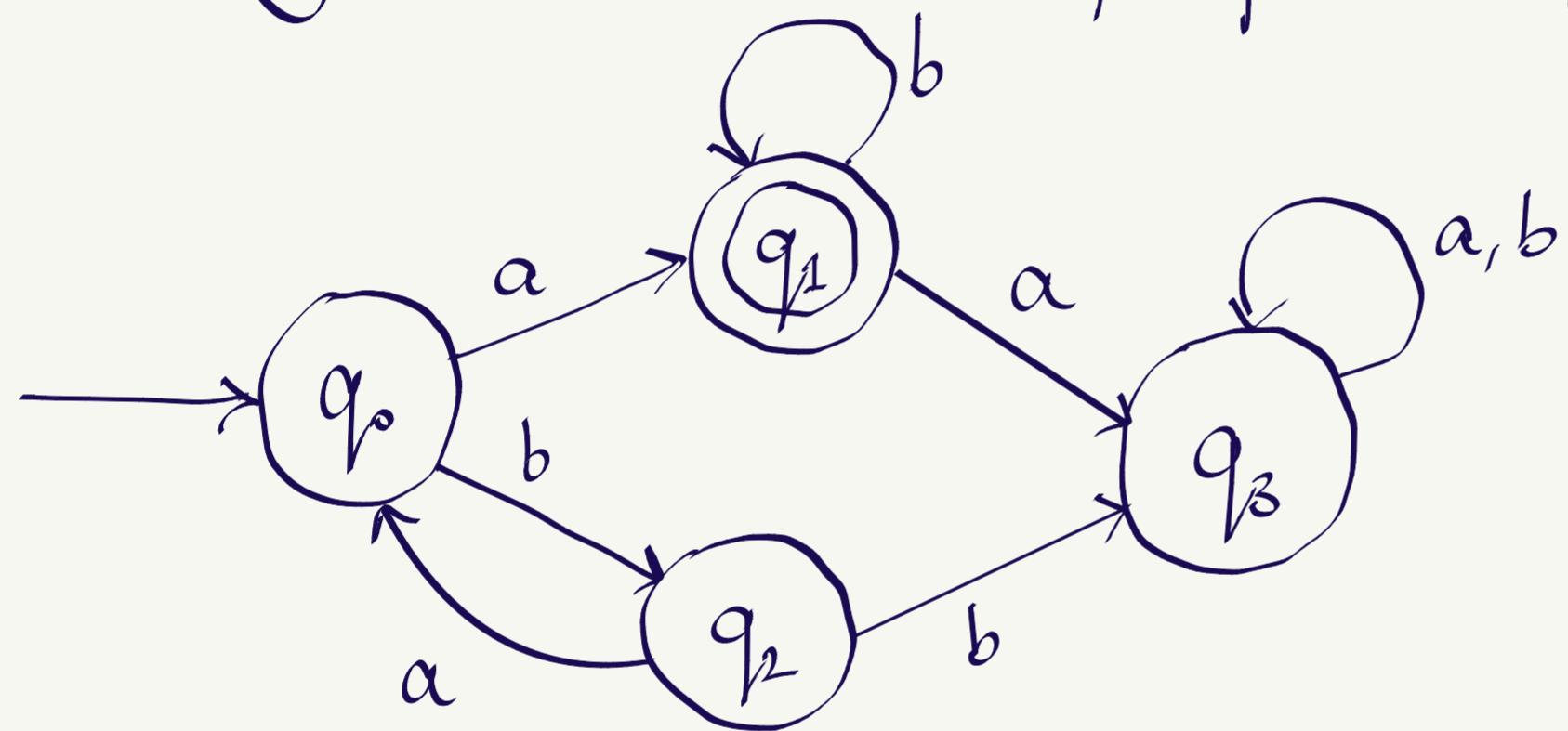


MACHINES As INPUT

Recall: Saw some equivalent models of TMs

Today: TMs can accept other machines as input; how do we use this?

