COL333/671: Introduction to AI

Semester I, 2024-25

Adversarial Search

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Outline

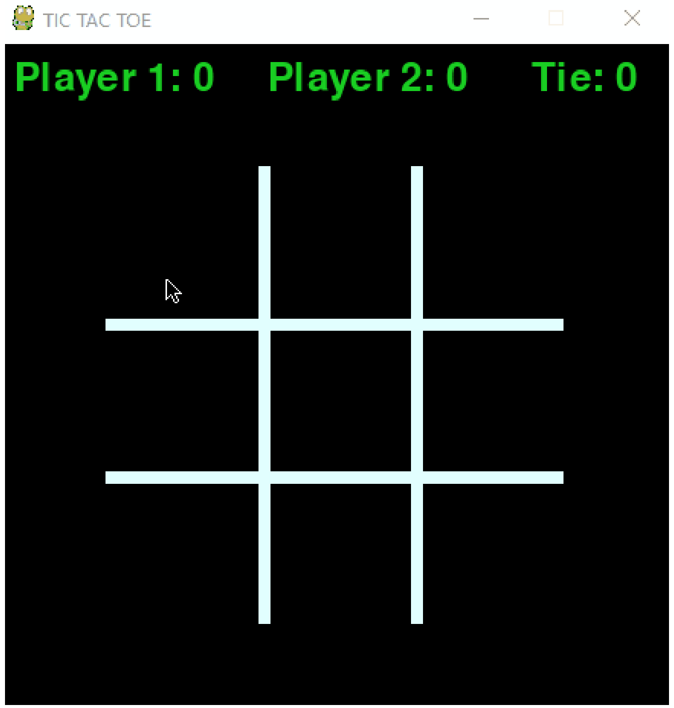
- Last Class
  - Constraint Satisfaction
- This Class
  - Adversarial Search
- Reference Material
  - AIMA Ch. 5 (Sec: 5.1-5.5)

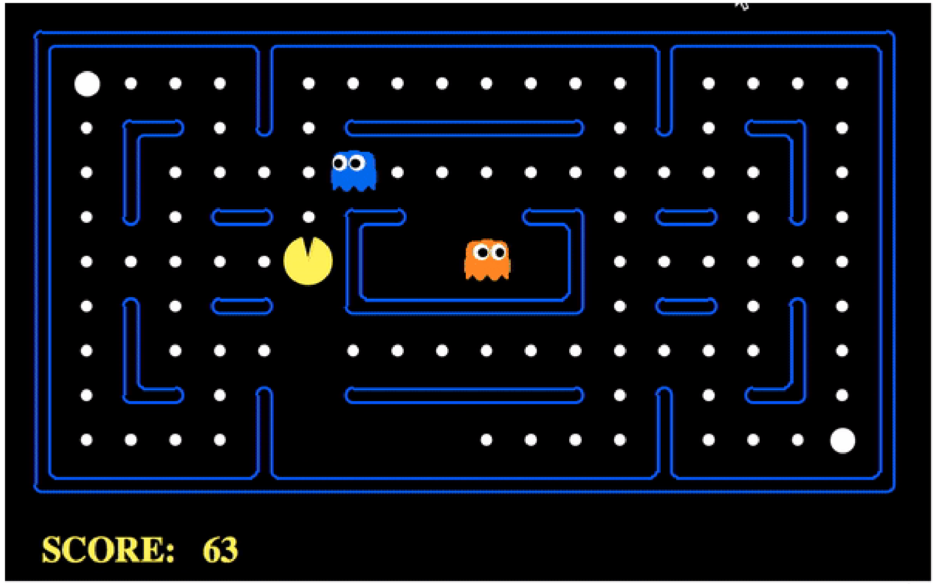
Acknowledgement

These slides are intended for teaching purposes only. Some material has been used/adapted from web sources and from slides by Doina Precup, Dorsa Sadigh, Percy Liang, Mausam, Dan Klein, Anca Dragan, Nicholas Roy and others.

Game Playing and AI

- **Games: challenging decision-making problems**
  - Incorporate the state of the other agent in your decision-making. Leads to a vast number of possibilities.
  - Long duration of play. Win at the end.
  - Time limits: Do not have time to compute optimal solutions.





Games: Characteristics

- Axes:
  - Players: one, two or more.
  - Actions (moves): deterministic or stochastic
  - States: fully known or not.

- Zero-Sum Games
  - Adversarial: agents have opposite utilities (values on outcomes)

- **Core: contingency problem**
  - The opponent's move is not known ahead of time. A player must respond with a move for every possible opponent reply.
- **Output**
  - Calculate a strategy (policy) which recommends a move from each state.

Playing Tic-Tac-Toe: *Essentially a search problem!*

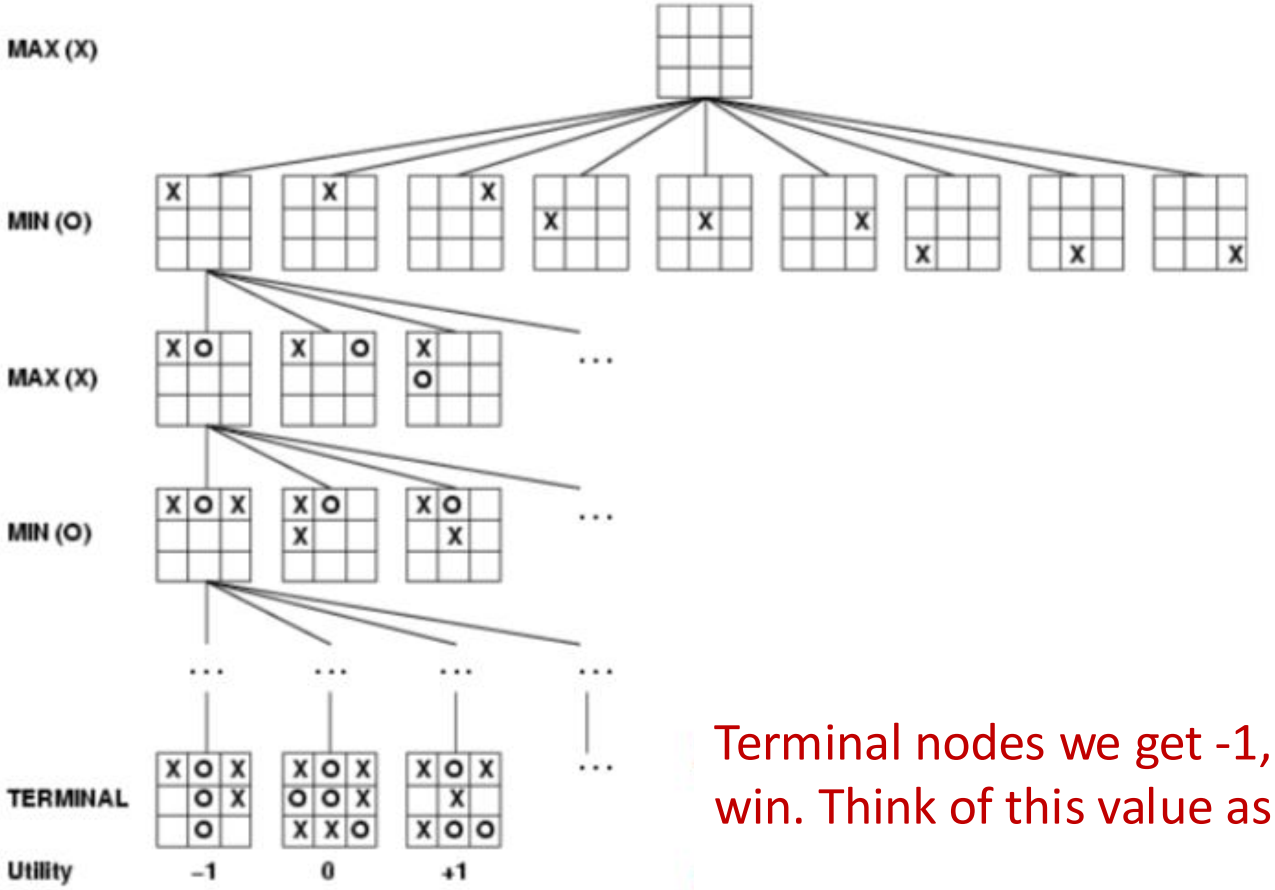
computer's turn

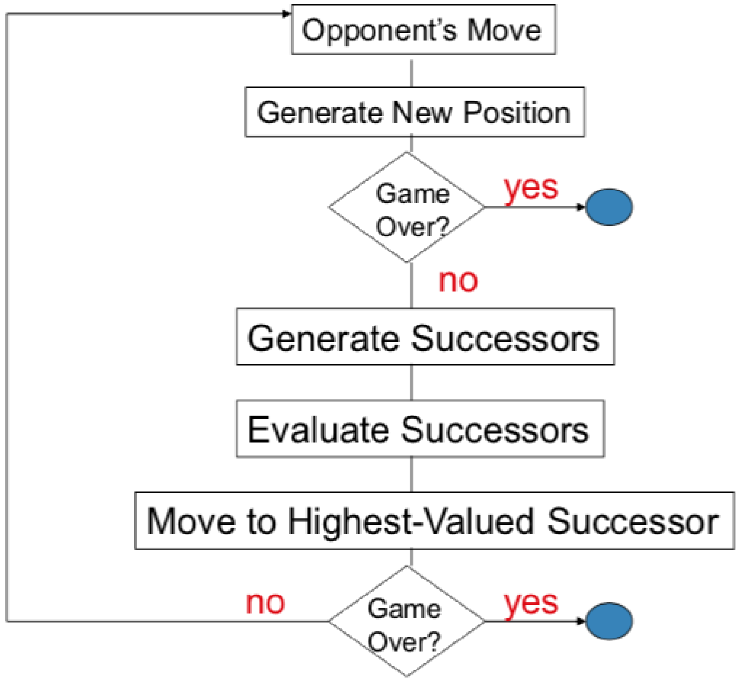
opponent's turn

computer's turn

opponent's turn

leaf nodes are evaluated

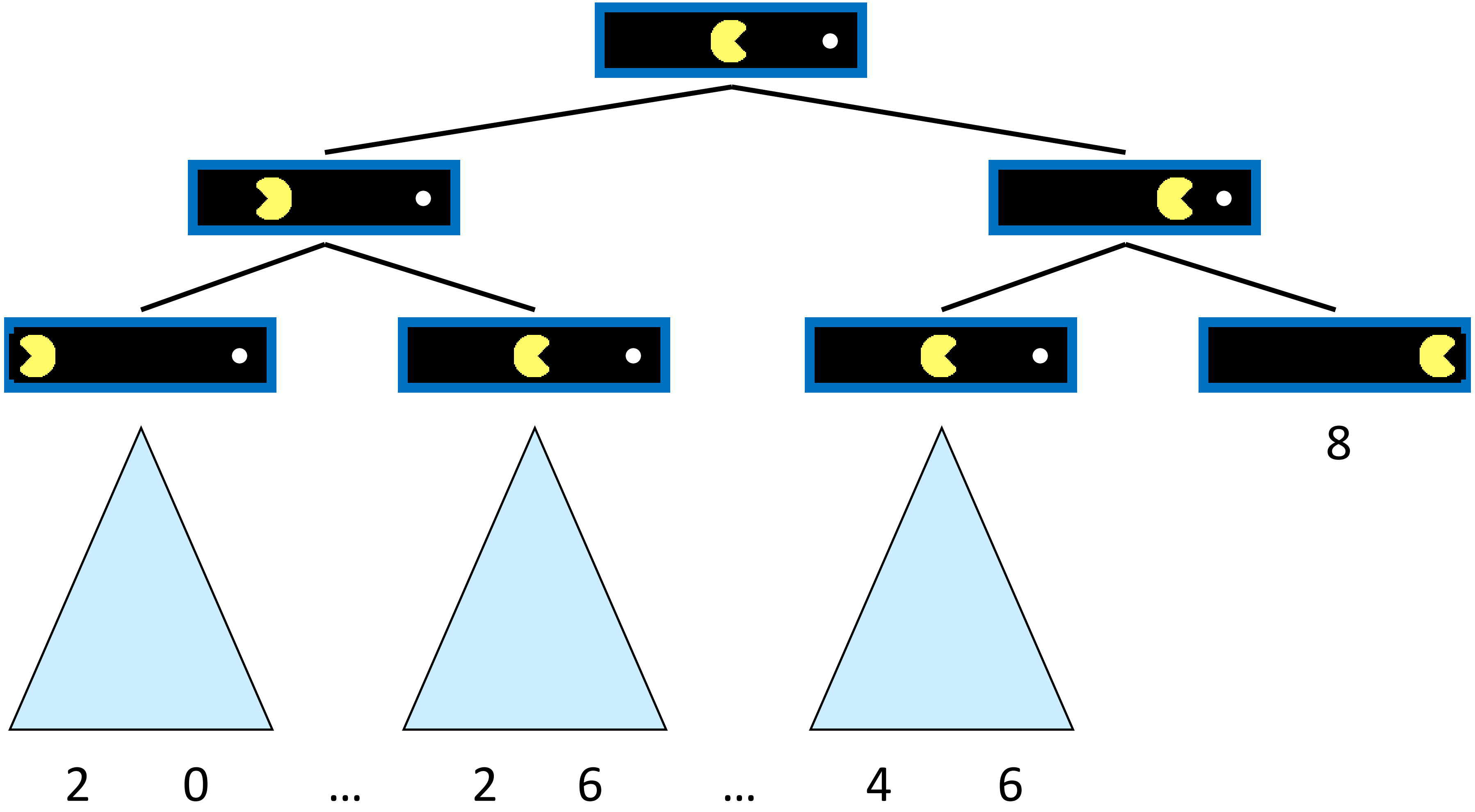




Terminal nodes we get -1, 0 or 1 for loss, tie or win. Think of this value as a "utility" of a state.

in and from Mausam

Single-Agent Trees

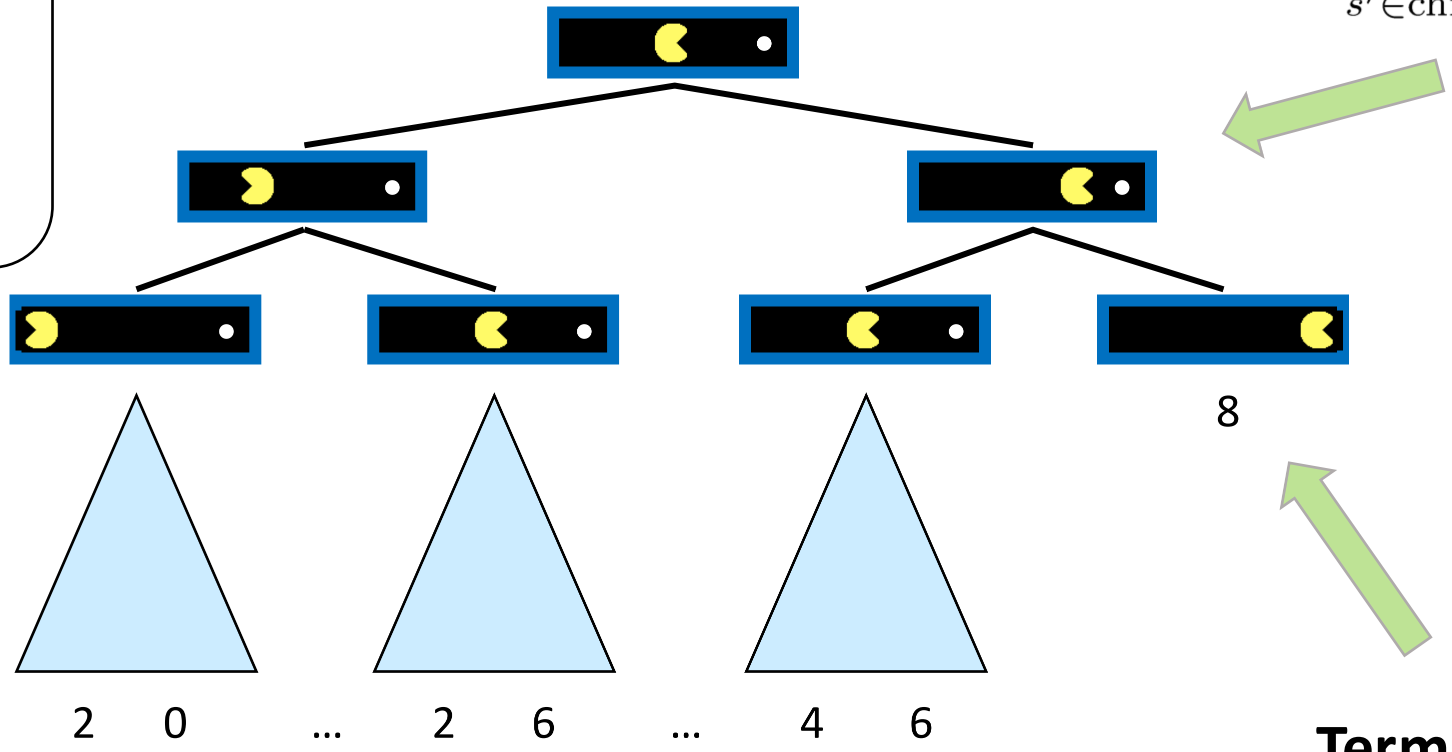


Computing “utility” of states to decide actions

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

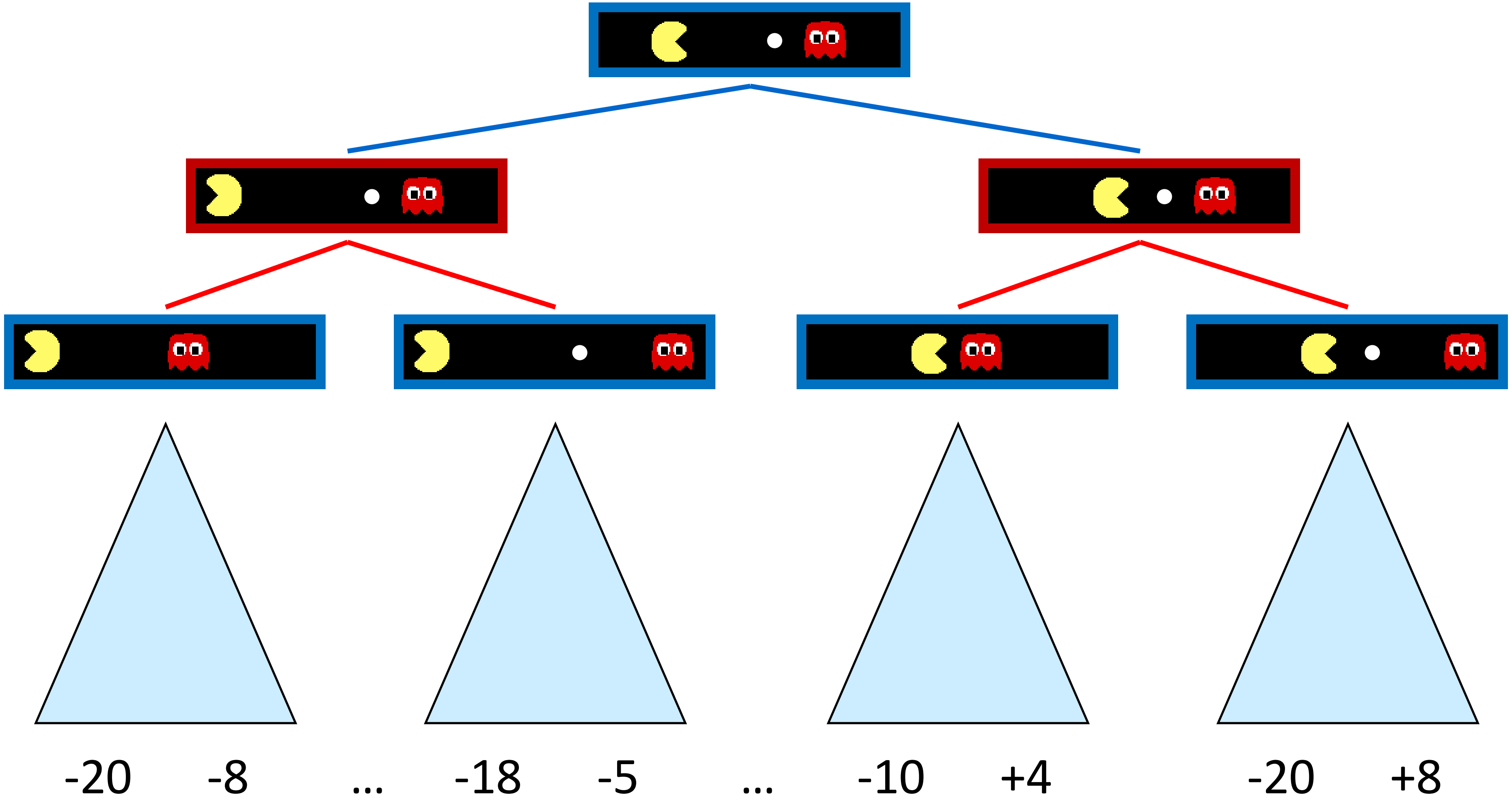
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

$$V(s) = \text{known}$$

Game Trees: Presence of an Adversary



The adversary's actions are not in our control. Plan as a contingency considering all possible actions taken by the adversary.

