IC CHARACTERSTICS

- Fan Out
- Propagation Delay
- Noise Margin
- Power Dissipation

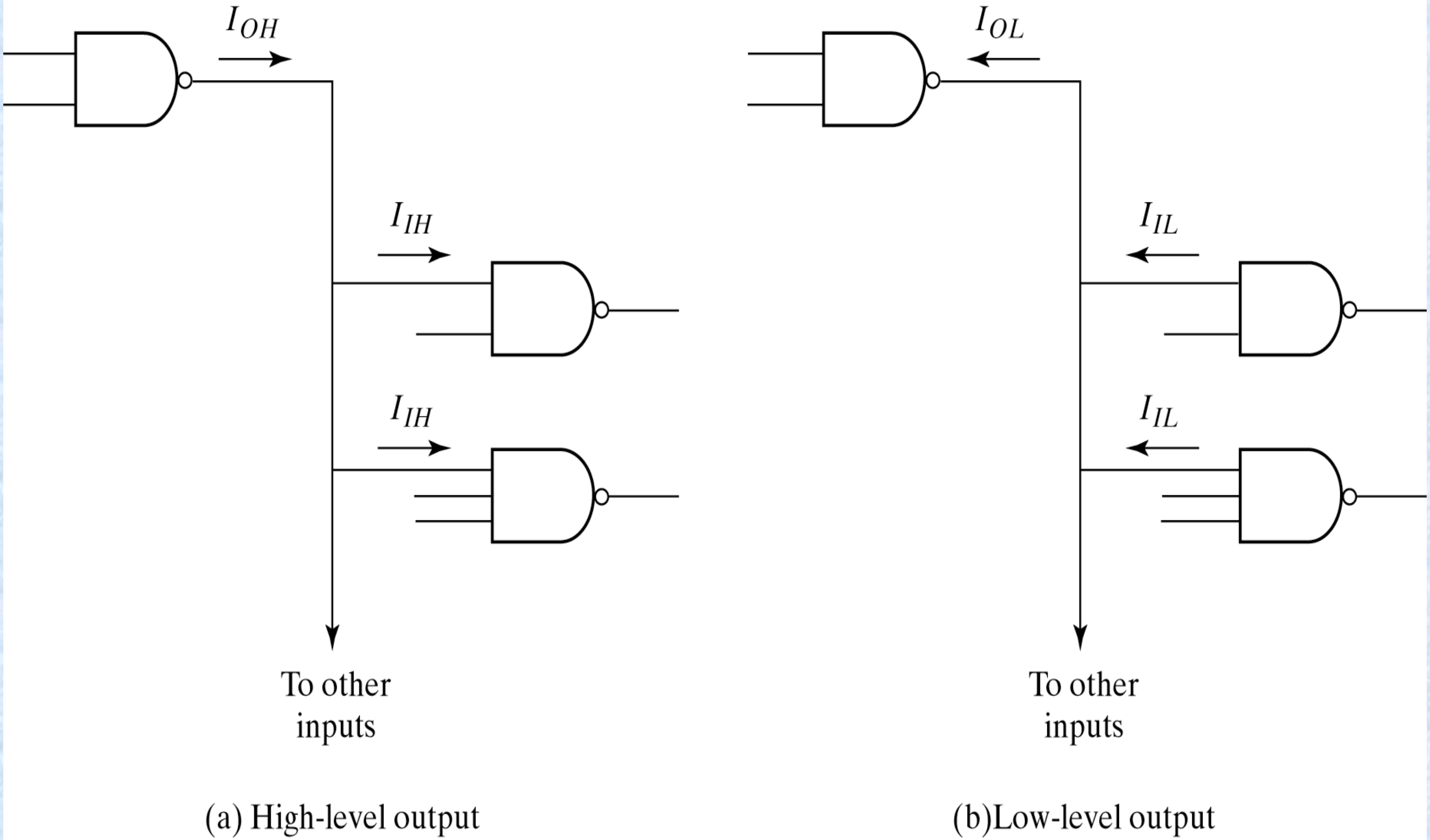


