Turing Machines

Recall: Defined computability via URMs ("infinite RAM + assembly")

Computable = partial recursive functions = URM-computable

Non-computable = ?

Today: Other models of computation

Church-Turing thesis: The following models of computation are equivalent

→ The lambda calculus

→ Combinatory logic

→ Partial recursive functions

→ Post systems

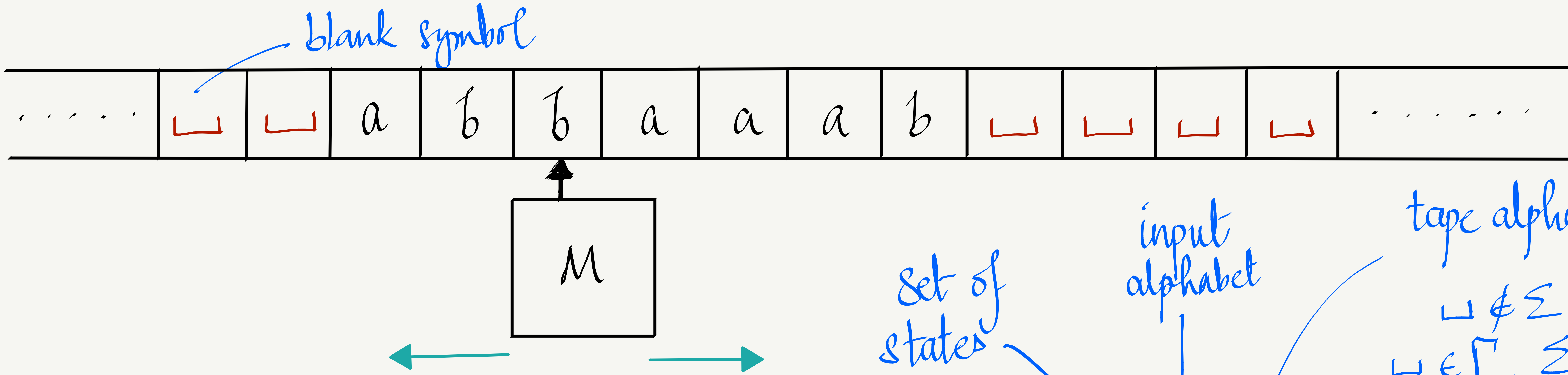
→ Turing machines

A Turing machine is basically a finite-state automaton with a global tape
Each cell of the tape can contain a letter from a finite alphabet
Start out with the input string, and the rest of the tape is empty.

What can the machine do?

* Read off the tape * Write on the tape * Move the head left or right

At each step, the head scans some cell of the tape, and the automaton is in some state, depending on which the automaton can move to some (other) state, the machine can write some new symbol on the tape cell, and move the head one cell to the left or to the right



Set of states

input alphabet

tape alphabet

$\sqcup \notin \Sigma$

$\sqcup \in \Gamma$, $\Sigma \subseteq \Gamma$.

A deterministic* Turing Machine: $M = (Q, \Sigma, \Gamma, \delta, s, t, r)$

Start state

reject state

accept state

$s, t, r \in Q$.

