

On Collatz Conjecture: Branching and Node Values

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Abstract

This paper delves into the Collatz Conjecture, specifically the generalized formula for nodes on a particular branch. This formula is derived from the pattern of the node values. It explores the relationship between the values of consecutive branches, both vertically and diagonally. The entire Collatz tree is divided into three main components: the Base Stem, Nodes, and Branches that emerge from the nodes. While the pattern for branches on the base stem is known, the pattern for other branches cannot be determined. The paper details these concepts and provides examples to illustrate them.

Keywords: Collatz Conjecture, Tree, Node, Branch

Introduction

Collatz Conjecture is a problem in which we end in a loop of 4-2-1 after applying two operations based on the number being odd or even.

$$a(n) = \begin{cases} 3n + 1, & n = \text{odd} \\ n/2, & n = \text{even} \end{cases}$$

-Eqn. 1

Ex: 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 15, 4, 2, 1, 4, 2, 1, 4, 2, 1, ...

In this example, which applies to all natural numbers, the sequence eventually falls into an infinite loop of 4-2-1. The number of steps required for an integer to either enter this 4-2-1 loop or reach 1 is known as the stopping time. This stopping time can vary widely, sometimes being a single digit or multiple digits, and appears random. Interestingly, it can be higher for smaller values and lower for larger values, or vice versa. For instance, it takes 12 steps for (n = 17), 111 steps for (n = 27), but only 10 steps for (n = 1024) to reach 1. For values of 2^n , it takes exactly n steps. Importantly, the stopping time is finite for any number.

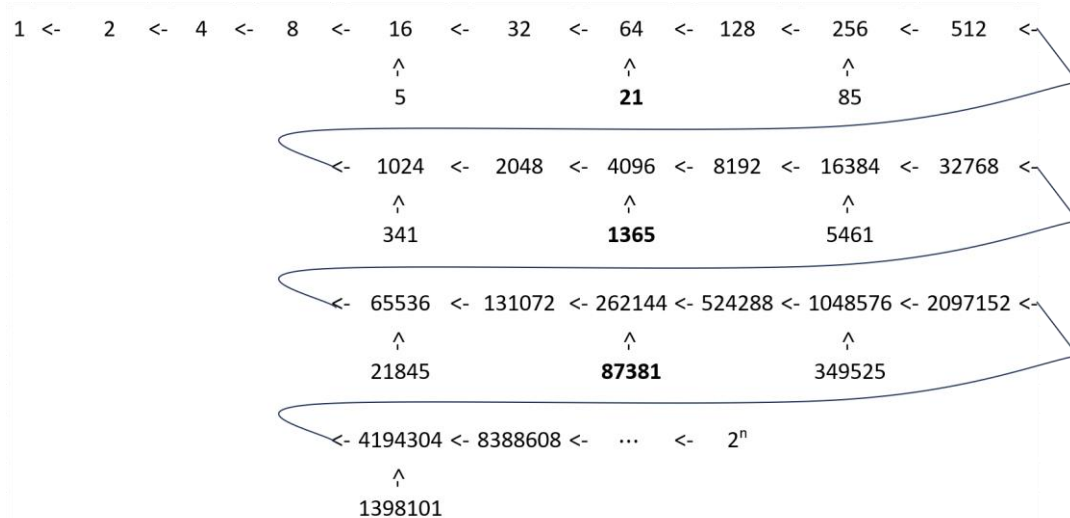
Base Stem and Powers of 2

If whole tree of Collatz conjecture is considered, a certain distinction can be made by looking at the pattern and the odd-even conditions. In the

examples presented below, see that every number leads to certain value which is power of 2 and from there it gets divided by 2 till it reaches 1.

- 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1 (15 steps)
- 16384, 8192, 4096, 2048, 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1 (14 steps)
- 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 (17 steps)

Additionally, consider other values: an even number can be obtained by dividing a number that is even or specifically a multiple of 2^n or by $(3x + 1)$. However, an odd number can only be achieved through division by 2. Therefore, values of 2^n should be considered as the primary branch or Base Stem since they require the fewest steps. Another reason for this is that it simplifies visualization, making it more linear and straightforward as the tree grows from the base stem, which then leads to multiple branches.



From here, R is considered as Root Term with following value and condition:

$$R = 2^{2n+4}, n \geq 1 \quad \text{-Eqn. 2}$$

Following observations can be drawn from this base stem:

1. The values $\frac{2^{n-1}}{3}$ when are divisible by 3, they lead to values or form branch which do not branch further. These are 2^{6n} values of the base stem.
2. Values 2^{4n} or powers of 2 which are multiples of 4 (i.e., 16^n) leads to branch which is multiple of 5 ($\frac{2^{4n-1}}{3}$ is multiple of 5).
3. Values which are 2^{6n-2} and 2^{6n+2} forms branch which further branch on odd and even nodes.

Visualization is crucial for understanding such problems, as it provides clearer explanations of arrangements. The Collatz conjecture can be visualized in various ways. Some visualizations depict stopping time, showing all values falling below a certain function's limit. Others illustrate a growing tree or display only odd numbers in a directed graph. A new visualization approach could involve creating a tree where each branch grows based on powers of 2. This new method can offer deeper insights into the values of nodes and their relationships with other branches.