Fig. 10-3 Fan-Out Computation

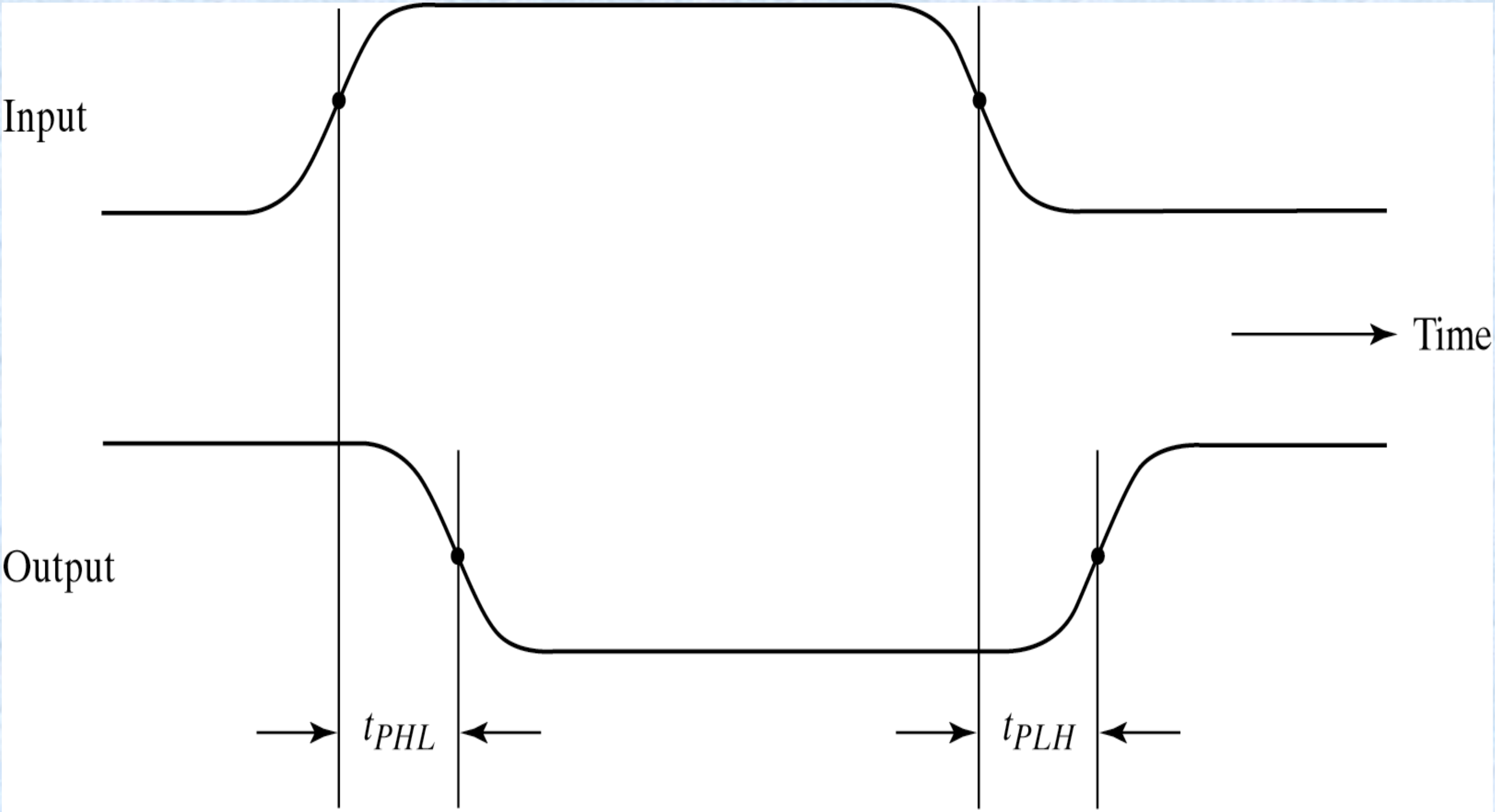


Fig. 10-4 Measurement