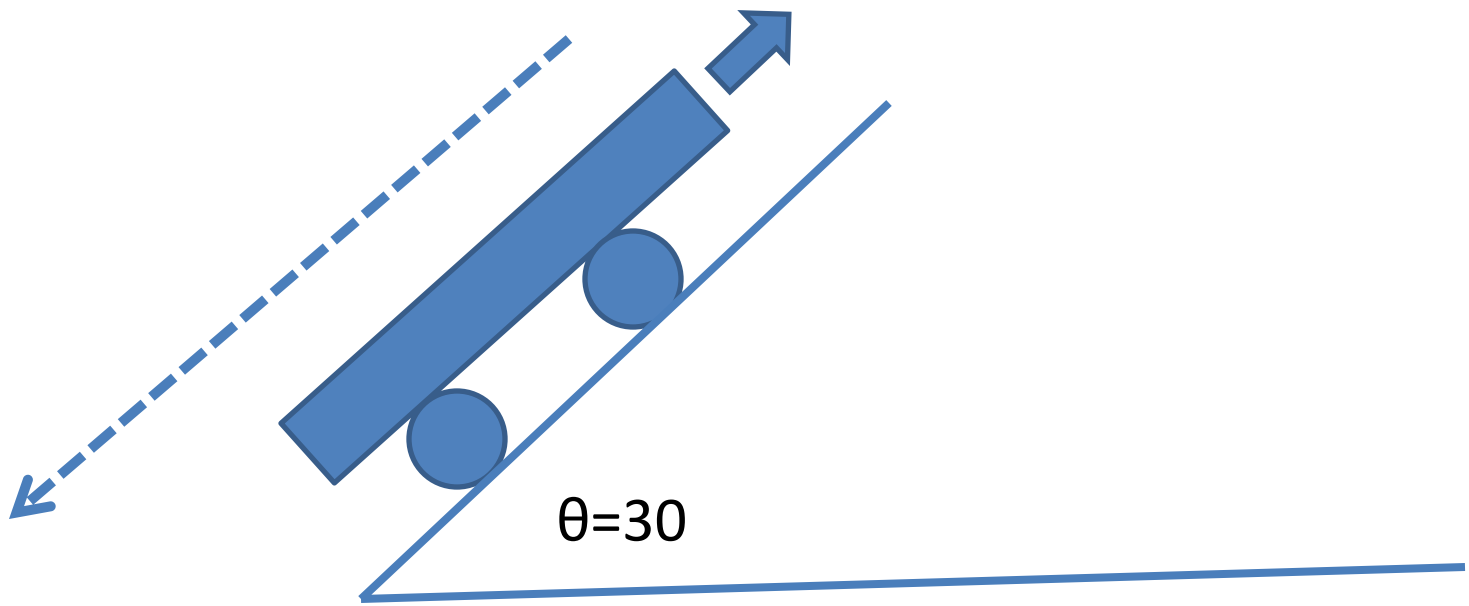
Problem: A sand-filled wagon slides down a smooth inclined plane, starting from rest at time t=0. The inclination of the plane to the horizontal is $\theta=30^0$. The initial mass of the wagon and its contents is M_0=5000 Kg. The sand is ejected backwards from the wagon at a speed equaling the wagon's speed, the rate of ejection being μ=50 kg/s. Find the speed of the wagon at t=4 s.



Chapter 4: Work and Energy

Integrating the Equation of Motion in One Dimension

$$m\frac{d^2x}{dt^2} = F(x)$$

$$m\frac{dv}{dt} = F(x)$$

$$m\int_{x_a}^{x_b}\frac{dv}{dt}dx = \int_{x_a}^{x_b}F(x)dx$$

$$m\int_{x_a}^{x_b}\frac{dv}{dt}v\,dt = \int_{x_a}^{x_b}F(x)dx$$

$$m\int_{t_a}^{t_b}\frac{d}{dt}\left(\frac{1}{2}v^2\right)dt = \int_{x_a}^{x_b}F(x)dx$$

$$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = \int_{x_a}^{x_b}F(x)dx$$

The Work-Energy Theorem in One Dimension

$$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = \int_{x_a}^{x_b} F(x)dx$$

$$W_{ba} = K_b - K_a$$

Integrating the Equation of Motion in Several Dimensions

$$\int_{r_a}^{r_b} \vec{F} \bullet d\vec{r} = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

