M-nacci Sequence and Generalisation.

Introduction:-

A fibonacci sequence is made by adding two previous terms to get further terms in the sequence. Representing n^{th} term as f^n, following is respective series: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
If we take two random variables x and y, we come with the following sequence: $x, y, x+y, x+2y, 2x+3y, 3x+5y, 5x+8y, \ldots$
Now, the sequence can be represented with term F_n as:

$$F_n = f_{n-2}\,x + f_{n-1}\,y \qquad (f_n = \text{fib. } n^{th} \text{ term})$$

ex: 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, 343, 555, 898, ...
Similarly, tribonacci, quadrobonacci and other sequences can be formed. But with ~~increase in numbers~~ other sequences there are different equations and dependencies of the coefficients on other coefficients. When taken ratios of coseecutive terms they follow the same routine as other standard sequences. Lucas Series is one such example.

Mnacci Sequence:-

Mnacci Sequence is made using (n-1) no. of zeroes and 1. Hence it can be said that i^{th} term is represented as:

$$M_i = \sum_{k=i-n-1}^{i-1} M_k \quad ; \quad M_i = i^{th} \text{ m-nacci term.}$$

First n-terms are: $\underbrace{0, 0, 0 \ldots\ldots 0}_{n-1 \text{ zeroes}}, 1$

Second n-terms are given by adding previous consecutive n terms. Hence following series is obtained:
$\rightarrow 1, 2, 4, 8, 16, \ldots\ldots, 2^{n-1}; \quad M_i = 2^{i-1}$ for $i \in [1, n]$.

Similarly third, fourth, fifth and consecutive n terms can be created. For third n-terms following series is obtained:

$$2^{n}-1,\ 2^{n+1}-3,\ 2^{n+2}-8,\ 2^{n+3}-20, \ldots\ldots 2^{n} - (n+1) \times 2^{n-2}$$

$$M_i = 2^{n+i} - (i+2)\,2^{i-1} \quad \text{for } i \in [0, n-1]$$

For ~~to~~ fourth n^{th} terms:

$$M_i = 2^{2n+i} - (n+i+2)2^{n+i-1} - \underline{i(i+3)2^{i-3}}$$

$$\left(= 2^{2n+i} - (n+i+2)2^{n+i-1} - ((i+2)(i+1)+i)2^{i-2}\right)$$

For fifth n^{th} terms:

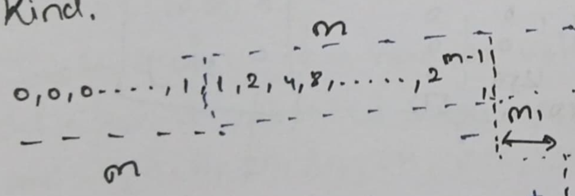
$$M_i = 2^{3n+i} - (2n+i+2)2^{2n+i-1} - (n+i)(2n+i+3)2^{n+i-3}$$
$$- \frac{(i+1)(i+2)(i+6)2^{i+2}}{3}$$

