DSTL (BCS-303) — Unit 1

Previous Year Questions with Step-by-Step Solutions

ABES Engineering College

Prepared for Practice and Revision

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Contents

How to Use

Each PYQ is followed by a line-by-line solution. For Hasse diagrams, we explain the construction steps (levels/covers). Redraw neatly in your answer booklet.

1 2018–19 (III Semester Theory Exam) Unit 1

2018–19 Q1: Power Sets

Find the power set of each of the following: (i) $\{a\}$ (ii) $\{a, b\}$ (iii) $\{\varnothing, \{\varnothing\}\}$ (iv) $\{a, \{a\}\}$.

Solution

Goal: List all subsets for each set.
Key fact: If $|S| = n$, then $|P(S)| = 2^n$.
(i) $S = \{a\}$:

- Subsets: $\varnothing$ (take nothing), $\{a\}$ (take a).
- Hence $P(S) = \{\varnothing, \{a\}\}$.

(ii) $S = \{a, b\}$:

- Subsets: $\varnothing, \{a\}, \{b\}, \{a, b\}$ (choose each element: in/out).
- Hence $P(S)$ is these 4 subsets.

(iii) $S = \{\varnothing, \{\varnothing\}\}$:

- Treat $\varnothing$ and $\{\varnothing\}$ as *two distinct elements.*
- Subsets: $\varnothing$, $\{\varnothing\}$, $\{\{\varnothing\}\}$, $\{\varnothing, \{\varnothing\}\}$.

(iv) $S = \{a, \{a\}\}$:

- Again two distinct elements: the symbol a and the set $\{a\}$.
- Subsets: $\varnothing$, $\{a\}$, $\{\{a\}\}$, $\{a, \{a\}\}$.

2018–19 Q2: Hasse Diagram of D_{30}

Draw the Hasse diagram for the divisibility poset D_{30} (positive divisors of 30 ordered by $|$).

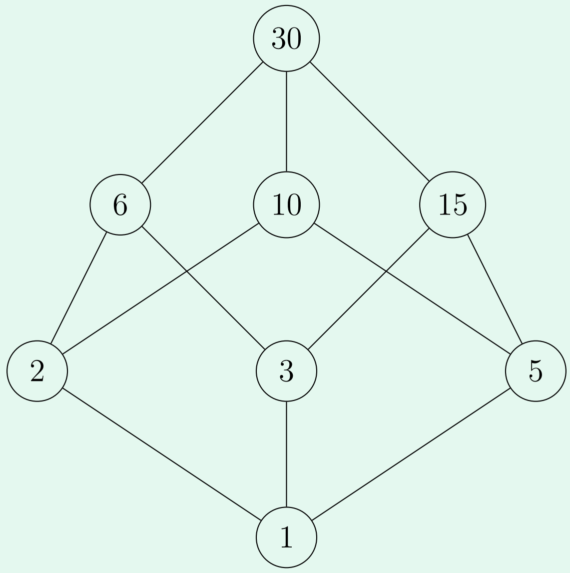
Solution

1. Elements: $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$.
2. Least element: 1, Greatest element: 30.
3. Covering relations (direct divisibility, no element strictly between):

$$1 \prec 2, 3, 5; \quad 2 \prec 6, 10; \quad 3 \prec 6, 15; \quad 5 \prec 10, 15; \quad 6, 10, 15 \prec 30.$$

4. Levels: $1 \mid (2, 3, 5) \mid (6, 10, 15) \mid 30$.
5. Draw by levels; connect only cover relations; omit transitive edges.

Hasse Diagram:



2018–19 Q3: Monotonicity in Lattices

Define a lattice. If $a \le b$ and $c \le d$ in a lattice $(A, \le)$, show $a \vee c \le b \vee d$ and $a \wedge c \le b \wedge d$.

Solution

Definition (Lattice). A poset $(A, \le)$ is a *lattice* if every pair $x, y \in A$ has a *least upper bound* $x \vee y$ (the *join*) and a *greatest lower bound* $x \wedge y$ (the *meet*).

