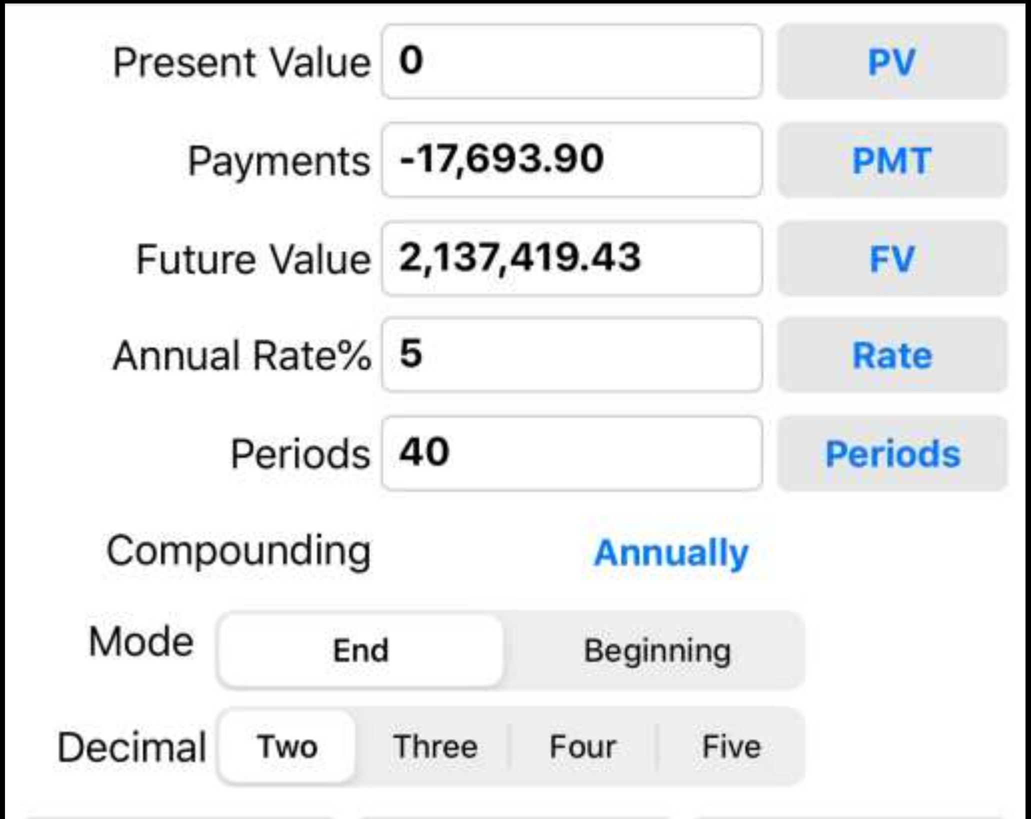
Example #1: Saving for Retirement – Part 2

In order to save that amount by your 65th birthday, you plan to save an equal amount each year for 40 years. We can use a future value annuity formula to calculate how much you need to save each year

$$FV = C/r\ [(1+r)^t - 1]$$

$$\$2{,}137{,}419.43 = \frac{C}{0.05}\left[(1.05)^{40} - 1\right] \Rightarrow C = \$17{,}693.90$$





Example 2: Mortgage Math

- Mortgage Basics
  - Loan Amount (Principal)
  - Down Payment (your “equity” contribution)
  - Amortization
    - Most typical: Fully Amortizing, Constant Payment Mortgage Loan
    - Fully amortizing simply means that there is no balloon payment at the end. After making all the payments, the mortgage will be fully paid off.
  - Fees: points, origination fees, prepayment fees

Example 2: Mortgage Math

A 30-year fully amortizing mortgage loan was originated 10 years ago for $1,000,000 at 5 percent interest (annually) where payments are made monthly.

(1) What is the monthly payment on the mortgage?

(2) The borrower has recently inherited some significant cash and would like to pay off the mortgage early (prepay the mortgage balance) in full. Assume that there is no prepayment penalty. How much would this cost today?

