DISCRETE STRUCTURE FOR COMP. SCI. (CS F222)

Practice Problems

Quiz-1

Recursion

1. A computer system considers a string of decimal digits a valid code-word if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let a_n be the number of valid n-digit code-words. Find a recurrence relation for a_n.

2. Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0's.

3. Solve the following recurrence relations
   (i) $a_n = 5a_{n-1} - 4a_{n-2}$, $a_0 = 1,\ a_1 = 0$
   (ii) $a_n = 3na_{n-1}$, $a_0 = 2$
   (iii) $a_n = 2a_{n-1} + 1$

4. Find a recurrence relation with initial condition(s) satisfied by the following sequences. Assume a_0 is the first term of the sequence.
   (i) $a_n = 2^n$.
   (ii) $a_n = 3n - 1$.

5. Consider a savings plan in which $10 is deposited per month, and a 6% / year interest rate given with payments made every month. If P_n represents the amount in the account after n months, find a recurrence relation for P_n.

6. Give the recursive definition with initial condition for the function f(n) = 5n + 2, n = 1,2,3, ...

7. Consider the following recursive function f(x; y) s.t. f(x; 0) = 0; for all x, and f(x; y) = f(x; y - 1) + x, where x; y are non-negative integers. What does f(x; y) calculate?

8. Let T(n) be the number of comparisons required to find the minimum and maximum integers from a list of n positive integers. Which of the following value of T(n) is correct?

   (A) T(n) = T(n - 1) + 1
   (B) T(n) = T(n - 1) + 2
   (C) T(n) = T(n/2) + 1
   (D) T(n) = T(n/2) + 2

Relations

1. The number of relations on an "n" element set that are symmetric is:
   (A) 2^{n^2} (B) $2^{n(n-1)}$ (C) $2^{\frac{n(n+1)}{2}}$ (D) $3^{\frac{n(n-1)}{2}}$

2. Let R be the relation on N given by xRy iff x divides y. Determine which of the following properties applies to each relation.
   (i) Reflexive (ii) Irreflexive (iii) Symmetric (iv) Antisymmetric (v) Asymmetric (vi) Transitive

3. Let R be the relation on R given by xRy if and only if x < y + 1.
   (A) Reflexive, but not symmetric and not transitive.
   (B) Reflexive, symmetric and not transitive.
   (C) Not Reflexive, not symmetric and not transitive.
   (D) Reflexive, but not symmetric and transitive.

4. Which of the relations on the given sets are antisymmetric?
   (S1) A = {1, 2, 3, 4, 5}, R = {(1, 3), (1, 1), (2, 4), (3, 2), (5, 4), (4, 2)}
   (S2) set of real numbers xRy iff $x^2 = y^2$.
   (A) Only S1 (B) Only S2 (C) Both S1 and S2 (D) Nither S1 nor S2

5. Which relation R is not transitive?
   (A) {(1, 1), (2, 2)} (B) {(1, 2), (2,3), (1, 3)}
   (C) {(1, 2), (2, 1), (1, 1), (2,2)} (D) {(1, 2), (3, 3)}

6. Which of the following relations are reflexive, symmetric and antisymmetric?

   R1 = {(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)}

   R2 = {(1,1),(1,2),(2,1)}

   R3 = {(1,1),(1,2), (1,4),(2,1),(2,2),(3,3),(4,1),(4,4)}

   R4 = {(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)}

   R5 = {(1,1),(1,2), (1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)}

   R1 = {(3,4)}

7. Let A and B be the set of all students and the set of all courses at a school, respectively. Suppose that R1 consists of all ordered pairs (a, b), where a is a student who has taken course b, and R2 consists of all ordered pairs (a, b), where a is a student who requires course b to graduate. What are the relations
    a. R1 ∪ R2
    b. R1 ∩ R2
    c. R1 ⊕ R2
    d. R1 − R2
    e. R2 − R1?

8. Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ∘ R.

9. Let R be the relation on the set of people consisting of pairs (a, b), where a is a parent of b. Let S be the relation on the set of people consisting of pairs (a, b), where a and b are siblings (brothers or sisters). What are S∘R and R∘S?

10. Suppose that the relation R on a set is represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

    Is R reflexive, symmetric, and/or antisymmetric?

11. Suppose that the relations R1 and R2 on a set A are represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

    What are the matrices representing R1 ∪ R2 and R1 ∩ R2?

