MATHEMATICS-I (MATH F111)

Dr. Krishnendra Shekhawat

BITS PILANI
Department of Mathematics

CHAPTER 11

Conic Sections and Polar Coordinates

Topics to be covered in this Chapter:

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1. Polar Coordinates

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2. Curve Tracing

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3. Faster Graphing

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4. Areas and Lengths of the Curves

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1. Polar Coordinates
2. Curve Tracing
3. Faster Graphing
4. Areas and Lengths of the Curves
5. Conic Sections

Section 11.3

Polar Coordinates

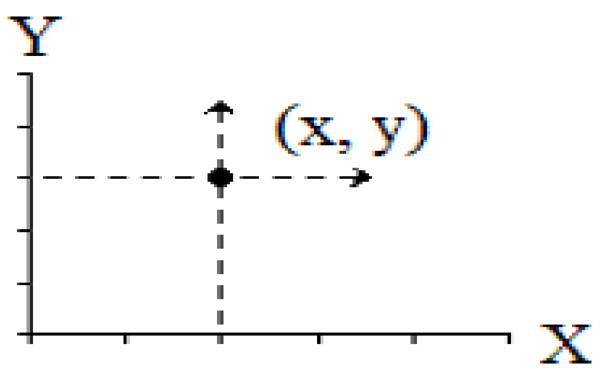
- How can be describe or represent a curve?

- How can be describe or represent a curve?
- Cartesian coordinates

- How can be describe or represent a curve?
- Cartesian coordinates
- Polar coordinates

Cartesian Coordinates

A point in cartesian coordinate or $x-y$ co-ordinate or rectangular co-ordinate is described in terms of horizontal and vertical distances from the origin $(0,0)$.



Polar Coordinates

To define polar coordinates in a plane, we start with an origin (called the pole) and an initial ray.

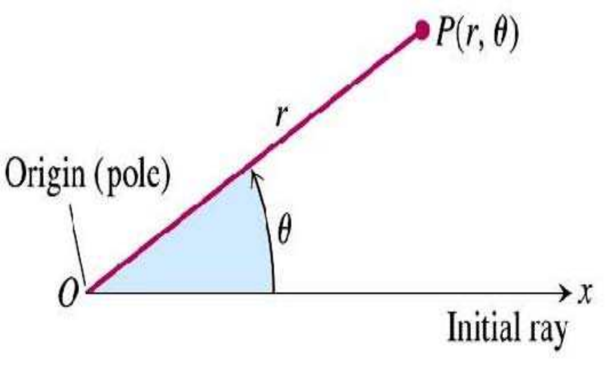
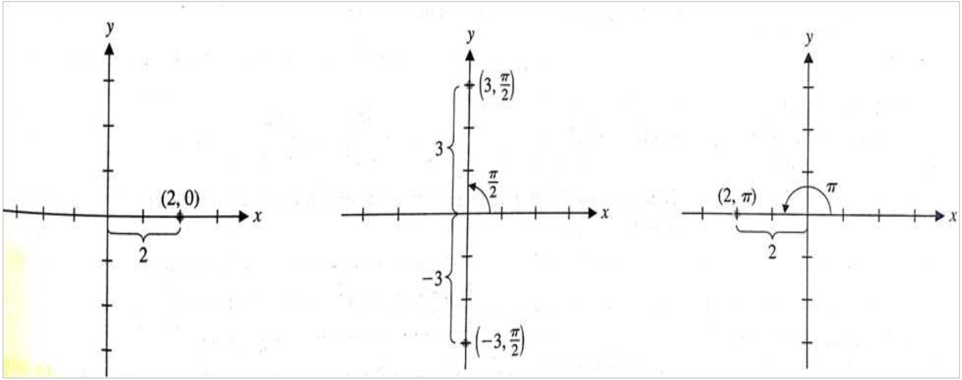


Figure: θ is an angle made by the ray OP with initial ray and r is the distance of P from O along ray OP.