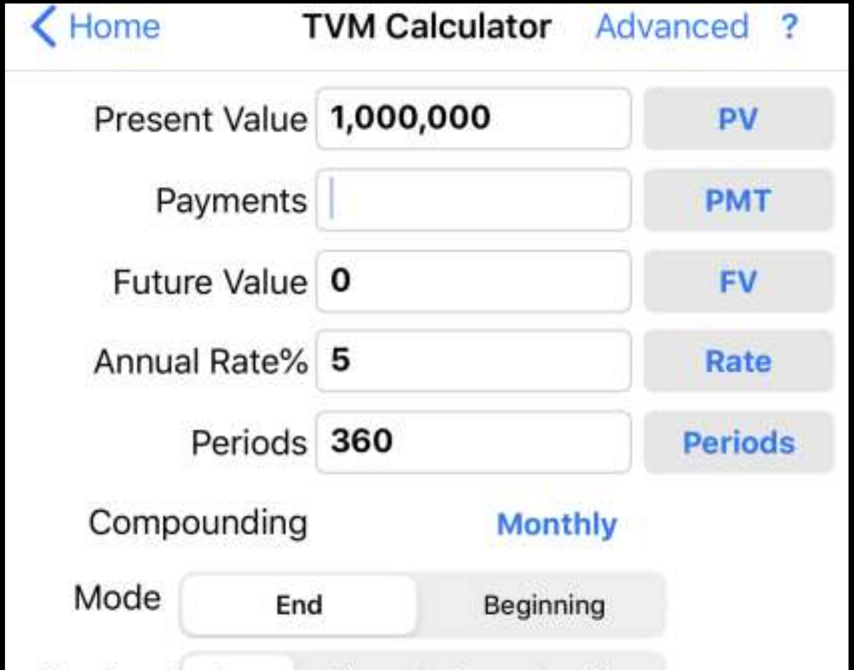
Example 2: Mortgage Math (continued)

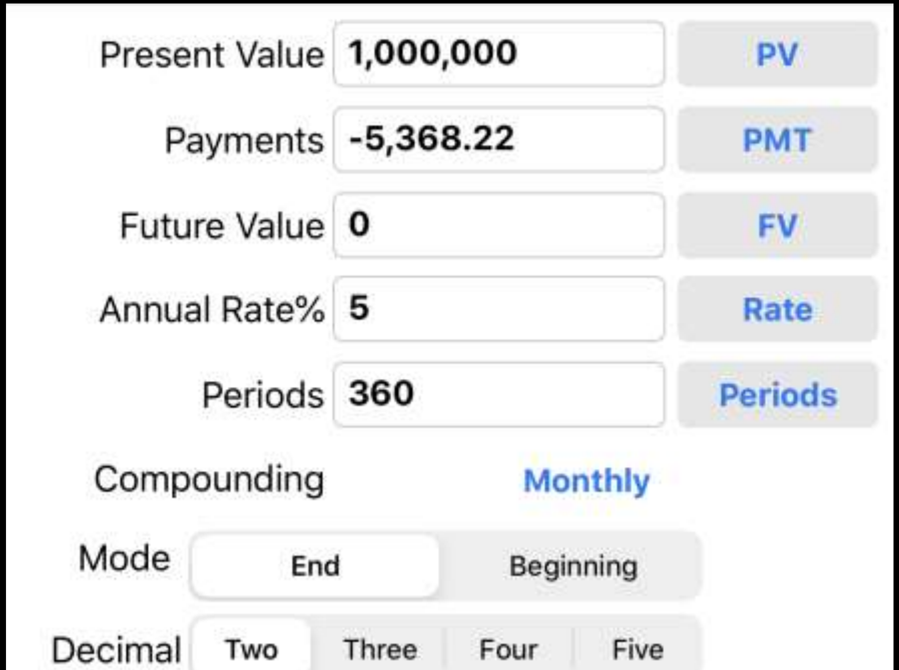
A 30-year fully amortizing mortgage loan was originated 10 years ago for $1,000,000 at 5 percent interest (APR simple annual rate) where payments are made monthly.

(1) What is the monthly payment on the mortgage?

$$\frac{C}{0.0041667}\left(1-\frac{1}{(1+0.0041667)^{360}}\right)=\$1{,}000{,}000$$

C=$5,368.22





Example 2, Part 2: Mortgage Math

- The borrower has recently inherited some significant cash and would like to pay off the mortgage early (prepay the mortgage balance) in full. How much would this cost today?

- You only pay interest to a bank on what you are borrowing.

- If you pay off the loan today, you will never owe any further interest.