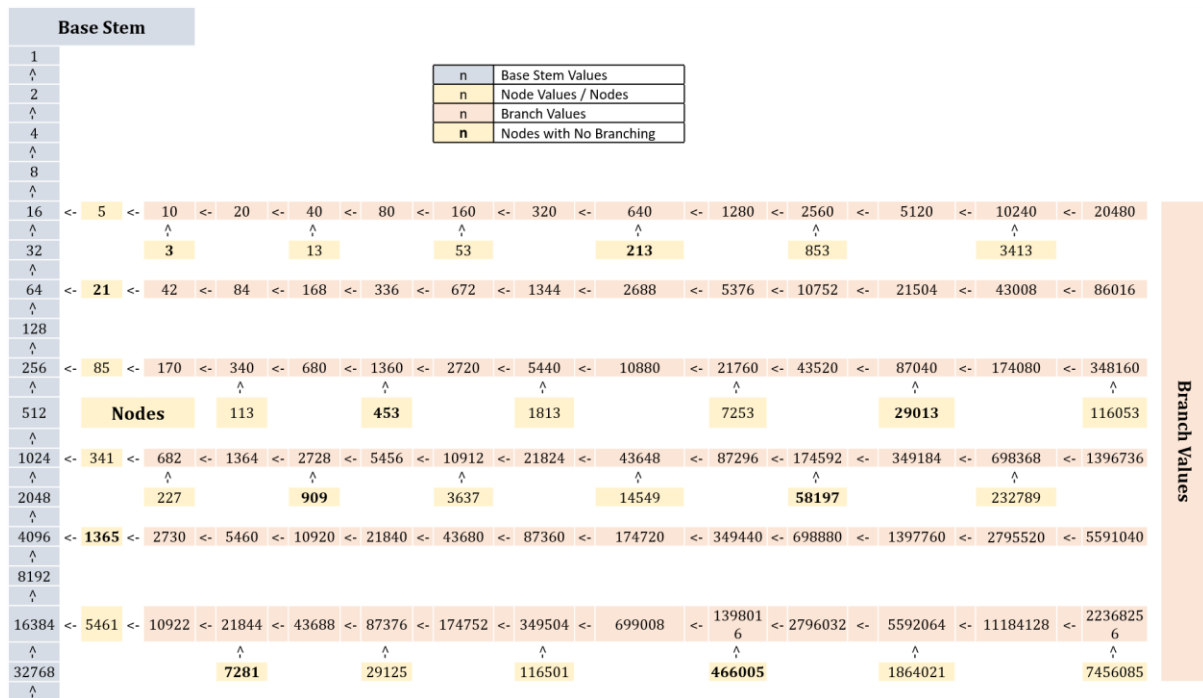


Fig. 1: Tree Structure

Nodes and Branches

It is assumed that all node values are at position $0(value * 2^0 = value)$. Nodes are value which raises whole branch. This can be made in two forms: First being by Base Stem values and other being formed from branch values.

$$Node = \frac{R - 1}{3} \text{ or } \frac{Branch Value - 1}{3}$$

-Eqn. 3

Branch Values are created from node values. An all these branch values are multiple of values of 2^n .

$$Branch Value = Node * 2^n$$

-Eqn. 4

From base stem, branches that occur are illustrated in following:

				1		2		3		4		5		6		7		8		9		10		11		12	
8																											
↑																											
16	<-	5	<-	10	<-	20	<-	40	<-	80	<-	160	<-	320	<-	640	<-	1280	<-	2560	<-	5120	<-	10240	<-	20480	<-
↑				↑				↑				↑				↑				↑				↑			
32				3				13				53				213				853				3413			
↑																											
64	<-	21	<-	42	<-	84	<-	168	<-	336	<-	672	<-	1344	<-	2688	<-	5376	<-	10752	<-	21504	<-	43008	<-	86016	<-
↑																											
128																											
↑																											
256	<-	85	<-	170	<-	340	<-	680	<-	1360	<-	2720	<-	5440	<-	10880	<-	21760	<-	43520	<-	87040	<-	174080	<-	348160	<-
↑						↑				↑				↑				↑				↑				↑	
512						113				453				1813				7253				29013				116053	
↑																											
1024	<-	341	<-	682	<-	1364	<-	2728	<-	5456	<-	10912	<-	21824	<-	43648	<-	87296	<-	174592	<-	349184	<-	698368	<-	1396736	<-
↑				↑				↑				↑				↑				↑			↑			↑	
2048				227				909				3637				14549				58197				232789			
↑																											
4096	<-	1365	<-	2730	<-	5460	<-	10920	<-	21840	<-	43680	<-	87360	<-	174720	<-	349440	<-	698880	<-	1397760	<-	2795520	<-	5591040	<-
↑																											
8192																											
↑																											
16384	<-	5461	<-	10922	<-	21844	<-	43688	<-	87376	<-	174752	<-	349504	<-	699008	<-	1398016	<-	2796032	<-	5592064	<-	11184128	<-	22368256	<-
↑						↑				↑				↑				↑				↑				↑	
32768						7281				29125				116501				466005				1864021				7456085	
↑																											

The branches that occur through 2^{6n-2} and 2^{6n+2} , cause further branching and hence they show similar pattern as either they will branch on even positions and odd positions or certain values will not branch. There are values that do not branch further and these nodes are formed through $R = 2^{6n}$ values.

