Collatz Conjecture and the Struture of Tree that it grows into.

Introduction:-

Collatz Conjecture is a problem in which we end in a loop of 4-2-1 after applying two operations based the number being odd or even.

$$a(n) = \begin{cases} 3n+1 & \ldots\ n = \text{odd} \\ n/2 & \ldots\ n = \text{even} \end{cases}$$

ex: 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, ...

In this example which occurs with all numbers (>0), the sequence gets into a infinite loop of 4-2-1. The number of steps taken by any integer to end up at 1 at last is random and could be in single-digit number to thousand or more. For instance, for $n = 17$, takes 12 steps to reach 1 while $n = 27$ takes 111 steps. 2^n values take minimum steps to reach 1 which is just n. These steps to reach 1 from initial n is called as stopping time. Stopping time is finite for all the integer values since they can be even and odd and will always reach 1 by means of two operations.

The visualization of collatz conjecture can be seen differently or can be created in different manner. Some visualisation can show number of steps taken to reach one, all ~~following~~ falling below ~~certain~~ limit of a certain function. Some visualisation show a growing a tree and some show only odd numbers in a directed graph. A new a tree can be created in a way where the tree can grow based on the number ~~of~~ being even and ~~certain~~ odds value can be used as nodes where new branches grow.

Theory:-

Considering 2^n takes minimum time to reach one we can have a bare stem as 2^n to 1.

$1 \leftarrow 2 \leftarrow 4 \leftarrow 8 \leftarrow 16 \leftarrow 32 \leftarrow 64 \leftarrow 128 \leftarrow 256 \leftarrow 512 \leftarrow 1024 \leftarrow 2048 \leftarrow \cdots 2^n \ldots$

If we consider other numbers, we see that before falling to 4-2-1, they all end up closer to multiples of 2 or something related to it. By going reverse from here, from the two basic operations, we can say that the even number can formed either by dividing a even number or by multiplying odd by 3 and adding 1.

$16 \leftarrow 32$ $\quad$ $64 \leftarrow 128$ $\quad$ $256 \leftarrow 512$ $\quad$ $16 \rightarrow 2^4$

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$ $\quad$ $64 \rightarrow 2^6$

$3(5)+1$ $\quad$ $3(21)+1$ $\quad$ $3(85)+1$ $\quad$ $256 \rightarrow 2^8$

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$

5 $\quad$ 21 $\quad$ 85

$1024 \leftarrow 2048$ $\quad$ $4096 \leftarrow 8192$ $\quad$ $16384 \leftarrow 32768$ $\quad$ $1024 \rightarrow 2^{10}$

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$ $\quad$ $4096 \rightarrow 2^{12}$

$3(341)+1$ $\quad$ $3(1365)$ $\quad$ $3(5461)+1$ $\quad$ $16384 \rightarrow 2^{14}$

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$

341 $\quad$ 1365 $\quad$ 5461

A generalisation can be drawn ~~the~~ for those values on ~~???~~ base stem we get ~~a~~ nodes which creates brances, i.e., 2^{6n-2}, 2^{6n}, 2^{6n+2} where 2^{6n} leading to a branch that has no branching. since $2^{6n}-1$ is divisible by 9. If we take any value of $\frac{2^{6n}-1}{3} \cdot 2^m$ we will find that it is not possible to have branching.

$$\begin{array}{ccccccccccccccc} & & & & 5 & & 21 & & 85 & & 341 & & 1365 & & \\ & & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ 1 \leftarrow 2 \leftarrow 4 \leftarrow 8 & \leftarrow & & & 16 \leftarrow 32 & \leftarrow & 64 \leftarrow 128 & \leftarrow & 256 \leftarrow 512 & \leftarrow & 1024 \leftarrow 2048 & \leftarrow & 4096 & \leftarrow & \cdots \end{array}$$

$$\leftarrow 8192 \leftarrow 16384 \leftarrow 32768 \leftarrow \cdots$$
$$\uparrow$$
$$5461$$

So the nodes with ~~no~~ branching can further have more branching and also they are not divisible by 9.

i.e., $\frac{2^{6n-2}-1}{3} \cdot 2^m$ and $\frac{2^{6n+2}-1}{3} \cdot 2^m$ are not divisibly 3 and hence will have branching. Anothe observation being $\frac{2^{4n}-1}{3}$ are divisible by 5.

If ~~a~~ we take nodes and make their branch we will end with their multiples of 2 with alternating values ~~starting~~ either odd powers of 2 or even powers of 2 from such branches.

