“Commuting graph associated with algebraic structure”

Special Project (MATH F491)

Presented by-

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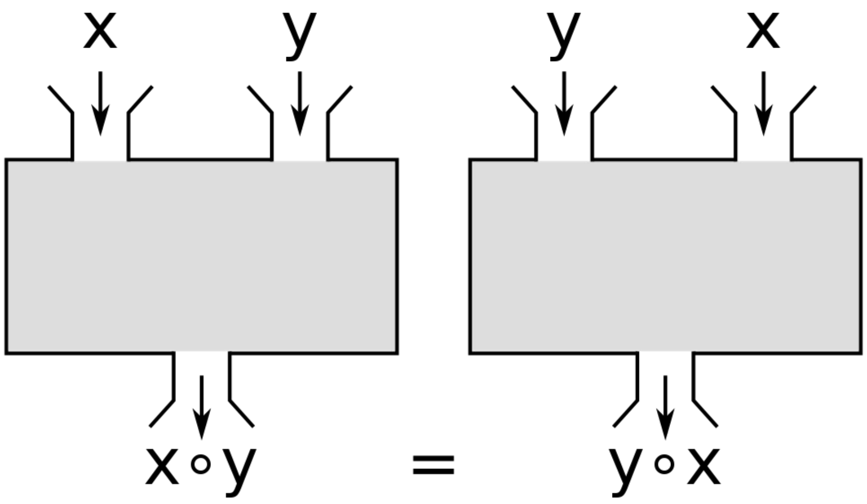
ABOUT THE PROJECT

PRE-MID-SEMESTER WORK

Commuting graph for group structure V_{8n}

Commutative property: Two elements (*x* and *y*) of any group G are said to **commute** iff $x \circ y = y \circ x$ where “o” is the binary operation associated with the group.



Commuting graph for group structure V_{8n}

Commuting Graph: Given a finite group G and a subset X of G, the commuting graph of G on X, denoted by C(G,X) is the graph that has X as its vertex set with x, y $\in$ X joined by an edge whenever $x \neq y$ and $x \circ y = y \circ x$ *(i.e. elements commute w.r.t. the binary operation* o)

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Commuting graph for group structure V_{8n}

V_{8n} group

The group V_{8n} is defined as:

$$\boxed{V_{8n} = \langle a, b \mid a^{2n} = b^4 = e,\ ba = a^{-1} b^{-1},\ b^{-1} a = a^{-1} b \rangle}$$

Note that $|V_{8n}| = 8n$.

Elements of $V_8 = \{ e, a, b, b^2, b^3, ab, ab^2, ab^3 \}$

(for n = 1)

Since, $ba = a^{-1}b^{-1}$ and $b^{-1}a = a^{-1}b$, one can observe that:

- $b^2a^i = a^ib^2$
- $ba^i = a^{2n-i}b$; if i is even

  $a^{2n-i}b^3$; if i is odd
- $b^3a^i = a^{2n-i}b^3$; if i is even

  $a^{2n-i}b$; if i is odd

Thus, every element of $V_{8n} \setminus \langle a \rangle$ is of the form a^ib^j for some i, j, where $1 \leq i \leq 2n$ and $1 \leq j \leq 3$

Commuting graph for group structure V_{8n}

<u>Center of a group:</u> The center of a group is the set of elements which commute with every element of the group. It is equal to the intersection of the centralizers of the group elements.

Examples:

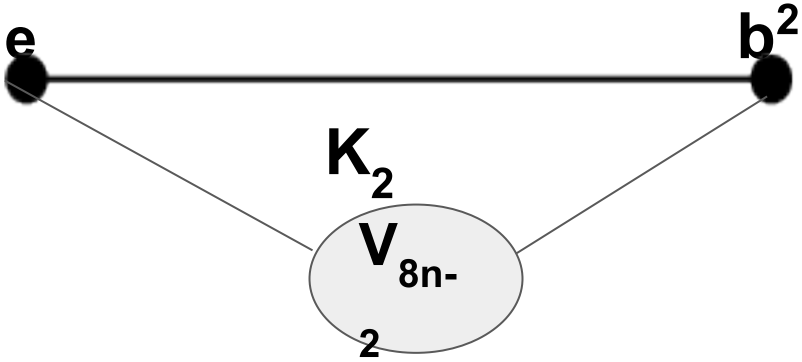
- **The center of an abelian group, *G*, is all of *G*.**
- **The center of a nonabelian simple group is trivial.**

Center of group V_{8n}

Case1: if n is odd

$$Z_{8n} = \{ e, b^2 \}$$

(Where e is the identity element)