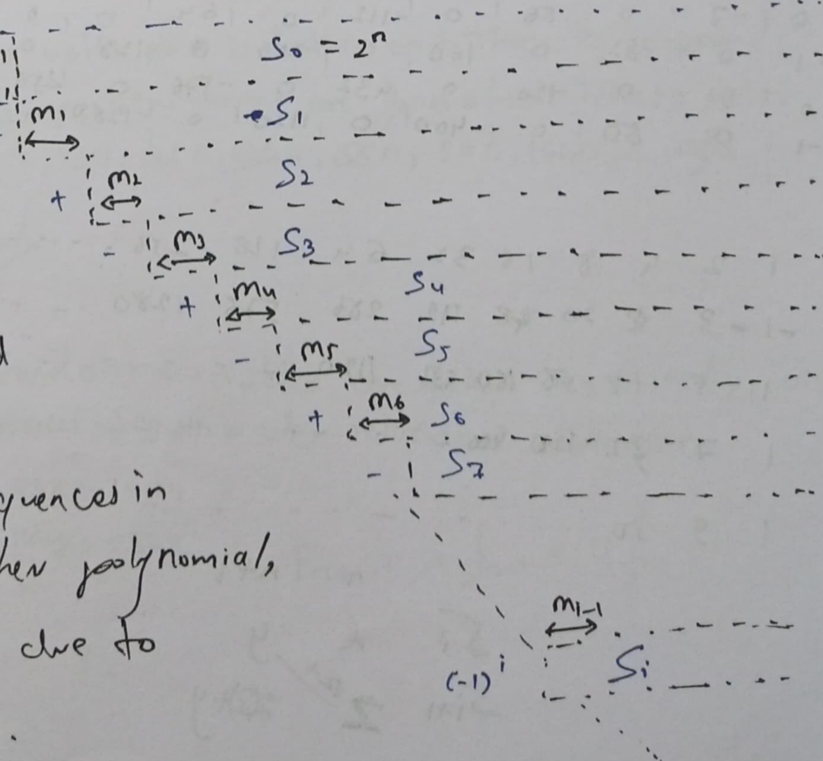
Similarly, various new subtractors get involved as the series rises. For ~~the~~ 3^{rd} its 1, for 4^{th} its 2, 5^{th} its 3, and so to extrapolate this m^{th} has $(m-2)$ subtractors. When done analysis, it can be find out that these values are surely coefficients of ~~$\left(\frac{1-x}{1-2x}\right)$~~ ~~power~~ $\frac{1-x}{(1-2x)^p}$ with $p \geq 2$. As it can be seen that the subtractors rise, the subtractors cand be said as.: $S_i \in$ Coefficients $\frac{1-x}{(1-2x)^{i-2}}$.

If generalised more over higher dimensions, we can say that $\frac{(\prod P_1\{x_j\})^m}{(\prod P_2\{x_j\})^n}$ could generate larger m-nacci of another kind.

So, it can be said that m-nacci sequences holds summation of various sequences, & it can be seen clearly when larger m-values are considered. And this can give more accurate results. While considering 8^{th} n^{th} terms, ~~as and~~ there is availability of chebeshev

If considered, more precisely, it can be said that the first m terms are of $(m-1)$ 0s and one 1. From $(m+1)^{th}$ term, recurrence takes place. Second m terms are powers of two power ranging from 0 to $m-1$. Second m terms are $1, 2, 4, 8, 16, \ldots, 2^{m-1}$. Hence sequence can be considered which is $0, 0, \ldots, 0, 1, 1, 2, \ldots, 2^{m-1}, \ldots\ldots 2^n \ldots\ldots$ This sequence is S_0. When compared with original sequence, the values start to deviate i.e., reduce from $(2m+1)^{th}$ term. Taking this difference, $d = 2^{2m+i-1} - m_{2m+i}$. This difference has general term of $(n+2)2^{n-1}$. This causes correction for 3^{rd} m terms and this is Sequence S_1. Again, if compared to original m-nacci, there is deviation after certain interval, i.e., it increases for $(S_0 - S_1)$ from $3m+1$. Now, again difference is taken which is ~~$d = 2^{3m+i-1} - 2^{2m-1}(2m+2)$~~. This difference is sequence S_2
$d = m_{3m+i} - 2^{3m+i-1} - 2^{2m-1}(2m+2)$. This difference is sequence S_2 which is added to $(S_0 - S_1)$ and has general term $n(n+3)2^{n-3}$. Now $S_0 - S_1 + S_2$ is compared after certain interval which shows deviation. $S_0 + \sum_{n=1}^{m} (-1)^n S_n$ is interval wise summation. This can be further checked either $d_i = \sum_{i=0}^{m} (-1)^i S_i - m_{nk+i}$ or $d_i = m_{nk+i} - \sum_{i=0}^{n} (-1)^i S_i$. And this alternative. Its simple, $d_1, d_2, d_3, d_4, d_5 \ldots\ldots$ Odd-numbered are subtracted while even-numbered are added. These differences and S_0 are actually part of one recurrent sequence itself. This is Chebyshev T Polynomial of First Kind.





Row-wise adjustments are done as per the generation of differences, if they are 0s and if not then added and subtracted.

Since there are sequences in coefficients of Chebyshev polynomial, they might be connected due to recurrence ~~itself~~ itself.

