Chapter 6:
Angular Momentum and Fixed Axis Rotation

Force, Linear Momentum and Center of Mass

For Translational Motion

Torque, Angular Momentum and Moment of Inertia

For Rotational Motion

The General Motion of an Extended Body:

Combination of :

(i) Center of Mass Motion

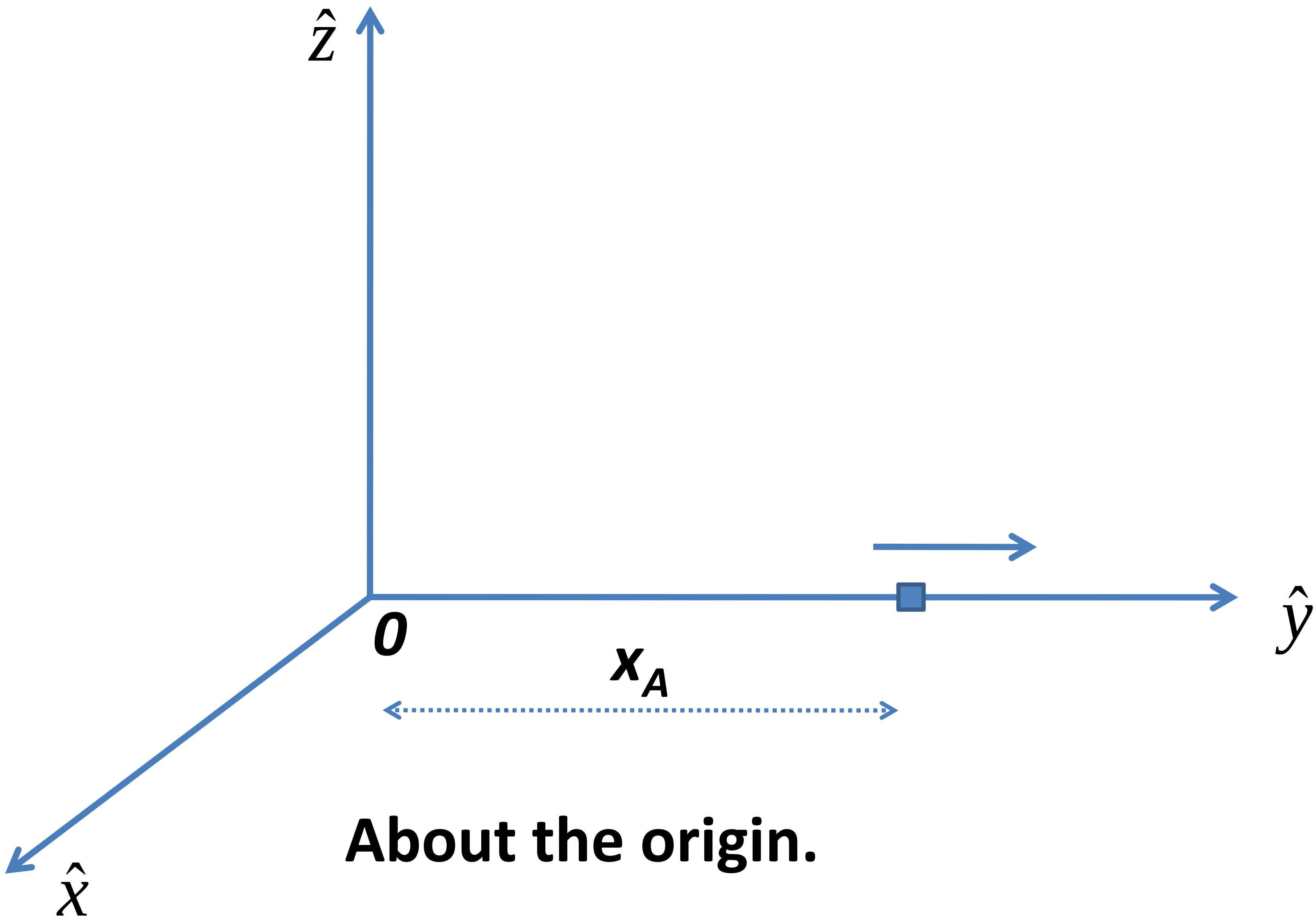
(ii) Rotation Motion about the C.M

Angular Momentum of a Particle:

$$\vec{L} = \vec{r} \times \vec{P}$$

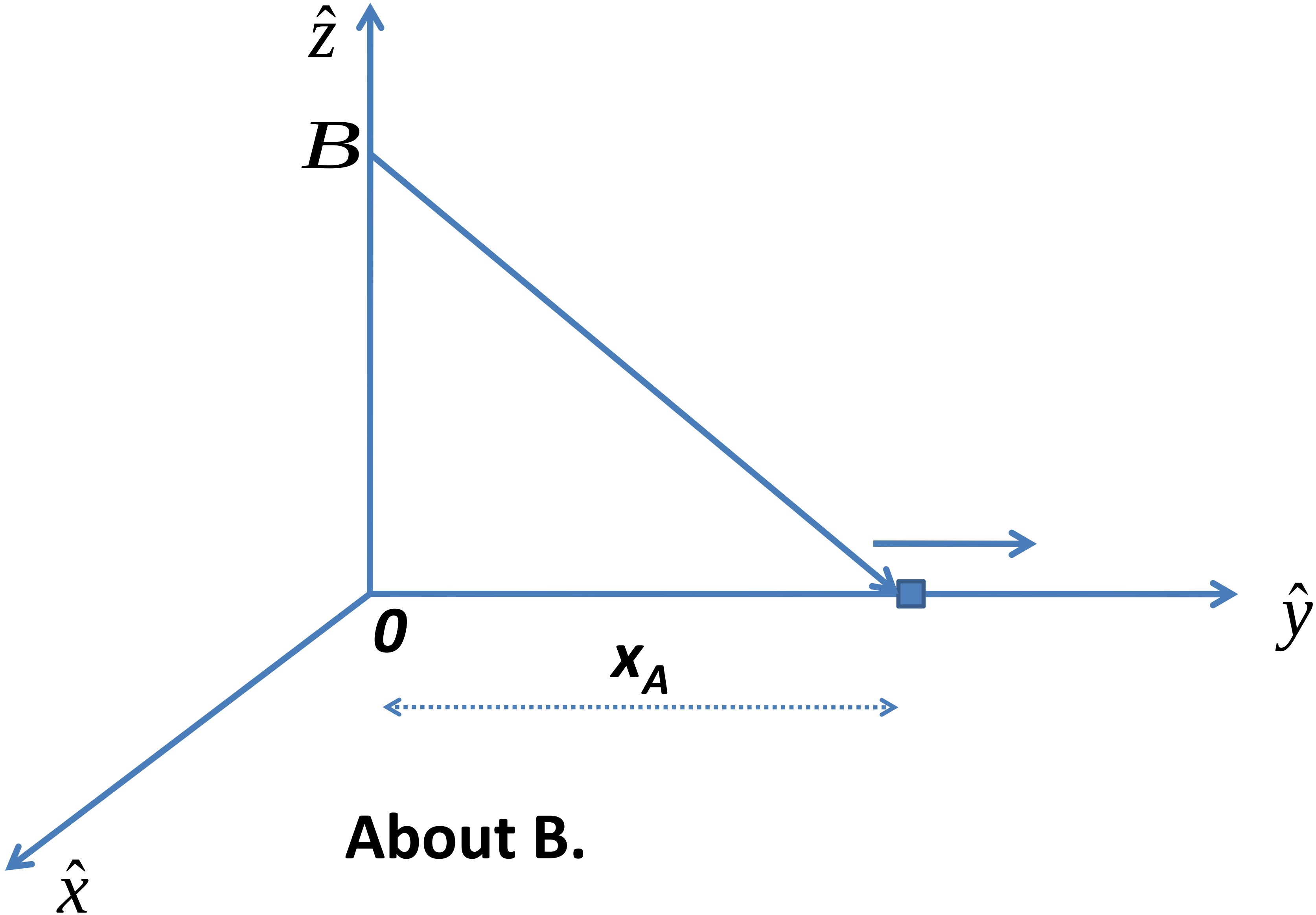
With respect to a given coordinate system.

Problem: Angular Momentum of a Sliding Block:



About the origin.

Problem: Angular Momentum of a Sliding Block:



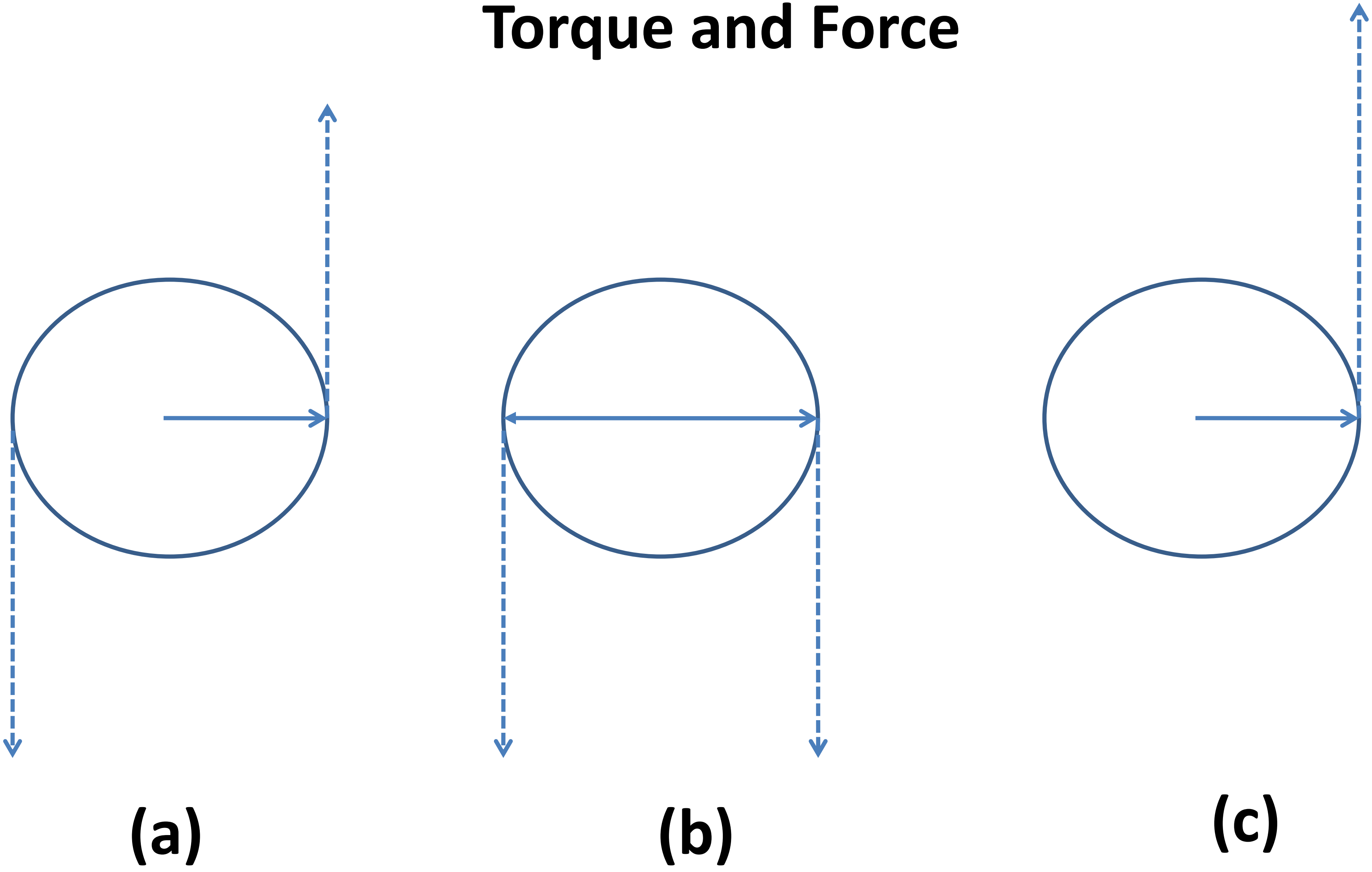
About B.