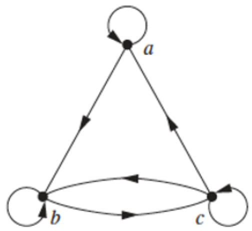
12. Find the matrix representing the relation S ∘R, where the matrices representing R and S are:

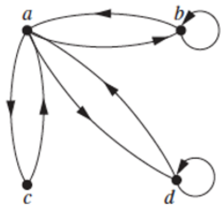
$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

13. Find the matrix representing the relation R^2, where the matrix representing R is:

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

14. Determine whether the relations for the directed graphs shown in the following figures are reflexive, symmetric, antisymmetric, and/or transitive.





15. Which of the following is equivalence relation?
    (A) <= on Z.
    (B) R = {(1, 2), (2, 3), (3, 1)} on the set {1, 2, 3}.
    (C) | on Z, i.e. divide on Z
    (D) R = {1, 2, 3} × {1, 2, 3} on the set {1, 2, 3}.

16. Which matrix represents an equivalence relation?

    (A) $\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}$ (C) $\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$ (D) $\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$

17. Let A = {2, 4, 5, 10}. Which relation R is an equivalence relation?

    (A) R = {(a,b) | a mod 2 = b mod 2} (B) R = {(a,b) | a mod 2 ≠ b mod 2}

    (C) R = {(a,b) | a mod b = 0} (D) R = {(a,b) | a mod b = 2}

18. Consider the equivalence relation R
    R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} on {1, 2, 3, 4}. The equivalence class for [1] is

    (A) {1} (B) {1, 2} (C) {1,2,3} (D) {1,2,4}

19. If R is the equivalence relation defined on the set B={1,2,3,4} by
    R = {(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4)} then the number of equivalence classes is:
    (A) 1 (B) 2 (C) 3 (D) 4

20. Circle all which are equivalence relation.

    (A) f(A, b) if a and b speak a common language.

    (B) f(x, y) if x and y are bit strings of length 3 or more that agree at 3 or more bits

    (C) f(f, g) if f(x) – g(x) = C for some constant C and for every x, where f and g are functions that map integers to integers

    (D) f(a, b) if a and b earn the same final letter grade, where a and b are students

21. Let A = {2,3,57,8}. Which relation is an equivalence relation?
    (A) R = {(a,b) | a < 2b} (B) R = {(a,b) | a mod 3 = b mod 2}

(C) R = {(a,b) | b mod a = 0}   (D) R = {(a,b) | a + b is even}

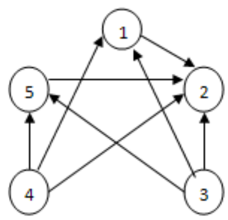
22. How many different equivalence relations are there on the set A = {a, b, c}

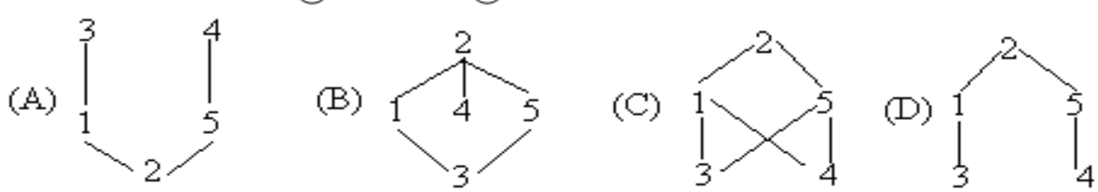
(A) 3   (B) 4   (C) 5   (D) 6

23. Which matrix represents a partial order relation?

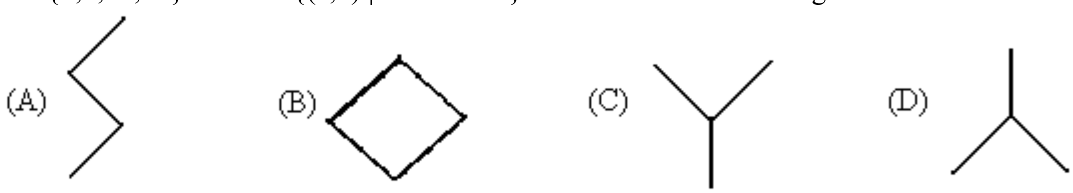
(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

24. Which one is the hasse diagram for the following digraph?





25. A = {4,8,12,24} and R = {(a,b) | a divides b} ⊆ A x A. The Hasse diagram is



26. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be partially ordered by the division relation (that is, for a, b ∈ A, we say that a R b if a is a divisor of b). How many maximal elements are there for this partial order relation?

(A) 5   (B) 2   (C) 3   (D) 4

27. Let A be a set. Consider the partial order ⊆ on P(A). Let C and D be subsets of A. Consider the following statement

S1: The least upper bound of {C,D} is CUD

S2: The greatest lower bound of {C,D} is C ∩ D.