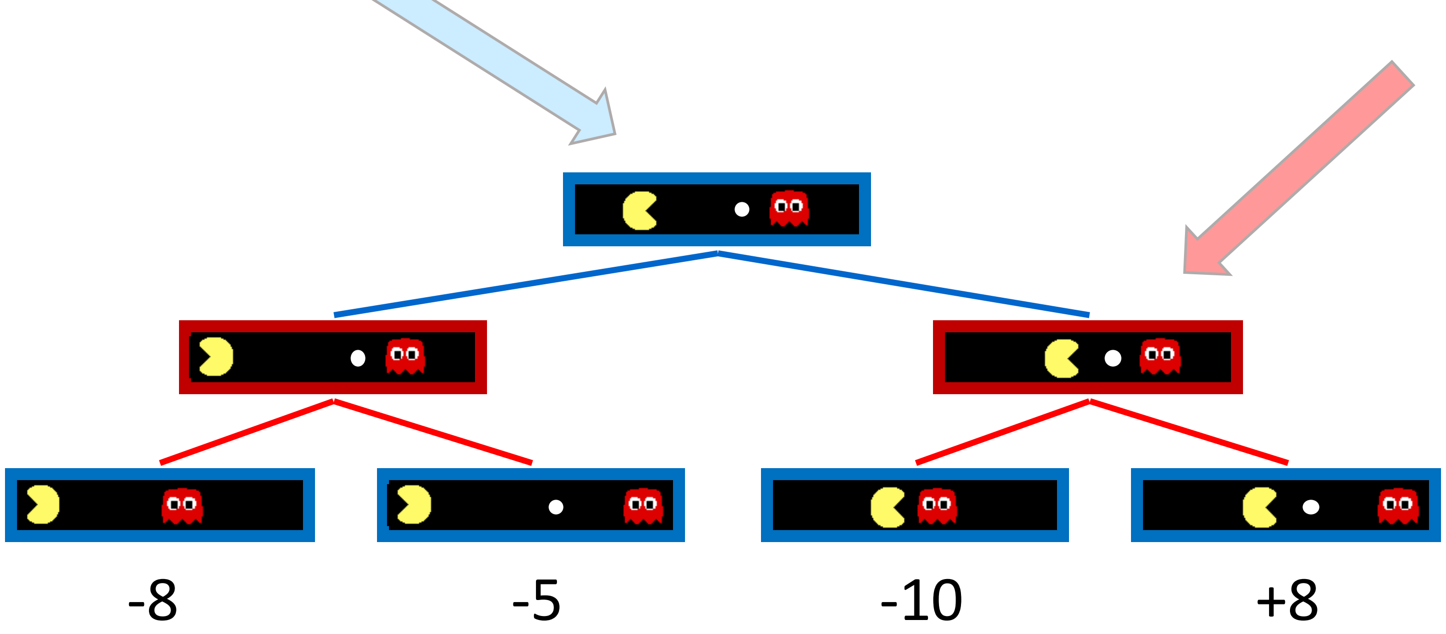
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

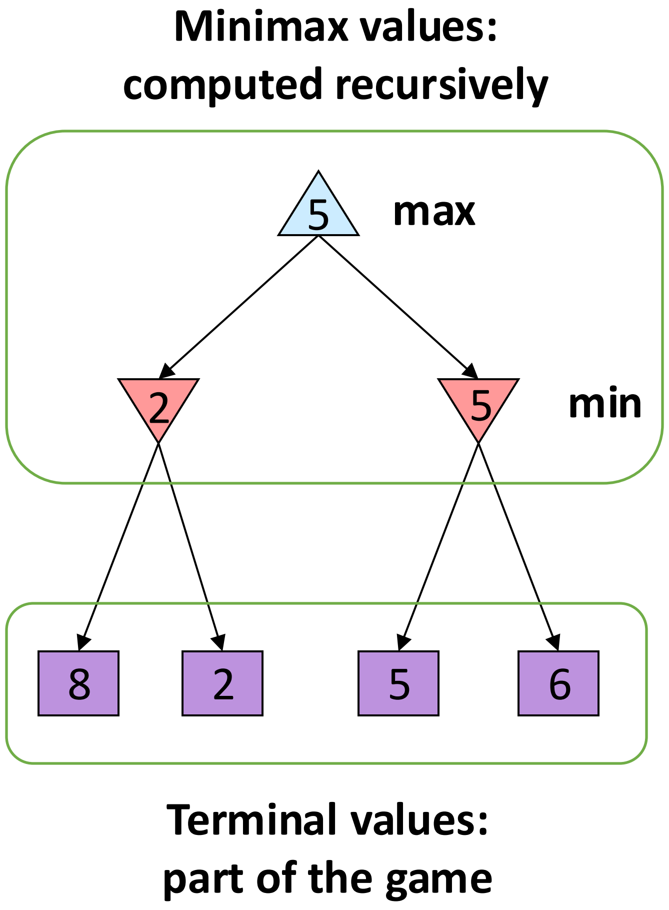
Adversarial Search (Minimax)

- Consider a deterministic, zero-sum game
  - Tic-tac-toe, chess etc.
  - One player maximizes result and the other minimizes result.

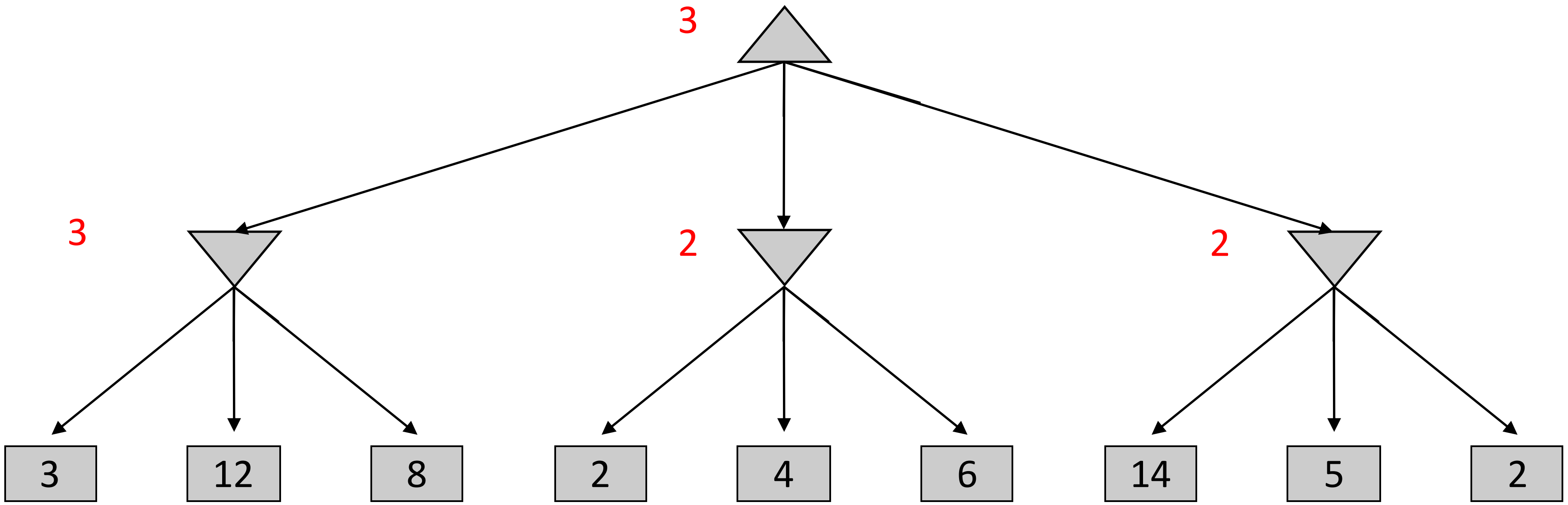
- Minimax Search
  - Search the game tree for best moves.
  - Select optimal actions that move to a position with the highest minimax value.
  - What is the minimax value?
    - It is the best achievable utility against the optimal (rational) adversary.
    - Best achievable payoff against the best play by the adversary.

Minimax Algorithm

- Ply and Move
  - Move: when action taken by both players.
  - Ply: is a half move.
- Backed-up value
  - of a MAX-position: the value of the largest successor
  - of a MIN-position: the value of its smallest successor.
- Minimax algorithm
  - Search down the tree till the terminal nodes.
  - At the bottom level apply the utility function.
  - Back up the values up to the root along the search path (compute as per min and max nodes)
  - The root node selects the action.



Minimax Example



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

Useful, when there are multiple adversaries.

Minimax Properties

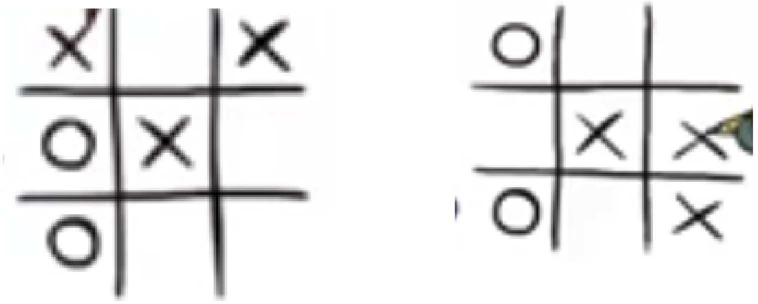
- Completeness
  - Yes
- Complexity
  - Time: $O(b^m)$
  - Space: $O(bm)$
- **Requires growing the tree till the terminal nodes.**
- **Not feasible in practice for a game like Chess.**

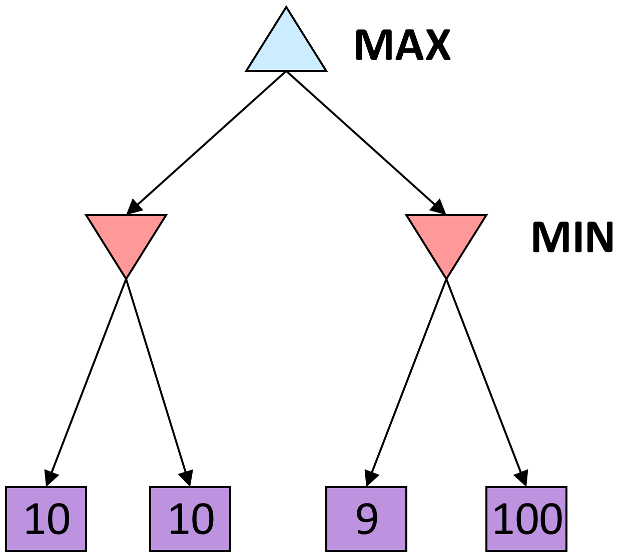
- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}

Minimax Properties

- Optimal
  - If the adversary is playing optimally (i.e., giving us the min value)
    - Yes
  - If the adversary is not playing optimally (i.e., <u>not</u> giving us the min value)
    - No. Why? It does not exploit the opponent’s weakness against a suboptimal opponent).

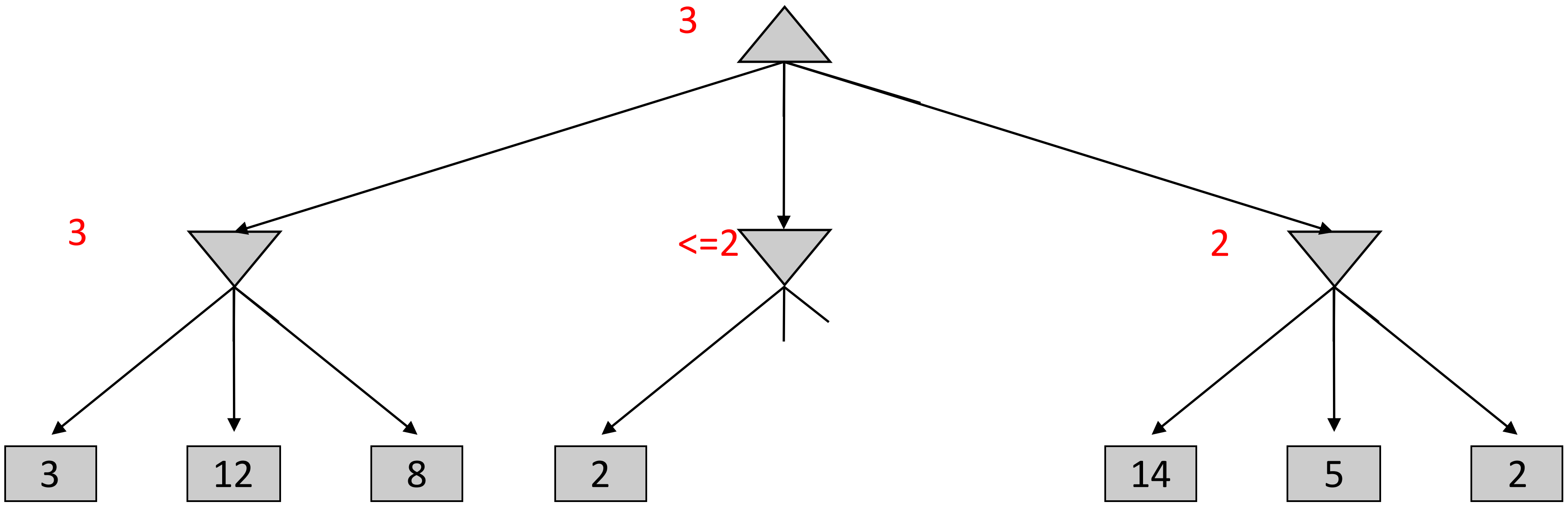
You: Cricle. Opponent: Cross





If min returns 9? Or 100?

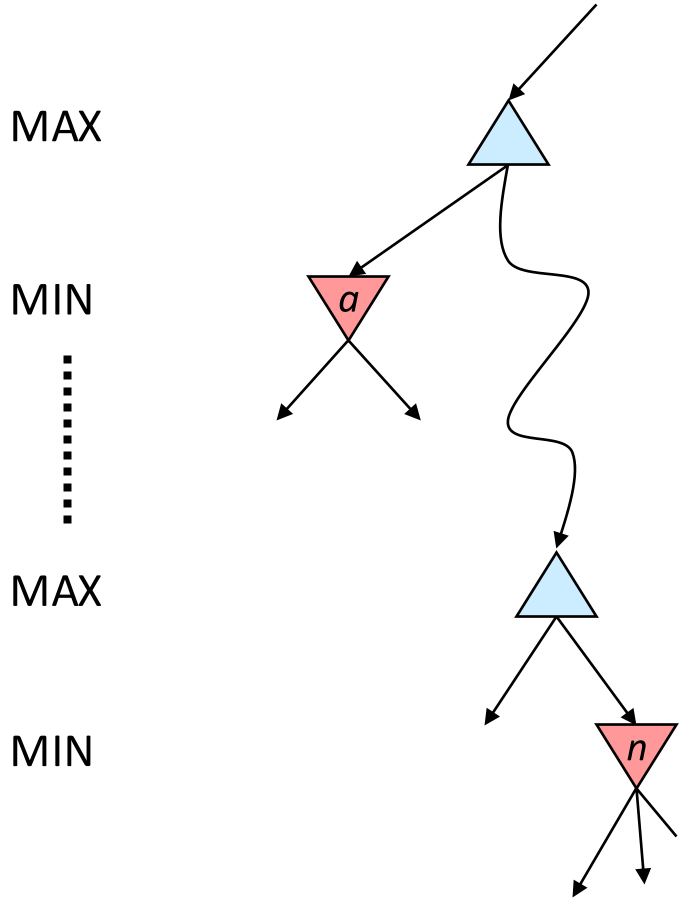
Necessary to examine all values in the tree?



Core Idea: Limit the search time can be limited to the 'more promising' subtree, and a deeper search can be performed in the same time.