Plotting of Polar co-ordinates

Plot the following points:

- $(2, 0)$
- $(3, \frac{\pi}{2})$
- $(-3, \frac{\pi}{2})$
- $(2, \pi)$



y
x
(2, 0)
2
y
x
$\left(3, \frac{\pi}{2}\right)$
3
$\frac{\pi}{2}$
−3
$\left(-3, \frac{\pi}{2}\right)$
y
x
(2, π)
π
2

Polar Coordinates with Negative r Values

The points $(-r, \theta)$ and (r, θ) lie on the same line through the pole O and at the distance $|r|$ from O, but on opposite sides of O.

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- If $r > 0$, the point (r, θ) is in the same quadrant as θ.

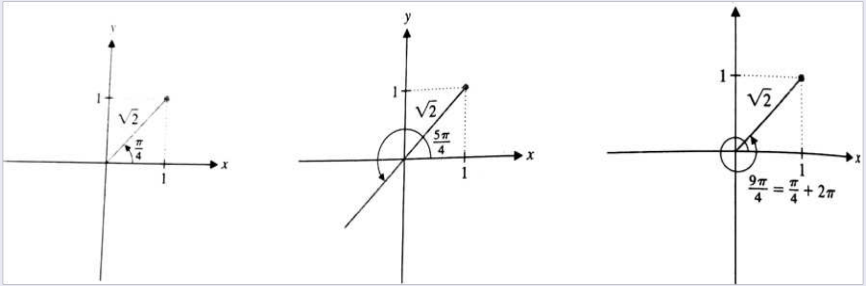
Polar Coordinates with Negative r Values

The points $(-r, \theta)$ and (r, θ) lie on the same line through the pole O and at the distance $|r|$ from O, but on opposite sides of O.

- If $r > 0$, the point (r, θ) is in the same quadrant as θ.
- If $r < 0$, the point (r, θ) is in the quadrant opposite of the angle θ, $i.e.$, opposite of pole.

Different representation of Polar co-ordinates

Plot $(\sqrt{2}, \pi/4)$



The number of Polar Coordinate Pairs

Q:. How many Polar Coordinate Pairs the point (r, θ) can have?

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Q:. How many Polar Coordinate Pairs the point (r,θ) can have?

Sol. Infinite

The different polar coordinates of a point (r, θ) are:

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- $(r, \theta + 2n\pi), \quad n = 0, \pm 1, \pm 2, \ldots\ldots$

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- $(r, \theta + 2n\pi), \quad n = 0, \pm 1, \pm 2, \ldots\ldots$
- $(-r, \theta + (2n+1)\pi), \quad n = 0, \pm 1, \pm 2, \ldots\ldots.$

Remark. If $P \equiv O$, then $r = 0$ and $(0, \theta)$ represents pole for any value of θ.

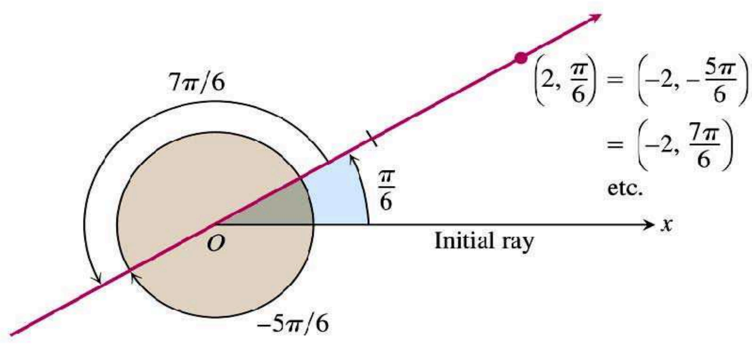

Figure: The point $\left(2, \frac{\pi}{6}\right)$ has infinitely many polar coordinate pairs

Q:. $0 \leqslant \theta \leqslant \frac{\pi}{6},\ r \geqslant 0.$

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Sol.