8

$\uparrow$

$16 \leftarrow 5 \leftarrow 10 \leftarrow 20 \leftarrow 40 \leftarrow 80 \leftarrow 160 \leftarrow 320 \leftarrow 640 \leftarrow 1280 \leftarrow 2560 \leftarrow \cdots 5 \cdot 2^n$

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$

32 $\quad$ 3 $\quad$ 13 $\quad$ 53 $\quad$ 213 $\quad$ 853

$\uparrow$ $\quad$ 1 $\quad$ 3 $\quad$ 5 $\quad$ 7 $\quad$ 9

$64 \leftarrow 21 \leftarrow 42 \leftarrow 84 \leftarrow 168 \leftarrow 336 \leftarrow 672 \leftarrow 1344 \leftarrow \cdots 21 \cdot 2^n$

$\uparrow$

↑

256 ← 85 ← 170 ← 340 ← 680 ← 1360 ← 2720 ← 5440 ← 10880 ← 21760 ← ⋯ 85.2^n

↑ 512   ↑ 113 (2)   ↑ 453 (4)   ↑ 1813 (6)   ↑ 7253 (8)

↑

1024 ← 341 ← 682 ← 1364 ← 2728 ← 5456 ← 10912 ← 21824 ← 43648 ← ⋯ 341.2^n

↑ 2048   ↑ 227 (1)   ↑ 909 (3)   ↑ 3637 (5)   ↑ 14549 (7)

↑

4096 ← 1365 ← 2730 ← 5460 ← 10920 ← 21840 ← 43680 ← 87360 ← ⋯ 1365.2^n

↑

8192

↑

16384 ← 5461 ← 10922 ← 21844 ← 43688 ← 87376 ← 174752 ← 349504 ← ⋯ 5461.2^n

↑   ↑ 7281 (2)   ↑ 29125 (4)   ↑ 116501 (6)

Hence, at this stage the pattern that can be considered is that the branches further branch at odd-power multiples or even-power multiples i.e. $2^{6n}, 2^{6n-2}, 2^{6n-4}$ or $2^{6n-5}, 2^{6n-3}, 2^{6n-1}$. Also, This can be observed in branches that leads to branching. $(n \geq 1)$.

5

↑

10 ← 3 ← 6 ← 12 ← 24 ← 48 ← 96 ← 192 ← 384 ⋯ ← 3.2^n

↑

20

↑

40 ← 13 ← 26 ← 52 ← 104 ← 208 ← 416 ← 832 ← 1664 ← 3328 ← 6656 ← ⋯ 13.2^n

↑ 80   ↑ 17 (2)   ↑ 69 (4)   ↑ 277 (6)   ↑ 1109 (8)

↑

160 ← 53 ← 106 ← 212 ← 424 ← 848 ← 1696 ← 3392 ← 6784 ← 13568 ← ⋯ 53.2^n

↑ 320   ↑ 35 (1)   ↑ 141 (3)   ↑ 565 (5)   ↑ 2261 (7)

↑

640 ← 213 ← 426 ← 852 ← 1704 ← 3408 ← 6816 ← 13632 ← ⋯ 213.2^n

↑

⋮

5.2^n

Occurance of nodes that lead to no branching over its branch ~~bet~~ is random. And so the odd nodes and even nodes also. Hence the patterm for this power values are quite hard to figure out. Over the branch the further branches, the nodes that lead to even or odd branching are in AP. But the position for it i.e. 1-3-5 and

2-4-6 cannot be patterned.

Position	Branching			Position
1	odd ⇌ even		NB	2
3	NB	odd ⇌ even		4
5	even	NB	odd	6

NB: No branching

Total Combination: 12

~~If we~~ It is observed that the placements are in AP which ~~can be shown~~ is is represented in following examples.

$x \leftarrow 2x \leftarrow 4x \leftarrow \overset{3}{8x} \leftarrow 16x \leftarrow \overset{5}{32x} \leftarrow 64x \leftarrow \overset{7}{128x} \leftarrow 256x \leftarrow \overset{9}{512x} \leftarrow 1024x \leftarrow \overset{11}{2048x}$

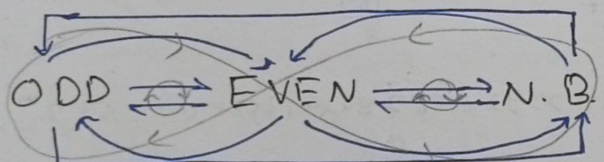
odd (1), NB (3), even (5), odd (7), NB (9), even (11)

$\overset{1}{y} \leftarrow \overset{2}{2y} \leftarrow 4y \leftarrow \overset{4}{8y} \leftarrow 16y \leftarrow 32y \leftarrow \overset{6}{64y} \leftarrow 128y \leftarrow \overset{8}{256y} \leftarrow 512y \leftarrow \overset{10}{1024y} \leftarrow 2048y \leftarrow \overset{12}{4096y} \leftarrow \dots y.2^n$

even, odd, ~~odd~~ NB, even, odd, NB

	Position	Branching			Position
Power or Nodes	6n−5	odd ⇌ even		NB	6n−4
	6n−3	even	NB	odd	6n−2
	6n−1	NB	odd ⇌ even		6n
Position		6n−5	6n−3	6n−1	
		6n−4	6n−2	6n	

This is a vague representation of the occurence of powers and nodes, and also ~~the~~ how randomly they appear in these 12 combinations throughout the trees. And hence there is no pattern to be recognized apart from the AP ~~or~~ of powers or the branching from the nodes.



