BITS Pilani
Pilani Campus

MATH F113
Probability and Statistics

Dr. Shivi Agarwal
Department of Mathematics

Chapter 8

Inferences on the mean and variance of a distribution

Estimation of variance

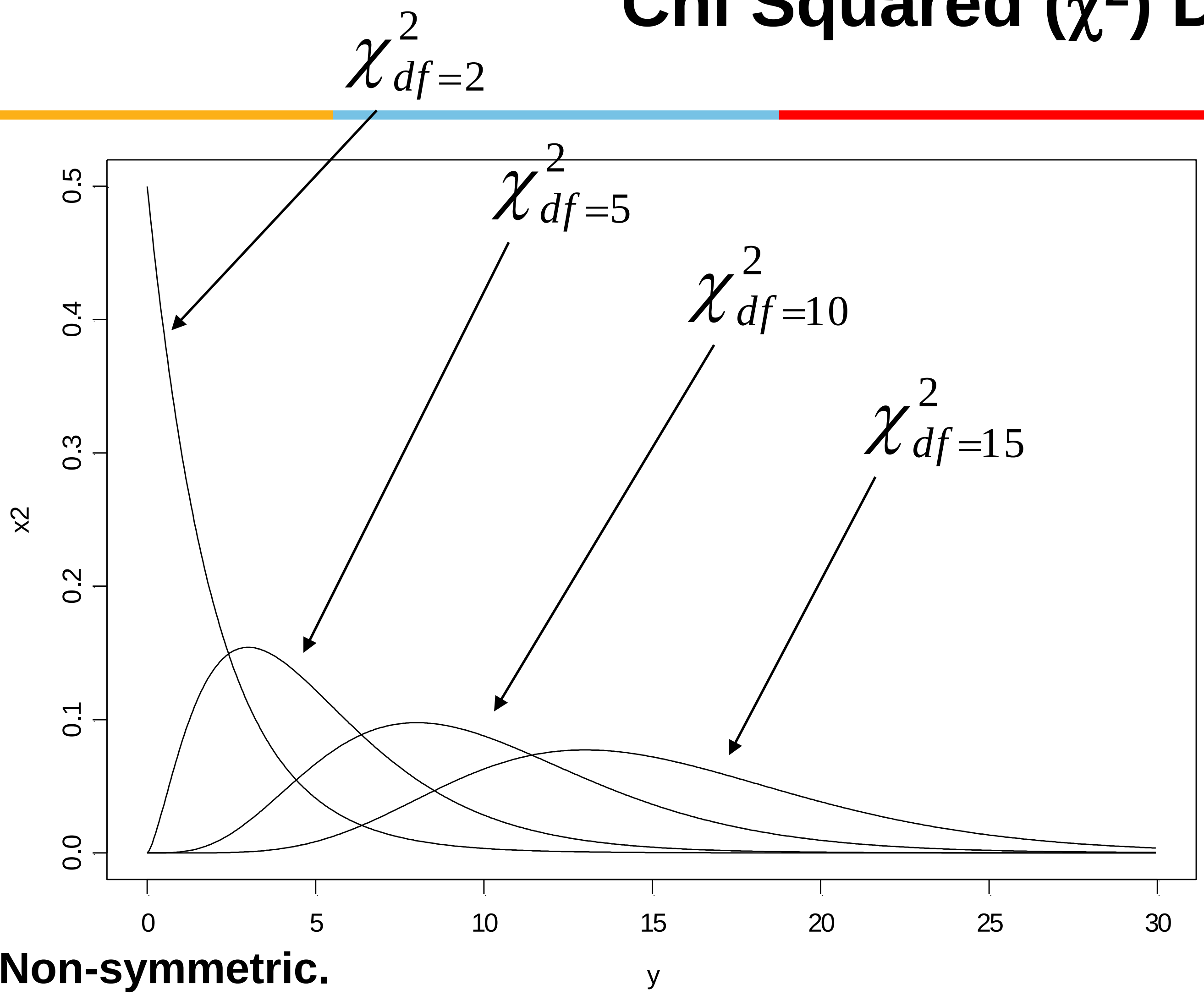
Recall : S^2 is an unbiased estimator for σ^2.

Theorem 8.1.1 : Let $X_1, \ldots, X_n$ be the random sample of size n from a normal population with mean μ and s.d. σ. Then $(n-1)\, S^2/\sigma^2 = \sum_{i=1}^{n}(X_i - \overline{X})^2/\sigma^2$

has chi-squared distribution with (n-1) degrees of freedom.

Recall : Chi-squared dist with (n-1) degrees of freedom is Gamma dist with $\alpha = (n-1)/2$, $\beta=2$.

Chi Squared (χ^2) Distribution



Non-symmetric.

Shape indexed by one parameter called the degrees of freedom (df).

Confidence interval for variance

Theorem 8.1.2: Let $X_1, \ldots, X_n$ be the random sample of size n from a normal population with mean μ and s.d. σ. Using the above theorem, the 100(1- α)% confidence interval for σ^2 is given by

