Unit 1 - Combinatorics - Basic Methods
Week 2 - Combinatorial Proof

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Reading: AC 2.4, rest of chapter 2

1

AC 2.9.21:
Give a combinatorial proof that

$$\sum_{j=0}^{k} \binom{m}{j}\binom{w}{k-j} = \binom{m+w}{k}$$

Proof. Let X = set of subsets of $\{1, \cdots, m+w\}$ with size k.
Let X_j = set of subsets of $\{1, \cdots, m+w\}$ with size k and j from $\{1, \cdots, m\}$.
Let A_j = set of subsets of $\{1, \cdots, m\}$ with size j.
Let B_j = set of subsets of $\{m, \cdots, m+w\}$ with size (k-j).
So, $X_j = \{a \bigcup b | a \in A_j, b \in B_j\}$
Therefore, $|X_j| = |A_j| \cdot |B_j|$
Also, $X \sqcup_j X_j$
Therefore $|X| = |X \sqcup_j X_j| = \sum_{j=0}^{k} |X_j|$
So $\sum_{j=0}^{k} |A_j| \cdot |B_j|$ As a result, $\sum_{j=0}^{k} \binom{m}{j}\binom{w}{k-j} = \binom{m+w}{k}$ $\square$

2

AC 2.9.26:
How many lattice paths go from (0, 0) to (14, 73) which do not go through (6, 37)?

3

AC 2.9.32:
How many ways are there to color a set of 27 objects such that 7 are painted white, 6 are painted gold, 2 are painted blue, 7 are painted yellow, and 5 are painted green.