From here, lets work on a generalisation of nodes. First of all, the base stem nodes will be treated as Root term but only for those that has branches i.e., 2^{6n} will be excluded.

$$R = \begin{cases} 2^{6n-2} \\ 2^{6n+2} \end{cases} ; n \geq 1.$$

So the values that will initiate the branching ~~one~~ is $\frac{R-1}{3}$ and the values of such branches are $\frac{R-1}{3} \cdot 2^{m_1}$ where $m_1 \in \mathbb{N}$ and is either odd or even.

From base stem to the branches, we can create new nodes of ~~the~~ these new branches as $\frac{\frac{R-1}{3}\cdot 2^m - 1}{3}$. Depending on the branch and the even-odd parity over it we can make new branches. Overall multiple branches and nodes, we can now develop the formula as :

$$\Rightarrow \boxed{\frac{\frac{\frac{\frac{R-1}{3}\cdot 2^{m_1}-1}{3}\cdot 2^{m_2}-1}{3}\cdot 2^{m_3}-1}{3}\cdots\cdots \cdot 2^{m_i}-1}{3}}$$

i = no. of branches. ~~the~~

$m_1, m_2, m_3 \ldots m_i \in \mathbb{Z}$ all depending on odd-even pattern.

For ~~further~~ simplification:

$$\rightarrow \frac{\frac{R-1}{3}\cdot 2^{m_1}-1}{3} = \cancel{\frac{R-1}{3^2}\cdot 2^{m_1}-1} \quad \frac{(R-1)\,2^{m_1}-3}{3^2}$$

$$\rightarrow \frac{\frac{(R-1)2^{m_1}-3}{3^2}\,2^{m_2}-1}{3} = \frac{((R-1)\,2^{m_1}-3)\,2^{m_2}-3^2}{3^3}$$

$$\rightarrow \frac{\frac{((R-1)2^{m_1}-3)\,2^{m_2}-3^2}{3^3}\cdot 2^{m_3}-1}{3} = \frac{(((R-1)\,2^{m_1}-3)2^{m_2}-3^2)\,2^{m_3}-3^3}{3^4}$$

$$\Rightarrow \boxed{\frac{(\ldots(((((R-1)2^{m_1}-3)\,2^{m_2}-3^2)2^{m_3}-3^3)\,2^{m_4}-3^4)\,2^{m_5}\cdots\cdots)2^{m_i}-3^i}{3^{i+1}}}$$

Conclusion:-

① The structure of the tree can be visualized at the base stem with power of 2, ~~while~~ the branches follow the same route.

② Pattern of position is either odd or even, but the nodes for even, odd and ~~NB~~ Non-branching branches ~~can be~~ are found to be in random.

③ The formula for generalisation of node values can be designed with root term with known pattern as it remain same while the branches have random values. The formula in division ladder is in simplest form while after multiplication a bracketed format is obtained with a further simplification can be done.

$$\rightarrow \cancel{(R-1)2^{\sum_{i=1}^{k} m_i} - 3.2^{\sum_{i=2}^{k} m_i} - \sum_{i=1}^{k} 3^i \cdot 2\sum}$$

$$\rightarrow \frac{(R-1)2^{\sum^{k} m} - 3.2^{\sum_{i=2}^{k} m_i} - \left(\sum_{i=1}^{k} 3^i \cdot 2^{\sum_{j=i}^{k} m_{j+2}}\right) - 3^{k-1}}{3^k}$$

$$\rightarrow \frac{(R-1)2^{\sum^{k} m}}{3^k} - \frac{2^{\sum_{i=2}^{k} m_i}}{3^{k-1}} - \sum_{i=1}^{k} \frac{2^{\sum_{j=i}^{k} m_{j+2}}}{3^{k-i}} - \frac{1}{3}$$

$\rightarrow R = 2^{6n-2}, 2^{6n}, 2^{6n+2}$, k = Number of branch

$n \geq 1$, $\{m_i\} \in \mathbb{N}$

— x —

<u>Note</u>:- m_i and k depends on the nodes selected or taken for various calculations for the validation of the formula / theory.

$$\rightarrow \boxed{\left(\frac{(R-1)2^{m_1}}{3} - 1\right)\frac{2^{\sum_{i=2}^{k} m_i}}{3^{k-1}} - \sum_{i=1}^{k} \frac{2^{\sum_{j=i}^{k} m_{j+2}}}{3^{k-i}} - \frac{1}{3}}$$

intro, Base stem, Nodes, branches, Consecutive branches, Generalized term, Conclusion, reference.

