BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: INTRODUCTION

A Brief – and Selective - History of Logic – Part III:
Godel and Incompleteness

Back to Hilbert Program

- Recall:
  - One of the challenges posed by Hilbert was to "formalize mathematics"

- **Kurt Godel** proved that:
  - *complex mathematical systems* (or *complex systems for formal reasoning*) **cannot be "fully formalized"**.

Proof Systems

- Consider a logic(al system) i.e. a proof system:
  - e.g. Euclid's geometry
- Recall that such a system consists of
  - Axioms
    - assumptions / facts that are given
      - e.g. Euclid's five axioms
  - Rules
    - steps / methods for proving results from axioms / other results
      - e.g. proof techniques such as Induction, PbC, LEM
  - Theorems
    - results proven in the systems

Proof Systems and Reality (or Truth)

- A ***proof system*** – we refer to such a system as ***a logic*** *in this course*) models (i.e. captures) a real-world or imaginary system:
  - Real-world systems can be physical systems or abstract systems.
- e.g.
  - Newtonian mechanics
  - Quantum mechanics
  - Indian constitution
  - Harry Potter's World
- Does a proof system capture the world exactly?
  - How do we define this notion?

Proof Systems – Soundness and Completeness

- Soundness
  - If everything that is provable in a proof system is "actually true", then the system is said to be ***sound***.
- Completeness
  - If everything that is "actually true" is provable in a proof system , then the system is said to be ***complete***
- Examples

Godel's Incompleteness Theorem

- There is no *sufficiently complex system* that is *sound and complete*.
  - e.g. Arithmetic is sufficiently complex.
- What is the implication for Hilbert's formalization problem?