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

current state

$Q \setminus \{t, r\}$

Contents of current cell

new state

new contents of current cell

direction in which tape head should move

Input $\omega_1 \omega_2 \cdots \omega_n$, each $\omega_i \in \Sigma$

Written on n cells of the tape

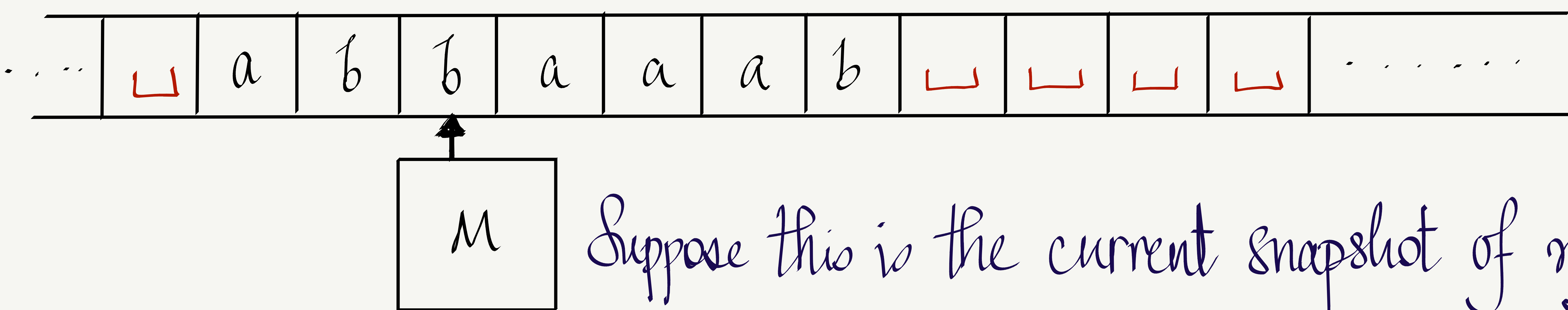
The first blank signals the end of input.

As part of its operation, the following parameters change:

- → current state
- → current tape contents
- → location of tape head

These form a configuration

$(ab, b, aaab, q)$



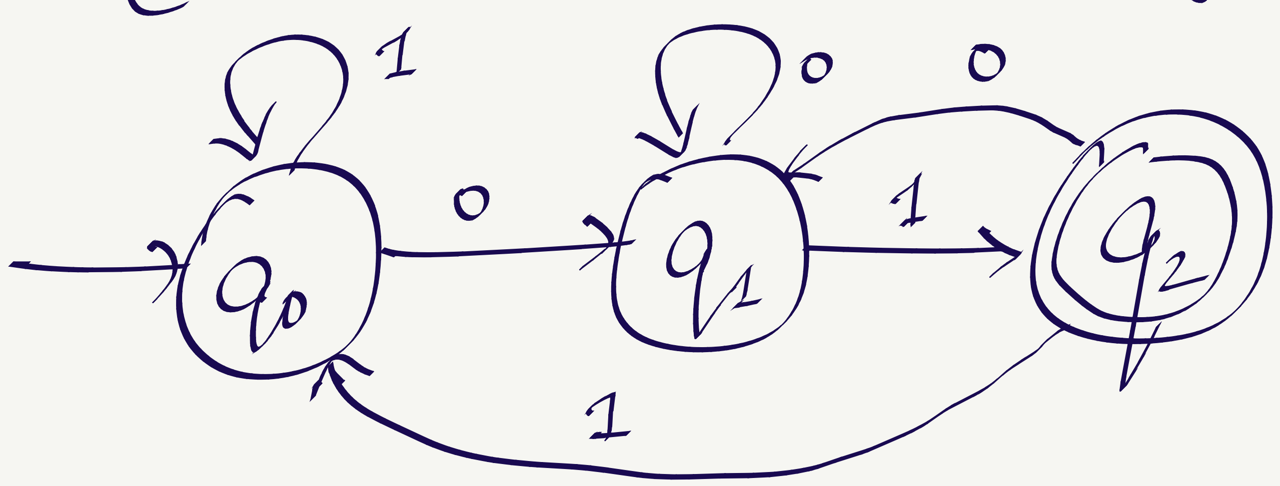
Suppose this is the current snapshot of my TM M, in state q.

We write the configuration as $abqbaaab$.

Formally, a configuration is uqv, where the machine is in state q, the current tape contents are uv, $u, v \in \Gamma^*$, and the head points to the cell containing the first letter of v.

