$\underline{Chapter\ 4}$

$L: 5, 13, 18, 19; T: 2, 14, 16, 21; S: 1, 3, 4, 6, 10, 11, 12, 15, 17, 20;$

$Not\ in\ Course: 9, 22-30\ and\ the\ problems\ related\ to\ article\ 4.14\ (oscillation\ related)$

4.2. At $t=0$, mechanical energy of the system: $\frac{1}{2}M\upsilon_0^2$. Let, x_0 is the compression in the spring.

$\therefore$ Work done against the firctional force: $\int_0^{x_0}\mu N dx = \int_0^{x_0} bxMgdx = \frac{1}{2}Mgbx_0^2$.

Energy stored in the spring: $\frac{1}{2}kx_0^2$; From energy conservation: $\frac{1}{2}M\upsilon_0^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}Mgbx_0^2$

$\therefore x_0^2 = \frac{M\upsilon_0^2}{(k+Mgb)}$; $\boxed{\text{Energy loss}: \frac{1}{2}Mgbx_0^2 = \frac{1}{2}Mgb\times\frac{M\upsilon_0^2}{(k+Mgb)} = \frac{1}{2}M\upsilon_0^2\Big/\left(1+\frac{k}{Mgb}\right)}$

4.4. From conservation of momentum: $m\upsilon + MV' = 0 \Rightarrow V' = -\frac{m\upsilon}{M}$

From conservation of energy: $\frac{1}{2}MV'^2 + \frac{1}{2}m\upsilon^2 = mgR \Rightarrow \frac{1}{2}M\left(\frac{m\upsilon}{M}\right)^2 + \frac{1}{2}m\upsilon^2 = mgR$

$\Rightarrow \frac{1}{2}\frac{m\upsilon^2}{M} + \frac{1}{2}m\upsilon^2 = mgR \Rightarrow \frac{1}{2}\frac{\upsilon^2}{M} + \frac{1}{2}\upsilon^2 = gR \Rightarrow \boxed{\upsilon = \sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}}$ Ans.

4.6. Let x be the vertical downward distance where the mass loses the contact.

Equation of motion of m: $mg\cos\theta - N = \frac{m\upsilon^2}{R} \Rightarrow N = mg\cos\theta - \frac{m\upsilon^2}{R}$

m loses contact when $N=0$, so, $\frac{1}{2}m\upsilon^2 = \frac{1}{2}Rmg\cos\theta ----(1)$

From energy conservation at the point where it will loses contact: $\frac{1}{2}m\upsilon^2 = mgx = \frac{1}{2}Rmg\cos\theta$

$\cos\theta = \frac{R-x}{R}$, so, $x = \frac{1}{2}R\left(\frac{R-x}{R}\right) \Rightarrow \boxed{x = \frac{R}{3}}$ Ans.

4.7. Force acting on the ring: Tension at the thread acting upward: T and the Mg force acting downward.

Normal reaction on the ring due to rotation of the beads: $2N$. And $2mg$ acting downward.

At any instant of time, the position of the beads are same w.r.t. the point of release.

$\therefore T - Mg + 2N\cos\theta - 2mg = 0.$

The ring will raise when vertical components of the normal reaction is such that $T=0$.

Now, $N = m\dot{\theta}^2 R$

If α is the angle at this moment, then,

$$-Mg + 2m\dot{\alpha}^2 R\cos\alpha - 2mg = 0 \Rightarrow \dot{\alpha}^2 = \frac{g(M+2m)}{2mR\cos\alpha}$$

4.10. *Equation of motion*: $(m+M)\ddot{x}=-kx \Rightarrow \omega=\sqrt{\dfrac{k}{m+M}}$ *and* $T=2\pi\sqrt{\dfrac{m+M}{k}}$

(a) *Let the mass pulled with am amplitude* A_0 *and then released. So at this position, at* $t=0,\ \upsilon=0$

$P.E.=\frac{1}{2}kA_0^2$; *When the putty of mass m stick to M, P.E. will remains same at the displaced position.*

So, $P.E.=\frac{1}{2}kA_0^2 \Rightarrow$ *unchanged and hence the amplitude and the mechanical energy.*

(b) *When m sticks to M at maximum velocity position: i.* $\omega=\sqrt{\dfrac{k}{m+M}}$ *and* $T=2\pi\sqrt{\dfrac{m+M}{k}}$

ii. Let the new amplitude is A. So, $\frac{1}{2}kA^2=\frac{1}{2}(M+m)V'^2$ *maximum K.E.*

