Prime Number Generation Using Primes with 2s and 3s only.

Introduction:-

Prime Numbers are number which are divisible only by themselves or one. These Numbers can be utilised to create composite numbers (numbers divisible by their prime factors and non-prime factors). The test by which a number can be determined as prime ~~or~~ ~~is~~ is called as primality test.

Prime Numbers can be determined by various method, but to generate a prime which is unknown is hard since ~~to carry out~~ its hard to find a pattern, apart from it being divisible by itself. Some methods being Mersenne's Prime, Wilson's formula, etc. Wilson's formula is based on Wilson's theorem which states that: "Any integer $n>1$ is a prime number if, and only if, $(n-1)!+1$ is divisible by n" i.e. $(n-1)! \equiv -1 \pmod{n}$

$$P_n = 1 + \sum_{i=1}^{2^n} \left[\left(\frac{n}{\sum_{j=1}^{i} \left[\left(\cos \frac{(j-1)!+1}{j} \pi \right)^2 \right]} \right)^{1/n} \right] \quad , M_p = 2^n - 1$$

These methods along with other methods are inefficient since all of these also generate composite number while Wilson's formula counts 1 for P_n times.

For example:

Primes are $2, 3, 5, 7, 11, 13, 17, 19, \ldots\ldots, P_n$

- $\cancel{M_1 = 2^1 - 1 = 1}$, $M_2 = 2^2 - 1 = 3$, $M_3 = 2^3 - 1 = 7$, $\cancel{M_4 = 16 - 1 = 15}$, $M_5 = 2^5 - 1 = 31$, $M_6 = 2^6 - 1 = 63$, $M_7 = 2^7 - 1 = 127, \ldots$

- $$P_5 = 1 + \sum_{i=1}^{2^5} \left[\left(\frac{5}{\sum_{j=1}^{i} \left[\left(\cos \frac{(j-1)!+1}{j} \pi \right)^2 \right]} \right)^{1/5} \right] = 1+1+1+1+1+1+1+1+1+1+1 \ldots + 0 \ldots\ldots (0 \ldots \infty) = 11.$$

Due to larger computational sizes and requirement of time and memory various methods ~~are~~ generally inefficient. And also to check whether a number is prime or not.

This method that I am proposing uses primes and its deconstruction to number of 2s and 3s to generate further primes.

Theory :-

A prime number can be partitioned as ~~a~~ sum of other primes or composite numbers like any other number. But partitioning them in 2s and 3s can give some fantastic results. Following is an example of it: $5 = 2+3$, $7 = 2(2)+3(1)$, $11 = 2(4)+3(1) = 2(1)+3(3)$, $13 = 2(5)+3(1) = 2(2)+3(3)$, $17 = 2(7)+3(1) = 2(1)+3(5)$, etc.

Similarly, a prime can be reprented in more than 2 ways for primes greater than 13.

Ex:

$17 \rightarrow 2(7)+3(1)$
$\rightarrow 2(4)+3(3)$
$\rightarrow 2(1)+3(5)$

$19 \rightarrow 2(8)+3(1)$
$\rightarrow 2(5)+3(3)$
$\rightarrow 2(2)+3(5)$

$31 \rightarrow 2(14)+3(1)$
$\rightarrow 2(11)+3(3)$
$\rightarrow 2(8)+3(5)$
$\rightarrow 2(5)+3(7)$
$\rightarrow 2(2)+3(9)$

$47 \rightarrow 2(22)+3(1)$
$\rightarrow 2(19)+3(3)$
$\rightarrow 2(16)+3(5)$
$\rightarrow 2(13)+3(7)$
$\rightarrow 2(10)+3(9)$
$\rightarrow 2(7)+3(11)$
$\rightarrow 2(4)+3(13)$
$\rightarrow 2(1)+3(15)$

T_2		T_3
$\frac{P_0-3}{2}$		1
	-3	3
	-6	5
	-9	7
	-12	9
⋮	⋮	⋮
$\frac{P_0-3}{2} \bmod 3$		$\frac{P_0 - 2\left(\frac{P_0-3}{2} \bmod 3\right)}{3}$

For the 2s and 3s in these deconstruction, the number of 2s decrement by 3 while number 3s increase by 2 ~~simultaneous~~ together. For the number of 2s in last deconstruction it is either 2 or 1 which can be known by remainder of number of 2s in first deconstruction when divided by 3.