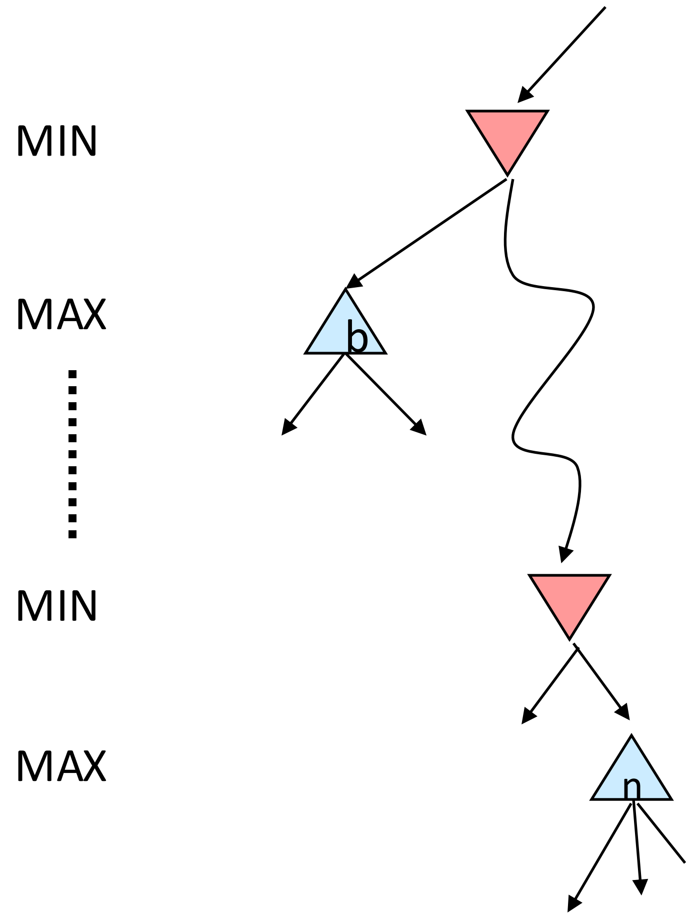
Alpha-Beta Pruning: General Idea

- **General Configuration (MIN version)**
  - Consider computing the MIN-VALUE at some node *n*, examining *n*'s children
  - *n*'s estimate of the childrens' min is reducing.
  - Who can use *n*'s value to make a choice? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If the value at *n* becomes worse than *a*, MAX will not pick this option, so we can stop considering *n*'s other children (any further exploration of children will only reduce the value further)

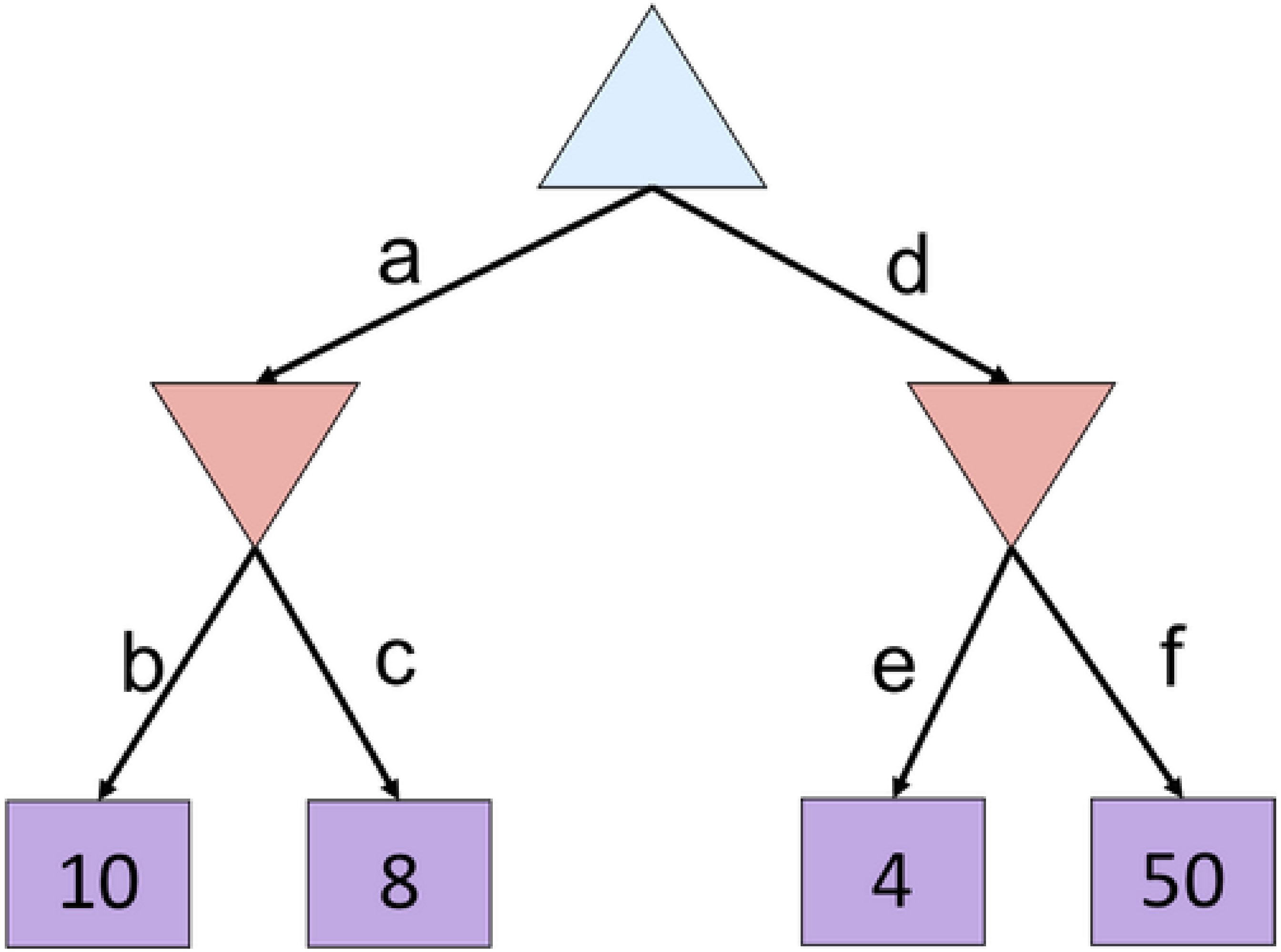


Alpha-Beta Pruning: General Idea

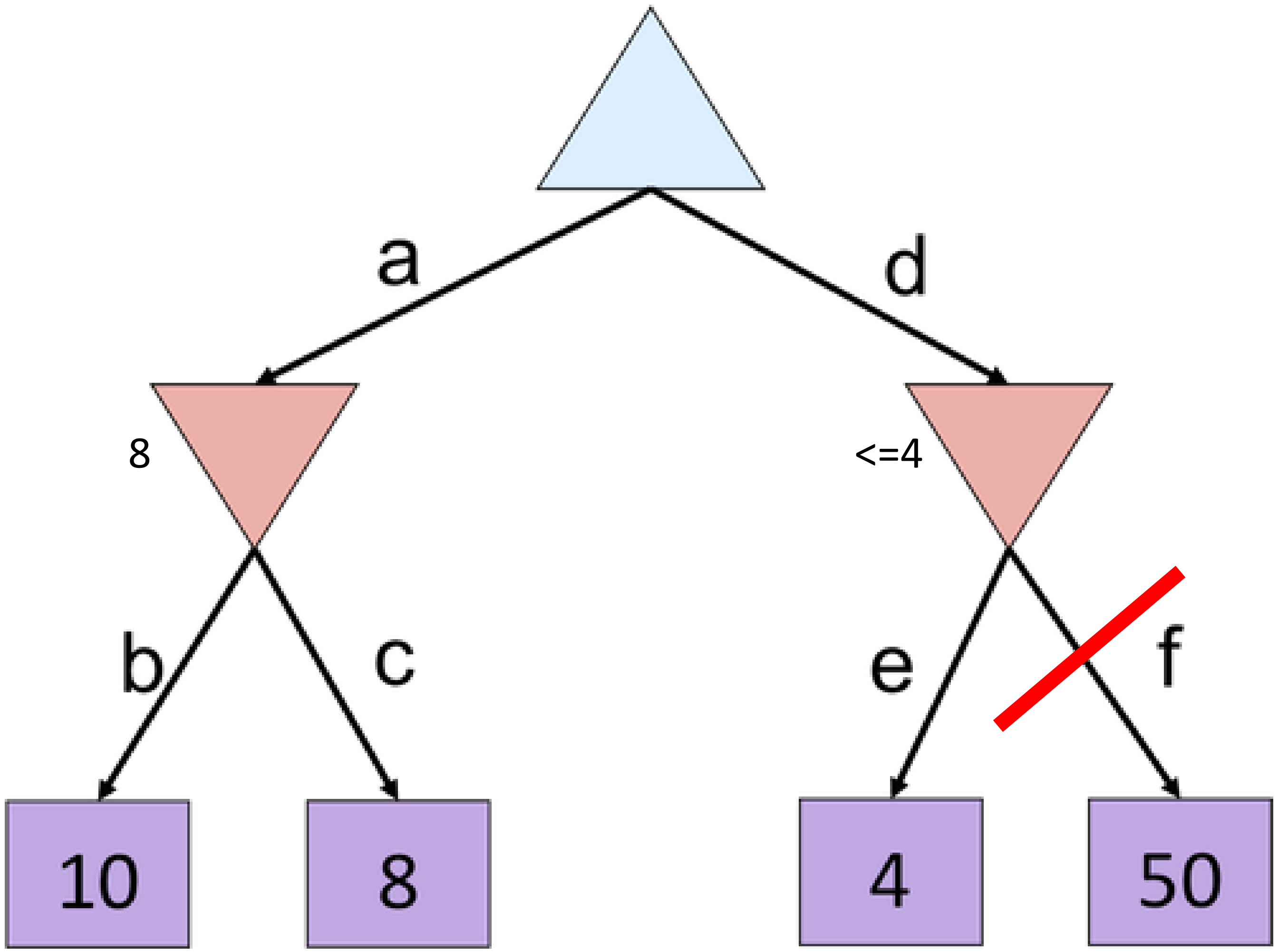
- **General Configuration (MAX version)**
  - Consider computing the MAX-VALUE at some node *n*, examining *n*'s children
  - *n*'s estimate of the childrens' max is increasing.
  - Who can use *n*'s value to make a choice? MIN
  - Let *b* be the lowest (best) value that MIN can get at any choice point along the current path from the root
  - If the value at *n* becomes higher than *b*, MIN will not pick this option, so we can stop considering *n*'s other children (any further exploration of children will only increase the value further)



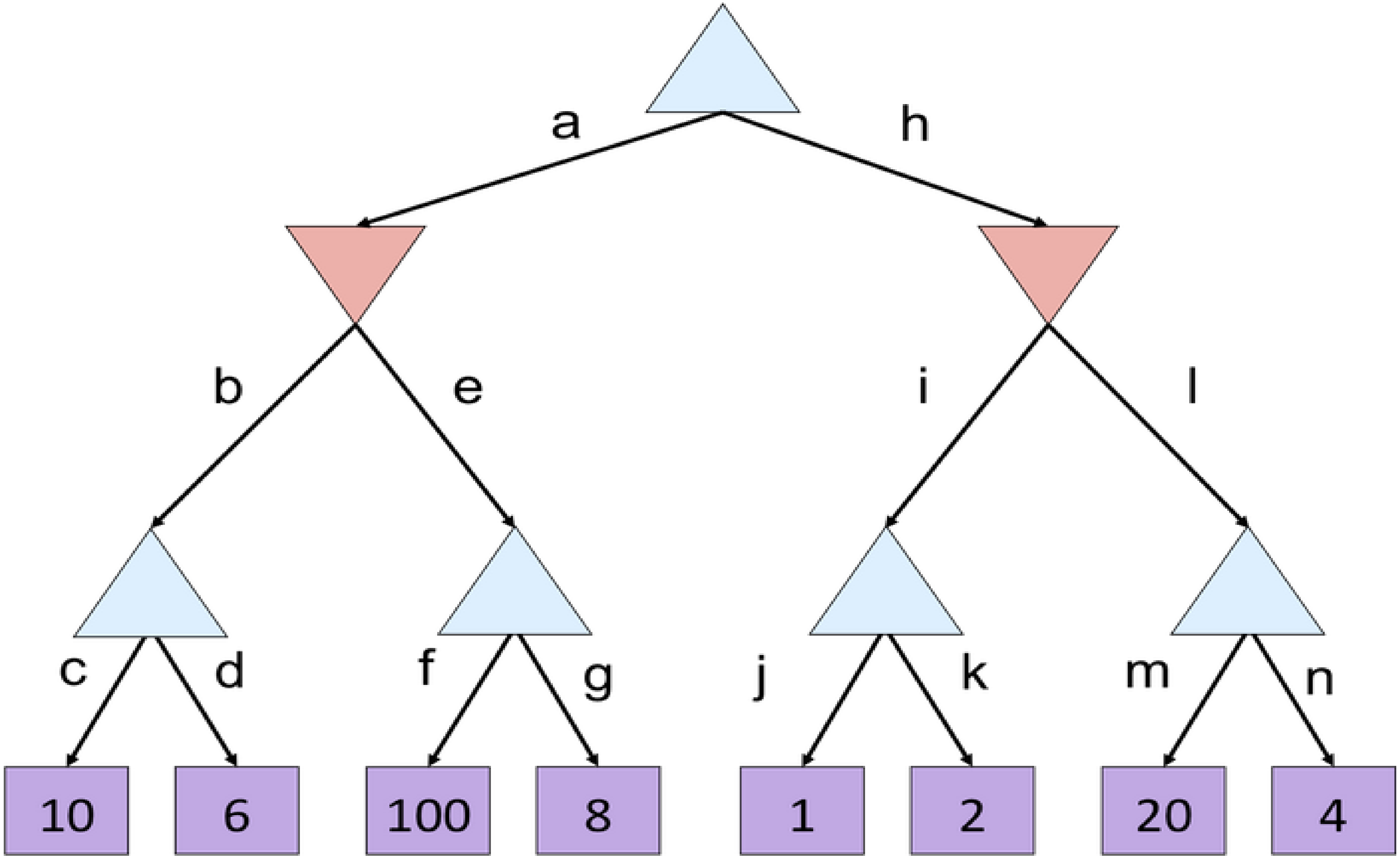
Pruning: Example



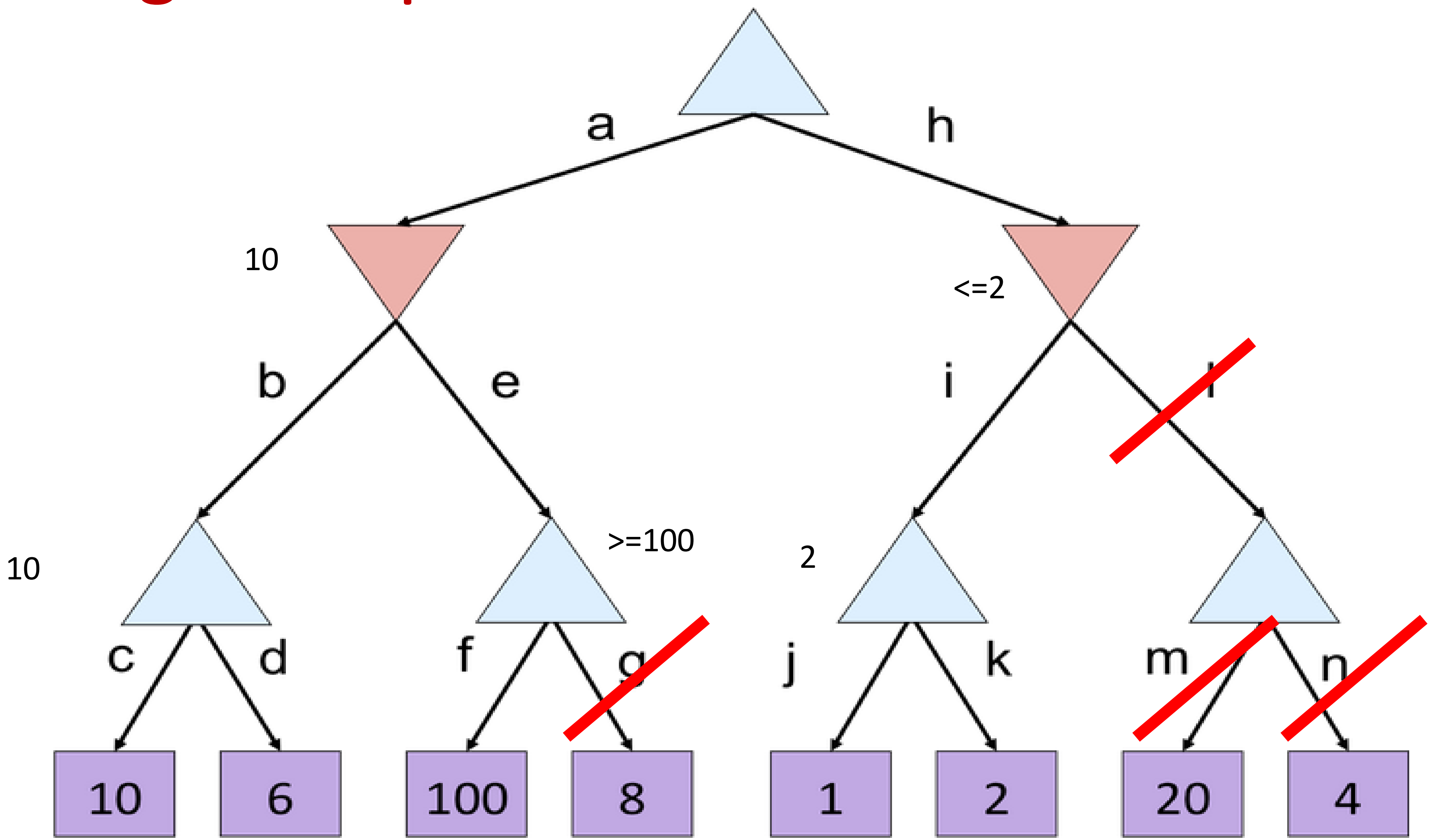
Pruning: Example



Pruning: Example



Pruning: Example



Alpha-Beta Implementation

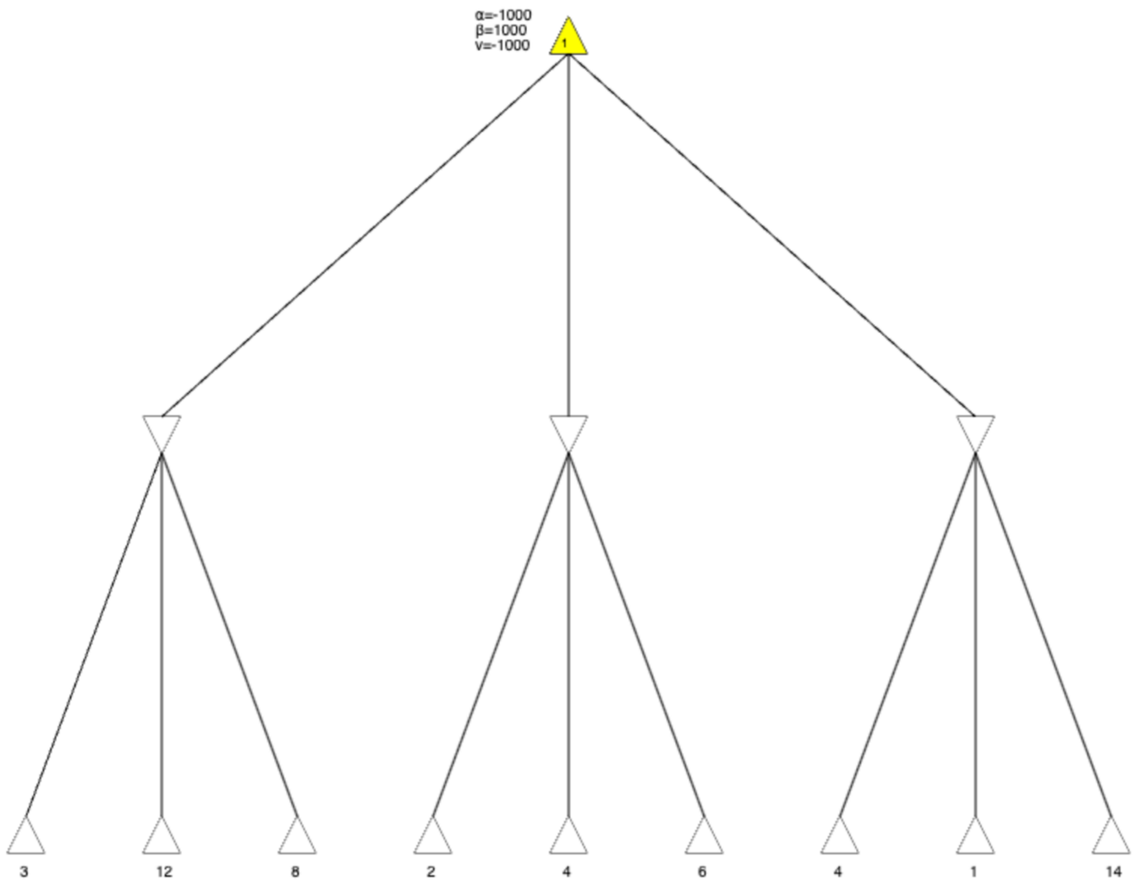
α: MAX's best option on path to root
β: MIN's best option on path to root

