ENGINEERING PHYSICS –I (UNIT-1)

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COURSE OBJECTIVE:

- To enhance the fundamental knowledge in Physics and its applications relevant to various streams of Engineering and Technology
- To give our students the ability to apply scientific and engineering principles to identify, formulate, and solve problems in analysis and design. In this pursuit, our students will need to understand the basis of modern engineering tools and the role of experimentation, and how to apply these methods appropriately.

COURSE LEARNING OUTCOMES

- An ability to analyze the engineering concepts based on fundamental physical concepts.
- Applications of the fundamental physical laws for better understanding of materials and their properties, for engineering applications.
- An ability to apply knowledge of science and engineering to design and conduct experiments, as well as to analyze and interpret data.
- An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability.

UNIT 1 - RELATIVISTIC MECHANICS

- Frame of reference, Inertial & non-inertial frames,
- Galilean transformations,
- Michelson-Morley experiment, Postulates of special theory of relativity,
- Lorentz transformations,
- Length contraction, Time dilation,
- Velocity addition theorem,
- Variation of mass with velocity,
- Einstein‟s mass energy relation, Relativistic relation between energy and momentum, Massless particle.

WHAT IS RELATIVITY?

- Position is relative
- Length is relative
- Motion is relative
- Time is relative
- Length, Position, Velocity , Time are not absolute

The Principle of Relativity

- Principle of Relativity
  - Every observer must experience the same natural laws
    - Special relativity: deals with reference frames in uniform motion relative to one another.
    - General relativity: applies to any reference frame whether or not it is accelerating relative to another
- *The speed of light, c, is the same in all reference frames*

Frames of Reference

Frames of Reference: Physical surroundings from which you observe and measure the world around you.

The forward motion of an adjacent bus can give you an impression that your own bus is moving backward.

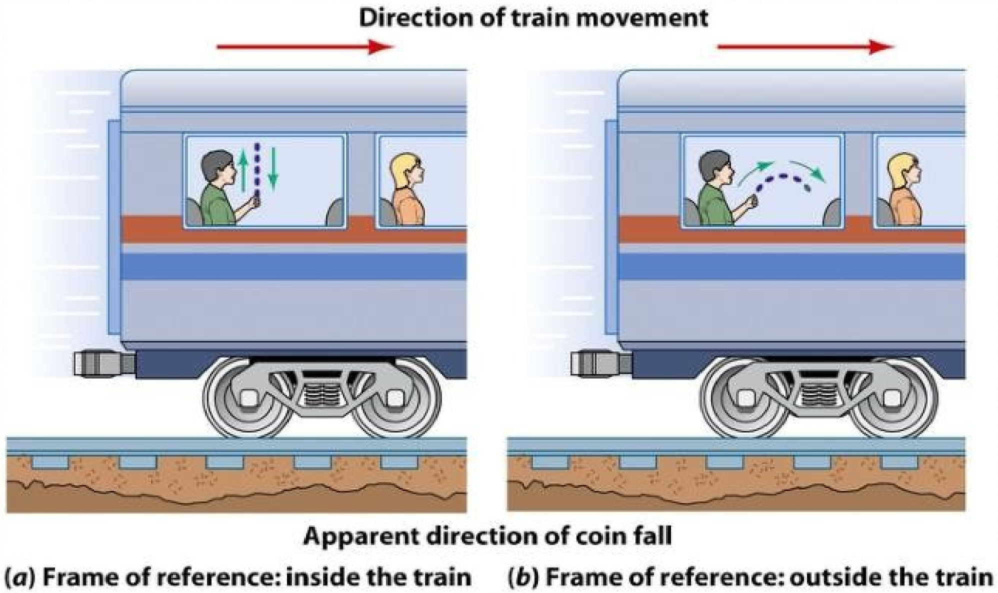
FRAME OF REFERENCE
INERTIAL & NON-INERTIAL

Position/Motion of the body is described with respect to some well defined coordinate system. This coordinate system is known as Frame of Reference. ***Thus any system relative to which the motion of an object can be described is called a Frame of Reference.***

- ***There are two types: Inertial and Non-Inertial***

- The frame of reference in which Newton's law of inertia and other laws of Newtonian mechanics hold good are called **inertial frame of reference.** Inertial frames are Unaccelerated frames.
- The frame of reference with respect to which an un accelerated body appears accelerated are called **non inertial frame of reference** or in other words the accelerated frames are called non inertial frames. In non inertial frames Newton's law does not hold.

Frames of Reference



- Coin's path appears different depending on your frame of reference

GALILEAN TRANSFORMATIONS

- It is used to transform the coordinates from one inertial frame of reference to another one. It is applicable when $v << c$.

  Assumptions:-

- Both the frames should be inertial one.
- Absolute time exists i.e. $t' = t$
- Initially both the frames coincides with each other.
- Consider two inertial frame of reference S and S', frame S' is moving with uniform velocity v with respect to frame S along +ve direction of X-axis.

From Fig. 1:

GALILEAN TRANSFORMATIONS CONTD.