→ Following are the symbols used in this calculation:

T_2 : Number of 2s (in first) | P_0 : Prime number input
T_3 : Number of 3s (in last) | P_n : next ~~prime or~~ number generated

$a \bmod b$: Remainder of a when divided by b

→ $P_n = 2(T_2) + 3(T_3)$: Here T_2 is the number of 2s in first and T_3 is the number of 3s in last deconstruction.

$$T_2 = \frac{P_0-3}{2}, \quad T_3 = \frac{P_0 - 2(T_2 \bmod 3)}{3}, \quad T_2 \bmod 3 = 2 \text{ or } 1.$$

$$P_n = P_0 - 3 + P_0 - 2(T_2 \bmod 3)$$

$$\rightarrow P_n = 2P_0 - 3 - 2(T_2 \bmod 3)$$

$$\rightarrow P_n = 2P_0 - 2\left(\frac{P_0-3}{2} \bmod 3\right) - 3$$

Ex:

$11 \rightarrow 17$	$23 \rightarrow 41$	~~$41 \rightarrow 77$~~
$13 \rightarrow 19$	$29 \rightarrow 53$	$43 \rightarrow 79$
$17 \rightarrow 29$	~~$31 \rightarrow 55$~~	$47 \rightarrow 89$
$19 \rightarrow 31$	$37 \rightarrow 67$	$53 \rightarrow 101$

~~Example~~: Explanation:

$11 \rightarrow T_2 = 4,\ T_3 = 3 \rightarrow 2(4)+3(3) = 17$
$13 \rightarrow T_2 = 5,\ T_3 = 3 \rightarrow 2(5)+3(3) = 19$
$17 \rightarrow T_2 = 7,\ T_3 = 5 \rightarrow 2(7)+3(5) = 29$
$19 \rightarrow T_2 = 8,\ T_3 = 5 \rightarrow 2(8)+3(5) = 31$
$41 \rightarrow T_2 = 19,\ T_3 = 13 \rightarrow 2(19)+3(13) = 77$
$97 \rightarrow T_2 = 42,\ T_3 = 31 \rightarrow 2(42)+3(31) = 187$
$101 \rightarrow T_2 = 49,\ T_3 = 33 \rightarrow 2(49)+3(33) = 197$

Using this method a series can be generated which involves generation of primes and ends with ~~an end~~ a non-prime. Some of the primes do not proceed into a series i.e., they generate a non-prime at first step.

Ex: $P_0 = 11 \rightarrow 17 \rightarrow 29 \rightarrow 53 \rightarrow 101 \rightarrow 197 \rightarrow 389 \rightarrow 773 \rightarrow 1541\ (+2 = 1543)$

$P_0 = 13 \rightarrow 19 \rightarrow 31 \rightarrow 55\ (+4 \rightarrow 59)$

$P_0 = 23 \rightarrow 41 \rightarrow 77\ (+2) = 79$

$P_0 = 37 \rightarrow 67 \rightarrow 127 \rightarrow 247\ (+4 = 251)$

× $P_0 = 31 \rightarrow 55\ (+4 = 59)$

× $P_0 = 41 \rightarrow 77\ (+2 = 79)$

The ratio of new term and old term i.e., $\frac{P_{n+1}}{P_n}$ is $\approx 1.99\ldots$

As the numbers increase, the ratio reaches to 1.99.... though it never leads to or touches 2.

$$\frac{P_n}{P_0} = \frac{2P_0 - 2\left(\frac{P_0-3}{2} \bmod 3\right) - 3}{P_0} = 2 - \frac{2}{P_0}\left(\frac{P_0-3}{2} \bmod 3\right) - \frac{3}{P_0} = 1.99\ldots$$

$2\left(\frac{P_0-3}{2} \bmod 3\right) + 3 = 5$ or 7.

This should be noted that when an integer multiplied by this ratio and either ceil or floor of value, there are chances of getting primes or numbers that close to these primes. The gaps or misses between the numbers generated and nearest primes are in the form $2n$.

For upto 170 first primes, the efficiency is of upto 53% with 79 numbers generated being non-primes.

Variations:-

① Using 2s and 3s of next prime :.

We can use 2s and 3s of prime alternatively in the formula.

→ $T_{21} = \frac{P_{01}-3}{2}$, $T_{22} = \frac{P_{02}-3}{2}$, $T_{31} = \frac{P_{02} - 2(T_{21} \bmod 3)}{3}$, $T_{32} = \frac{P_{02} - 2(T_{22} \bmod 3)}{3}$

→ $P_{n1} = 2(T_{21}) + 3(T_{32})$, $P_{n2} = 2(T_{22}) + 3(T_{31})$

→ Ratio $= \frac{2P_{n1}}{P_{01}+P_{02}}$, $\frac{2P_{n2}}{P_{01}+P_{02}}$

→ Similar approach can be applied using previous prime too.

② Using Two different Primes :

Similar formula can be used as previous.

③ Use of odd and even number.