Which of the following statement is correct?

(A) Only Si is true   (B) Only S2 is true

(C) Both S1 & S2 are true   (D) Neither S1 nor S2 are true

28. Let A = {1, 2, 3, 4,5}. Which of the following is a partial order relation on A?

(A) R = {(a, b) | b mod a = 3}   (B) R = {(a, b) | a mod b = 0}

(C) R = {(a, b) | a + b is even}   (D) R = {(a, b) | a mod 3 = b}

29. Let be a relation R is defined as all even number are less than all odd numbers and the usual ordering is applied between the evens and the odds. Is R a total ordering relations. Also, give the order of the elements.

30. For which sets A of P(A) with set inclusion (⊆) a total ordering?

(i) ∅   (ii) {a}   (iii) {a, b}   (iv) {a, b, c}

(A) i & ii   (B) ii and iii   (C) iii and iv   (D) i, ii, iii, iv

31. Let (S, ≤) be a partial order with two minimal elements a and b, and a maximum element c. Let P : S →{True, False} be a predicate defined on S. Suppose that P(a) = True, P(b) = False and P (x) ⇒ P(y) for all x, y ∈S satisfying x ≤ y, where ⇒ stands for logical implication. Which of the follwong statements CANNOT be true?

(A) P(x) = True for all x ∈S such that x ≠ b

(B) P (x) = False for all x ∈S such that x ≠ a and x ≠ c

(C) P(x) = False for all x ∈S such that b ≤ x and x ≠ c

(D) P(x) = False for all x ∈S such that a ≤ x and b ≤ x

32. A relation R is defined on ordered pairs of integers as follows (x, y) R(u, v) if x < u and y> v. Then R is

Equivalence relation, Total Order relation, Partial Order relation

33. Consider the set S = {a, b, c, d}. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$, on

S : $\pi_1 = \overline{\{abcd\}}$, $\pi_2\{\overline{ab}, \overline{cd}\}$, $\pi_3 = \{\overline{abc}, \overline{d}\}$, $\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$

Let ≺ be the partial order on the set of partitions S' = ($\pi_1, \pi_2, \pi_3, \pi_4$) defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for (S', ≺) is

34. Draw the hasse diagram of relation R = {(1,1), (2,2), (3,3), (4,4), (5,5), (3,1), (1,4), (5,1), (3,4), (5,4)}

35. In a partially ordered set, a chain is a totally ordered subset. For example, in the set 1, 2, 3, 4, 5, 6, the divisibility relation is a partial order and 1, 2, 4 and 1, 3, 6 are chains. What is the longest chain on the set {1, 2 . . . n} using the divisibility relation?

36. What is the longest chain on the power set of a set A with |A| = n with the ⊆ relation?

37. Let R be a binary relation on the set of all strings of 0's and 1's such that R={(a,b)| a and b are strings

that have same number of 0s}. Which of the following statement is correct?

A) R is reflexive but not symmetric
B) R symmetric but not anti-symmetric
C) R is anti-symmetric but not transitive
D) None of the above

38. Let A be the set of all strings of 0′s, 1′s, and 2′s of length 4. Define a relation R on the set A as follows: ∀s,t∈A,sRt iff the sum of the characters in s equals the sum of the characters in t. For example, the string "0121" is related to "2200". Which of the following statement is correct?

A) R is reflexive but not symmetric and transitive
B) R is reflexive and symmetric but not transitive
C) R is reflexive, symmetric and transitive
D) None of the above

39. The number of relation on a three element set that are both symmetric and antisymmetric is:

A) 3^2
B) 7
C) 2^6
D) 2^3

40. The number of relation on a three element set that are both symmetric and antisymmetric is:

A) 3^
B) 2^6
C) 0^
D) 5^

41. Which of the following statement is true?

A) the transitive closure of a symmetric relation is symmetric
B) the symmetric closure of a transitive relation is transitive
C) the reflexive closure of a transitive relation is transitive
D) the transitive closure of an antisymmetric relation is antisymmetric

42. Let r(R), s(R) and t(R) be the reflexive, symmetric and transitive closures of a relation R respectively. Which of the following statement is NOT correct?