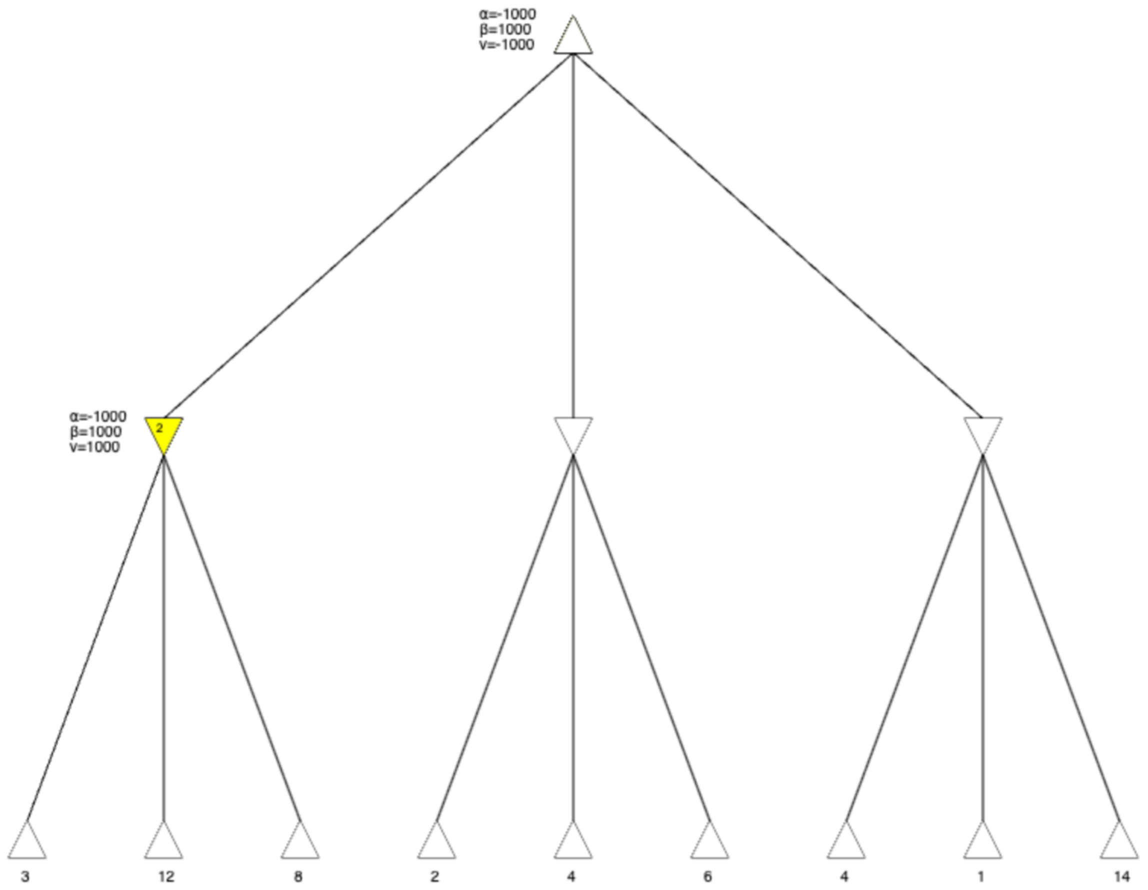
```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v,
          min_value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v,
          max_value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

MAX node will maximise the values obtained from its children (updates v value).
MAX's value matters to MIN, hence value is compared with beta for pruning.
If not pruned then value is returned.

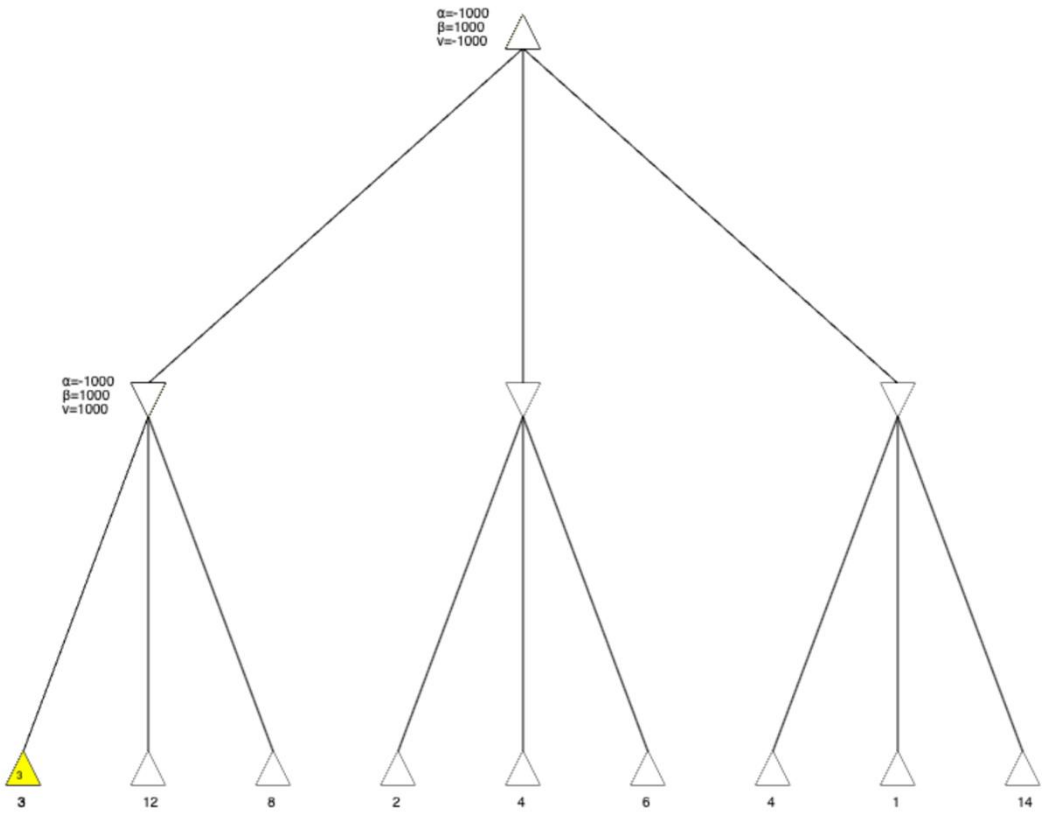
Example (1/7)

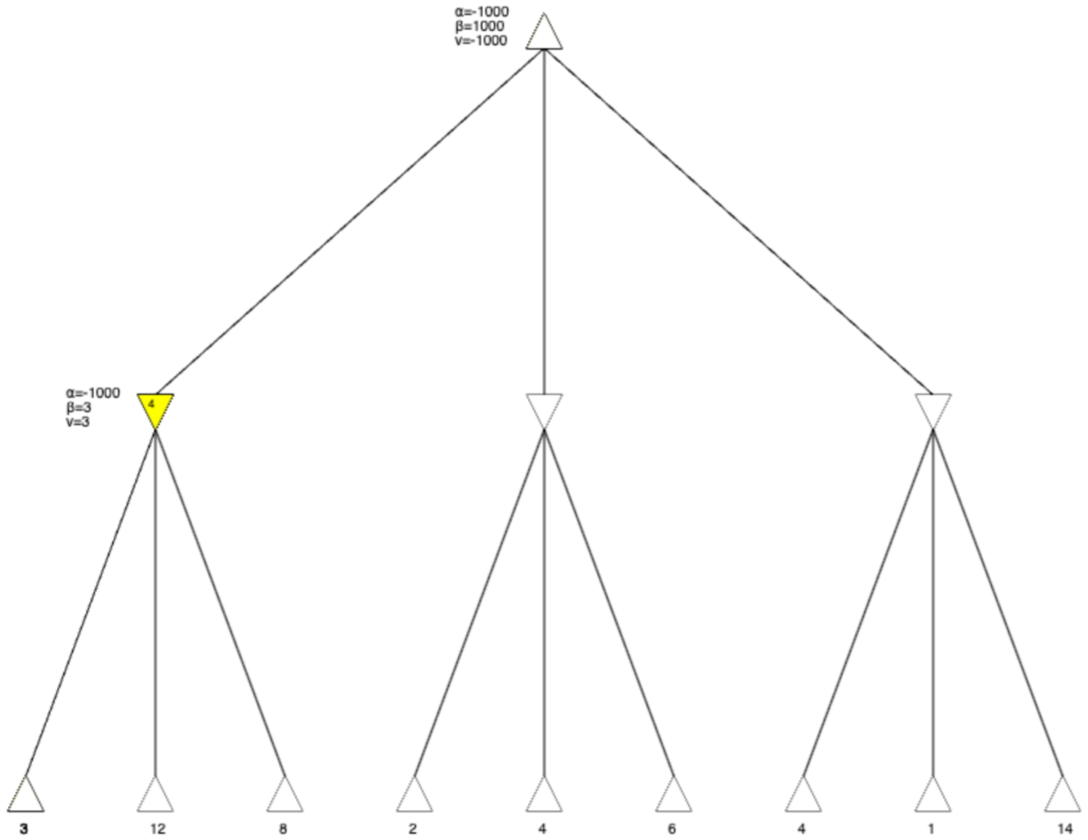




Initialize: alpha large negative, beta large positive

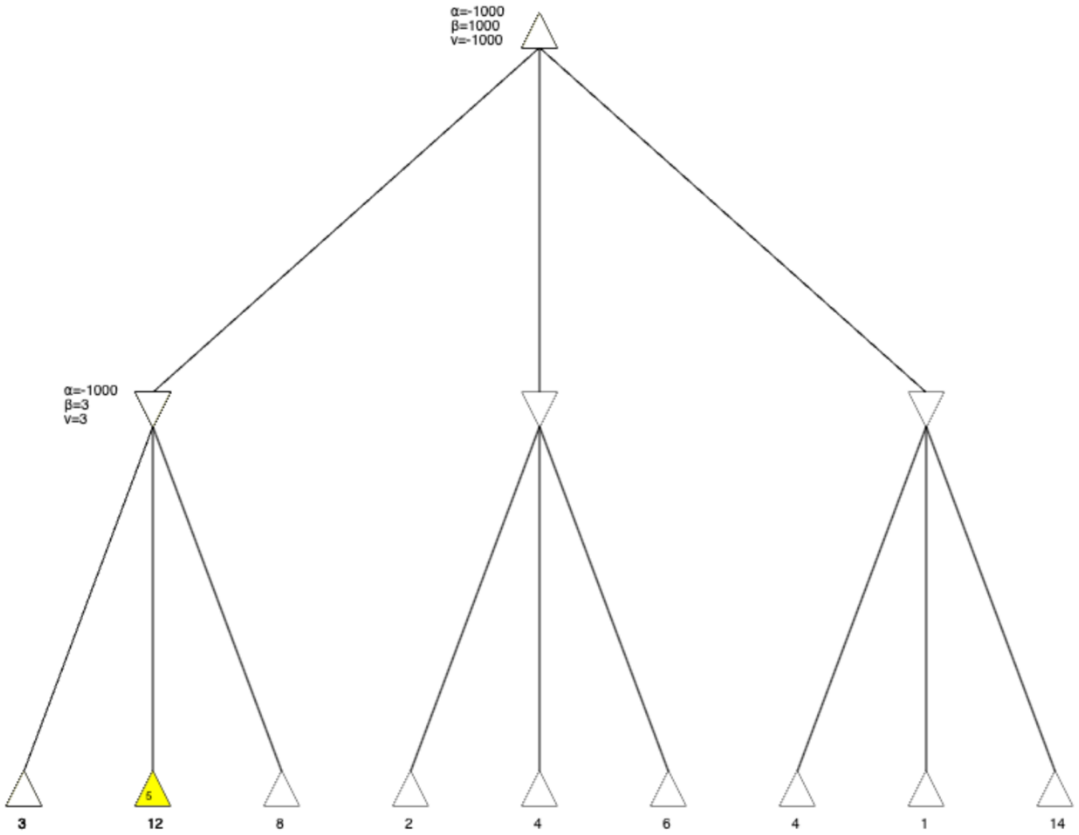
Example (2/7)

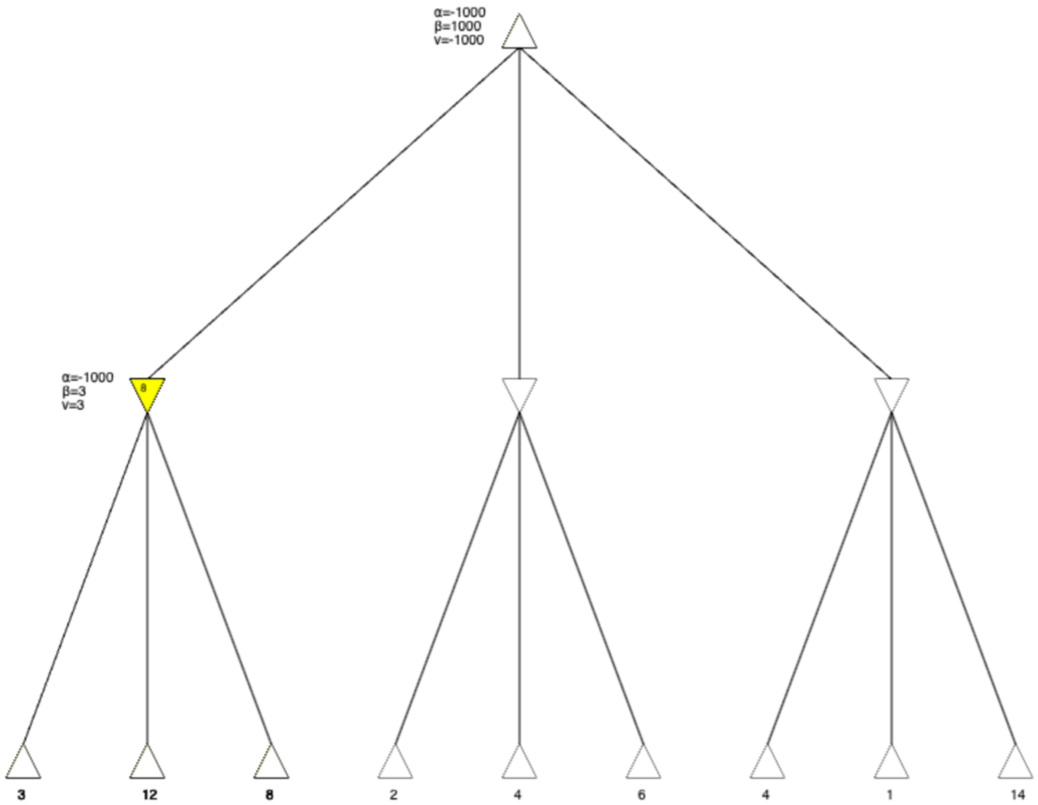




Min node updates beta

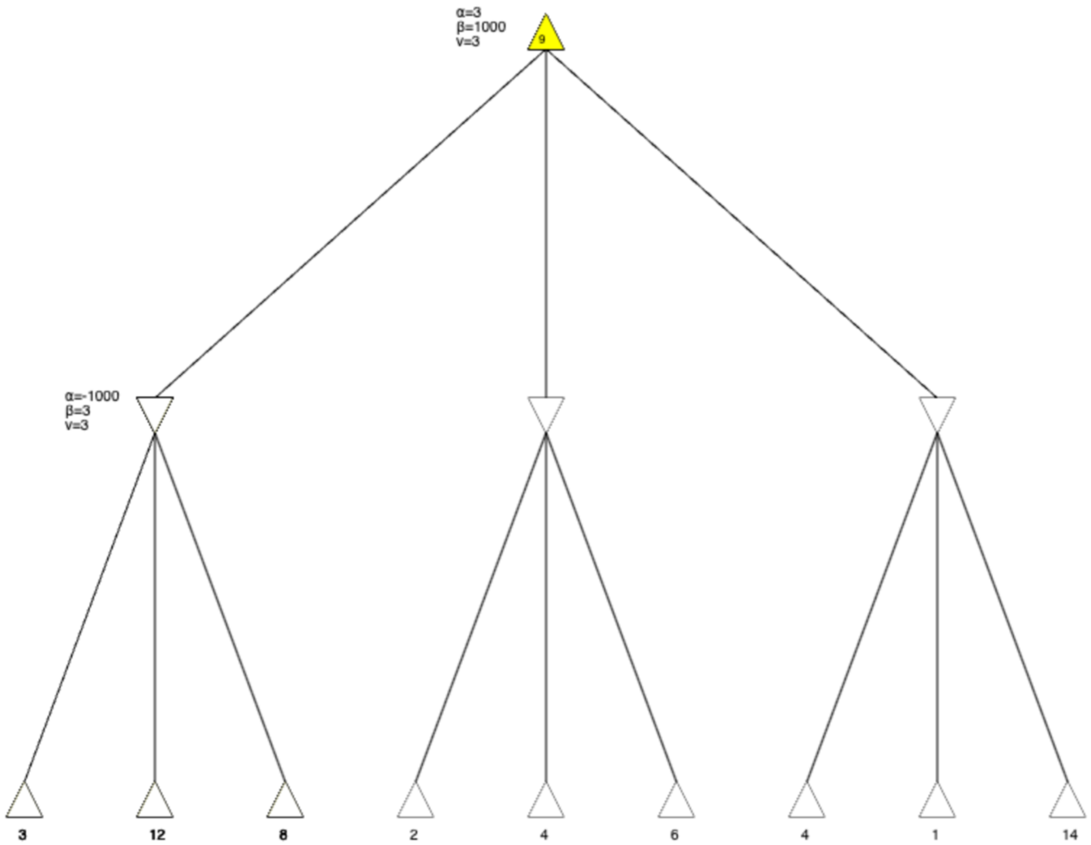
Example (3/7)



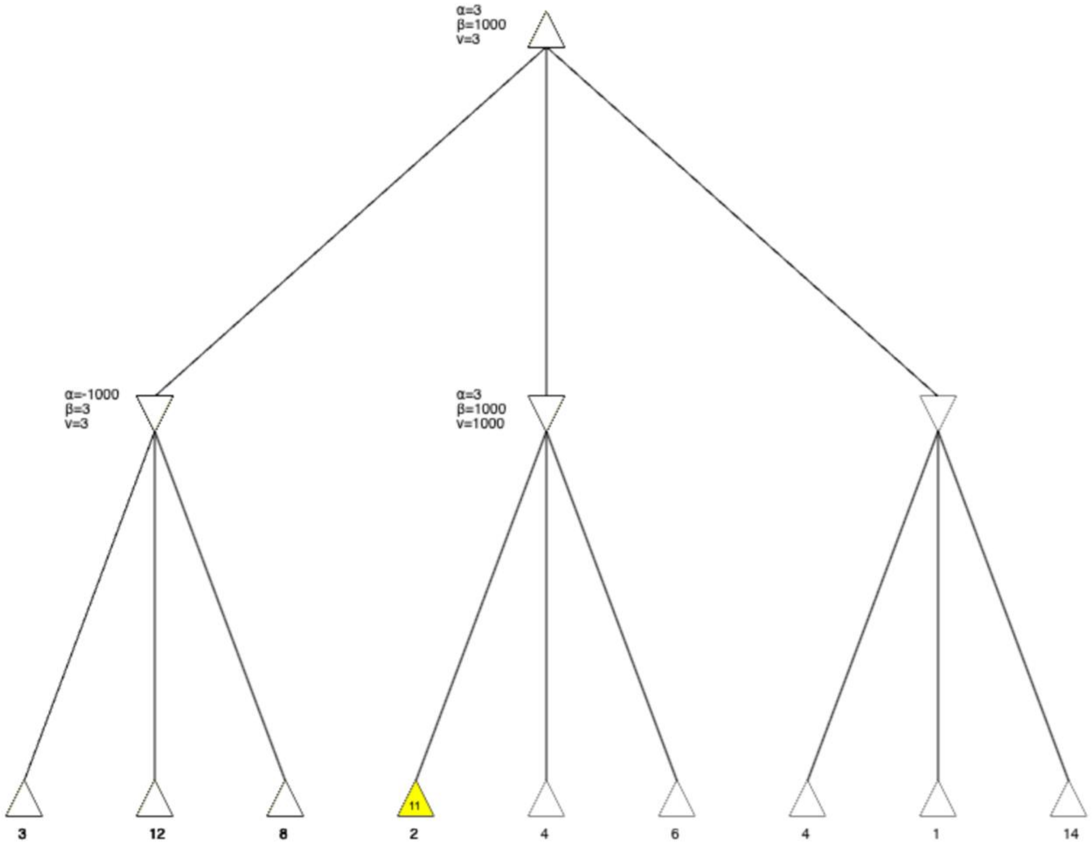


Other values are examined at Min node but they don’t change the beta value.

Example (4/7)

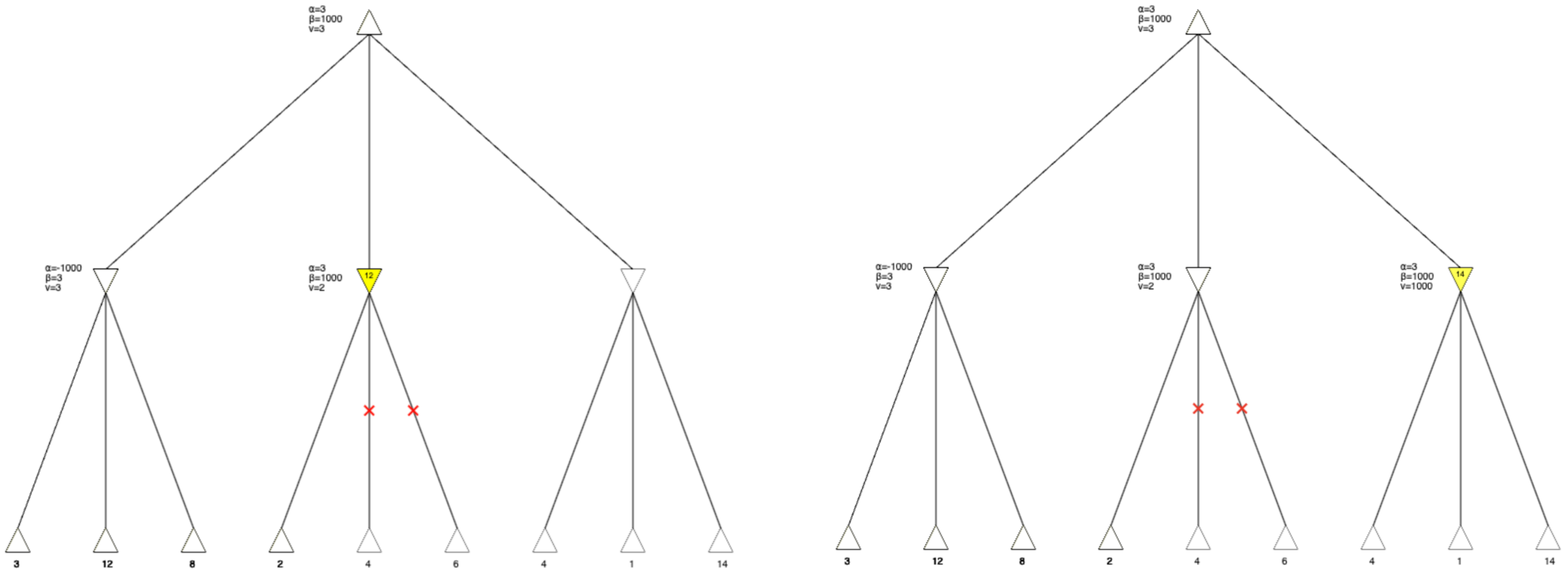


The value returned by the Min node updates the alpha value of the Max node.



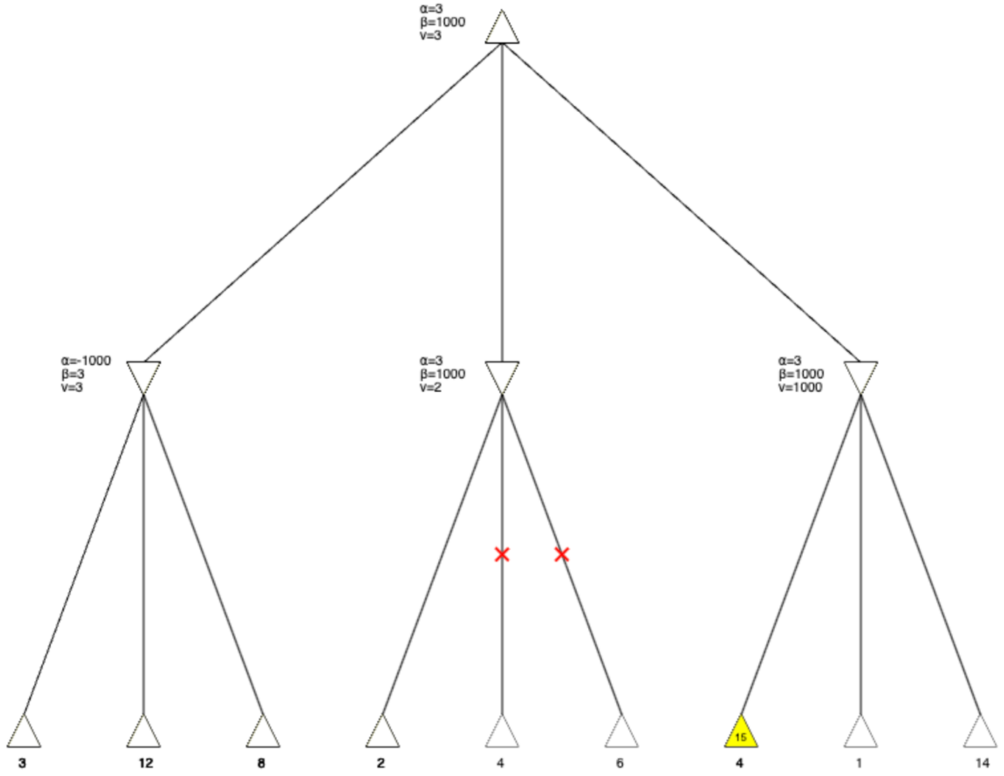
Search proceeds to the next sub-tree.

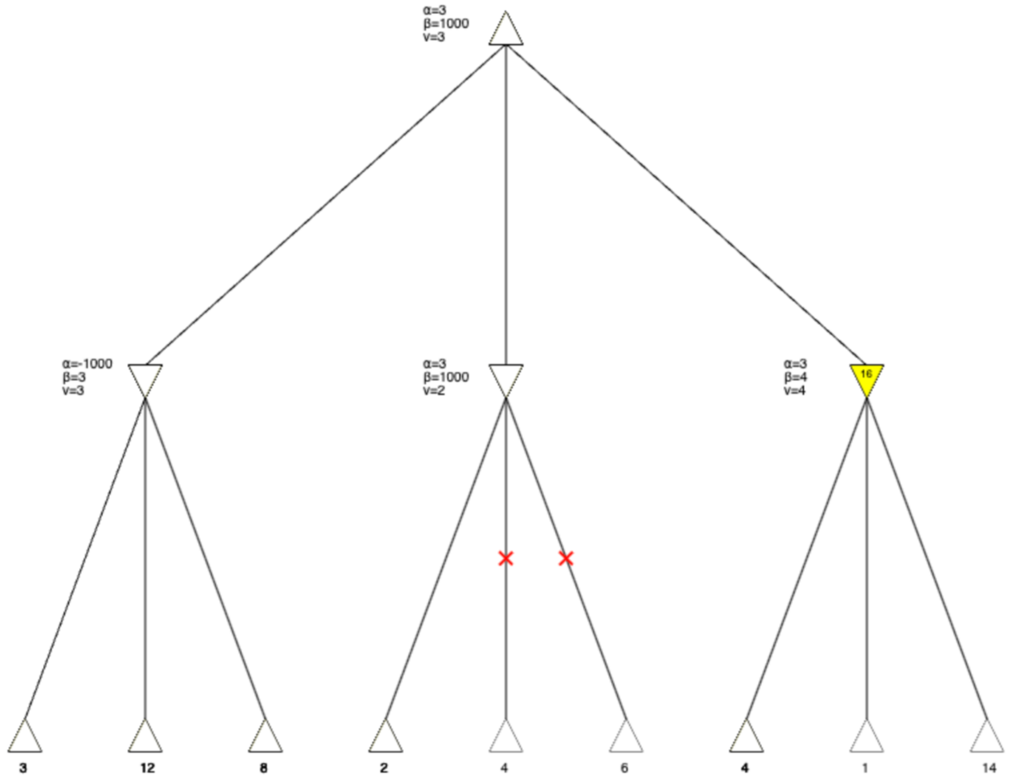
Example (5/7)



The value at the min node is smaller than alpha
