Student Answer Script View

MIT MPL - BTech-M Sc-MCA - II-IV and VI Semester - Midterm Examination - Mar 2025 Answer Sheet

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Course:	MIT-OPE
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Subject Name:	APPLIED GRAPH THEORY
Exam Date:	08-Mar-2025

Q.No : 1)

If (4, 3, 2, 2, x) is a degree sequence of a simple graph, then the value of x is

0	1	2	3

Q.No : 2)

In a social network, if the sum of degrees of all users is100, how many connections (edges) are there in the network?

50	52	54	56

Q.No : 3)

If G is a disconnected graph of order 101, then the minimum degree of G is

- less than 50
- less than 51
- less than 52
- less than 53

Q.No : 4)

In a self-complementary graph with 101 vertices, the number of edges is

2525	2526	2527	2528

Q.No : 5)

The number of cut vertices in a Hamiltonian graph of order 10 is

0	2	4	6

Q.No : 6)

The vertex independence number of a path graph with 15 vertices is

8	6	4	2

Q.No : 7)

The vertex covering number of a cycle with 30 vertices is

14	15	16	17

Q.No : 8)

The number of cut edges in a tree with 24 vertices is

22	23	24	25

Q.No : 9)

If G is a connected graph with diameter 10, then diameter of the complement graph of G is at most

4	3	5	6

Q.No : 10)

Maximum number of edges in a bipartite graph G with 14 vertices is

- 49
- 51
- 54
- 58

Q.No : 11)

. (i) Prove that a connected graph on n vertices with $n-1$ edges is a tree.

(ii) A tree T with 50 pendant vertices has an equal number of vertices of degree 2, 3, 4 and 5 and no vertices of degree greater than 5. Determine the order of T.

(ii) Let no of vertices of degree 2, 3, 4, 5 x.

Let total no of vertices be n.

$\Rightarrow n = 50 + x + x + x + x$

$n = 50 + 4x$ —— ①

$$\sum_{i=1}^{n} \deg(v_i) = 2(n-1)$$

$50 + 2x + 3x + 4x + 5x = 2(n-1)$

$50 + 14x = 2n - 2$

$52 + 14x = 2n$

$26 + 7x = n$ —— ②

From ① and ②,

$50 + 4x = 26 + 7x$

$24 = 3x$

$\boxed{x = 8}$

$\Rightarrow n = 50 + 4(8)$

$\boxed{n = 82}$

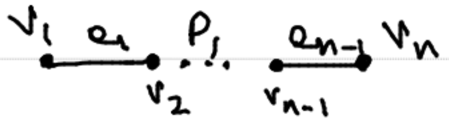
order of $T = 50 + 14x$

$= \underline{\underline{162}}$

(i) A connected graph on n vertices is tree if there are no cycles present in the graph.

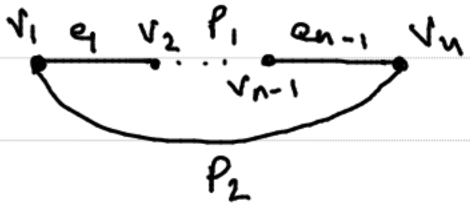
A graph with n vertices needs at least edges to make a cycle.

Let e_1 and e_{n-1} be two edges with vertices v_1 on the with path P_1 between v_2 and v_{n-1}.



It has $n-1$ edges a no cycles.

Let P_2 be another path between ther



$v_1 \xrightarrow{e_1} v_2 \xrightarrow{P_1} v_{n-1} \xrightarrow{e_{n-1}} v_n \xrightarrow{P_2} v_1$ makes cycle with n edges.

This is a contradiction.

Hence a connected graph on n vertices with $n-1$ edges is a tree.

Q.No : 12)

Test whether the vector (3, 3, 2, 2, 2, 2, 2) is graphical. If so, draw a simple graph with this vector as a degree vector.