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}.$$

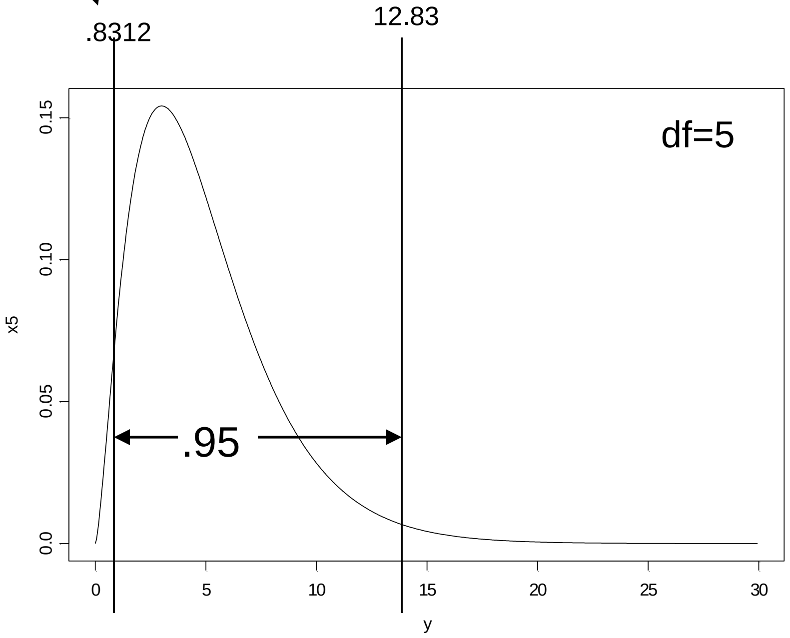
Confidence Interval for σ^2

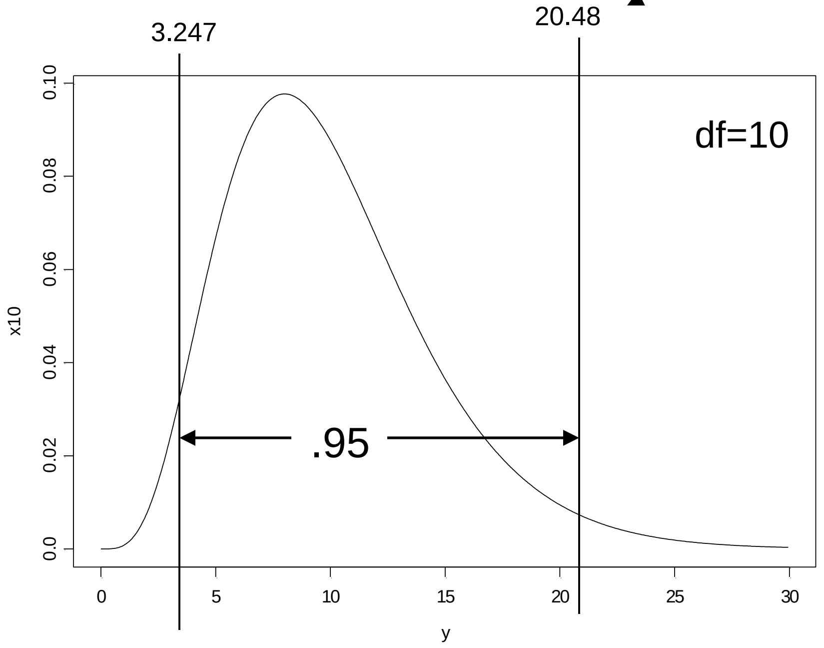
If $\dfrac{(n-1)s^2}{\sigma^2}$ has a Chi Squared Distribution, then a 100(1-α)% CI can be computed by finding the upper and lower α/2 critical values from this distribution.

$$P\left(\chi^2_{df,1-\alpha/2} < \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{df,\alpha/2}\right) = 1-\alpha$$