A) r(s(R)) = s(r(R))
B) s(t(R)) = t(s(R))
C) r(t(R)) = t(r(R))
D) t(s(r(R))) = r(t(s(R)))

43. Given Poset ({3,5,9,15,24,45},/).

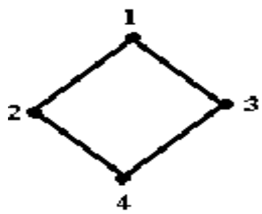
a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {3,5}.

f) Find the least upper bound of {3,5}, if it exists.
g) Find all lower bounds of {15,45}.
h) Find the greatest lower bound of {15,45}, if it exists.

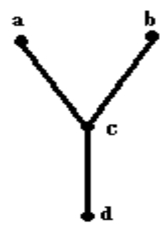
44. Consider R with usual order ≤ :
a). Find lub{x ∈ R : x < 73} b). Find lub{x ∈ R : x2 < 73} c). Is [R; ≤] a lattice?

45. For the elements x, y, z in a poset, show that if lub[x,y] = a and lub[a,z] = b, then lub[x,y,z] = b.
46. We are having poset of the 7 elements then of possible length of anti-chain is
(A) 2 (B) 3 (C) either 2 or 3 (D) can't be predicted

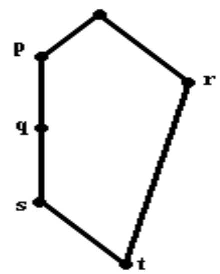
47. Which of the following Hasse diagrams represent lattices?



(I)



(II)



(III)

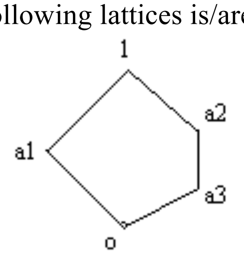
(IV)

(A) I and III only
(B) I, III and IV only
(C) II and IV only
(D) I, II, III and IV only.

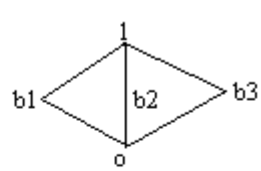
48. Let A be any set such that A = {1, 2, 3, 4, 5, 6} and R be any relation defined as (a, b) ∈R if a divides b then

(A) R forms lattice over A
(B) R doesn't form lattice over A but R forms poset
(C) Neither R form lattice over A nor it forms poset over A
(D) None of the above

49. Which of the following lattices is/are distributive?



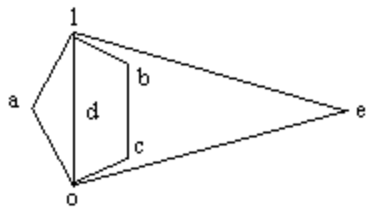
I



II

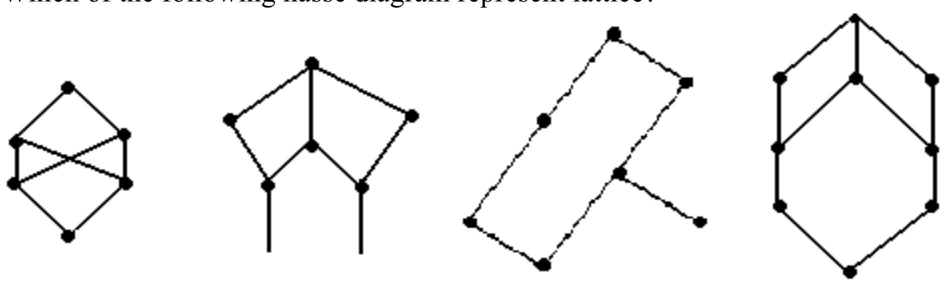
(A) I (B) II (C) both (D) none

50. The complement(s) of the element 'a' in the lattice shown in fig.



(A) e only (B) b, c and e only (C) b, c, d and e only (D) none

51. Which of the following hasse diagram represent lattice?



(A) (B) (C) (D)

52. List all the binary relations on the set {0, 1}.

53. A company makes four kinds of products. Each product has a size code, a weight code, and a shape code. The following table shows these codes:

	Size Code	Weight Code	Shape Code
#1	42	27	42
#2	27	38	13
#3	13	12	27
#4	42	38	38

Find which of the three codes is a primary key. If none of the three codes is a primary key, explain why.

54. If X =(Fran Williams, 617885197, MTH 202, 248B West), find the projections P1,3(X) and P1,2,4(X).

55. R and S are relations on {a, b, c, d}, where R = {(a, b), (a, d), (b, c), (c, c), (d, a)} and S = {(a, c), (b, d), (d, a)}.
Find the following combination of relations.
i) R^2, ii) $R^{3,}$. iii) S^2. iv) S^3. v) R : S. vi) S : R.

56. Calculate R^{-1}, where R is the relation on {1, 2, 3, 4} such that a R b means |a – b| <= 1.