BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: PREDICATE LOGIC

Predicate Logic
– Proofs and Proof Rules: Introduction

Universally Quantified Variables and Inference

- Given a statement of the form:
    - **∀X (bird(X) --> fly(X))**
  - one can infer the following:
    - **bird(fancy_piggy) --> fly(fancy_piggy)**
    - **bird(seagull(jonathon)) --> fly(seagull(jonathon))**

- This amounts to an *inference rule*:
  - If a statement with a universally quantified variable, say **X**, is true
    - then any instance of that statement – i.e. *the statement with any term substituted for* **X** – should be true.

Inference Rule for Universally Quantified Formulas

- The inference rule can be stated formally as follows:
  - Proof Rule (*Elimination of universal quantifier*):

$$\frac{\forall X\ \varphi}{\varphi\,[t/X] \quad \mathbf{t}\ \textit{is a term}}\ \forall e$$

  - where $\varphi[t/X]$ denotes
    - *the formula* φ *in which each occurrence of variable* **X** *has been replaced with term* **t**.

Inference Rule for Universally Quantified Formulas

- Proof Rule (*Elimination of universal quantifier*):

$$\frac{\forall \mathbf{X}\ \varphi}{\varphi\,[\mathbf{t}/\mathbf{X}]}\ \forall\mathbf{e}$$

t *is a term*

- φ**[t/X]** is usually read as:
  - φ with **t** (*substituted*) for **X**
- *Note that* φ**[t/X]** *is not a formula (in our language):*
  - it denotes the substitution operation that results in a formula

Existentially Quantified Variables and Inference

- Given a statement of the form:
    - **mammal(platypus) $\wedge$ lays_eggs(platypus)**
  - one can infer the following:
    - **$\exists$X (mammal(X) $\wedge$ lays_eggs(X))**
- Given a statement of the form:
    - **ugly(duckling) $\wedge$ becomes(swan(duckling))**
  - one can infer the following:
    - **$\exists$X (ugly(X) $\wedge$ becomes(swan(X)))**

Existentially Quantified Variables and Inference

- From: **mammal(platypus) ∧ lays_eggs(platypus)**
  - infer : **∃X (mammal(X) ∧ lays_eggs(X))**
- From: **ugly(duckling) ∧ becomes(swan(duckling))**
  - infer : **∃X (ugly(X) ∧ becomes(swan(X)))**
- This kind of reasoning amounts to an *inference rule*:
  - If a statement is true for a concrete value
    - then the same statement existentially quantified by a variable, say **X**, is true
      - where **X** replaced by the concrete instance yields the original statement.

Inferring Existentially Quantified Formulas

- The inference rule can be stated formally as follows:
  - Proof Rule (Introduction of existential quantifier):

$$\boxed{\dfrac{\varphi[t/X] \qquad \mathbf{t} \textit{ is a term}}{\exists X\, \varphi}} \quad \exists\mathbf{i}$$

  - where denotes φ**[t/X]** the formula φ in which each occurrence of variable **X** has been replaced with term **t**.

- Note that the ***substitution*** operation referred here is the same one in the proof rule $\forall$**e**
  - but this proof rule uses it in reverse