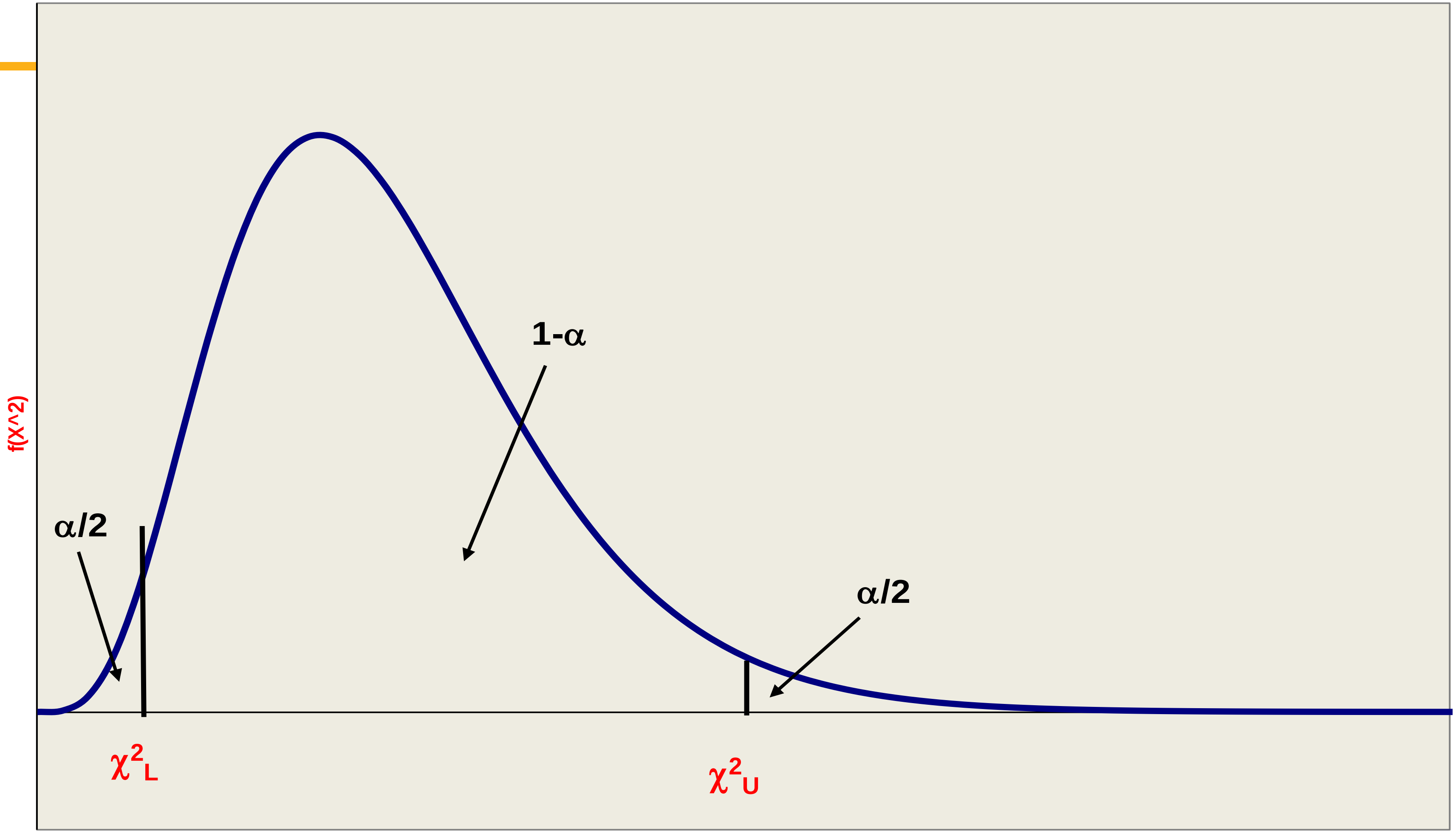
$\chi^2_{df,1-\alpha/2}$

$\chi^2_{df,\alpha/2}$





Chi-Squared distribution



$$P\left(\chi^2_{df,1-\alpha/2} < \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{df,\alpha/2}\right) = 1-\alpha$$

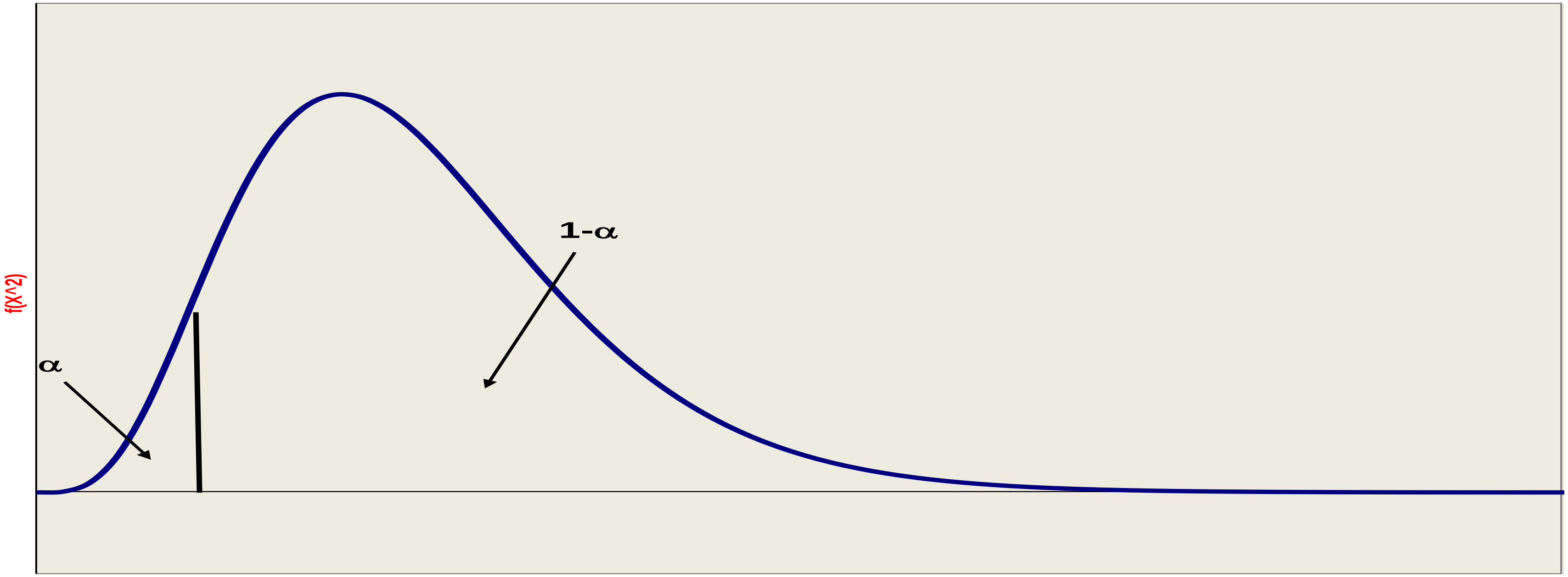
$$\frac{(n-1)s^2}{\chi^2_{df,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{df,1-\alpha/2}}$$

Q 7. Recent research indicates that heating and cooling commercial buildings with ground water source heat pumps is economically sound. The crucial random variable being studied is the water temperature which is normally distributed. A sample of 15 wells in the state of California yields a sample standard deviation of 7.5^o F. Find a 95% confidence interval on the standard deviation in temperature of wells in California.

df	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995	0.999
	P													
1	0.000	0.000	0.001	0.004	0.016	0.102	0.455	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.010	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.072	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	0.989	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.647	2.180	2.733	3.490	5.071	7.344	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	5.578	7.584	10.34	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	3.074	3.571	4.404	5.226	6.304	8.438	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	7.041	9.299	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	7.790	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	8.547	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	9.312	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.31

Q 4. Since variance is a measure of consistency, it is usually hoped that σ^2 will be small.

One sided confidence interval on σ^2=[0,L]



$$\text{where} \quad L = \frac{(n-1)S^2}{\chi^2_{1-\alpha}}$$

- Example: X is actual length of 63mm nails, Use the given data to find a 95% one side confidence interval on the variance in length

63.0	63.1	63.0	63.0	62.9	63.0	63.0
63.1	62.8	63.1	63.1	63.0	62.9	63.2

The manufacturer wants to check to be sure that the population variance of the length of nails being produced does not exceed 0.03. Assume that the length is normally distributed. Does this sample indicate that this is in the case? Explain.

df	P													
	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1	0.000	0.000	0.001	0.004	0.016	0.102	0.455	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.010	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.072	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	0.989	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.647	2.180	2.733	3.490	5.071	7.344	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	5.578	7.584	10.34	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	3.074	3.571	4.404	5.226	6.304	8.438	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	7.041	9.299	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	7.790	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	8.547	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	9.312	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.31

Confidence interval of mean-variance unknown.

- If sample size is large and variance known, then using central limit theorem, we have given confidence interval involving σ^2.

