MATHEMATICS-II (MATH F112)

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Section 4.7

Coordinatization

Ordered Basis

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For example, (e_1, e_2) and (e_2, e_1) are two ordered bases for $\mathbb{R}^2$.

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Similarly, $[11,13] = 2[4,2]+3[1,3]$. Hence, $[11,13]_B = [2,3]$.

Example 2

Q:. Let $B = ([-4, 5, -1, 0, -1], [1, -3, 2, 2, 5], [1, -2, 1, 1, 3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$.

Example 2

Q:. Let $B = ([-4,5,-1,0,-1],[1,-3,2,2,5],[1,-2,1,1,3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$. Compute $[-23,30,-7,-1,-7]_B, [1,2,3,4,5]_B$.

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Sol. To find $[-23,30,-7,-1,-7]_B$, we solve the following equation

$$[-23,30,-7,-1,-7] = a[-4,5,-1,0,-1] + b[1,-3,2,2,5] + c[1,-2,1,1,3]$$

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-4a + b + c &= -23 \\
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To solve this system, we row reduce

$$\left[\begin{array}{ccc|c} -4 & 1 & 1 & -23 \\ 5 & -3 & -2 & 30 \\ -1 & 2 & 1 & -7 \\ 0 & 2 & 1 & -1 \\ -1 & 5 & 3 & -7 \end{array}\right]$$

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Hence, the unique solution for the system is
$a = 6, b = -2, c = 3 \implies$

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Hence, the unique solution for the system is
$a = 6, b = -2, c = 3 \implies$
$[-23, 30, -7, -1, -7]_B = [6, -2, 3]$.

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This system has no solutions,

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This system has no solutions, implying the vector $[1, 2, 3, 4, 5]$ is not in span$(B) = V$.

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- Form an augmented matrix $[A|v]$ by using the vectors in B as the columns of A, in order, and using v as a column on the right.
- Row reduce $[A|v]$ to obtain $\mathrm{RREF}[C|w]$.

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- Eliminate all rows consisting entirely of zeros in $[C|w]$ to obtain $[I_k|y]$. Then, $[v]_B = y$, the last column of $[I_k|y]$.

Example 3

Q:. Let $B = \left(\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix} \right)$ be an ordered basis of the subspace V of M_{22}.

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Sol. Now

$$[A|v] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & -3 \\ -2 & -1 & -1 & -2 \\ 0 & 1 & 3 & 0 \\ 1 & 0 & 1 & 3 \end{array} \right] \Longrightarrow$$

$$\text{RREF}[A|v] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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- $[a_1 w_1]_B = a_1 [w_1]_B$
- $[a_1 w_1 + a_2 w_2 + \ldots a_k w_k]_B = a_1 [w_1]_B + a_2 [w_2]_B + \cdots + a_k [w_k]_B$

Exercises

Q:. Let $B = (3x^2 - x + 2, x^2 + 2x - 3, 2x^2 + 3x - 1)$ be an ordered basis of the subspace V of P_2.

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Sol. $[v]_B = [4, -5, 3]$.

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$B = ([-4, 5, -1, 0, -1], [1, -3, 2, 2, 5], [1, -2, 1, 1, 3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$.

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$x = [1, 0, -1, 0, 4], y = [0, 1, -1, 0, 3], z = [0, 0, 0, 1, 5]$.
Compute $[2x - 7y + 3z]_B$.

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Sol. $[2x - 7y + 3z]_B = [-2, 9, -15]$.

Example 4

Q:. Let $C = ([-4, 5, -1, 0, -1], [1, -3, 2, 2, 5], [1, -2, 1, 1, 3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$.

Example 4

Q:. Let $C = ([-4,5,-1,0,-1],[1,-3,2,2,5],[1,-2,1,1,3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$. Using simplified span method on C, compute an ordered basis $B = (x,y,z)$ for V.

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Example 4

Q:. Let $C = ([-4,5,-1,0,-1],[1,-3,2,2,5],[1,-2,1,1,3])$ be an ordered basis of the subspace V of $\mathbb{R}^5$. Using simplified span method on C, compute an ordered basis $B = (x,y,z)$ for V. Also, compute $[x]_C,[y]_C,[z]_C$.

Sol. Using simplified span method, we have $B = ([1,0,-1,0,4],[0,1,-1,0,3],[0,0,0,1,5])$.