$$V_{8n-2} = V_{8n} \sim Z_{8n}$$

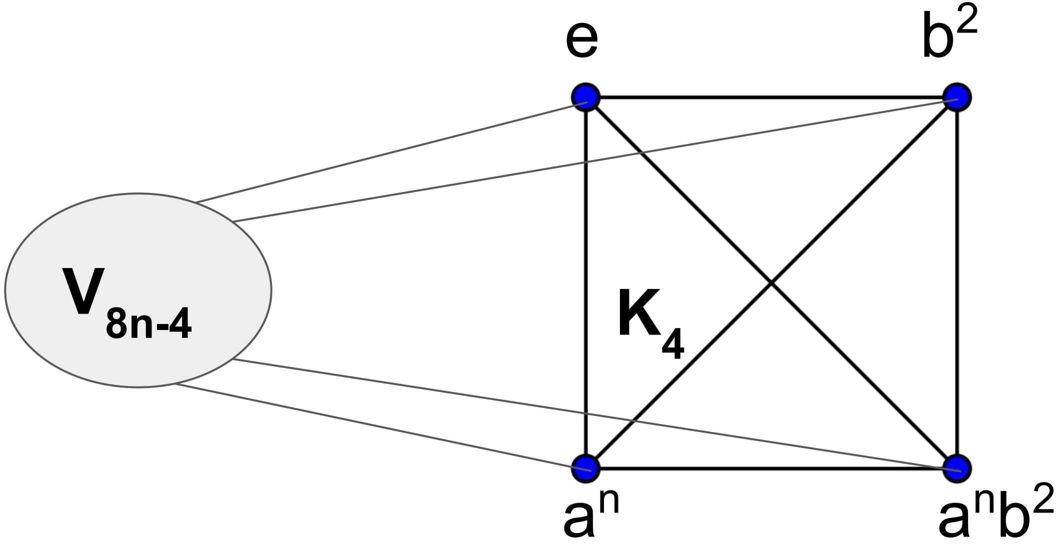
K_2 : Complete graph with two nodes

Case2: if n is even

$$Z_{8n} = \{ e, b^2, a^n, a^nb^2 \}$$

(Where e is the identity element)

$$\mathbf{V_{8n-4} = V_{8n} \sim Z_{8n}}$$



K_4 : Complete graph with four nodes

POST-MID SEMESTER

Commuting graph for group structure V_{8n}

<u>The neighbourhood of a vertex:</u> The **neighbourhood** of a vertex *v* in a graph *G* is the subgraph of *G* induced by all vertices adjacent to *v*, i.e., the graph composed of the vertices adjacent to *v* and all edges connecting vertices adjacent to *v. (N[x])*

$N[e] = V_{8n}$ (for both n even and odd)

(Where e is the identity element)

Case 1: n is ODD

Center of V_{8n} for n odd: $Z_{8n} = \{ e, b^2 \}$

- $N[e] = V_{8n}$
- $N[b] = \{e, b, b^2, b^3\}$
- $N[b^2] = V_{8n}$ (Since, $b^2 \in Z_{8n}$)
- $N[b^3] = \{e, b, b^2, b^3\}$
- $N[a^i b] = \{ e, b^2 \} \cup \{a^i b\} \cup \{a^i b^3\}$
- $N[a^i b^2] = \{ e, b^2 \} \cup \{a^i\} \cup \{a^i b^2\}$
- $N[a^i b^3] = \{ e, b^2 \} \cup \{a^i b^3\} \cup \{a^i b\}$
- $N[a^i] = \{ e, b^2 \} \cup \{ a^i \} \cup \{ a^i b^2 \}$

Case 2: n is EVEN

Center of V_{8n} for n even: $Z_{8n} = \{ e, b^2 , a^n, a^nb^2 \}$

- $N[e] = V_{8n}$
- $N[b] = \{e, b, b^2, b^3, a^nb, a^nb^2, a^nb^3, a^n\}$
- $N[b^2] = V_{8n}$ (Since, $b^2 \in Z_{8n}$)
- $N[b^3] = \{e, b, b^2, b^3, a^nb, a^nb^2, a^nb^3, a^n \}$
- $N[a^ib] = \{ e, b^2, a^n, a^nb^2 \} \cup \{a^ib\} \cup \{a^ib^3\}$
- $N[a^ib^2] = V_{8n}$; for i = n

  $= \{ e, b^2 \} \cup \{a^i\} \cup \{a^ib^2\}$; otherwise
- $N[a^ib^3] = \{ e, b^2, a^n, a^nb^2 \} \cup \{a^ib\} \cup \{a^ib^3\}$
- $N[a^i] = V_{8n}$; for i = n

  $= \{ e, b^2, a^n, a^nb^3 \} \cup \{ a^ib^2 \} \cup \{ a^i \}$; otherwise

PROPERTIES OF V_{8n}

<u>Edge connectivity:</u>

Edge connectivity: The minimum number of edges whose detention from a graph disconnects the graph.

Case 1: n even <u>Edge connectivity = 4</u>

Case 2: n odd <u>Edge connectivity = 3</u>

PROPERTIES of V_{8n}

Vertex connectivity

Vertex connectivity: The minimum number of vertices whose deletion from a graph disconnects the graph. Also referred to as point connectivity.

Case 1: n even

Vertex connectivity = 4

Case 2: n odd

Vertex connectivity = 2

PROPERTIES of V_{8n}

<u>Clique Number</u>

Clique number: The size of the largest complete subgraph(clique) in the given graph.

<u>**Clique number = 4n (for both odd and even)**</u>

<u>Independence Number</u>

Independent set: A set of vertices of the graph, no two of which are adjacent.

Independence number: The cardinality of the largest independent vertex set.

Case 1: n even

<u>Independence number = 2n -1</u>

Case 2: n odd

<u>Independence number = 2n+1</u>

PROPERTIES of V_{8n}

Vertex covering number

Vertex covering number: The size of the minimum vertex cover (A vertex cover is a set of vertices such that each edge of the graph is incident to at least one vertex of the set)

Vertex cover number = 6n (for both odd and even)

Matching Number

Matching number: The size of maximum vertex cover (A vertex cover is a set of vertices such that each edge of the graph is incident to at least one vertex of the set)

Case 1: n even

Matching number = 4n-2

Case 2: n odd: 4n-1

Matching number = 4n-1

Properties of V_{8n}

Edge cover number

Edge covering number: The size of the minimum edge cover (An edge cover of a graph is a set of edges such that every vertex of the graph is incident to at least one edge of the set)

Edge cover number = 4n (for both n even and odd)

Eulerian Graph

Eulerian graph: An Eulerian trail (or Eulerian path) is a trail in a finite graph that visits every edge exactly once. Similarly, a Eulerian circuit or Eulerian cycle is a Eulerian trail that starts and ends on the same vertex. A Eulerian graph is a graph that contains a Eulerian cycle.

References

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THANK YOU