- If sample size is large and variance unknown, then we can replace σ^2 by its estimate s^2 in this formula.

- What if sample is small?

T-distribution

What is the distribution of

$$\frac{\bar{X} - \mu}{S/\sqrt{n}}?$$

Need assumption of normality on population X to find it.

Let Z be a standard normal r.v and X_γ^2 be an independent chi-squared r.v. with γ degrees of freedom. Then distribution of T is called T distribution with γ degrees of freedom where

$$T = \frac{Z}{\sqrt{X_\gamma^2/\gamma}}.$$

Theorem 8.2.1

Let $X_1,\ldots,X_n$ be the random sample of size n from a normal r.v X mean μ (unknown) and variance σ^2 (unknown). Then

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim T_{n-1}$$

More about T dist.

innovate achieve lead

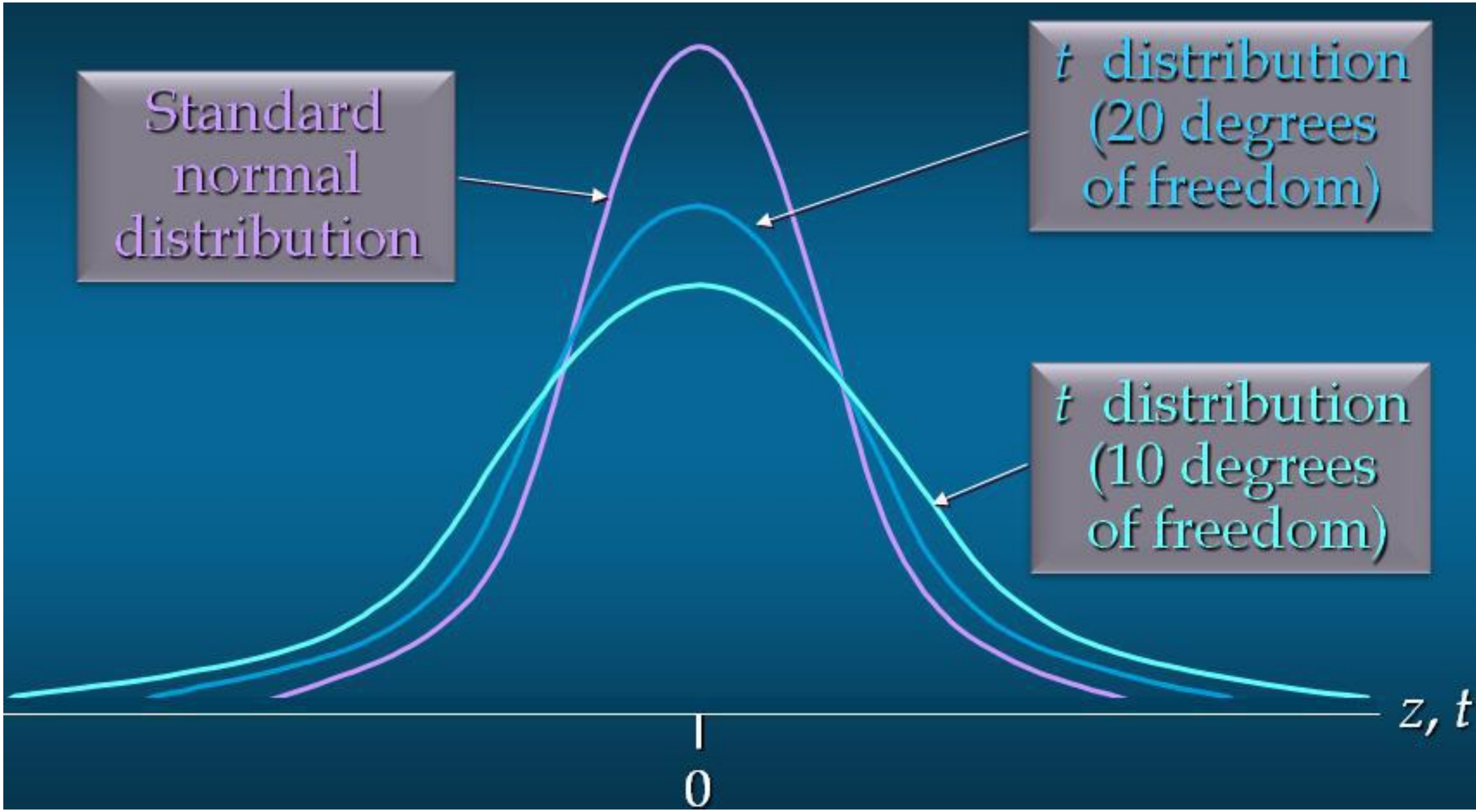
- T random variable with γ degrees of freedom (called parameter) is a continuous r.v. with density

$$f(t) = \frac{\Gamma(\gamma+1)/2}{\Gamma(\gamma/2)\sqrt{\pi\gamma}}\left(1+\frac{t^2}{\gamma}\right)^{-(\gamma+1)/2} \quad ; -\infty < t < \infty.$$

