

Quine-McCluskey Method

K-Maps Difficult in Visualization for
six variables and above

Not software adaptable

Quine-McCluskey Method

An Example

1. Find all the prime implicants

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Group the minterms according to the number of 1s in the minterm.

This way we only have to compare minterms from adjacent groups.

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Combining
group 0 and
group 1:

Quine-McCluskey Method


An Example

Column I

Column II

group 0 0 0000 

0,1 000-

group 1 { 1 0001 
 2 0010
 8 1000

group 2 { 5 0101
 6 0110
 9 1001
 10 1010

group 3 { 7 0111
 14 1110

Combining
group 0 and
group 1:




Quine-McCluskey Method

An Example

Column I

Column II

group 0 0 0000 

group 1 { 1 0001 
 2 0010 
 8 1000 

group 2 { 5 0101
 6 0110
 9 1001
 10 1010

group 3 { 7 0111
 14 1110

0,1 000-

0,2 00-0

Combining
group 0 and
group 1:

Quine-McCluskey Method

An Example

Column I

Column II

Does it make sense to combine group 0 with group 2 or 3?

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
0,2 00-0
0,8 -000

No, there are at least two bits that are different.

Quine-McCluskey Method

An Example

Column I

Column II

Does it make sense to no combine group 0 with group 2 or 3?

No, there are at least two bits that are different.

Thus, next we combine group 1 and group 2.

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
0,2 00-0
0,8 -000

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01

Combine group 1
 and group 2.

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	
	9	1001	
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01

Combine group 1
 and group 2.

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
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	8	1000	☐
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	6	0110	
	9	1001	☐
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001

Combine group 1
and group 2.

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	
	9	1001	☐
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001

Combine group 1
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Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
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	2	0010	☐
	8	1000	☐
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	6	0110	
	9	1001	☐
	10	1010	
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001

Combine group 1
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Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
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	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001

Combine group 1
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Quine-McCluskey Method

An Example

Column I

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	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001
 2,6 0-10

Combine group 1
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Column II

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	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001
 2,6 0-10

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	6	0110	☐
	9	1001	☐
	10	1010	☐
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	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001
 2,6 0-10
 2,10 -010

Combine group 1
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Quine-McCluskey Method

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Column I

Column II

group 0	0	0000	☐
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	2	0010	☐
	8	1000	☐
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	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1 000-
 0,2 00-0
 0,8 -000
 1,5 0-01
 1,9 -001
 2,6 0-10
 2,10 -010

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group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010

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Column II

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group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-

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	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
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	14	1110	

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0

Quine-McCluskey Method

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group 1	1	0001	☐
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	8	1000	☐
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	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	
	14	1110	

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0

Lets try to combine
group 2 with
group 3.

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1

Combine group 2
and group 3.

Quine-McCluskey Method

An Example

Combine group 2
and group 3.

	Column I	Column II
group 0	0 0000 ☐	0,1 000-
group 1	1 0001 ☐	0,2 00-0
	2 0010 ☐	0,8 -000
	8 1000 ☐	1,5 0-01
group 2	5 0101 ☐	1,9 -001
	6 0110 ☐	2,6 0-10
	9 1001 ☐	2,10 -010
	10 1010 ☐	8,9 100-
group 3	7 0111 ☐	8,10 10-0
	14 1110	5,7 01-1

Quine-McCluskey Method

An Example

Combine group 2
and group 3.

	Column I	Column II
group 0	0 0000 ☐	0,1 000-
group 1 {	1 0001 ☐	0,2 00-0
	2 0010 ☐	0,8 -000
	8 1000 ☐	1,5 0-01
group 2 {	5 0101 ☐	1,9 -001
	6 0110 ☐	2,6 0-10
	9 1001 ☐	2,10 -010
	10 1010 ☐	8,9 100-
group 3 {	7 0111 ☐	8,10 10-0
	14 1110	5,7 01-1
		6,7 011-

Quine-McCluskey Method

An Example

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Column II

group 0	0	0000	☐
group 1	1	0001	☐
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	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

Combine group 2
and group 3.