Collatz Conjecture:-

→ $1-2-4\ldots 2^n$: Primary stem.

→ 2^{4+2n} : Node of Branches; $n \geq 0$

→ 2^{6n} : Branches with no branching, multiple of 3 branches.

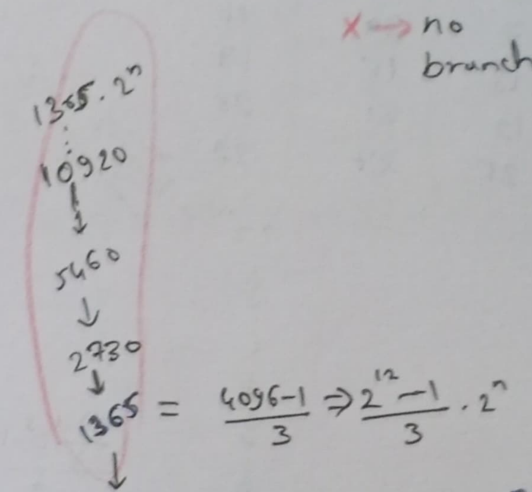
→ 2^{6n+2} ; 2^{6n+4} : Branches with Branching $\Big\}$ $n \geq 1$

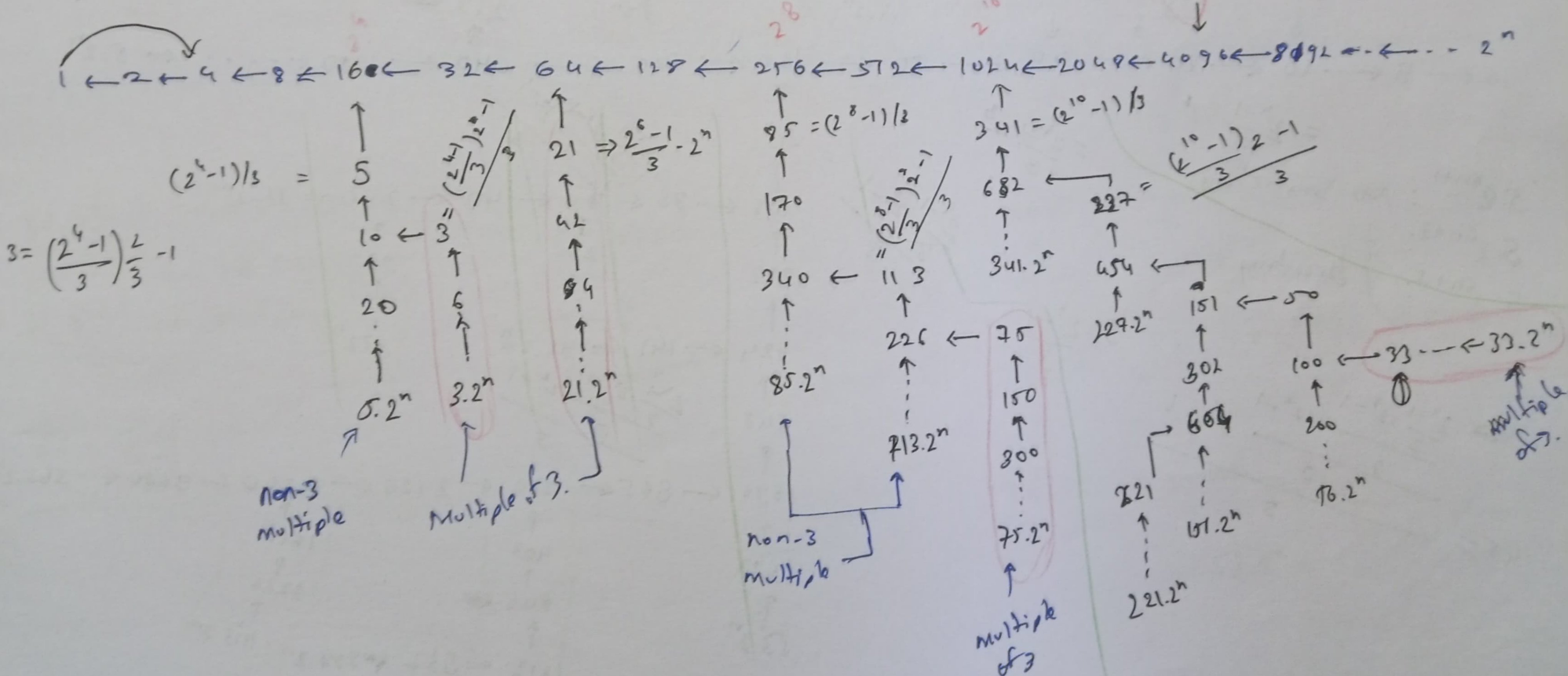
→ 16^n : Nodes of branches of multiple of 5.