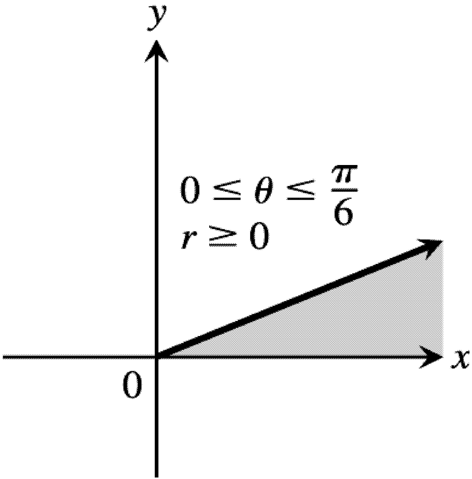

Figure: An infinite region

Q:. $0 \leqslant \theta \leqslant \frac{\pi}{6},\ r \geqslant 0.$

Sol.

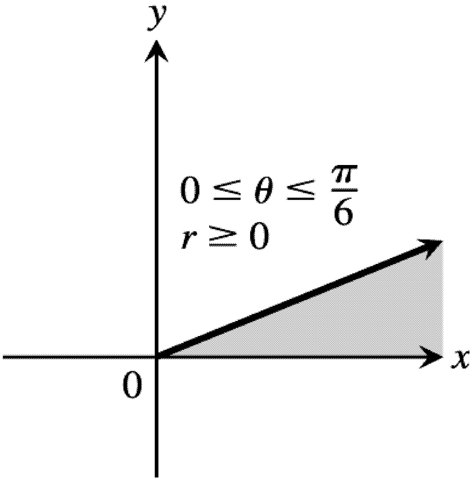

Figure: An infinite region

Q:. $\theta = \frac{\pi}{3}, \; -1 \leqslant r \leqslant 3.$

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Sol.

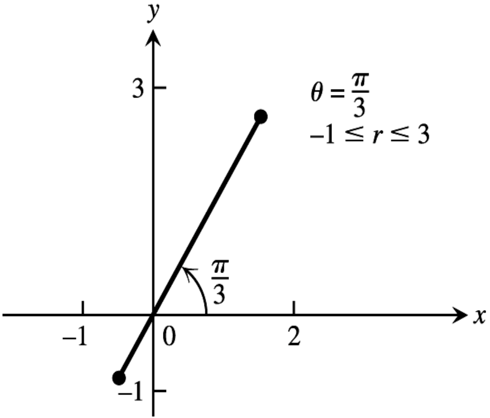
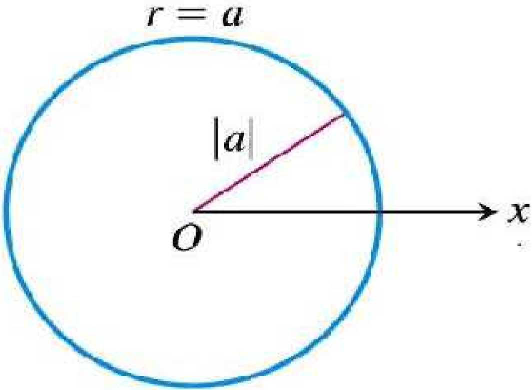


Figure: A line segment

Q:. $r = a$

Q:. $r = a$ (The Polar Equation of a Circle with radius $|a|$ centered at O)



Q:. Graph the set of points whose polar coordinates satisfy the inequality $1 \leqslant r \leqslant 2$.

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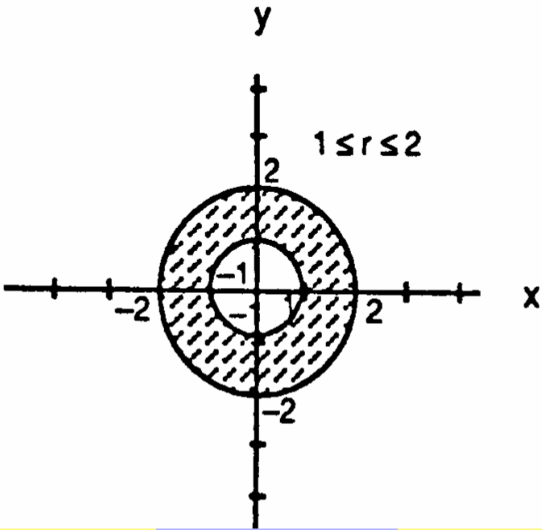
Sol.