of Propagation Delay

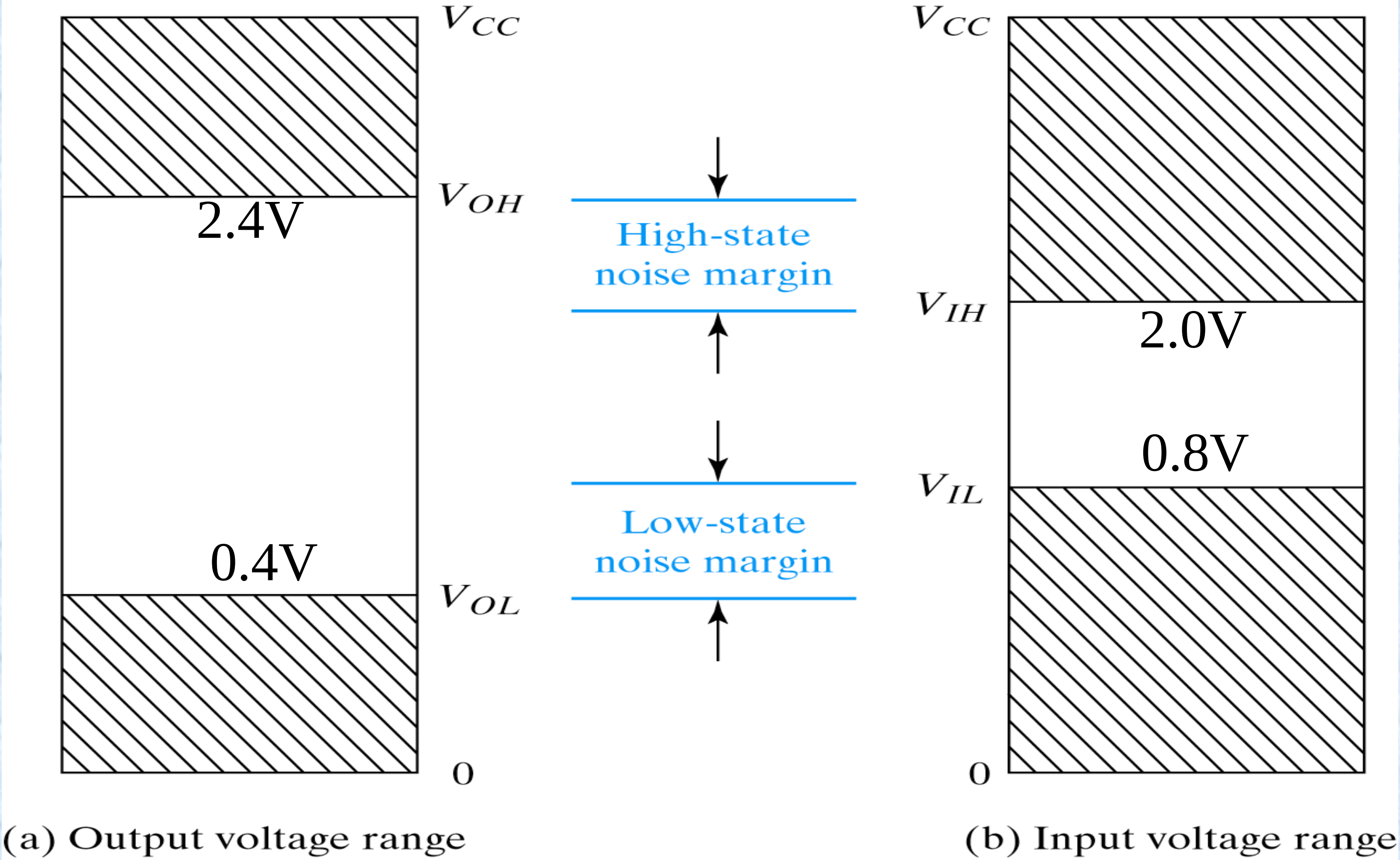
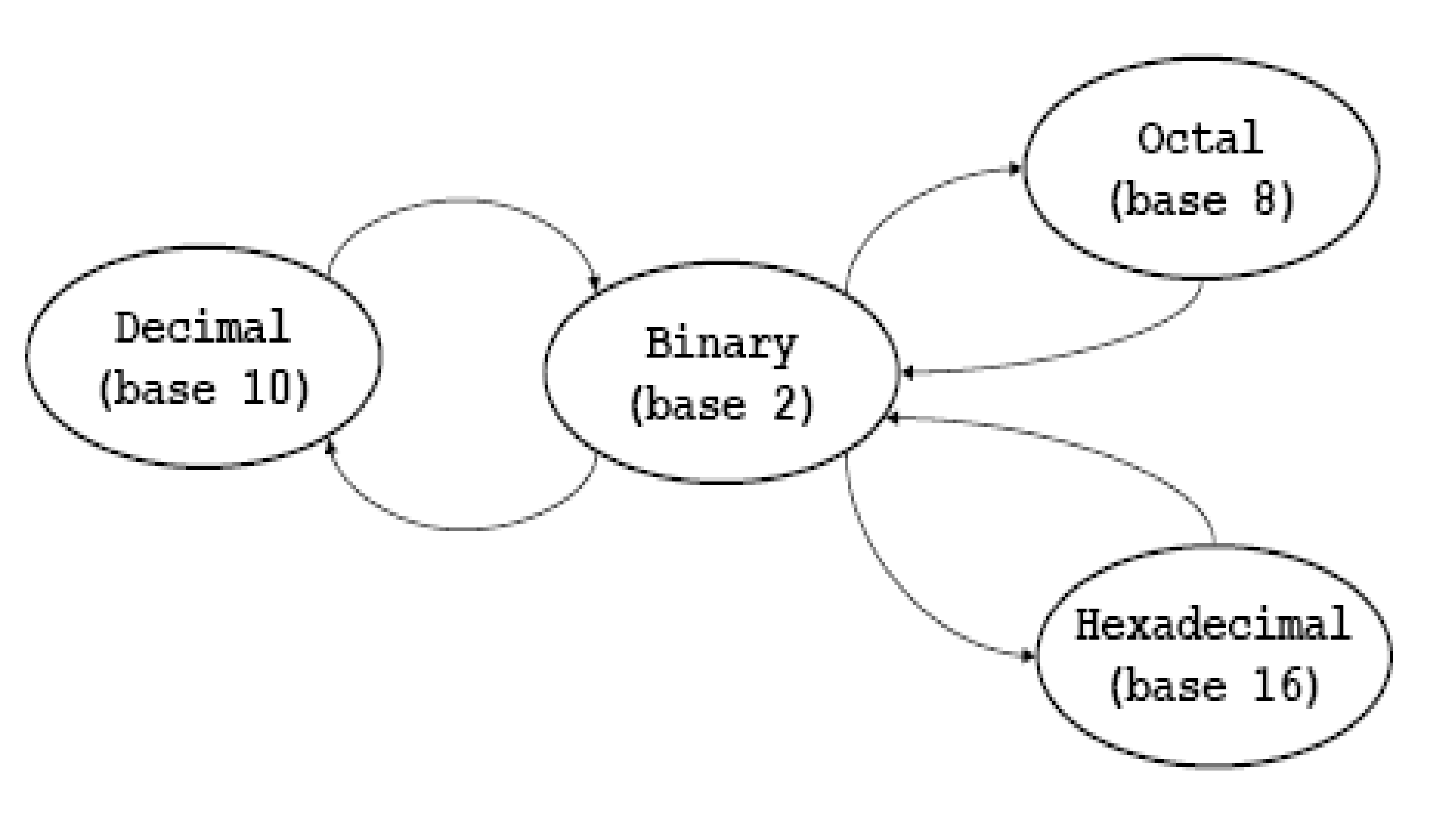