→ 2^{2n+1} : Branchless Nodes, $n \geq 2$

6n	6n+2	6n+4
0	2	4
6	8	10
12	14	16
18	20	22
24	26	28
30	32	34
36	38	40
42	44	46
48	50	52

X → no branching





6n+1	6n+3	6n+5
1	3	5
7	9	11
13	15	17
19	21	23
25	27	29

$13 \cdot 2^n \to \cdots \to 208 \to 104 \to 52 \to 26 \to 13 \to 40$

$208 \leftarrow 67 \leftarrow 134 \leftarrow 268 \leftarrow 536 \leftarrow 1072 \leftarrow 2144 \leftarrow 4288 \leftarrow 8576 \leftarrow \cdots 67 \cdot 2^n$

$67 = \dfrac{\left(\dfrac{\dfrac{2^4-1}{3} \cdot 2^3 - 1}{3}\right) 2^4 - 1}{3}$

$268 \leftarrow 89$, $1072 \leftarrow 357$

$52 \leftarrow 17 \leftarrow 34 \leftarrow 68 \leftarrow 136 \leftarrow 272 \leftarrow 544 \leftarrow 1088 \leftarrow \cdots 17 \cdot 2^n$

$17 = \dfrac{\left(\dfrac{\dfrac{2^4-1}{3} \cdot 2^3 - 1}{3}\right) 2^2 - 1}{3}$

$34 \leftarrow 11 \leftarrow 22 \leftarrow \cdots 11 \cdot 2^n$

$22 \leftarrow 7 \leftarrow 14 \leftarrow 28 \leftarrow 56$

$136 \leftarrow 45 \leftarrow \cdots 45 \cdot 2^n$

$28 \leftarrow 9 \leftarrow \cdots 9 \cdot 2^n$

$544 \leftarrow 181 \leftarrow 362 \leftarrow 213$

$213 = \dfrac{\left(\dfrac{2^4-1}{3}\right) 2^7 - 1}{3}$

$3 \cdot 2^n \to \cdots \to 6 \to 3 \to 10$

$3 = \dfrac{\dfrac{2^4-1}{3} \cdot 2^1 - 1}{3}$

$13 = \dfrac{\left(\dfrac{2^4-1}{3} 2^3 - 1\right)}{3}$

$16 \leftarrow 5 \leftarrow 10 \leftarrow 20 \leftarrow 40 \leftarrow 80 \leftarrow 160 \leftarrow 320 \leftarrow 640 \leftarrow 1280 \leftarrow \cdots \leftarrow 5 \cdot 2^n$

1 $= \dfrac{2^4-1}{3} \cdot 2^1$ 3 $= \dfrac{2^4-1}{3} \cdot 2^3$ 5 $=$ 7

$160 \leftarrow 53 \leftarrow 106 \leftarrow 212 \leftarrow 424 \leftarrow 848 \leftarrow 1696 \leftarrow 3392 \leftarrow \cdots 53 \cdot 2^n$

$53 = \dfrac{\dfrac{2^4-1}{3} \cdot 2^5 - 1}{3}$

$106 \leftarrow 35 \leftarrow 70 \leftarrow 140 \leftarrow 280 \leftarrow \cdots 35 \cdot 2^n$

$35 = \dfrac{\left(\dfrac{\dfrac{2^4-1}{3} \cdot 2^5 - 1}{3}\right) 2^2 - 1}{3}$

$70 \leftarrow 23$

$280 \leftarrow 93 \leftarrow \cdots 93 \cdot 2^n$

$424 \leftarrow 141 \leftarrow 141 \cdot 2^n$

$141 = \dfrac{\dfrac{\dfrac{2^4-1}{3} \cdot 2^5 - 1}{3} \cdot 2^2 - 1}{3}$

$1696 \leftarrow 565 \leftarrow 1210 \leftarrow 2420 \leftarrow 4840 \leftarrow 9680 \leftarrow \cdots 565 \cdot 2^n$

$1210 \leftarrow 403 \leftarrow 806 \leftarrow 1612 \leftarrow \cdots$

$1612 \leftarrow 537 \leftarrow 537 \cdot 2^n$

$4840 \leftarrow 1613 \leftarrow 3226 \leftarrow \cdots 1613 \cdot 2^n$

$5 \cdot 2^{6n+1}$: No-branching.

$5 \cdot 2^{6n+3}$:
$5 \cdot 2^{6n+5}$: } branching.

