CS F214 Logic in Computer Science - End-Semester Test (Open Book) – Solutions

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Time: 120 minutes **Marks:57**

1. Natural Deduction Proof

1.	$\exists x\ p(x,z) \wedge q(x,x)$	Premise
2.	fresh x_0	
3.	$p(x_0,z) \wedge q(x_0,x_0)$	Assumption
4.	$\exists y\ p(y,z) \wedge q(y,y)$	$\exists_i$ 3
5.	$\exists y\ p(y,z) \wedge q(y,y)$	$\exists_e$ 1, 2-4
6.	$(\exists y\ p(y,z) \wedge q(y,y))\ [t/x]$	Copy 5
7.	$\exists x\ \exists y\ p(y,z) \wedge q(y,y)$	$\exists_i$ 6
8.		
9.		
10.		

2.

(i) G F ϕ	(ii) F G ϕ	(iii) F G ϕ

3. (i) Model: Universe = N
Meaning of p is '>'

(ii) Model: Universe = {1,2,3}
Meaning of p: {1,2}
Meaning of q: {3}

4. (i) $\forall x\ \forall y\ \neg(x = y) \rightarrow \neg\big(g(x) = g(y)\big)$

(ii) $\forall x\ \exists y\ g(y) = x$

5.

$$\forall g\ \Big(\Big(\forall x\ \forall y\ \neg(x = y) \rightarrow \neg\big(g(x) = g(y)\big)\Big) \wedge (\forall x\ \exists y\ g(y) = x)$$
$$\rightarrow \exists g_1(\forall x\ \forall y\ (g(x) = y) \rightarrow g_1(y) = x)\Big)$$

Note: g(x)= y can be written as g(x,y)

6. Proof:

1.	p --> ((p --> p)-->p)	Rule B
2.	(p --> (p --> p) -->p) --> ((p-->(p-->p)) --> (p-->p))	Rule C
3.	((p-->(p-->p)) --> (p-->p)	Rule D 1,2
4.	p-->(p-->p)	Rule B
5.	p --> p	Rule D 3,4
6.		
7.		

7. Possible answers

1. M(ϕ_1 , ϕ_2, F) = NAND(ϕ_1 , ϕ_2)
NAND is now adequate.

2. M(ϕ_1 , ϕ_2, T) = NOR(ϕ_1 , ϕ_2)
NOR is now adequate.

3. M(F,F,F) = TRUE
M(ϕ, ϕ,T) = $\neg\phi$
$\neg$M(ϕ_1 , ϕ_2, T) = OR (ϕ_1 , ϕ_2)
$\neg$M(ϕ_1 , ϕ_2, F) = AND (ϕ_1 , ϕ_2) Only one of these two is required as we have negation.

8. (i) $\exists v_1 \forall v\ (v = v_1)$

	(ii) $\exists v_1 \exists v_2 \neg(v_1 = v_2)$
	(iii) $\exists x\ \left(p(x) \wedge \left(\forall y \neg(y = x) \rightarrow \neg p(y)\right)\right)$
9.	Post-Condition: $\forall i\ \left(i \geq 0 \wedge i < len(X)\right) \rightarrow \left(\left(isPrime(X[i]) \wedge \left(\forall k\ isPrime(X[k]) \rightarrow (k < i)\right)\right) \rightarrow (j = i)\right)$
	Pre-Condition: TRUE
	Loop Invariant: $\forall n\ (n \geq 0 \wedge n < i) \rightarrow \left(\left(isPrime(X[n]) \wedge \left(\forall k\ isPrime(X[k]) \rightarrow (k < n)\right)\right) \rightarrow (j = n)\right)$
10.	
11.	Proof:

1.	$\forall x\ p(x) \rightarrow q(x)$	Premise
2.	$\exists x\ p(x)$	Assumption
3.	Fresh x_0	
4.	$p(x_0)$	Assumption
5.	$p(x_0) \rightarrow q(x_0)$	$\forall_e 1$
6.	$q(x_0)$	MP 4,5
7.	$\exists x\ q(x)$	$\exists i$ 6
8.	$\exists x\ q(x)$	$\exists e$ 2, 3-7
9.	$\exists x\ p(x) \rightarrow \exists x\ q(x)$	$\rightarrow i$ 2…8
10.		
11.		
12		

12.	(i) Model that satisfies Universe: R or Q	
	(i) Model that doesn't satisfy Universe: N or Z	
	(ii) No model satisfies this formula (because z < z is always FALSE with the usual meaning of <)	
	(ii) Any model would not satisfy this formula.	
13.	A is FALSE	B is TRUE
14.	$(\neg(c_1 = c_2) \wedge \neg(c_1 = c_3) \wedge \neg(c_1 = c_4) \wedge \neg(c_2 = c_3) \wedge \neg(c_2 = c_4) \wedge \neg(c_3 = c_4)) \wedge$ $\left(\forall v\ (c(v) = c_1) \vee (c(v) = c_2) \vee (c(v) = c_3) \vee (c(v) = c_4)\right) \wedge$ $\left(\forall v_1\ \forall v_2\ E(v_1, v_2) \rightarrow \neg c(v_1) = c(v_2)\right)$	
15.	(i) $\phi \rightarrow F\ \psi$	
	(ii) $\phi \rightarrow X(\chi\ U\ \psi)$	