Q:. $1 \leqslant |r| \leqslant 2, 0 \leqslant \theta \leqslant \pi/2$

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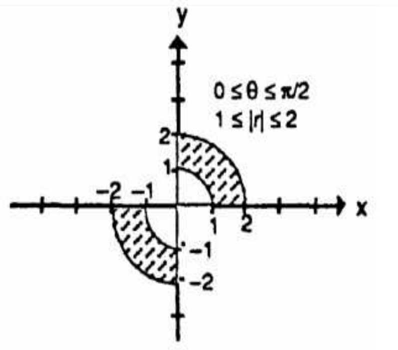
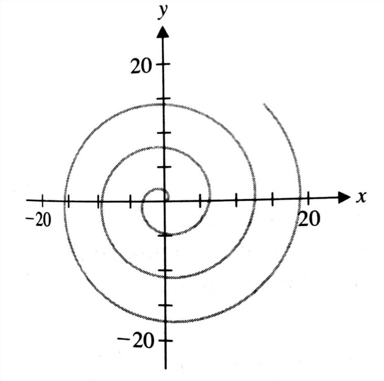
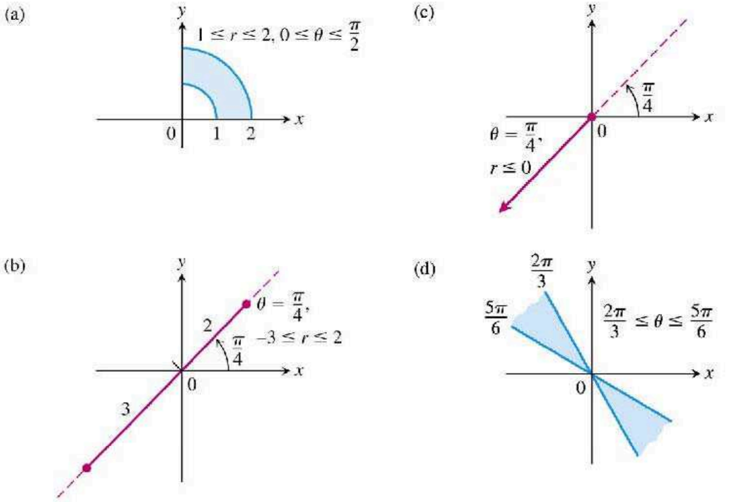
Q:. $r = \theta, \theta \geqslant 0$


y
0 ≤ θ ≤ π/2
1 ≤ |r| ≤ 2
2
1
−2
−1
1
2
x
−1
−2



y
20
x
−20
20
−20

The Graph of Some Inequalities



Homework. Graph the set of points whose polar coordinates satisfy the inequality:

- $-2 \leqslant r \leqslant -1$.

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- $-2 \leqslant r \leqslant -1$.(An annulus).

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- $-1 \leqslant r \leqslant 2$.(Interior of circle $r = 2$ including boundary).

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- $-2 \leqslant r \leqslant -1$.(An annulus).
- $-1 \leqslant r \leqslant 2$.(Interior of circle $r = 2$ including boundary).
- $r \leqslant 1$.

Homework. Graph the set of points whose polar coordinates satisfy the inequality:

- $-2 \leqslant r \leqslant -1$.(An annulus).
- $-1 \leqslant r \leqslant 2$.(Interior of circle $r = 2$ including boundary).
- $r \leqslant 1$.(Whole plane).

Homework. Graph the set of points whose polar coordinates satisfy the inequality:

- $-2 \leqslant r \leqslant -1$.(An annulus).
- $-1 \leqslant r \leqslant 2$.(Interior of circle $r = 2$ including boundary).
- $r \leqslant 1$.(Whole plane).
- $r \geqslant -1$. (Whole plane).