Example :

$L = \{ \omega \mid \omega \text{ ends with } 01 \}$



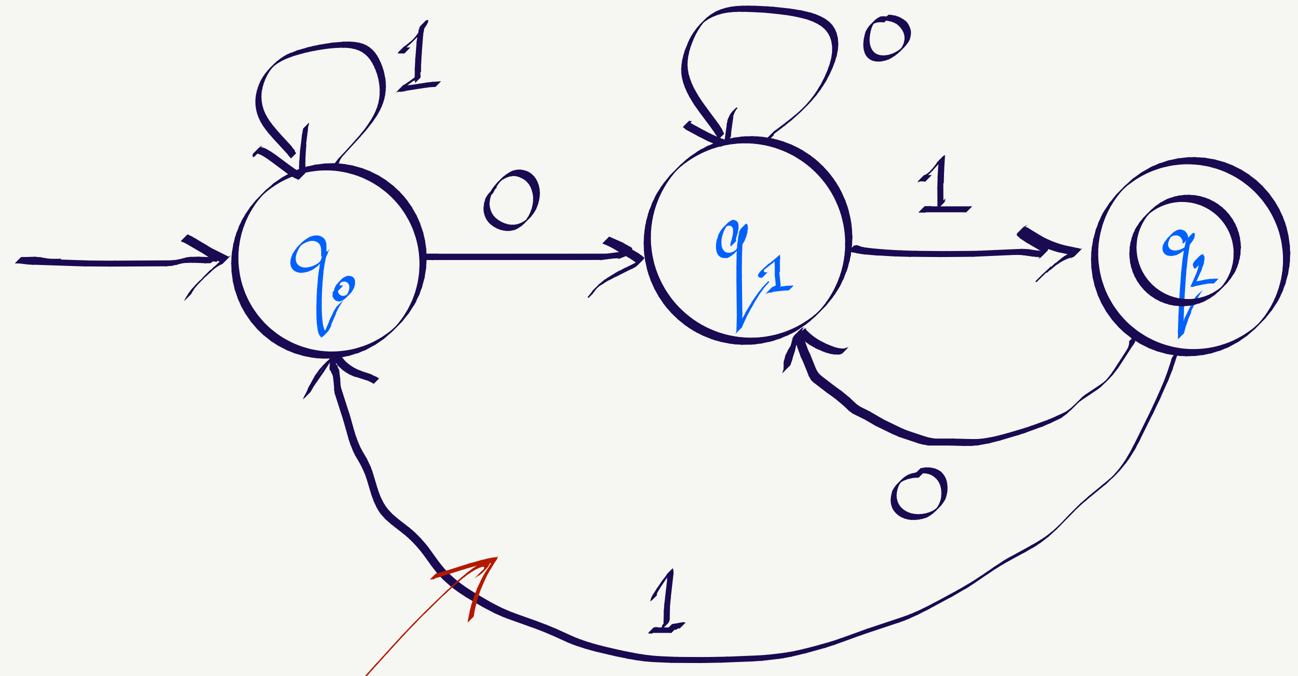
$M = (Q, \Sigma, \Gamma, \delta, s, t, r)$

$\Sigma = \{0, 1\}$ $\quad \Gamma = \{0, 1, \sqcup\}$

$s = q_0$ $\quad Q = \{q_0, q_1, q_2, t, r\}$

Example:

$L = \{ \omega \mid \omega \text{ ends with } 01 \} \in \{0,1\}^*$ $\Sigma = \{0,1\}$