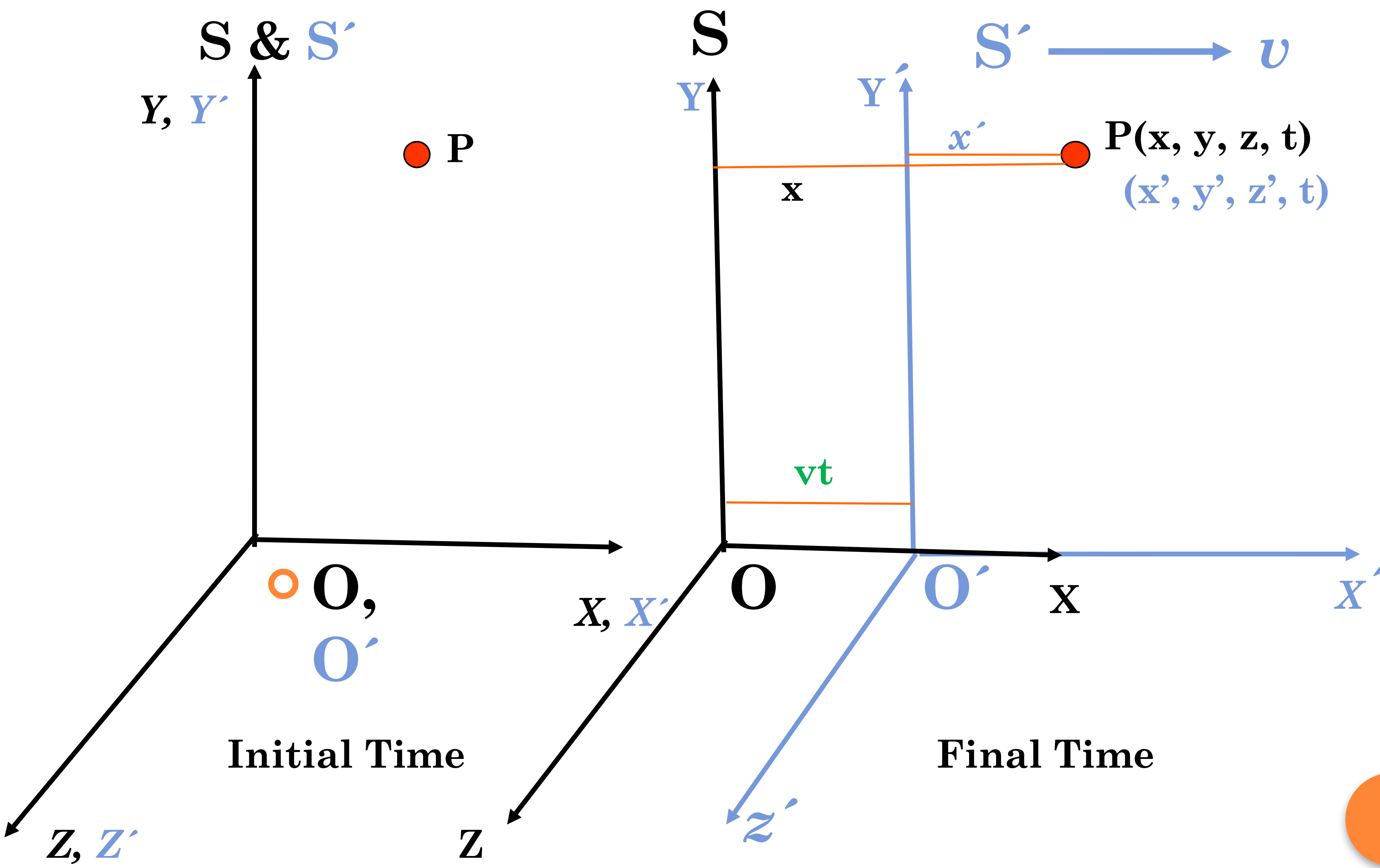


Fig. 1

GALILEAN TRANSFORMATIONS CONTD.

- From Fig. 1:

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \quad (1)$$

Similarly for Inverse transformation

$$\left.\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}\right\} \quad (2)$$

In vector notation

$$\vec{r'} = \vec{r} - \vec{v}t \quad (3)$$

Eqn. (1), (2) and (3) represents transformation eqn. of position under Galilean transformation.

GALILEAN TRANSFORMATIONS CONTD.

<u>Transformation equation of velocity</u>

Diffg. Eqn. (1) with respect to time

$$\left.\begin{aligned} \frac{dx'}{dt'} &= \frac{dx}{dt} \\ \frac{dy'}{dt'} &= \frac{dy}{dt} \\ \frac{dz'}{dt'} &= \frac{dz}{dt} \end{aligned}\right] \Rightarrow \left.\begin{aligned} u_x' &= u_x - v \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}\right] \quad \text{as } dt' = dt \qquad \text{———} \quad \textbf{(4)}$$

In vector notation

$$\overrightarrow{u'} = \vec{u} - \vec{v} \qquad \text{———} \quad \textbf{(5)}$$

- Eqn. (4) and (5) represents transformation eqn. of velocity under Galilean transformation.
- It is clear from above eqns. That transformation eqn. of position and velocity are variant under the Galilean transformation.

GALILEAN TRANSFORMATIONS CONTD.

Transformation equation of acceleration

Diffg. Eqn. (4) with respect to time

$$\frac{du_x'}{dt'} = \frac{du_x}{dt} - v$$

$$\frac{du_y'}{dt'} = \frac{du_y}{dt}$$

$$\frac{du_z'}{dt'} = \frac{du_z}{dt}$$

$$\Rightarrow \left[\begin{array}{l} a_x' = a_x \\ a_y' = a_y \\ a_z' = a_z \end{array}\right] \quad \text{(6)}$$

In vector notation

$$\vec{a}' = \vec{a} \quad \text{(7)}$$

Eqn. (6) and (7) represents transformation eqn. of acceleration under Galilean transformation. This shows that acceleration is invariant under the Galilean transformation.