Chebyshev T Polynomials :-

Chebyshev T Polynomial of first kind is defined by recurrence as:

$T_1(x) = 1, \quad T_2(x) = x, \quad T_{n+1} = 2x\,T_n(x) - T_{n-1}(x).$

$T_0 = 1$
$T_1 = x$
$T_2 = 2x^2 - 1$
$T_3 = 4x^3 - 3x$
$T_4 = 8x^4 - 8x^2 + 1$
$T_5 = 16x^5 - 20x^3 + 5x$
$T_6 = 32x^6 - 48x^4 + 18x^2 - 1$
$T_7 = 64x^7 - 112x^5 + 56x^3 - 7x$
$T_8 = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$
$T_9 = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$
$T_{10} = 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1$

$i \geq 0, i \in \mathbb{Z}.$

→ Even has $c = (-1)^{i/2}$ $\quad i \Rightarrow T_i(x)$

→ Highest powers have coefficients 2^{i-1}

→ Total terms:
for i = even, $\frac{i}{2} + 1$
for i = odd, $\frac{i-1}{2} + 1$

i and $i+1$ have same ~~term~~ number of terms.

$$\boxed{T_n(x) = 2^{n-1} \prod_{k=1}^{n} \left\{ x - \cos\left[\frac{(2k-1)\pi}{2n}\right] \right\}}$$

T	C	1	2	3	4	5	6	7	8	9	10
0	1	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0
2	-1	0	2	0	0	0	0	0	0	0	0
3	0	-3	0	4	0	0	0	0	0	0	0
4	1	0	-8	0	8	0	0	0	0	0	0
5	0	5	0	-20	0	16	0	0	0	0	0
6	-1	0	18	0	-48	0	32	0	0	0	0
7	0	-7	0	56	0	-112	0	64	0	0	0
8	1	0	-32	0	160	0	-256	0	128	0	0
9	0	9	0	-120	0	432	0	-576	0	256	0
10	-1	0	50	0	-400	0	1120	0	-1280	0	512

$\boxed{A028297}$

i	$i+1$
0	y
x	0
0	$2x-y$

S_0 1 2 4 8 16 32 64 128 256 ...... 2^n
$-S_1$ -1 -3 8 20 48 112 288 576 1280 ....
S_2 1 5 18 56 160 432 1120 ......
$-S_3$ 1 7 32 120 400 ......
S_4 1 9 50 ......

coefficients of $\frac{1-x}{(1-2x)^n}$

$n \geq 2.$

	n	$n+1$
S_i	x	y
S_{i+1}	z	$2z+y$

Analysis Over Proposed Theory:-

By row-wised adjustment at regular interval of each row, calculations can be made by alternate ~~alt~~ subtraction and addition for main M-nacci generation. This can be corrected when proper results are not obtained. Hence, if the irregularity is obtained in series', we can be adjust them and All of these can be corrected. Hence, the generalisation to be proposed is based on the coefficients of ~~Poly~~ Chebyshev T Polynomial of first kind.

$$m_{j0} = \sum_{i=0}^{n} S_{ji} \qquad (S_j : \text{Columns of coefficient triangle.})$$

For computational result, and analysis, the Chebyshev T Polynomial of ~~the~~ first kind and its ~~co~~ coefficient triangle is used. All the columns of the triangle are shifted regular interval ~~base~~ based on m-value and the ~~a~~ by above eqn new terms are calculated. Then the results are compared with m-nacci sequence, obtained by recurrence, and the difference is taken. Then at first non-zero value it is ~~sent~~ checked whether the sequence need to be shifted or not. It is to be shifted based on the polarity of difference.

Fibonacci Sequence with Random Variable:-

$(0,1) \Rightarrow (0,n)$:

Instead of 1, if a random value n is taken and then by recurrence definition of fibonacci sequence, new terms are generated. These terms are 0, n, n, 2n, 3n, 5n, 8n, 13n, 21n, 34n, 55n, 89n, 144n, ... $n f_n$.

$$F_n = n^* f_n.$$

$(0,1) \Rightarrow (x,y)$:

Instead of 0 and 1, two random variables are taken i.e., x and y, we create sequence like Fibonacci Sequence by recurrence definition.