Note for Students: What is Isotony?

Isotony is just a fancy word for *order-preserving*.
- If $a \le b$, then applying the same operation with another element c keeps the order. - For joins: $a \vee c \le b \vee c$. - For meets: $a \wedge c \le b \wedge c$.

Everyday example: - Adding the same number to two others keeps their order. If $2 \le 5$, then $2 + 3 \le 5 + 3$. - Multiplying by a positive number also keeps order: If $3 \le 7$, then $3 \cdot 2 \le 7 \cdot 2$.

In lattices: Isotony means that join ($\vee$) and meet ($\wedge$) never "break" the order — replacing an element with a bigger one will not make the result smaller.

Key fact (Isotony of $\vee$ and $\wedge$). For any $x \le y$ and any $z \in A$:

$$x \vee z \le y \vee z \quad \text{and} \quad x \wedge z \le y \wedge z.$$

Proof of the fact for $\vee$: Since $x \le y \le y \vee z$ and $z \le y \vee z$, the element $y \vee z$ is an upper bound of $\{x, z\}$; hence the least upper bound $x \vee z$ satisfies $x \vee z \le y \vee z$. The meet case is dual: because $x \wedge z$ is a lower bound of $\{x, z\}$ and any lower bound of $\{y, z\}$ is $\le y \wedge z$, from $x \le y$ we get $x \wedge z \le y \wedge z$.

Now prove the two inequalities.

1. *Join inequality $a \vee c \le b \vee d$:*

$$\begin{aligned} a \le b &\implies a \vee c \le b \vee c \quad &&\text{(isotony of } \vee) \\ c \le d &\implies b \vee c \le b \vee d \quad &&\text{(isotony of } \vee) \\ &\implies a \vee c \le b \vee d \quad &&\text{(transitivity).} \end{aligned}$$

2. *Meet inequality $a \wedge c \le b \wedge d$:*

$$\begin{aligned} c \le d &\implies a \wedge c \le a \wedge d \quad \text{(isotony of } \wedge\text{)} \\ a \le b &\implies a \wedge d \le b \wedge d \quad \text{(isotony of } \wedge\text{)} \\ &\implies a \wedge c \le b \wedge d \quad \text{(transitivity)}. \end{aligned}$$

Conclusion. In any lattice, join and meet are *monotone* in each argument; therefore $a \le b$ and $c \le d$ imply $a \vee c \le b \vee d$ and $a \wedge c \le b \wedge d$.

2018–19 Q4: $(P(S), \subseteq)$ as Poset and Lattice

Show $\subseteq$ is a partial order on $P(S)$. Draw the Hasse diagram for $(P(S), \subseteq)$ where $S = \{a, b, c, d\}$. Decide if it is a lattice.

Solution

1. Reflexive: $X \subseteq X$.
2. Antisymmetric: $X \subseteq Y$ and $Y \subseteq X \Rightarrow X = Y$.
3. Transitive: $X \subseteq Y$ and $Y \subseteq Z \Rightarrow X \subseteq Z$.
4. Hence $(P(S), \subseteq)$ is a poset.
5. Lattice: For $X, Y \subseteq S$, $X \vee Y = X \cup Y$, $X \wedge Y = X \cap Y \in P(S)$.
6. Hasse: Layer by set size $0 \to 4$; covers differ by one element.

2 2019–20 (III Semester Theory Exam) Unit 1

2019–20 Q1: Counting Relations

How many symmetric and reflexive relations are possible on a set A with $|A| = n$?

Solution

We are asked to count how many relations can be formed on a set A (with $|A| = n$) when the relation is:

1. Any relation,
2. Reflexive,
3. Symmetric.

Step 1: Total number of relations.

- A relation on A is any subset of $A \times A$.
- Since $|A \times A| = n^2$, each of the n^2 pairs can either be chosen or not chosen.
- Hence, the total number of possible relations is:

$$2^{n^2}.$$

Step 2: Number of reflexive relations.