CONCEPT OF ETHER

"In the later part of the 19th century, the theory of light assumed the existence of a medium called the ether, whose vibrations produce the phenomenon of heat and light, and which is supposed to fill all space. According to Fresnel, the ether, which is enclosed in optical media, partakes of the motion of these media, to an extent depending on their indices of refraction. Assuming then that the ether is at rest, the Earth moving through it, the time required for light to pass from one point to another on the Earth's surface, will depend on the direction in which it travels."

MICHELSON MORLEY EXPERIMENT

- **Objective** – To confirm the existence of ether as an absolute frame of reference.
- **Principle** – Based on theory of relativity

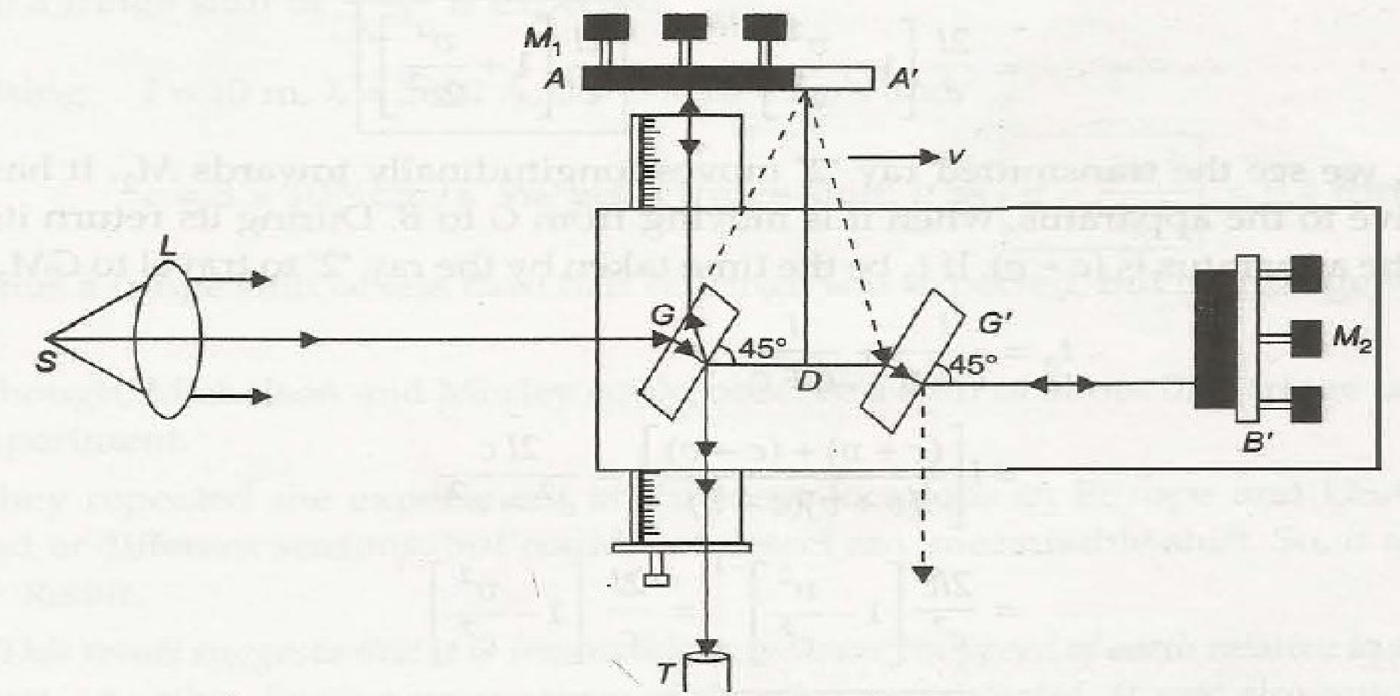


FIGURE 8.2 Michelson-Morley Experiment

MICHELSON MORLEY EXPERIMENT CONTD.

Time taken by reflected ray

$$t_1 = \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Time taken by transmitted ray

$$t_2 = \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Time difference

$$\Delta t = \frac{lv^2}{c^3}$$

Corresponding path difference

$$\Delta = \frac{lv^2}{c^2}$$

MICHELSON MORLEY EXPERIMENT CONTD.

Path difference in terms of order of fringes

$$n_1 = \frac{lv^2}{c^2\lambda}$$

Let the whole apparatus is rotated through 90º,

Path difference in terms of order of fringes

Total fringe shift

$$\Delta n = 2\frac{lv^2}{c^2\lambda}$$

MICHELSON MORLEY EXPERIMENT CONTD.

Put the values of l = 11m, v = 3 x 10^4 m/sec, c = 3x 10^8 m/sec and λ = 5893Å

Δ n = 0.37

Thus negligible fringe shift is observed. Ether was discarded as fixed frame of reference.

Explanation and Interpretation of Negative Result

- Speed of light is constant in all inertial frame of reference.
- There is no relative motion between the ether and the earth as ether was carried away by material bodies like earth.
- According to Lorentz Fitzerald contraction hypothesis, length of a material body is contracted by a factor al $\sqrt{1-\frac{v^2}{c^2}}$ rection of motion.