$5 = \dfrac{2^4-1}{3}$

$\dfrac{\dfrac{\dfrac{2^4-1}{3} \cdot 2^3 - 1}{3} \cdot 2^2 - 1}{3} \cdot 2^3 - 1$

$45 = \dfrac{}{3}$

6n-4	6n-2	6n
2	4	6
8	10	12
14	16	18
20	22	24
26	28	30

$113 \cdot 2^n \to \dots \to 7232 \to 3616 \to 1808 \to 904 \to 452 \to 226 \to 113$

$803 \to 2410$; $3616 \leftarrow 1205 \leftarrow 2410 \leftarrow$

$401 \to 1204$; $904 \leftarrow 301 \leftarrow 602 \leftarrow 1204 \leftarrow$

$226 \leftarrow 75 \dots \leftarrow 75 \cdot 2^n = \dfrac{\frac{2^8-1}{3}2^2-1}{3} \cdot 2 \cdot 2^n$

$113 \leftarrow \dfrac{\frac{2^8-1}{3}2^2-1}{3}$

$256 \leftarrow 85 \leftarrow 170 \leftarrow 340 \leftarrow 680 \leftarrow 1360 \leftarrow 2720 \leftarrow 5440 \leftarrow 10880 \leftarrow 21760 \leftarrow 43520 \leftarrow \dots 85 \cdot 2^n$

2 4 6 8

$453 \cdot 2^n \to \dots \to 453 \to 1360$

$= \dfrac{\frac{2^8-1}{3}2^4-1}{3} \cdot 2^n$

$1813 \cdot 2^n \to \dots \to 58016 \to 29008 \to 14504 \to 7252 \to 3626 \to 1813 \to 5440$

$29008 \leftarrow 9669$

$6444 \to 3222 \to 1611 \to 4834$

$25780 \leftarrow 8553$; $25780 \to 12890 \to 6445 \to 19336$

$7252 \leftarrow 2417 \leftarrow 4834 \leftarrow 9668 \leftarrow 19336 \leftarrow$

$1813 = \dfrac{\frac{2^8-1}{3}2^6-1}{3}$

$7253 \cdot 2^n \to 232096 \to 116048 \to 58024 \to 29012 \to 14506 \to 7253 \to 21760$

$58024 \leftarrow 19341$

$4835 \cdot 2^n \to \dots \to 9670 \to 4835 \to 14506$

$3223 \to 9670$

$77365 \to 232096$

$7253 \cdot 2^n = \dfrac{\frac{\frac{2^8-1}{3}2^6-1}{3} \cdot 2^8-1}{3} 2^n$

$85 \cdot 2^{6n-2} \to$ no branching

$85 \cdot 2^{6n-4} \to$ branching

$85 \cdot 2^{6n} \to$ branching

$85 = \dfrac{2^8-1}{3}$

6n+1	6n+3	6n+5
1	3	5
7	9	11
13	15	17
19	21	23
25	27	29
31	$\cancel{33}$	35

$341 \cdot 2^{6n+1}$; $341 \cdot 2^{6n+5}$: } branching

$341 \cdot 2^{6n+3}$: No branching

$341 = \frac{2^{10}-1}{3}$

$\frac{\frac{2^{10}-1}{3} 2^3 - 1}{3} \cdot 2^n$

11

$\boxed{909 \cdot 2^n \; \vdots \; \downarrow \; 909}$

↓

1 2 3 4 5 6 7 8 9

1024 ← 341 ← 682 ← 1364 ← 2728 ← 5456 ← 10912 ← 21824 ← 43648 ← 87296 ← 174592 ← 341·2^n

↑ (from 682)

$227 = \frac{\frac{2^{10}-1}{3} 2 - 1}{3}$

↑

454 ← 151 ← 302 ← 604 ← 1208 ← 151·2^n

11

$\frac{\frac{\frac{2^{10}-1}{3} 2 - 1}{3} 2 - 1}{3}$

↑

908

↑

1816 ← 605

↑ ↑

3632 1210 ← 403

↑ ↑

227·2^n 2420

↑

805·2^n

$\boxed{201 \; \uparrow \; \vdots \; 201 \cdot 2^n}$ = $\frac{\frac{\frac{2^{10}-1}{3} 2 - 1}{3} 2 - 1}{3}$

↑ (from 10912)

$36377 = \frac{\frac{2^{10}-1}{3} 2^5 - 1}{3}$

↑

7274

↑

14548 ← 4849 ← 9698 ← 19396 ← 38792 ← 77584 ←

$\boxed{6465}$ ↓ (to 19396)

↑

29096

↑

58192 ← 19397 ← 38794 ← 77588 ← 155176 ← 310352 ← 620704

↑ 12931 ↑ 51725 ↑ 201901

↑

116384

↑

232768 ← $\boxed{77589..}$ = $\frac{\frac{\frac{2^{10}-1}{3} 2^5 - 1}{3} 2^6 \cdot 1}{3} \cdot 2^n$

↑ ⋮

3637·2^n

$29125 \cdot 2^n$

$\downarrow$

$1864000 \leftarrow \boxed{621333}$

$\downarrow$

932000

$\downarrow$

$466000 \leftarrow 155333 \leftarrow 310666 \leftarrow 621332 \leftarrow 1242664 \leftarrow 2485328 \leftarrow \cdots 155333 \cdot 2^n$

(103555 $\to$ 310666; 414221 $\to$ 1242664)

