Latex Homework 10th Grade
Unit 3 - Abstract Algebra - Group Homomorphisms
Week 1 - Examples and Definitions

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1

Gallian Ch 3.30:
Prove that the dihedral group of order 6 does not have a subgroup of order 4.

Proof. Say we H is the subgroup of D_3 of order 4. If $\rho \in H$, there are too many elements in H. If $\rho \notin H$, then $H = \{Id., \tau, \tau\rho, \tau\rho^2\}$. However, $\tau \cdot \tau\rho = \rho$ but $\rho \notin H$. Therefore, H is not a subgroup of D_3 with order 4. □

2

Gallian Ch 1-4 Suplement, problem 41, page 98:
Give an example of a group G with infinitely many distinct subgroups H_1, H_2, H_3, . . . such that $H_1 \subseteq H_2 \subseteq H_3 \subseteq \ldots$

$$(\mathbb{Q}, +) \subseteq (\mathbb{Q}[\sqrt{2}], +) \subseteq (\mathbb{Q}[\sqrt{2}, \sqrt{3}], +) \subseteq \ldots \subseteq (\mathbb{R}, +)$$

3

Gallian Ch 10.7: If ϕ is a homomorphism from G to H and σ is a homomorphism from H to K, show that $\sigma\phi$ is a homo- morphism from G to K. How are Kerϕ and Ker$\sigma\phi$ related? If ϕ and σ are onto and G is finite, describe $[\ker(\sigma\phi) : \ker(\phi)]$ in terms of $|H|$ and $|K|$.
$\ker(\phi) = \{a|\phi(a) = 0\}$
$\ker(\sigma\phi) = \{a|\sigma\phi(a) = 0\}$
$\ker(\sigma\phi) \subseteq \ker(\phi)$
$\ker(\sigma\phi) : \ker(\phi)] = \frac{|\ker(\sigma\phi)|}{|\ker(\phi)|} = \frac{|K|/|G|}{|H|/|G|} = \frac{|K|}{|H|}$