④ $2\left(\frac{P_0 - 3d}{2} \bmod 3\right) + 3$:

→ This can be either 5 or 7. Another term can added to the formula by using $T_2 \bmod 5$, $T_2 \bmod 7$, $T_3 \bmod 5$ or $T_3 \bmod 5$.

→ Using different deconstructions of prime and use of T_2 and T_3.

⑤ Decomposition in the form of smaller primes: 2, 3, 5, 7.

→ $11 \to 2(4)+5 \to 2(2)+7$
$13 \to 3+2(5) \to 2(3)+7$
$17 \to 2(5)+7 \to 2(7)+3$ ... etc.

} This is possible when it has decomposition more than 1.

⑥ Use of Different prime as combination to define a prime which larger. (~~Odd numbered primes~~).

Conclusion:-

To generate a formula for generating primes is hard as it gets random since they are the most fundamental in number theory. Numbers that can be used to create other numbers cannot be defined with formulas but with more of a chaotic random way.

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Note:- There are more or can be more variation as more of the numbers are taken ~~just~~ just the numbers become more large.

Prime Numbers:-

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, - - - - P

1. 5 → 2(1) + 3(1)
2. 7 → 2(2) + 3(1)
3. 11 → 2(4) + 3(1)
   → 2(1) + 3(3)
4. 13 → 2(5) + 3(1)
   → 2(2) + 3(3)
5. 17 → 2(4) + 3(3)
   → 2(7) + 3(1)
   → 2(1) + 3(5)
6. 19 → 2(2) + 3(5)
   → 2(8) + 3(1)
   → 2(5) + 3(3)
7. 23 → 2(10) + 3(1)
   → 2(7) + 3(3)
   → 2(4) + 3(5)
   → 2(1) + 3(7)
8. 29 → 2(13) + 3(1)
   → 2(10) + 3(3)
   → 2(7) + 3(5)
   → 2(4) + 3(7)
   → 2(1) + 3(9)
9. 31 → 2(14) + 3(1)
   → 2(11) + 3(3)
   → 2(8) + 3(5)
   → 2(5) + 3(7)
   → 2(2) + 3(9)
10. 37 → 2(17) + 3(1)
   → 2(14) + 3(3)
   → 2(11) + 3(5)
   → 2(8) + 3(7)
   → 2(5) + 3(9)
   → 2(2) + 3(11)
11. 41 → 2(19) + 3(1)
   → 2(16) + 3(3)
   → 2(13) + 3(5)
   → 2(10) + 3(7)
   → 2(7) + 3(9)
   → 2(4) + 3(11)
   → 2(1) + 3(13)
12. 43 → 2(20) + 3(1)
   → 2(17) + 3(3)
   → 2(14) + 3(5)
   → 2(11) + 3(7)
   → 2(8) + 3(9)
   → 2(5) + 3(11)
   → 2(2) + 3(13)
13. 47 → 2(22) + 3(1)
   → 2(19) + 3(3)
   → 2(16) + 3(5)
   → 2(13) + 3(7)
   → 2(10) + 3(9)
   → 2(7) + 3(11)
   → 2(4) + 3(13)
   → 2(1) + 3(15)
14. 53 → 2(25) + 3(1)
   → 2(22) + 3(3)
   → 2(19) + 3(5)
   → 2(16) + 3(7)
   → 2(13) + 3(9)
   → 2(10) + 3(11)
   → 2(7) + 3(13)
   → 2(4) + 3(15)
   → 2(1) + 3(17)
15. 59 → 2(28) + 3(1)
   → 2(25) + 3(3)
   → 2(22) + 3(5)
   → 2(19) + 3(7)
   → 2(16) + 3(9)
   → 2(13) + 3(11)
   → 2(10) + 3(13)
   → 2(7) + 3(15)
   → 2(4) + 3(17)
   → 2(1) + 3(19)
16. 61 → 2(29) + 3(1)
   → 2(26) + 3(3)
   → 2(23) + 3(5)
   → 2(20) + 3(7)
   → 2(17) + 3(9)
   → 2(14) + 3(11)
   → 2(11) + 3(13)
   → 2(8) + 3(15)
   → 2(5) + 3(17)
   → 2(2) + 3(19)
17. 67 → 2(32) + 3(1)
   → 2(29) + 3(3)
   → 2(26) + 3(5)
   → 2(23) + 3(7)
   → 2(20) + 3(9)
   → 2(17) + 3(11)
   → 2(14) + 3(13)
   → 2(11) + 3(15)
   → 2(8) + 3(17)
   → 2(5) + 3(19)
   → 2(2) + 3(21)
18. 71 → 2(34) + 3(1)
   → 2(31) + 3(3)
   → 2(28) + 3(5)
   → 2(25) + 3(7)
   → 2(22) + 3(9)
   → 2(19) + 3(11)
   → 2(16) + 3(13)
   → 2(13) + 3(15)
   → 2(10) + 3(17)
   → 2(7) + 3(19)
   → 2(4) + 3(21)
   → 2(1) + 3(23)