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1
6,7	011-
6,14	-110

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

Combine group 2
and group 3.

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1
6,7	011-
6,14	-110

Quine-McCluskey Method

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Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

Combine group 2
and group 3.

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1
6,7	011-
6,14	-110

Quine-McCluskey Method

An Example

Column I

Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

Combine group 2
and group 3.

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1
6,7	011-
6,14	-110

Quine-McCluskey Method

An Example

We have now completed the first step. All minterms in column I were included.

We can divide column II into groups.

Column I		Column II	
group 0	0 0000 ☐	0,1 000-	
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100-	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

Column I











Column II

group 0	0	0000	☐
group 1	1	0001	☐
	2	0010	☐
	8	1000	☐
group 2	5	0101	☐
	6	0110	☐
	9	1001	☐
	10	1010	☐
group 3	7	0111	☐
	14	1110	☐

0,1	000-
0,2	00-0
0,8	-000
1,5	0-01
1,9	-001
2,6	0-10
2,10	-010
8,9	100-
8,10	10-0
5,7	01-1
6,7	011-
6,14	-110
10,14	1-10

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 	0,1 000-	
group 1	1 0001 	0,2 00-0	
	2 0010 	0,8 -000	
	8 1000 	1,5 0-01	
group 2	5 0101 	1,9 -001	
	6 0110 	2,6 0-10	
	9 1001 	2,10 -010	
	10 1010 	8,9 100-	
group 3	7 0111 	8,10 10-0	
	14 1110 	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	











Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000-	
group 1	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
		1,9 -001	
group 2	5 0101 ☐	2,6 0-10	
	6 0110 ☐	2,10 -010	
	9 1001 ☐	8,9 100-	
	10 1010 ☐	8,10 10-0	
group 3	7 0111 ☐	5,7 01-1	
	14 1110 ☐	6,7 011-	
		6,14 -110	
		10,14 1-10	











Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 	0,1 000-	
group 1	1 0001 	0,2 00-0	
	2 0010 	0,8 -000	
	8 1000 	1,5 0-01	
group 2	5 0101 	1,9 -001	
	6 0110 	2,6 0-10	
	9 1001 	2,10 -010	
	10 1010 	8,9 100-	
group 3	7 0111 	8,10 10-0	
	14 1110 	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	











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	Column I	Column II	Column III
group 0	0 0000 	0,1 000-	
group 1	1 0001 	0,2 00-0	
	2 0010 	0,8 -000	
	8 1000 	1,5 0-01	
group 2	5 0101 	1,9 -001	
	6 0110 	2,6 0-10	
	9 1001 	2,10 -010	
	10 1010 	8,9 100-	
group 3	7 0111 	8,10 10-0	
	14 1110 	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 	0,1 000-	
group 1	1 0001 	0,2 00-0	
	2 0010 	0,8 -000	
	8 1000 	1,5 0-01	
group 2	5 0101 	1,9 -001	
	6 0110 	2,6 0-10	
	9 1001 	2,10 -010	
	10 1010 	8,9 100-	
group 3	7 0111 	8,10 10-0	
	14 1110 	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0	
	2 0010 ☐	0,8 -000	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {	7 0111 ☐	8,10 10-0	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	<u>0,8 -000</u>	
	8 1000 ☐	<u>1,5 0-01</u>	
group 2 {	5 0101 ☐	1,9 -001	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {		8,10 10-0 ☐	
	7 0111 ☐	<u>5,7 01-1</u>	
	14 1110 ☐	6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	<u>0,8 -000</u>	
	8 1000 ☐	1,5 0-01	
group 2 {	5 0101 ☐	<u>1,9 -001</u>	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3 {		8,10 10-0 ☐	
	7 0111 ☐	<u>5,7 01-1</u>	
	14 1110 ☐	6,7 011-	
		6,14 -110	
		10,14 1-10	

















Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01	
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 	0,1 000- 	0,1,8,9 -00-
group 1	1 0001 	0,2 00-0 	0,2,8,10 -0-0
	2 0010 	0,8 -000 	0,8,1,9 -00-
	8 1000 	1,5 0-01	
group 2	5 0101 	1,9 -001 	
	6 0110 	2,6 0-10	
	9 1001 	2,10 -010	
	10 1010 	8,9 100- 	
group 3		8,10 10-0 	
	7 0111 	5,7 01-1	
	14 1110 	6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	2,10,6,14 --10
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	2,10,6,14 --10
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	2,10,6,14 --10
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	2,10,6,14 --10
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1	
		6,7 011-	
		6,14 -110 ☐	
		10,14 1-10 ☐	

No more combinations are possible, thus we stop here.

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	0,8,1,9 -00-
	8 1000 ☐	1,5 0-01 ☐	0,8,2,10 -0-0
group 2	5 0101 ☐	1,9 -001 ☐	2,6,10,14 --10
	6 0110 ☐	2,6 0-10 ☐	2,10,6,14 --10
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

We can eliminate repeated combinations


Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1 {	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	<u>2,6,10,14 --10</u>
	8 1000 ☐	1,5 0-01 ☐ ←	
group 2 {	5 0101 ☐	1,9 -001 ☐	Now we form f with the terms not checked
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3 {		8,10 10-0 ☐	f = a'c'd
		5,7 01-1	
		6,7 011-	
		6,14 -110 ☐	
		10,14 1-10 ☐	

Quine-McCluskey Method


An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	<u>2,6,10,14 --10</u>
	8 1000 ☐	1,5 0-01 ☐	
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

$$f = a'c'd + a'bd$$

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	<u>2,6,10,14 --10</u>
	8 1000 ☐	1,5 0-01 ☐	
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- 	
		6,14 -110 ☐	
		10,14 1-10 ☐	

$$f = a'c'd + a'bd + a'bc$$

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00- ←
group 1 {	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	<u>2,6,10,14 --10</u>
	8 1000 ☐	1,5 0-01 ☐	
group 2 {	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3 {		8,10 10-0 ☐	
	7 0111 ☐	5,7 01-1 ☐	
	14 1110 ☐	6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

$$f = a'c'd + a'bd + a'bc + b'c'$$

Quine-McCluskey Method

An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0 ←
	2 0010 ☐	0,8 -000 ☐	2,6,10,14 --10
	8 1000 ☐	1,5 0-01 ☐	
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

$$f = a'c'd + a'bd + a'bc + b'c' + b'd'$$

Quine-McCluskey Method






















An Example

	Column I	Column II	Column III
group 0	0 0000 ☐	0,1 000- ☐	0,1,8,9 -00-
group 1	1 0001 ☐	0,2 00-0 ☐	0,2,8,10 -0-0
	2 0010 ☐	0,8 -000 ☐	<u>2,6,10,14 --10</u> ←
	8 1000 ☐	1,5 0-01 ☐	
group 2	5 0101 ☐	1,9 -001 ☐	
	6 0110 ☐	2,6 0-10 ☐	
	9 1001 ☐	2,10 -010 ☐	
	10 1010 ☐	8,9 100- ☐	
group 3	7 0111 ☐	8,10 10-0 ☐	
	14 1110 ☐	5,7 01-1 ☐	
		6,7 011- ☐	
		6,14 -110 ☐	
		10,14 1-10 ☐	

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

Quine-McCluskey Method

An Example

Column I			Column II			Column III		
group 0	0		0,1			0,1,8,9		
group 1	1		0,2			0,2,8,10		
	2		0,8			<hr/>		
	8		1,5			2,6,10,14		
group 2	5		1,9					
	6		2,6					
	9		2,10					
	10		8,9					
group 3			8,10					
	7		<hr/>					
	14		5,7					
			6,7					
			6,14					
			10,14					