x	$5x+8y$	$89x+144y$
y	$8x+13y$	$144x+233y$
$x+y$	$13x+21y$	$\vdots$
$x+2y$	$21x+34y$	
$2x+3y$	$34x+55y$	F_n
$3x+5y$	$55x+89y$	

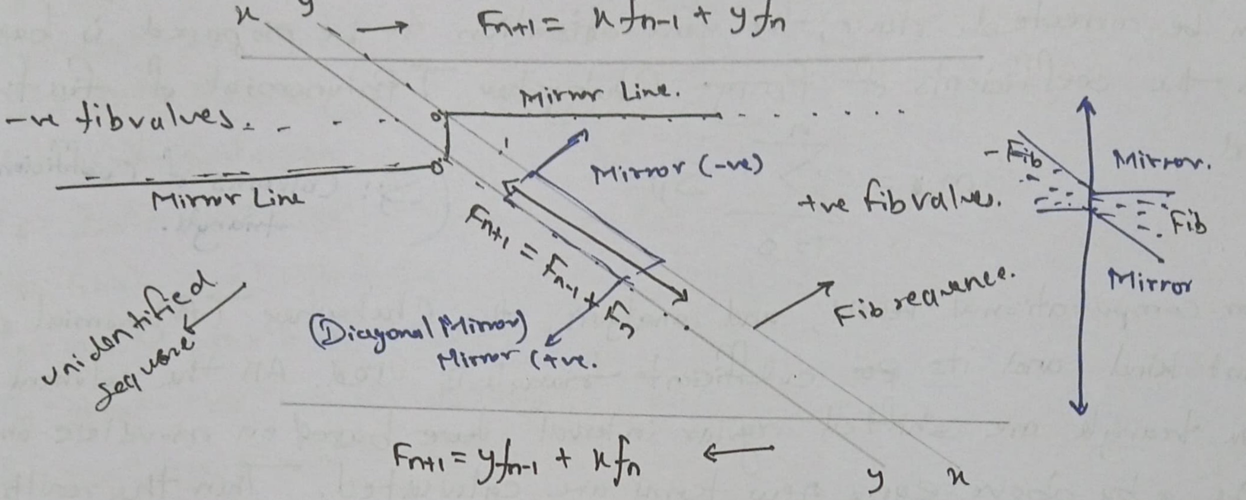
$$F_{n+1} = x f_{n-1} + y f_{n}$$

When taken in other direction i.e., y and x, the sequence looks like:

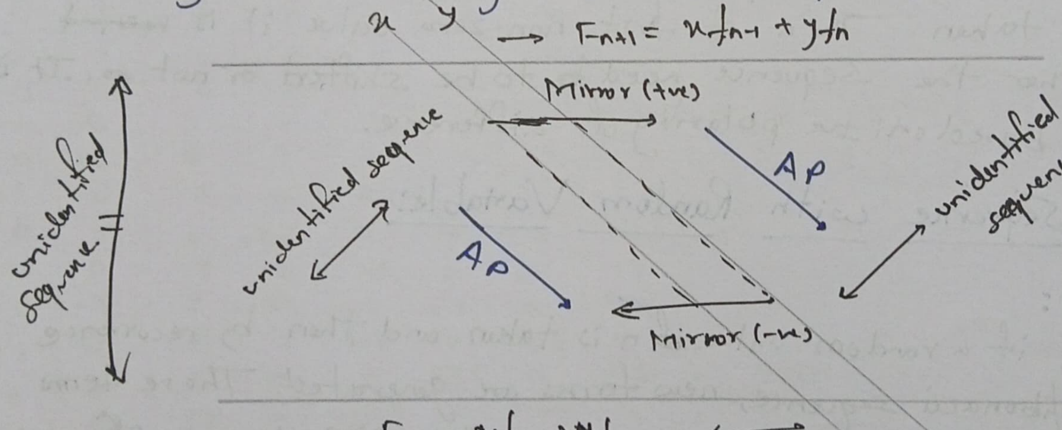
y	$3y+5x$	$34y+55x$
x	$5y+8x$	$55y+89x$
$y+x$	$8y+13x$	$89y+144x$
$y+2x$	$13y+21x$	$\vdots$
$2y+3x$	$21y+34x$	F_n

$$F_{n+1} = y f_{n-1} + x f_n$$

Fibmat: Using Fibonacci and its negative values. (Row-wise sequence)



Fibmat: Using consecutive integers. (Row-wise Sequence)



Tribonacci Sequence With Random Variables:

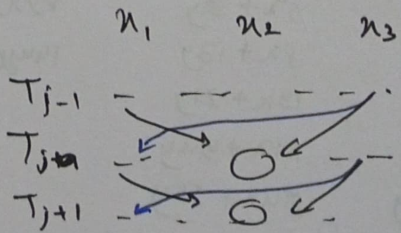
Sequence: 0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, ...