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Vector $v = (3, 3, 2, 2, 2, 2, 2)$

① $$v = \left(\begin{array}{ccccccc} \underline{3}, & 3, & 2, & 2, & 2, & 2, & 2 \\ v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array}\right)$$

$$v_1 = \left(\begin{array}{cccccc} v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ 2, & 1, & 1, & 2, & 2, & 2 \end{array}\right) \longrightarrow v_1$$

② $$v = \left(\begin{array}{cccccc} v_2 & v_5 & v_6 & v_7 & v_3 & v_4 \\ 2, & 2, & 2, & 2, & 1, & 1 \end{array}\right)$$

$$v_1 = \left(\begin{array}{ccccc} v_5 & v_6 & v_7 & v_3 & v_4 \\ 1, & 1, & 2, & 1, & 1 \end{array}\right) \longrightarrow v_2$$

③ $$v = \left(\begin{array}{ccccc} v_7 & v_5 & v_6 & v_3 & v_4 \\ 2, & 1, & 1, & 1, & 1 \end{array}\right)$$

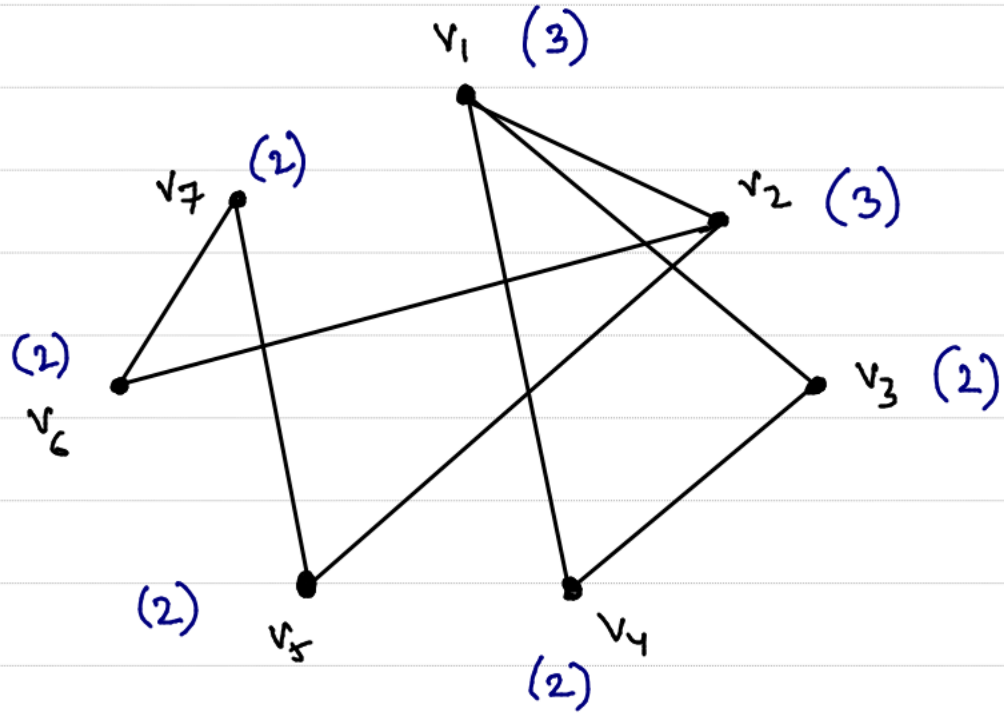
$$v_1 = \left(\begin{array}{cccc} v_5 & v_6 & v_3 & v_4 \\ 0, & 0, & 1, & 1 \end{array}\right) \longrightarrow v_7$$

④ $$v = \left(\begin{array}{cccc} v_3 & v_4 & v_5 & v_6 \\ 1, & 1, & 0, & 0 \end{array}\right)$$

$$v_1 = \left(\begin{array}{ccc} v_4 & v_5 & v_6 \\ 0, & 0, & 0 \end{array}\right) \longrightarrow v_3$$

Since all are 0, the vector is graphica

Graph:


v_1 (3)
v_2 (3)
v_3 (2)
v_4 (2)
v_5 (2)
v_6 (2)
v_7 (2)

