MAE598/494 Design Optimization
Homework 1

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Problem 1.a)

Use initial point: x0 = (2, 2, 2, 2, 2) to solve:

minimize:
$(x1 - x2)^2 + (x2 + x3 - 2)^2 + (x4 - 1)^2 + (x5 - 1)^2$

subject to:
$x1 + 3x2 = 0$
$x3 + x4 - 2x5 = 0$
$x2 - x5 = 0$
$-10 <= xi <= 10, i = 1, ..., 5$

(Refer next page for solution using the Excel Solver and Matlab's *fmincon* solver.)

Microsoft Excel 15.0 Answer Report
Worksheet: [aviprada_hw1.xlsx]Sheet1
Report Created: 18/01/2016 8:06:10 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.062 Seconds.
Iterations: 1 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$7	(x1 - x2)^2 + (x2 + x3 - 2)^2 + (x4 - 1)^2 + (x5 - 1)^2	6.00000	4.09302

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$1	x1 =	2.00000	-0.76744	Contin
$C$2	x2 =	2.00000	0.25581	Contin
$C$3	x3 =	2.00000	0.62791	Contin
$C$4	x4 =	2.00000	-0.11628	Contin
$C$5	x5 =	2.00000	0.25581	Contin

Constraints

Cell	Name	Cell Value	Formula	Status
$C$10	x3 + x4 - 2*x5 = 0	0.00000	$C$10=$E$10	Binding
$C$11	x2 - x5 = 0	0.00000	$C$11=$E$11	Binding
$C$12	x1 >= -10, x1 <= 10	-0.76744	$C$12<=$G$12	Not Binding
$C$12	x1 >= -10, x1 <= 10	-0.76744	$C$12>=$E$12	Not Binding
$C$13	x2 >= -10, x2 <= 10	0.25581	$C$13<=$G$13	Not Binding
$C$13	x2 >= -10, x2 <= 10	0.25581	$C$13>=$E$13	Not Binding
$C$14	x3 >= -10, x3 <= 10	0.62791	$C$14<=$G$14	Not Binding
$C$14	x3 >= -10, x3 <= 10	0.62791	$C$14>=$E$14	Not Binding
$C$15	x4 >= -10, x4 <= 10	-0.11628	$C$15<=$G$15	Not Binding
$C$15	x4 >= -10, x4 <= 10	-0.11628	$C$15>=$E$15	Not Binding
$C$16	x5 >= -10, x5 <= 10	0.25581	$C$16<=$G$16	Not Binding
$C$16	x5 >= -10, x5 <= 10	0.25581	$C$16>=$E$16	Not Binding
$C$9	x1 + 3*x2 = 0	0.00000	$C$9=$E$9	Binding

```
%Name: Aditya Vipradas
%ASURITE User ID: aviprada
%ASU ID: 1209435588
%Homework 1 Problem 1.a
%clear screen
clc;

%Define the objective function
fun1 = @(x)(x(1)-x(2))^2 + (x(2)+x(3)-2)^2 + (x(4)-1)^2 + (x(5)-1)^2;

%Define the initial guess
x0 = [2 2 2 2 2];

%Define the equality constraints
Aeq = [1 3 0 0 0;0 0 1 1 -2;0 1 0 0 -1];
beq = [0; 0; 0];

%Define the inequality constraints
A = [];
b = [];

%Define the upper and lower bounds
lb = [-10 -10 -10 -10 -10];
ub = [10 10 10 10 10];

x = fmincon(fun1, x0, A, b, Aeq, beq, lb, ub);
fx = (x(1)-x(2))^2 + (x(2)+x(3)-2)^2 + (x(4)-1)^2 + (x(5)-1)^2;

str1 = sprintf('The function minimizes at \n x1 = %0.5f \n x2 = %0.5f \n x3 = %0.5f \n x4 =
%0.5f \n x5 = %0.5f', x(1),x(2),x(3), x(4), x(5));
disp(str1);
str2 = sprintf('\n The minimum function value is %0.5f',fx);
disp(str2);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.



The function minimizes at
 x1 = -0.76744
 x2 = 0.25581
 x3 = 0.62791
 x4 = -0.11628
 x5 = 0.25581

 The minimum function value is 4.09302
```

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Problem 1.b)

Use initial point: x0 = (1, 1, 1, 1) to solve:

minimize:
$24.55x1 + 26.75x2 + 39.00x3 + 40.50x4$

subject to:
$2.3x1 + 5.6x2 + 11.1x3 + 1.3x4 - 5 >= 0$
$12x1 + 11.9x2 + 41.8x3 + 52.1x4 - 21 - 1.645(0.28x1^2 + 0.19x2^2 + 20.5x3^2 + 0.62x4^2)^{1/2} >= 0$
$x1 + x2 + x3 + x4 - 1 = 0$
$0 <= xi, i = 1, ..., 4$

(Refer next page for solution using the Excel Solver and Matlab's *fmincon* solver.)