For node value 5:

↓																											
50480				5512				0101				30402				142051				285482				5350041			
↓				↓				↓				↓				↓				↓			↓				
10540	<-	3413	<-	0850	<-	13025	<-	51304	<-	24008	<-	100510	<-	518435	<-	430804	<-	813158	<-	1144420	<-	3404015	<-	0080854	<-	13010048	<-
↓																											
2150				1131				4240				18101				15180				501121				1104050			
↓				↓				↓				↓				↓				↓			↓				
5200	<-	023	<-	1100	<-	3415	<-	0854	<-	13048	<-	51500	<-	24205	<-	100184	<-	518308	<-	430130	<-	813415	<-	1144044	<-	3403888	<-
↓																											
1580																											
↓																											
040	<-	513	<-	450	<-	025	<-	1104	<-	3408	<-	0810	<-	13035	<-	51504	<-	24258	<-	100020	<-	518115	<-	430554	<-	815448	<-
↓																											
350				32				141				202				5501				0042				30181			
↓				↓				↓				↓				↓				↓			↓				
100	<-	23	<-	100	<-	515	<-	454	<-	048	<-	1000	<-	3305	<-	0104	<-	13208	<-	51130	<-	24515	<-	108244	<-	511088	<-
↓																											
80				11				00				511				1100				4431				11140			
↓				↓				↓				↓				↓				↓			↓				
40	<-	13	<-	50	<-	25	<-	104	<-	508	<-	410	<-	835	<-	1004	<-	3358	<-	0020	<-	13315	<-	50054	<-	23548	<-
↓																											
50																											
↓																											
10	<-	3	<-	0	<-	15	<-	54	<-	48	<-	00	<-	105	<-	384	<-	108	<-	1230	<-	3015	<-	0144	<-	15588	<-
↓																											
2				1		5		3		4		2		0		1		8		0		10		11		15	

			1		2		3		4		5		6		7		8		9		10		11		12		
7																											
↑																											
14																											
↑																											
28	<-	9	<-	18	<-	36	<-	72	<-	144	<-	288	<-	576	<-	1152	<-	2304	<-	4608	<-	9216	<-	18432	<-	36864	<-
↑																											
56																											
↑																											
112	<-	37	<-	74	<-	148	<-	296	<-	592	<-	1184	<-	2368	<-	4736	<-	9472	<-	18944	<-	37888	<-	75776	<-	151552	<-
↑						↑				↑				↑				↑				↑				↑	
224						49				197				789				3157				12629				50517	
↑																											
448	<-	149	<-	298	<-	596	<-	1192	<-	2384	<-	4768	<-	9536	<-	19072	<-	38144	<-	76288	<-	152576	<-	305152	<-	610304	<-
↑				↑				↑				↑				↑				↑			↑				
896				99				397				1589				6357				25429			101717				
↑																											
1792	<-	597	<-	1194	<-	2388	<-	4776	<-	9552	<-	19104	<-	38208	<-	76416	<-	152832	<-	305664	<-	611328	<-	1222656	<-	2445312	<-
↑																											
3584																											
↑																											
7168	<-	2389	<-	4778	<-	9556	<-	19112	<-	38224	<-	76448	<-	152896	<-	305792	<-	611584	<-	1223168	<-	2446336	<-	4892672	<-	9785344	<-
↑						↑				↑				↑				↑				↑				↑	
14336						3185				12741				50965				203861				815445				3261781	
↑																											
28672	<-	9557	<-	19114	<-	38228	<-	76456	<-	152912	<-	305824	<-	611648	<-	1223296	<-	2446592	<-	4893184	<-	9786368	<-	19572736	<-	39145472	<-
↑				↑				↑				↑				↑				↑			↑				
				6371				25485				101941				407765				1631061			6524245				

	1	2	3	4	5	6	7	8	9	10	11	12		
25														
↑														
50														
↑														
100	<- 33	<- 66	<- 132	<- 264	<- 528	<- 1056	<- 2112	<- 4224	<- 8448	<- 16896	<- 33792	<- 67584	<- 135168	<-
↑														
200														
↑														
400	<- 133	<- 266	<- 532	<- 1064	<- 2128	<- 4256	<- 8512	<- 17024	<- 34048	<- 68096	<- 136192	<- 272384	<- 544768	<-
↑			↑		↑		↑		↑		↑		↑	
800			177		709		2837		11349		45397		181589	
↑														
1600	<- 533	<- 1066	<- 2132	<- 4264	<- 8528	<- 17056	<- 34112	<- 68224	<- 136448	<- 272896	<- 545792	<- 1091584	<- 2183168	<-
↑		↑		↑		↑		↑		↑		↑		
3200		355		1421		5685		22741		90965		363861		
↑														
6400	<- 2133	<- 4266	<- 8532	<- 17064	<- 34128	<- 68256	<- 136512	<- 273024	<- 546048	<- 1092096	<- 2184192	<- 4368384	<- 8736768	<-
↑														
12800														
↑														
25600	<- 8533	<- 17066	<- 34132	<- 68264	<- 136528	<- 273056	<- 546112	<- 1092224	<- 2184448	<- 4368896	<- 8737792	<- 17475584	<- 34951168	<-
↑			↑		↑		↑		↑		↑		↑	
51200			11377		45509		182037		728149		2912597		11650389	
↑														
102400	<- 34133	<- 68266	<- 136532	<- 273064	<- 546128	<- 1092256	<- 2184512	<- 4369024	<- 8738048	<- 17476096	<- 34952192	<- 69904384	<- 139808768	<-
↑		↑		↑		↑		↑		↑		↑		
		22755		91021		364085		1456341		5825365		23301461		

As it can be seen from these examples presented above, that any number m can be taken and a branch can be created in this format with values $m \cdot 2^n$, $n > 0$. This will lead to further branching on either odd or even nodes. An important thing to notice is that m will always be an odd number

as $\left(\frac{\text{Branch Value}-1}{3}\right)$ is always odd. So, the branch obtained from the Branch Value will have values:

$$\text{Node} * 2^n = \frac{\text{Branch Value} - 1}{3} * 2^n$$

Branches can be formed at odd or even positions. The nodes that lead to branching at even or odd positions appear randomly, although for a specific branch, this pattern will always repeat.