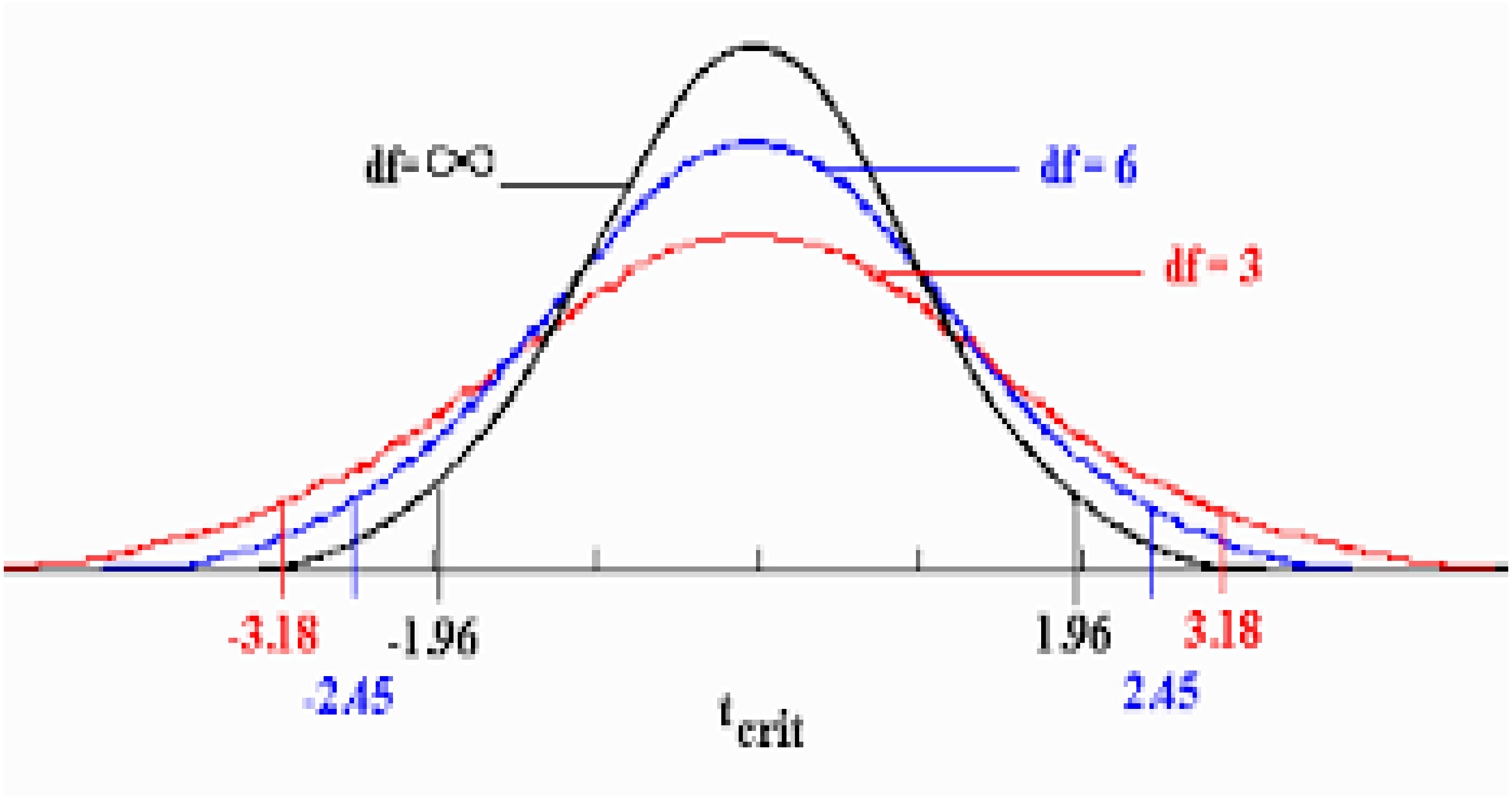
- Graph of the density is bell shaped sym about 0.

- Variance of T decreases as γ increases. In fact T approximately is standard normal for large γ.

