Birla Institute of Technology and Science, Pilani
Second Semester 2017–2018,
MATH F112 (Mathematics-II)
Assignment-II

Q.1 Show that the function $f(z) = z^k$ is continuous at all points in the finite complex plane for any positive integer k.

Q.2 Find the constant values a, b, c such that the function $f(z) = (x - 2ay) + i(bx - cy)$ is analytic. Also, express $f(z)$ in the terms of z.

Q.3 (a) Show that $|\exp(z^2)| \leq \exp|z|^2$.

(b) Find out the subset S of $\mathbb{C}$ where the function $f(z) = \frac{Log(z+1+i)}{z^4 - i}$ is not analytic.

Q.4 Solve for z:

(i) $sinz = cosh4$, (ii) $cosz = 2$, (iii) $tanz = 2$.

Q.5 (i) Find all values of $(1+i)^{2-i}$.

(ii) If $z = i^z$, then show that $|z|^2 = e^{-(4n+1)y\pi}$, $z = x + iy, n$ is any integer.

Q.6 Consider the function $h(x, y) = \sinh x \sin y$. Without using Laplace equation and the continuity of the derivatives of h show that h is harmonic.

Q.7 Find the value of z for which $\sin z = e^2$.

Q.8 Consider the function $f(z) = \dfrac{1}{z-2}$ defined on a contour C. Evaluate the integral $\int_C f(z)dz$ for the set of contours:

(a) the circle, $|z-2| = 4$ **(b)** the circle, $|z| = 1$ and **(c)** the square with vertices at $3 \pm 3i$, $-3 \pm 3i$.

Q.9 Using Liouville's theorem, show that an entire function having its real part non-positive in $\mathbb{C}$ is necessarily a constant.

Q.10 Let C is the contour defined as $z = 2e^{i\theta}$, $0 \le \theta \le \pi/3$ and $f(z) = \dfrac{e^{iz}(z^2+3)Log(z)}{z^2-2}$ is a function on C. Use ML inequality to determine the upper bound of $\left| \int_C f(z)dz \right|$.

Q.11 Evaluate the integral

$$\oint_C \frac{z^2+1}{z(2z-1)} dz \quad C: |z| = 1.$$

Q.12 Give the Laurent series expansion in powers of z of the function $f(z)$, when $0 < |z| < 1$.

$$f(z) = \frac{1}{z(1+z^2)}$$

Q.13 Let a function f be analytic throughout the finite plane except for a finite number of singular points $z_1, z_2, \ldots, z_n$. Show that $\sum_{j=1}^{n} Res_{z=z_j} f(z) + Res_{z=\infty} f(z) = 0$.

Q.14 Suppose C_n denote the positively oriented boundary of the square whose edges lie along the lines $x = \pm\left(n+\frac{1}{2}\right)\pi$ and $y = \pm\left(n+\frac{1}{2}\right)\pi$, where n is a positive integer. Show that

$$\int_{C_n} \frac{dx}{z^2 sinz} = 2\pi i \left[\frac{1}{6} + 2\sum_{j=1}^{n} \frac{(-1)^n}{j^2\pi^2}\right].$$

Hence deduce that

$$\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j^2} = \frac{\pi^2}{12}.$$

Q.15 Use residue to evaluate the definite integral $\int_0^{2\pi} \dfrac{\sin^2\theta}{a+b\cos\theta} d\theta$,$0 < b < a$

Q.16 Use residue to evaluate the improper integral $\int_{-\infty}^{\infty} \dfrac{1}{(x^2+b^2)(x^2+c^2)^2} dx$, $b > c > 0$