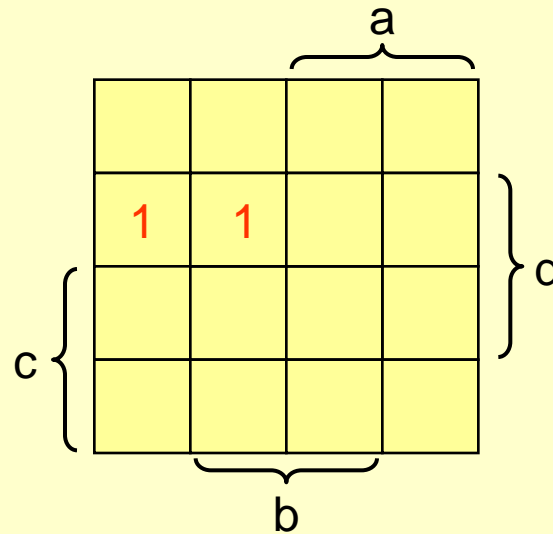
$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = \underbrace{a'c'd + a'bd}_{\text{group 1}} + a'bc + b'c' + b'd' + cd'$$

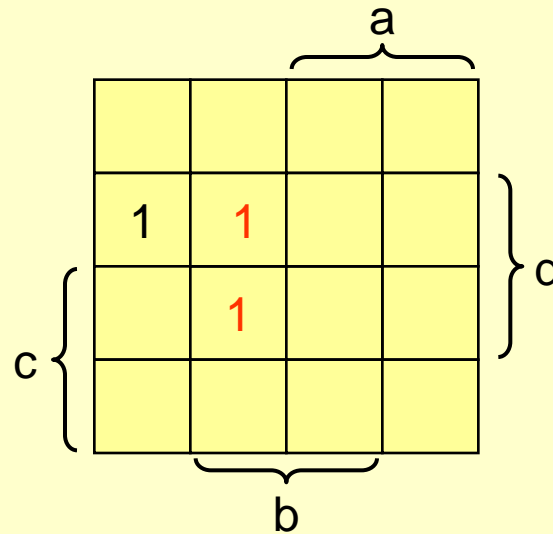


Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + \underbrace{a'bd}_{\text{red}} + a'bc + b'c' + b'd' + cd'$$

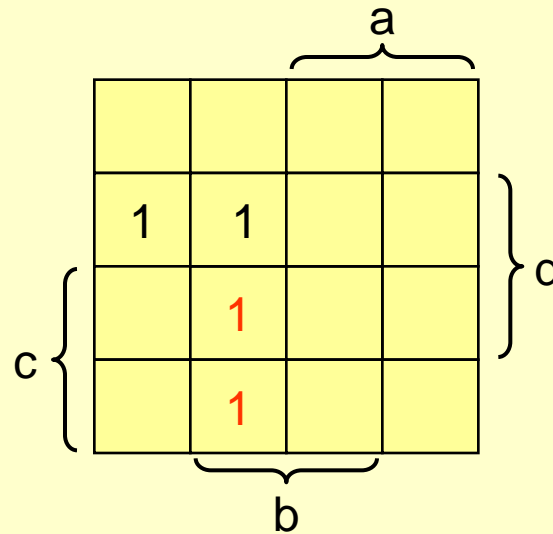


Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + \underbrace{a'bc}_{\text{red}} + b'c' + b'd' + cd'$$