Torque

Torque due to a force F

$$\tau = \vec{r} \times \vec{F}$$

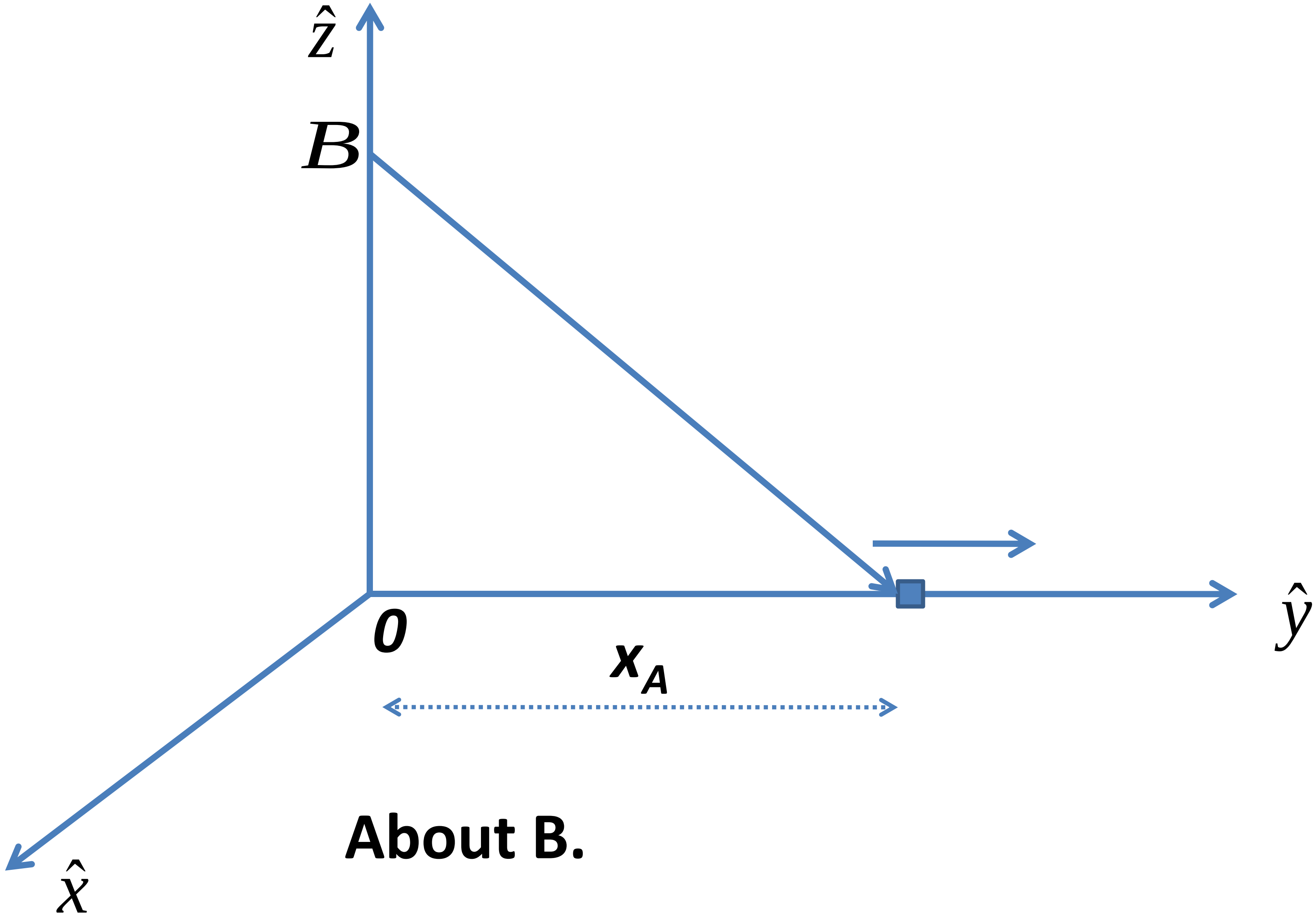
Torque and Force



Torque is important because it is intimately related to the rate of change of Momentum.

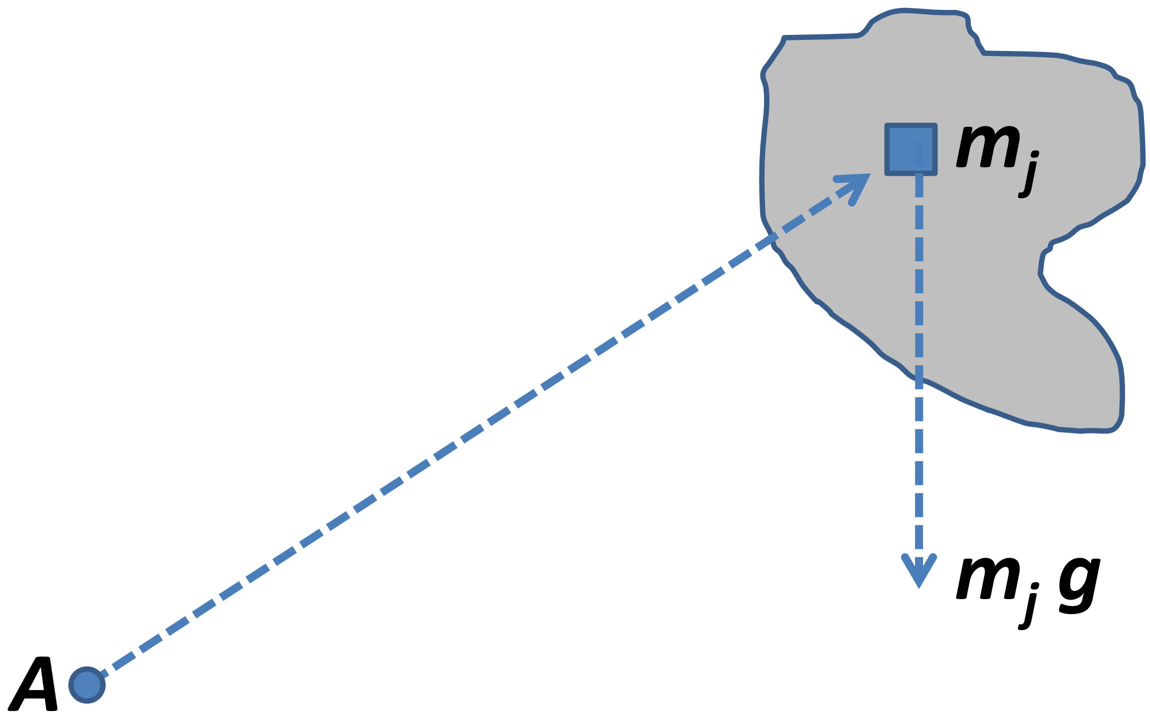
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Problem: Torque on a Sliding Block:



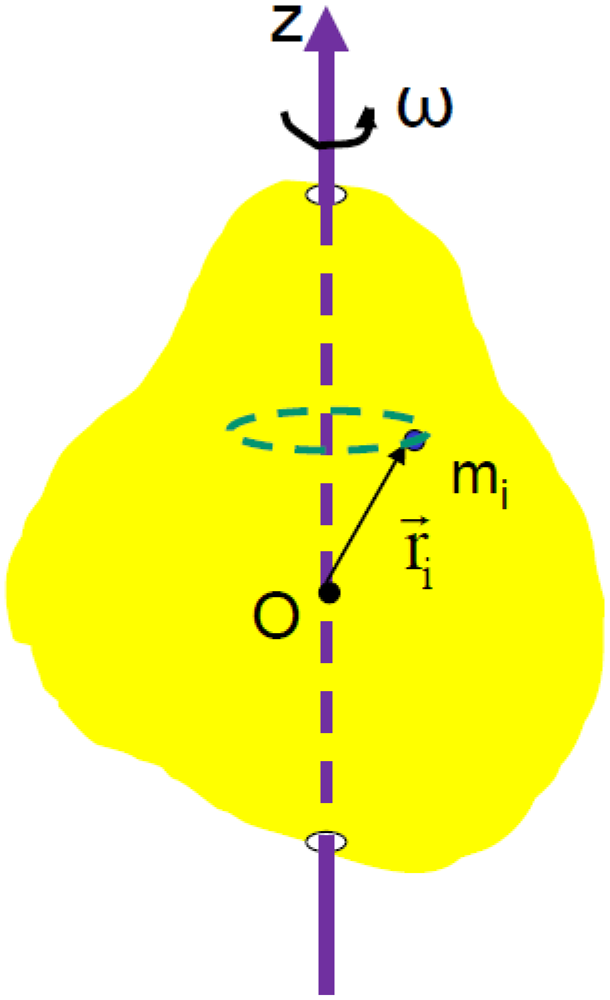
About B.

Torque due to Gravity

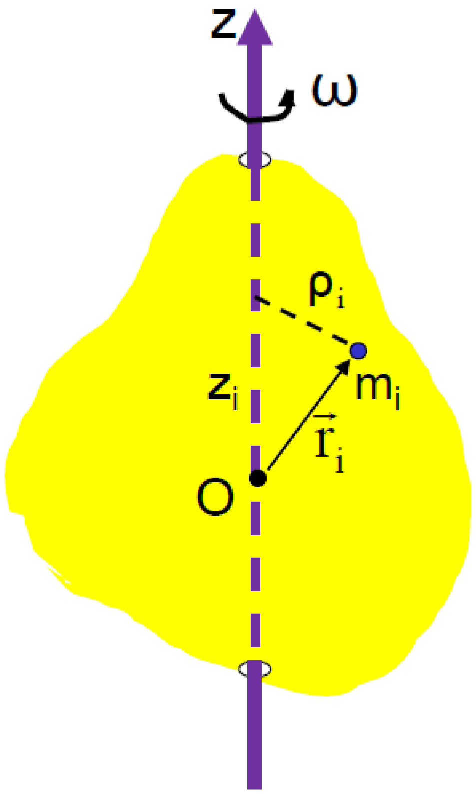


Angular Momentum and Fixed Axis Rotation

$$\vec{L} = \sum_i m_i (\vec{r}_i \times \vec{v}_i)$$



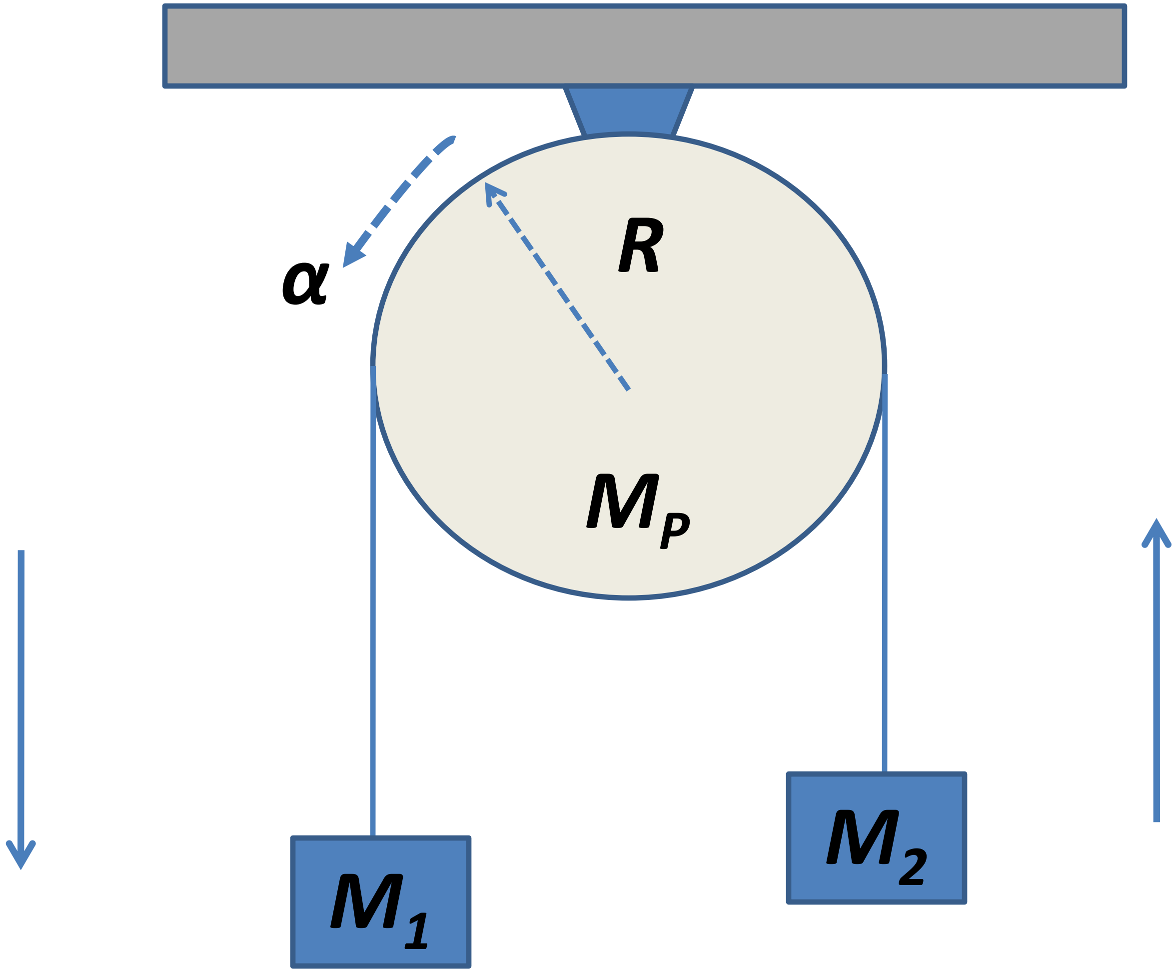
Here, we are concerned only with the component along the axis of rotation