We have the following augmented matrix

$$\left[\, A \,\middle|\, x \quad y \quad z \,\right] = \left[\begin{array}{rrr|rrr} -4 & 1 & 1 & 1 & 0 & 0 \\ 5 & -3 & -2 & 0 & 1 & 0 \\ -1 & 2 & 1 & -1 & -1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ -1 & 5 & 3 & 4 & 3 & 5 \end{array}\right]$$

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Row reduce above augmented matrix to obtain

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -5 & -4 & -3 \\ 0 & 0 & 1 & 10 & 8 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Clearly, $[x]_C = [1, -5, 10]$, $[y]_C = [1, -4, 8]$ and $[z]_C = [1, -3, 7]$.

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Here, $P = \begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & -3 \\ 10 & 8 & 7 \end{bmatrix}$ is called transition matrix from B-coordinates to C-coordinates.

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to produce

$$\left[\begin{array}{r|r} I_k & P \\ \hline rows of & zeroes \end{array}\right]$$

Example 5

Q:. For the ordered bases $B = \left(\begin{bmatrix} 7 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right)$

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Sol. Row reduce

$$\left[\begin{array}{ccc|ccc} 22 & 12 & 33 & 7 & 1 & 1 \\ 7 & 4 & 12 & 3 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & -1 & 1 \end{array}\right]$$

to obtain

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & -4 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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Example 6

Q:. Let $C = (a, b, c) = ([1,0,1],[1,1,0],[0,0,1])$ and

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Sol. $x = 1.a + 2.b - 1.c = [3, 2, 0]$.

Example 6

Q:. Let $C = (a, b, c) = ([1, 0, 1], [1, 1, 0], [0, 0, 1])$ and $B = (x, y, z)$ are ordered bases of $\mathbb{R}^3$. Let $P = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ be the transition matrix from B to C. Find the basis B.

Sol. $x = 1.a + 2.b - 1.c = [3, 2, 0]$.

Similarly, $y = 1.a + 1.b - 1.c = [2, 1, 0]$ and $z = 2.a + 1.b + 1.c = [3, 1, 3]$.

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Sol. $x = 1.a + 2.b - 1.c = [3, 2, 0]$.

Similarly, $y = 1.a + 1.b - 1.c = [2, 1, 0]$ and $z = 2.a + 1.b + 1.c = [3, 1, 3]$.

Hence, $B = ([3, 2, 0], [2, 1, 0], [3, 1, 3])$.

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Also, it should be noted that $\boldsymbol{C}P = \boldsymbol{B}$, where matrix $\boldsymbol{C}$ is obtained by considering vectors of basis $\boldsymbol{C}$ as columns of the matrix.

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Theorem: Suppose that V is a nontrivial n-dimensional vector space with ordered bases B and C. Let P be an $n \times n$ matrix. Then P is the transition matrix from B to C if and only if for every $v \in V$, $P[v]_B = [v]_C$.

Example 7

Q:. For the ordered bases $B = \left(\begin{bmatrix} 7 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right)$

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Example 7

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Sol. Clearly,

$$\begin{bmatrix} 25 & 24 \\ 0 & -9 \end{bmatrix} = 4\begin{bmatrix} 7 & 3 \\ 0 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} - 6\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

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Hence, $[v]_B = [4, 3, -6]^T$.

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Hence, $[v]_B = [4, 3, -6]^T$.

Now $P = \begin{bmatrix} 1 & -2 & 1 \\ -4 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \implies$

$[v]_C = P[v]_B = [-8, -19, 13]^T$.

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Theorem: Let B and C be be ordered bases for a nontrivial finite dimensional vector space V, and let P be the transition matrix from B to C. Then P is nonsingular, and P^{-1} is the transition matrix from C to B.

Example 8

Q:. For an ordered basis $B = ([1, -4, 1, 2, 1], [6, -24, 5, 8, 3], [3, -12, 3, 6, 2])$ of a subspace V of $\mathbb{R}^5$,

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- Use the Simplified Span Method to find a second ordered basis C.

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Q:. For an ordered basis $B = ([1, -4, 1, 2, 1], [6, -24, 5, 8, 3], [3, -12, 3, 6, 2])$ of a subspace V of $\mathbb{R}^5$, perform the following steps:

- Use the Simplified Span Method to find a second ordered basis C.
- Find the transition matrix P from B to C.

