Birla Institute of Technology and Science, Pilani
Second Semester 2017–2018,
MATH F112 (Mathematics-II)
Assignment-I

Q.1 The general equation for the circle is given by $(x-a)^2+(y-b)^2=c^2$, find the equation for the circle which passes through (-1,1), (7,1) and (8,4).

Q.2 Let A and B are $m \times n$ matrices then prove that

$$rank(A+B) \leq rank(A) + rank(B).$$

Q.3 Let λ be an eigenvalue of a matrix A, such that $A = A^3$, then find the value(s) of λ.

Q.4 Find the eigenvalues and eigenvectors for the matrix:

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix}$$

Q.5 Let V be a vector space of all 2×2 matrices. Show that $W_1 = \left\{ \begin{bmatrix} x & -x \\ y & z \end{bmatrix} : x, y, z \text{ are real} \right\}$ and $W_2 = \left\{ \begin{bmatrix} a & b \\ -a & c \end{bmatrix} : a, b, c \text{ are real} \right\}$ are two subspaces of V, then find $\dim(W_1 \cap W_2)$.

Q.6 Let V be the vector space of functions from $\mathbb{R}$ into $\mathbb{R}$. Show that the set $S = \{f, g, h\} \subseteq V$ is linearly independent where $f(t) = e^{2t}, g(t) = t^2, h(t) = t$.

Q.7 Let $\{v_1, \cdots, v_n\}$ be a basis for a vector space V. Is $\{v_1, v_1+v_2,\ v_1+v_2+v_3, \cdots, v_1+\cdots+v_n\}$ is also a basis for V? Justify.

Q.8 Suppose $L: R^3 \to R^3$ is a linear operator and $L([1,0,0]) = [-2,1,0]$, $L([0,1,0]) = [3,-2,1]$ and $L([0,0,1]) = [0,-1,3]$. Find $L([-3,2,4])$ and give a formula for $L([x,y,z])$ for any $[x,y,z] \in R^3$.

Q.9 (a) Suppose that $L: V \to W$ is a linear transformation. Show that if $\{L(v_1), L(v_2), \dots, L(v_n)\}$ is a linearly independent set of n distinct vectors in W for some vectors $v_1, \dots, v_n \in V$, then $\{v_1, v_2, \dots, v_n\}$ is a linearly independent set in V.

(b) Examine whether the converse of part (a) is true or not?

Q.10 Consider $L: \mathbb{P}_2 \to \mathbb{P}_4$ given by $L(p(x)) = x^2 p(x)$ what is *ker* (L)? What is *range* (L)? Verify that *dim* (*ker* (L)) + *dim* (*range* (L)) = *dim* ($\mathbb{P}_2$)

Q.11 Let $L: R^5 \to R^4$ be given by $L(X) = AX$, where

$$A = \begin{bmatrix} 8 & 4 & 16 & 32 & 0 \\ 4 & 2 & 10 & 22 & -4 \\ -2 & -1 & -5 & -11 & 7 \\ 6 & 3 & 15 & 33 & -7 \end{bmatrix}$$

Determine ker (L).

Q.12 Suppose $L: R^3 \to R^3$ is a linear operator and $L([1,0,0]) = [2,1,3]$, $L([0,1,0]) = [0,1,1]$ and $L([1,0,-1]) = [1,1,0]$. Find $L([x,y,z])$. Show that L is invertible. Find $L^{-1}([p,q,r])$.

Q.13 $T: R^3 \to P_3$ is a linear transformation defined as $T(a,b,c) = a + (b+c)x + (c-a)x^2 + cx^3$. $B_1 = \{(1,0,0), (0,1,0), (0,0,1)\}$ is an ordered basis of R^3 and $B_2 = \{1+x, x+x^2, x^2+x^3, x^3\}$ is an ordered basis of P_3. Find the associated matrix of transformation.

Q.14 Let $T: R^3 \to P_2$ be a linear transformation. Matrix corresponding to T be $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & -2 \\ 1 & -1 & 3 \end{bmatrix}$, where $B_1 = \{(1,1,0), (0,1,1), (1,0,1)\}$ is a basis of R^3, and $B_2 = \{1+x, x+x^2, x^2+1\}$ is a basis of P_2. Find $T(x,y,z)$.