- A relation is reflexive if all diagonal pairs (a, a) for $a \in A$ are included.
- There are n such diagonal pairs, and we have no choice (they must be in the relation).
- The remaining $n^2 - n$ pairs can be chosen freely.
- Hence, the total number of reflexive relations is:

$$2^{n^2-n}.$$

Step 3: Number of symmetric relations.

- A relation is symmetric if whenever (a, b) is included, then (b, a) is also included.
- Think of two cases:
  1. *Diagonal pairs:* (a, a) for $a \in A$. There are n of them, and each can either be included or excluded. $\Rightarrow 2^n$ choices.
  2. *Off-diagonal pairs:* (a, b) and (b, a) with $a \neq b$. These come in unordered pairs $\{a, b\}$, and for each such pair, either both are included or both are excluded. The number of such unordered pairs is $\binom{n}{2} = \frac{n(n-1)}{2}$. Each pair gives 2 choices $\Rightarrow 2^{\frac{n(n-1)}{2}}$.
- Therefore, the number of symmetric relations is:

$$2^{n+\frac{n(n-1)}{2}} = 2^{\frac{n(n+1)}{2}}.$$

For students:

- For *total relations*, every pair in $A \times A$ is independent, so each can be "in or out" $\Rightarrow 2^{n^2}$.
- For *reflexive relations*, diagonal pairs must always be there, so choices remain only for the others $\Rightarrow 2^{n^2-n}$.
- For *symmetric relations*, we group (a, b) with (b, a). This reduces the number of free choices to half for off-diagonals. That's why the formula becomes $2^{n(n+1)/2}$.

2019–20 Q2: Transitivity of $|$; Closures of $R = \{(a, b) : a > b\}$

Is "divides" transitive? Find reflexive and symmetric closures of R on positive integers.

Solution

1. Transitivity of $|$: $a \mid b$, $b \mid c \Rightarrow b = ak$, $c = b\ell = ak\ell \Rightarrow a \mid c$.
2. Reflexive closure: $R \cup \{(x, x) : x \in \mathbb{Z}^+\}$.
3. Symmetric closure: $R \cup R^{-1} = \{(x, y) : x \neq y\}$ (since for $x \neq y$, either $x > y$ or $y > x$).

2019–20 Q3: Distributive Lattice Uniqueness

Let $(L, \vee, \wedge)$ be distributive, $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$. Prove $b = c$.

Solution

1. $b = b \wedge (a \vee c) = (b \wedge a) \vee (b \wedge c) = (a \wedge b) \vee (b \wedge c)$.
2. Replace $a \wedge b$ by $a \wedge c$: $b = (a \wedge c) \vee (b \wedge c) = c \wedge (a \vee b)$.
3. Replace $a \vee b$ by $a \vee c$: $b = c \wedge (a \vee c) = c$. Hence $b = c$.

3 2020–21 (III Semester Theory Exam) Unit 1

2020–21 Q1: Counting Reflexive & Symmetric Relations

Let R be a relation on A, $|A| = n$. Find number of reflexive and symmetric relations on A.

Solution

Same as 2019–20 Q1: Reflexive $= 2^{n^2-n}$; Symmetric $= 2^{\frac{n^2+n}{2}}$.

2020–21 Q2: Hasse Diagram on Divisibility

Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ with order $|$. Draw the Hasse diagram.

Solution

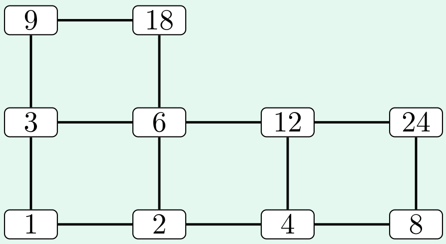


Figure 1: Hasse diagram of $(A, |)$ for $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$. Edges show covers under divisibility.