EINSTEIN'S POSTULATES OF SPECIAL THEORY OF RELATIVITY

Fundamental Postulates of Special theory of relativity :-

- Principle of Equivalence:-

  The fundamental laws of physics are same in all inertial frame of references.
- Principle of constancy of speed of light:-

  Speed of light is constant in all inertial frame of references.

LORENTZ TRANSFORMATIONS

- Lorentz transformation equation of position and time – used to transform the coordinates from one inertial frame of reference to another one.
- It is based on Einstein's postulates of special theory of relativity.

Assumptions:-

- Both the frames should be inertial one.
- Speed of light is constant in all inertial frame of references.
- Absolute time does not exist i.e.
- Consider two inertial frames of reference S and S', frame S' is moving with uniform velocity with respect to frame S.

LORENTZ TRANSFORMATIONS CONTD.

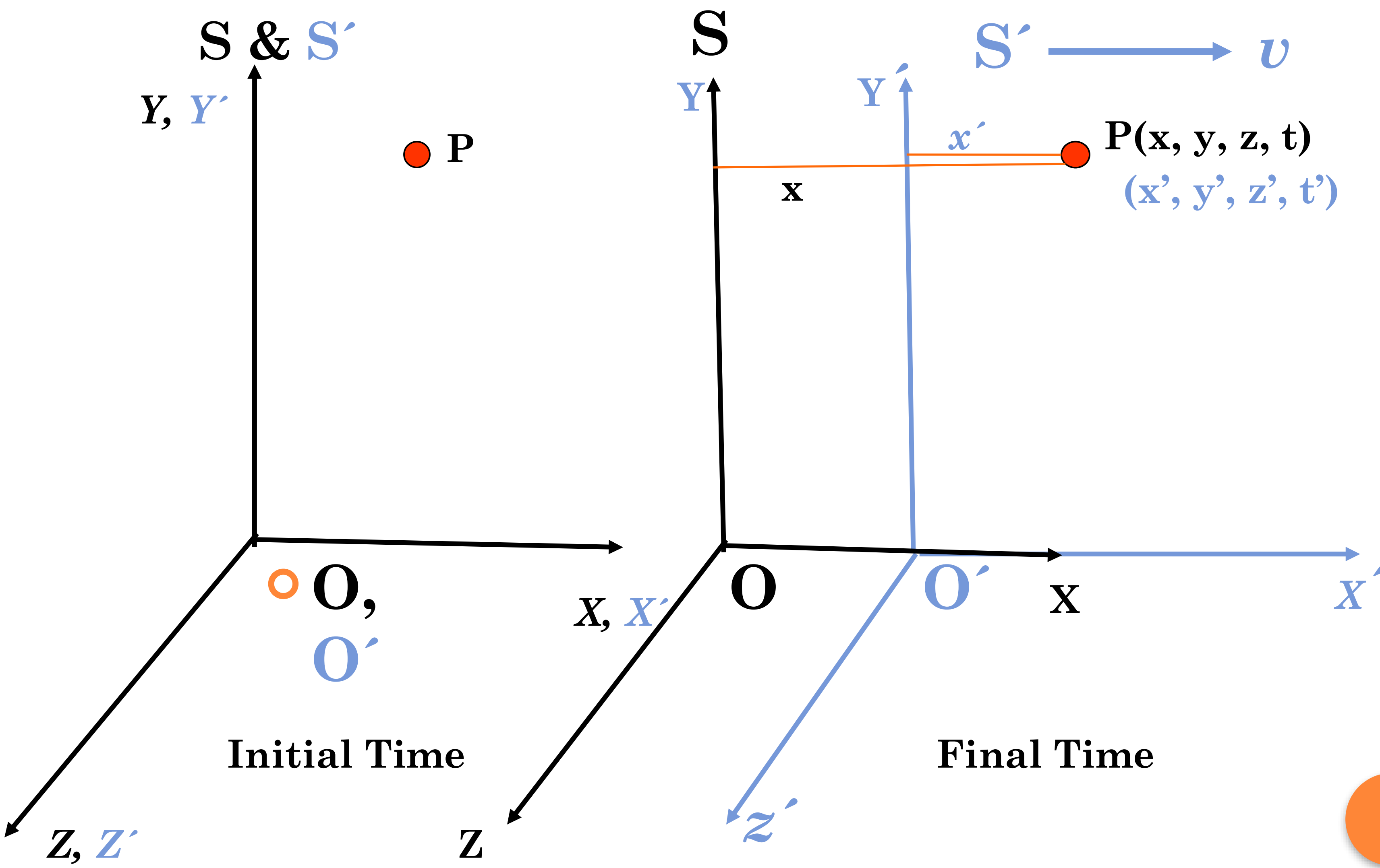


Fig. 1

LORENTZ TRANSFORMATIONS CONTD.

In Frame S

$$\text{Speed of light} = \frac{\text{distance travelled } OP}{\text{time taken}}$$

$$c = \frac{\sqrt{(x^2 + y^2 + z^2)}}{t}$$

$$x^2 + y^2 + z^2 = c^2t^2 \quad \ldots\ldots(1)$$

In Frame S'

$$\text{Speed of light} = \frac{\text{distance travelled } O'P}{\text{time taken}}$$

$$c = \frac{\sqrt{(x'^2 + y'^2 + z'^2)}}{t'}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \quad \ldots\ldots(2)$$

As frame S' is moving along the X-axis, therefore

y = y' and z = z' ……(3)

From eqn. (1), (2) and (3) we have

$$x^2 - c^2t^2 = x'^2 - c^2t'^2 \quad \ldots\ldots\ldots(4)$$

LORENTZ TRANSFORMATIONS CONTD.