T- distribution

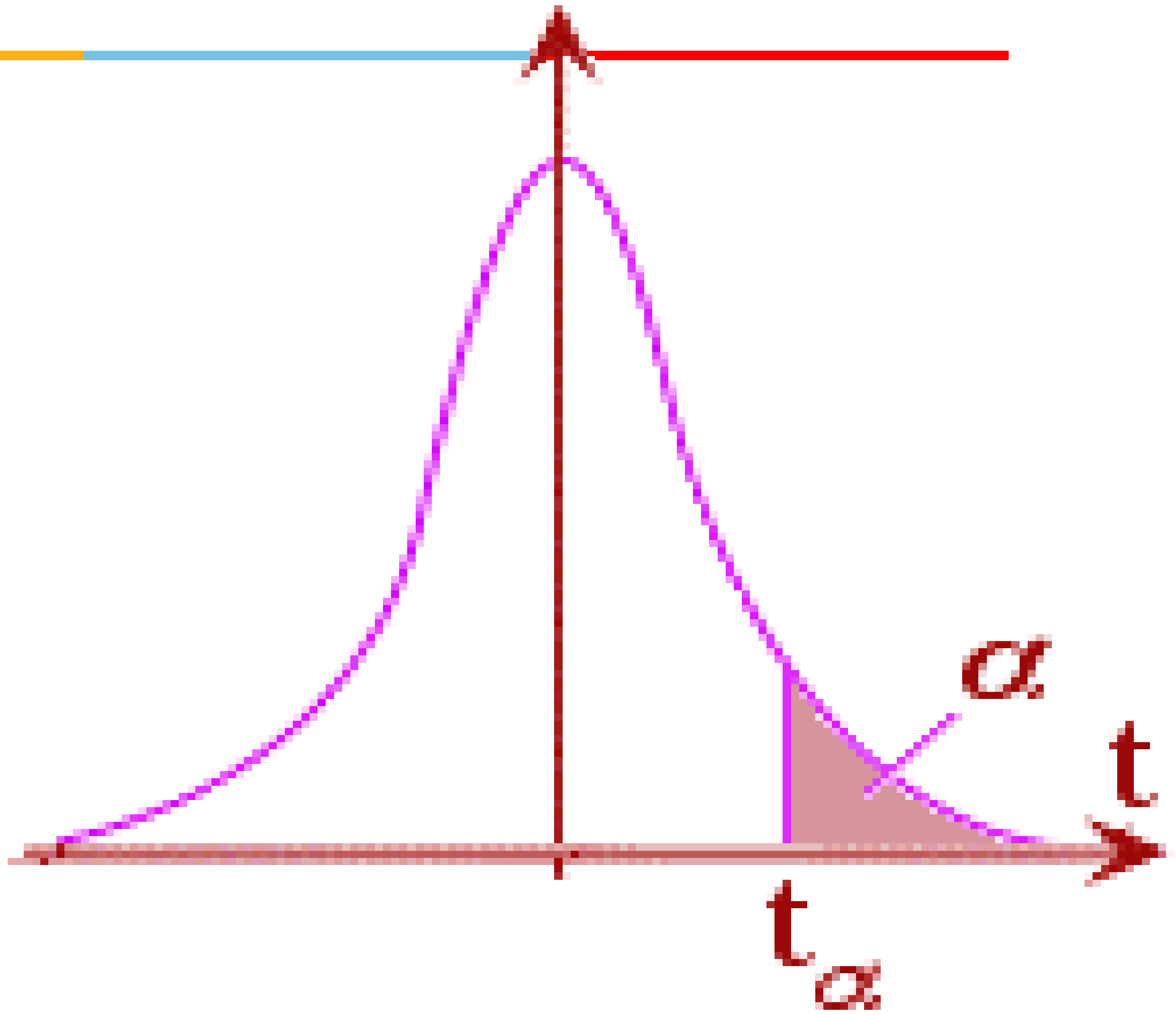


T- distribution



c.d.f. of T dist.

- Cdf is given by tables (Table VI, p.699).
- As for chi-square table, γ is the label of rows, F(t) is label of columns, at their intersection is value (t) for γ degrees of freedom.
- By t_r we denote the value of the t-variable such that area under its density to its right is r. (The degrees of freedom must be mentioned separately).


α
t
t_α

df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
					P					
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$t_{0.10}\,(\gamma = 10) = 1.372$$

					P					
df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$t_{0.05}\ (\gamma = 8) \ = ?$$

df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
					P					
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$t_{0.975}(\gamma = 12) = ?$$

$t_{0.05}(\gamma = 150) = ?$

df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
37	0.255	0.681	0.851	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	0.255	0.681	0.851	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	0.255	0.681	0.851	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	0.255	0.681	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.255	0.679	0.849	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.254	0.679	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
70	0.254	0.678	0.847	1.294	1.667	1.994	2.381	2.648	3.211	3.435
80	0.254	0.678	0.846	1.292	1.664	1.990	2.374	2.639	3.195	3.416
90	0.254	0.677	0.846	1.291	1.662	1.987	2.368	2.632	3.183	3.402
100	0.254	0.677	0.845	1.290	1.660	1.984	2.364	2.626	3.174	3.390
∞	0.253	0.674	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.290

Ex. 9

(g) Using T-table find t such that $P[-t \leq T_{25} \leq t] = 0.90$. By symmetry, need to find $t_{0.05}$ for 25 degrees of freedom. Equivalently, $F(t_{0.05}) = 0.95$ for 25 degrees of freedom.

Look in T-table in row corresponding to and column corresponding to 0.95 .

This gives $t = 1.708$.

(k) $P[T_{16} \leq -t] = 0.05$ implies $P[T_{16} \geq t] = 0.05$, i.e. $P[T_{16} \leq t] = 0.95$, from table, $t = 1.746$.

$P[-t \leq T_{25} \leq t] = 0.90$

df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
21	0.257	0.686	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.256	0.683	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.256	0.683	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646

	p									
df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$P[T_{16} \le -t] = 0.05; \; t = ?$$

C.I. for mean-variance unknown

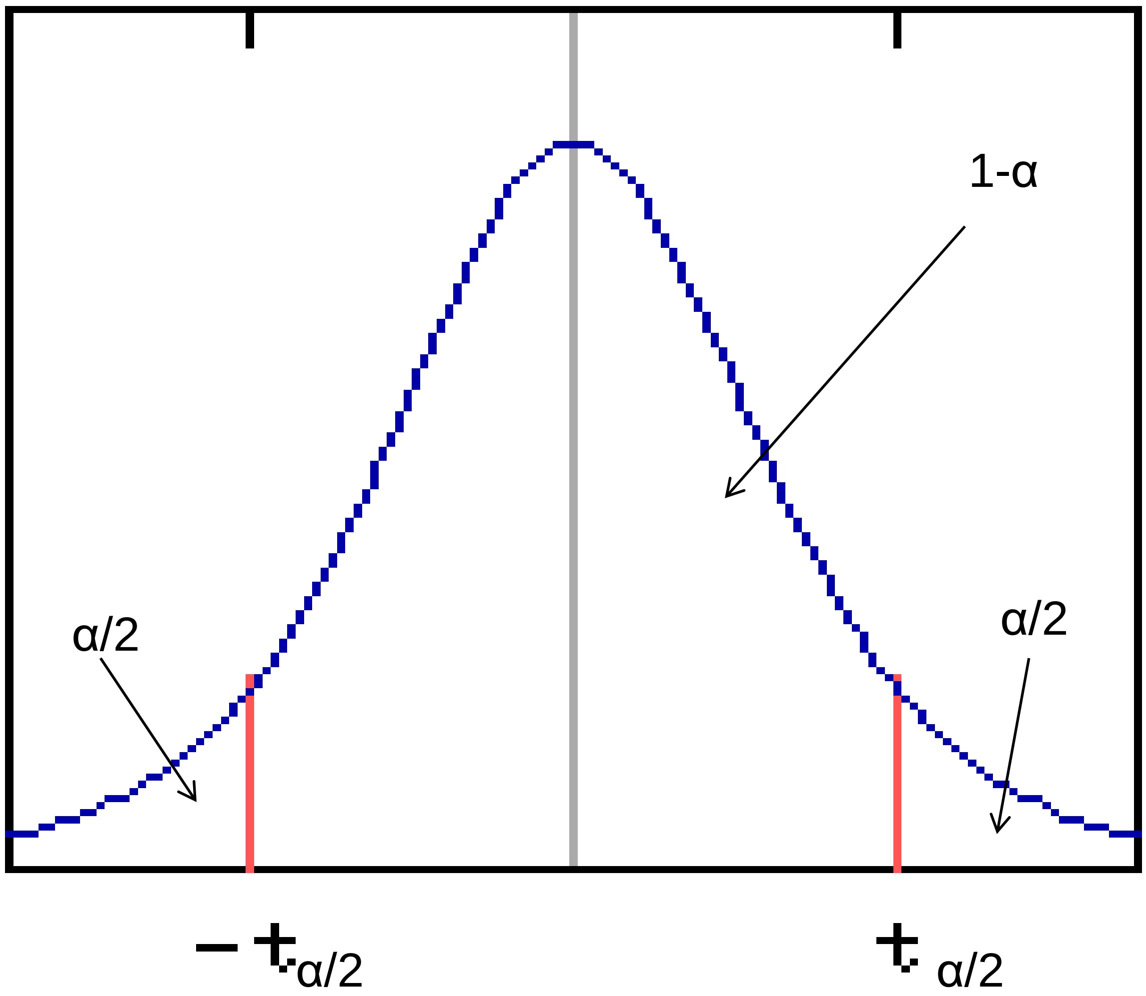
Theorem 8.2.1 : Let $X_1,\ldots,X_n$ be the random sample of size n from a normal r.v X. Then