Fig. 10-5 Signals for Evaluating Noise Margin

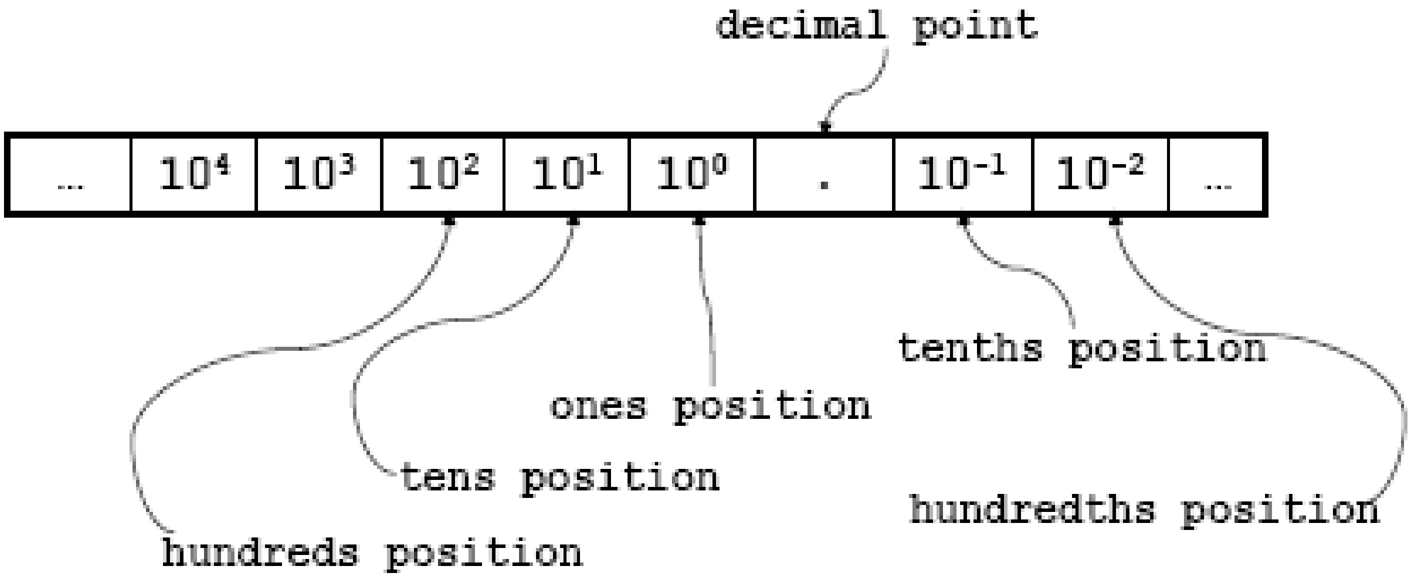
Power Dissipation (P_D)

- Expressed in Milliwatts
- $PD= V_{CC} * I_{CC}$
- $I_{CC}(avg)=(I_{CCH}+I_{CCL})/2$