Assuming that Lorentz transformation eqn. is simple, linear

$$x' = \lambda\,(x - vt) \qquad \dots\dots\dots\dots(5)$$

For Inverse transformation

$$x = \lambda'(x' + vt') \qquad \dots\dots\dots\dots(6)$$

Put the value of x' from eqn. (5) in eqn. (6)

$$x = \lambda'[\lambda\,(x - vt) + vt']$$

$$\frac{x}{\lambda'} = [\lambda\,(x - vt) + vt']$$

$$vt' = \frac{x}{\lambda'} - \lambda\,(x - vt)$$

$$t' = \lambda\left[t - \frac{x}{v}\left(1 - \frac{1}{\lambda\lambda'}\right)\right] \dots\dots\dots\dots(7)$$

LORENTZ TRANSFORMATIONS CONTD.

Put the value of x' and t' in eqn. (4), we get

$$x^2 - c^2t^2 = \{\lambda\,(x - vt)\}^2 - c^2\left[\lambda\left\{t - \frac{x}{v}\left(1 - \frac{1}{\lambda\lambda'}\right)\right\}\right]^2$$

$$x^2 - c^2t^2 - \{\lambda\,(x - vt)\}^2 + c^2\left[\lambda\left\{t - \frac{x}{v}\left(1 - \frac{1}{\lambda\lambda'}\right)\right\}\right]^2 = 0 \qquad \dots\dots(8)$$

On solving the coefficient of t^2, xt and x^2 we get

$$\lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \lambda' \qquad \dots\dots(9)$$

LORENTZ TRANSFORMATIONS CONTD.

From eqn. (3), (5) and (7), we have

$$\left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}\right\} \quad \textit{(10)}$$

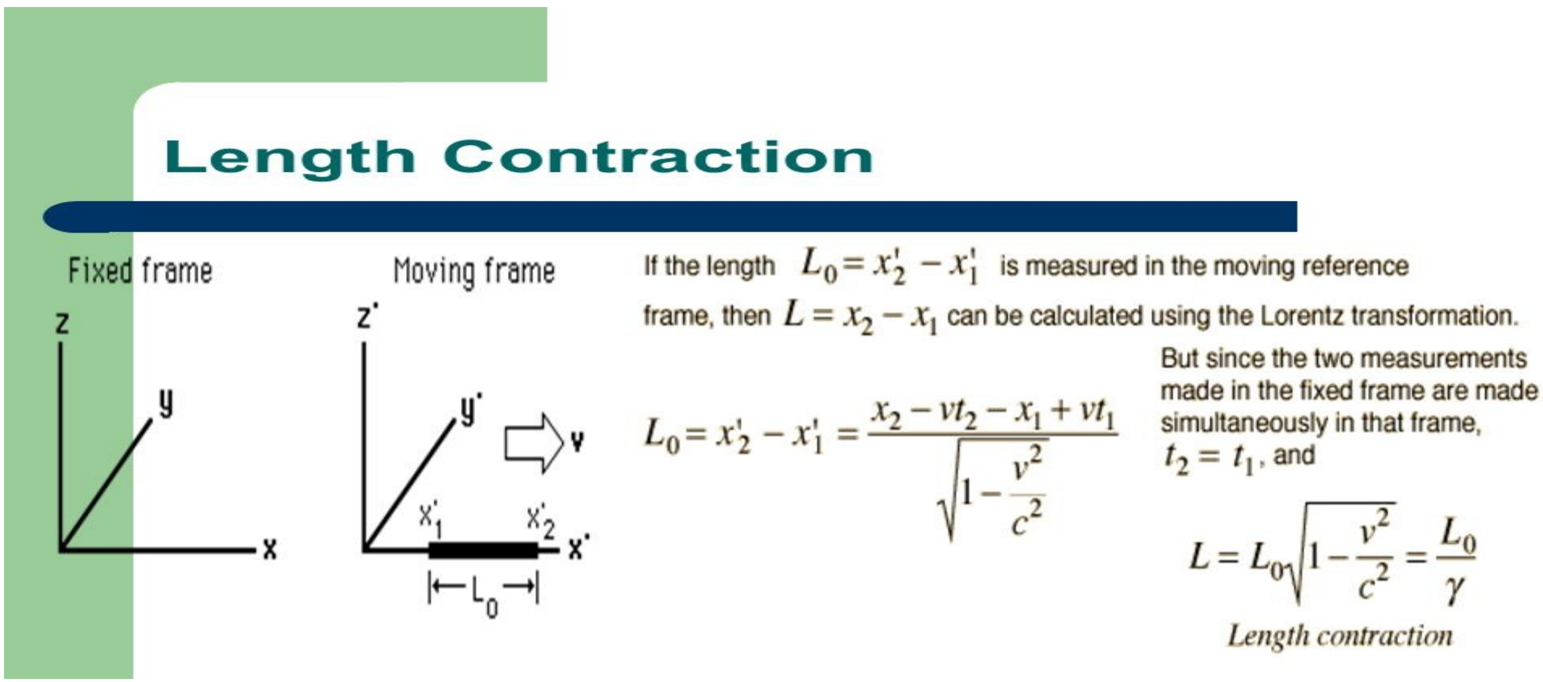
Eqn. (10) represents Lorentz transformation equation of position and time. Galilean transformation eqn. For

v << c, it reduces to Galilean transformation eqn.

