BITS Pilani
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CS/IS F214
Logic in Computer Science

MODULE: TEMPORAL LOGICS

Linear Temporal Logic – Binary Temporal Operators

Formulas and Interpretation

- Now let us consider some binary ***temporal*** connectives (and formulas) :
  - ϕ U ψ /* *read* ϕ **Until** ψ */
  - ϕ W ψ /* *read* ϕ **Weak-until** ψ
    or ϕ **Wait** ψ */
  - ϕ R ψ /* *read* ϕ **Release** ψ */

Formulas and Interpretation

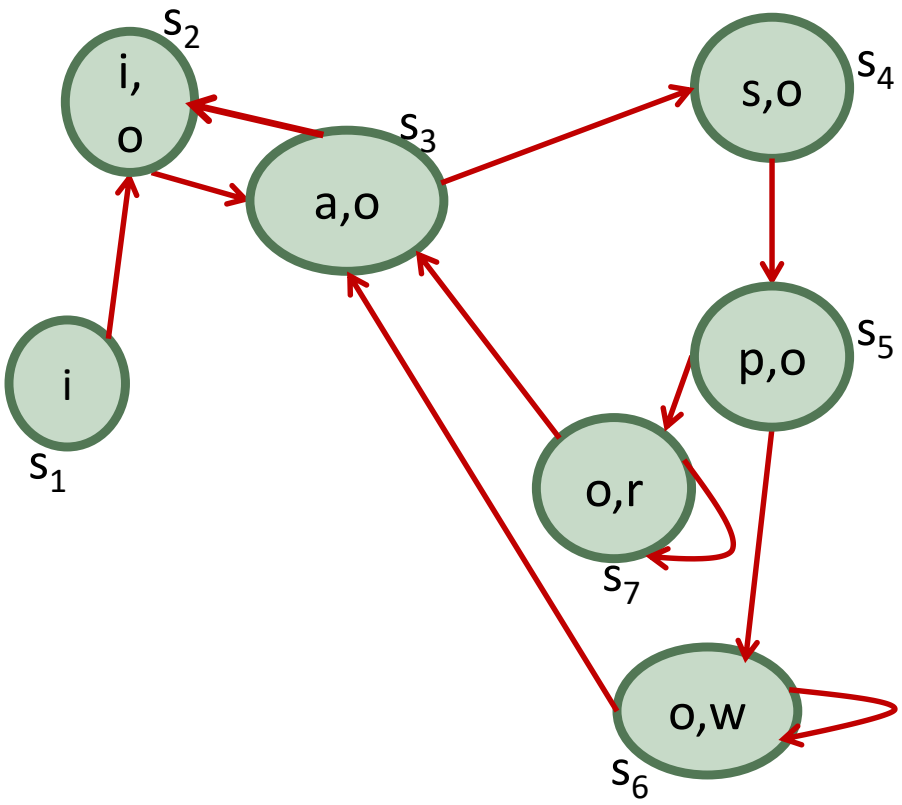
- Semantics of these temporal operators:
  - Let $M = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in *M.*
  - Then define the *satisfaction relation* $\models$ as follows:
    - $\pi \models \phi \, \mathbf{U} \, \psi$ iff there is some i such that $\pi^i \models \psi$ and for all j = 1,..., i-1, $\pi^j \models \phi$
    - $\pi \models \phi \, \mathbf{W} \, \psi$ iff there is some i such that $\pi^i \models \psi$ and for all j = 1,..., i-1, $\pi^j \models \phi$; or all k>=1 $\pi^k \models \phi$
    - $\pi \models \phi \, \mathbf{R} \, \psi$ iff for some i>=1 $\pi^i \models \phi$ and for all j = 1,..., i, $\pi^j \models \psi$; or all k>=1 $\pi^k \models \psi$
- Exercise:
  - Express **U** in terms of **W** and vice-versa

Examples

How do you read the following formulas?

1) **X G o**
2) **F (w $\vee$ r)**
3) **X (o U w)**
4) **X X (a U w)**
5) **G F s**
6) **G F (w $\vee$ r)**
7) **G F ((w $\vee$ r) $\wedge$ (X a))**

Examples



How do you read the following formulas?

1) **X G o**
2) **F (w ∨ r)**
3) **X (o U w)**
4) **X X (a U w)**
5) **G F s**
6) **G F (w ∨ r)**
7) **G F ((w ∨ r) ∧ (X a))**

Evaluate each formula under the path

$\pi_w = s_2 \rightarrow s_3 \rightarrow s_4 \rightarrow s_5 \rightarrow s_6 \rightarrow s_3 \ldots$,

where the sequence $s_3 \rightarrow s_4 \rightarrow s_5 \rightarrow s_6 \rightarrow$ repeats ad infinitum.

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LTL Syntax

Syntax of LTL

- Write the grammar for LTL
- Precedence and Associativity
  - One convention:
    - Propositional Operators ($\wedge$, $\vee$, -->, and $\neg$) have higher precedence over Temporal Operators (F, G, X, U, W, and R).
      - Unary Operators have higher precedence over Binary Operators within each category.
    - All binary operators are right-associative i.e.
      - ϕ U ψ U χ would be associated as ϕ U (ψ U χ)
  - Rewrite the grammar for LTL using these precedence and associativity conventions.