The Work-Energy Theorem:

$$W_{ba} = K_b - K_a$$

Integrating the Equation of Motion in Several Dimensions for Translational Motion of an extended system:

$$\vec{F} = M\ddot{\vec{R}}_{CM} = M\frac{d\vec{V}_{CM}}{dt}$$

$$\int_{R_a}^{R_b} \vec{F} \bullet d\vec{R} = \frac{1}{2}MV_b^{\,2} - \frac{1}{2}MV_a^{\,2}$$

$$W_{ba} = K_b - K_a$$

Applying the Work-Energy Theorem:

$$W_{ba} = K_b - K_a$$

Applying the Work-Energy Theorem:

- Applying it in the case of conservative force field
- Applying it in the case of constrained motion

Problem: Work Done by a Uniform Force

$$F = F_0 \hat{n}$$

Problem: Work Done by a Central Force

$$\vec{F}\left(\vec{r}\right) = f\left(\vec{r}\right)\hat{r}$$

Problem: Path Dependent Integral

$$\vec{F} = A\left(xy\hat{x} + y^2\hat{y}\right)$$

Potential

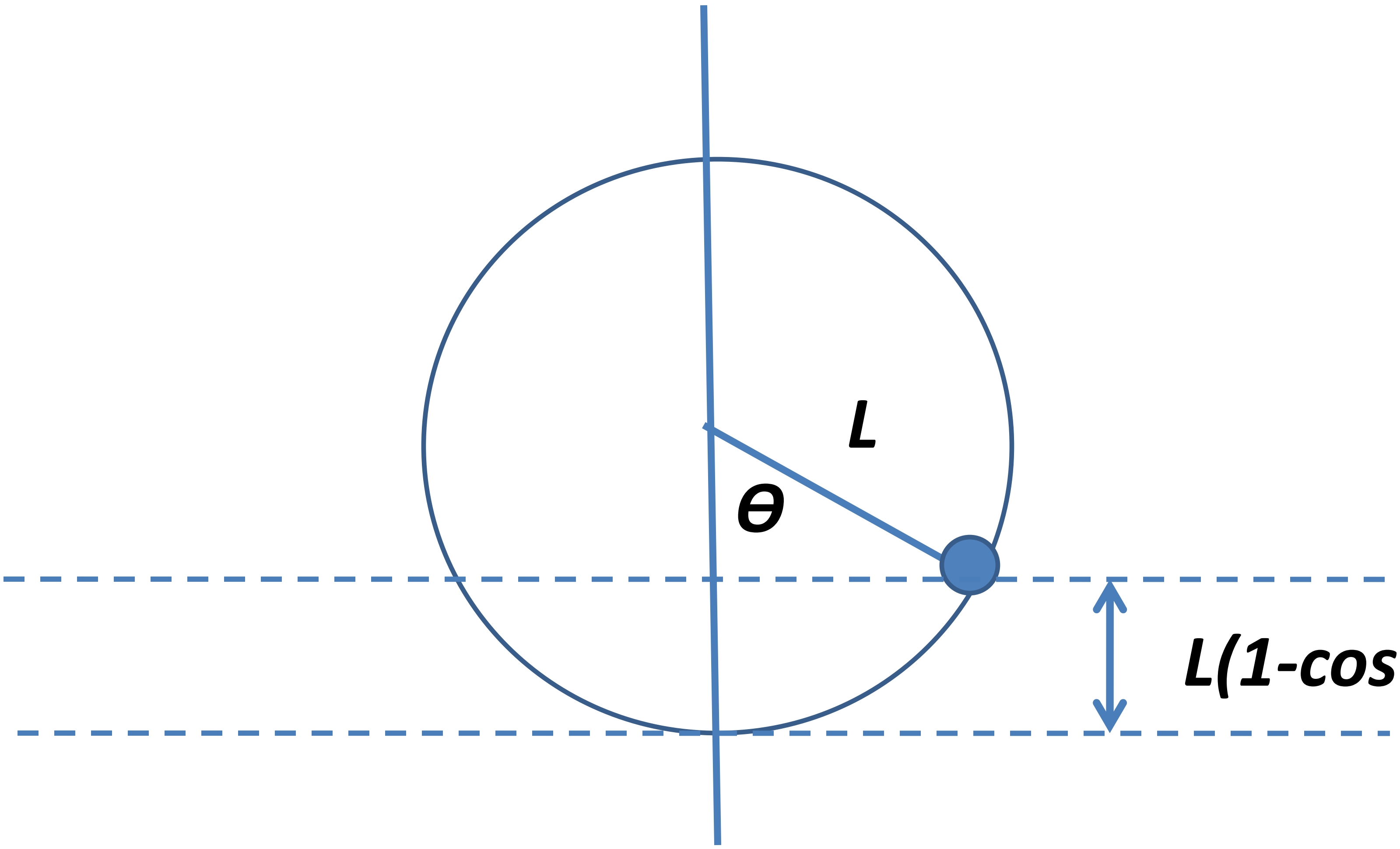
Potential Energy of an Inverse Square Force Field