T_i = i^{th} term of Sequence. $T_i = \sum_{i=1}^{3} C_i x_i$, x_1, x_2, x_3 : Variables.

C_{j3} = Main tribonacci Sequence $= t_j$.

$C_{j1} = t_{j-1}$

$C_{j2} = C_{(j-1)1} + C_{(j-1)3} = t_{j-1} + t_{j-2}$



Tetranacci Sequence with Random Variables:-

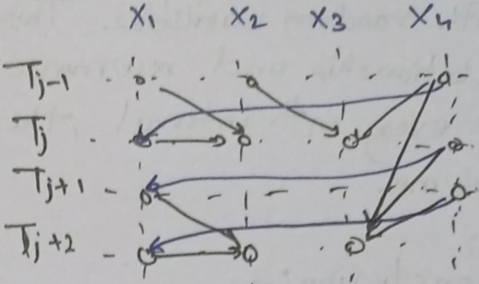
Sequence : 0, 0, 0, 1, 1, 2, 4, 8, 15, 29, 56, 108, 208, 401, 773, ...

q_i = ith tetrabonacci terms.

$C_{j4} = C_{(j+1)1} = q_j \qquad C_{j1} = C_{(j-1)4} = q_{j-1}$

$C_{j2} = C_{(j-1)1} + C_{(j-1)4} = q_{j-1} + q_{j-2}$

$C_{j3} = C_{(j-1)2} + C_{(j-1)4} = q_{j-1} + q_{j-2} + q_{j-3}$



Mnacci with Random Variables : Expansion to m-variables:-

m = number of variables. $\{x_i\}$ = set of variables.

m_i = main m-nacci sequence.

M_j = jth m-nacci term.

1st Pingalmat: Identity matrix of m×m.

As coefficient matrix:-

$$\begin{array}{c} x_1 \quad \cdots \quad x_i \quad \cdots \quad x_m \\ \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & 1 & & & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \end{array}$$

2nd Pingalmat: m×m Squarematrix.

Diagonal and rightside coefficients. $\rightarrow 2^{i-1}$

For values left of diagonal = $\sum_{j=0}^{i-1} 2^{m-j-2} = 2^{m-1} - 2^{m-i-1}$; $i = 2$ to $m-1$.
(for ith colum)

nth Pingalmat:

$$C_{ji} = \sum_{k=1}^{m} m_{j-k} \;, \quad C_{ji} = C_{j(i-1)} + m_{j-1} \;, \quad C_{jm} = C_{(j+1)1} = m_j.$$

Colum 1

m_{j-1}

$m_{j-1} + m_{j-2}$

$m_{j-1} + m_{j-2} + m_{j-3}$

$m_{j-1} + m_{j-2} + m_{j-3} + m_{j-4}$

Coefficient in (i)

Row j ⟷

Colum m. $m_{j-1} + m_{j-2} + \cdots\cdots m_{j-i-1} \cdots\cdots m_{j-m-1}$

Intermediate Colums:

Intermediate Colums can be taken as special cases of m-nacci with random variables. This can be a good logic for its ~~recurrence~~ relationship and recurrence. Here, it can be observed that at every m^{th} interval, the relationship can be observed with other column.

Conclusion:-

① Chebyshev T Polynomial of first kind plays an important role in mnacci sequences, as they are itself the solution.

② ~~Every~~ ~~Mnacc~~. Mnacci Sequence with random variables and coefficient of each variable can be represented in pingalmats.

③ Every coefficient is dependent on main ~~M-nacci~~ mnacci sequence.

