ME598/494 Homework 2

1. (5 points) Find if the following problem is well constrained:

$$\begin{aligned}
&\text{maximize: } f = x_1 - x_2 \\
&\text{subject to: } g_1 = 2x_1 + 3x_2 - 10 \leq 0, \\
&g_2 = -5x_1 - 2x_2 + 2 \leq 0, \\
&g_3 = -2x_1 + 7x_2 - 8 \leq 0.
\end{aligned}$$

2. (10 points) Using both monotonicity principles, solve the following for positive finite $x_i, i = 1, 2, 3$:

$$\begin{aligned}
&\text{maximize: } x_1 \\
&\text{subject to: } \exp(x_1) \leq x_2, \exp(x_2) \leq x_3, x_3 \leq 10.
\end{aligned}$$

3. (Optional for MAE494, 30 points) Show that the stationary point of the function

$$f = 2x_1^2 - 4x_1x_2 + 1.5x_2^2 + x_2$$

is a saddle. Find the directions of downslopes away from the saddle. Hint: Use Taylor's expansion at the saddle point. Find directions that reduce f.

4. (a) (10 points) Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in $\mathbb{R}^3$ that is nearest to the point $(-1, 0, 1)^T$. Hint: Convert the problem into an unconstrained problem using $x_1 + 2x_2 + 3x_3 = 1$.

   (b) (40 points) Implement the gradient descent and Newton's algorithm to solve the problem. Attach your codes in the report, along with a short summary of your findings. The summary should include: (1) The initial points tested; (2) corresponding solutions; (3) A log-linear convergence plot. Based on your results, which algorithm do you think is better? Why?

5. (Optional for MAE494, 5 points) Prove that a hyperplane is a convex set. Hint: A hyperplane in $\mathbb{R}^n$ can be expressed as: $\mathbf{a}^T\mathbf{x} = c$ for $\mathbf{x} \in \mathbb{R}^n$, where $\mathbf{a}$ is the normal direction of the hyperplane and c is some constant.