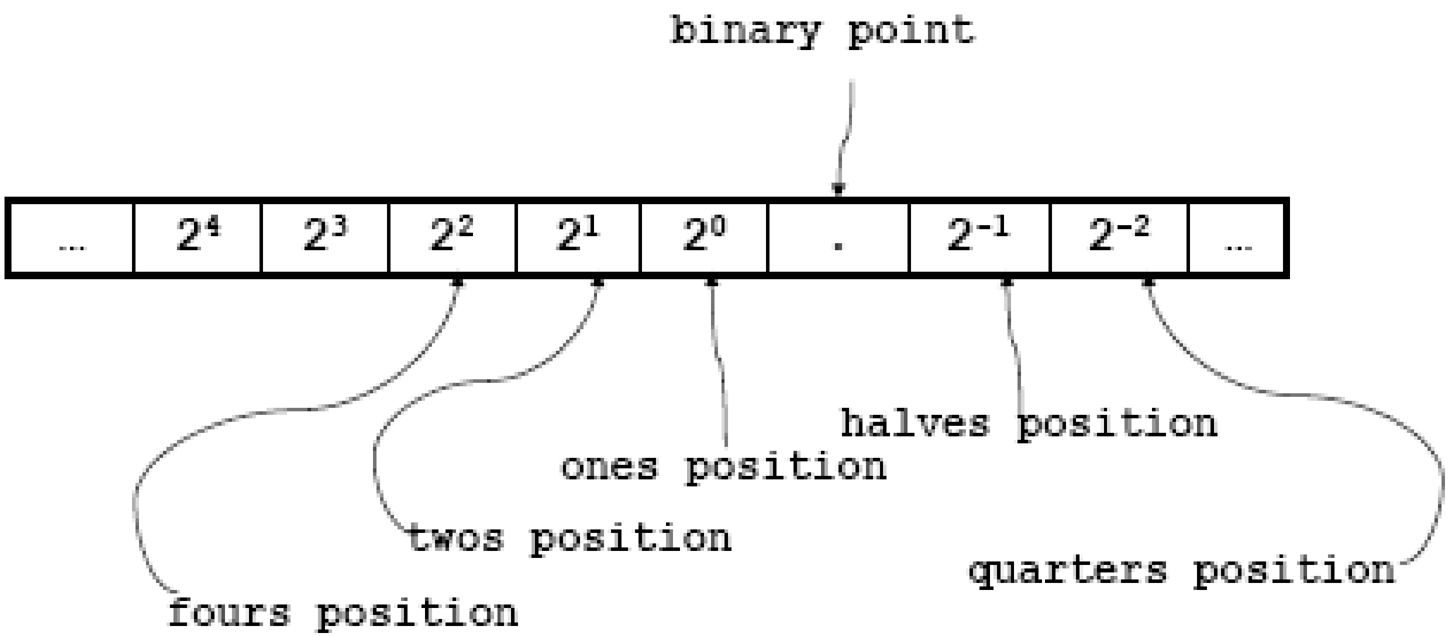
Number Systems and Codes



Decimal Positional System
(Base 10 or radix 10)



Binary Positional System
(Base 2 or radix 2)



Example

Decimal Example

$$346.17_{10} = (3\times10^{2}) + (4\times10^{1}) + (6\times10^{0}) + (1\times10^{-1}) + (7\times10^{-2})$$
$$= 300 \quad + 40 \quad + 6 \quad + 0.1 \quad + 0.07$$

Binary Example

$$1101.01_{2} = 1\times2^{3} + 1\times2^{2} + 0\times2^{1} + 1\times2^{0} + 0\times2^{-1} + 1\times2^{-2}$$
$$= 8 \quad + 4 \quad + 0 \quad + 1 \quad + 0 \quad + .25$$
$$= 13.25_{10}$$

- **Diminished Radix Complement**

  Number N in base r having n digits

  (r-1)'s complement = $(r^n - 1) - N$

- **Radix Complement**

  Number N in base r having n digits

  (r)'s complement = $(r^n - N)$

Ex: base 2: 2's complement, 1's complement

base10: 10's compliment, 9's compliment

Some possible binary codes are illustrated below.

Decimal digits	BCD (8421)	Excess-3	84-2-1	2421	Bi-quinary (5043210)
0	0000	0011	0000	0000	0100001
1	0001	0100	0111	0001	0100010
2	0010	0101	0110	0010	0100100
3	0011	0110	0101	0011	0101000
4	0100	0111	0100	0100	0110000
5	0101	1000	1011	1011	1000001
6	0110	1001	1010	1100	1000010
7	0111	1010	1001	1101	1000100
8	1000	1011	1000	1110	1001000
9	1001	1100	1111	1111	1010000

The Reflected Code

The advantage of the reflected code over pure binary numbers is that a number in the reflected code changes by only one bit as it proceeds from one number to the next. The reflected code is also known as the *Gray* code.

Generation of Gray Codes:

an n-bits code is generated by reflecting the (n-1)-bit code.