This represents the occurrence of powers and nodes and their random appearance in these six combinations throughout the tree. Consequently, no discernible pattern exists apart from the powers of 2 originating from the base stem

Current Locations of Branches	6n-5	6n-3	6n-1	Branching from new branches
	EVEN	ODD	NB	
	ODD	EVEN	NB	
	EVEN	NB	ODD	
	ODD	NB	EVEN	
	NB	EVEN	ODD	
	NB	ODD	EVEN	
	6n-4	6n-2	6n	

Fig. 2: Different Permutations of Further Branching on a current Branch

Approximation of Generalised Term

For base stem, values that have branching can be given as follows:

$$R = \begin{cases} 2^{6n-2} \\ 2^{6n+2} \end{cases}, n \geq 1 \quad \text{-Eqn. 5}$$

Here 2^{6n} values are not taken since these values create branches with no branching. For branches, it can be said that they cause branching when an odd value is obtained.

$$\frac{\text{Branch Value} - 1}{3} * 2^n$$

Again, Nodes are at zeroth position. The first value of branch is node*2, and so on given by node* 2^n , where n is position of each branch value. So, a

branch is to be created from m_1 position of branch of R. This node can be expressed as:

$$\frac{\left(\frac{R-1}{3} * 2^{m_1} - 1\right)}{3} = \frac{(R-1)2^{m_1} - 3}{3^2} = \frac{(R-1)2^{m_1}}{3^2} - \frac{1}{3}$$

Now for m_2 position of this branch, node value can be expressed as:

$$\begin{aligned} & \frac{\left(\frac{\left(\frac{R-1}{3} * 2^{m_1} - 1\right)}{3} * 2^{m_2} - 1\right)}{3} \\ &= \frac{((R-1)2^{m_1} - 3)2^{m_2} - 3^2}{3^3} \\ &= \frac{(R-1)}{3^3} 2^{m_1+m_2} - \frac{2^{m_2}}{3^2} - \frac{1}{3} \end{aligned}$$

For m_3 position of this branch, node value can be expressed as:

$$\begin{aligned} & \frac{\left(\frac{\left(\frac{\left(\frac{R-1}{3} * 2^{m_1} - 1\right)}{3} * 2^{m_2} - 1\right)}{3} * 2^{m_3} - 1\right)}{3} \\ &= \frac{(((R-1)2^{m_1} - 3)2^{m_2} - 3^2)2^{m_3} - 3^3}{3^4} \\ &= \frac{(R-1)}{3^4} 2^{m_1+m_2+m_3} - \frac{2^{m_2+m_3}}{3^3} - \frac{2^{m_3}}{3^2} - \frac{1}{3} \end{aligned}$$

For m_4 position of a particular branch, node value can be expressed as:

$$\begin{aligned} & \frac{\left(\frac{\left(\frac{\left(\frac{\left(\frac{R-1}{3} * 2^{m_1} - 1\right)}{3} * 2^{m_2} - 1\right)}{3} * 2^{m_3} - 1\right)}{3} * 2^{m_4} - 1\right)}{3} \\ &= \frac{\left(\left(\left(\left((R-1)2^{m_1} - 3\right)2^{m_2} - 3^2\right)2^{m_3} - 3^3\right)2^{m_4} - 3^4\right)}{3^5} \\ &= \frac{(R-1)}{3^5} 2^{m_1+m_2+m_3+m_4} - \frac{2^{m_2+m_3+m_4}}{3^4} - \frac{2^{m_3+m_4}}{3^3} - \frac{2^{m_4}}{3^2} - \frac{1}{3} \end{aligned}$$

A generalisation can be made using this division ladder:

$$\begin{array}{r}
 \left(\frac{R-1}{3}\right) 2^{m_1} - 1 \\
 \hline
 \frac{\quad}{3} 2^{m_2} - 1 \\
 \hline
 \frac{\quad}{3} 2^{m_3} - 1 \\
 \hline
 \frac{\quad}{3} 2^{m_4} - 1 \\
 \hline
 \frac{\quad}{3} \dots \dots 2^{m_k} - 1 \\
 \hline
 \vdots \\
 \hline
 3
 \end{array}$$

After simplification, following expression is obtained:

$$\begin{aligned}
 & \frac{\left(\left(\left(\left(\left((R-1)2^{m_1} - 3\right)2^{m_2} - 3\right)2^{m_3} - 3\right)2^{m_4} - 3\right)2^{m_5} - \dots \dots \right)2^{m_k} - 3^k}{3^{k+1}} \\
 &= \frac{1}{3^{k+1}} \left((R-1)2^{\sum_{i=1}^k m_i} + 3 * 2^{\sum_{i=2}^k m_i} \right. \\
 & \quad \left. - \sum_{i=1}^k (3^i * 2^{\sum_{j=i}^k m_{j+2}}) - 3^k \right)
 \end{aligned}$$