Relation between Polar and Cartesian Coordinates

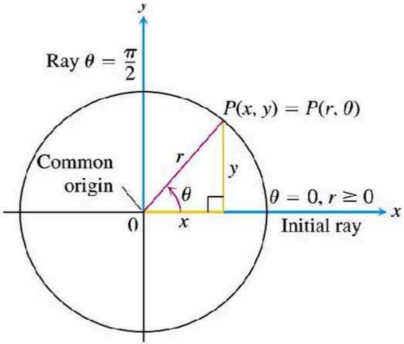

Figure: The usual way to relate polar and cartesian coordinates

From figure, we have

$$x = r\cos\theta, \quad y = r\sin\theta,$$

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on squaring and adding:

$$x^2 + y^2 = r^2,$$

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$$x = r\cos\theta, \quad y = r\sin\theta,$$

on squaring and adding:

$$x^2 + y^2 = r^2,$$

on dividing:

$$\theta = \tan^{-1}\frac{y}{x}.$$

Q:. Replace $r = \sin\theta$ by an equivalent cartesian equation.

Q:. Replace $r = \sin\theta$ by an equivalent cartesian equation.

Sol.

$$
\begin{aligned}
r &= \sin\theta \\
\Rightarrow r^2 &= r\sin\theta \\
\Rightarrow x^2 + y^2 &= y \\
\Rightarrow x^2 + y^2 - y &= 0 \\
\Rightarrow x^2 + \left(y - \frac{1}{2}\right)^2 &= \left(\frac{1}{2}\right)^2.
\end{aligned}
$$

Q:. Replace $r^2(1+\sin 2\theta) = 1$ by an equivalent cartesian equation.

Q:. Replace $r^2(1+\sin 2\theta) = 1$ by an equivalent cartesian equation.

Sol.

$$
\begin{aligned}
r^2 + 2r^2 \sin\theta \cos\theta &= 1 \\
\Rightarrow r^2 + 2(r\sin\theta)(r\cos\theta) &= 1 \\
\Rightarrow x^2 + y^2 + 2xy &= 1 \\
\Rightarrow x + y &= \pm 1.
\end{aligned}
$$

Q:. Replace $x^2 - y^2 = 9$ by an equivalent polar equation.

Q:. Replace $x^2 - y^2 = 9$ by an equivalent polar equation.

Sol.

$$\begin{aligned} x^2 - y^2 &= 9 \\ \Rightarrow r^2 \cos^2\theta - r^2 \sin^2\theta &= 9 \\ \Rightarrow r^2\left(\cos^2\theta - \sin^2\theta\right) &= 9 \\ \Rightarrow r^2 \cos 2\theta &= 9 \\ \Rightarrow r &= \pm 3\sqrt{\sec 2\theta}. \end{aligned}$$

Q:. Replace $(x-5)^2+y^2=25$ by an equivalent polar equation.

Q:. Replace $(x-5)^2+y^2=25$ by an equivalent polar equation.

Sol.

$$\begin{aligned}(x-5)^2+y^2&=25\\ \Rightarrow x^2-10x+25+y^2&=25\\ \Rightarrow r^2-10r\cos\theta&=0\\ \Rightarrow r&=10\cos\theta.\end{aligned}$$

Section 11.4

Graphing in Polar Coordinates

How to Trace a Curve in Polar Coordinate

How to Trace a Curve in Polar Coordinate

Trace the Curve $r = 1 - cos\theta$

How to Trace a Curve in Polar Coordinate

Trace the Curve $r = 1 - cos\theta$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	0	0.29	1	1.71	2	1.71	1	0.29	0