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

LENGTH CONTRACTION

- **Proper length** is the length of rod measured by an observer in the frame in which the rod is at rest.



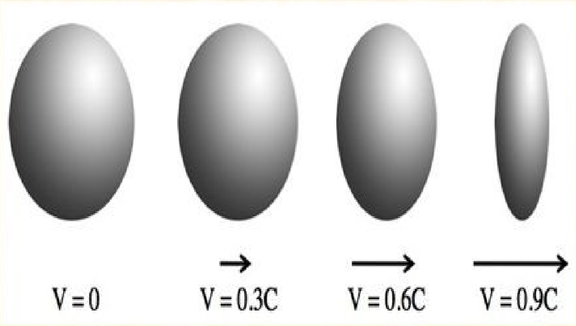
Length Contraction

- According to the theory of Special Relativity, objects appear to contract in the direction they are traveling when they are moving as fast as the speed of light.

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

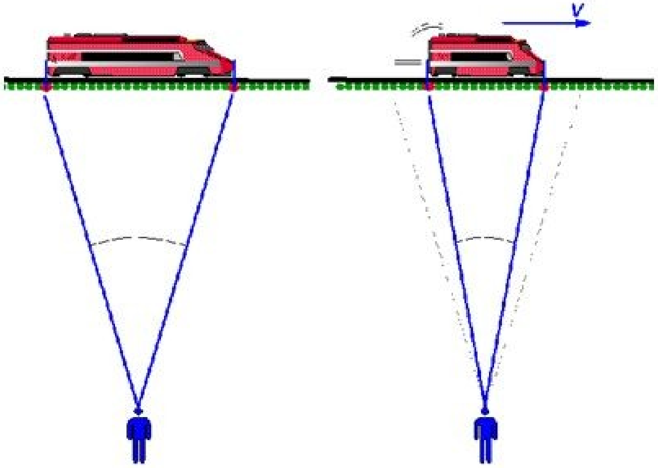
L = length of object moving at speed v

L_o = length of object at rest



Length Contraction

Similarly, it can be shown that lengths parallel to the direction of motion are contracted:



$$L' = L\sqrt{1-\frac{v^2}{c^2}} \quad or \quad L = \frac{L'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L' = \frac{L}{\gamma} \quad or \quad L = \gamma L'$$

TIME DILATION

- TIME DILATION: It is the effect in which the clock moving with a uniform velocity v relative to an observer appears to him to go slow by $\sqrt{1-\frac{v^2}{c^2}}$, than when at rest relative to him. Consider two inertial frames of reference S and S', frame S' is moving with uniform velocity with respect to frame S. Let a clock is placed in the frame S.

TIME DILATION CONTD.:

Proper time interval $\Delta t_o = t_2 - t_1$

Apparent time interval $\Delta t' = t_2' - t_1'$

From Lorantz transformation equation of time

$$t_1' = \frac{t_1 - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and } t_2' = \frac{t_2 - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore \Delta t' = \frac{t_2 - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1 - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Since v << c, $\Delta t' > \Delta t_o$

TIME DILATION IS A REAL EFFECT

- The experimental evidence for time dilation is observed when μ - mesons are created at high altitudes in the earth atmosphere at the height of about 10 km by the interaction of fast cosmic ray photons and are projected towards the earth surface with a very speed of about 0.998 c. μ - mesons are unstable and decay into electrons or positrons with an average life time of about 2 μs.
- Thus the distance travelled by μ - mesons without relativistic effect

TIME DILATION IS A REAL EFFECT CONTD.

Thus the distance travelled by μ - mesons without relativistic effect

$d = v \times t = 0.998c \times 2 \times 10^{-6} \cong 600$ m

Delayed life time of μ - mesons

$$\Delta t' = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2 \times 10^{-6}}{\sqrt{1 - \frac{(0.998\ c)^2}{c^2}}} = 3.17 \times 10^{-5}\ s$$

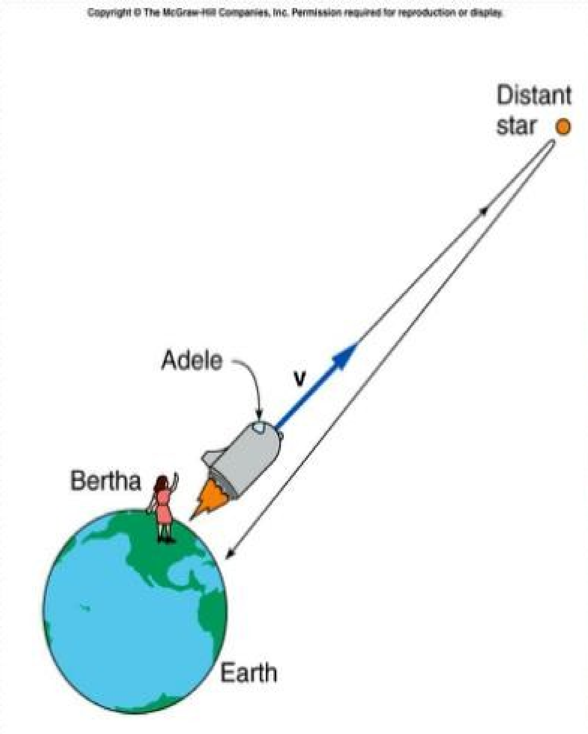
Thus the distance travelled by μ - mesons with relativistic effect

$$d' = v \times \Delta t' = 0.998 \times 10^8 \times 3.17 \times 10^{-5} = 9600\ m$$

$$d' \cong 10\ km$$

This explains the presence of μ - mesons on the earth surface, i.e. time dilation is a real effect.