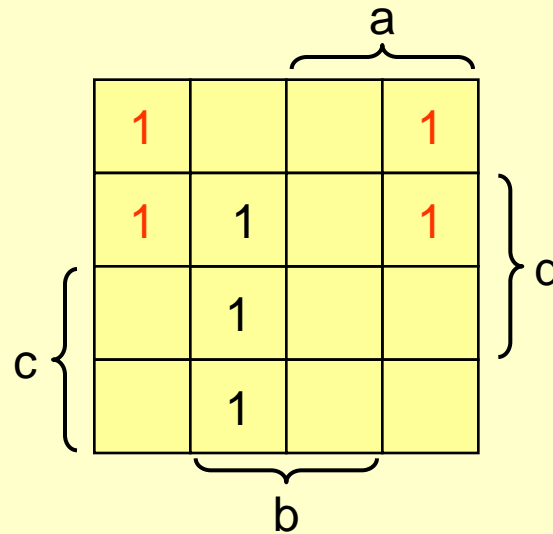


Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + \underbrace{b'c'} + b'd' + cd'$$

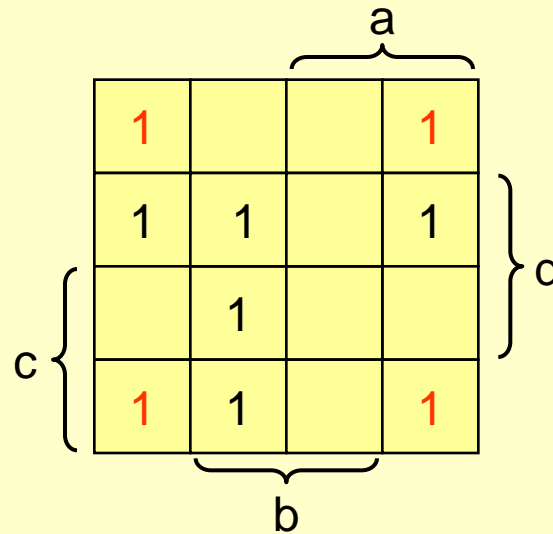


Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + \underbrace{b'd'} + cd'$$



Quine-McCluskey Method

An Example

But, the form below is not minimized. Using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + \underbrace{cd'}$$

$$f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$$

A 4x4 Karnaugh map for the function f(a,b,c,d). The map is a 4x4 grid of cells. The top row has 1s in the first and fourth columns. The second row has 1s in the first and second columns. The third row has a 1 in the second column. The bottom row has 1s in all four columns. The bottom row's 1s are colored red. Brackets indicate groupings: a horizontal bracket labeled 'a' above the top row; a vertical bracket labeled 'c' to the left of the first column; a vertical bracket labeled 'd' to the right of the second column; and a horizontal bracket labeled 'b' below the bottom row.

1			1
1	1		1
	1		
1	1	1	1

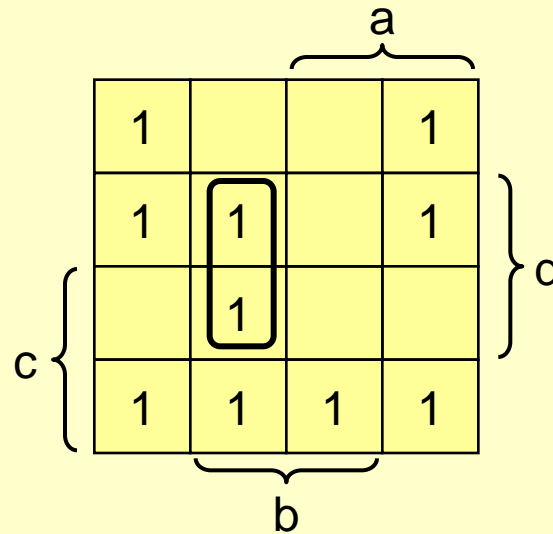
Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd$$



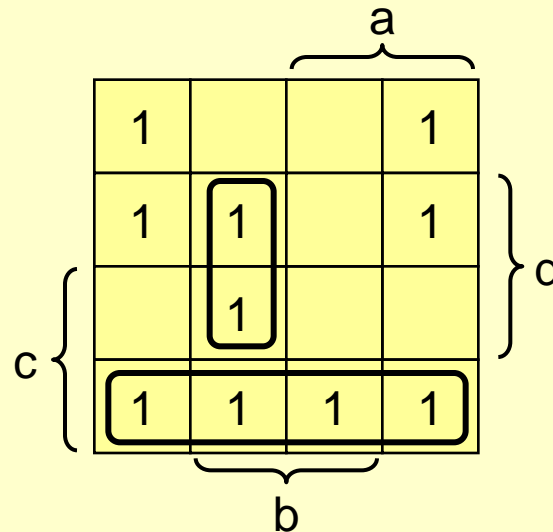
Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd + cd'$$