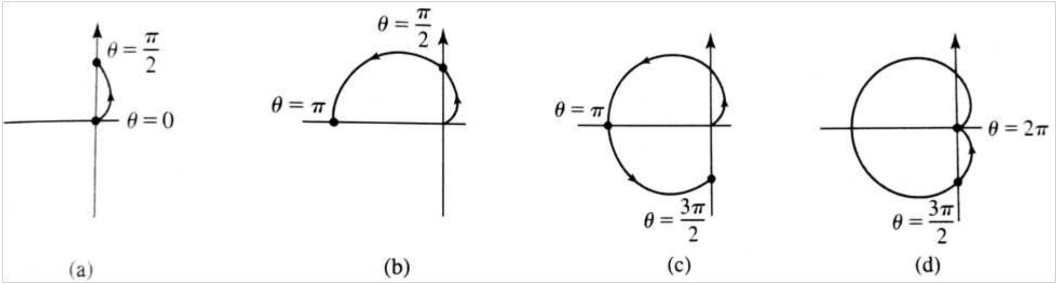

Figure: $r = 1 - cos\theta$

Q:. Do you see symmetry in above Figure ?

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Q:. Where can be draw tangent in the above Figure

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Q:. Where can be draw tangent in the above Figure (slope of a curve) ?

Curves in Polar Coordinates

- For polar coordinates (r, θ), the equation $f(r, \theta) = 0$ (implicit form) or $r = f(\theta)$ (explicit form) defines a curve C in the plane.

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- A point P lies on C if and only if for at least one polar coordinate (r_0, θ_0) of P, $f(r_0, \theta_0) = 0$.

Curves in Polar Coordinates

- For polar coordinates (r,θ), the equation $f(r,\theta)=0$ (implicit form) or $r=f(\theta)$ (explicit form) defines a curve C in the plane.
- A point P lies on C if and only if for at least one polar coordinate (r_0,θ_0) of P, $f(r_0,\theta_0)=0$.

In this section, our aim is to trace the curve $r=f(\theta)$ or $f(r,\theta)=0$.

Symmetry Test 1

- **Symmetry about x-axis:** If the point (r, θ) lies on the graph, the point $(r, -\theta)$ or $(-r, \pi - \theta)$ also lies on the graph.

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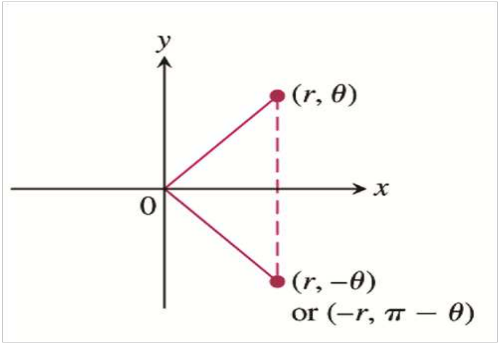

Symmetry Test 2

- **Symmetry about the y-axis:** If the point (r,θ) lies on the graph, the point $(-r,-\theta)$ or $(r,\pi-\theta)$ also lies on the graph.

Symmetry Test 2

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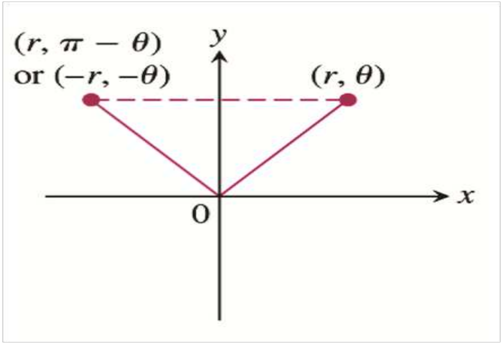

Symmetry Test 3 and 4

- **Symmetry about the origin:** If the point (r,θ) lies on the graph, the point $(-r,\theta)$ or $(r,\pi+\theta)$ also lies on the graph.

Symmetry Test 3 and 4

- **Symmetry about the origin:** If the point (r, θ) lies on the graph, the point $(-r, \theta)$ or $(r, \pi + \theta)$ also lies on the graph.
- **Symmetry about the line $y = x$:** If the point (r, θ) lies on the graph, the point $\left(r, \frac{\pi}{2} - \theta\right)$ or $\left(-r, -\frac{\pi}{2} - \theta\right)$ also lies on the graph.