Following is the generalised term:

$$T(k, \{m_i\}) = \left(\frac{R-1}{3} 2^{m_1} - 1\right) \frac{2^{\sum_{i=2}^k m_i}}{3^k} - \sum_{i=1}^{k-2} \frac{2^{\sum_{j=i}^k m_{j+2}}}{3^{k-i}} - \frac{1}{3}$$

-Eqn. 7

Here:

k = number of branches to be traversed for the node value

$\{m_i\}$ = set of values of positions of nodes in each branch for traversing (total k values)

Following are two illustrations for values 9 and 267, with a follow up of visual representation of Generalised term.

$$T(5, \{3, 2, 1, 1, 2\}) = 9$$

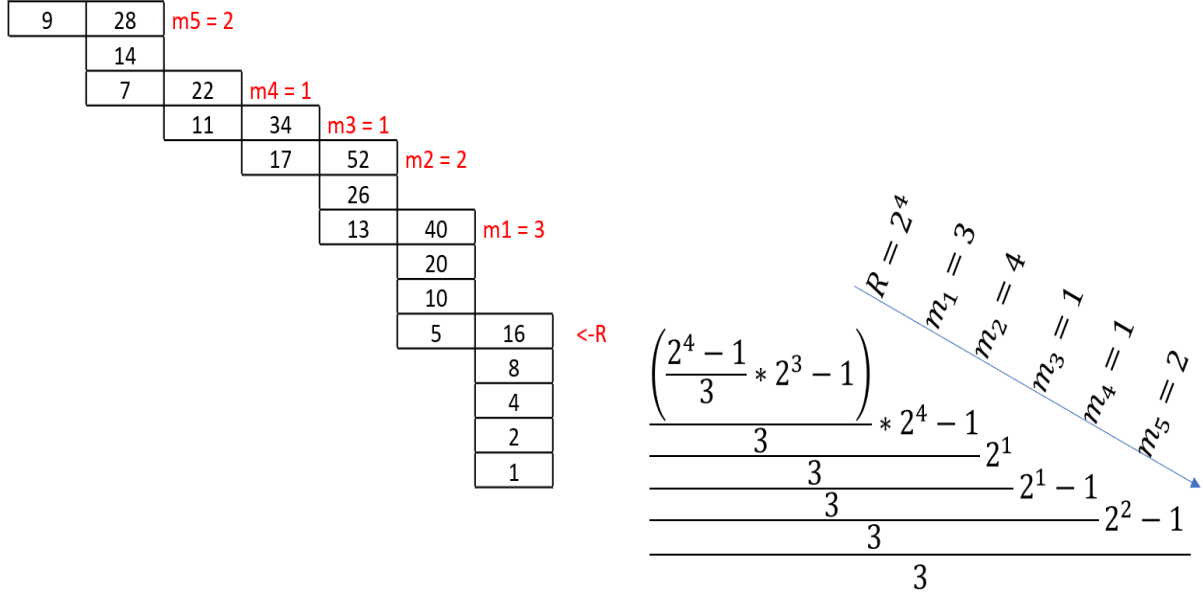


Fig. 4(a): Illustration of Formula for Value 9

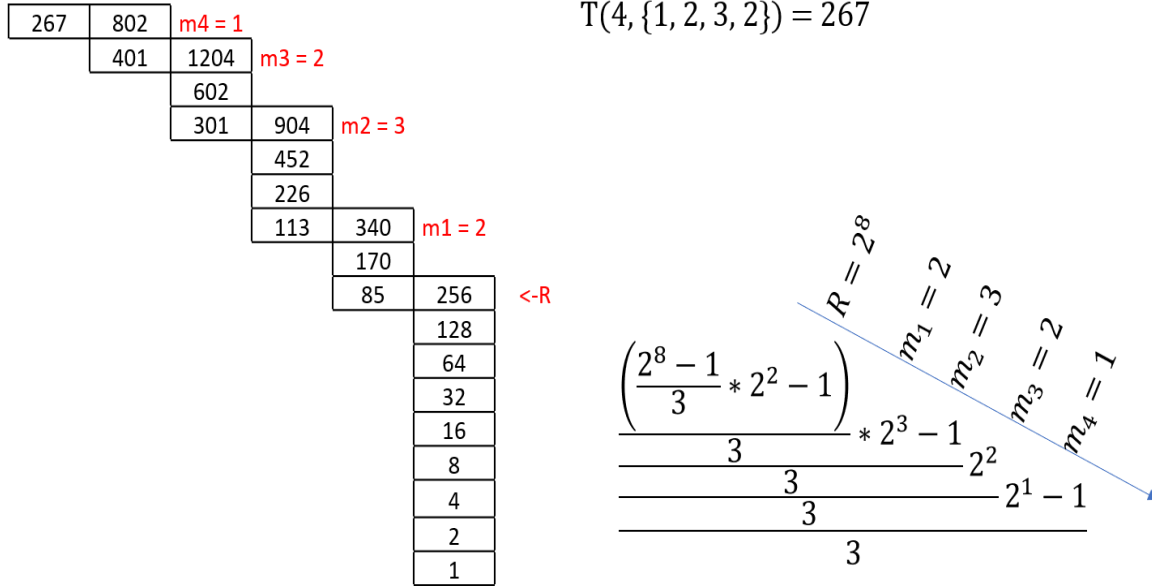


Fig. 4(b): Illustration of Formula for Value 267

The values of m_i are random as there is not pattern for positions of branching over other nodes and the branches form on alternative nodes and hence only pattern available is for R.