1. Levels: $1 \,|\, (2, 3) \,|\, (4, 6, 8, 9) \,|\, (12, 18) \,|\, 24$.
2. Covers (non-transitive edges only):

$$1 \prec 2, 3; \quad 2 \prec 4, 6, 8; \quad 3 \prec 6, 9; \quad 4 \prec 12; \quad 6 \prec 12, 18; \quad 8 \prec 24; \quad 9 \prec 18; \quad 12 \prec 24.$$

3. Note: $18 \nmid 24$, so no edge $18 \to 24$.

2020–21 Q3: $a \wedge b = a \iff a \le b$

In a lattice L, prove $a \wedge b = a$ iff $a \le b$.

Solution

1. $(\Rightarrow)$ If $a \wedge b = a$, then a is a lower bound of $\{a, b\}$ equal to the meet, hence $a \le b$.
2. $(\Leftarrow)$ If $a \le b$, then greatest lower bound of $\{a, b\}$ is a, so $a \wedge b = a$.

2020–21 Q4: Distributive Inequalities in Any Lattice

Prove: (i) $a \wedge (b \vee c) \ge (a \wedge b) \vee (a \wedge c)$; (ii) $a \vee (b \wedge c) \le (a \vee b) \wedge (a \vee c)$.

Solution

1. Since $b \le b \vee c$ and $c \le b \vee c$, by isotony of $\wedge$: $a \wedge b \le a \wedge (b \vee c)$ and $a \wedge c \le a \wedge (b \vee c)$.
2. Taking join: $(a \wedge b) \vee (a \wedge c) \le a \wedge (b \vee c)$.

3. Duality gives (ii): $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$.

4 2021–22 (III Semester Theory Exam) Unit 1

2021–22 Q1: Relation Properties

Let $A = \{1, 2, 3, 4, 5, 6\}$ and

$$R = \{(1,1),(1,5),(2,2),(2,3),(2,6),(3,2),(3,3),(3,6),(4,4),(5,1),(5,5),(6,2),(6,3),(6,6)\}.$$

Check reflexive, symmetric, antisymmetric, transitive.

Solution

1. Reflexive: (x, x) for all x present $\Rightarrow$ Yes.
2. Symmetric: every (a, b) has (b, a) (e.g., $1 \leftrightarrow 5$, $2 \leftrightarrow 3$, $2 \leftrightarrow 6$, $3 \leftrightarrow 6$) $\Rightarrow$ Yes.
3. Antisymmetric: fails since $(1, 5)$ and $(5, 1)$ with $1 \neq 5 \Rightarrow$ No.
4. Transitive: blocks $\{1, 5\}$, $\{2, 3, 6\}$, and $\{4\}$ show equivalence-like structure; sample checks: $(2, 3)$ & $(3, 6) \Rightarrow (2, 6)$, $(2, 6)$ & $(6, 3) \Rightarrow (2, 3)$, $(1, 5)$ & $(5, 1) \Rightarrow (1, 1)$. All compositions required are present $\Rightarrow$ Yes.

2021–22 Q2: Complemented vs Distributive Lattice

Differentiate complemented lattice and distributive lattice.

Solution

1. Complemented lattice: bounded; each x has some y with $x \wedge y = 0$, $x \vee y = 1$ (complement may be non-unique).
2. Distributive lattice: satisfies distributive laws; if a complement exists, it is unique.
3. Examples: Boolean algebra $(P(S), \subseteq)$ is both; M_3 is complemented but not distributive.

2021–22 Q3: Duality; Equivalence Relation

State Principle of Duality. Prove $(a, b)R(c, d) \iff a^2 + b^2 = c^2 + d^2$ is an equivalence on $\mathbb{R}^2$.

Solution

1. Duality: swap $\wedge \leftrightarrow \vee$, $0 \leftrightarrow 1$, reverse order—valid theorems remain valid.
2. Reflexive: $a^2 + b^2 = a^2 + b^2$.
3. Symmetric: equality is symmetric.
4. Transitive: equality is transitive.

2021–22 Q4: Modular Lattice & Law

Define a modular lattice. Prove the modular law with example.