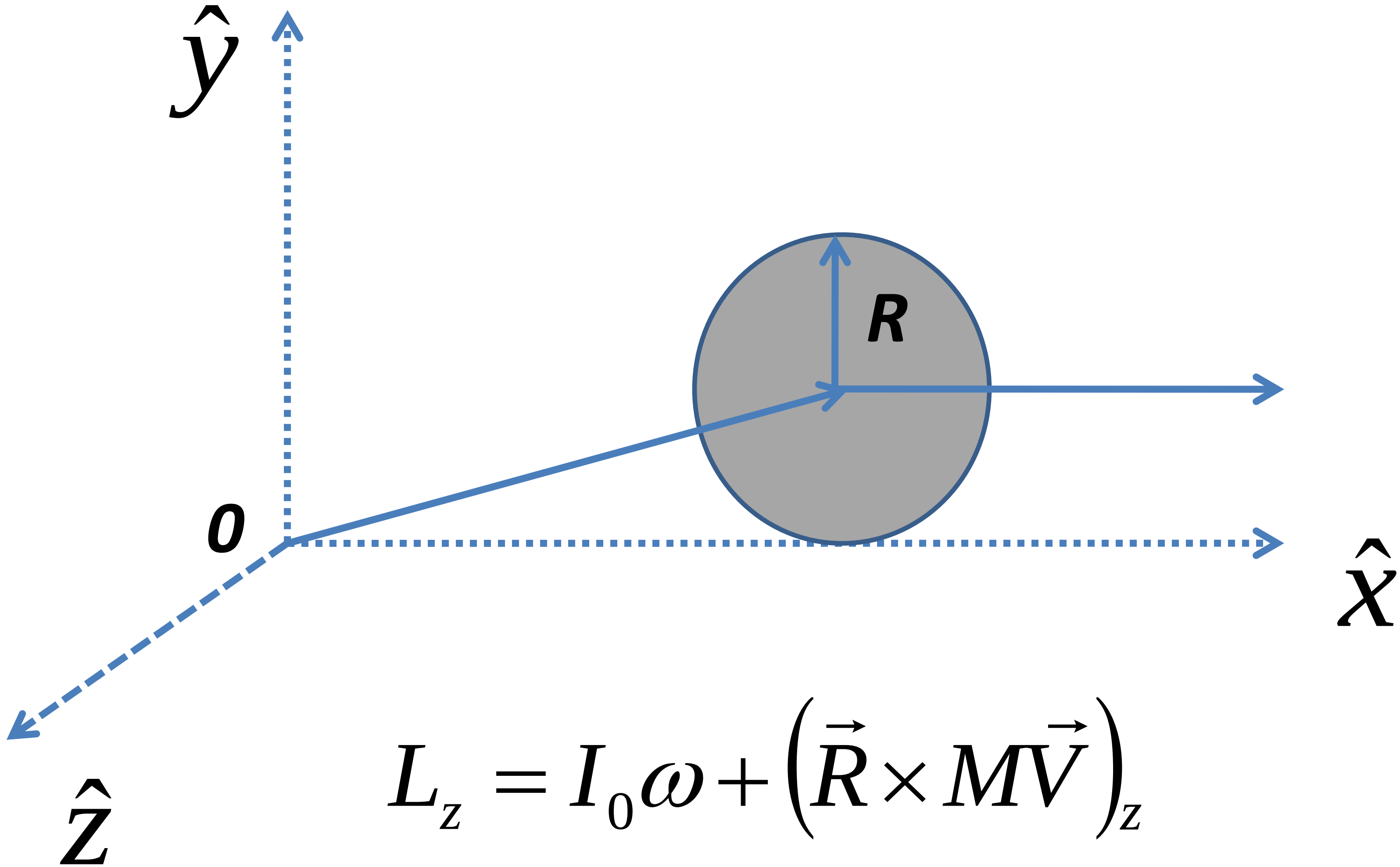
Dynamics of Pure Rotation about an Axis:

The Axis of Rotation is at Rest.

Problem: Atwood's Machine with Massive Pulley

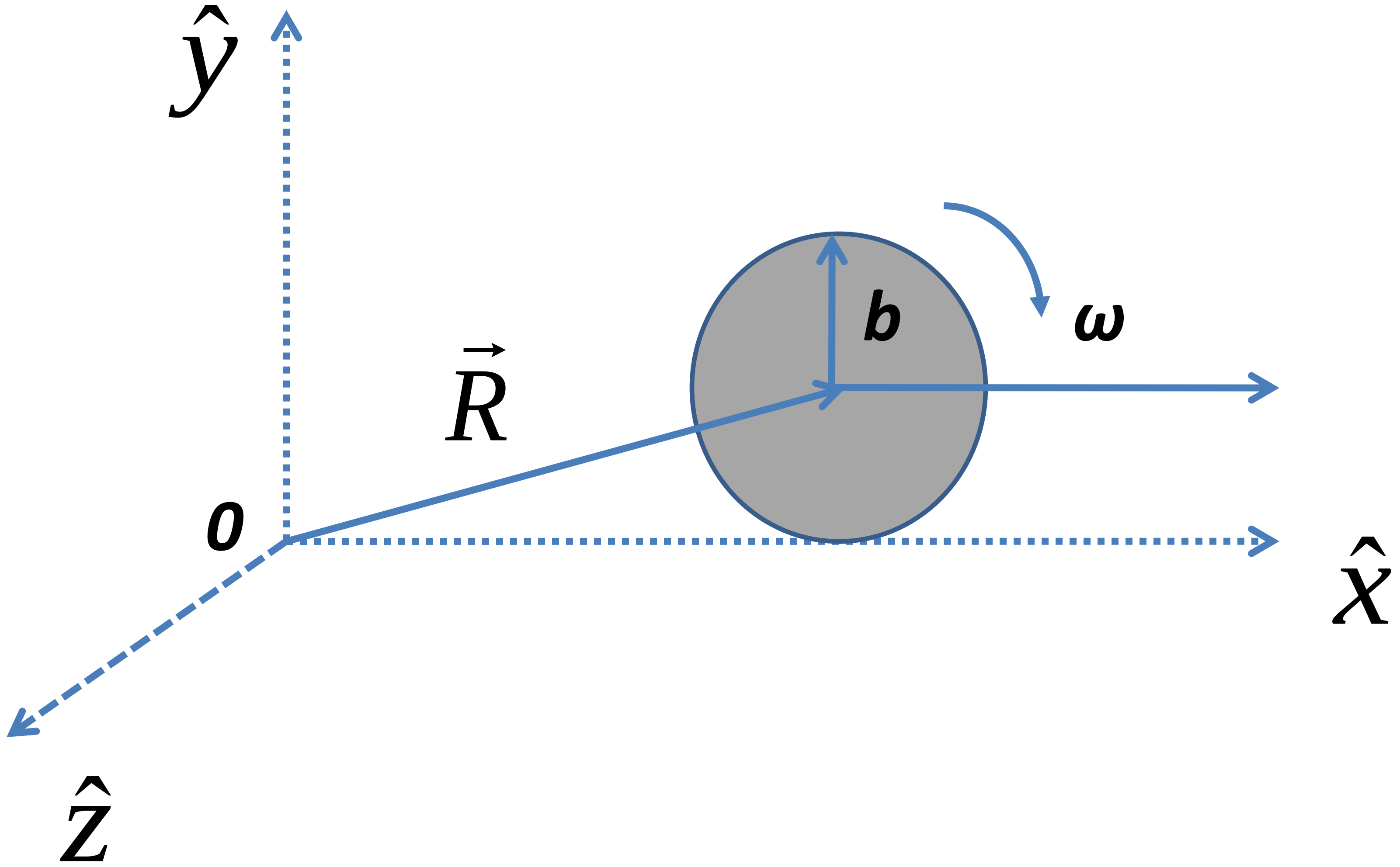


Motion Involving Both Translation & Rotation



$$L_z = I_0\omega + \left(\vec{R} \times M\vec{V}\right)_z$$

Angular Momentum of a Rolling Wheel

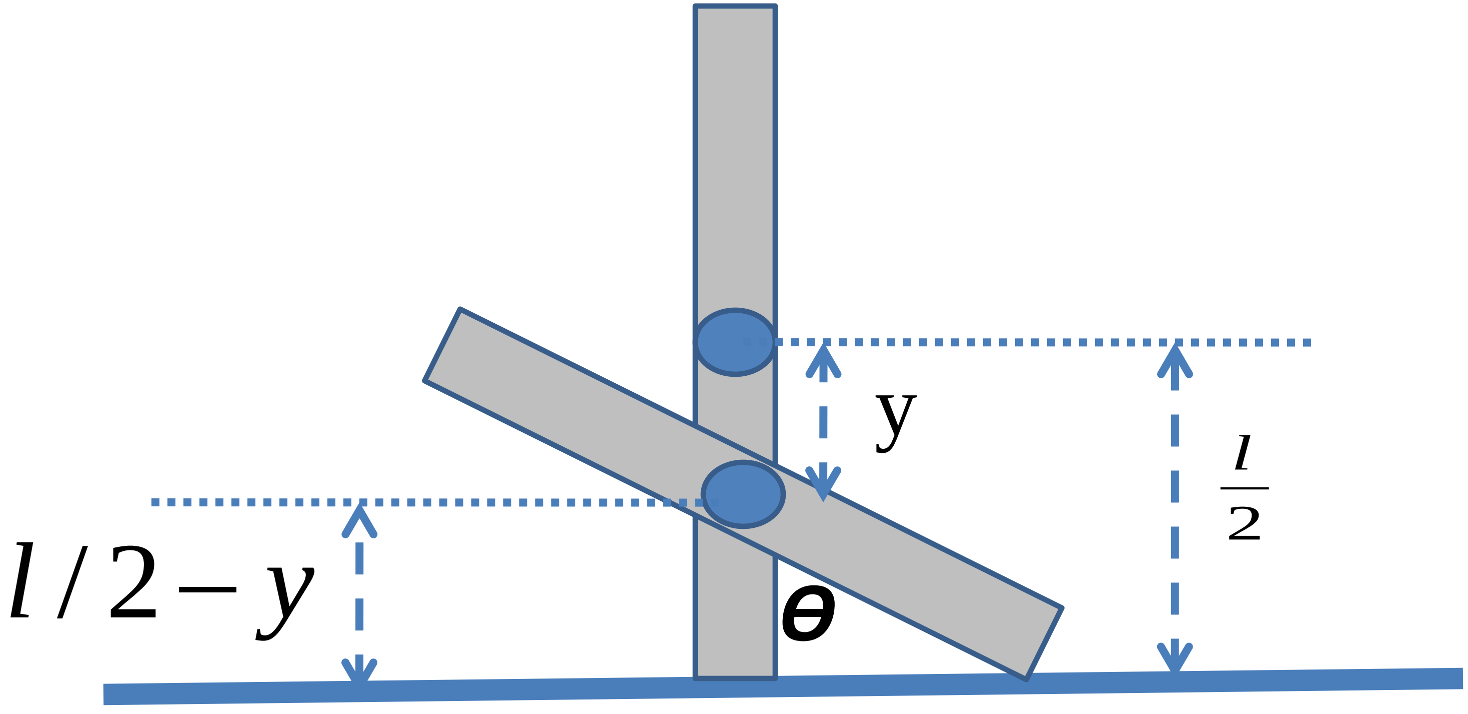


Torque also naturally divides itself into two components

Kinetic Energy

The Work Energy Theorem

Problem: The Falling Stick :