We already saw some example problems for which we described algorithms



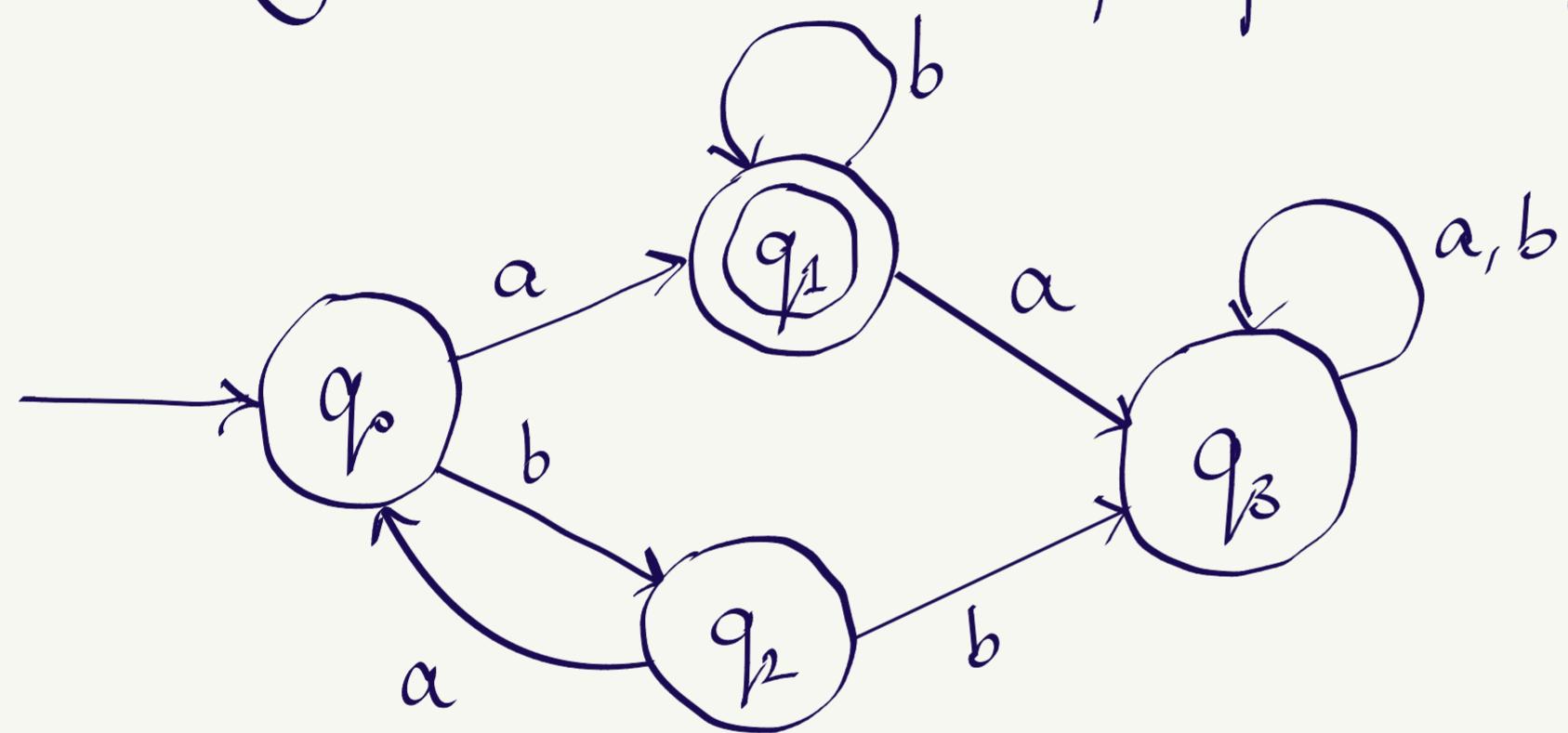
Does this machine accept
abbaaabbb?

What is an algorithm to check if this machine accepts this string?

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Does this machine accept
abbaaabbb?

What is an algorithm to check if this machine accepts this string?

What about this algorithm is specific to this machine or this string?

Essentially, the algorithm decides the following language:

$$L_{DFA} = \{ D \# \omega \mid D \text{ is a DFA which accepts } \omega \}$$

D has to be of the form $(Q, \Sigma, \delta, q_p, F)$
 ω is a string over Σ

Suppose we give $D \# \omega$
as input to a Turing Machine M

- Check if D is of the form $(Q, \Sigma, \delta, q_p, F)$
- Run D on ω → How do we implement this?
Use the tape to keep track of state and symbol currently being read
- See if D ends up in an accepting state

Is $\bar{L}_{DFA} = \{ D \# \omega \mid D \text{ is a DFA which does not accept } \omega \}$ decidable? [In general, complements of decidable languages...]

$L_{NFA} = \{ N \# \omega \mid N \text{ is an NFA which accepts } \omega \}$

→ Check if N is of the form $(Q, \Sigma, \Delta, Q_0, F)$

→ Run N on ω

→ See if N ends up in an accepting state

Another option is to use the earlier M .

→ Convert N to its equivalent DFA D_N

→ Run M on D_N  Incorporate the functionality of M

→ Accept if M accepts

Is the following language decidable?

$$L_{CFG} = \{ G \# \omega \mid G \text{ is a CFG which generates } \omega \}$$

What about the following?

$$L_{TM} = \{ M \# \omega \mid M \text{ is a Turing Machine which accepts } \omega \}$$

Whether one has an algorithm for deciding this language or not, one needs a Turing Machine which can simulate an arbitrary Turing Machine on arbitrary input.

Such a TM is called a Universal Turing Machine (UTM).

L_{TM} is Turing-recognizable, or recursively enumerable (r.e.)

Use a UTM to do the following on input $M \# \omega$:

- Simulate M on ω
- If M accepts, accept.

, might loop forever, in which case this UTM also loops forever

We said that if L is decidable, so is \bar{L} .

What if both L and \bar{L} are r.e.?

Can we say something more?

M_1 : γ if $x \in L$, _____ if $x \notin L$

M_2 : γ if $x \in \bar{L}$, _____ if $x \notin \bar{L}$

Then, L is decidable