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CS/IS F214
Logic in Computer Science

MODULE: **PREDICATE LOGIC**

Semantics

- Soundness and Completeness

Predicate Logic: Proofs

- Recall:
  - Given sets ***F*** and ***P*** of functions and predicates,
    - Let ψ be a formula and
    - let Γ be a sequence of formulas $\varphi_1, \varphi_2, \ldots, \varphi_n$

    in predicate logic over ***F*** and ***P*** .
    - Then
      - Γ |- ψ denotes that ψ is provable given $\varphi_1, \varphi_2, \ldots, \varphi_n$ as premises
        - using proof rules (say, in Natural Deduction).

Predicate Logic: Soundness

- Claim (**Soundness**):
  - <u>*Predicate Logic is sound*</u> i.e.
    - *if a formula can be proved it is true:*
      - Let ψ be a formula and
      - let Γ be a sequence of formulas $\varphi_1, \varphi_2, \ldots, \varphi_n$ in predicate logic over ***F*** and ***P*** .
      - Then,
        - if $\Gamma \mid\!- \psi$ then $\Gamma \mid= \psi$

- Proof omitted.

Predicate Logic: Completeness

- Claim (***Completeness***):
  - *Predicate Logic is complete* i.e.
    - *if a formula is true then it can be proved:*
      - Let ψ be a formula and
      - let Γ be a sequence of formulas $\varphi_1, \varphi_2, \ldots, \varphi_n$

      in predicate logic over ***F*** and ***P*** .
      - Then,
        - if $\Gamma \models \psi$ then $\Gamma \vdash \psi$

- Proof omitted.

Godel's Incompleteness Results

- Godel defined a set of axioms for natural numbers (including +, *, and induction) and
  - argued that there exists a formula ϕ such that neither ϕ nor $\neg\phi$ can be proved from the axioms.
- In other words the system was incomplete!

- What is the gap between vanilla first order logic and this system?