(Max has a better value) -> Pruning occurs

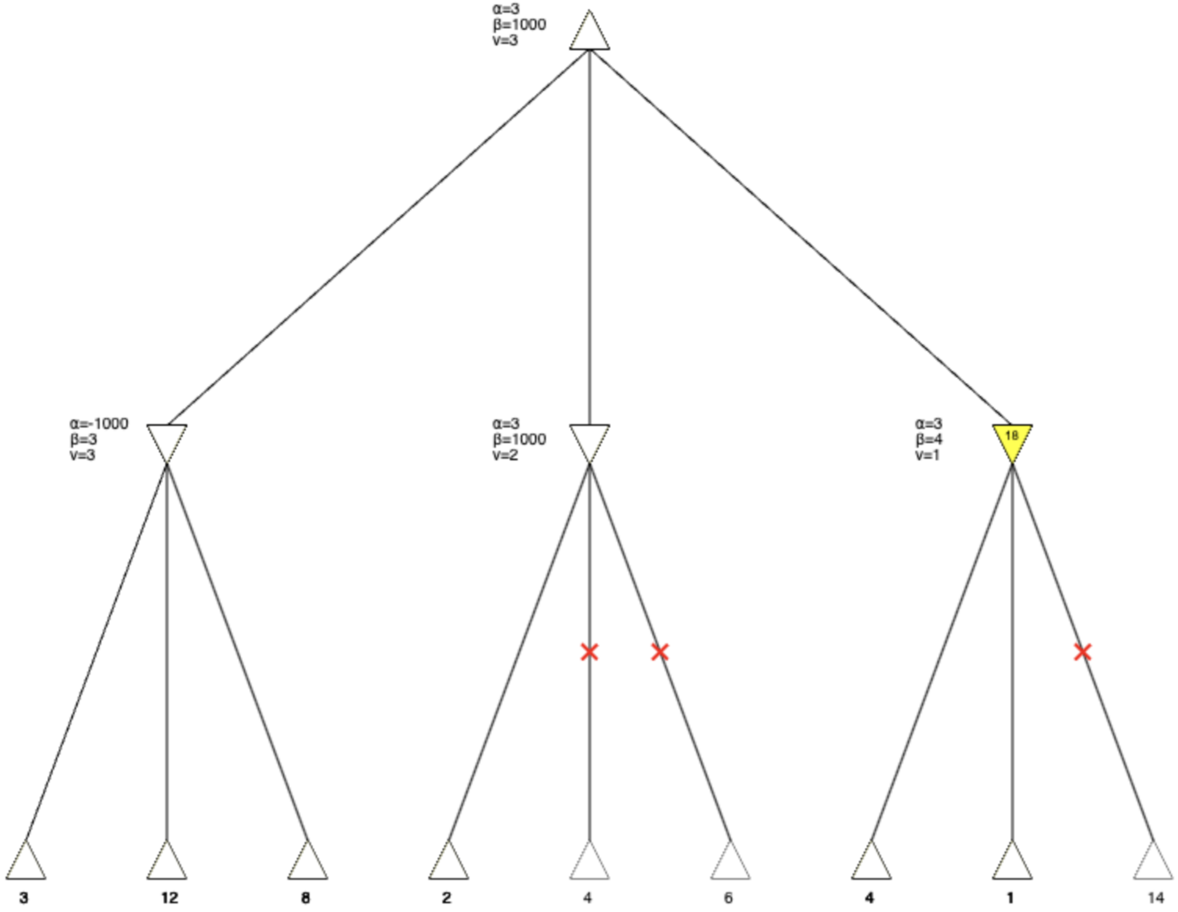
Example (6/7)

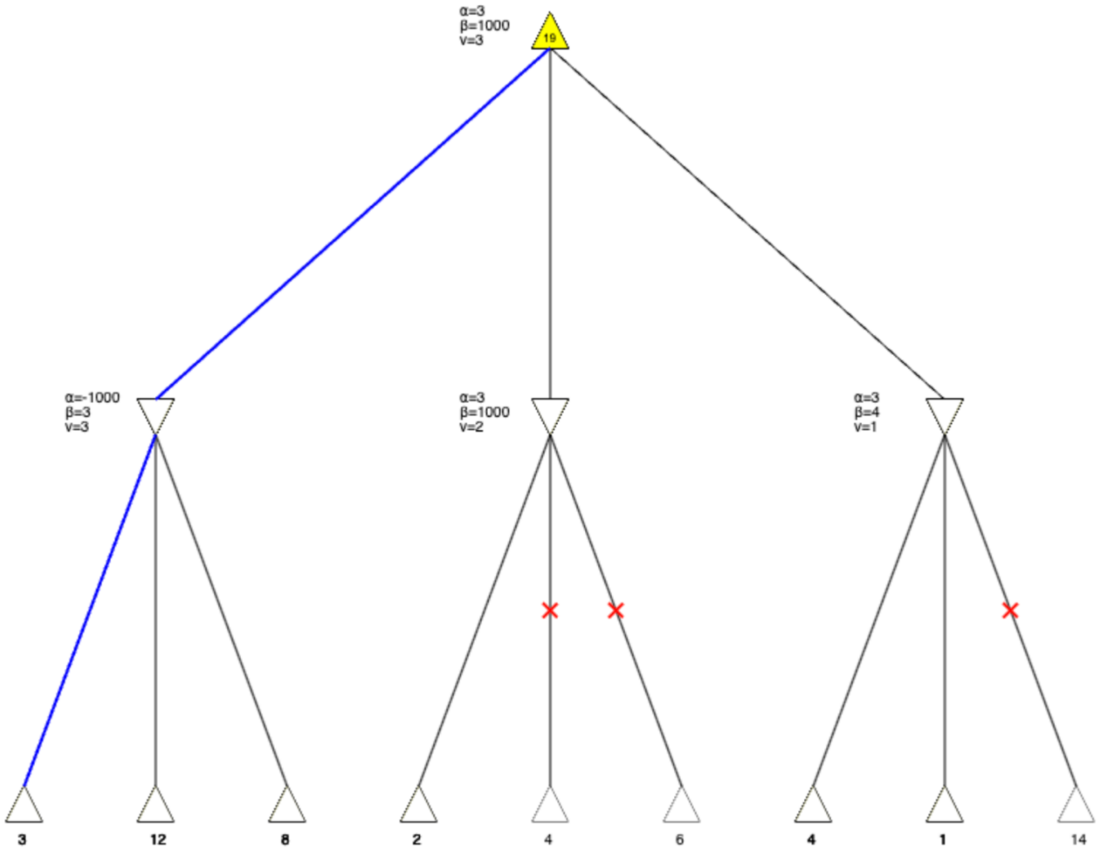




The value at MIN node is higher than the alpha value, cannot prune.

Example (7/7)





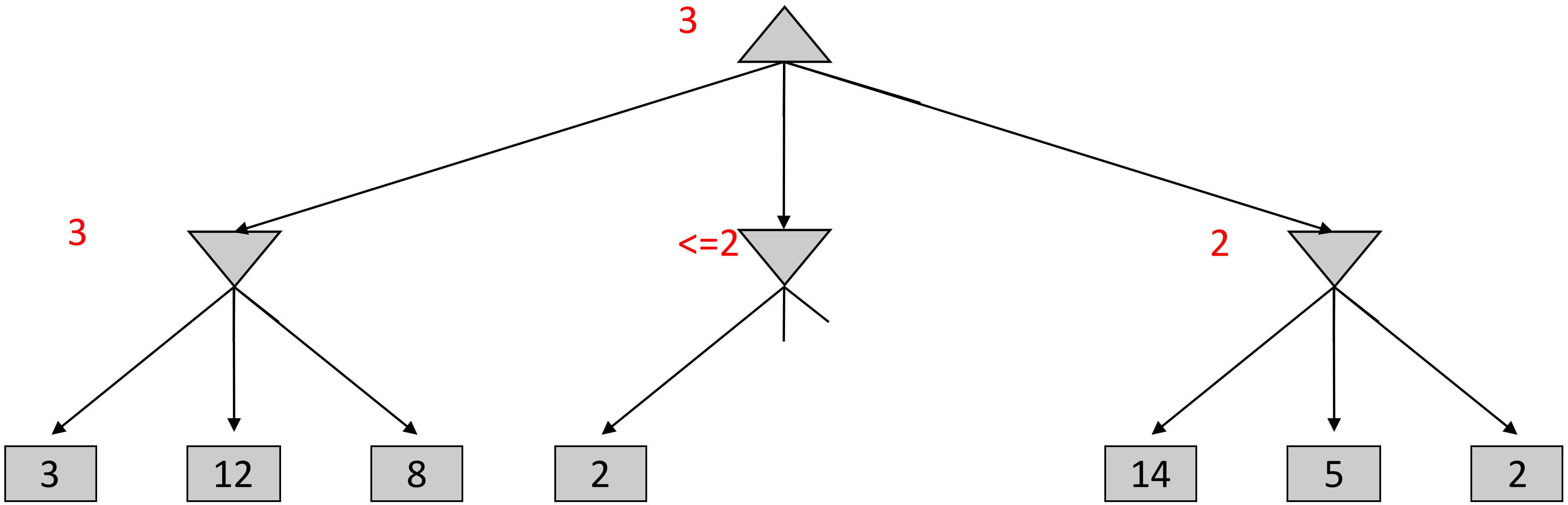
After examining the node with terminal utility of 1, the value is now lower than alpha. Allow pruning.

Game actions selected.

Alpha-Beta Pruning - Properties

1. Pruning has **no effect** on the minimax value at the root.
   - Pruning does not affect the final action selected at the root.
2. A form of **meta-reasoning** (computing what to compute)
   - Eliminates nodes that are irrelevant for the final decision.

Alpha-Beta Pruning – Order of nodes matters



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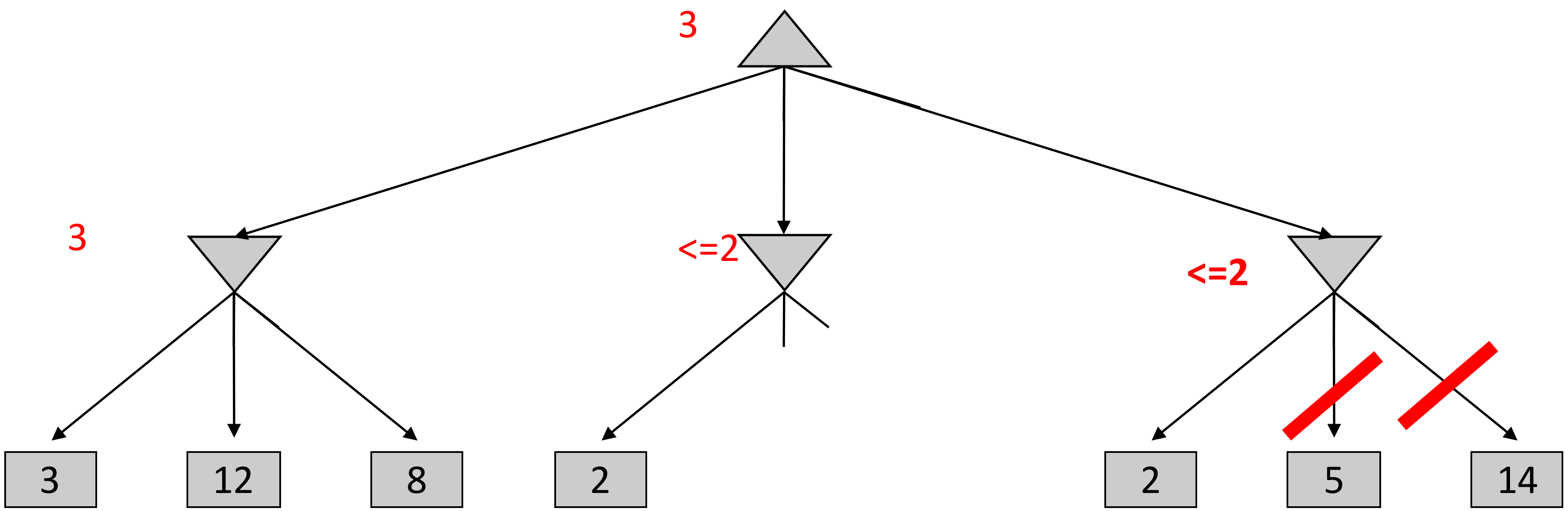

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2. A form of **meta-reasoning** (computing what to compute)
    - Eliminates nodes that are irrelevant for the final decision.
3. The alpha-beta search cuts the largest amount off the tree when we examine the **best move first**
    - However, best moves are typically **not** known. Need to make estimates.

Alpha-Beta Pruning – Order of nodes matters