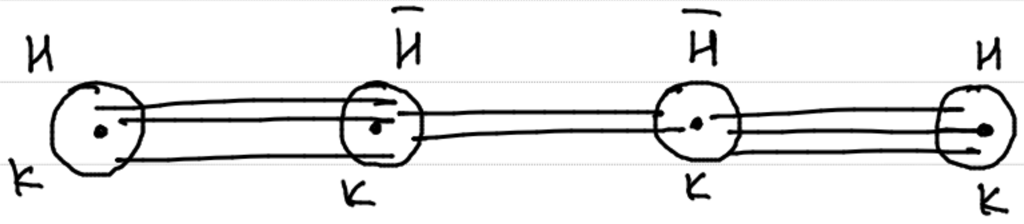
Q.No : 13)

Show that a self-complementary graph is of order 4k or 4k+1 for some integer k. Draw a self-complementary graph with 9 vertices.

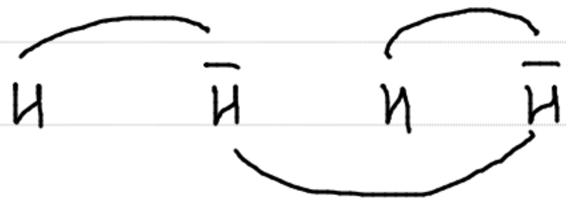
A self-complementary graph can only be o order 4k or 4k+1 for some integer.

If order = 4k,

Let H be a graph and $\bar{H}$ be its complementary graph.

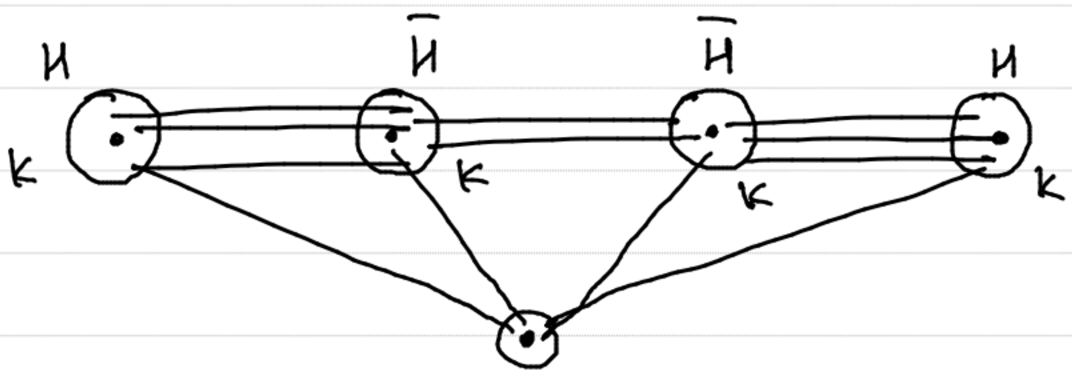


Thus, the graph is



If order = 4k+1,

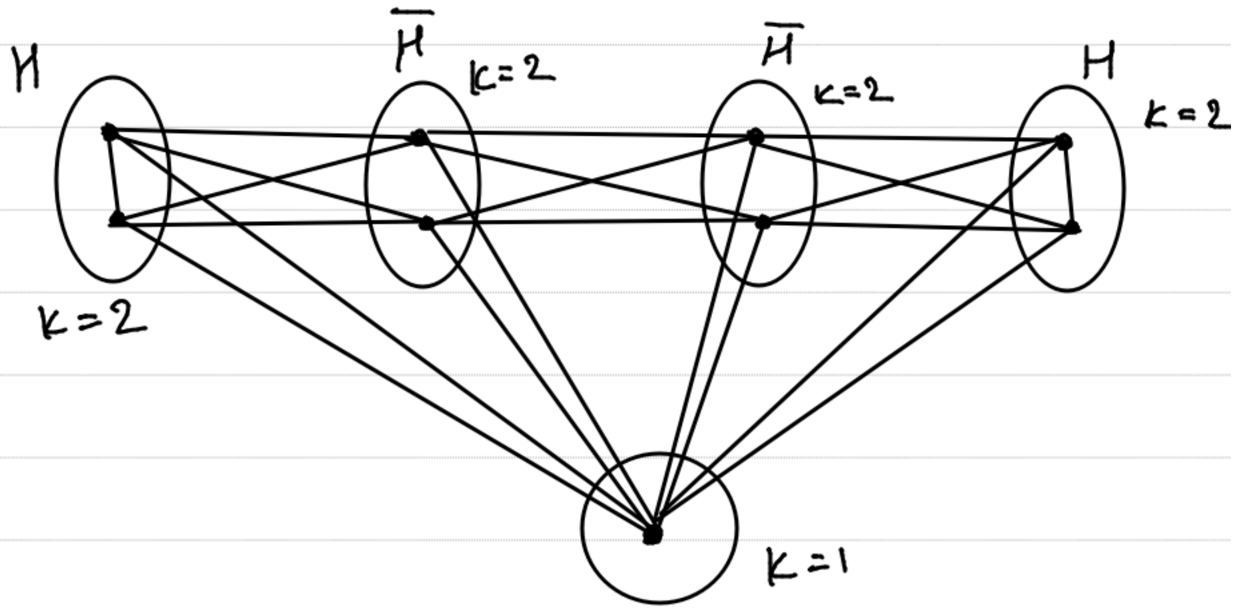
Let H be a graph and $\bar{H}$ be its complementary graph.