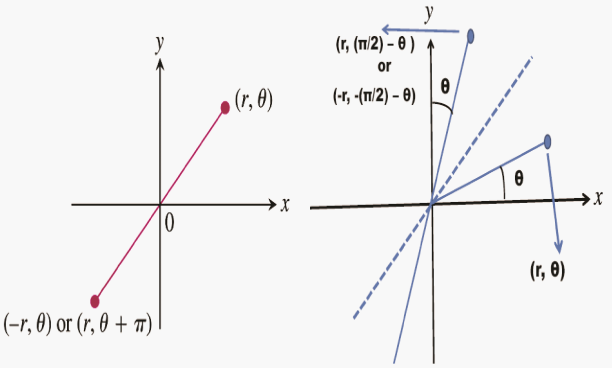

Figure: Symmetry about the pole and line $y = x$ respectively

Q:. Identify the symmetries of the following curves.

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- $r = 2 + \sin\theta$

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- $r = 2 + \sin\theta$ (symmetry about y-axis)

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- $r = 2 + \sin\theta$ (symmetry about y-axis)
- $r^2 = 4\sin 2\theta$

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Q:. Identify the symmetries of the following curves.

- $r = 2 + \sin\theta$ (symmetry about y-axis)
- $r^2 = 4\sin 2\theta$ (symmetry about the origin)
- $r^2 = 4\cos 2\theta$ (symmetry about x-axis, y-axis, the origin)

Points to remember about Symmetry

- If a curve is symmetric about x-axis and y-axis, then it is symmetric about the pole.

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- If a curve is symmetric about x-axis and y-axis, then it is symmetric about the pole.
- If a curve is symmetric about x-axis & pole, then it is symmetric about y-axis.
- If a curve is symmetric about y-axis & pole, then it is symmetric about x-axis.

Points to remember about Symmetry

- If a curve is symmetric about x-axis and y-axis, then it is symmetric about the pole.
- If a curve is symmetric about x-axis & pole, then it is symmetric about y-axis.
- If a curve is symmetric about y-axis & pole, then it is symmetric about x-axis.
- Thus if a curve is symmetric about x-axis but not symmetric about y-axis (or if a curve is symmetric about y-axis but not symmetric about x-axis) then it can not be symmetric about the pole.

Slope of a Polar Curve

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- The parametric equations of $r = f(\theta)$ are
  $x = r\cos\theta,\ y = r\sin\theta.$

Slope of a Polar Curve

- The parametric equations of $r = f(\theta)$ are $x = r\cos\theta,\ y = r\sin\theta$.
- The slope of the curve $r = f(\theta)$ at any point (r,θ) is given by

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \left.\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

provided $\frac{dx}{d\theta} \neq 0$ at any point (r,θ).

Example

Find the slope of the curve $r = 4\sin 3\theta$ at $\theta = \frac{\pi}{6}$.

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Sol.

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$
$$= \frac{12\cos 3\theta\sin\theta + 4\sin 3\theta\cos\theta}{12\cos 3\theta\cos\theta - 4\sin 3\theta\sin\theta}.$$

Example

Find the slope of the curve $r = 4\sin 3\theta$ at $\theta = \frac{\pi}{6}$.

Sol.

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$
$$= \frac{12\cos 3\theta\sin\theta + 4\sin 3\theta\cos\theta}{12\cos 3\theta\cos\theta - 4\sin 3\theta\sin\theta}.$$

Thus

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = -\sqrt{3}.$$

Q:. What does the slope of a curve represent.

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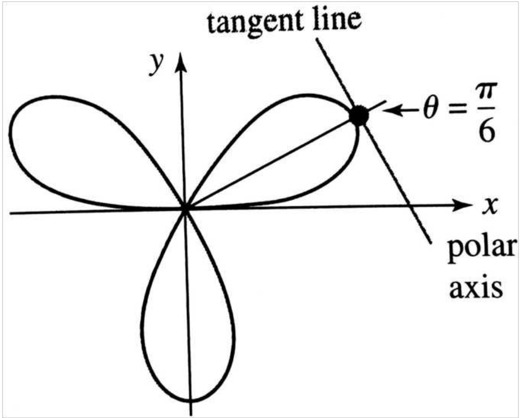
At $\theta = \frac{\pi}{6}$, $r = 4$ and $\tan\theta_1 = -\sqrt{3} \implies \theta_1 = \frac{2\pi}{3}$.