Solution

We want to show the **Modular Law**: If $a \leq c$ in a lattice, then

$$a \vee (b \wedge c) \ = \ (a \vee b) \wedge c.$$

Step 1. The left-hand side is $a \vee (b \wedge c)$. Since $a \leq c$, and $b \wedge c \leq c$, their join must also be $\leq c$. So $a \vee (b \wedge c) \leq c$.
Step 2. On the right-hand side, $(a \vee b) \wedge c$ is the greatest element that is $\leq c$ and $\leq a \vee b$. But $a \vee (b \wedge c)$ is also $\leq c$ and $\leq a \vee b$. Hence,

$$a \vee (b \wedge c) \ \leq \ (a \vee b) \wedge c.$$

Step 3. For the reverse inequality, note that $(a \vee b) \wedge c$ is $\geq a$ and also $\geq b \wedge c$. Therefore it is $\geq a \vee (b \wedge c)$.
Conclusion. Both inequalities together give

$$a \vee (b \wedge c) = (a \vee b) \wedge c.$$

Example. In the lattice of sets $(\mathcal{P}(X), \subseteq)$, join = union, meet = intersection. Take $a = \{1\}$, $b = \{2, 3\}$, $c = \{1, 2, 3, 4\}$. Then

$$a \cup (b \cap c) = \{1, 2, 3\} = (a \cup b) \cap c.$$

Thus modular law holds. Another important example: subspaces of a vector space with join = span, meet = intersection always form a modular lattice.

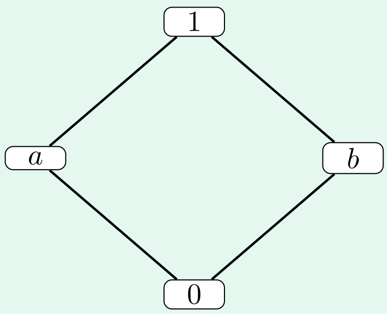


Figure 2: A distributive (hence modular) lattice B_2. For every $a \leq c$, the modular law $a \vee (b \wedge c) = (a \vee b) \wedge c$ holds.

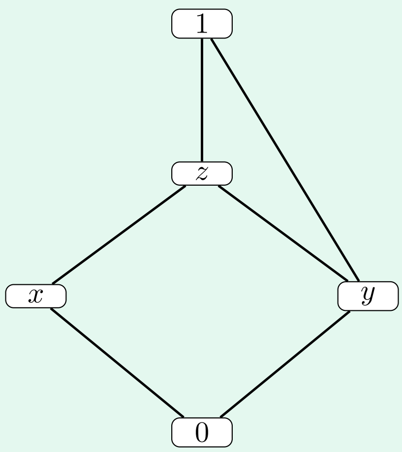


Figure 3: The lattice N_5 is *not* modular. One counterexample: take $a = x$, $b = y$, $c = z$ (note $a \leq c$). Then $b \wedge c = y \wedge z = y$, so $a \vee (b \wedge c) = x \vee y = 1$. But $a \vee b = x \vee y = 1$ and $(a \vee b) \wedge c = 1 \wedge z = z \neq 1$. Hence $a \vee (b \wedge c) \neq (a \vee b) \wedge c$.

5 2022–23 (III Semester Theory Exam) Unit 1

2022–23 Q1: Extremal Elements from a Hasse Diagram

From the given Hasse diagram, find all maximal and minimal elements.

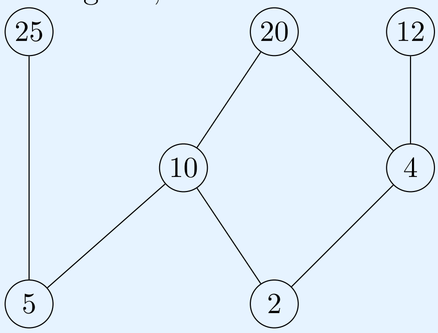


Figure 4: Hasse diagram for the divisibility poset on $\{2, 4, 5, 10, 12, 20, 25\}$.

Solution

Method to read any Hasse diagram:

- Minimal elements: vertices with no downward edge (no lower cover).
- Maximal elements: vertices with no upward edge (no upper cover).
- If exactly one minimal (resp. maximal) exists, it is the least (resp. greatest) element.