④ Each column in these pingalmats are special cases of mnacci sequence with random-variables.

x

M-nacci Sequence :-

Fibonacci Sequence :- $0, \boxed{\overset{f_1}{1}, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots, (\overset{f_n}{f_{n-1} + f_{n-2}})}$

Tribonacci Sequence :- $0, 0, \boxed{\underset{t_1}{1}, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, \ldots, (\overset{t_n}{t_{n-1} + t_{n-2} + t_{n-3}})}$

→ Fibonacci Sequence Using two random variables:

let x and y be the two random variables.

$F_1 = x \quad F_2 = y$

$F_3 = x + y = f_1 x + f_2 y$

$F_4 = x + 2y = f_2 x + f_3 y$

$F_5 = 2x + 3y = f_3 x + f_4 y$

$F_6 = 3x + 5y = f_4 x + f_5 y$

$F_7 = 5x + 8y = f_5 x + f_6 y$

$F_8 = 8x + 13y = f_6 x + f_7 y$

$F_9 = 13x + 21y = f_7 x + f_8 y$

$F_{10} = 21x + 34y = f_8 y + f_9 y$

$F_{11} = 34x + 55y = f_9 x + f_{10} y$

$F_{12} = 55x + 89y = f_{10} x + f_{11} y$

$\vdots$

$$\boxed{F_n = f_{n-2} x + f_{n-1} y}$$

$$\boxed{\frac{f_n}{f_{n-1}} = \frac{F_n}{F_{n-1}} = \phi}$$

if $x = 0$, $y = 1$: fibonacci.

if $x = 1$, $y = 0$:

→ 1, 0, 1, 1, 2, 3, ...

→ 1, fibonacci.

→ Tribonacci Sequence using three random variables.

let a, b and c be the three random variables.

$T_1 = a \quad T_2 = b \quad T_3 = c$

$T_4 = a + b + c = t_1 a + b + t_2 c$

$T_5 = a + 2b + 2c = t_2 a + (t_1 + t_2) b + t_3 c$

$T_6 = 2a + 3b + 4c = t_3 a + (t_2 + t_3) b + t_4 c$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

$T_7 = 4a + 6b + 7c = t_4 a + (t_3 + t_4) b + t_5 c$

$T_8 = 7a + 11b + 13c = t_5 a + (t_4 + t_5) b + t_6 c$

$T_9 = 13a + 20b + 24c = t_6 a + (t_5 + t_6) b + t_7 c$

$T_{10} = 24a + 37b + 44c = t_7 a + (t_6 + t_7) b + t_8 c$

$T_{11} = 44a + 68b + 81c = t_8 a + (t_7 + t_8) b + t_9 c$

$T_{12} = 81a + 125b + 149c = t_9 a + (t_8 + t_9) b + t_{10} c$

$\vdots$

$$\boxed{T_n = t_{n-3}\, a + (t_{n-3} + t_{n-4})\, b + t_{n-2}\, c}$$

if $a = 0$, $b = 0$, $c = 1$: tribonacci. → 0, 0, 1, 1, 2, 4, 7, 13, ...

if $a = 0$, $b = 1$, $c = 0$: 0, 1, 0, 1, 2, 3, 6, 11, 20, ...

if $a = 1$, $b = 0$, $c = 0$: 1, 0, 0, 1, 1, 2, 4, 7, 13, ... 1, tribonacci.

→ Quadrobonacci Sequence using four random variables:

$$0,0,0,[\overset{q_1}{1},1,2,4,8,15,29,56,108,208,401,773,1490,\dots \leftarrow \text{Tetranacci}$$

let a, b, c, d be four random variables.

$Q_1 = a,\ Q_2 = b,\ Q_3 = c,\ Q_4 = d$

$$\left.\begin{aligned}
Q_5 &= a+b+c+d\\
Q_6 &= a+2b+2c+2d\\
Q_7 &= 2a+3b+4c+4d\\
Q_8 &= 4a+6b+7c+8d
\end{aligned}\right\} = \left\{\begin{matrix} 1 & 1 & 1 & 1\\ 1 & 2 & 2 & 2\\ 2 & 3 & 4 & 4\\ 4 & 6 & 7 & 8\end{matrix}\right\}_{2^n}$$