From momentum conservation, $MV=(M+m)V' \Rightarrow V'=\dfrac{MV}{M+m}$

$$\therefore A^2=\frac{(M+m)V'^2}{k}=\frac{M^2V^2}{k(M+m)} \Rightarrow \boxed{A=\frac{MV}{\sqrt{k(M+m)}}}$$

$$But, \frac{1}{2}mV^2=\frac{1}{2}kA_0^2 \Rightarrow A_0=V\sqrt{\frac{m}{k}};\ So,\ \boxed{A=\frac{MV}{\sqrt{k(M+m)}}=\sqrt{\frac{M}{M+m}}A_0}$$

$$iii.\ Change\ in\ mechanical\ energy: \Delta E_{Mech}=\frac{1}{2}(M+m)V'^2-\frac{1}{2}MV^2=\frac{1}{2}(M+m)\left(\frac{MV}{M+m}\right)^2-\frac{1}{2}MV^2$$

$$\Rightarrow \boxed{\Delta E_{Mech}=-\frac{1}{2}kA_0^2\left(\frac{m}{m+M}\right)} \Rightarrow Energy\ loss$$

4.14. (a) *Potential energy of the bead*: $\boxed{\dfrac{-2GMm}{\left(x^2+a^2\right)^{\frac{1}{2}}}}$.

(b) *Following the energy conservation*:

$$\overbrace{\frac{1}{2}m\upsilon_0^2+\frac{-2GMm}{\left((3a)^2+a^2\right)^{\frac{1}{2}}}}^{Initial\ energy}=\overbrace{\frac{1}{2}m\upsilon^2+\frac{-2GMm}{a}}^{Energy\ when\ it\ will\ pass\ the\ origin} \Rightarrow \boxed{\upsilon=\sqrt{\upsilon_0^2+\frac{4GM}{a}\left(1-\frac{1}{\sqrt{10}}\right)}}$$

$$(c)\ Force\ on\ the\ bead: 2F\cos\theta=-2\frac{GMm}{\left(x^2+a^2\right)}\cos\theta=-2\frac{GMm}{\left(x^2+a^2\right)}.\frac{x}{\left(x^2+a^2\right)^{\frac{1}{2}}}=\frac{-2GMmx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$$

$$\therefore m\frac{d^2x}{dt^2}+\frac{2GMmx}{\left(x^2+a^2\right)^{\frac{3}{2}}}=0\approx m\frac{d^2x}{dt^2}+\left(\frac{2GMm}{a^3}\right)x=0;\therefore \boxed{\omega=\sqrt{\frac{2GM}{a^3}}}$$

$2.16. Power = Force \times Velocity; Final\ velocity: 60m/hr = 88\,ft/\sec$

$Average\ velocity: 88/2 = 44\,ft/\sec;\ Acceleration: 88/8 = 11\,ft/sec^2$

$$\boxed{Force: \frac{1800}{32}\times 11\ and\ \ power: \frac{1800}{32}\times 11\times 44 = 27225\ \ ft - Ib/s}$$

$4.21.\ (a)$ *Let at any instant of time y is the height of the rope above the table.*

If F is the external pulling force on the rope, then the net external force:

$$F_{ext} = F - \lambda gy = M\frac{dV}{dt} - V_{rel}\frac{dM}{dt}; V = 0\, and\ V_{rel} = 0 - u = -\upsilon_0$$

$$\therefore F - \lambda gy = \upsilon_0\frac{dM}{dt} = \upsilon_0\frac{d}{dt}(\lambda y) = \lambda\upsilon_0\frac{dy}{dt} = \lambda\upsilon_0^2.$$

$F = \lambda gy + \lambda\upsilon_0^2;$ *Hence power*: $\boxed{P = F\upsilon_0 = \lambda gy\upsilon_0 + \lambda\upsilon_0^3}$

$$(b)\frac{dE}{dt} = \frac{d}{dt}(K+V) = \frac{d}{dt}\left(\frac{1}{2}\lambda y\upsilon_0^2 + \frac{y}{2}\lambda gy\right) = \frac{1}{2}\lambda\upsilon_0^3 + \lambda gy\upsilon_0$$

$$\therefore \boxed{P = \lambda gy\upsilon_0 + \lambda\upsilon_0^3 = \frac{dE}{dt} + \frac{1}{2}\lambda\upsilon_0^3}\ Ans$$