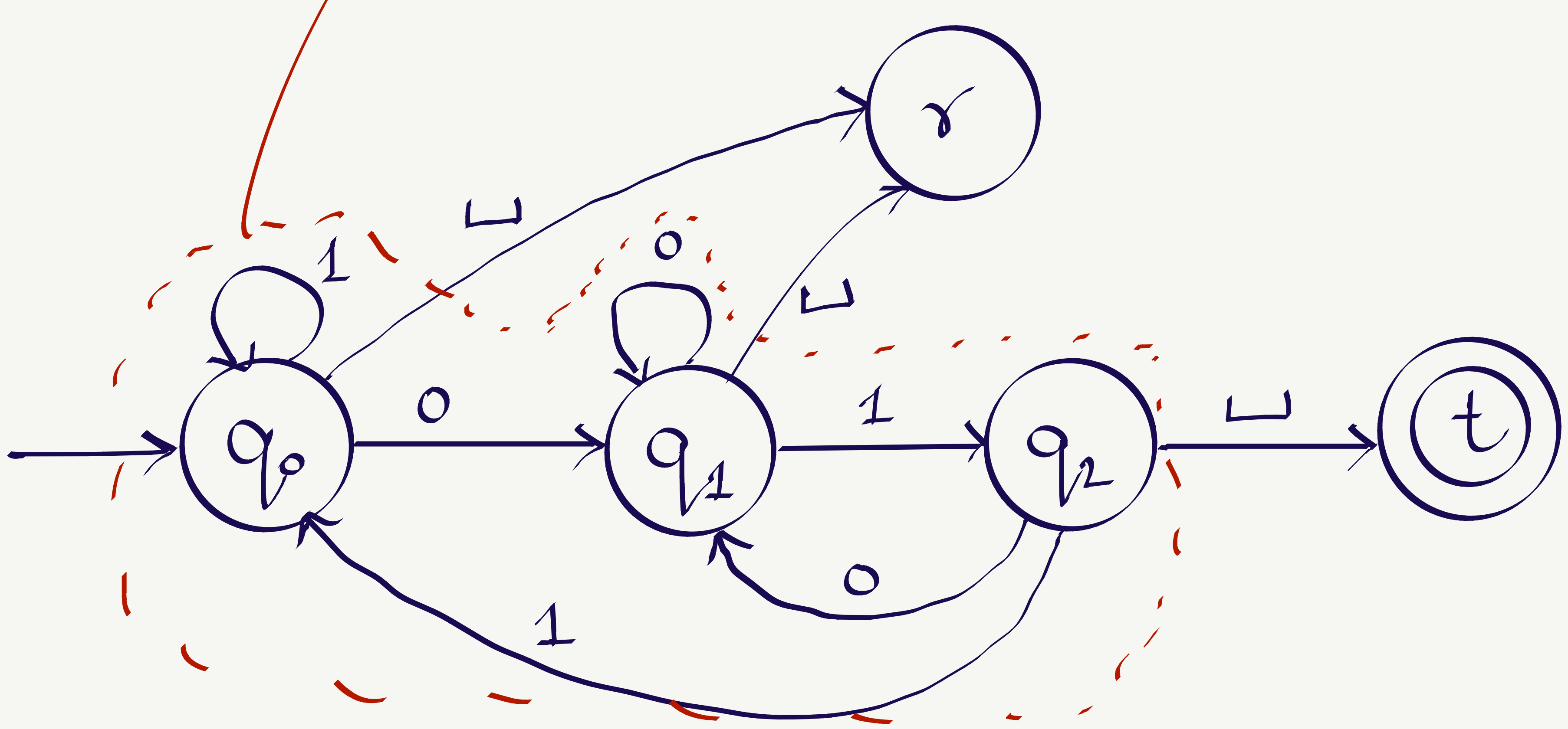


$Q = \{q_0, q_1, q_2, t, r\}$

$\Gamma = \{0, 1, \sqcup\}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, t, r)$