Conditions of Equilibrium for a Rigid Body

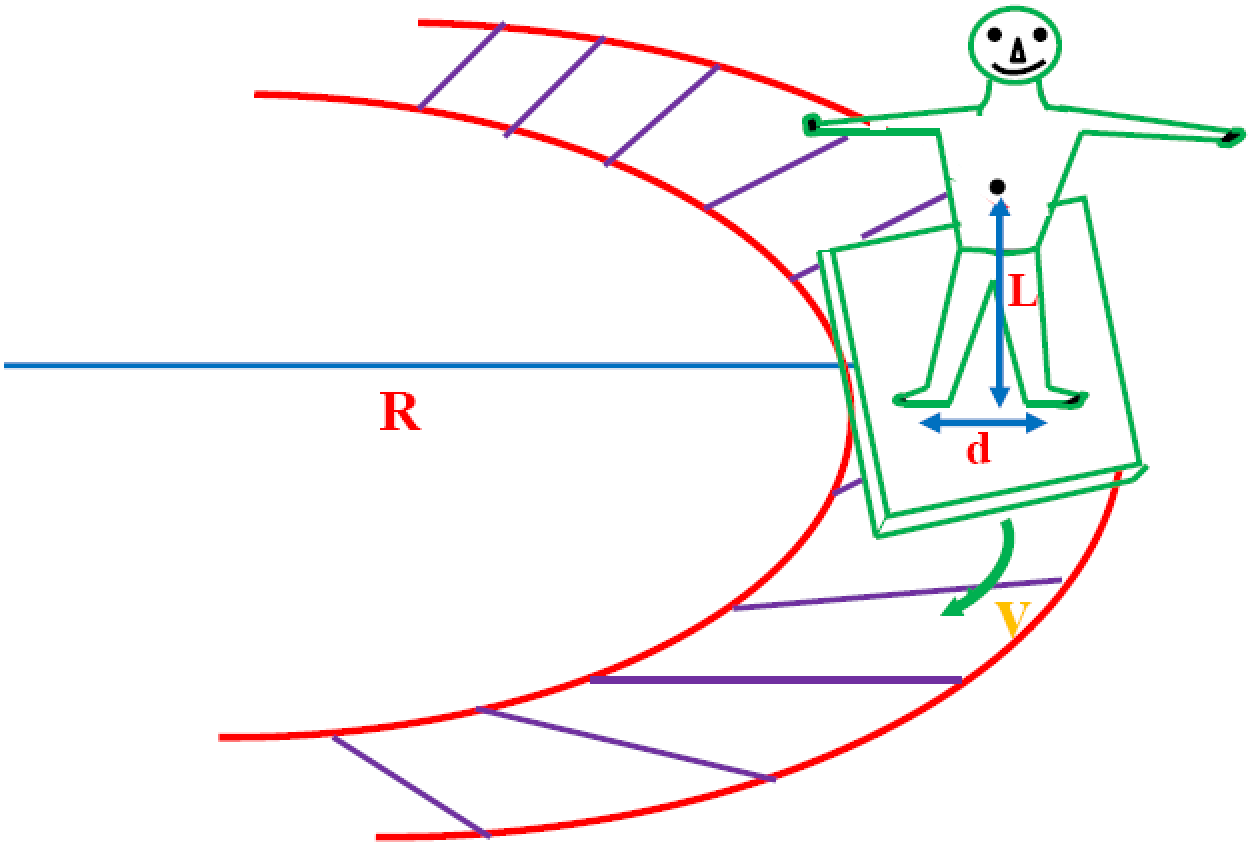
$$\sum \vec{F}_{ext} = 0$$

$$\sum \vec{\tau}_{ext} = 0$$

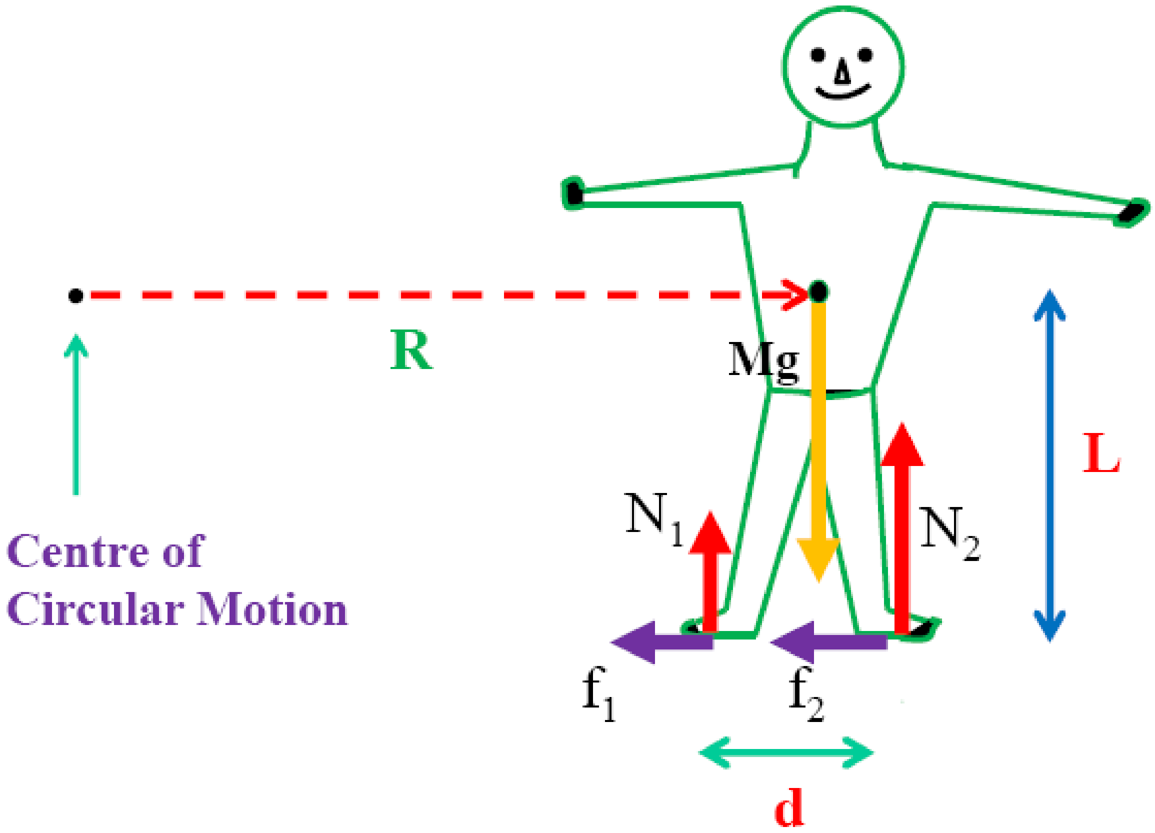
(About an Inertial Point)

Problem: 6.6 A man of mass M stands on a railroad car which is rounding an unbanked turn of radius R at a speed v. His center of mass is height L above the car, and his feet are distance d apart. The man is facing the direction of motion. How much weight is on each of his feet?

Problem: 6.6

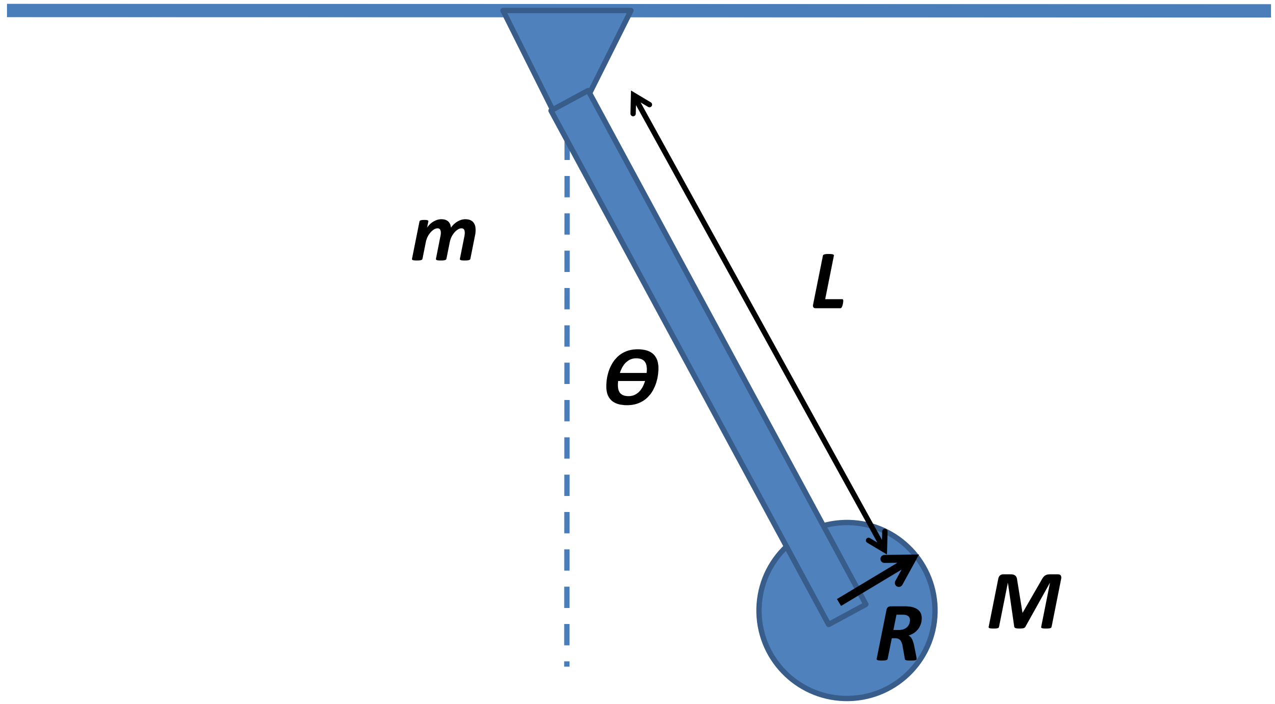


Problem: 6.6



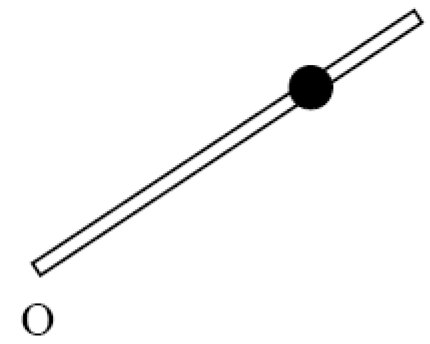
Problem: 6.18 Find the period of a pendulum consisting of a disc of mass M and radius R fixed to the end of a rod of length L and mass m. How does the period change if the disk is mounted to the rod by a frictionless bearing so that it is perfectly free to rotate?

Problem: 6.18



Prob. 6.22

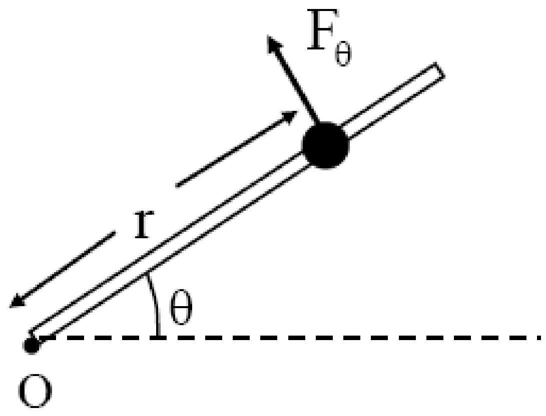
A bead of mass m slides without friction on a rod that is made to rotate at a constant angular velocity ω. Neglect gravity



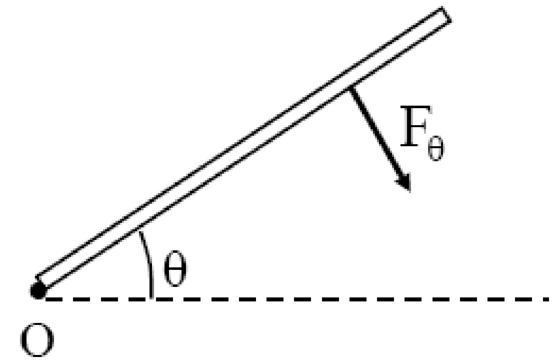
a) Show that $r = r_0 e^{\omega t}$ is a possible motion of the bead.

b) For the motion described in part a, find the force exerted on the bead by the rod.

c) Find the power exerted by the agency that is turning the rod and show that it is the rate of change of KE of the bead.



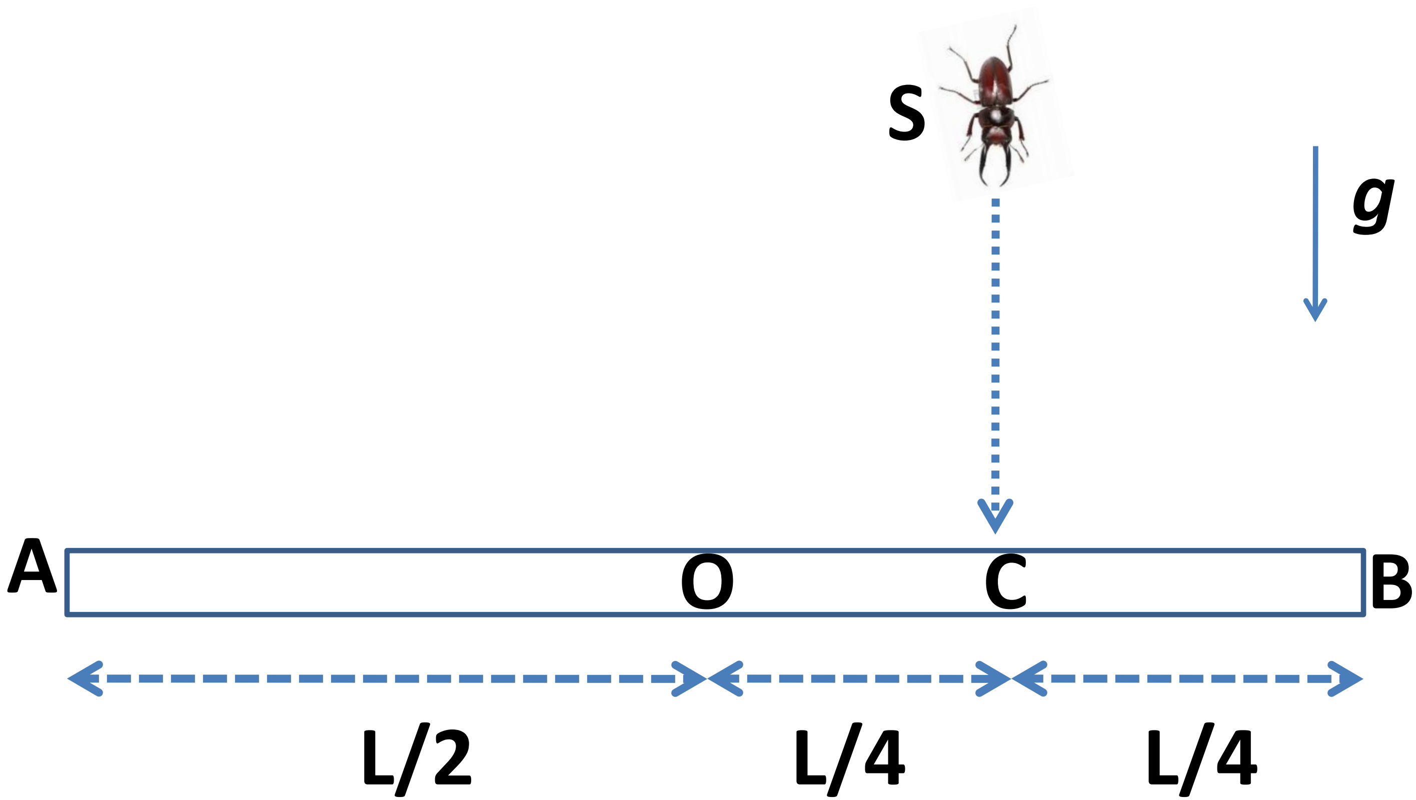
Force diagram of bead



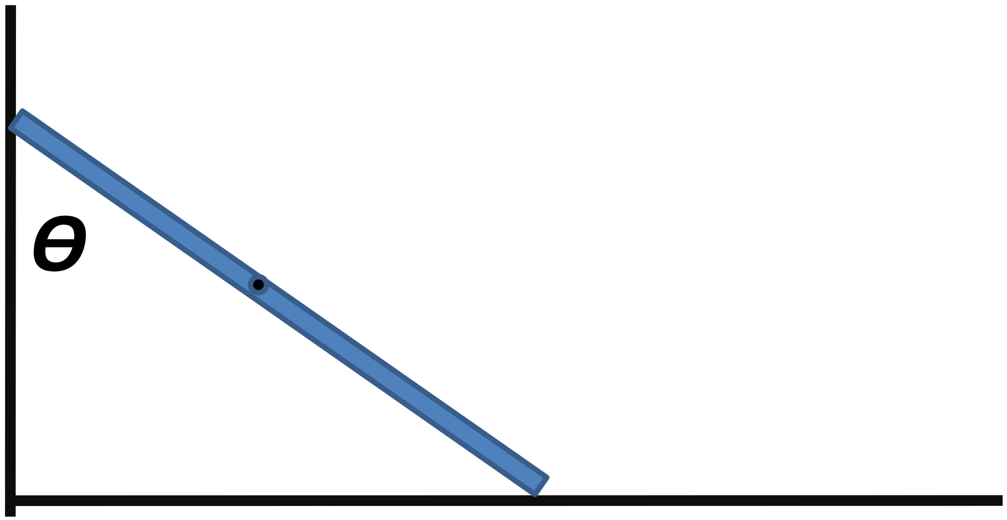
Force diagram of rod

Problem: A rod AB of length L=1.8m and Mass M is pivoted at the center O in such a way that it can rotate freely the vertical plane. The rod is initially in the horizontal , position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling, the insect moves toward the end B such that the rod rotates with a constant angular velocity ω.