Example 8

Q:. For an ordered basis $B = ([1, -4, 1, 2, 1], [6, -24, 5, 8, 3], [3, -12, 3, 6, 2])$ of a subspace V of $\mathbb{R}^5$, perform the following steps:

- Use the Simplified Span Method to find a second ordered basis C.
- Find the transition matrix P from B to C.
- Find the transition matrix Q from C to B.

Example 8

Q:. For an ordered basis $B = ([1,-4,1,2,1],[6,-24,5,8,3],[3,-12,3,6,2])$ of a subspace V of $\mathbb{R}^5$, perform the following steps:

- Use the Simplified Span Method to find a second ordered basis C.
- Find the transition matrix P from B to C.
- Find the transition matrix Q from C to B.
- For the given vector $v = [2,-8,-2,-12,3] \in V$, calculate $[v]_B$ and $[v]_C$.

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Sol. Here

$$B = \begin{bmatrix} 1 & -4 & 1 & 2 & 1 \\ 6 & -24 & 5 & 8 & 3 \\ 3 & -12 & 3 & 6 & 2 \end{bmatrix} \Longrightarrow$$

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$$B = \begin{bmatrix} 1 & -4 & 1 & 2 & 1 \\ 6 & -24 & 5 & 8 & 3 \\ 3 & -12 & 3 & 6 & 2 \end{bmatrix} \Longrightarrow$$

$$\mathrm{REF}(B) = \begin{bmatrix} 1 & -4 & 0 & -2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$

- Use the Simplified Span Method to find a second ordered basis C.

Sol. Here

$$B = \begin{bmatrix} 1 & -4 & 1 & 2 & 1 \\ 6 & -24 & 5 & 8 & 3 \\ 3 & -12 & 3 & 6 & 2 \end{bmatrix} \Longrightarrow$$

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$$C = ([1,-4,0,-2,0],[0,0,1,4,0],[0,0,0,0,1])$$

- Find the transition matrix $\boldsymbol{P}$ from $\boldsymbol{B}$ to $\boldsymbol{C}$.

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The transition matrix P from B to C is

$$P = \begin{bmatrix} 1 & 6 & 3 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

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- Find the transition matrix Q from C to B.

The transition matrix Q from C to B is

$$Q = P^{-1} = \begin{bmatrix} 1 & -3 & 3 \\ 1 & -1 & 0 \\ -2 & 3 & -1 \end{bmatrix}$$

- For the given vector $v = [2, -8, -2, -12, 3] \in V$, calculate $[v]_B$ and $[v]_C$.

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$$[B|v] = \left[\begin{array}{ccc|c} 1 & 6 & 3 & 2 \\ -4 & -24 & -12 & -8 \\ 1 & 5 & 3 & -2 \\ 2 & 8 & 6 & -12 \\ 1 & 3 & 2 & 3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Longrightarrow$$

$$[v]_B = [17, 4, -13]$$

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Now $P[v]_B = [v]_C \implies$

$[v]_B = [17, 4, -13]$

Now $P[v]_B = [v]_C \implies [v]_C = [2, -2, 3]$

Exercises

Q:. For the ordered bases
$B = \left(2x^2+3x-1, 8x^2+x+1, x^2+6\right)$ and
$C = \left(x^2+3x+1, 3x^2+4x+1, 10x^2+17x+5\right)$ of P_2, find the transition matrix P from B to C.

Exercises

Q:. For the ordered bases $B = \left(2x^2+3x-1, 8x^2+x+1, x^2+6\right)$ and $C = \left(x^2+3x+1, 3x^2+4x+1, 10x^2+17x+5\right)$ of P_2, find the transition matrix P from B to C.

Sol. $P = \begin{bmatrix} 20 & -30 & -69 \\ 24 & -24 & -80 \\ -9 & 11 & 31 \end{bmatrix}$

Q:. Let $P = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ be the transition matrix from $\boldsymbol{B}$ to C. If $C = \left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$, find the basis $\boldsymbol{B}$.

Q:. Let $P = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ be the transition matrix from B to C. If $C = \left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$, find the basis B.

Sol. $B = \left\{ \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix} \right\}$

Q:. For an ordered basis
$B = ([3,-1,4,6],[6,7,-3,-2],[-4,-3,3,4],[-2,0,1,2])$
of a subspace V of $\mathbb{R}^4$, perform the following steps:

- Use the Simplified Span Method to find a second ordered basis C.
- Find the transition matrix P from B to C.
- Find the transition matrix Q from C to B.
- For the given vector $v = [10,14,3,12] \in V$, calculate $[v]_B$ and $[v]_C$.