Functions in Discrete Mathematics

Prepared for 2nd-Year Students

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What is a Function?

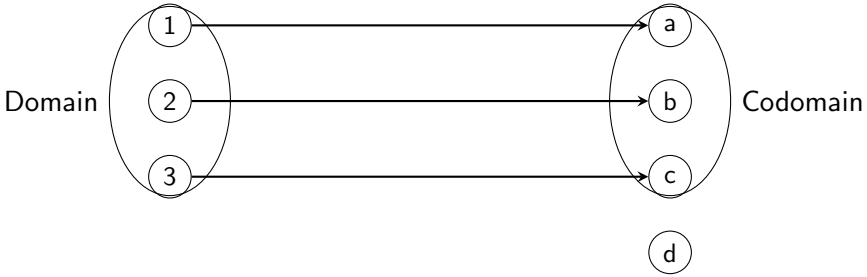
- A function $f : A \to B$ maps each $a \in A$ to exactly one $b \in B$.
- **Domain:** set of inputs.
- **Codomain:** set of possible outputs.
- **Range:** actual outputs obtained.

Injective (One-to-One)

Definition

A function is injective if different inputs map to different outputs.

- $f(x) = 2x + 3$ on $\mathbb{R}$
- Roll number $\rightarrow$ student
- $f : \{1, 2, 3\} \rightarrow \{a, b, c, d\}, f(1) = a, f(2) = b, f(3) = c$

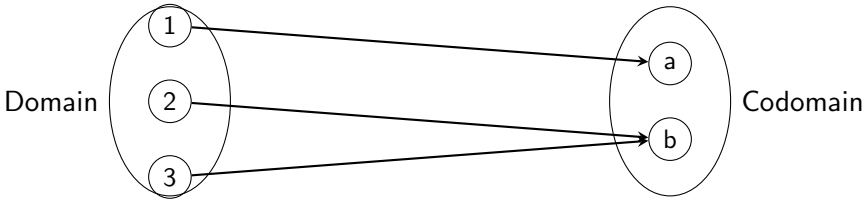


Surjective (Onto)

Definition

A function is surjective if every element of the codomain has a preimage.

- $f(x) = x^3 : \mathbb{R} \to \mathbb{R}$
- Students $\to$ birth months
- $f : \mathbb{Z} \to \{0, 1\}, f(n) = n \bmod 2$

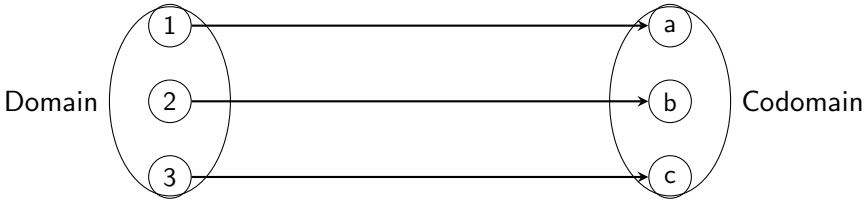


Bijective (One-to-One Correspondence)

Definition

A function is bijective if it is both injective and surjective.

- $f(x) = x + 5$ on $\mathbb{R}$
- Students $\leftrightarrow$ Seats
- $f : \{1, 2, 3\} \to \{a, b, c\}, f(1) = a, f(2) = b, f(3) = c$



Identity Function

Definition

The identity function returns each element unchanged: $f(x) = x$.

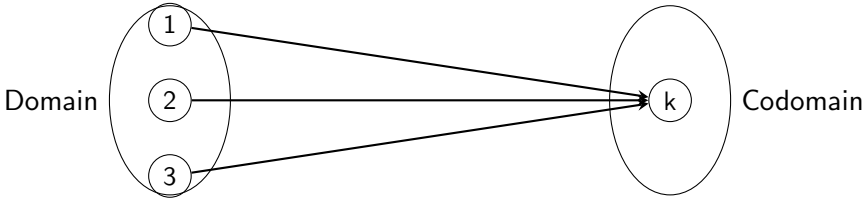
- $f(x) = x$ on $\mathbb{R}$
- Student $\rightarrow$ same student
- On $\{a, b, c\}$: $f(a) = a, f(b) = b, f(c) = c$

Constant Function

Definition

A constant function maps all inputs to the same output.

- $f(x) = 5$
- Every student $\rightarrow$ grade A
- $\{1, 2, 3\} \rightarrow \{k\}$



Projection Function

Definition

A projection function selects one component from a tuple.

- $\pi_1(x, y) = x$
- $(x, y, z) \mapsto y$
- $(roll, name) \mapsto roll$

Inverse Function

Definition

If f is bijective, its inverse f^{-1} reverses the mapping.

- $f(x) = 2x + 3$, inverse $f^{-1}(y) = (y - 3)/2$
- Celsius $\leftrightarrow$ Fahrenheit
- Seat number $\leftrightarrow$ Student