Microsoft Excel 15.0 Answer Report
Worksheet: [aviprada_hw1.xlsx]Sheet1
Report Created: 18/01/2016 8:02:46 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.032 Seconds.
Iterations: 0 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegat

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$24	24.55*x1 + 26.75*x2 + 39.00*x3 + 40.50*x4	130.80000	29.89438

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$19	x1 =	1.00000	0.63552	Contin
$C$20	x2 =	1.00000	0.00000	Contin
$C$21	x3 =	1.00000	0.31270	Contin
$C$22	x4 =	1.00000	0.05178	Contin

Constraints

Cell	Name	Cell Value	Formula	Status
$C$26	2.3*x1 + 5.6*x2 + 11.1*x3 + 1.3*x4 - 5 >= 0	0.00000	$C$26>=$E$26	Binding
$C$27	12*x1 + 11.9*x2 + 41.8*x3 + 52.1*x4 - 21 - 1.645*(0.28*x1^2 + 0.19*x2^2 + 20.5*x3^2 + 0.62*x4^2)^(1/2) >= 0	0.00000	$C$27>=$E$27	Binding
$C$28	x1 + x2 + x3 + x4 - 1 = 0	0.00000	$C$28=$E$28	Binding
$C$29	x1 >= 0	0.63552	$C$29>=$E$29	Not Binding
$C$30	x2 >= 0	0.00000	$C$30>=$E$30	Binding
$C$31	x3 >= 0	0.31270	$C$31>=$E$31	Not Binding
$C$32	x4 >= 0	0.05178	$C$32>=$E$32	Not Binding

```
%Name: Aditya Vipradas
%ASURITE User ID: aviprada
%ASU ID: 1209435588
%Homework 1 Problem 1.b

%Define nonlinear constraints in nconstraints.m file
function [c, ceq] = nconstraints(x)
    c = -1 * (12*x(1) + 11.9*x(2) + 41.8*x(3) + 52.1*x(4) - 21 - 1.645 ...
        *(0.28*x(1)^2 + 0.19*x(2)^2 + 20.5*x(3)^2 + 0.62*x(4)^2)^(1/2));
    ceq = [];
end
%clear screen
clc;

%Define the objective function
fun2 = @(x)24.55*x(1) + 26.75*x(2) + 39.00*x(3) + 40.50*x(4);
%Define the initial guess
x0 = [1 1 1 1];
%Define the equality constraints
Aeq = [1 1 1 1];
beq = [1];
%Define the inequality constraints
A = [-2.3 -5.6 -11.1 -1.3];
b = [-5];
%Define lower and upper bounds
lb = [0 0 0 0];
ub = [];

nonlcon = @nconstraints;
x = fmincon(fun2, x0, A, b, Aeq, beq, lb, ub, nonlcon);
fx = 24.55*x(1) + 26.75*x(2) + 39.00*x(3) + 40.50*x(4);

str1 = sprintf('The function minimizes at \n x1 = %0.5f',x(1));
str2 = sprintf('x2 = %0.5f \n x3 = %0.5f \n x4 = %0.5f',x(2),x(3),x(4));
disp(str1);
disp(str2);
str3 = sprintf('\n The minimum function value is %0.5f',fx);
disp(str3);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.



The function minimizes at
 x1 = 0.63552
 x2 = 0.00000
 x3 = 0.31270
 x4 = 0.05178

 The minimum function value is 29.89438
```

Published with MATLAB® R2014b

Problem 2

Optimization problem to design a cylindrical cola can of volume V:

1. *Design variables:* radius of top face of the can (r) and height of the can (h)
2. *Objective function:* minimize the material usage in the cola can by minimizing its surface area i.e. $S = 2\pi rh + 2\pi r^2$
3. *Constraints:* volume of the can $= V$ i.e. $\pi r^2 h = V$, $r, h > 0$
4. *Assumptions:*
   (a) The cola can is assumed to have negligible thickness
   (b) The cola can is assumed to be perfectly cylindrical i.e. without any opening, surface defects and dents

Assume the volume of a standard Coca Cola can to be $V = 355mL$. Solving the optimization problem by substituting $V/(\pi r^2)$ for h from the constraint equation in the objective function. Consequently, differentiating the objective function in r with respect to r and equating to zero, the relation $2r = h$ is obtained. Thus, in order to minimize the material usage in a cola can, for a given volume V, the radius r of the top face should be half of that of its height h. The dimensions of a standard $355mL$ Coca Cola are approximately as follows:

As observed, for a given diameter of $6.5cm$, the height of the can is $11cm$. But from the relation obtained above i.e. $2r = h$, for a volume of $355mL$, the height and diameter of the can should be 7.67 cm. Thus, the obtained optimal solution is not close to the reality. The practical reasons for this are as follows:

1. The dimensions of the cola can should be ergonomic. The consumer should be able to grasp the can in his hand. It would be cumbersome for the user to hold a can 7.67 cm long using all his fingers. Hence, the height is generally bigger than the can diameter, here 11 cm for the user to easily hold it in his hand.

2. If not negligible, the cola can has some finite thickness which will increase the surface area and in turn the material usage.

3. Moreover, the base of the can is not perfectly flat. It has a dent for it to get perfectly located on the can tray. The top face is also not flat. It has extra material that goes in the opener. Thus, more material than the one predicted by the optimal solution is used to manufacture the cola can.

4. Furthermore, the can has to sustain the pressure produced in it. Thus, additional material than the one obtained from the optimal solution is used in the appropriate locations to preserve its strength.