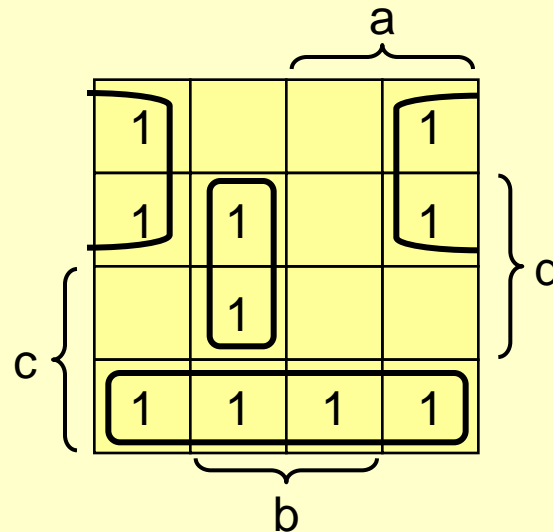
Quine-McCluskey Method

An Example

But, the form below is not minimized, using a Karnaugh map we can obtain:

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

$$F = a'bd + cd' + b'c'$$



Quine-McCluskey Method

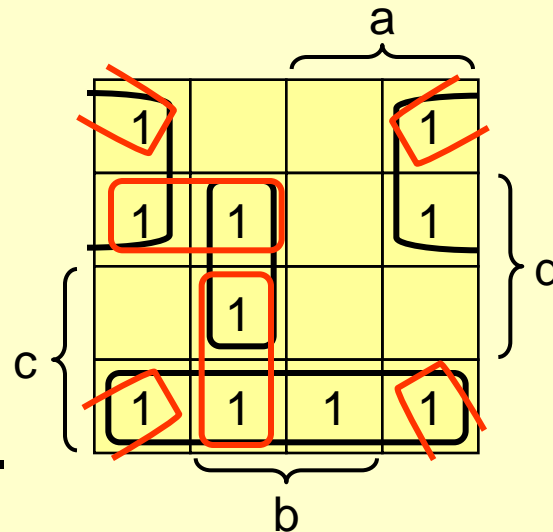
An Example

What are the extra terms in the solution obtained with the Quine-McCluskey method?

$$f = \underbrace{a'c'd}_{\text{prime implicant}} + a'bd + \underbrace{a'bc}_{\text{prime implicant}} + b'c' + \underbrace{b'd'}_{\text{prime implicant}} + cd'$$

$$F = a'bd + cd' + b'c'$$

Thus, we need a method to eliminate this redundant terms from the Quine-McCluskey solution.



The Prime Implicant Chart

The prime implicant chart is the second part of the Quine-McCluskey procedure.

It is used to select a minimum set of prime implicants.

Similar to the Karnaugh map, we first select the essential prime implicants, and then we select enough prime implicants to cover all the minterms of the function.

Prime Implicant Chart (Example)

		minterms											
		0	1	2	5	6	7	8	9	10	14		
Prime Implicants	(0,1,8,9) $b'c'$	X	X					X	X				
	(0,2,8,10) $b'd'$	X		X				X		X			
	(2,6,10,14) cd'			X		X				X	X		
	(1,5) $a'c'd$		X		X								
	(5,7) $a'bd$				X		X						
	(6,7) $a'bc$					X	X						

Question: Given the prime implicant chart above, how can we identify the essential prime implicants of the function?