For graphs of order $4k+2$ and $4k+3$, a self-complementary graph cannot be drawn

9 vertices $= 4k+1$

$k=2$



Q.No : 14)
If G is Eulerian, then show that all its vertices have even degree. Give an example of a Hamiltonian graph that is not Eulerian.

Eulerian graph is when a graph contains an Eulerian cycle.

Hamiltonian graph is when a spanning set the graph exists.

→ If any vertice has an odd degree, it wi
have a cycle.
⇒ It won't have an Eulerian ~~cyle~~ cycle.
⇒ It is a contradiction.
Hence, all vertices must

Q.No : 15)

Let G be graph on n vertices, with vertex covering number α_0 and vertex independence number β_0 . Prove that $\alpha_0 + \beta_0 = n$. .

Vertex covering number α_0 is the minimum cardin
of a vertex covering set. A vertex covering set
when all the edges are connected to at leas
one of the vertices.

Vertex independence number β_0 is the maximum cardin
of a vertex independence set. A vertex independence set
when no two vertices in the set are ad
to each other.

Using an example:

vertex cover = $\{ v_2, v_4 \}$

$\alpha_0 = 2$

Vertex independence set = $\{$

$\beta_0 = 3$

$n = 5$

$\alpha_0 + \beta_0 = 5$

Hence, $\alpha_0 + \beta_0 = n$

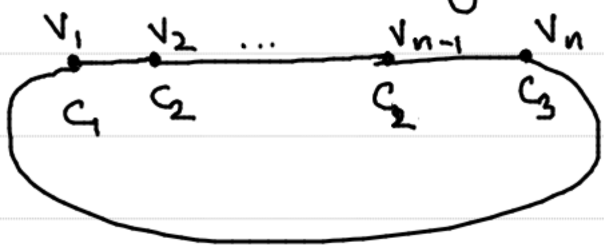
Q.No : 16)

Prove that the chromatic polynomial $P_{\chi}(C_n, x)$ of a cycle C_n is $(x-1)^n + (-1)^n(x-1)$ for $n \geq 3$. .

For an even cycle, no of colours used =
For an odd cycle, no of colours used = 3

Even cycle, n = even
v_1 would get colour 1.
$\Rightarrow$ v_2 would get colour 2.
$\Rightarrow$ v_3 would get colour 1.
Similarly, v_n would get colour 2.
v_1 and v_n are adjacent but have different

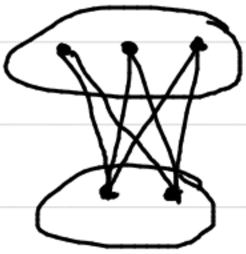
Odd cycle, n = odd
v_1 would get colour 1.
$\Rightarrow$ v_2 would get colour 2.
$\Rightarrow$ v_3 would get colour 1.
Similarly, v_{n-1} would get colour 2.
Similarly, v_n would get colour 1.
v_1 and v_n are adjacent but have same col
This would be a contradiction.
Hence we assign a third colour to v_n.