Time Dilation: Twin Paradox



At a speed of 0.995c for v, the difference in aging of the twin sisters could be quite striking: 10 vs 100 years.

RELATIVISTIC VELOCITY ADDITION THEOREM

Let velocity of body A in frame S as observed by an observer O is $\vec{u} = \hat{\imath}u_x + \hat{\jmath}u_y + \hat{k}u_z$ … … … … [1]

Where $u_x = \dfrac{dx}{dt}$; $u_y = \dfrac{dy}{dt}$ and $u_z = \dfrac{dz}{dt}$

Let velocity of body A in frame S' as observed by an observer O' is $\overrightarrow{u'} = \hat{\imath}u'_x + \hat{\jmath}u'_y + \hat{k}u'_z$ … … … … [2]

Where $u'_x = \dfrac{dx'}{dt}$; $u'_y = \dfrac{dy'}{dt}$ and $u'_z = \dfrac{dz'}{dt}$

Let u_x , u_y , u_z and u'_x , u'_y , u'_z be the velocity components in X, Y and Z directions in S and S' frames.

RELATIVISTIC VELOCITY ADDITION THEOREM

Then by Lorentz Transformation equations,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots \quad (A); \qquad y' = y \quad \ldots \quad (B);$$

$$z' = z \quad \ldots \quad (C); \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots \ (D) \ldots [3]$$

RELATIVISTIC VELOCITY ADDITION THEOREM

Differentiating eqn. [3(A), (B), (C) & (D)]

$$dx' = \frac{dx - vdt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \dots \dots \dots \quad (4)$$

$$dy' = dy \qquad \dots \dots \dots \quad (5)$$

$$dz' = dz \qquad \dots \dots \dots \quad (6)$$

$$dt' = \frac{dt - \frac{vdx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \dots \dots \dots \quad (7)$$

RELATIVISTIC VELOCITY ADDITION THEOREM

Dividing (4) by (7):

$$\frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{vdx}{c^2}}$$

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v}{1 - \frac{vdx}{c^2 dt}}$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} u_x} \qquad \dots\dots\dots\dots \quad (8)$$

RELATIVISTIC VELOCITY ADDITION THEOREM

Dividing (5) by (7)

$$\frac{dy'}{dt'} = \frac{\sqrt{1-\frac{v^2}{c^2}}\,dy}{dt-\frac{vdx}{c^2}}$$

$$u_y' = \frac{\sqrt{1-\frac{v^2}{c^2}}\,u_y}{dt-\frac{vu_x}{c^2}} \qquad \dots\dots\dots\dots \quad (9)$$

Similarly we get, by dividing (6) by (7)

:

$$u_z' = \frac{\sqrt{1-\frac{v^2}{c^2}}\,u_z}{dt-\frac{vu_x}{c^2}} \qquad \dots\dots\dots\dots \quad (10)$$

RELATIVISTIC VELOCITY ADDITION THEOREM

Consistency with Einstein's second postulate:

The inverse velocity addition theorem can be written as:

$$u_x = \frac{u_x' + v}{1 + \frac{v}{c^2} u_x'} \quad \dots\dots\dots\dots (11)$$

CASE 1: If a particle velocity $u_x' = c$ (for a photon)

Putting in eqn. (11)

$$u_x = \frac{c + v}{1 + \frac{vc}{c^2}}$$

Solving we get

$$u_x = c$$

So, the observer in frame S and S' observe the same value of the velocity of photon. This is the second postulate of Einstein's theory of relativity.

RELATIVISTIC VELOCITY ADDITION THEOREM

CASE 2: If a particle and frame both moving with c:

If $u_x' = c$ (for a photon) and $v = c$ (for a photon)

Putting in eqn. (11)

$$u_x = \frac{c + c}{1 + \frac{c.c}{c^2}} = \frac{2c}{2}$$

Solving, we get $u_x = c$

Therefore, the relativistic velocity addition theorem is consistent with Einstein's second postulate of special theory of relativity according to which speed of light is constant in all inertial frame of reference.

VARIATION OF MASS WITH VELOCITY

Assumptions:-

- There exists an entity momentum of a body, the direction of momentum of a body is same as that of velocity of a body.
- Law of conservation of mass and momentum holds good in relativity.
- Collision is elastic.

  Consider two inertial frames of reference S and S', frame S' is moving with uniform velocity with respect to frame S.

  Let two identical bodies A and B having mass m_1 and m_2 are moving with velocities u' and –u' respectively along x-axis are kept in frame S'
-

VARIATION OF MASS WITH VELOCITY CONTD.