Apart from 2^{6n} , there will be values in $\{m_i\}$ which will fall on branchless nodes and will cause failure as there will not be integer result but a fraction.

This function or generalised term (Eqn. 7) is like proposition 7^[4] given by Bohm et. Al.:

$\forall x \in Z$ we have: $x \in U$ if and only if $\exists \{v_0, v_1, \dots, v_m\} \in N^{m-1}$ with $v_{i-1} < v_i$ for $1 \leq i < m$ such that

$$x = \frac{1}{3^m} \left(2^{v_m} - \sum_{k=0}^{m-1} 3^{m-k-1} * 2^{v_k} \right)$$

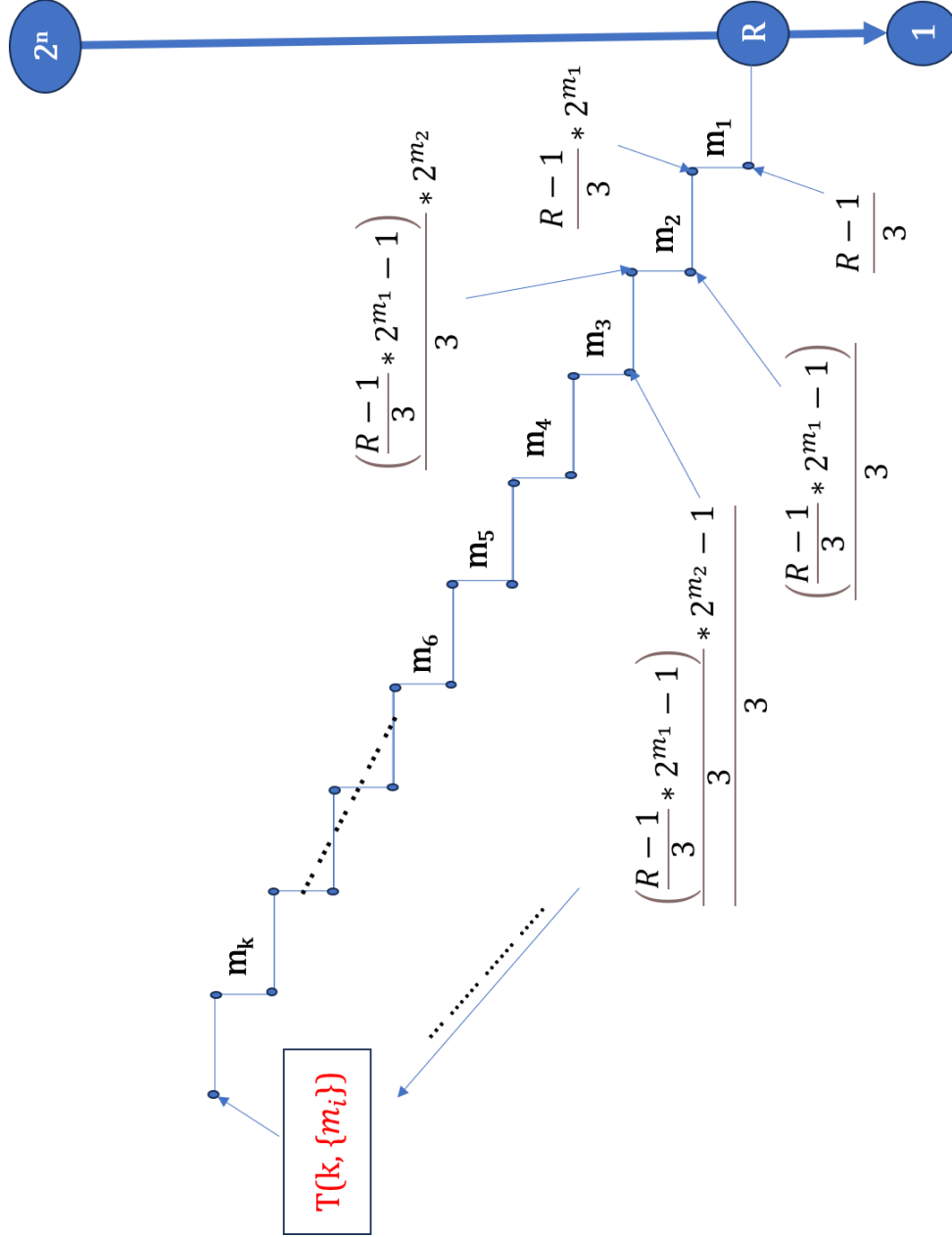


Fig. 5: Node Positions w.r.t. formula

Node Values and Consecutive Branches

If we take a branch of any value m , this branch will be related to the next branch in the manner of $4 * (\text{branch value at position } n) + 2^n$. For example, base stem and their branches can be expressed as:

		1		2		3		4		5		6		7		8		9		10		11		12			
8	↑																										
16	↖	5	↖	10	↖	20	↖	40	↖	80	↖	160	↖	320	↖	640	↖	1280	↖	2560	↖	5120	↖	10240	↖	20480	↖
32	↑	$4 \times +1$	↓	$4 \times +2$	↓	$4 \times +4$	↓	$4 \times +8$	↓	$4 \times +16$	↓	$4 \times +32$	↓	$4 \times +64$	↓	$4 \times +128$	↓	$4 \times +256$	↓	$4 \times +512$	↓	$4 \times +1024$	↓	$4 \times +2048$	↓	$4 \times +4096$	↓
64	↖	21	↖	42	↖	84	↖	168	↖	336	↖	672	↖	1344	↖	2688	↖	5376	↖	10752	↖	21504	↖	43008	↖	86016	↖
128	↑	$4 \times +1$	↓	$4 \times +2$	↓	$4 \times +4$	↓	$4 \times +8$	↓	$4 \times +16$	↓	$4 \times +32$	↓	$4 \times +64$	↓	$4 \times +128$	↓	$4 \times +256$	↓	$4 \times +512$	↓	$4 \times +1024$	↓	$4 \times +2048$	↓	$4 \times +4096$	↓
256	↖	85	↖	170	↖	340	↖	680	↖	1360	↖	2720	↖	5440	↖	10880	↖	21760	↖	43520	↖	87040	↖	174080	↖	348160	↖
512	↑	$4 \times +1$	↓	$4 \times +2$	↓	$4 \times +4$	↓	$4 \times +8$	↓	$4 \times +16$	↓	$4 \times +32$	↓	$4 \times +64$	↓	$4 \times +128$	↓	$4 \times +256$	↓	$4 \times +512$	↓	$4 \times +1024$	↓	$4 \times +2048$	↓	$4 \times +4096$	↓
1024	↖	341	↖	682	↖	1364	↖	2728	↖	5456	↖	10912	↖	21824	↖	43648	↖	87296	↖	174592	↖	349184	↖	698368	↖	1396736	↖
2048	↑	$4 \times +1$	↓	$4 \times +2$	↓	$4 \times +4$	↓	$4 \times +8$	↓	$4 \times +16$	↓	$4 \times +32$	↓	$4 \times +64$	↓	$4 \times +128$	↓	$4 \times +256$	↓	$4 \times +512$	↓	$4 \times +1024$	↓	$4 \times +2048$	↓	$4 \times +4096$	↓
4096	↖	1365	↖	2730	↖	5460	↖	10920	↖	21840	↖	43680	↖	87360	↖	174720	↖	349440	↖	698880	↖	1397760	↖	2795520	↖	5591040	↖
8192	↑	$4 \times +1$	↓	$4 \times +2$	↓	$4 \times +4$	↓	$4 \times +8$	↓	$4 \times +16$	↓	$4 \times +32$	↓	$4 \times +64$	↓	$4 \times +128$	↓	$4 \times +256$	↓	$4 \times +512$	↓	$4 \times +1024$	↓	$4 \times +2048$	↓	$4 \times +4096$	↓
16384	↖	5461	↖	10922	↖	21844	↖	43688	↖	87376	↖	174752	↖	349504	↖	699008	↖	1398016	↖	2796032	↖	5592064	↖	11184128	↖	22368256	↖
32768	↑																										

For nodes itself: $21 = 4 \cdot 5 + 2^0$ $85 = 4 \cdot 21 + 2^0$ $341 = 4 \cdot 85 + 2^0$

For branch values: Value at position n of current branch = $4 \cdot \text{value at position n of previous branch} + 2^n$

		1		2		3		4		5		6		7		8		9		10		11		12			
5	↑																										
10	↖	3	↖	6	↖	12	↖	24	↖	48	↖	96	↖	192	↖	384	↖	768	↖	1536	↖	3072	↖	6144	↖	12288	↖
20	↑	4x+1	↓	4x+2	↓	4x+4	↓	4x+8	↓	4x+16	↓	4x+32	↓	4x+64	↓	4x+128	↓	4x+256	↓	4x+512	↓	4x+1024	↓	4x+2048	↓	4x+4096	↓
40	↖	13	↖	26	↖	52	↖	104	↖	208	↖	416	↖	832	↖	1664	↖	3328	↖	6656	↖	13312	↖	26624	↖	53248	↖
80	↑	4x+1	↓	4x+2	↓	4x+4	↓	4x+8	↓	4x+16	↓	4x+32	↓	4x+64	↓	4x+128	↓	4x+256	↓	4x+512	↓	4x+1024	↓	4x+2048	↓	4x+4096	↓
160	↖	53	↖	106	↖	212	↖	424	↖	848	↖	1696	↖	3392	↖	6784	↖	13568	↖	27136	↖	54272	↖	108544	↖	217088	↖
320	↑	4x+1	↓	4x+2	↓	4x+4	↓	4x+8	↓	4x+16	↓	4x+32	↓	4x+64	↓	4x+128	↓	4x+256	↓	4x+512	↓	4x+1024	↓	4x+2048	↓	4x+4096	↓
640	↖	213	↖	426	↖	852	↖	1704	↖	3408	↖	6816	↖	13632	↖	27264	↖	54528	↖	109056	↖	218112	↖	436224	↖	872448	↖
1280	↑	4x+1	↓	4x+2	↓	4x+4	↓	4x+8	↓	4x+16	↓	4x+32	↓	4x+64	↓	4x+128	↓	4x+256	↓	4x+512	↓	4x+1024	↓	4x+2048	↓	4x+4096	↓
2560	↖	853	↖	1706	↖	3412	↖	6824	↖	13648	↖	27296	↖	54592	↖	109184	↖	218368	↖	436736	↖	873472	↖	1746944	↖	3493888	↖
5120	↑	4x+1	↓	4x+2	↓	4x+4	↓	4x+8	↓	4x+16	↓	4x+32	↓	4x+64	↓	4x+128	↓	4x+256	↓	4x+512	↓	4x+1024	↓	4x+2048	↓	4x+4096	↓
10240	↖	3413	↖	6826	↖	13652	↖	27304	↖	54608	↖	109216	↖	218432	↖	436864	↖	873728	↖	1747456	↖	3494912	↖	6989824	↖	13979648	↖
20480	↑																										