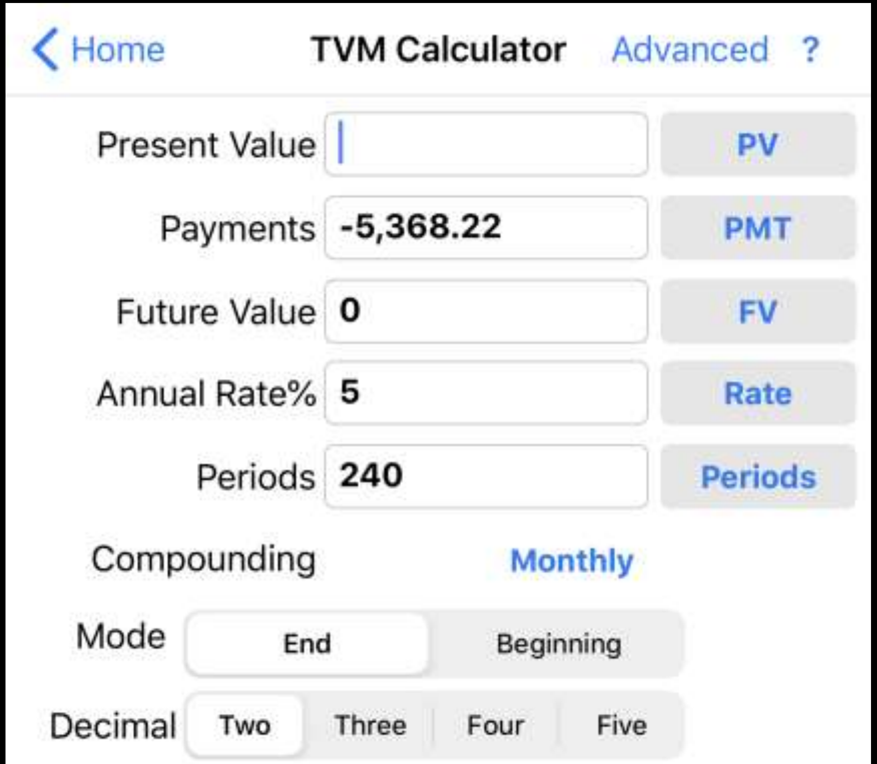
- So what we really need to know is how much of the original $1,000,000 loan principal is remaining after 10 years

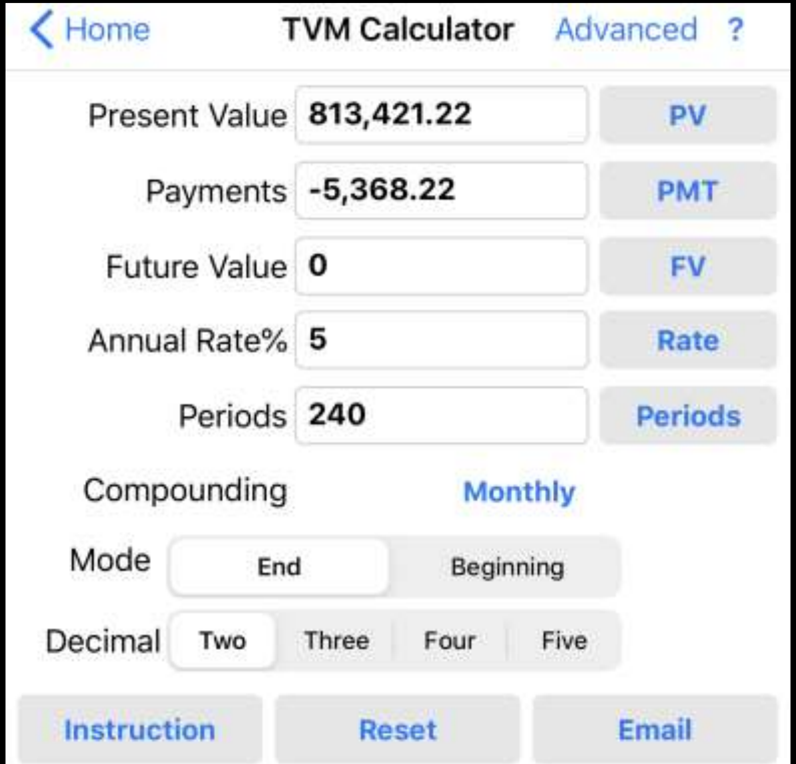
- If we just multiplied the monthly payment amount of $5,368.22 by the 240 remaining months on the loan (120 of the 360 payments have been made), that is a huge number. $1,288,372.80. Bigger than where we started.

Example 2: Mortgage Math (continued)

The bank that lent you the money expects 240 more monthly checks for $5,368.22 each. What is the present value of their asset? That would be the principal amount of your loan.

$$PV = \frac{\$5{,}368.22}{0.0041667}\left(1 - \frac{1}{(1+0.0041667)^{240}}\right) = \$813{,}421.22$$





Example 2: Mortgage Math

OBSERVATIONS:

1. This means that the borrower has only paid off \$1,000,000-\$813,421.22 = \$186,578.78 of the \$1,000,000 original loan principal in 10 years! But the borrower has made 120 payments of \$5,368.22 = \$644,186.40. \$457,607.62 has been interest to the bank.

2. The interest component of the first monthly payment on this \$1,000,000 mortgage was 0.0041667•\$1,000,000=\$4,166.7 and the remaining \$1,201.52 was retirement of principal

3. Because the principal on the loan slowly declines, the interest also declines.

Summary

- Finance is about time travel for money
- Determining the value of future cash flows requires discounting (due to time preference, inflation and credit risks)
- $PV = FV/(1+r)^t$
- Time to double   Rule of 72 $T = 72/r$
- Accept projects with a positive NPV (the sum of discounted future cash flows minus costs is positive)
- Nominal Interest rate = real interest rate + expected rate of inflation
- Annuity formula
- Practice with our financial calculator app