$\downarrow$

233000

$\downarrow$

$116500 \leftarrow 38833 \leftarrow \cancel{51777}$ 77666 $\leftarrow 155332 \leftarrow 310664 \leftarrow 621328 \leftarrow \cdots 388332^n$

($\boxed{51777} \to 155332$; $207109 \to 621328$)

$\downarrow$

58250

$\downarrow$

29125

$\downarrow$

$16384 \leftarrow 5461 \leftarrow 10922 \leftarrow 21844 \leftarrow 43688 \leftarrow 87376 \leftarrow 174752 \leftarrow 349504 \leftarrow 699008 \leftarrow 1398016 \leftarrow \cdots 5461 \cdot 2^n$

$= \frac{2^{14}-1}{3}$

$\boxed{466005} \to 1398016$

$\boxed{7281 \cdot 2^n \to \cdots \to 7281} \to 21844$

$7281 \cdot 2^n = \dfrac{\frac{2^{14}-1}{3} \cdot 2^2 - 1}{3} \cdot 2^n$

$116501.2^n \to \cdots \to 7458064 \to 3728032 \to 1864016 \to 932008 \to 466004 \to 233002 \to 116501 \to 349504$

$3728032 \leftarrow 1242677 \leftarrow 2485353 \leftarrow$ (828451 $\to$ 2485353)

$932008 \leftarrow 310669 \leftarrow 621338 \leftarrow 1242676 \leftarrow \cdots$ ($\boxed{414225} \to 1242676$)

$233002 \leftarrow \boxed{77667}$

$5461 \cdot 2^{6n+2}$: No branching

$5461 \cdot 2^{6n+4}$:

$5461 \cdot 2^{6n+6}$: branching

6n+2	6n+4	6n+6
2	4	6
8	10	12
14	16	18
20	22	24
26	28	30
32	34	36

Generalized terms

$R_1 = \frac{2^{6n-2}-1}{3}$

$R_2 = \frac{2^{6n-4}-1}{3}$

$\boxed{R_3 = \frac{2^{6n}-1}{3}}$

$\frac{\left(\frac{2^8-1}{3}\right)2^2-1}{3}$

$\frac{\left(\frac{2^8-1}{3}\right)2^4-1}{3} \cdot 2^n$

$\left(\frac{2^8-1}{3}\right) \leftarrow \left(\frac{2^8-1}{3}\right)2 \leftarrow \left(\frac{2^8-1}{3}\right)2^2 \leftarrow \left(\frac{2^8-1}{3}\right)2^3 \leftarrow \left(\frac{2^8-1}{3}\right)2^4 \leftarrow \cdots$

809

$\frac{2^{10}-1}{3} \leftarrow 341 \leftarrow 682 \leftarrow 1364 \leftarrow 2728 \leftarrow$

$\left(\frac{2^6-1}{3}\right) \cdot 2^n$

$2^0 \leftarrow 2^1 \leftarrow 2^2 \leftarrow 2^3 \leftarrow 2^4 \leftarrow 2^5 \leftarrow 2^6 \leftarrow 2^7 \leftarrow 2^8 \leftarrow 2^9 \leftarrow 2^{10} \leftarrow 2^{11} \leftarrow 2^{12} \leftarrow 2^{13} \leftarrow 2^{14} \leftarrow 2^{15} \leftarrow 2^{16} \leftarrow 2^{17} \cdots \leftarrow 2^n$

$\frac{\left(\frac{2^4-1}{3}\right)2^5-1}{3}$

$\frac{2^4-1}{3} \leftarrow \left(\frac{2^4-1}{3}\right)2 \leftarrow \left(\frac{2^4-1}{3}\right)2^2 \leftarrow \left(\frac{2^4-1}{3}\right)2^3 \leftarrow \left(\frac{2^4-1}{3}\right)2^4 \leftarrow \left(\frac{2^4-1}{3}\right)2^5 \leftarrow \left(\frac{2^4-1}{3}\right)2^6 \leftarrow \cdots$

$\frac{\left(\frac{2^4-1}{3}\right)2^3-1}{3} \leftarrow$

$\frac{\left(\frac{2^4-1}{3}\right)2-1}{3}$

Root term

$$\cfrac{\cdots\left(\cfrac{\left(\cfrac{\left(\cfrac{\boxed{\left(\frac{2^n-1}{3}\right)}2^{a_1}-1}{3}\right)2^{a_2}-1}{3}\right)2^{a_3}-1}{3}\cdots\right)2^{a_n}-1}{3}$$

$$\underbrace{\{a_1, a_2, a_3 \ldots a_n\}}_{}\in \mathbb{N}.$$

follows patter

$n \in \mathbb{N}.$

← Generalised Term.

$$\cfrac{\left(\cfrac{\left(\cfrac{\left(\cfrac{2^n-1}{3}\right)2^{a_1}-1}{3}\right)2^{a_2}-1}{3}\right)\cdots\cdots -1}{3}\,2^{a_m}-1$$

$$3$$

Root term: $\frac{2^{6n-2}-1}{3}$, $\frac{2^{6n-4}-1}{3}$, $\frac{2^{6n}-1}{3}$ ×

This can be solved in base '6N' number system. Also, by reducing the set of natural numbers to odd and then involving compression the chart to more linear arrangements. Also, it can be seen with this that various such ifinite trees can be conquered via base change and tree compression or reduction multiple bifuractions / multifurcations.