Q:. What does the slope of a curve represent.

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = -\sqrt{3}.$$

At $\theta = \frac{\pi}{6}$, $r = 4$ and $\tan\theta_1 = -\sqrt{3} \implies \theta_1 = \frac{2\pi}{3}$. Here θ_1 is the angle the tangent at $(r, \theta) = (4, \frac{\pi}{6})$ makes with the $x-$axis.


tangent line
y
$\theta = \frac{\pi}{6}$
x
polar
axis

Slope at Pole

How to compute Slope at Pole?

Slope at Pole

How to compute Slope at Pole?
If the curve $r = f(\theta)$ passes through the pole at $\theta = \theta_0$, then $r = f(\theta_0) = 0$.

Slope at Pole

How to compute Slope at Pole?

If the curve $r = f(\theta)$ passes through the pole at $\theta = \theta_0$, then $r = f(\theta_0) = 0$. Hence the slope of the curve $r = f(\theta)$ at pole is given by

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \tan\theta_0$$

.

Slope at Pole

How to compute Slope at Pole?
If the curve $r = f(\theta)$ passes through the pole at $\theta = \theta_0$, then $r = f(\theta_0) = 0$. Hence the slope of the curve $r = f(\theta)$ at pole is given by

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \tan\theta_0$$

.
Hence the line $\theta = \theta_0$ is the tangent to the curve at the pole.

Q:. Find the slope of the curve $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{2}$.

Q:. Find the slope of the curve $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{2}$.

Sol.

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = 1.$$

Q:. Trace the curve $r = 1 + \cos\theta$.

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Step 1. Check for symmetries (it will reduce the work for tracing).

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- Since $(r, -\theta)$ lies on the curve, it is symmetric about x-axis.

Q:. Trace the curve $r = 1 + \cos\theta$.

Step 1. Check for symmetries (it will reduce the work for tracing).

- Since $(r, -\theta)$ lies on the curve, it is symmetric about x-axis. Hence, it is enough to consider the steps for $0 \leqslant \theta \leqslant \pi$.

Step 2. Solve the equation $r = 0$ for θ.

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- $r = 0$ gives $\cos\theta = -1 \implies \theta = \pi$. Thus $\theta = \pi$ is a tangent to the curve at pole.

Step 3 Find $\frac{dr}{d\theta}$.

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- $\frac{dr}{d\theta} = -\sin\theta.$

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- $\frac{dr}{d\theta} = -\sin\theta$.

Step 3.1 Find the values of θ for which $\frac{dr}{d\theta} > 0$. This would be an interval (or no value of θ). In this interval r is an increasing function. Find max/min values of r and the associated θ values in this interval.

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- $\frac{dr}{d\theta} > 0 \Rightarrow \sin\theta < 0$, thus no value of θ in between 0 and π.

Step 3.2 Similarly find the values of θ for which $\frac{dr}{d\theta} < 0$. This would be an interval (or no value of θ). In this interval r is a decreasing function. Find max/min values of r and the associated θ values in this interval.

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  Clearly $\max r = 2$ at $\theta = 0$ and $\min r = 0$ at $\theta = \pi$.

Step 4. Find slopes at some special points (like at end points of the intervals obtained in step 3).

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At $\theta = \frac{\pi}{2}$, $r = 1$ and $\tan\theta_1 = 1 \implies \theta_1 = \frac{\pi}{4}$.

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θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π
r	2	$1+\frac{1}{\sqrt{2}}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$1-\frac{1}{\sqrt{2}}$	0

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Step 6. Plot the curve while considering the above steps.