```
                 4                  MAX
       +---------+---------+
       2         3         4        MIN
    +--+--+   +--+--+   +--+--+
    6  4  2   7  5  3   8  6  4     MAX
  +-+ +-+ +-+ +-+-+ +-+ +-+ +-+ +-+ +-+-+
  6 5 4 3 2 1 1 3 7 4 5 2 3 8 2 1 6 1 2 4
```

If the nodes were indeed encountered as “worst moves first” – then no pruning is possible

```
                 4                  MAX
       +---------+---------+
       4         3         2        MIN
    +--+--+   +--+--+   +--+--+
    4  6  8   3  x  x   2  x  x     MAX
  +-+ +-+ +-+ +-+       +-+-+
  4 2 6 x 8 x 3 2       1 2 1
```

If the nodes were encountered as “best moves first” – then pruning is possible

Note: In reality, we don’t know the ordering.

Slide adapted from Prof. Mausam

Alpha-Beta Pruning - Properties

1. Pruning has **no effect** on the minimax value at the root.
    - Pruning does not affect the final action selected at the root.
2. A form of **meta-reasoning** (computing what to compute)
    - Eliminates nodes that are irrelevant for the final decision.
3. The alpha-beta search cuts the largest amount off the tree when we examine the **best move first**
    - Problem: However, best moves are typically **not** known.
    - Solution: Perform iterative deepening search and evaluate the states.

Alpha-Beta Pruning - Properties

1. Time Complexity
   - **Best ordering - O($b^{m/2}$).** *Can double the search depth for the same resources*. Effective branching factor becomes b^{1/2} instead of b.
     - Why? Pruning edge evaluations, i.e., some states are not required to be evaluated.
   - On average – O($b^{3m/4}$) if we expect to find the min or max after b/2 expansions.
2. Intuition – why branching factor is effectively reduced?
   - Standard game tree size: O(b x b x b, …. b), that is, m-times b is multiplied, hence O(b^m)
   - Best case alpha-beta pruning: O(b x 1 x b x 1, b x 1 … …. 1), m/2 times b is multiplied, hence O($b^{(m/2)}$)
     - All the first player's moves must be studied to find the best one.
     - But for each move, only the second player's best move is needed to refute all but the first (and best) first player move—no other second player moves are considered.
3. Implication
   - For the same resources overall to store the tree, one can go deeper into the search.

Minimax for Chess

- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}

Alpha-Beta for Chess

- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^{m/2} \approx 35^{50} \approx 10^{77}$

Slide adapted from Prof. Mausam

In 1997, an IBM computer beat a chess world champion for the first time

"He can't believe it," were the words commentators had for a speechless Garry Kasparov, a world chess champion, after he lost to IBM's computer named Deep Blue. Watch how CNN covered the historic event 25 years ago.

02:18 - Source: CNN

Deep Blue

Gary Kasparov

IBM's computer checkmated a human chess champion in a computing tour de force

- Overview
- Building a digital chess master
- Challenging Garry Kasparov
- Deep Blue's legacy

In 1997, IBM's Deep Blue did something that no machine had done before. In May of that year, it became the first computer system to defeat a reigning world chess champion in a match under standard tournament controls. Big Blue's victory in the six-game marathon against Garry Kasparov marked an inflection point in computing, heralding a future in which supercomputers and artificial intelligence could simulate human thinking.

Deep Blue derived its chess prowess through brute force computing power. It used 32 processors to perform a set of coordinated, high-speed computations in parallel. Deep Blue was able to evaluate 200 million chess positions per second, achieving a processing speed of 11.38 billion floating-point operations per second, or flops. By comparison, IBM's first supercomputer, Stretch, introduced in 1961, had a processing speed of less than 500 flops.

Deep Blue wasn't just a breakthrough for the world of chess. Its underlying technology advanced the ability of supercomputers to tackle the complex calculations needed to discover new pharmaceuticals, assess financial risk, uncover patterns in massive databases, and explore the inner workings of human genes.

Deep Blue computing power

32
processors

200 million
chess positions evaluated per second

11.38 billion
floating point operations per second (flops) of processing speed

Links:

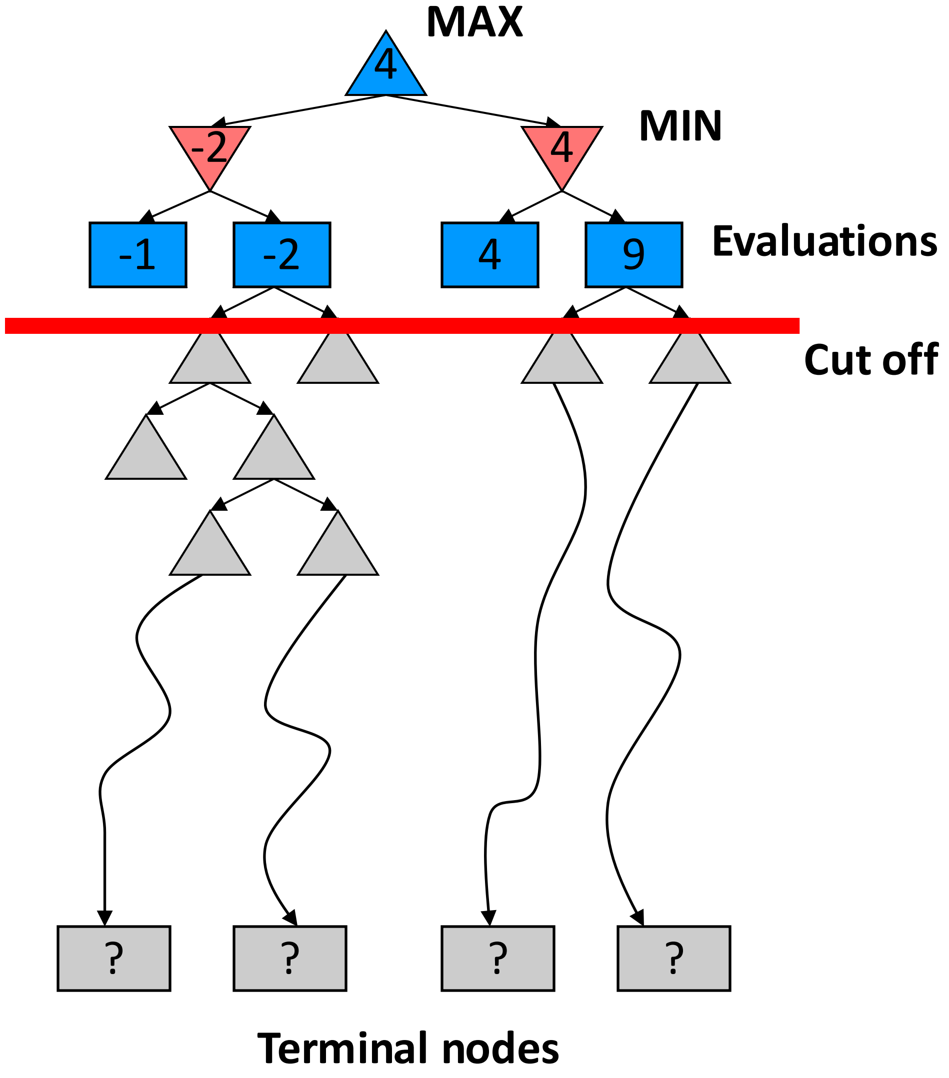
https://www.youtube.com/watch?v=NJarxpYyoFI

https://www.ibm.com/history/deep-blue

Cutting-off Search

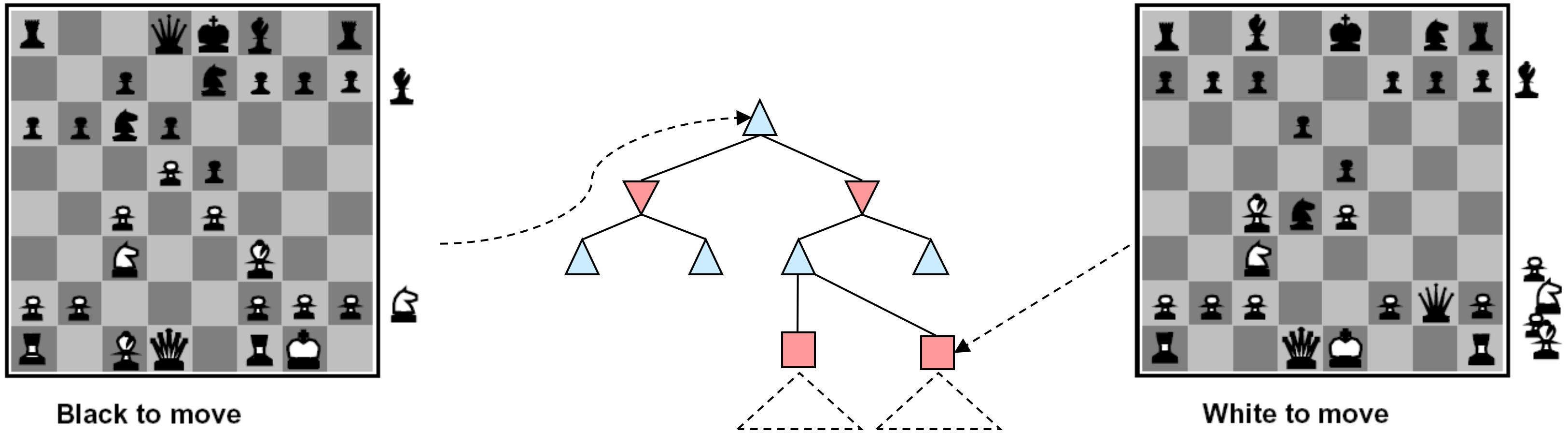
- Problem (Resource constraint):
  - Minimax search: full tree till the terminal nodes.
  - Alpha-beta prunes the tree but still searches till the terminal nodes.
    - *Even with alpha-beta pruning still we may not be able to search till the terminal nodes.*
- Solution:
  - Depth-limited Search (H-Minimax)
  - Search only to a limited depth (cutoff) in the tree
  - **Replace the terminal utilities with an evaluation function for non-terminal positions.**

$$\text{H-MINIMAX}(s,d) = \begin{cases} \boxed{\text{EVAL}(s) \quad \text{if CUTOFF-TEST}(s,d)} \\ \max_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MIN}. \end{cases}$$



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search.
- Estimate the chances of winning.



Black to move

White slightly better

White to move

Black winning

- Ideal function: returns the actual **minimax** value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

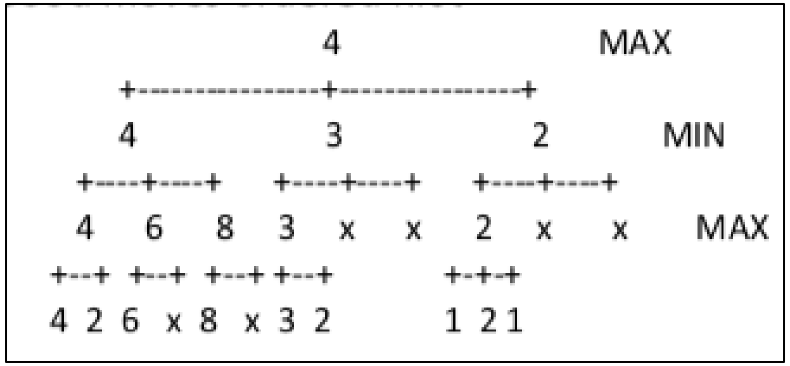
- e.g. $f_i(s)$ = (number of pieces of type i), each weight w_i etc.

Evaluation Functions

- Evaluation functions take a state and output an estimate of the true minimax value of that node.
  - Typically, “better” states will be assigned higher values by a good evaluation function in comparison to ”worse” states. Evaluation functions serve a similar purpose as heuristics in classical search.
- Depth-limited search applies evaluation function at the maximum solvable depth
  - Gives them mock terminal utilities by the evaluation function.
- Evaluation functions require features (some aspect of the current state).