(a) Determine ω in terms of v and L.
(b) If the insect reaches the end B when the rod has turned through an angle of 90^0, determine v.


S
g
A
O
C
B
L/2
L/4
L/4

Problem: A uniform ladder of length 2a is supported by a smooth horizontal floor and leans against a smooth vertical wall. The ladder is released from rest in a position making an angle of 60^0 with the downward vertical. Find the energy conservation equation for the ladder.

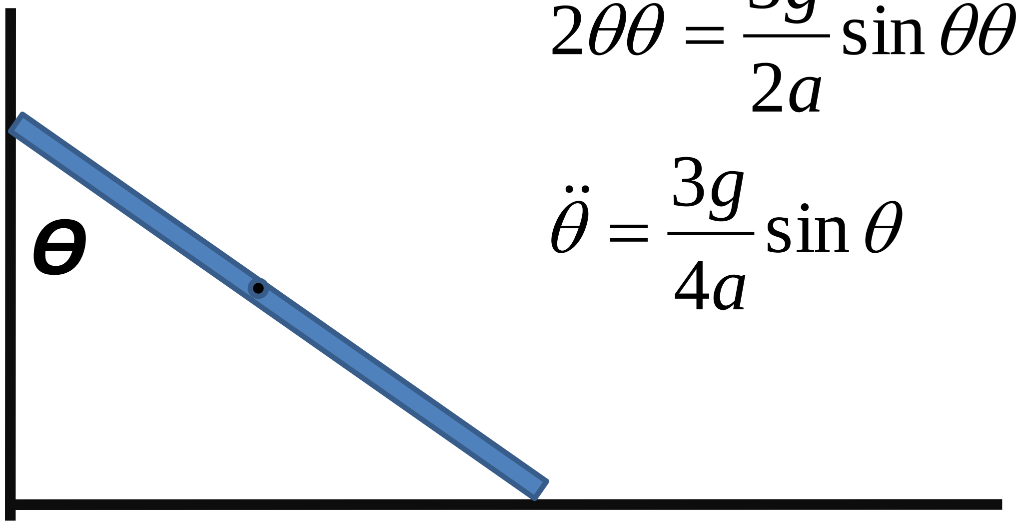


Problem: Show that the rod doesn't maintain contact with the wall all the way down, but leaves the wall when Θ becomes equal to $\cos^{-}(1/3) \approx 71^0$

$$\dot{\theta}^2 = \frac{3g}{4a}\left(1 - 2\cos\theta\right)$$

$$2\dot{\theta}\ddot{\theta} = \frac{3g}{2a}\sin\theta\dot{\theta}$$

$$\ddot{\theta} = \frac{3g}{4a}\sin\theta$$



The ladder will lose its contact when:

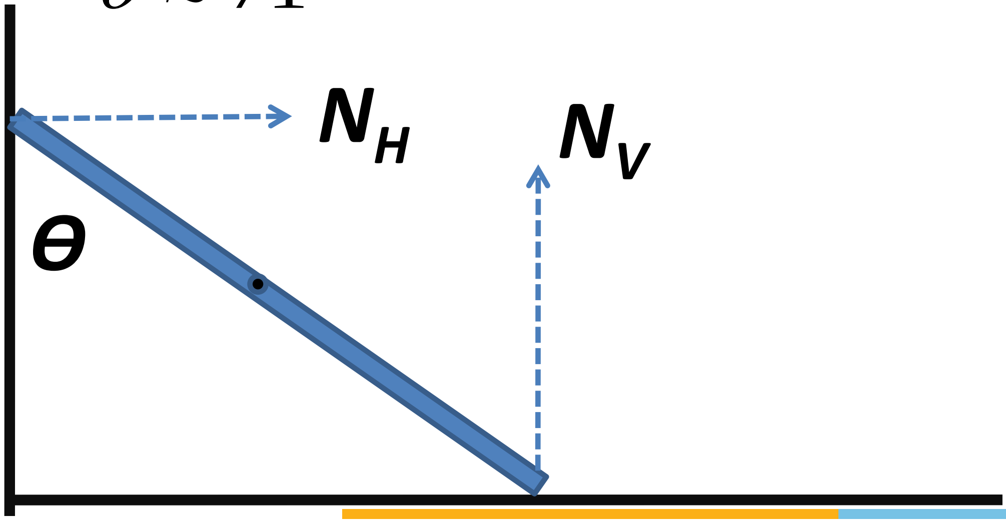
$$N_H = M\ddot{x} = 0$$

$$\ddot{x} = -a(\sin\theta)\dot{\theta}^2 + a(\cos\theta)\ddot{\theta} = 0$$

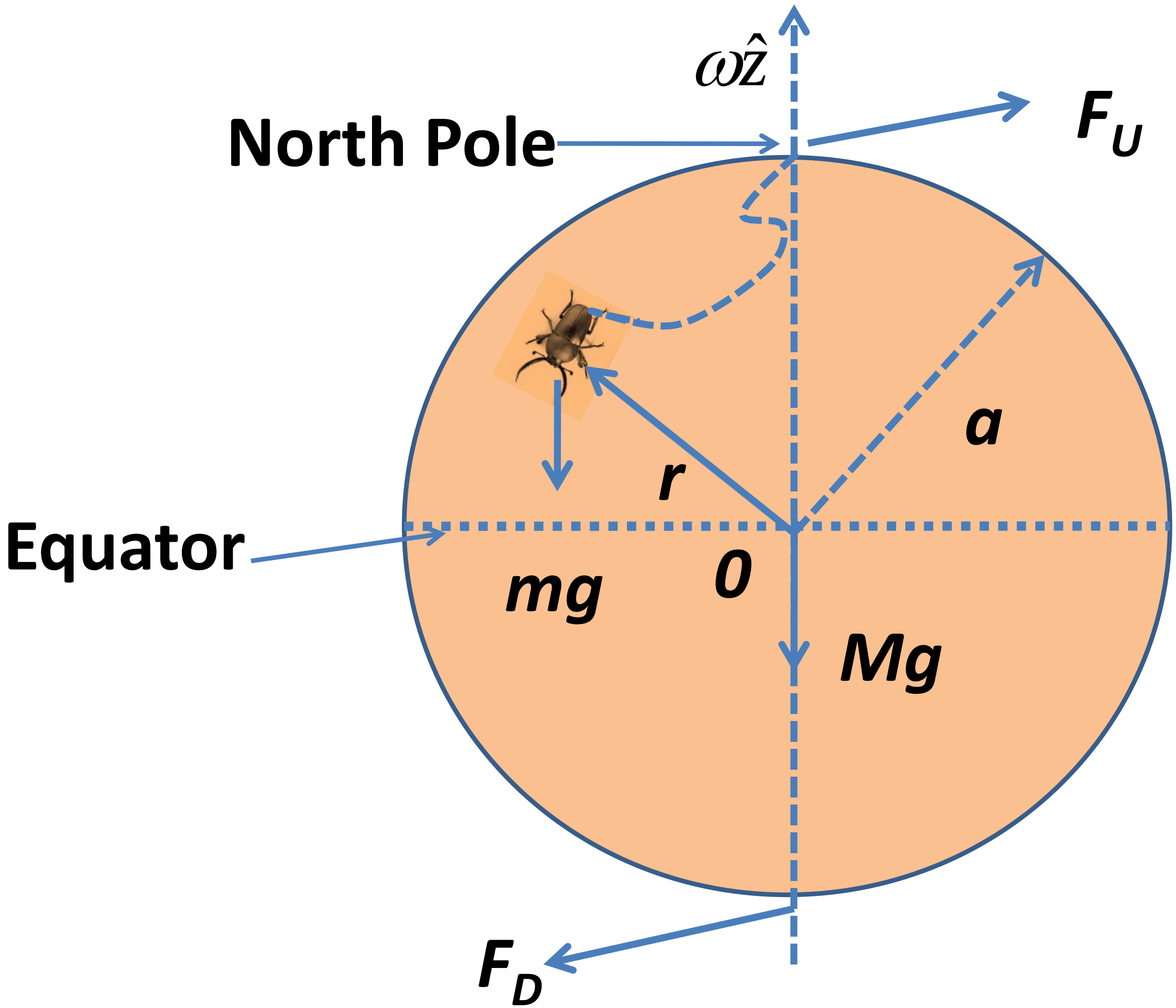
$$a(-\sin\theta)\left[\frac{3g}{4a}(1-2\cos\theta)\right] + a\cos\theta\left(\frac{3g}{4a}\sin\theta\right) = 0$$

$$\cos\theta = \frac{1}{3}$$

$$\theta \approx 71^0$$



Problem: A uniform ball of mass M and radius a is pivoted so that it can turn freely about one of its diameters which is fixed in a vertical position. A beetle of mass m can crawl on the surface of the ball. Initially the ball is rotating with angular speed ω with the beetle at the North Pole. The beetle then walks in any manner to the equator of the ball and sits down. What is the angular speed of the ball now?



$\omega\hat{z}$
North Pole
F_U
a
r
Equator
mg
0
Mg
F_D

Summary of Dynamical Formulas for Fixed Axis Motion

- Pure Rotation About an Axis-No Translation

$$L = I\omega$$

$$\tau = I\alpha$$

$$K = \frac{1}{2} I\omega^2$$

➢Rotation and Translation (Subscript 0 refers to Center of Mass)

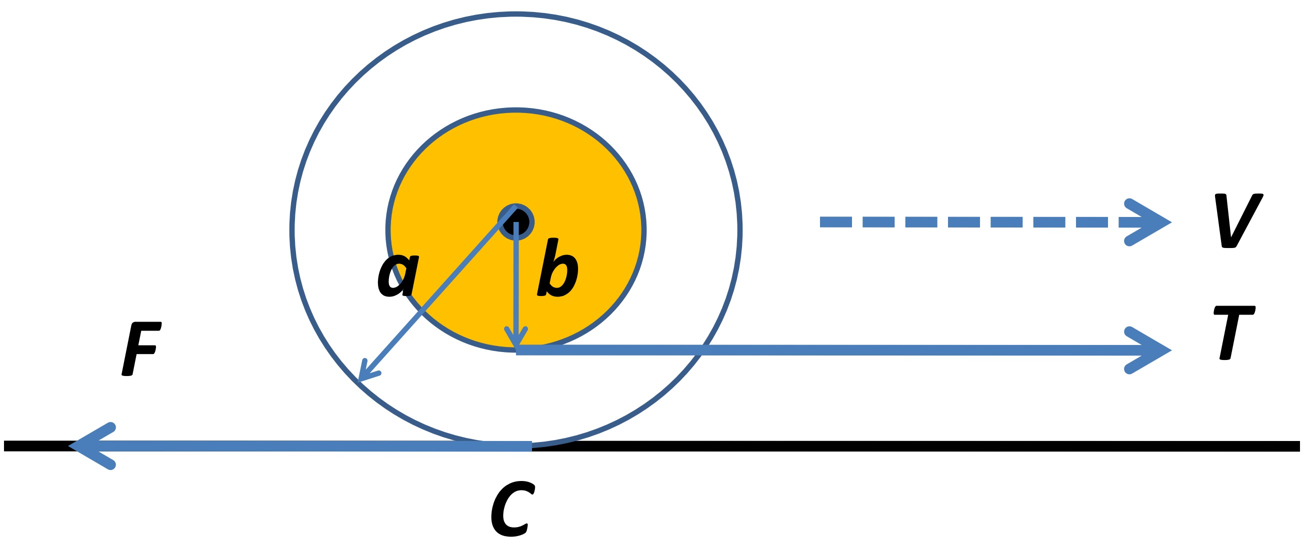
$$L_z = I_0\omega + \left(\vec{R} \times M\vec{V}\right)_z$$

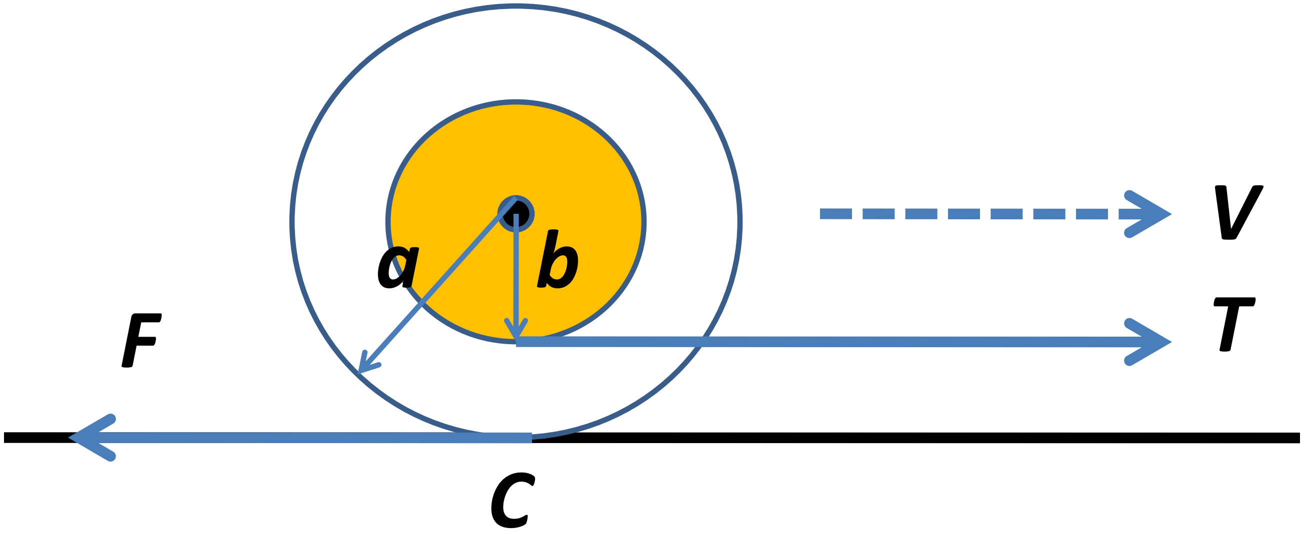
$$\tau_z = \tau_0 + \left(\vec{R} \times \vec{F}\right)_z$$

$$\tau_0 = I_0\alpha$$

$$K = \frac{1}{2} I_0\omega^2 + \frac{1}{2} MV^2$$

Problem: A cotton flat tape reel is at rest on a rough horizontal table when the free end of the flat tape is pulled horizontally with a constant force T. Given that the reel undergoes planer motion, how does it move?