- Some branches have odd ~~shoots~~ position shoots while some have even positioned shoots.
- Shoots also follow same nature of having their shoots on even or odd positioned.
- This could be 1-3-5 or 2-4-6, any of these positions can have a branchless branch or shoots, it is difficult to recognize a pattern to find how it could be a non-random thing.
- Also, the inbetween positions do not create shoots or branch.

58252

↓

29126

↓

$14563 = \dfrac{\left(\dfrac{2^{16}-1}{3}\right)2^{1}-1}{3}$

↓

$2^{16} \leftarrow 21845 \leftarrow 43690 \leftarrow 87380 \leftarrow 174760 \leftarrow 349520 \leftarrow 699040 \leftarrow 1398080 \leftarrow 2796160 \leftarrow 5592320 \leftarrow \cdots 21845 \cdot 2^{n}$

$233013 = \dfrac{\left(\dfrac{2^{16}-1}{3}\right)2^{5}-1}{3}$

$58253 = \dfrac{\left(\dfrac{2^{16}-1}{3}\right)2^{3}-1}{3}$

$\dfrac{\left(\dfrac{2^{16}-1}{3}\right)2^{7}-1}{3}$

$\left.\begin{array}{l} 21845 \cdot 2^{6n+1} : \\ 21845 \cdot 2^{6n+3} : \end{array}\right\}$ Branching

$21845 \cdot 2^{6n+5}$: No branching.

- Apart from stem where power of 2's ~~are~~ that generate branches are even, other branches have ~~their~~ even or odd placement of shoot i.e. $\dfrac{\text{node value}-1}{3} \cdot 2^{\text{odd}}$ or $\dfrac{\text{node value}-1}{3} \cdot 2^{\text{even}}$.
- Shoot/Branches with no branching ~~are~~ are found at regular interval ~~or~~ with respect to the ~~~~ patterns with each ~~pattern~~ branch.

Pattern:

① From ~~original root~~ stem, 2^{6n} values produce ~~branches~~ branching while 16^n nodes are divisible by 5, 2^{6n+2} and 2^{6n+4} produce branching. This nodes that produce also have ~~certain~~ branches at repetition that doe not branch. Such ~~branches~~ Nodes that produce no branching with their branch are actually ~~divisible by~~. have neighbouring node divisible by 3 and hence no branches. $(n \geq 1)$.

$$\begin{array}{c} 32 \\ \uparrow \\ 64 \\ \uparrow \\ 128 \end{array} \leftarrow \frac{64-1}{3} = 21 \leftarrow 42 \leftarrow 64 \ldots$$

Stem

Neighbouring node

$$\leftarrow 454 \leftarrow 151 \leftarrow \begin{array}{c} \boxed{50} \\ \uparrow \\ 100 \\ \uparrow \\ 200 \end{array} \leftarrow \frac{100-1}{3} = 33 \leftarrow 66 \leftarrow 132 \leftarrow \ldots$$

Neighbouring node

branch

Neighbouring node.

②

$6n+1$	$6n+2$	$6n$	$6n-5$	~~$6n$~~
$6n+3$	$6n+4$	$6n-2$	$6n-3$	~~$6n+2$~~
$6n+5$	$6n+6$	$6n-4$	$6n-1$	~~$6n+4$~~
$n \geq 0$	$n \geq 0$	$n \geq 1$	$n \geq 1$	~~$n \geq 0$~~

These are the patterns that are followed by branches and the nodes of ~~stem~~ branches for further branching.

In one of these power of 2, there will be a node that produces a branch with no of-shoots.

And there ~~are~~ is random pattern to of-shoots on even and odd positions.

③ No-branching criteria:

$\frac{\text{Node value} - 1}{9} \in I$ i.e., $(\text{Node value} - 1) \bmod (9) = 0$ i.e., divisible by 9 → will lead to no branching from ~~to~~ such nodes.

$(64-1) \bmod 9 = 0$ $\quad$ $(699040-1) \bmod 9 = 0$ $\quad$ $(4091-1) \bmod 9 = 0$. $\quad$ $(640-1) \bmod 9 = 0$.

$100-1 \bmod 99 = 0$. $\quad$ $(2728-1) \bmod 9 = 0$ $\quad$ $(226-1) \bmod 9 = 0$. $\quad$ $(136-1) \bmod 9 = 0$.