  - Functions may or may not be linear. Require considerable thought and experimentation for designing.
- The better the evaluation function is, the closer the agent will come to behaving optimally.
  - Going deeper into the tree before using the evaluation function also tends to give better results. Reduces the compromise of optimality.

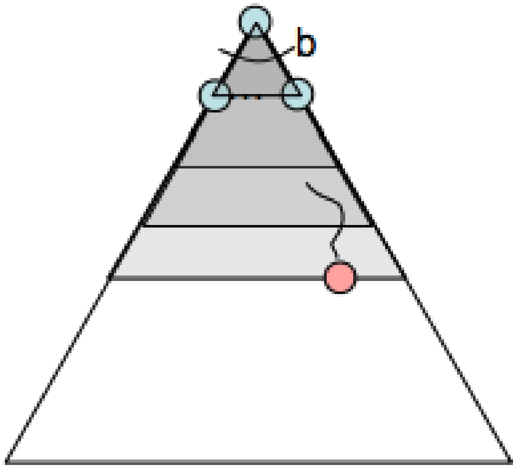
Determining "good" node orderings

- The ordering of nodes helps alpha-beta pruning.
  - Worst ordering $O(b^m)$. Best ordering $O(b^{m/2})$.

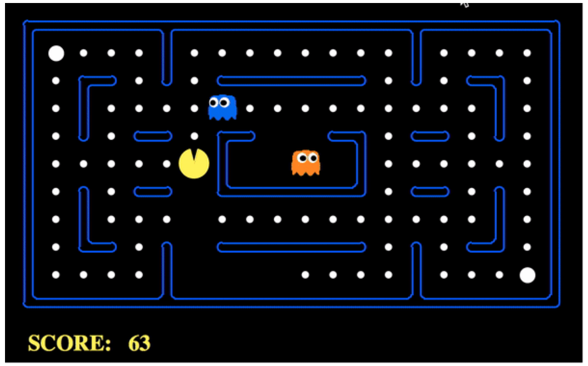


- How to find good orderings
  - Problem: we only know them when we evaluate the nodes.

- One approach – iterative deepening to determine evaluations for nodes
  - What if we can do iterative deepening to a certain depth. Use the evaluation function at the set depth and then compute the values for the nodes in the tree that is generated.
  - Next time, use the evaluations of the previous search to order the nodes. Use them for pruning.
  - Use evaluations of the previous search for order.



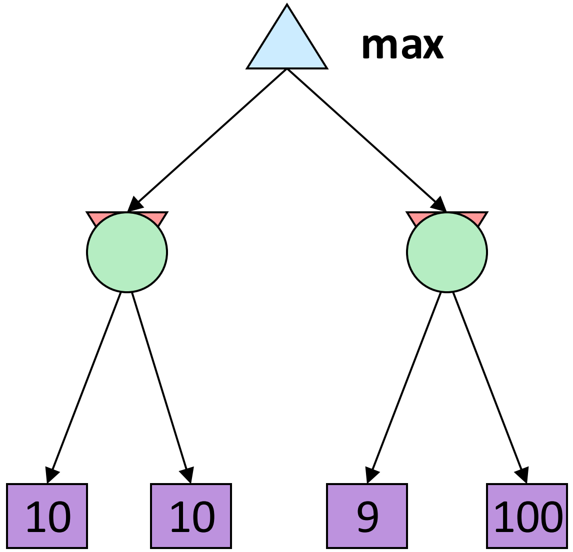
Game of Chance: Expectimax



- When the result of an action is not exactly known. Need a notion of uncertainty or chance in action selection.

- Explicit randomness in the opponent's action selection
  - Unpredictable opponents: the ghosts move randomly in Pacman.
  - Rolling dice by a player in a game.

- Pessimistic assumption is not valid for the adversary
  - The adversary may not be that bad. May not provide the worst value. Optimal response may not be guaranteed.



Expectimax:

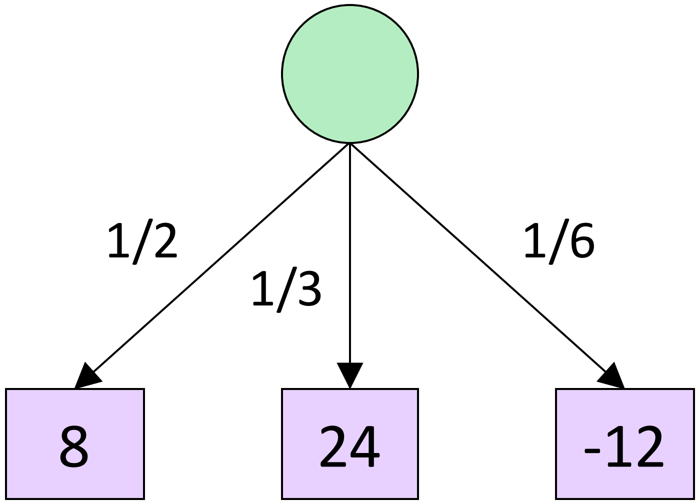
At chance nodes the outcome is uncertain. Calculate the ***expected utilities:*** weighted average (expectation) of children

Expectimax Search

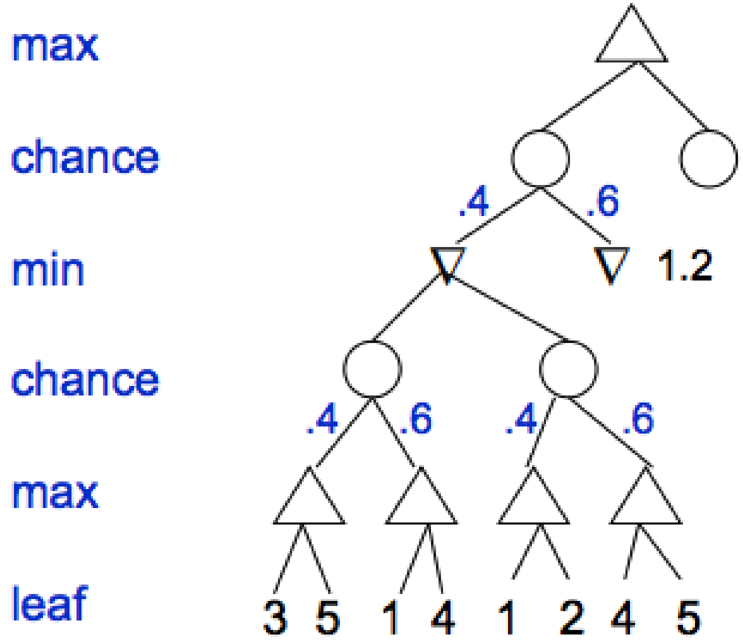
$$\forall \text{ agent-controlled states, } V(s) = \max_{s' \in successors(s)} V(s')$$

$$\forall \text{ chance states, } V(s) = \sum_{s' \in successors(s)} p(s'|s)V(s')$$

$$\forall \text{ terminal states, } V(s) = \text{ known}$$

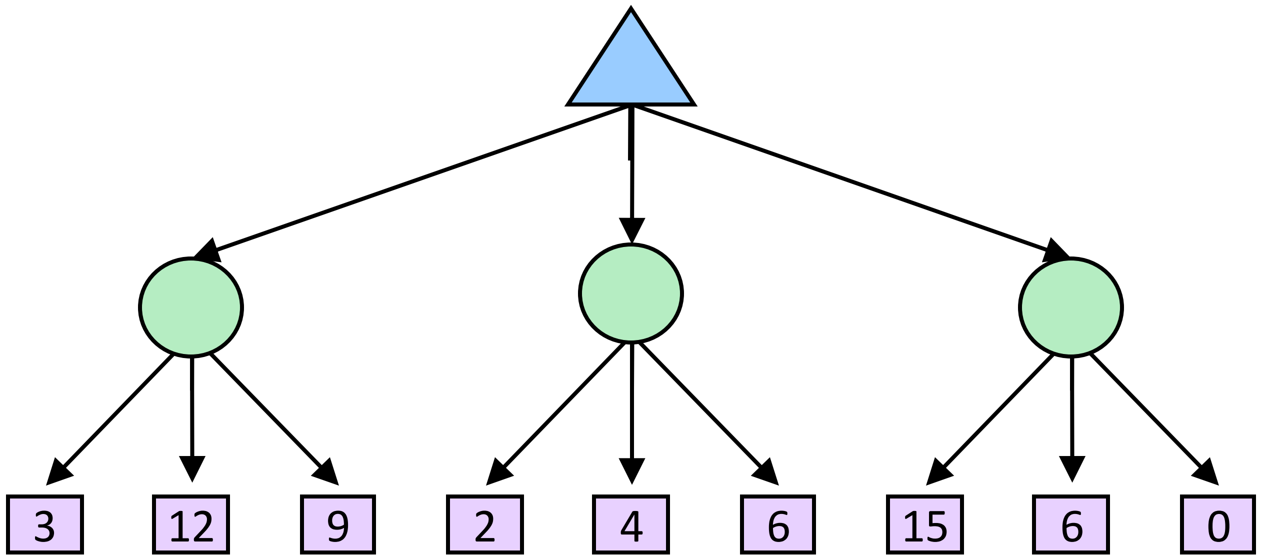


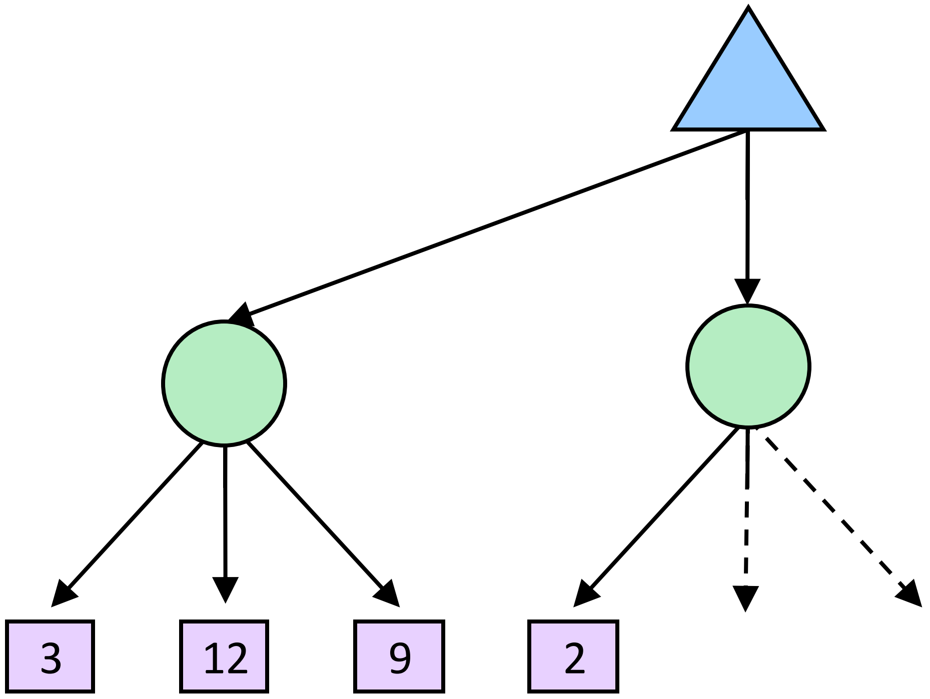
v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10



Mixed-type layers in a game tree are also possible. More than two agents.

Expectimax Search





Can we perform pruning?

Expectimax Search

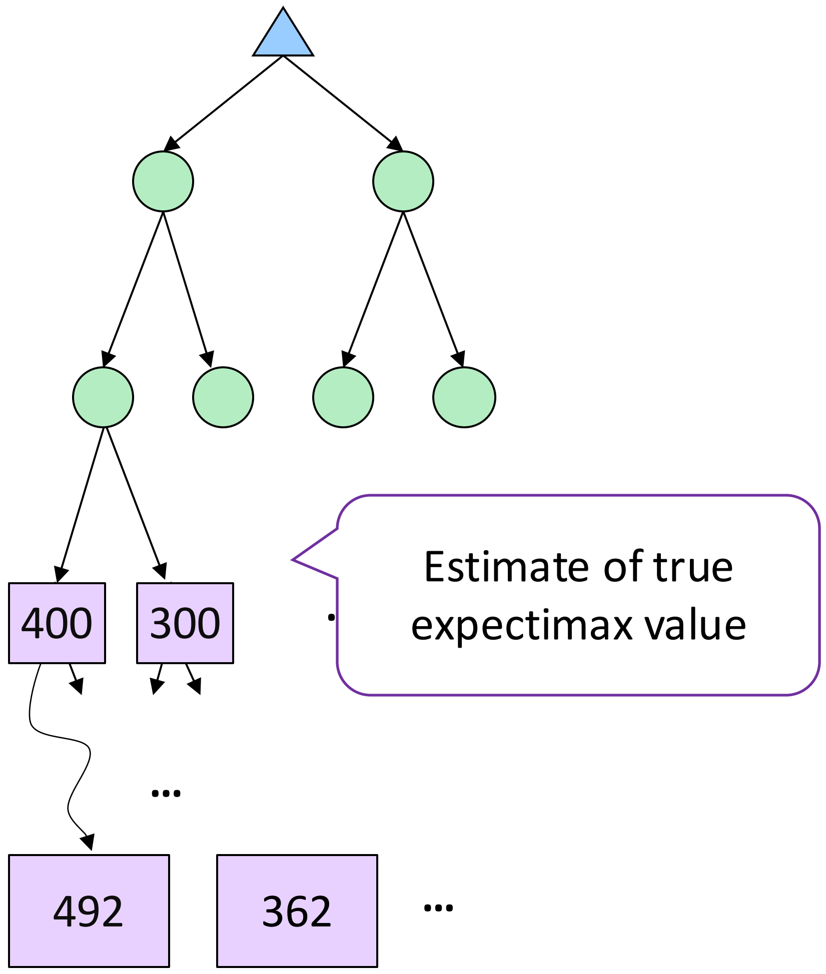
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

Depth-Limited Expectimax

- Depth-limit can also be applied in Expectimax search.
- Use heuristics to estimate the values at the depth limit.



Example: Game of Go

- The game of Go originated in China more than 2000 years ago.
- Usually played on 19x19, also 13x13 or 9x9 board
- Black and white place down stones alternately.
- Surrounded stones are captured and removed.
- The player with more territory wins the game.
- Complex strategy for capturing and creating a territory.
- Grand challenge in AI game playing because of its complexity.

	Chess	Go
Size of board	8 x 8	19 x 19
Average no. of moves per game	100	300
Avg branching factor per turn	35	235
Additional complexity		Players can pass

Example: Game of Go

Challenges posed by Game of Go

- **Large branching factor (larger than Chess)**
  - Not easy to evaluate all the action outcomes.
  - Alpha-beta pruning/minimax does not scale.
- **Design of a heuristic function is difficult**
  - Most positions are in a flux till the end game. Value not a strong indicator of winning.
  - Essentially, need to play the game till the end to determine the utility of a state.

Monte-Carlo Tree Search – Core Idea

- Select which moves (nodes) to evaluate for what effect they have on winning the game.
- Use simulations to determine the value of state when it is a new state.
- Remember and update the values.
- "Monte-carlo" – the name indicates sampling.

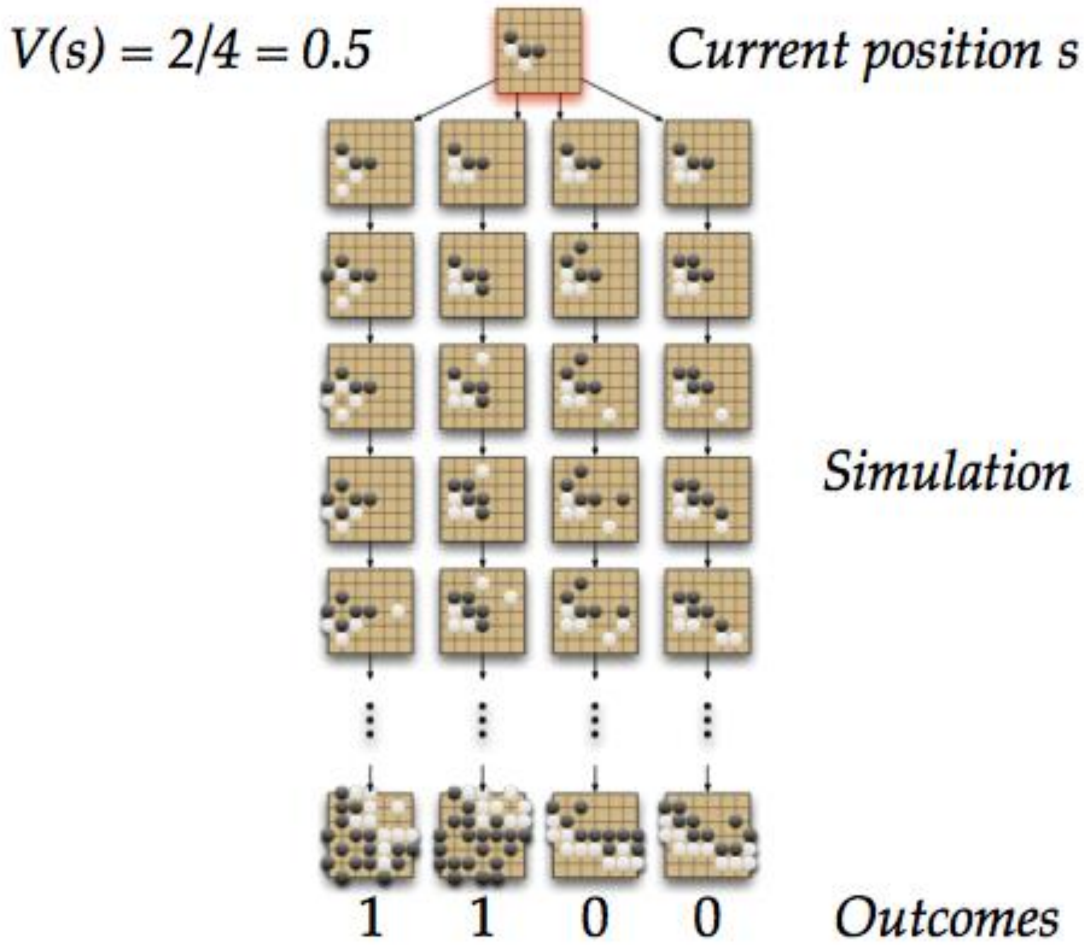
Popularized by Alpha Go
https://www.deepmind.com/research/highlighted-research/alphago

Monte Carlo Tree Search (MCTS)

1. Simulations/Rollouts

- Evaluation of a state V(s) using roll outs or simulating what will happen from this state on wards.
  - From state s play many times using a policy (e.g., random) and count wins and losses.

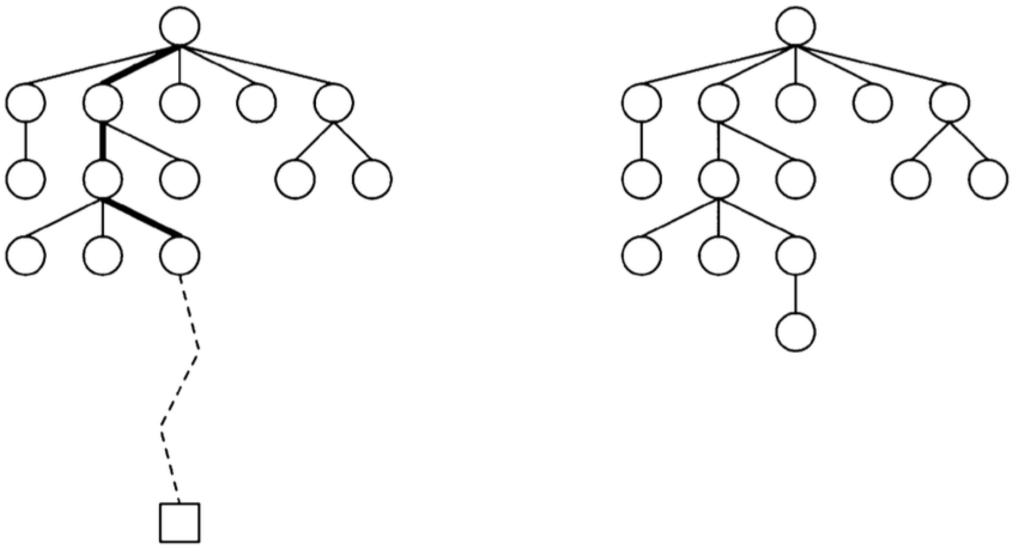
- For games in which the only outcomes are a win or a loss,
  - The “win percentage” approximates the “average utility”.

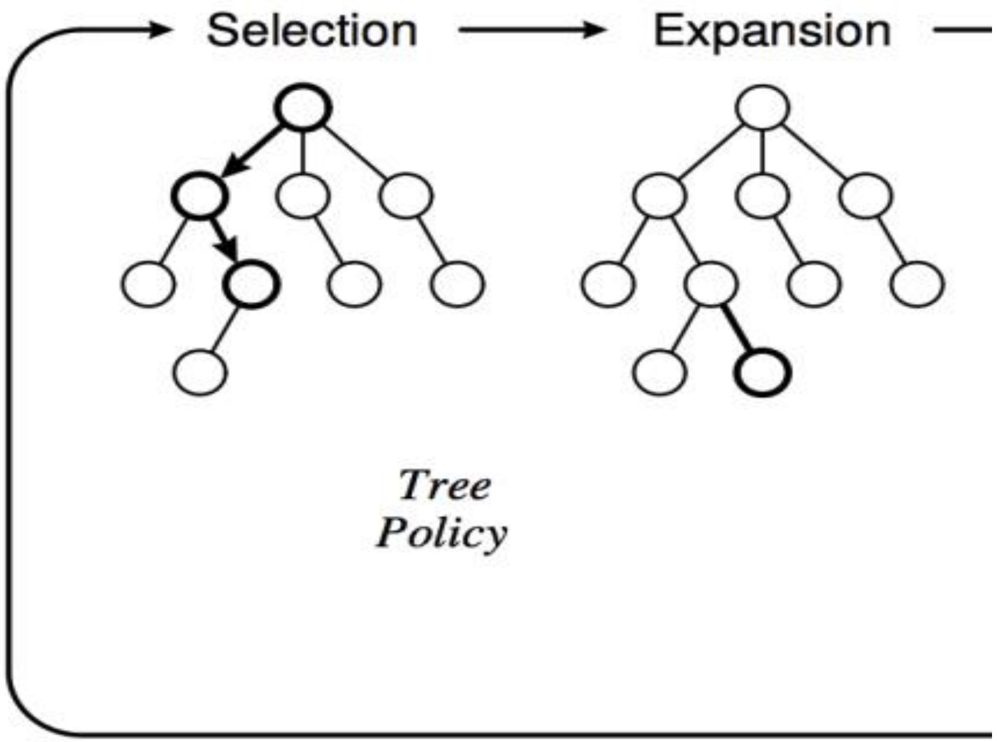


Monte Carlo Tree Search (MCTS)

2. Selective Search

- May not evaluate all states.
    - Be selective with evaluations on more promising actions/states.
- Explore parts of the tree (without an explicit depth for exploration) that will
    - Improve the decision at the root (improve the estimation of the value function)
    - Grow the tree of states as needed to improve the value estimates of a state.



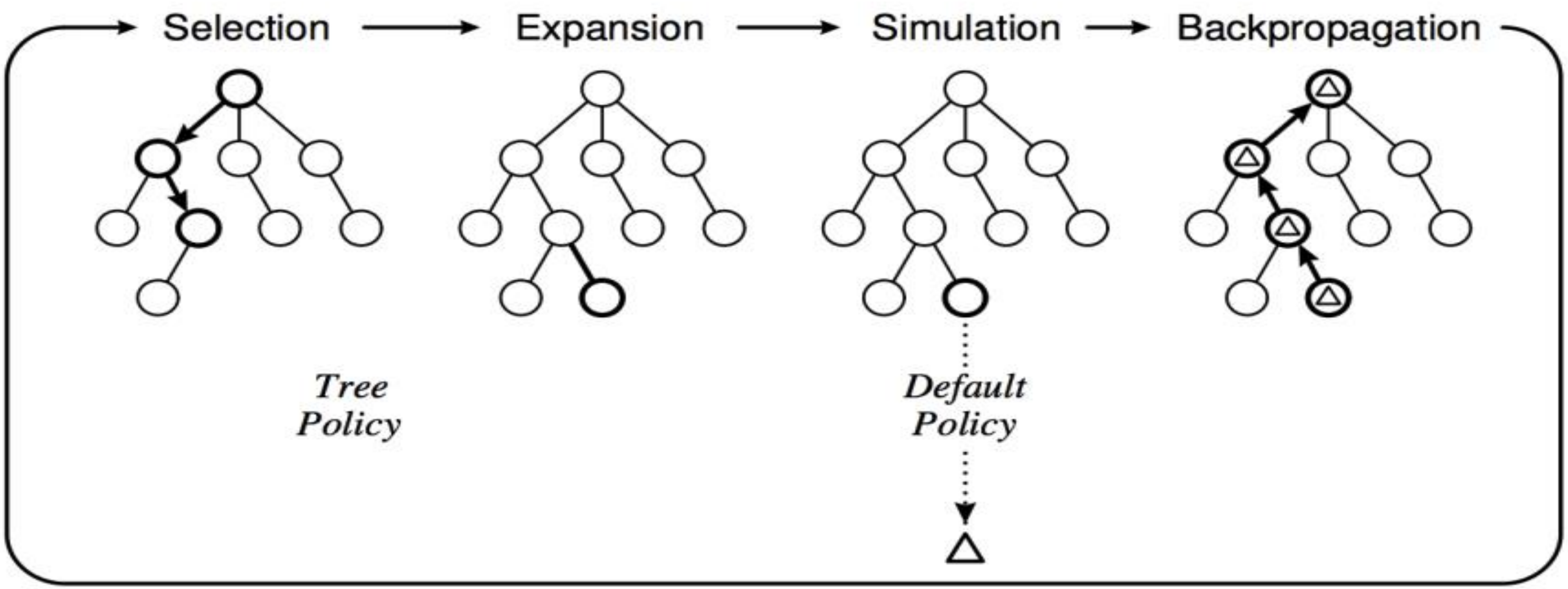


Selection

- Start from the root and select a move (via a selection/tree policy).
- Used for nodes we *have* seen before

Expansion

- When we reach the frontier, grow the search tree by generating a new child node of the node selected from the frontier.



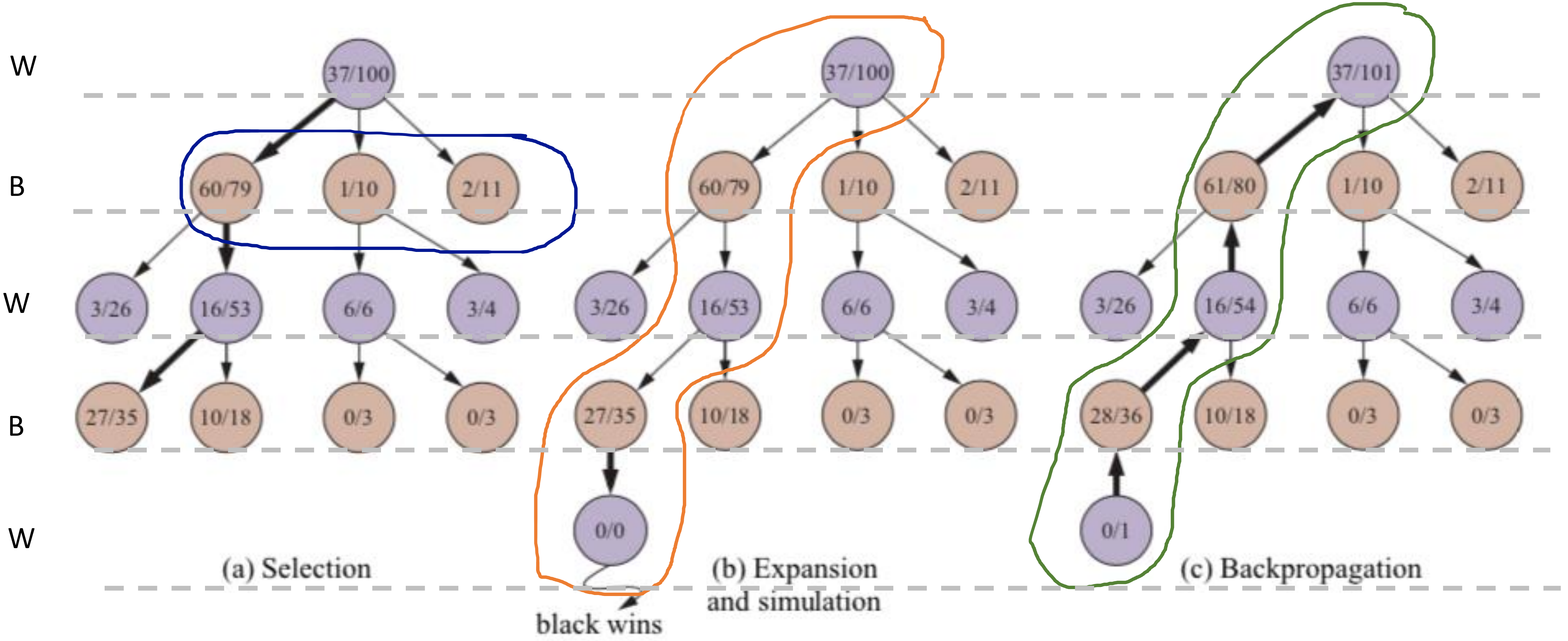
Selection
- Start from the root and select a move (via a selection/tree policy).
- Used for nodes we *have* seen before

Expansion
- When we reach the frontier, grow the search tree by generating a new child node of the node selected from the frontier.

Simulation
- Perform playout from the newly generated child node.
- Select moves for both players according to a playout policy (also called default policy) such as random action selection.
- Do not record the nodes in the tree.

Backpropagation
- After reaching a terminal node
- Update value and visits for states expanded in selection and expansion

MCTS Example



MCTS Procedure- Meta Routine

function MONTE-CARLO-TREE-SEARCH(*state*) **returns** *an action*
  tree ← NODE(*state*)
  while IS-TIME-REMAINING() **do**
    leaf ← SELECT(*tree*)
    child ← EXPAND(*leaf*)
    result ← SIMULATE(*child*)
    BACK-PROPAGATE(*result*, *child*)
  return the move in ACTIONS(*state*) whose node has highest number of playouts

Monte-Carlo Tree Search