Apply this checklist to the given diagram and list the nodes accordingly.

2022–23 Q2: Hasse Diagram for $(L, \subseteq)$

$L = \{S_0, \ldots, S_7\}$ with
$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$,
$S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$, $S_7 = \{a\}$. Draw Hasse diagram.

Solution

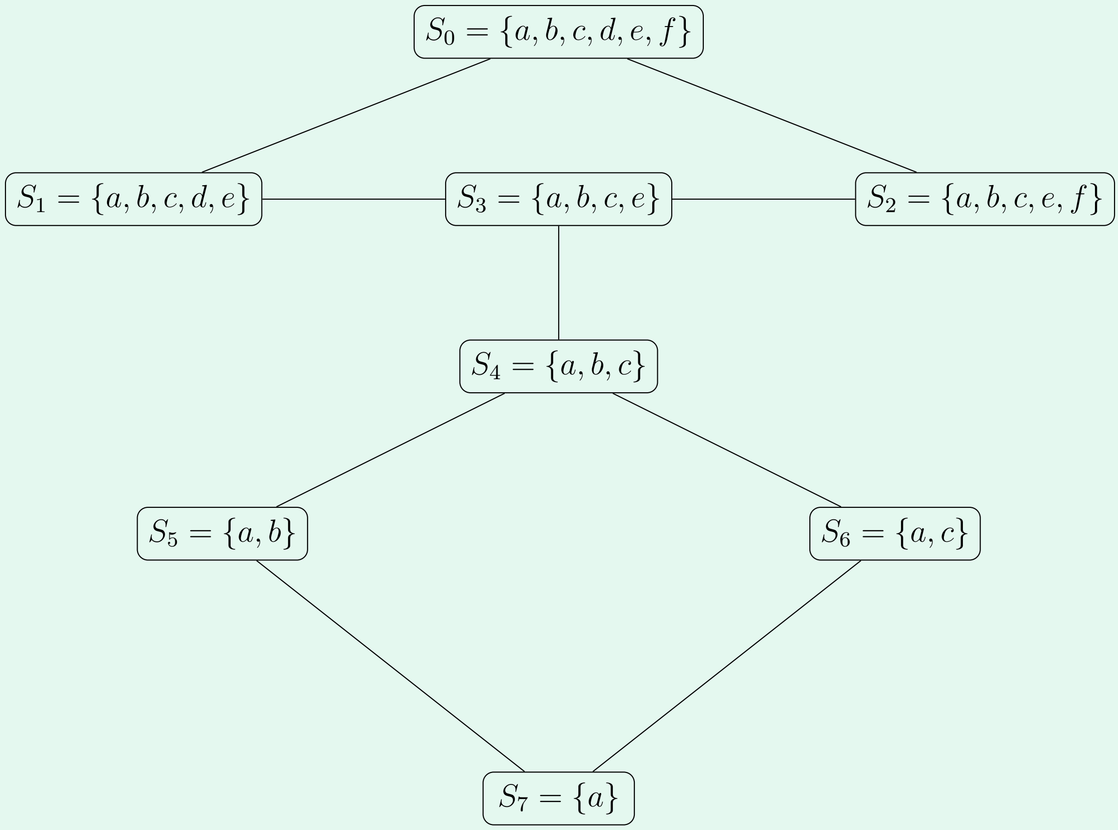


Figure 5: Hasse diagram of $(L, \subseteq)$ drawn by cardinality layers; edges are covers.

Layer by size (cardinality):

$$|S_7| = 1; \quad |S_5| = |S_6| = 2; \quad |S_4| = 3; \quad |S_3| = 4; \quad |S_1| = |S_2| = 5; \quad |S_0| = 6.$$

Covers under $\subseteq$:

$$S_7 \prec S_5, S_6; \quad S_5 \prec S_4; \quad S_6 \prec S_4; \quad S_4 \prec S_3, S_1; \quad S_3 \prec S_2; \quad S_1, S_2 \prec S_0.$$

Draw tip: Arrange by layers from bottom (S_7) to top (S_0) and connect only the covers above.