$$\frac{\bar{X}-\mu}{S/\sqrt{n}}$$

has T distribution with ***(n - 1)*** degrees of freedom.

This gives 100(1-α)% confidence interval for the mean from ***normal*** population of unknown variance as

$\left[\bar{X}-t_{\alpha/2}S/\sqrt{n}, \bar{X}+t_{\alpha/2}S/\sqrt{n}\right]$ where T dist. has (n-1) degrees of freedom.


1-α
α/2
α/2
− t α/2
t α/2

Q 10. The “supergopher” is a device invented to drill through arctic pack ice. It is a cone shaped apparatus 5 feet high, 4 feet wide, and wound with a copper coil. Water heated to 180°F is pumped through the coil. This allows the gopher to melt a vertical round shaft through the ice. Let X denote the distance or depth that the gopher can drill per hour which is normally distributed. These data are obtained on 10 test holes (depth is in feet):

2.0	1.7	2.6	1.5	1.4
2.1	3.0	2.5	1.8	1.4

(a) Use these data to find $\overline{x}, s^2$, and s.

(b) Find a 90% confidence interval on the average distance that can be drilled in an hour.

	p									
df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$t_{0.05}(\gamma = 9) = ?$$

Q 14. To estimate the average number of pounds of copper recovered per ton of ore mine, a sample of 150 tons of ores is monitored. A sample mean of 11 pounds with a sample s.d. of 3 pounds was obtained. Construct a 95% confidence interval on the mean number of pounds of copper recovered per ton of ore mined. Assume normality of the population.

$T_{0.025} = 1.96$ for 149 $\sim\infty$ degrees of freedom

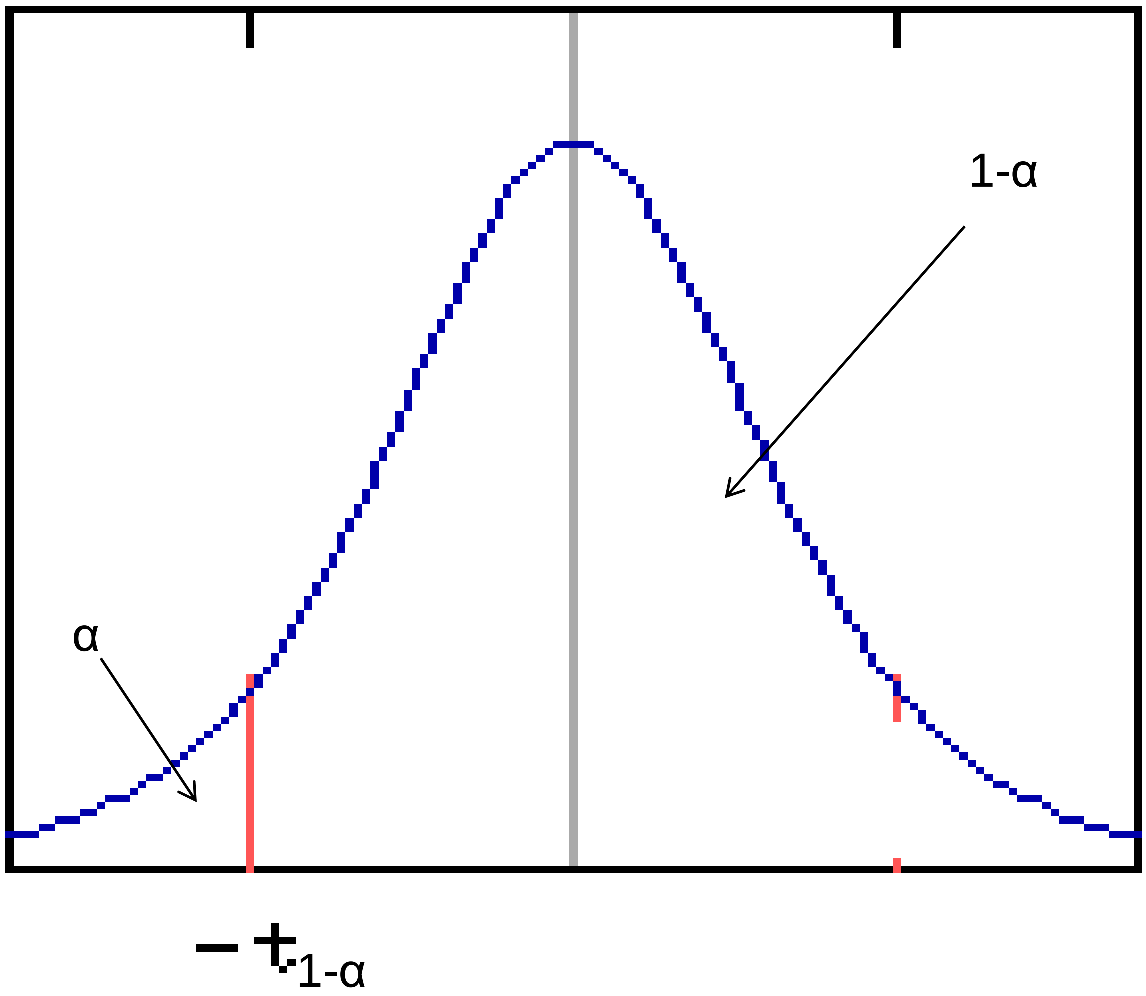
(See in row for ∞ and col. 0.975 in T-table.)