0	0	00	000	0000
1	1	01	001	0001
	1	11	011	0011
	0	10	010	0010
		10	110	0110
		11	111	0111
		01	101	0101
		00	100	0100
			100	1100
			101	1101
			111	1111
			110	1110
			010	1010
			011	1011
			001	1001
			000	1000

Reflected Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	2
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Four-bit reflected code

- Boolean expressions must be evaluated with the following order of operator precedence
  - parentheses
  - NOT
  - AND
  - OR

Example:

$$F = (A(\overline{C + \overline{B}D}) + \overline{B\overline{C}})\overline{E}$$

$$\Downarrow$$

$$F = \underbrace{\underbrace{\left(\underbrace{A\underbrace{\left(\underbrace{\overline{C + \underbrace{\underbrace{\overline{B}}D}}}\right)}} + \underbrace{\underbrace{\overline{B\underbrace{\overline{C}}}}}\right)}\;\underbrace{\overline{E}}}$$

$X + 0 = X$	$X \cdot 1 = X$	Identity
$X + 1 = 1$	$X \cdot 0 = 0$	
$X + X = X$	$X \cdot X = X$	Idempotent Law
$X + X' = 1$	$X \cdot X' = 0$	Complement
$(X')' = X$		Involution Law
$X + Y = Y + X$	$XY = YX$	Commutativity
$X + (Y + Z) = (X + Y) + Z$	$X(YZ) = (XY)Z$	Associativity
$X(Y + Z) = XY + XZ$	$X + YZ = (X + Y)(X + Z)$	Distributivity
$X + XY = X$	$X(X + Y) = X$	Absorption Law
$X + X'Y = X + Y$	$X(X' + Y) = XY$	Simplification
$(X + Y)' = X'Y'$	$(XY)' = X' + Y'$	DeMorgan's Law
$XY + X'Z + YZ = XY + X'Z$	$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$	Consensus Theorem

- Example: Simplify the following expression

$$F = BC + B\overline{C} + BA$$

- Simplification

$$F = B(C + \overline{C}) + BA$$

$$F = B \cdot 1 + BA$$

$$F = B(1 + A)$$

$$F = B$$

- Example: Simplify the following expression

$$F = A + \overline{A}B + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}E$$

- Example: Simplify the following expression

$$F = A + \overline{A}B + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}E$$

- Simplification

$$F = A + \overline{A}(B + \overline{B}C + \overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D}E)$$

$$F = A + B + \overline{B}C + \overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D}E$$

$$F = A + B + \overline{B}(C + \overline{C}D + \overline{C}\,\overline{D}E)$$

$$F = A + B + C + \overline{C}D + \overline{C}\,\overline{D}E$$

$$F = A + B + C + \overline{C}(D + \overline{D}E)$$

$$F = A + B + C + D + \overline{D}E$$

$$F = A + B + C + D + E$$

Complementing a truth table

- The complement of a function should output 0 when the original function outputs 1, and vice versa.
- In a truth table, we can just exchange 0 and 1 in the output column.
  - On the left is a truth table for $f(x,y,z) = (x + y')z + x'$.
  - On the right is the table for the complement of f, denoted $f'(x,y,z)$.

x	y	z	f(x,y,z)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x	y	z	f'(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Complementing an expression

- To complement an expression, you can use DeMorgan's Laws to keep "pushing" the NOT operator inwards, all the way to the literals.

$$f(x,y,z) = (x + y')z + x'$$

$$\begin{aligned} f'(x,y,z) &= ((x + y')z + x')' && [\text{ complementing both sides }] \\ &= ((x + y')z)' \bullet (x')' && [\text{ because } (x + y)' = x'y' \text{ }] \\ &= ((x + y')' + z') \bullet x && [\text{ } (xy)' = x' + y'\text{, and } (x')' = x \text{ }] \\ &= (x'y + z') \bullet x && [\text{ } (x + y)' = x'y' \text{ again }] \end{aligned}$$

- Another clever method of complementing an expression is to take the dual of the expression, and then complement each literal.
  - The dual of $(x + y')z + x'$ is $(xy' + z) \bullet x'$.
  - Complementing each literal yields $(x'y + z') \bullet x$.
  - So $f'(x,y,z) = (x'y + z') \bullet x$.

- **Complement of a Function.**

  $$F = x'yz' + x'y'z$$

- **Complement of a Function.**

  $F = x'yz' + x'y'z$

  dual of F is $(x'+y+z')(x'+y'+z)$

  complement each literal

  - $(x+y'+z)(x+y+z') = F'$

Sum of products expressions

- There are many equivalent ways to write a function, but some forms turn out to be more useful than others.
- A sum of products or SOP expression consists of:
  – One or more terms *summed* (OR'ed) together.
  – Each of those terms is a *product of literals*.