$\ldots \sqcup \sqcup 100 \sqcup \sqcup \ldots$

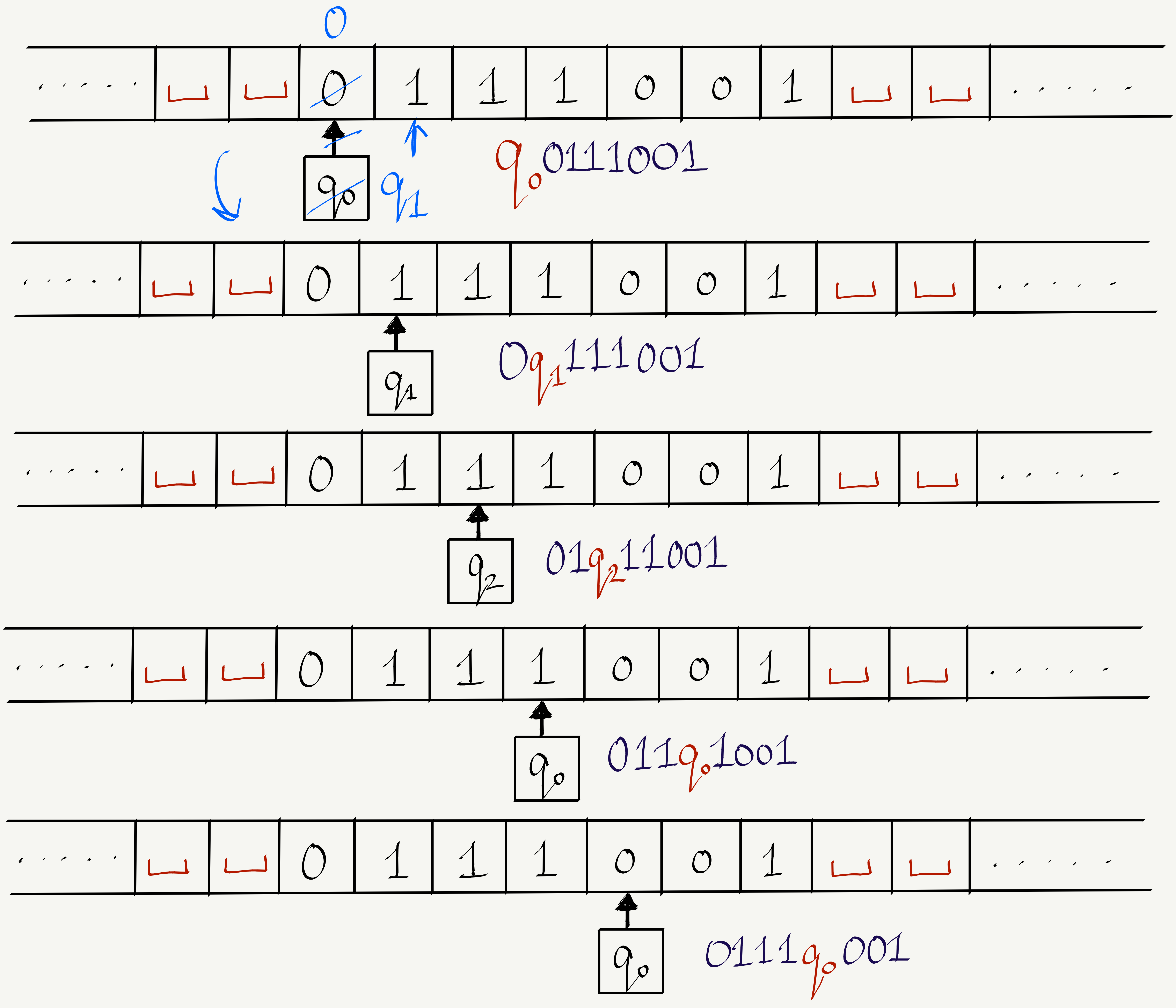


$\delta(q_0, 0) = (q_1, 0, R)$

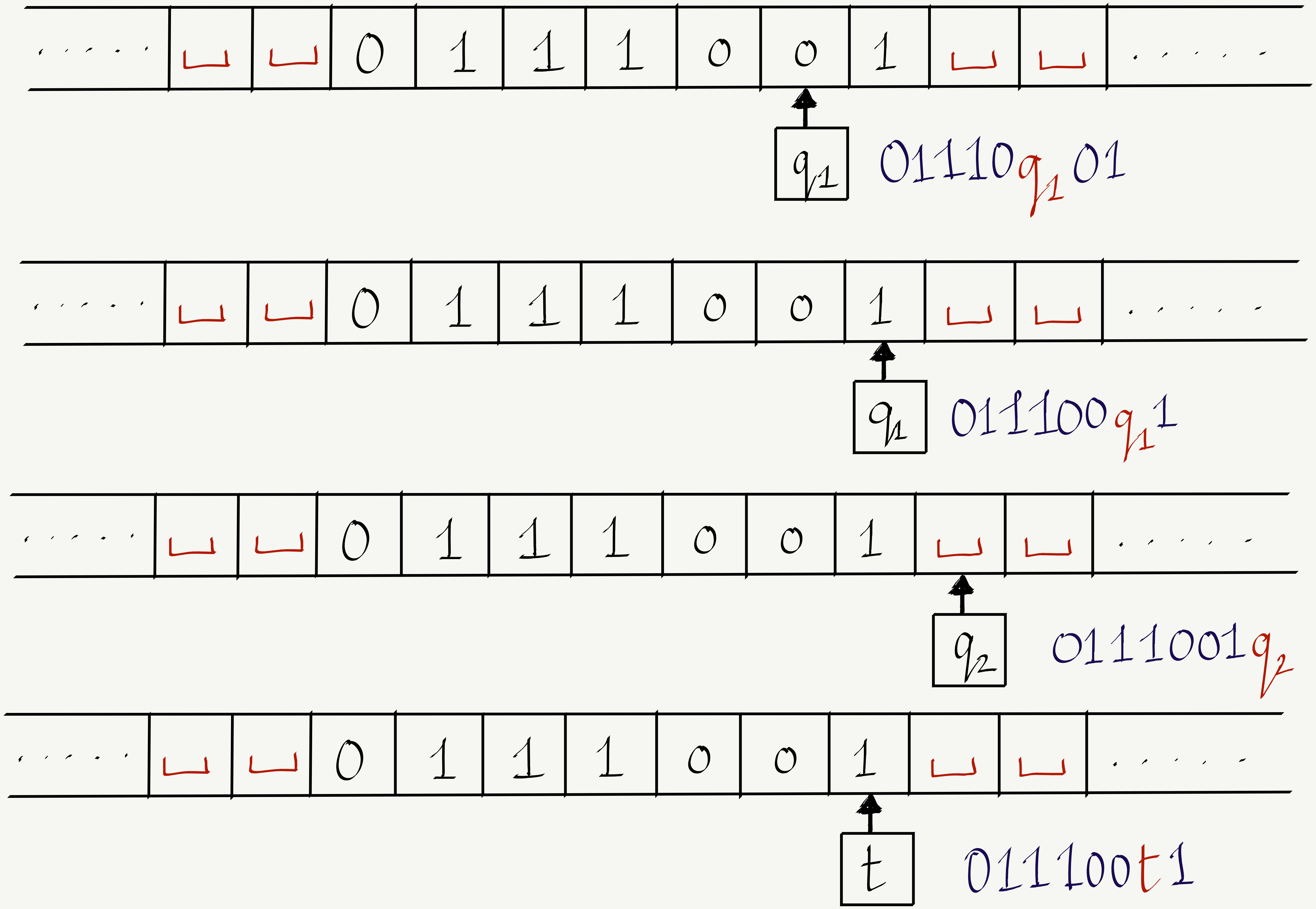
$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_2, \sqcup) = (t, \sqcup, L)$



0
0 1 1 1 0 0 1
q0 q1
q0 0111001
0 1 1 1 0 0 1
q1
0q1 111001
0 1 1 1 0 0 1
q2
01q2 11001
0 1 1 1 0 0 1
q0
011q0 1001
0 1 1 1 0 0 1
q0
0111q0 001



q1
01110q1 01
q1
011100q1 1
q2
0111001q2
t
011100t1

Let $u, v \in \Gamma^*$, and $a, b, c \in \Gamma$. Then,

$$uaqbv \xrightarrow[M]{1} uacq'v \quad \text{iff} \quad \delta((q,b)) = (q', c, R), \quad \text{and}$$

$$uaqbv \xrightarrow[M]{1} uq'acv \quad \text{iff} \quad \delta((q,b)) = (q', c, L)$$

The initial configuration is of the form $s\omega$, where ω is the input.

ω is accepted by M if $s\omega \xrightarrow[M]{*} utv$, for some $u, v \in \Gamma^*$.

$\underbrace{utv}_{\text{accepting configuration}}$

Similarly,

ω is rejected by M if $s\omega \xrightarrow[M]{*} urv$, for some $u, v \in \Gamma^*$.

$\underbrace{urv}_{\text{rejecting configuration}}$

M does not proceed beyond an accepting or a rejecting configuration.

A halting configuration is either an accepting or rejecting configuration.

$\mathcal{L}(M) = \{ w \mid s w \xrightarrow[M]{*} u t v, \text{ for some } u, v \in \Gamma^* \}$

$\mathcal{L}$ is Turing-recognizable if there exists a TM M s.t. $\mathcal{L} = \mathcal{L}(M)$.