73 → 2(35) + 3(1)
→ 2(32) + 3(3)
→ 2(29) + 3(5)
→ 2(26) + 3(7)
→ 2(23) + 3(9)
→ 2(20) + 3(11)
→ 2(17) + 3(13)
→ 2(14) + 3(15)
→ 2(11) + 3(17)
→ 2(8) + 3(19)
→ 2(5) + 3(21)
→ 2(2) + 3(23)

79 → 2(38) + 3(1)
→ 2(2) + 3(25)

83 → 2(40) + 3(1)
→ 2(1) + 3(27)

89 → 2(43) + 3(1)
→ 2(1) + 3(29)

~~91 → 2(44) + 3(1)~~
~~→ 2(2) + 3(29)~~

97 → 2(47) + 3(1)
→ 2(2) + 3(31)

$$T(2) = \frac{P_o - 3}{2}$$

$$T(3) = \frac{P_o - 2[T(2) \% 3]}{3}$$

~~P_{new}~~

$P_{original}$		P_{new}	
11		17	
13		19	
17		29	
19		31	
23		41	
29		53	
31	X	55	$\xrightarrow{-2}$ 53, $\xrightarrow{+4}$ 59
37		67	
41	X	77	$\xrightarrow{-6}$ 71, $\xrightarrow{-4}$ 73
43		79	
47		89	
53		101	
59		113	-2
61	X	115	+12
67		127	
71		137	
73		139	
79		151	
83	X	161	$\xrightarrow{-4}$ 157, $\xrightarrow{+2}$ 163, $\xrightarrow{+6}$ 167
89		173	
97	X	187	$\xrightarrow{-6}$ 181, $\xrightarrow{+4}$ 191

Total 16 new primes

$$\boxed{P_{new} = 2(T(2)) + 3(T(3))}$$

- This formula generates numbers using prime.
- The generated numbers can be closer to primes or itself be prime.
- The new numbers are generally larger than original primes or almost close to the double of original primes.

$$\frac{P_{new}}{P_{original}} \approx 1.92784 < 2$$

$$\boxed{P_{new} = (P_o - 3) + \left(P_o - 2\left[\frac{P_o - 3}{2} \% 3\right]\right)}$$

$$\boxed{P_{new} = 2P_o - 3 - 2\left(\left(\frac{P_o - 3}{2}\right) \% 3\right)}$$

Prime Series Generator.

101 → 2(49) + 3(1)
→ 2(1) + 3(33)

103 → 2(50) + 3(1)
→ 2(2) + 3(33)

107 → 2(52) + 3(1)
→ 2(1) + 3(35)

109 → 2(53) + 3(1)
→ 2(2) + 3(35)

113 → 2(55) + 3(1)
→ 2(1) + 3(37)

127 → 2(62) + 3(1)
→ 2(2) + 3(41)

131 → 2(64) + 3(1)
→ 2(1) + 3(43)

137 → 2(67) + 3(1)
→ 2(1) + 3(45)

139 → 2(68) + 3(1)
→ 2(2) + 3(45)

149 → 2(73) + 3(1)
→ 2(1) + 3(49)

151 → 2(74) + 3(1)
→ 2(2) + 3(49)

Poriginal		Pnew	
101		197	
103		199	
107		209	
109		211	
113	×	221	+2 → 223
127	×	247	+4 → 251
131		257	
137		269	
139		271	
149		293	
151	×	295	+12 → 307
157		307	
163	×	319	-2 → 317
167	×	329	+2 → 331
173	×	341	+6 → 347
179		353	
181	×	355	+4 → 359
191	×	377	+2 → 379
193		379	
197		389	
199	×	391	+6 → 397

Total 12 New primes.

157 → 2(77) + 3(1)
→ 2(2) + 3(51)

163 → 2(80) + 3(1)
→ 2(2) + 3(53)

167 → 2(82) + 3(1)
→ 2(1) + 3(55)

173 → 2(85) + 3(1)
→ 2(1) + 3(57)

179 → 2(88) + 3(1)
→ 2(1) + 3(59)

181 → 2(89) + 3(1)
→ 2(2) + 3(59)

191 → 2(94) + 3(1)
→ 2(1) + 3(63)

193 → 2(95) + 3(1)
→ 2(2) + 3(63)

197 → 2(97) + 3(1)
→ 2(1) + 3(65)

199 → 2(98) + 3(1)
→ 2(2) + 3(65)

This fails, as the series grows or some/half of the primes don't produce the primes, just nearest integers.