$$f(x, y, z) = y' + x'yz' + xz$$

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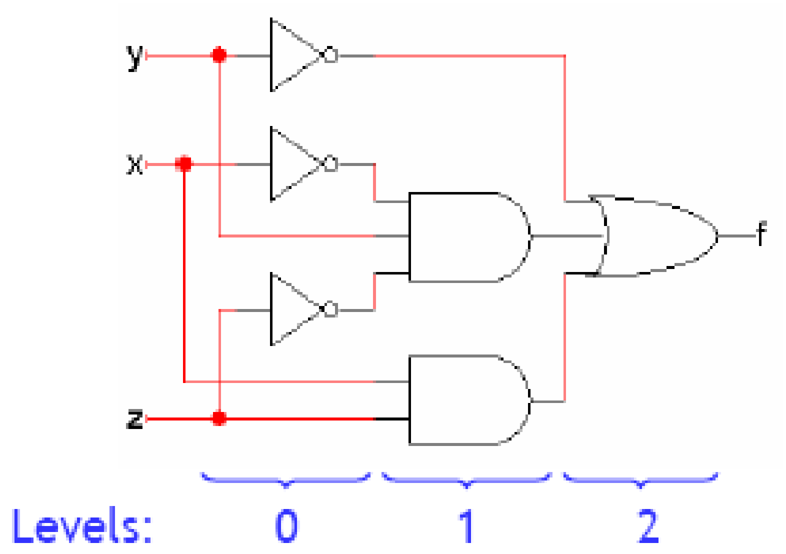
- Sum of products expressions can be implemented with two-level circuits.



Minterms

- A minterm is a special product of literals, in which each input variable appears exactly once.
- A function with n input variables has 2^n possible minterms.

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- A function with n input variables has 2^n possible minterms.
- For instance, a three-variable function f(x,y,z) has 8 possible minterms:

$x'y'z'$	$x'y'z$	$x'y z'$	$x'y z$
$x y'z'$	$x y'z$	$x y z'$	$x y z$

- Each minterm is true for exactly one combination of inputs.

Minterm	True when	Shorthand
$x'y'z'$	xyz = 000	m_0
$x'y'z$	xyz = 001	m_1
$x'y z'$	xyz = 010	m_2
$x'y z$	xyz = 011	m_3
$x y'z'$	xyz = 100	m_4
$x y'z$	xyz = 101	m_5
$x y z'$	xyz = 110	m_6
$x y z$	xyz = 111	m_7

Sum of minterms expressions

- A sum of minterms is a special kind of sum of products.
- Every function can be written as a *unique* sum of minterms expression.
- A truth table for a function can be rewritten as a sum of minterms just by finding the table rows where the function output is 1.

Sum of minterms expressions

- A sum of minterms is a special kind of sum of products.
- Every function can be written as a *unique* sum of minterms expression.
- A truth table for a function can be rewritten as a sum of minterms just by finding the table rows where the function output is 1.

x	y	z	C(x,y,z)	C'(x,y,z)
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$C = x'yz + xy'z + xyz' + xyz$$
$$= m_3 + m_5 + m_6 + m_7$$
$$= \Sigma m(3,5,6,7)$$

$$C' = x'y'z' + x'y'z + x'yz' + xy'z'$$
$$= m_0 + m_1 + m_2 + m_4$$
$$= \Sigma m(0,1,2,4)$$

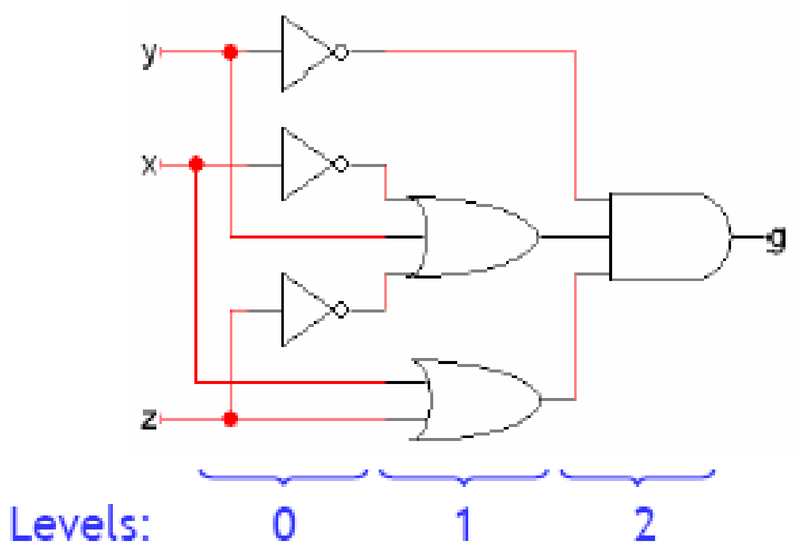
C' contains all the minterms *not* in C, and vice versa.

Product of sums expressions

- As you might expect, we can work with the duals of these ideas too.
- A product of sums or POS consists of:
  - One or more terms *multiplied* (AND'ed) together.
  - Each of those terms is a *sum of literals*.

$$g(x, y, z) = y'(x' + y + z')(x + z)$$

- Products of sums can also be implemented with two-level circuits.