Prime Implicant Chart (Example)

		minterms											
		0	1	2	5	6	7	8	9	10	14		
Prime Implicants	(0,1,8,9) $b'c'$	X	X					X	X				
	(0,2,8,10) $b'd'$	X		X				X		X			
	(2,6,10,14) cd'			X		X				X	X		
	(1,5) $a'c'd$		X		X								
	(5,7) $a'bd$				X		X						
	(6,7) $a'bc$					X	X						

Similar to the Karnaugh map, all we have to do is to look for minterms that are covered by a single term.

Prime Implicant Chart (Example)

		minterms											
		0	1	2	5	6	7	8	9	10	14		
Prime Implicants	(0,1,8,9) $b'c'$	X	X					X	X				
	(0,2,8,10) $b'd'$	X		X				X		X			
	(2,6,10,14) cd'			X		X				X	X		
	(1,5) $a'c'd$		X		X								
	(5,7) $a'bd$				X		X						
	(6,7) $a'bc$					X	X						

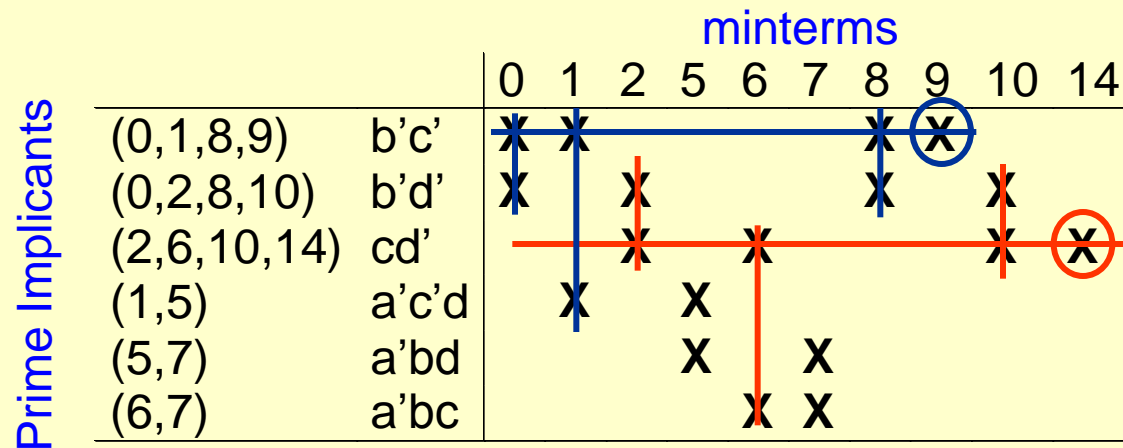
Once a term is included in the solution, all the minterms covered by that term are covered.

Therefore we may now mark the covered minterms and find terms that are no longer useful.

Prime Implicant Chart (Example)

		minterms											
		0	1	2	5	6	7	8	9	10	14		
Prime Implicants	(0,1,8,9) $b'c'$	X	X					X	X				
	(0,2,8,10) $b'd'$	X		X				X		X			
	(2,6,10,14) cd'			X		X				X	X		
	(1,5) $a'c'd$		X		X								
	(5,7) $a'bd$				X		X						
	(6,7) $a'bc$					X	X						

Prime Implicant Chart (Example)



