MAE 404/598 Finite Elements in Engineering
Written Homework assignment #1

Due 2/4/2016 (in class)

Do problems 3.13 and 3.14 (a,b only) from the textbook. You may use symbolic math tool, e.g. Mathematica if you like, but you must clearly explain your work (i.e. do not assume that the grader will know Mathematica commands/syntax).

<u>Problem 3.13 is optional for undergraduates taking MAE 404.</u>

Problem 3.13

Find the weak form for the following strong form:

$$x\frac{d^2u}{dx^2}+\frac{du}{dx}-x=0, \qquad 0\le x\le 1,$$

subject to $u(0)=u(1)=0$.

Problem 3.14

Consider a bar in Figure 3.12 subjected to linear body force $b(x)=cx$. The bar has a constant cross-sectional area A and Young's modulus E. Assume quadratic trial solution and weight function

$$u(x)=\alpha_1+\alpha_2x+\alpha_3x^2, \qquad w(x)=\beta_1+\beta_2x+\beta_3x^2,$$

where α_i are undetermined parameters.

a. For what value of α_I is $u(x)$ kinematically admissible?

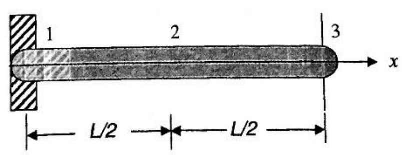

Figure 3.12 Elastic bar subjected to linear body force of Problem 3.14.

b. Using the weak form, set up the equations for α_I and solve them. To obtain the equations, express the principle of virtual work in the form $\beta_2(\cdots)+\beta_3(\cdots)=0$. By the scalar product theorem, each of the parenthesized terms, i.e. the coefficients of β_I, must vanish.

c. Solve the problem in Figure 3.12 using two 2-node elements considered in Chapter 2 of equal size. Approximate the external load at node 2 by integrating the body force from $x=L/4$ to $x=3L/4$. Likewise, compute the external at node 3 by integrating the body force from $x=3L/4$ to $x=L$.