Q.No : 17)
Show that a bipartite graph G has no odd cycles.

A bipartite graph is one where all the v
can be divided into two sets where the
vertices in each set are not adjacent to t
other vertices in the same set.

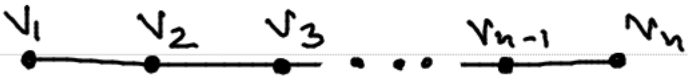
$K_{3,2}$



If a cycle graph has odd cycles,

$\Rightarrow$ no of vertices $=$ odd $= n$
$\Rightarrow$ no of edges $= n$

Let G be a graph with vertices v_1, v_2
Let the two bipartite sets be S_1, S_2



Let v_1 belongs to S_1.
$\Rightarrow v_2$ belongs to S_2.
$\Rightarrow v_3$ belongs to S_1.

Similarly, $v_{2n} \in S_2$ and $v_{2n+1} \in S_1$

If $n =$ odd,

v_n would belong to S_1

v_1 and v_n are adjacent and v_1 is al

in S_1.

Thus,

$$S_1 = \{ v_1, v_3 \dots v_n \}$$

$$S_2 = \{ v_2, v_4, v_6 \dots v_{n-1} \}$$

Thus, v_1 and v_n would form a cycl

since they are in the same set.

This is a contradiction.

Hence, a bipartite graph cannot h

odd cycles.

Q.No : 18)

Draw a simple graph with vertex connectivity 2, edge connectivity 3 and minimum degree 4.

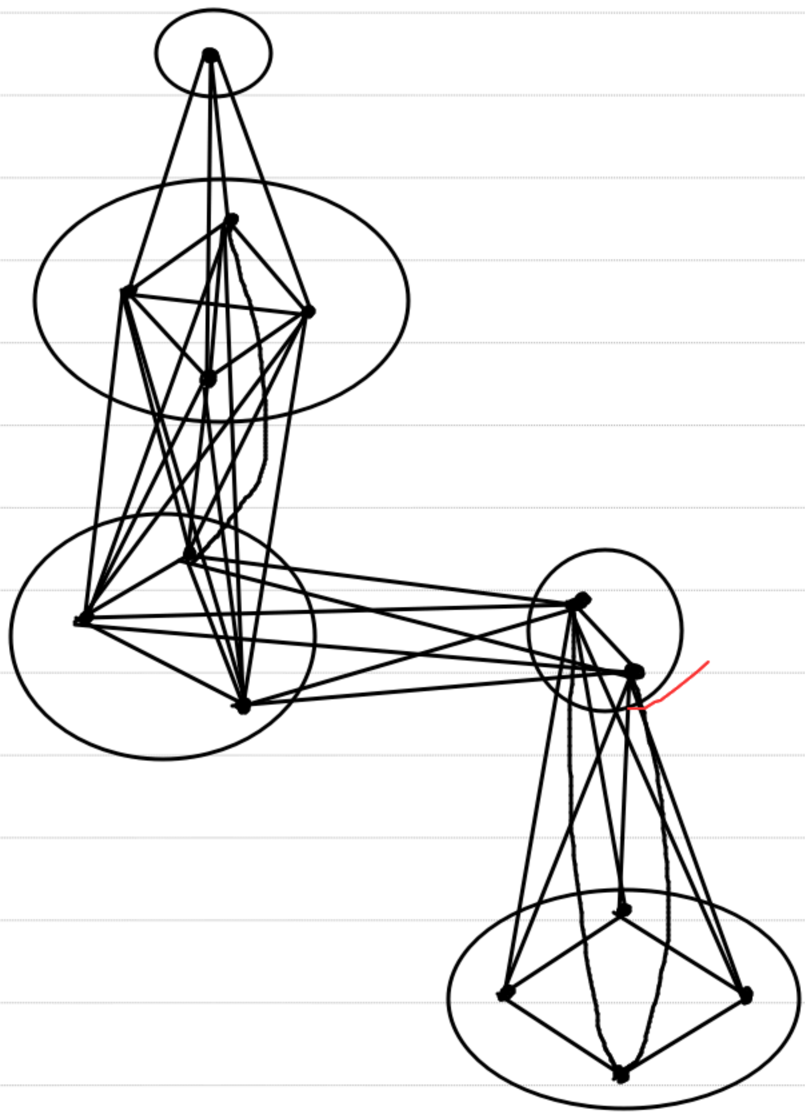
Vertex connectivity = $\kappa(G) = 2 = K_a$

