



**BITS Pilani**  
Pilani Campus



# CS/IS F214 Logic in Computer Science

## MODULE: PREDICATE LOGIC

### Syntax of Predicate Logic

# Predicate Logic: Grammar using precedence

- **Gr-Pred-Prec-0.1:**

1.  $\text{PredF} \rightarrow \forall \text{Var PredF}$
2.  $\text{PredF} \rightarrow \exists \text{Var PredF}$
3.  $\text{PredF} \rightarrow \text{QFPredF}$

- Quantifiers extend over the rest of the formula i.e.  
*they have the lowest precedence*

- **QFPredF** denotes Quantifier Free Predicate Formula

- **Var** denotes a variable symbol  
(Convention – adopted from Prolog : *a variable symbol is an alpha-numeric string starting with an upper-case letter*)  
e.g. X, X1, XY, Xy, Y, Ya, Yak, YzX123

# Predicate Logic: Grammar using precedence

## • Gr-Pred-Prec-0.7:

1.  $\text{PredF} \rightarrow \forall \text{ Var PredF}$
2.  $\text{PredF} \rightarrow \exists \text{ Var PredF}$
3.  $\text{PredF} \rightarrow \text{QFPredF}$
4.  $\text{QFPredF} \rightarrow \text{QFDisF} \rightarrow \text{QFPredF}$
5.  $\text{QFPredF} \rightarrow \text{QFDisF}$
6.  $\text{QFDisF} \rightarrow \text{QFConC} \vee \text{QFDisF}$
7.  $\text{QFDisF} \rightarrow \text{QFConC}$
8.  $\text{QFConC} \rightarrow \text{QFSingle} \wedge \text{QFConC}$
9.  $\text{QFConF} \rightarrow \text{QFSingle}$
10.  $\text{QFSingle} \rightarrow \neg \text{ Predicate}$
11.  $\text{QFSingle} \rightarrow \text{ Predicate}$
12.  $\text{QFSingle} \rightarrow ( \text{ PredF } )$

• **Quantifier Free Predicate Formulas** use the conventional precedence order (*from low to high*) :

$\rightarrow, \vee, \wedge$ , and  $\neg$

and precedence is overridden using parentheses.

## Predicate Logic: Grammar

## Predicate: Syntax

A **predicate** is a *predicate symbol* followed by a *comma-separated list of terms* in parenthesis: i.e.

Predicate  $\rightarrow$  PredSym ( Terms )

Predicate  $\rightarrow$  PredSym ( )

Terms  $\rightarrow$  Term ‘,’ Terms

Terms  $\rightarrow$  Term

A predicate symbol (**PredSym**) is *any string starting with a lower-case letter*.

e.g. f, p, father, pred, succ,  
grandFather, maternalGrandMother,  
foo123, bar123, chachi420

## Term: Examples

Examples:

- 123
  - abc
  - aB12C
- } constants

- X
  - X100
  - Ls
- } variables

- f(X, Y, Z)
  - succ(123)
  - succ(N1)
  - mother(kavi)
  - graph(V,E)
  - tree(plus,  
tree(star, tree(plus, 1, 2), 3),  
tree(star, tree(plus, 4, 5), 6))
- } function terms

## Term: Syntax

A **term** is one of the following:

- a **constant** i.e.
  - *a string starting with a lower case alphabet or*
  - *a number*
- a **variable**
- a **function term** of the form:
  - a ***function symbol*** followed by a list of comma-separated terms in parentheses
  - and a **function symbol** is any string starting with a lower case alphabet

# Predicate Logic: Grammar using precedence

## • Gr-Pred-Prec:

1.  $\text{PredF} \rightarrow \forall \text{ Var PredF}$
  2.  $\text{PredF} \rightarrow \exists \text{ Var PredF}$
  3.  $\text{PredF} \rightarrow \text{QFPredF}$
  4.  $\text{QFPredF} \rightarrow \text{QFDisF} \rightarrow \text{QFPredF}$
  5.  $\text{QFPredF} \rightarrow \text{QFDisF}$
  6.  $\text{QFDisF} \rightarrow \text{QFConC} \vee \text{QFDisF}$
  7.  $\text{QFDisF} \rightarrow \text{QFConC}$
  8.  $\text{QFConC} \rightarrow \text{QFSingle} \wedge \text{QFConC}$
  9.  $\text{QFConF} \rightarrow \text{QFSingle}$
  10.  $\text{QFSingle} \rightarrow \neg \text{Predicate}$
  11.  $\text{QFSingle} \rightarrow \text{Predicate}$
  12.  $\text{QFSingle} \rightarrow ( \text{PredF} )$
  13.  $\text{Predicate} \rightarrow \text{PredSym} ( \text{Terms} )$
  14.  $\text{Predicate} \rightarrow \text{PredSym} ( )$
  15.  $\text{Terms} \rightarrow \text{Term} ; \text{Terms}$
  16.  $\text{Terms} \rightarrow \text{Term}$
  17.  $\text{Term} \rightarrow \text{Const}$
  18.  $\text{Term} \rightarrow \text{Var}$
  19.  $\text{Term} \rightarrow \text{FunSym} ( \text{Terms} )$
- A **Const** or **FunSym** or **PredSym** is an alpha-numeric string starting with
- a lower-case letter or a number

# Predicate Logic: Examples

- Consider the following “Definition of Natural Numbers”:  
 $\forall X \text{ equals}(X,0) \vee (\exists Z \text{ natural}(Z) \wedge \text{equals}(X,\text{succ}(Z))) \rightarrow \text{natural}(X)$
- Parse this formula using **Gr-Pred-Prec**.
  - Draw the parse tree.

