Latex Homework 10th Grade
Unit 3 - Abstract Algebra - Group Homomorphisms
Week 3 - Quotients and Isomorphism Theorems

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1

Prove that if $N \trianglelefteq G$ and $S \leq G$, then $S \cap N \trianglelefteq S$. This is used in part 2.
Assume $N \trianglelefteq G$ and $S \leq G$.
$n \in S \cap N, s \in S$
$sns^{-1} \in S$, cause it's closed.
$sns^{-1} \in N$, cause all elements of S are in G meaning it's normal to N. Therefore,

$$S \cap N \trianglelefteq S$$

2

Prove the second isomorphism theorem: If $N \trianglelefteq G$, and $S \leq G$, then $S/(N \cap S) \cong SN/N$.
Assume $N \trianglelefteq G$ and $S \leq G$.
$s_3, s_2 \in S, s_2(N \cap S)$
$\phi(s_2(N \cap S)) = s_2N$
$s_2(N \cap S) = s_3(N \cap S)$
$s_2 = s_3n, n \in N \cap S$
$s_2N = s_3N$
$\phi(s_2(N \cap S)) = \phi(s_3(N \cap S))$
$n = s_3^{-1}s_2$
So $n \in S$ cuase is product of 2 elements of S, so injective. Therefore, $S/(N \cap S) \cong SN/N$.

3

Prove the third isomorphism theorem: If $A \trianglelefteq B \trianglelefteq C$ and $A \trianglelefteq C$ then $A/B \cong (A/C)/(B/C)$.
Assume $A \trianglelefteq B \trianglelefteq C$ and $A \trianglelefteq C$
Let $\phi : A/C \to A/B$ given by $\phi(aC) = aB$
$(A/C)/ker(\phi) \cong Im(\phi)$
$ker(\phi) = \{aC|\phi(aC) = B\} = \{aC|aB = B\} = \{aC|a \in B\} = \{bC\}$
$\phi(aC) = aB = Im(\phi)$
$A/B \cong (A/C)/(B/C)$