Maxterms

- A maxterm is a *sum* of literals where each input variable appears once.
- A function with n input variables has 2^n possible maxterms.
- For instance, a function with three variables x, y and z has 8 possible maxterms:

$$x + y + z \qquad x + y + z' \qquad x + y' + z \qquad x + y' + z'$$
$$x' + y + z \qquad x' + y + z' \qquad x' + y' + z \qquad x' + y' + z'$$

- Each maxterm is *false* for exactly one combination of inputs.

Maxterm	False when	Shorthand
$x + y + z$	xyz = 000	M_0
$x + y + z'$	xyz = 001	M_1
$x + y' + z$	xyz = 010	M_2
$x + y' + z'$	xyz = 011	M_3
$x' + y + z$	xyz = 100	M_4
$x' + y + z'$	xyz = 101	M_5
$x' + y' + z$	xyz = 110	M_6
$x' + y' + z'$	xyz = 111	M_7

Product of maxterms expressions

- Every function can also be written as a unique product of maxterms.
- A truth table for a function can be rewritten as a product of maxterms just by finding the table rows where the function output is 0.

x	y	z	C(x,y,z)	C'(x,y,z)
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$\begin{aligned} C &= (x + y + z)(x + y + z')\\ &\quad (x + y' + z)(x' + y + z)\\ &= M_0\, M_1\, M_2\, M_4\\ &= \prod M(0,1,2,4) \end{aligned}$$

$$\begin{aligned} C' &= (x + y' + z')(x' + y + z')\\ &\quad (x' + y' + z)(x' + y' + z')\\ &= M_3\, M_5\, M_6\, M_7\\ &= \prod M(3,5,6,7) \end{aligned}$$

C' contains all the maxterms *not* in C, and vice versa.

- Now we've seen two different ways to write the function C, as a sum of minterms $\Sigma m(3,5,6,7)$ and as a product of maxterms $\Pi M(0,1,2,4)$.
- Notice the product term includes maxterm numbers whose corresponding minterms do not appear in the sum expression.

x	y	z	C(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned} C &= x'yz + xy'z + xyz' + xyz \\ &= m_3 + m_5 + m_6 + m_7 \\ &= \Sigma m(3,5,6,7) \end{aligned}$$

$$\begin{aligned} C &= (x + y + z)(x + y + z') \\ &\quad (x + y' + z)(x' + y + z) \\ &= M_0\, M_1\, M_2\, M_4 \\ &= \prod M(0,1,2,4) \end{aligned}$$

The relationship

- Every minterm m_i is the *complement* of its corresponding maxterm M_i.

Minterm	True when
(m_0) $x'y'z'$	xyz = 000
(m_1) $x'y'z$	xyz = 001
(m_2) $x'y\,z'$	xyz = 010
(m_3) $x'y\,z$	xyz = 011
(m_4) $x\,y'z'$	xyz = 100
(m_5) $x\,y'z$	xyz = 101
(m_6) $x\,y\,z'$	xyz = 110
(m_7) $x\,y\,z$	xyz = 111

Maxterm	False when
(M_0) $x + y + z$	xyz = 000
(M_1) $x + y + z'$	xyz = 001
(M_2) $x + y' + z$	xyz = 010
(M_3) $x + y' + z'$	xyz = 011
(M_4) $x' + y + z$	xyz = 100
(M_5) $x' + y + z'$	xyz = 101
(M_6) $x' + y' + z$	xyz = 110
(M_7) $x' + y' + z'$	xyz = 111

- For example, $m_4' = M_4$ because $(xy'z')' = x' + y + z$.

Converting between standard forms

- We can convert sums of minterms to products of maxterms algebraically.

C	$= \Sigma m(3,5,6,7)$	
C'	$= \Sigma m(0,1,2,4)$	[C' contains the minterms not in C]
	$= m_0 + m_1 + m_2 + m_4$	
$(C')'$	$= (m_0 + m_1 + m_2 + m_4)'$	[complementing both sides]
C	$= m_0' \, m_1' \, m_2' \, m_4'$	[DeMorgan's law]
	$= M_0 \, M_1 \, M_2 \, M_4$	[from the previous page]
	$= \prod M(0,1,2,4)$	

- The easy way is to replace minterms with maxterms, using the maxterm numbers that do not appear in the sum of minterms.

$$C = \Sigma m(3,5,6,7) = \prod M(0,1,2,4)$$

- Example

$$F(A,B,C) = AB + \overline{B}(\overline{A} + \overline{C}) = AB + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$
$$= AB(C + \overline{C}) + \overline{A}\,\overline{B}(C + \overline{C}) + (A + \overline{A})\overline{B}\,\overline{C}$$
$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$
$$= \sum m(0,1,4,6,7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as 1s in Truth Table

- **Canonical Form**

  - Boolean Expression expressed in sum of minterms or product of maxterms

- **Standard form**

  F1 = y' + xy + x'yz'

- **F = (AB + CD)(A'B'+C'D')**