In frame S,

Velocity of body A is $$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \ldots\ldots\ldots\ldots\ldots (i)$$

Velocity of body B is $$u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad \ldots\ldots\ldots\ldots\ldots (ii)$$

According to law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \qquad \ldots\ldots (iii)$$

$$m_1 \left(\frac{u' + v}{1 + \frac{u'v}{c^2}} \right) + m_2 \left(\frac{-u' + v}{1 - \frac{u'v}{c^2}} \right) = (m_1 + m_2) v$$

VARIATION OF MASS WITH VELOCITY CONTD.

On further Solving

$$m_1\left(\frac{u^{'}+v}{1+\frac{u'v}{c^2}}-\mathrm{v}\right)=m_2\left(\mathrm{v}-\frac{-u^{'}+v}{1-\frac{u'v}{c^2}}\right)$$

$$m_1\left(\frac{u^{'}+v-v-\frac{u'\mathrm{v}^2}{c^2}}{1+\frac{u'v}{c^2}}\right)=m_2\left(\frac{v-\frac{u'\mathrm{v}^2}{c^2}-u^{'}+v}{1-\frac{u'v}{c^2}}\right)$$

$$m_1\left(\frac{u^{'}+v-v-\frac{u'\mathrm{v}^2}{c^2}}{1+\frac{u'v}{c^2}}\right)=m_2\left(\frac{v-\frac{u'\mathrm{v}^2}{c^2}-u^{'}+v}{1-\frac{u'v}{c^2}}\right)$$

$$\frac{m_1}{m_2}=\frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \quad \dots\dots\dots\dots\dots (\mathrm{iv})$$

VARIATION OF MASS WITH VELOCITY CONTD.

Now squaring eqn. (i), dividing it by c^2 and subtracting it from unity, we get

$$1 - \frac{{u_1}^2}{c^2} = 1 - \frac{1}{c^2}\left(\frac{u' + v}{1 + \frac{u'v}{c^2}}\right)$$

On solving we get,

$$1 + \frac{u'v}{c^2} = \sqrt{\frac{\left(1 - \frac{u'^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{{u_1}^2}{c^2}}} \quad \ldots\ldots\ldots (v)$$

And from eqn. (ii),

we have

$$1 - \frac{u'v}{c^2} = \sqrt{\frac{\left(1 - \frac{u'^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{{u_2}^2}{c^2}}} \quad \ldots\ldots\ldots (vi)$$

VARIATION OF MASS WITH VELOCITY CONTD.

Put the above values in eqn. (iv)

$$\frac{m_1}{m_2} = \frac{\sqrt{\dfrac{\left(1 - \dfrac{u'^2}{c^2}\right)\left(1 - \dfrac{v^2}{c^2}\right)}{1 - \dfrac{{u_1}^2}{c^2}}}}{\sqrt{\dfrac{\left(1 - \dfrac{u'^2}{c^2}\right)\left(1 - \dfrac{v^2}{c^2}\right)}{1 - \dfrac{{u_2}^2}{c^2}}}}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{1 - \dfrac{{u_2}^2}{c^2}}}{\sqrt{1 - \dfrac{{u_1}^2}{c^2}}} \quad \ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

VARIATION OF MASS WITH VELOCITY CONTD.

Assuming that both the bodies having same mass and are moving with same velocity $v(\approx c)$ but in opposite direction in frame S'. Therefore from eqn. (i) and (ii) we have

$$u_1 = \frac{v+v}{1+\frac{v.v}{c^2}} \cong v \quad and\, m_1 = m\ (apparent\, mass)$$

$$u_2 = \frac{-v+v}{1-\frac{v.v}{c^2}} = 0 \quad and\, m_2 = m_o \ \ (rest\, mass)$$

Put the above values in eqn. (vii)

$$\mathrm{m} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \ldots\ldots\ldots\ldots\ldots \text{(viii)}$$

Eqn. (viii) represents an expression for variation of mass with velocity.

EINSTEIN'S MASS ENERGY RELATION

Relativistic Force is given by

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(mv)}{dt}$$ as m & v are variables

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \ldots\ldots\ldots\ldots\ldots(i)$$

Let a particle is displaced under the action of force through a small distance dx, then work done (or increase in kinetic energy) is

$$dK = dW = F.dx \quad \ldots\ldots\ldots(ii)$$

$$dK = m\frac{dv}{dt}dx + v\frac{dm}{dt}\,dx$$

$$dK = mv\,dv + v^2\,dm \quad \ldots\ldots\ldots(iii)$$

EINSTEIN'S MASS ENERGY RELATION

We have

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_o c}{\sqrt{c^2 - v^2}}$$

Squaring above eqn.

$$m^2 c^2 - m^2 v^2 = m_o{}^2 c^2$$

diffg. above eqn.

$$c^2 2m\,dm - m^2 2v\,dv - v^2 2m\,dm = 0$$

$$c^2 dm - mv\,dv - v^2\,dm = 0$$

$$c^2 dm = mv\,dv - v^2\,dm \quad \ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

From eqn. (i) and (iv) $dK = c^2 dm$

EINSTEIN'S MASS ENERGY RELATION

If particle is displaced from its rest mass to m, then change in kinetic energy is from 0 to K

$$\int_0^K dK = \int_{m_0}^{m} c^2 dm$$

Relativistic Kinetic Energy is

$$K = (m - m_o)\, c^2 \qquad \ldots\ldots\ldots\ldots (v)$$

So Total Energy can be written as-

E = K.E+ P.E.

$$\boxed{E = mc^2}$$