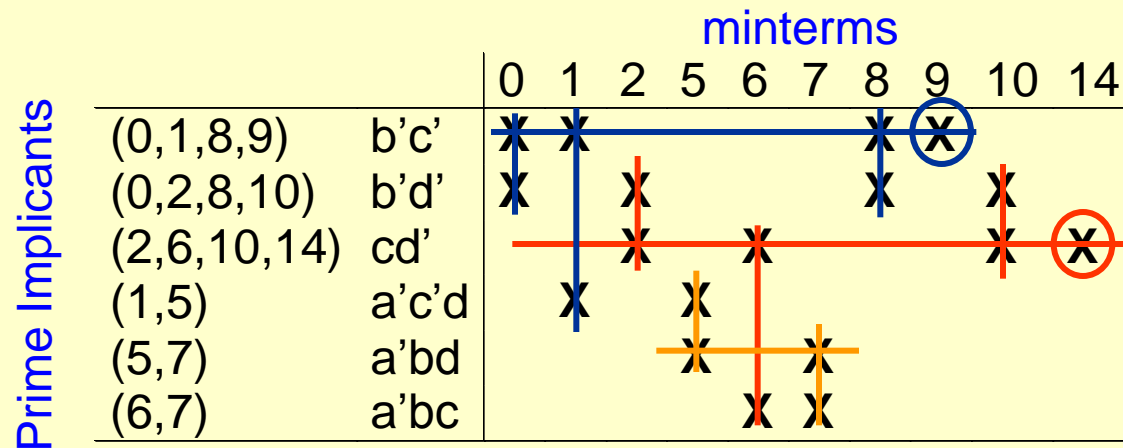
As we have not covered all the minterms with essential prime implicants, we must choose enough non-essential prime implicants to cover the remaining minterms.

Prime Implicant Chart (Example)

		minterms										
		0	1	2	5	6	7	8	9	10	14	
Prime Implicants	(0,1,8,9) $b'c'$	*	*					*	*			
	(0,2,8,10) $b'd'$	*	*		*			*	*		*	
	(2,6,10,14) cd'	*		*	*				*	*		
	(1,5) $a'c'd$	*		*								
	(5,7) $a'bd$			*		*						
	(6,7) $a'bc$					*		*				

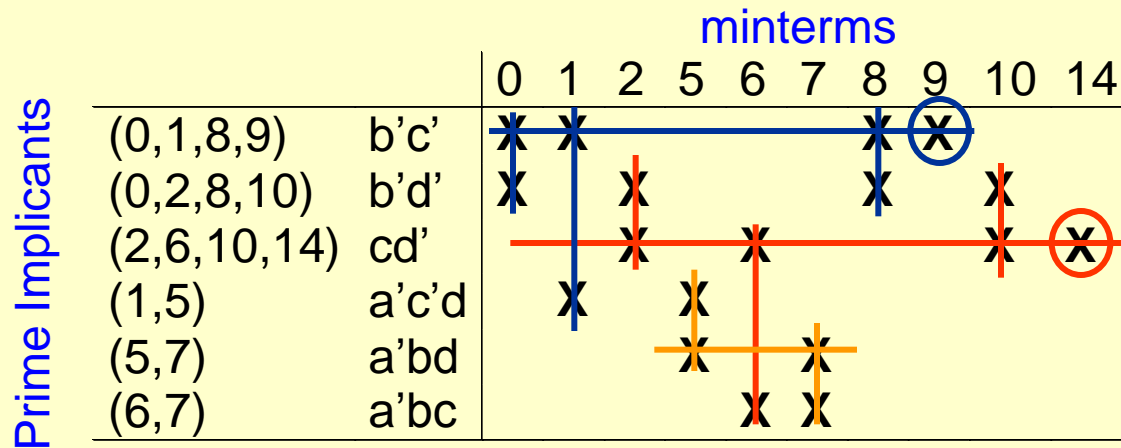
What strategy should we use to find a minimum cover for the remaining minterms?

Prime Implicant Chart (Example)



We choose first prime implicants that cover the most minterms.

Prime Implicant Chart (Example)



Therefore our minimum solution is:

$$f(a,b,c,d) = b'c' + cd' + a'bd$$

To Simplify in POS Form:

- Take 0's as initial list of minterms**
- Simplify to get the compliment of the function in SOP form**
- Compliment to get the function in POS form**

Function with Don't care inputs

- Don't cares included while computing Prime implicants**
- In the Selection of Essential Prime implicants don't cares not used.**

Simplify Using QM Method

$$\mathbf{F(A,B,C,D) = \Sigma(6,7,14)}$$

$$\mathbf{d(A,B,C,D) = \Sigma(0,8,15)}$$