Another relation that can be drawn from these values is that in current branch a value at a position n is that it has addition of 2^n to the value at n+2 position of previous branch. Again taking example of branches of base stem:

		1	2	3	4	5	6	7	8	9	10	11	12														
8																											
↑																											
16	<-	5	<-	10	<-	20	<-	40	<-	80	<-	160	<-	320	<-	640	<-	1280	<-	2560	<-	5120	<-	10240	<-	20480	<-
↑																											
32																											
↑																											
64	<-	21	<-	42	<-	84	<-	168	<-	336	<-	672	<-	1344	<-	2688	<-	5376	<-	10752	<-	21504	<-	43008	<-	86016	<-
↑																											
128																											
↑																											
256	<-	85	<-	170	<-	340	<-	680	<-	1360	<-	2720	<-	5440	<-	10880	<-	21760	<-	43520	<-	87040	<-	174080	<-	348160	<-
↑																											
512																											
↑																											
1024	<-	341	<-	682	<-	1364	<-	2728	<-	5456	<-	10912	<-	21824	<-	43648	<-	87296	<-	174592	<-	349184	<-	698368	<-	1396736	<-
↑																											
2048																											
↑																											
4096	<-	1365	<-	2730	<-	5460	<-	10920	<-	21840	<-	43680	<-	87360	<-	174720	<-	349440	<-	698880	<-	1397760	<-	2795520	<-	5591040	<-
↑																											
8192																											
↑																											
16384	<-	5461	<-	10922	<-	21844	<-	43688	<-	87376	<-	174752	<-	349504	<-	699008	<-	1398016	<-	2796032	<-	5592064	<-	11184128	<-	22368256	<-
↑																											
32768																											
↑																											

Similarly, for other node values it can be shown in the same way. Following is example for node value 5:

		1	2	3	4	5	6	7	8	9	10	11	12														
5																											
↑																											
10	<-	3	<-	6	<-	12	<-	24	<-	48	<-	96	<-	192	<-	384	<-	768	<-	1536	<-	3072	<-	6144	<-	12288	<-
↑																											
20																											
↑																											
40	<-	13	<-	26	<-	52	<-	104	<-	208	<-	416	<-	832	<-	1664	<-	3328	<-	6656	<-	13312	<-	26624	<-	53248	<-
↑																											
80																											
↑																											
160	<-	53	<-	106	<-	212	<-	424	<-	848	<-	1696	<-	3392	<-	6784	<-	13568	<-	27136	<-	54272	<-	108544	<-	217088	<-
↑																											
320																											
↑																											
640	<-	213	<-	426	<-	852	<-	1704	<-	3408	<-	6816	<-	13632	<-	27264	<-	54528	<-	109056	<-	218112	<-	436224	<-	872448	<-
↑																											
1280																											
↑																											
2560	<-	853	<-	1706	<-	3412	<-	6824	<-	13648	<-	27296	<-	54592	<-	109184	<-	218368	<-	436736	<-	873472	<-	1746944	<-	3493888	<-
↑																											
5120																											
↑																											
10240	<-	3413	<-	6826	<-	13652	<-	27304	<-	54608	<-	109216	<-	218432	<-	436864	<-	873728	<-	1747456	<-	3494912	<-	6989824	<-	13979648	<-
↑																											
20480																											
↑																											

Value of n^{th} position of branch = value at $(n+2)^{\text{th}}$ position at previous branch + 2^n (including nodes at $n = 0$)

Similarly other relations can be made by looking at the values which are:

- Value of n^{th} position of branch = $2 * \text{value at } (n+1)^{\text{th}} \text{ position at previous branch} + 2^n$

- Value of $(n+1)^{\text{th}}$ position of next branch = $8 * \text{value at } n^{\text{th}} \text{ position at current branch} + 2^{n+1}$
- There are similar relations diagonally.

Conclusion

Here are the key points from this work:

- The tree's structure can be visualized starting from the base stem, which follows the powers of 2, and the branches follow a similar pattern.
- The position pattern is either odd or even, but the nodes for even, odd, and no-branching branches appear randomly.
- The criteria for no branching: $(value - 1) \bmod 9 \equiv 0$
- Base stem and branch values are even (except for 1), while node values are odd.
- Consecutive branches of the same node are related either vertically by $4x+2^n$ or diagonally by $(\text{previous branch value at } n^{\text{th}} \text{ position}) + 2^{n-2}$.
- The generalization of node values can be designed with a root term that follows a known pattern, while the branches have random values. The formula in the division ladder is in its simplest form.

References

1. Collatz conjecture, https://en.wikipedia.org/wiki/Collatz_conjecture
2. The Ultimate Challenge: The $3x+1$ Problem; Jeffery C. Lagarias
3. On the existence of cycles of given length in integer sequences like $x_{n+1} = x_n/2$ if x_n even, and $x_{n+1} = 3x_n + 1$; Corrado Böhm, Giovanna Sontacchi