2022–23 Q3: Classify Relations on $X = \{1, 2, 3, 4\}$

(i) $R_1 = \{(1, 1), (1, 2), (2, 1)\}$; (ii) $R_2 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$; (iii) $R_3 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$. Decide reflexive, symmetric, transitive, antisymmetric.

Solution

(i) $R_1 = \{(1, 1), (1, 2), (2, 1)\}$:

- Reflexive: No (missing $(2, 2), (3, 3), (4, 4)$).
- Symmetric: Yes ($1 \leftrightarrow 2$).
- Transitive: No ($2, 1$ and $1, 2$ would require $(2, 2)$).
- Antisymmetric: No ($1 \leftrightarrow 2$ with $1 \neq 2$).

(ii) $R_2 = \{(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)\}$:

- Reflexive: Yes (all loops present).
- Symmetric: Yes ($1 \leftrightarrow 2$ and $1 \leftrightarrow 4$).
- Transitive: No (from $(2,1)$ and $(1,4)$ we need $(2,4)$, absent).
- Antisymmetric: No (has $1 \leftrightarrow 2$).

(iii) $R_3 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$:

- Reflexive: No (no loops).
- Symmetric: No (e.g., $(2,1)$ but not $(1,2)$).
- Transitive: Yes (all needed compositions such as $(3,2)$ with $(2,1) \Rightarrow (3,1)$ are present).
- Antisymmetric: Yes (no pair with both directions for $a \neq b$).

2022–23 Q4: Complement Uniqueness in Distributive Lattice

Define complemented lattice. Show that in a distributive lattice, if an element has a complement, then it is unique.

Solution

Assume x has complements y and z:

$$x \wedge y = 0, \quad x \vee y = 1, \qquad x \wedge z = 0, \quad x \vee z = 1.$$

Then, using distributivity and absorption,

$$y = y \wedge 1 = y \wedge (x \vee z) = (y \wedge x) \vee (y \wedge z) = 0 \vee (y \wedge z) = y \wedge z,$$

$$y = y \vee 0 = y \vee (x \wedge z) = (y \vee x) \wedge (y \vee z) = 1 \wedge (y \vee z) = y \vee z.$$

The only way to have $y = y \wedge z$ and $y = y \vee z$ is $y = z$. Hence the complement (if it exists) is unique.

2022–23 Q5: $(D_{36}, |)$ is a Lattice; $D_6 \simeq P(\{a,b\})$

Justify $(D_{36}, |)$ is a lattice. Show D_6 is isomorphic to $(P(\{a,b\}), \subseteq)$.

Solution

(i) Lattice property. For divisors d_1, d_2 of 36, both $\gcd(d_1,d_2)$ and $\operatorname{lcm}(d_1,d_2)$ are also divisors of 36. Under $|$, meet $=$ gcd and join $=$ lcm. Thus every pair has meet and join $\Rightarrow$ a lattice.

(ii) Isomorphism $D_6 \cong P(\{a,b\})$.

- $D_6 = \{1,2,3,6\}$ with order $|$. $P(\{a,b\}) = \{\varnothing, \{a\}, \{b\}, \{a,b\}\}$ with $\subseteq$.
- Define $\phi : 1 \mapsto \varnothing,\ 2 \mapsto \{a\},\ 3 \mapsto \{b\},\ 6 \mapsto \{a,b\}$.
- ϕ is bijective; and $d_1 \mid d_2 \iff \phi(d_1) \subseteq \phi(d_2)$ (preserves order).

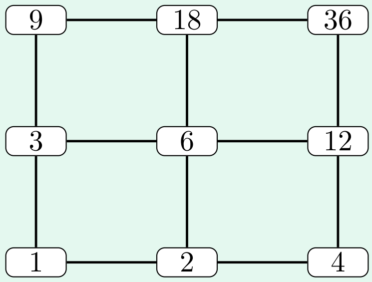


Figure 6: Hasse diagram of $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ under divisibility.

Therefore D_6 and $P(\{a, b\})$ are isomorphic lattices.