$Q_9 = 8a+12b+14c+15d$

$Q_{10} = 15a+23b+27c+29d$

$Q_{11} = 29a+44b+52c+56d$

⋮

$$\boxed{Q_n = (q_{n-4})\cdot a + (q_{n-5}+q_{n-4})\cdot b + (q_{n-5}+q_{n-6}+q_{n-4})\cdot c + (q_{n-3})\cdot d}$$

→ Pentabonacci Sequence using five random variables.

$$0,0,0,0,[\overset{p_1}{1},1,2,4,8,16,31,61,120,236,464,912,1793,\dots$$

let a, b, c, d, e be five random variables.

$$\left.\begin{aligned}
P_6 &= a+b+c+d+e\\
P_7 &= a+2b+2c+2d+2e\\
P_8 &= 2a+3b+4c+4d+4e\\
P_9 &= 4a+6b+7c+8d+8e\\
P_{10} &= 8a+12b+14c+15d+16e
\end{aligned}\right\} = \left\{\begin{matrix} 1 & 1 & 1 & 1 & 1\\ 1 & 2 & 2 & 2 & 2\\ 2 & 3 & 4 & 4 & 4\\ 4 & 6 & 7 & 8 & 8\\ 8 & 12 & 14 & 15 & 16\end{matrix}\right\}_{2^n}$$

$P_{11} = 16a+24b+28c+30d+31e$

$P_{12} = 31a+47b+55c+59d+61e$

$P_{13} = 61a+92b+108c+116d+120e$

⋮

$$\boxed{\begin{aligned}P_n = {}&(p_{n-5})a + (P_{n-4})e\\ &+ (p_{n-5}+P_{n-6})b\\ &+ (p_{n-5}+p_{n-6}+2^{n-9})c\\ &+ (p_{n-5}+p_{n-6}+p_{n-4}+2^{n-9})d\end{aligned}}$$

$192 = 256-64$
$= 2^8 - 2^6$

$384 = 512-128$
$= 2^9 - 2^7$

M-nacci Sequence:-

M_1 to M_n:

$$\begin{bmatrix} M_1 & 0 & 0 & \cdots & 0 \\ 0 & M_2 & 0 & \cdots & \vdots \\ 0 & 0 & M_3 & \cdots & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n \end{bmatrix} = I_n \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_n \end{bmatrix}$$

↑ Identity Matrix ↑ Value Column vector

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	...	M_n
M_{n+1}	1	1	1	1	1	1	1	1	1	1	1	...	1
M_{n+2}	1	2	2	2	2	2	2	2	2	2	2	...	2
M_{n+3}	2	3	4	4	4	4	→					← 4	4
M_{n+4}	4	6	7	8	→								8
M_{n+5}	8	12	14	15	16	→						→	16
M_{n+6}	16	24	28	30	31	32	→						32
M_{n+7}	32	48	56	60	62	63	64	→				→	64
M_{n+8}	64	96	112	120	124	126	127	128	→				128
M_{n+9}	128	192	224	240	248	252	254	255	256	→			256
M_{n+10}	256	384	448	480	496	504	508	510	511	512	→		512
M_{n+11}	512	768						1020	1022	1023	1024	→	1024

Upper Triangle Rows: 2^{n-1}

$c \to$ column number

$j \Rightarrow$ column number

$$\sum 2^{n-2-i}$$

for $i=0$ to $i=j-2$

$$= \sum_{i=1}^{c} 2^{n-2-(i-1)}$$

$$= 2^{n-1} - 2^{n-(j+1)}$$

$1 : j : n-1$

2^*

	1	2	3	4	5	6	7	8	9	10	11	12	...	...	n-4	n-3	n-2	n-1	n
M_{2n}	2^{n-2}	$2^{n-1}-2^{n-3}$	$2^{n-1}-2^{n-4}$	$2^{n-1}-2^{n-5}$	$2^{n-1}-2^{n-6}$	$2^{n-1}-2^{n-7}$	$2^{n-1}-2^{n-8}$	$2^{n-1}-2^{n-9}$	$2^{n-1}-2^{n-10}$	$2^{n-1}-2^{n-11}$	$2^{n-1}-2^{n-12}$	..	..	..	$2^{n-1}-2^3$	$2^{n-1}-2^2$	$2^{n-1}-2^1$	$2^{n-1}-2^0$	2^{n-1}

S1: $a(n) = (n+2)2^{n-1}$, $n \geq 0$.