```
function UCB_sample(node):
   weights = []
   for child of node:
      w = child.value
      w += C*sqrt(ln(node.visits) / child.visits)
      add w to weights
   distribution = normalize weights to sum to 1
   return child sampled according to distribution
```

Monte-Carlo Tree Search

```
function MCTS_sample(node)
    if all children expanded:#selection
        next = UCB_sample(node)
        outcome = MCTS_sample(next)
    else:#expansion
        next = random unexpanded child
        create node for next, add to tree
        #simulation
        outcome = random_playout(next.state)
    #backpropagation
    update_value(node, outcome)
```

For every state within the search tree we bookkeep # of visits and # of wins

Monte-Carlo Tree Search (helper functions)

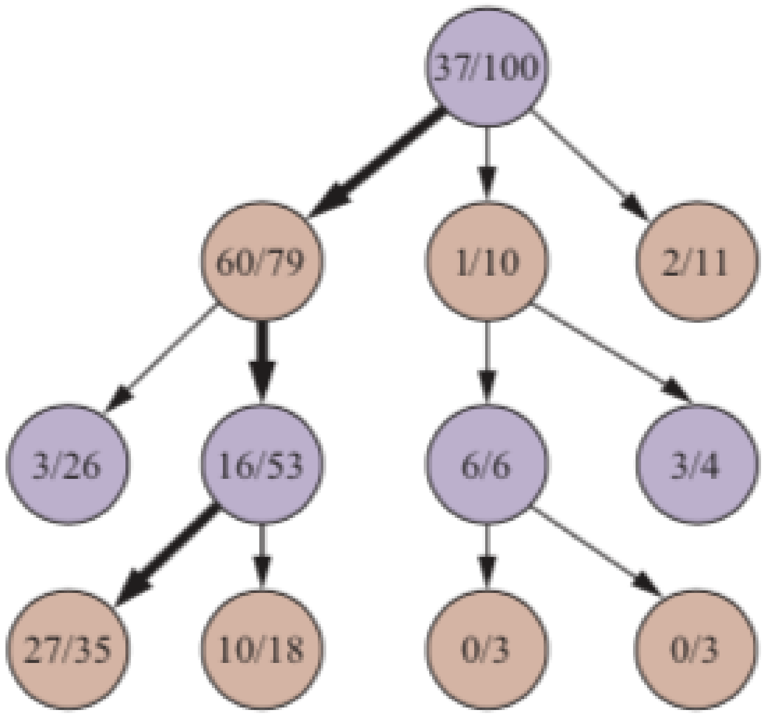
```
function random_playout(state):
    while state is not terminal:
        state = make a random move from state
    return outcome


function update_value(node, outcome):
    #combine the new outcome with the average value
    node.value *= node.visits
    node.visits++
    node.value += outcome
    node.value /= node.visits
```

Exploration vs. Exploitation

Selection Strategy

- How to select moves/actions in the tree?
- Bias the moves towards those providing higher value.
- But we may not know about the value of certain states or may be very uncertain about them. Hence, sometimes we should explore too.
- Fundamental trade-off between exploration and exploitation.



How to select the moves balancing exploration and exploitation.

Upper Confidence Bound applied to Trees

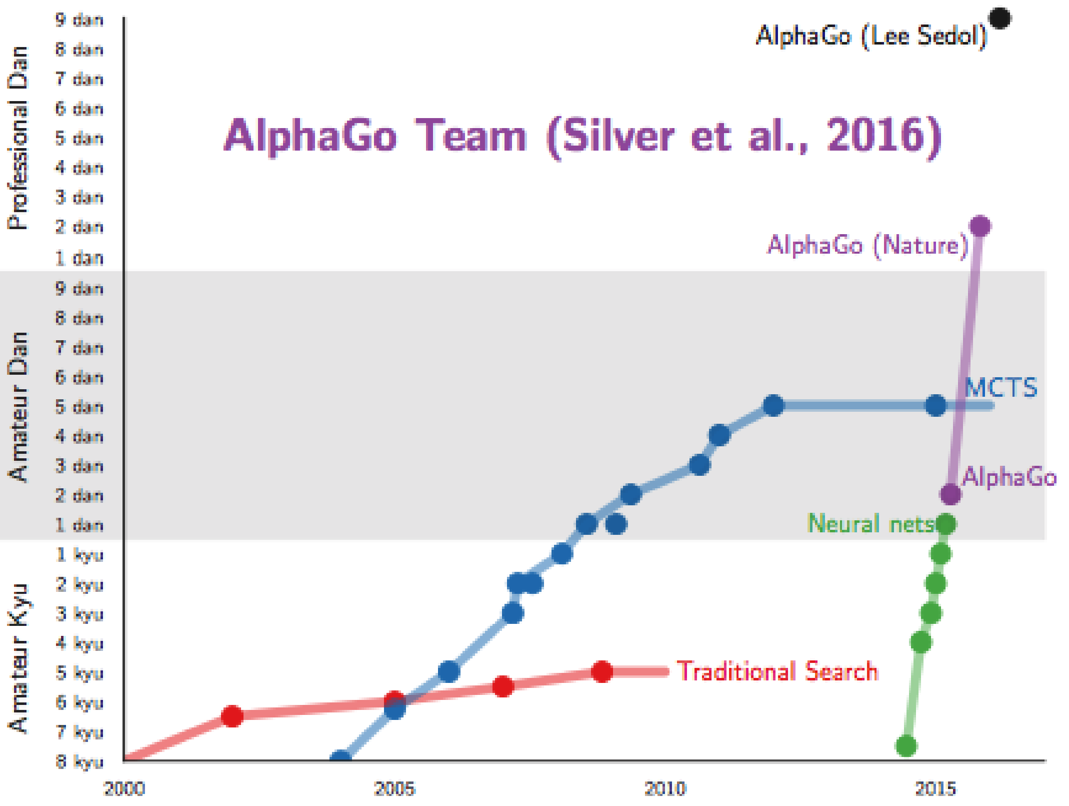
$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\textsc{Parent}(n))}{N(n)}}$$

- $N(n)$ = number of rollouts from node n
- $U(n)$ = total utility of rollouts (e.g., # wins) for Player(Parent(n))
- C is the tunable parameter.

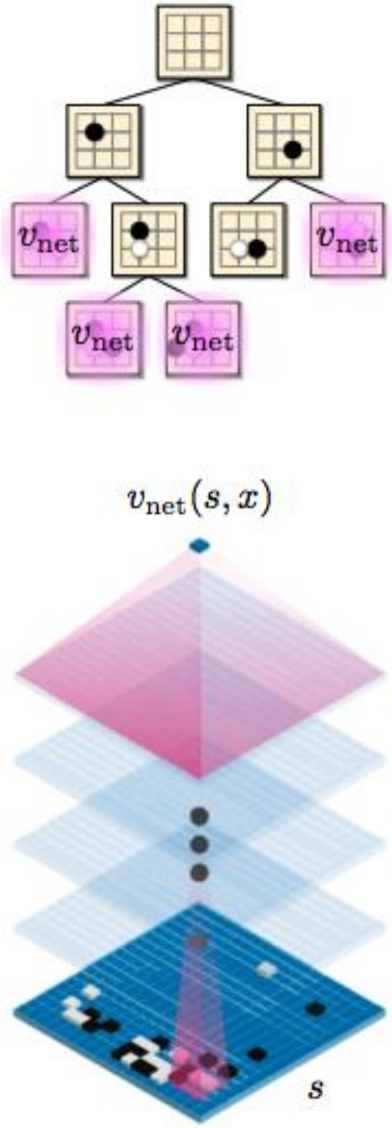
- The first term is the exploitation term: the average utility of node n.
- The second term is the exploration term: how uncertain we are about the node's utility.
  - The denominator is the number of visits to the states, so states visited less often are preferred.
  - The numerator is the log of the number of times the parent is explored.
  - If we are selecting n for some non-zero percentage of times then the exploration term goes to zero as the counts increase.
- We will revisit this concept in the discussion on Reinforcement Learning later.

MCTS Popularized by Alpha Go

https://www.deepmind.com/research/highlighted-research/alphago



adapted from Sylvain Gelly & David Silver, Test of Time Award ICML 2017



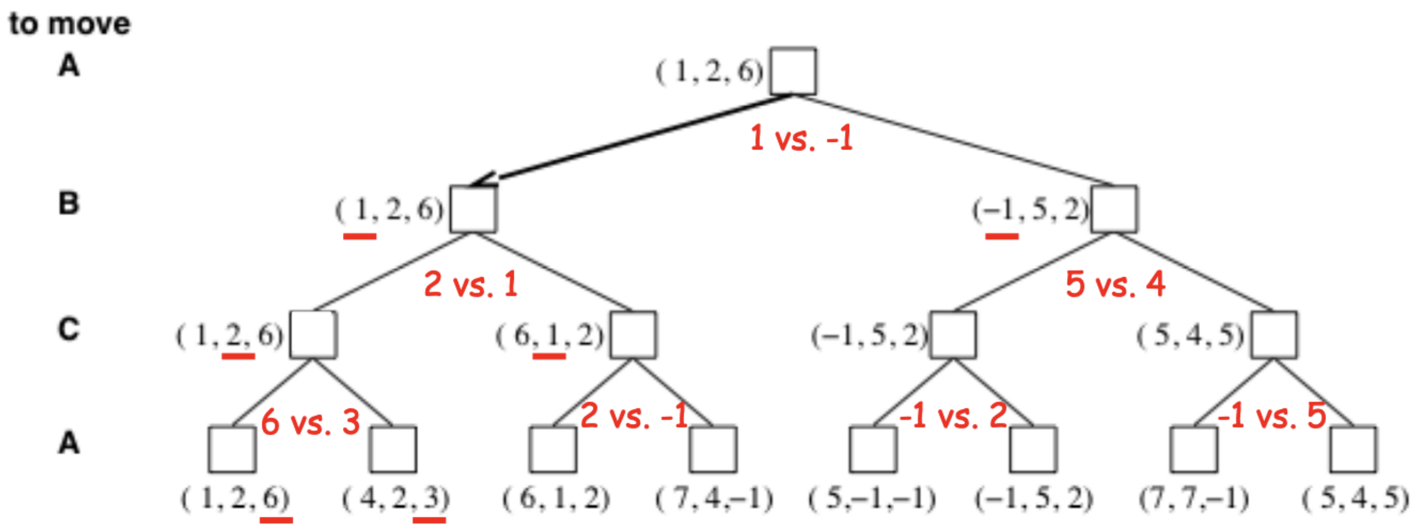
(Silver et al., 2016)

Alpha Go combined learning with MCTS (used a neural network (which we will discuss later) to predict values/utilities of states). Employed self play etc.

Multiple players and other games

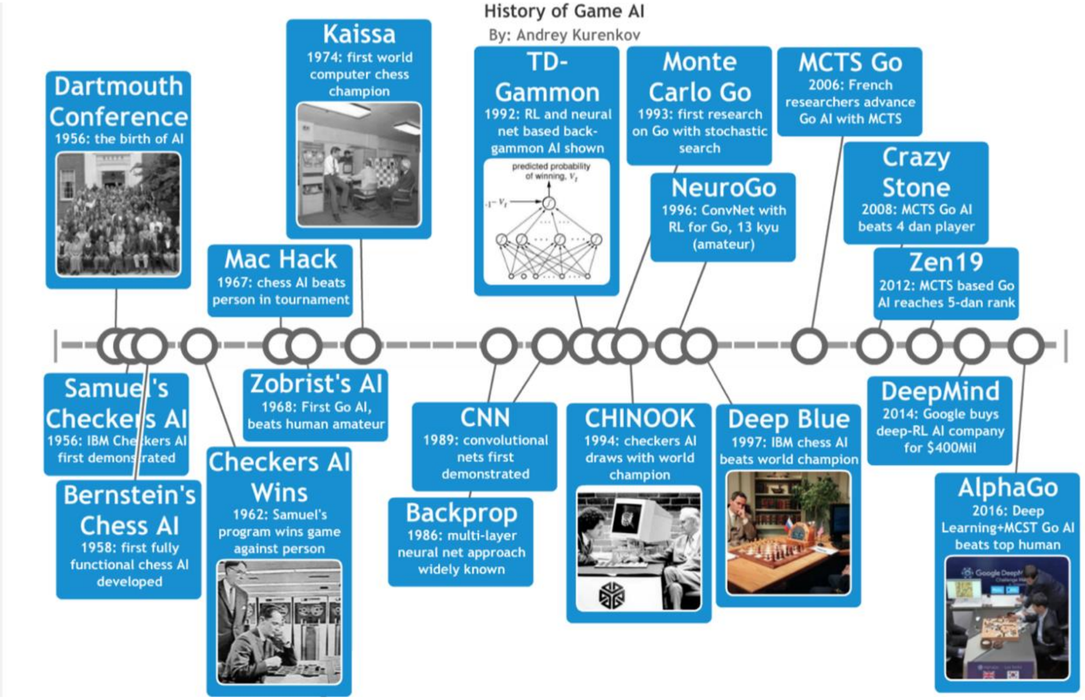
- Not all games are zero sum.
  - Loss for one agent may not be win for the other agent.
  - Different agents may have different tasks in the game that don't directly involve strictly competing against each other.

- Multi-agent utilities.
  - Generalization of minimax.
  - Each player maximizes its own utility at each node they control and ignore the utilities of the other agents.
- General gams with multi-agent utilities
  - Can invoke cooperation
  - The utility selected at the root tends to yield a reasonable utility for all participating agents.



Game Playing AI: Wrap up

- Game playing domains
  - Very large amount of contingency reasoning.
- Exact decision making is nearly impossible.
  - Approximate evaluation functions etc.
  - Force efficient use of computation (alpha-beta pruning.)
- An important test bed for AI algorithms.
  - We play games intuitively, used to reasoning.
  - Easy to compare human and computer performance.
- Game playing has produced important research ideas
  - Reinforcement learning (checkers)
  - Iterative deepening (chess)
  - Monte Carlo tree search (chess, Go)
  - Related ideas of game theory, cooperation etc. Also studied in economics.



"Games are to AI as grand prix is to automobile design"
Games viewed as an indicator of intelligence.