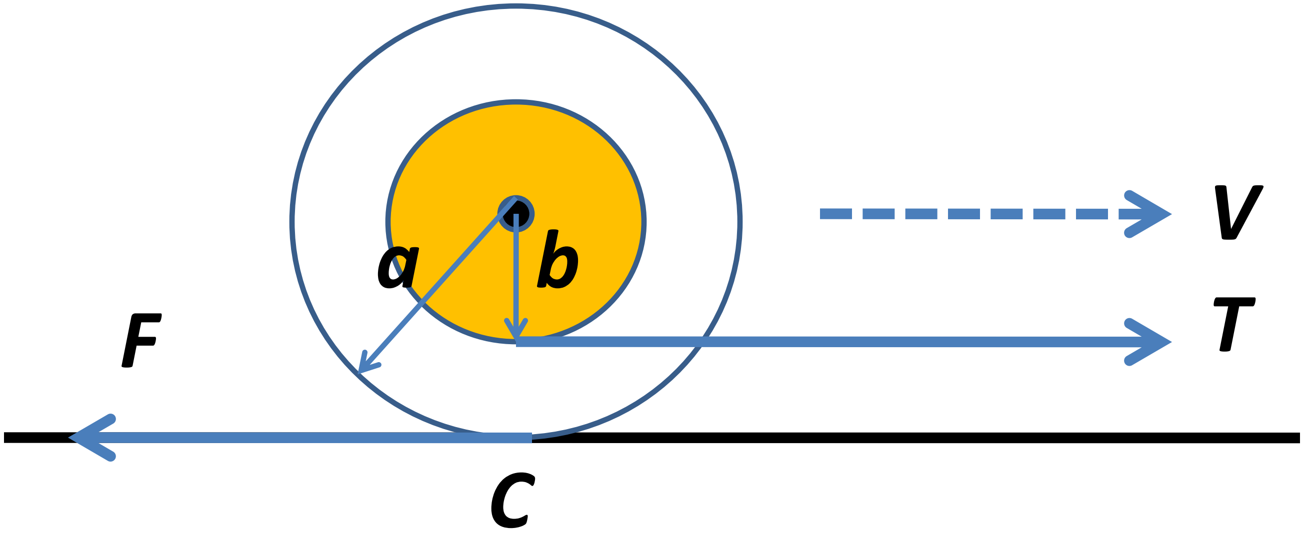




The Equation of Motions:

$$M\frac{dV}{dt} = T - F$$

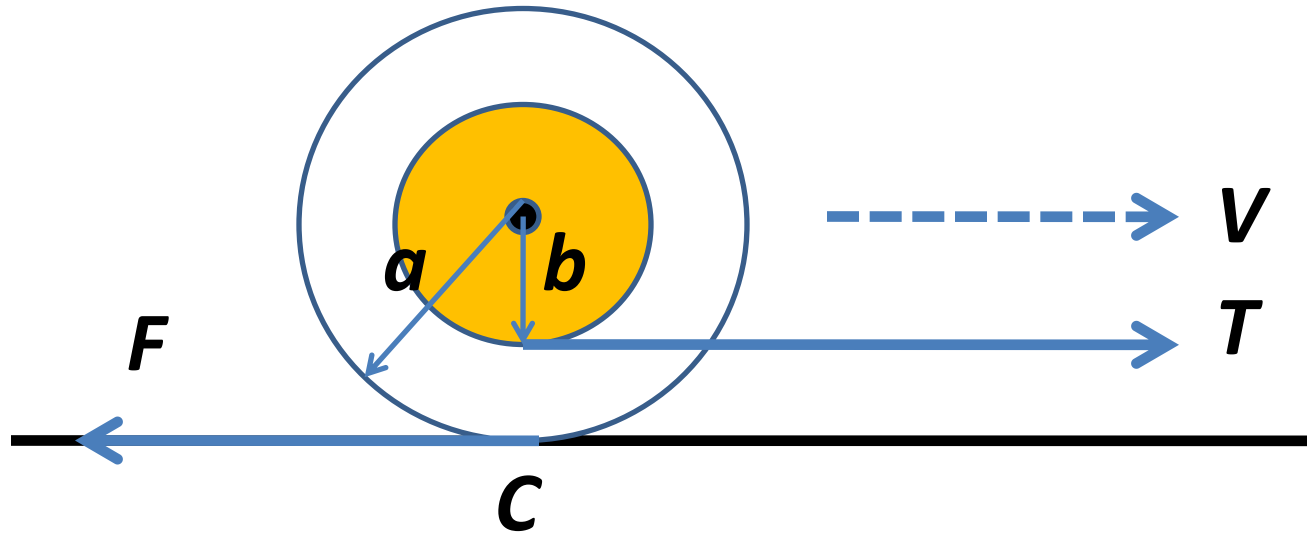
$$Mk^2\frac{d\omega}{dt} = aF - bT$$



Let's don't make any assumption: Whether the Reel slides or rolls.

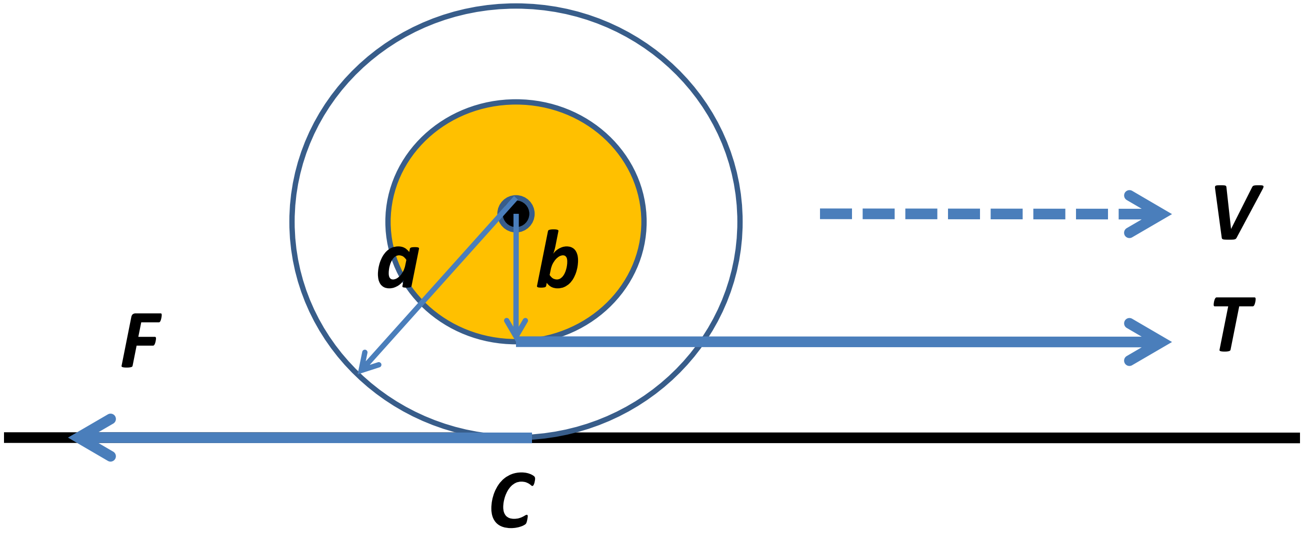
Also, lets presume that the frictional force bounded in magnitude by some maximum F^{Max}

$$-F^{Max} \leq F \leq F^{Max}$$



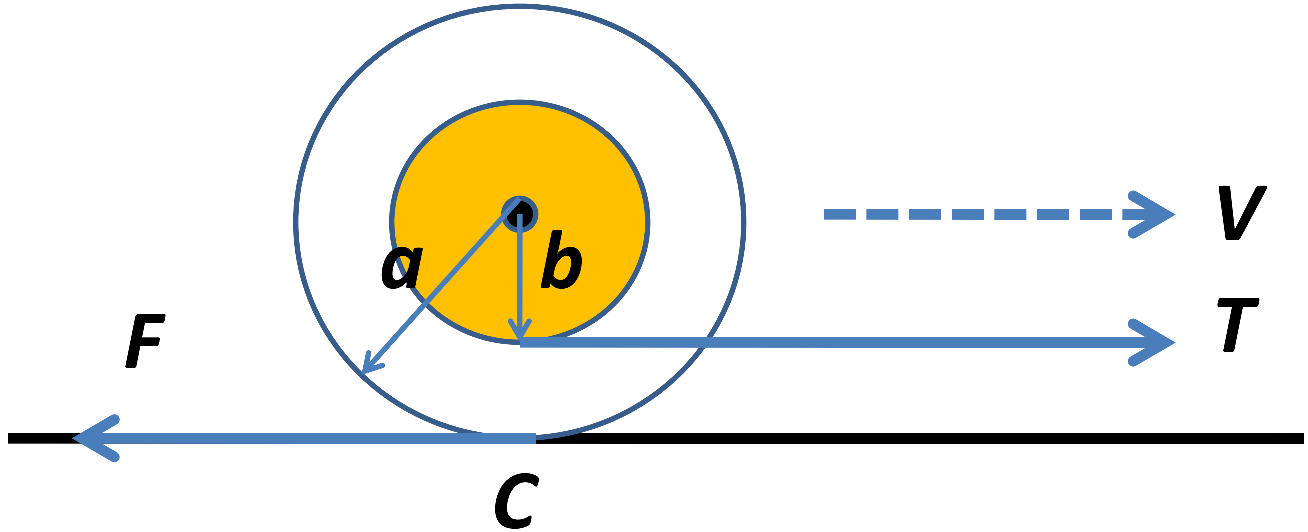
Now, the reel slides or rolls depends on V^C; the velocity of the contact particle C.

$$V^C = V - \omega a$$



$$V^C = V - \omega a$$

$$\frac{dV^C}{dt} = \frac{dV}{dt} - a\frac{d\omega}{dt}$$

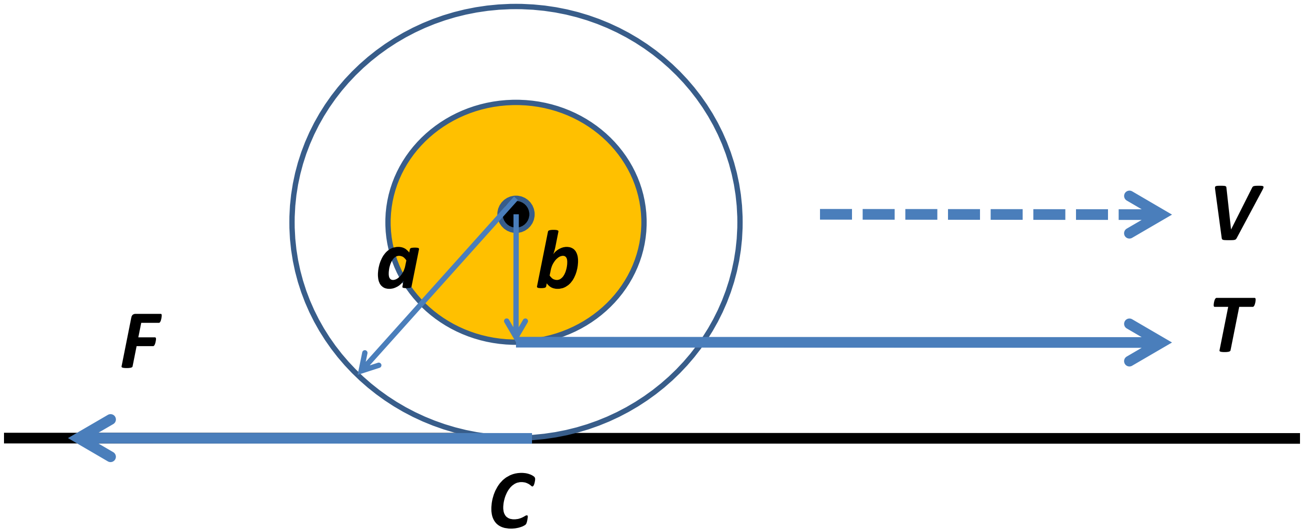


The Equation of Motions:

$$M\frac{dV}{dt} = T - F$$

$$Mk^2\frac{d\omega}{dt} = aF - bT$$

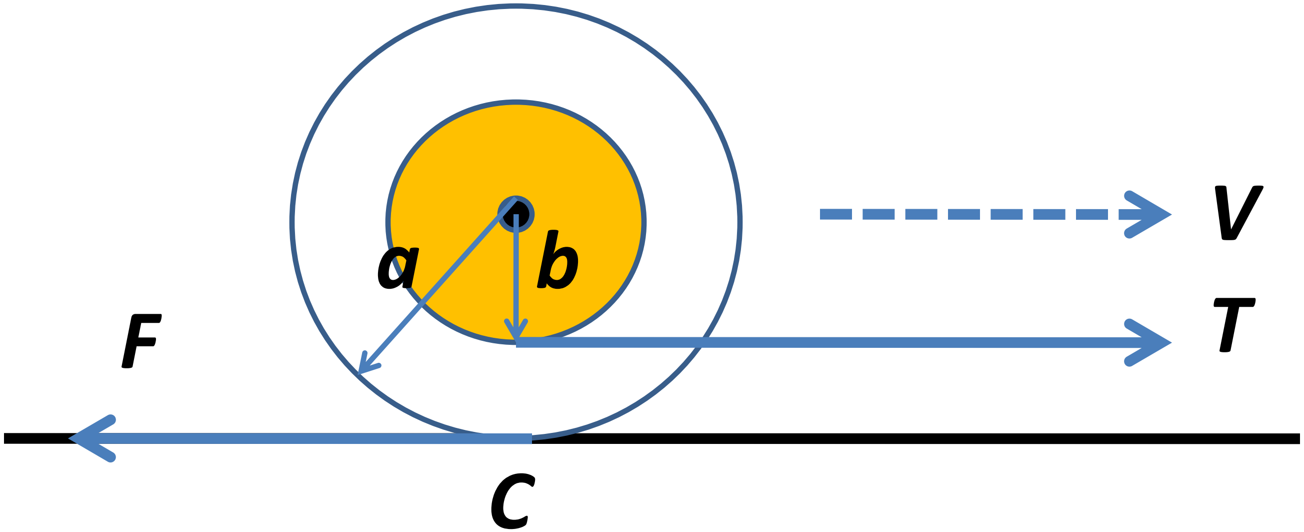
$$\frac{dV^C}{dt} = \frac{dV}{dt} - a\frac{d\omega}{dt}$$



$$\frac{Mk^2}{k^2+ab}\frac{dV^C}{dt}=T-\gamma F$$

where:

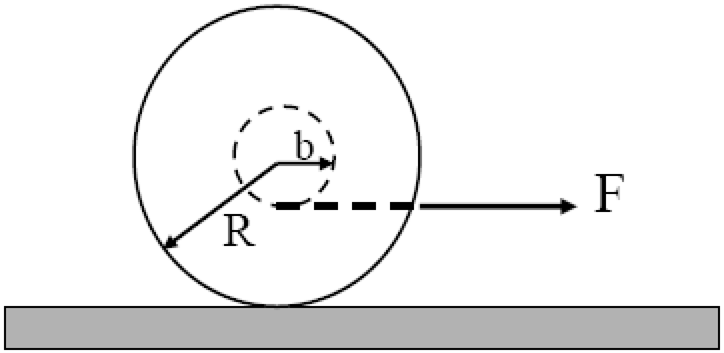
$$\gamma=\frac{k^2+a^2}{k^2+ab}$$



(i) Hard Pull: $T > \gamma F^{Max}$

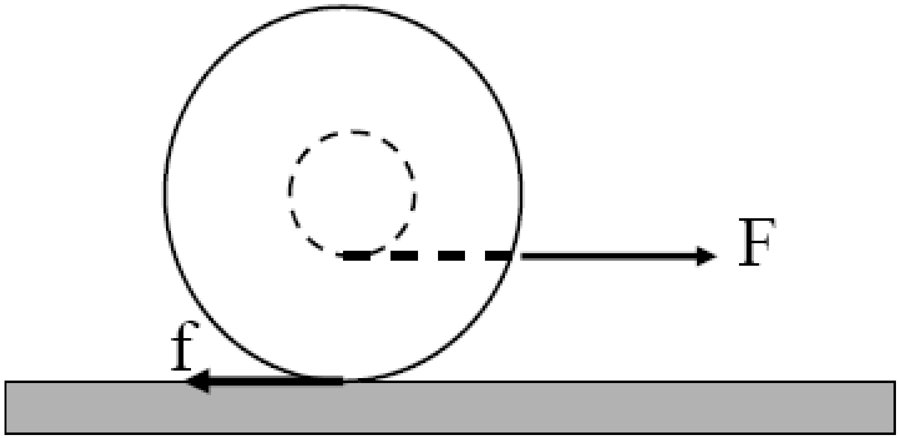
(ii) Gentle Pull: $T < \gamma F^{Max}$

Prob. 6.26



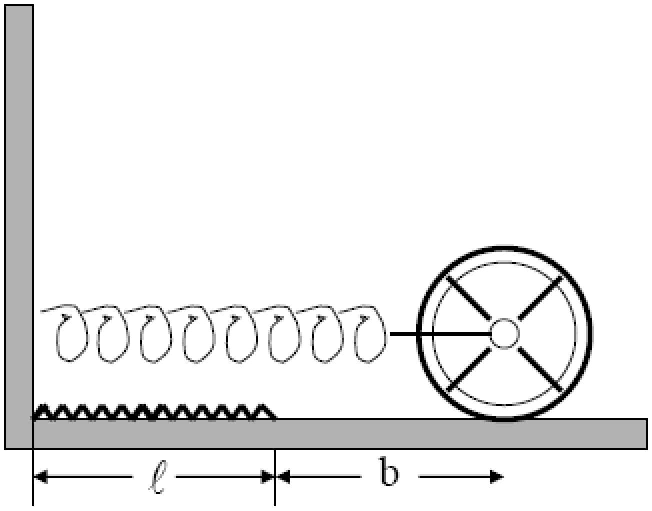
A Yo-Yo of mass M has an axle of radius b and a spool of radius R. Its MI can be taken to be $MR^2/2$. The Yo-Yo is placed upright on a table and the string is pulled with a force F. The coeff. of friction between the Yo-Yo and the table is μ.

What is the maximum value of F for which the Yo-Yo will roll without slipping?


F
f

Prob. 6.40

A wheel with fine teeth is attached to the end of a spring with spring constant k and unstretched length ℓ .

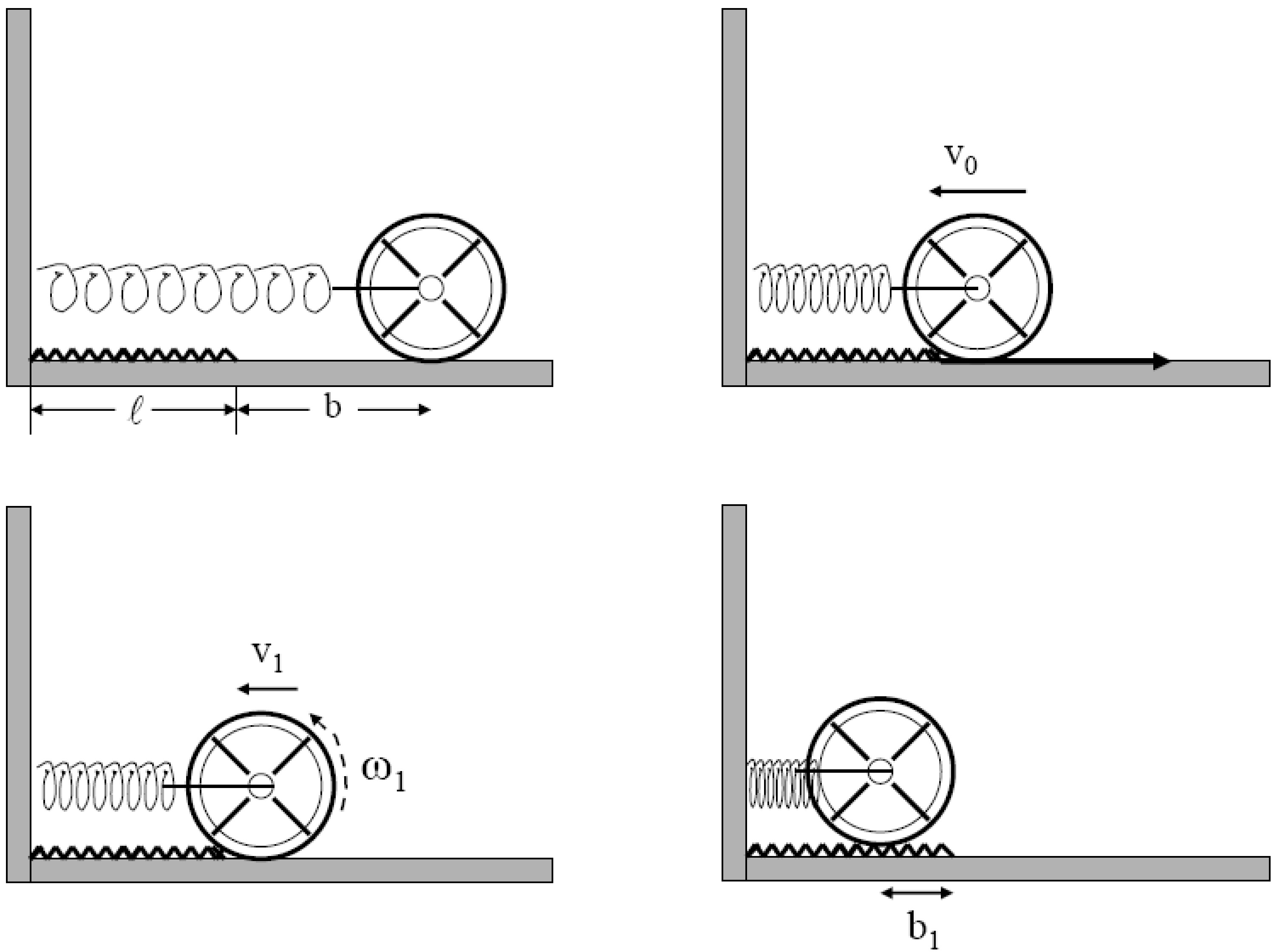


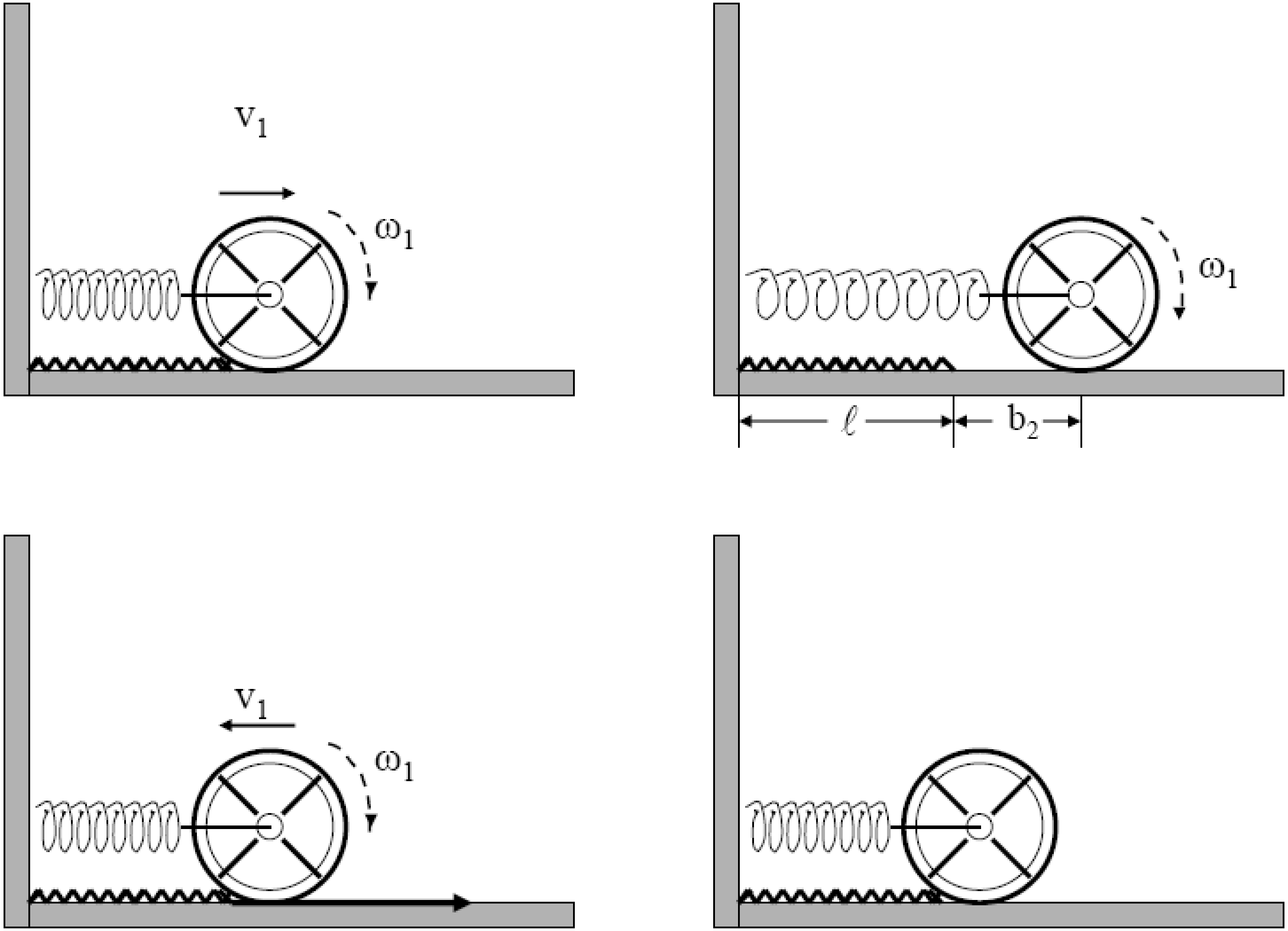
For $x > \ell$, the wheel slips freely on the surface, but for $x < \ell$, the teeth mesh with the teeth on the ground so that it cannot slip. Assume that all the mass of the wheel is in its rim.

a) The wheel is pulled to $x = \ell + b$ and released. How close will it come to the wall on its first trip?

b) How far out will it go as it leaves the wall?

c) What happens when the wheel hits the gear track?