$a(0) = 1$
$a(1) = 3$ $\quad 2^{n-2}(n+1)(n+4)$
$a(2) = 8$
$a(3) = 20$ $\quad - 2^{n-3}(n)(n+3)$
$a(4) = 48$ $\quad 2^{n-3}(2n^2+5n+4 - n^2 3n)$
$a(5) = 112$
$a(6) = 256$ $\quad \rightarrow 2^{n-3}(n^2+2n+4)$
$a(7) = 576$
$a(8) = 1280$
$a(9) = 2816$
$a(10) = 6144$

S2: $a(n) = n(n+3)2^{n-3}$, $n \geq 0$

$a(0) = 0$
$a(1) = 1$
$a(2) = 5$
$a(3) = 18$
$a(4) = 56$
$a(5) = 160$
$a(6) = 482$
$a(7) = 1120$
$a(8) = 2816$
$a(9) = 6912$
$a(10) = 16640$

S3: $a(n) = \frac{(n+1)(n+2)(n+6)\,2^{n-2}}{3}$

$$\boxed{(n+2)^* 2^{n-1} - (n+1)^* 2^{n-2}}$$

$\rightarrow n(2^{n-1} - 2^{n-2}) + 2^n - 2^{n-2}$

$\rightarrow n 2^{n-2}(1) + 2^{n-2}(4-1)$

$\rightarrow 2^{n-2}(n+3)$.

$S(4) = \longrightarrow \frac{n}{2} 2^{n-2}(n+3)$

$\longrightarrow 2^{n-3}\, n(n+3)$.

S3: 0, 0, 1, 7, 32, 120, 400, 1232, 3584, 9984 $\leftarrow (1-x)\cancel{\left(\frac{2}{(1-2x)^4}\right)} \leftarrow \frac{2^{(n-2)}}{3}\left((n+1)(n+2)(n+6)\right)$

0 0 1 7 32 120 400 1232 3584 9984
0 1 6 25 88 280 832 2352 6400 $\leftarrow a(n) = \frac{2^{n-3}}{3}\left((n+1)(n+3)(n+8)\right)$
1 5 19 63 192 552 1520 4048 $\leftarrow a(n+1) = \frac{2^n \cdot n^3}{48} + \frac{5 \cdot 2^n \cdot n^2}{16} + \frac{7 \cdot 2^n \cdot n}{6} + 2^n$
4 14 44 129 360 968 2528 $\quad n \geq 1$.
10 30 85 231 608 1560

$$\frac{(n+5)(n^2+13n+18)\,2^{n-5}}{3} = \left(\frac{1-x}{1-2x}\right)^4$$

$$\frac{\lambda(1-x)^3}{(1-2x)^4}$$

This takes $\frac{1-x}{(1-2x)^n}$ and its coeff. for calculations ~~solution~~ of m-nacci terms.

$$\frac{2^{in-mi} - \sum 2^{(i-1)(n-j(i+1))}\prod(x-i)}{2}$$

→ M-nacci sequence holds summation of various sequences. If $(m-1)$ 0s and just one 1 is taken then it can be utilised the each terms of mnacci matrix.

→ But the case with m-nacci using 0s and 1 is that for each n^{th} term taken consecutively, there is presence of other terms for subtraction from 2^n.

→ First m-terms are $(m-1)$ 0s and one 1.

→ Second m-terms are of 2^n from 1 to 2^{m-1}.

→ Third m-terms are of 2^m with subtraction from 1, 3, 8, 20, 48 .... $(m+2)2^{m-1}$. → $2^{m+1} - (m+2)2^{m-1}$

→ Fourth m-terms are of 2^n with subtraction using $(2m+1)2^{m-1}$ and $m(m+3)^{m-3}$.

i.e. $2^{m+1} - (2m+1)2^{2m-1} - m(m+3)^{m-3}$

→ Similarly other m-terms can be solved via studying those difference and n^{th} m-terms include subtraction of chebyshev negated coefficients of specific column.

→ Studying ~~these~~ these terms we will find more terms for fundamental m-nacci and then utilised it for m-nacci matrix.