Edge connectivity = $\lambda(G) = 3 = K_b$

Min degree = $\delta(G) = 4 = K_d$

Graph:

$$K_1 \geq K_d \geq K_b \geq K_a \geq K_d$$



Q.No : 19)

Define a cut edge in a graph. What is the total number of cut vertices in a tree with 10 vertices and 6 pendant vertices.

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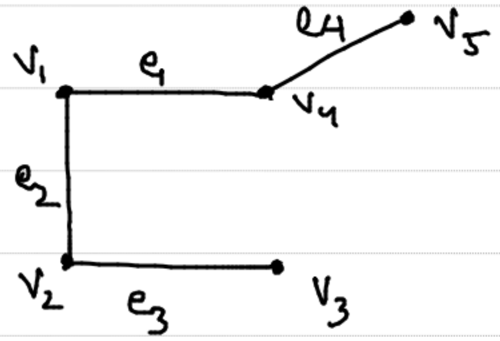
A cut edge in a graph is any edge wh
on deleting increases the number of componen
in the graph.

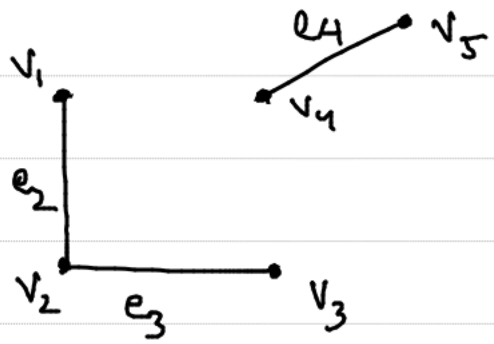
Eg:

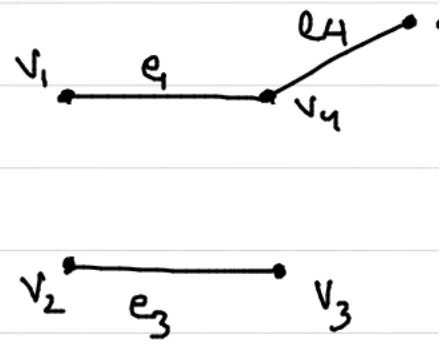


Here either e_1 or e_2 is a cut edge.

cut edge $= e_1$

cut edge $= e_2$

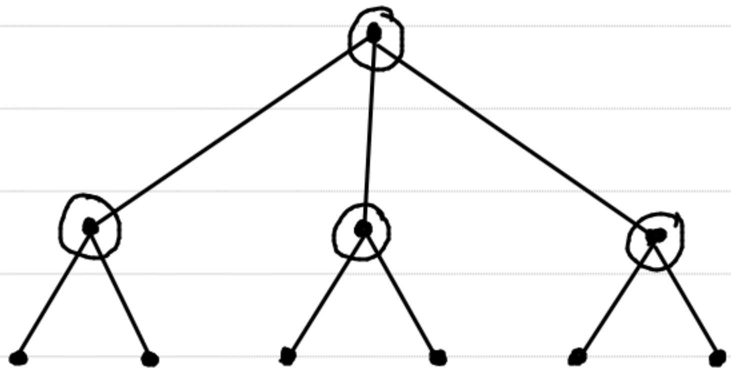




No of vertices $= 10$

No of pendant vertices $= 6$

Tree :



There are 4 cut vertices.

Since there are 6 pendant vertices, they can't be cut vertices.

Hence no of cut vertices = 10 − 6 = 4