And substitute this and $\bar{x}=11$, ,s=3, n=150 in C.I. formula.

Q 17. One sided confidence interval can be used to approximate the maximum and minimum value of the population mean.

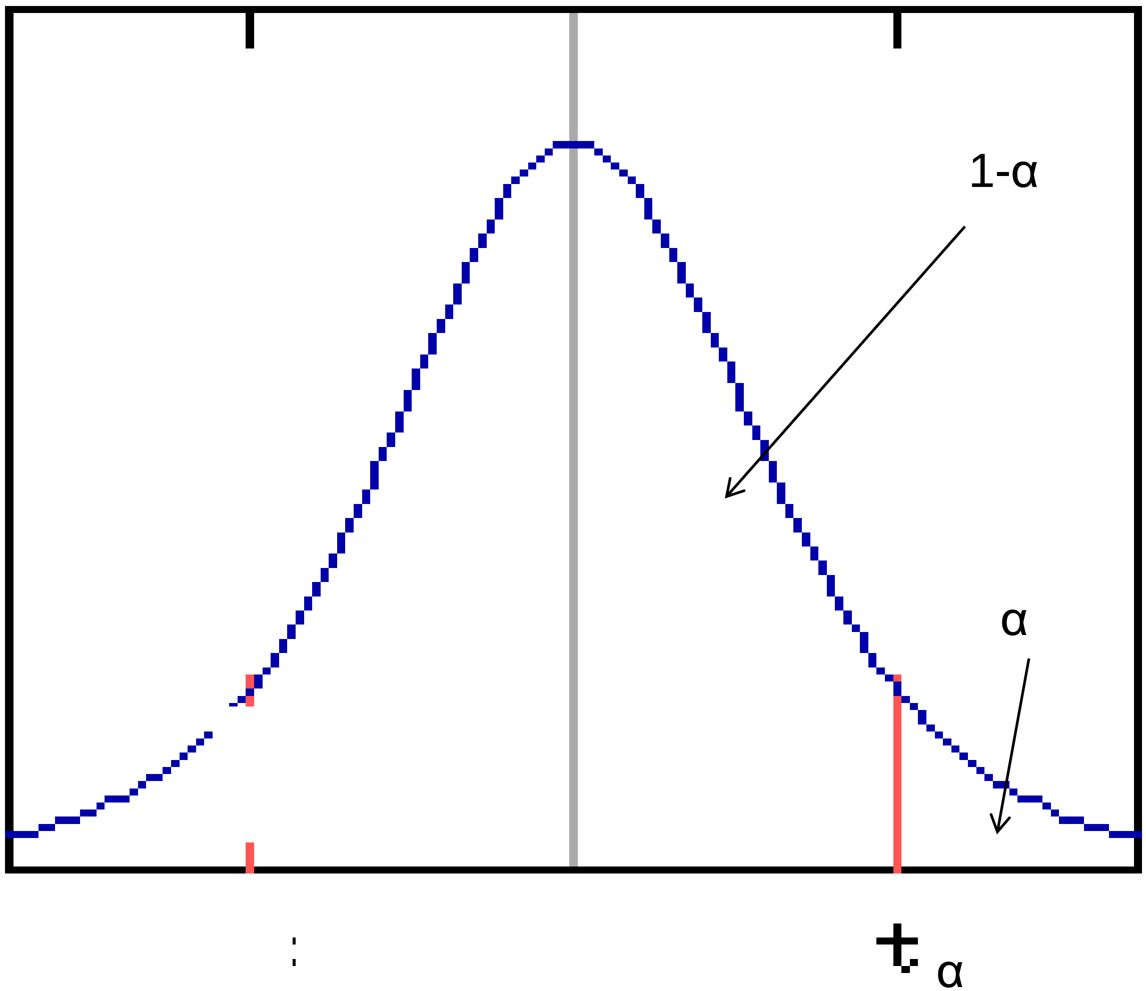
An interval $(-\infty, L_1]$ such that $P(\mu \le L_1) = 1-\alpha$ allows us to place bounds on the maximum value of population mean

$$\text{where } L_1 = \bar{X} + t_\alpha S / \sqrt{n}$$


1-α
α
$-t_{1-\alpha}$

An interval $[L_2, \infty)$ such that $P(\mu \geq L_2) = 1-\alpha$ allows us to place bounds on the minimum value of population mean

$$\text{where } L_2 = \bar{X} - t_\alpha S / \sqrt{n}$$


1-α
α
t α

Use the following data on X, the time that a commercial airliner stays at the gate during a through flight, to find a 95% one sided confidence interval that puts a bound on the minimum time in minutes for μ:

25 29 32 37 40 27 30 35 38 41

42 45 45 47 49 50 55 53 60

	p									
df	0.600	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9990	0.9995
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850

$$t_{0.05}(\gamma = 18) = ?$$

Sample size required to estimate μ with specified error

Q 19. (A consequence of central limit theorem) We can assert with 100(1-α)% confidence that sample mean from a sample of size n differs from population mean by at most d if the sample size $n \geq (z_{\alpha/2})^2 \sigma^2/d^2$, i.e., using the sample mean of any sample of such size, we can estimate the population mean within d units with 100(1-α)% confidence.

Q 19. (b) A preliminary pilot study is run, and an estimated standard deviation of 500 units is obtained. How large a sample is needed to estimate μ to within 50 units with 95% confidence?

(c) If rough estimate of σ is 0.75. How large a sample is needed to estimate μ to within 0.1 with 90% confidence?