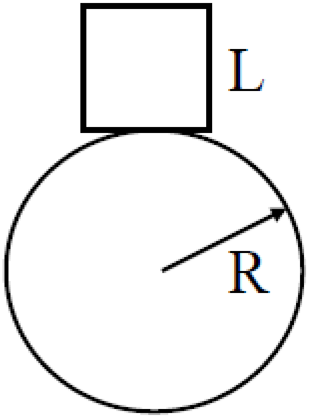

ℓ
b
v0
v1
ω1
b1

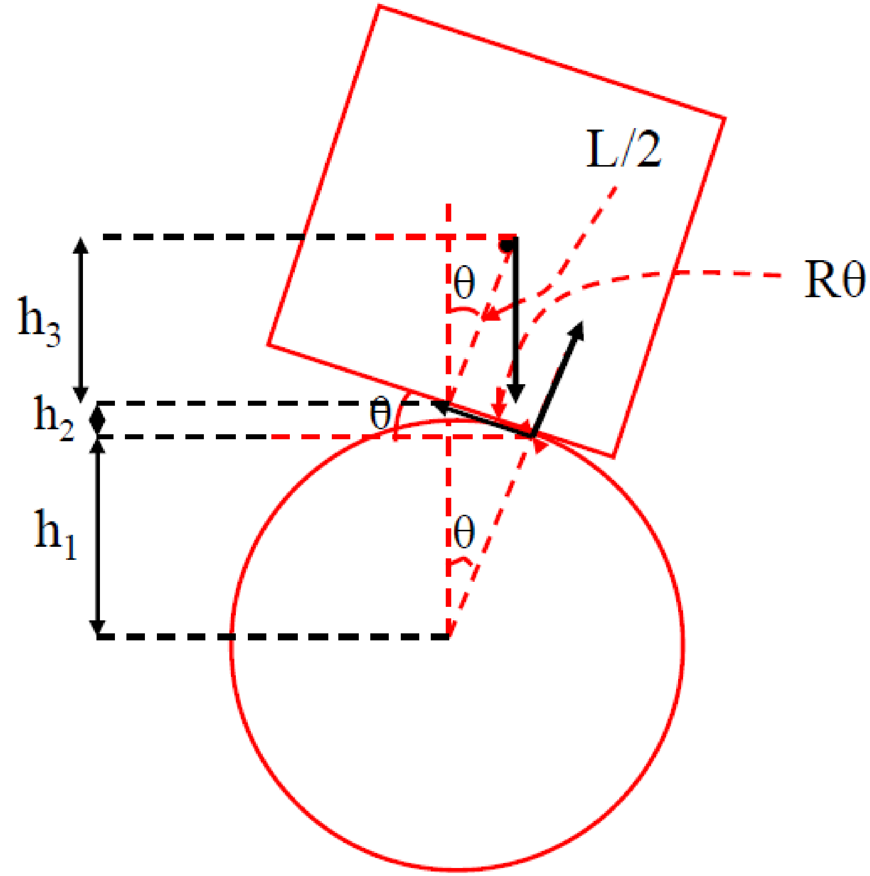


v_1
ω_1
ω_1
ℓ
b_2
v_1
ω_1

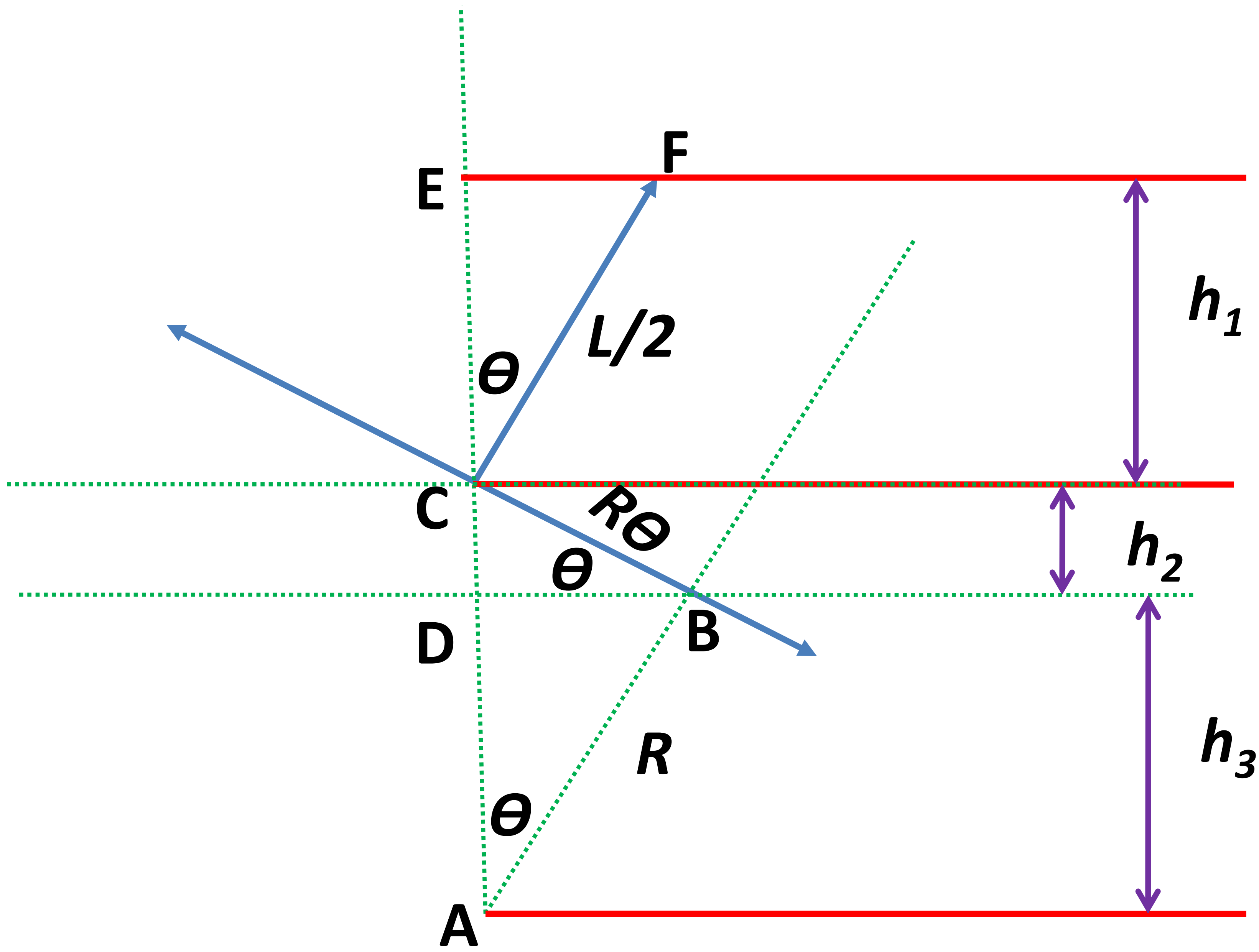
Prob. 6.35

A cubical block of side L rests on a fixed cylindrical drum of radius R. Find the largest value of L for which the block is stable.



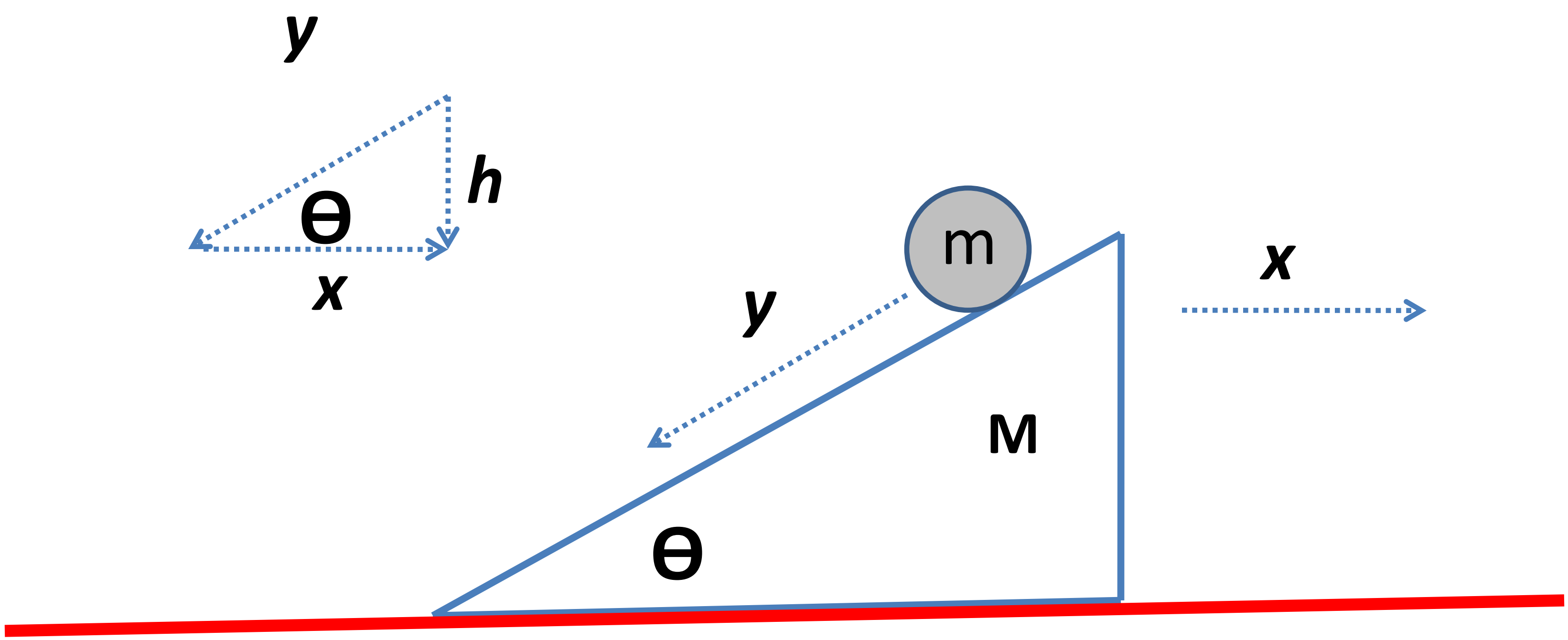


$$h = h_1 + h_2 + h_3$$

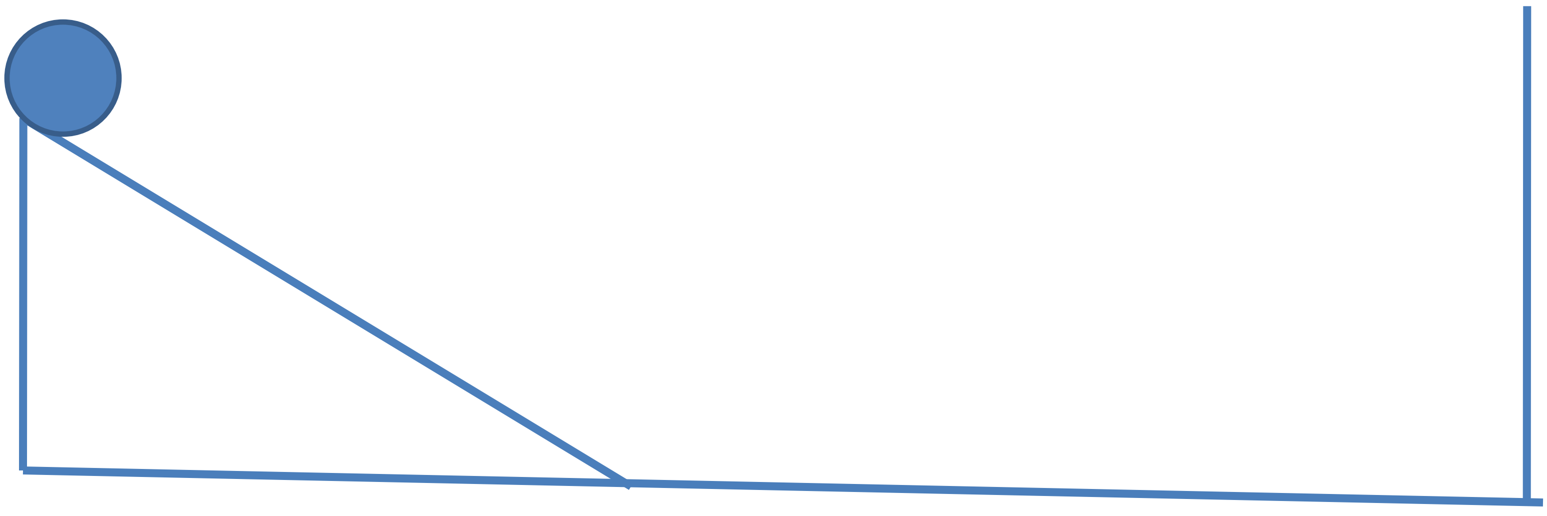

E
F
L/2
θ
h1
C
Rθ
θ
h2
D
B
R
θ
h3
A

Problem: A carpet of mass M. made from an inextensible material, is rolled along its length in the form of a solid cylinder of radius R and kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/3.

Problem: A spherical shell rolls down without slipping along the inclined plane of the wedge which is kept on smooth horizontal plane. Calculate the expression of the acceleration of the wedge.



Problem: A roller with $I_0=(1/3)MR^2$ is released from rest at the top of an incline of height h. It rolls without slipping down the incline, and then a long way along a horizontal road until it collides elastically with a smooth rigid vertical wall. When the roller again Begins to roll without slipping, what is the speed of the centre of mass?



Problem 6.34 A marble of radius b rolls back and forth in a shallow dish of radius R. Find the frequency of small oscillations. R>>b