$$\vec{F}(\vec{r}) = \frac{A}{r^2}\hat{r}$$

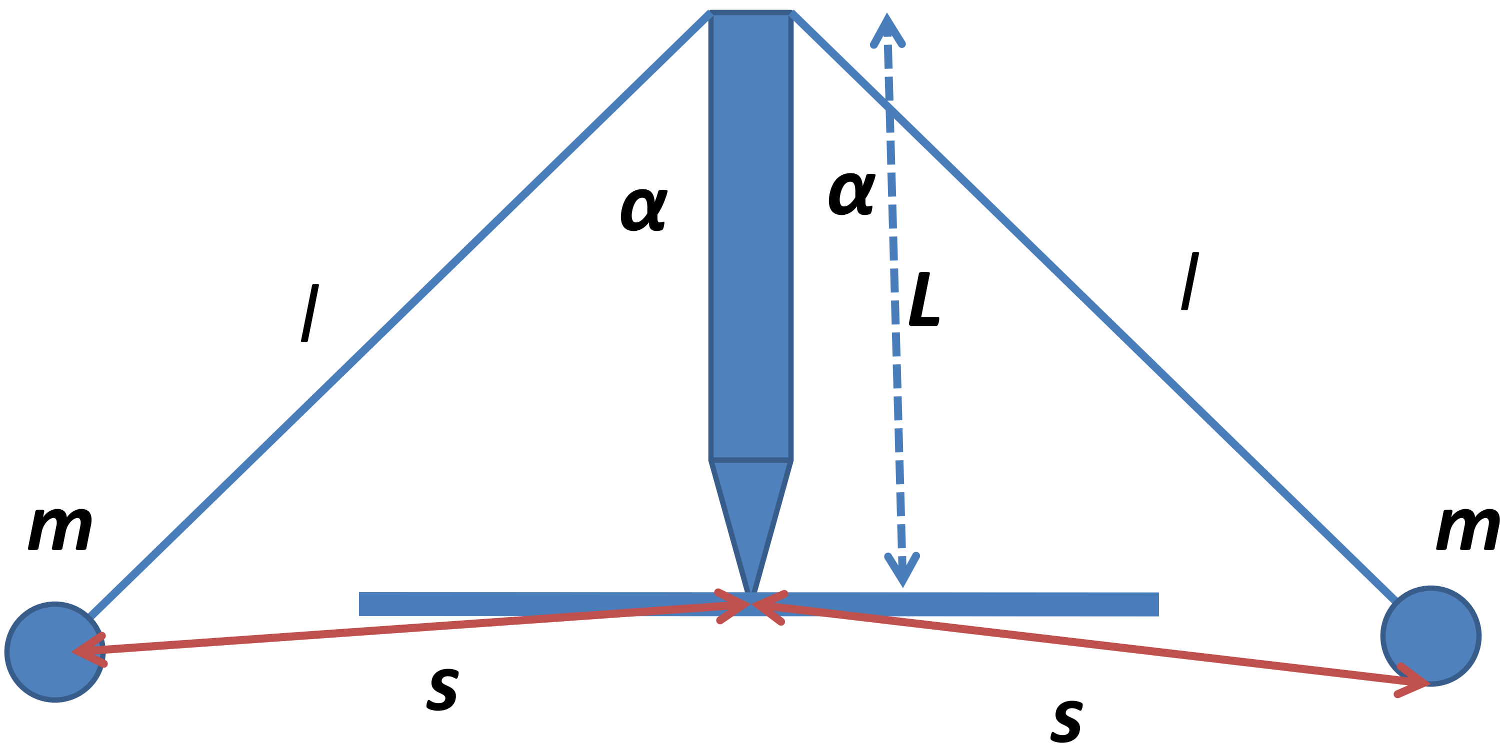
Potential Energy of a Restoring Force

What Potential Energy Tells Us about Force?

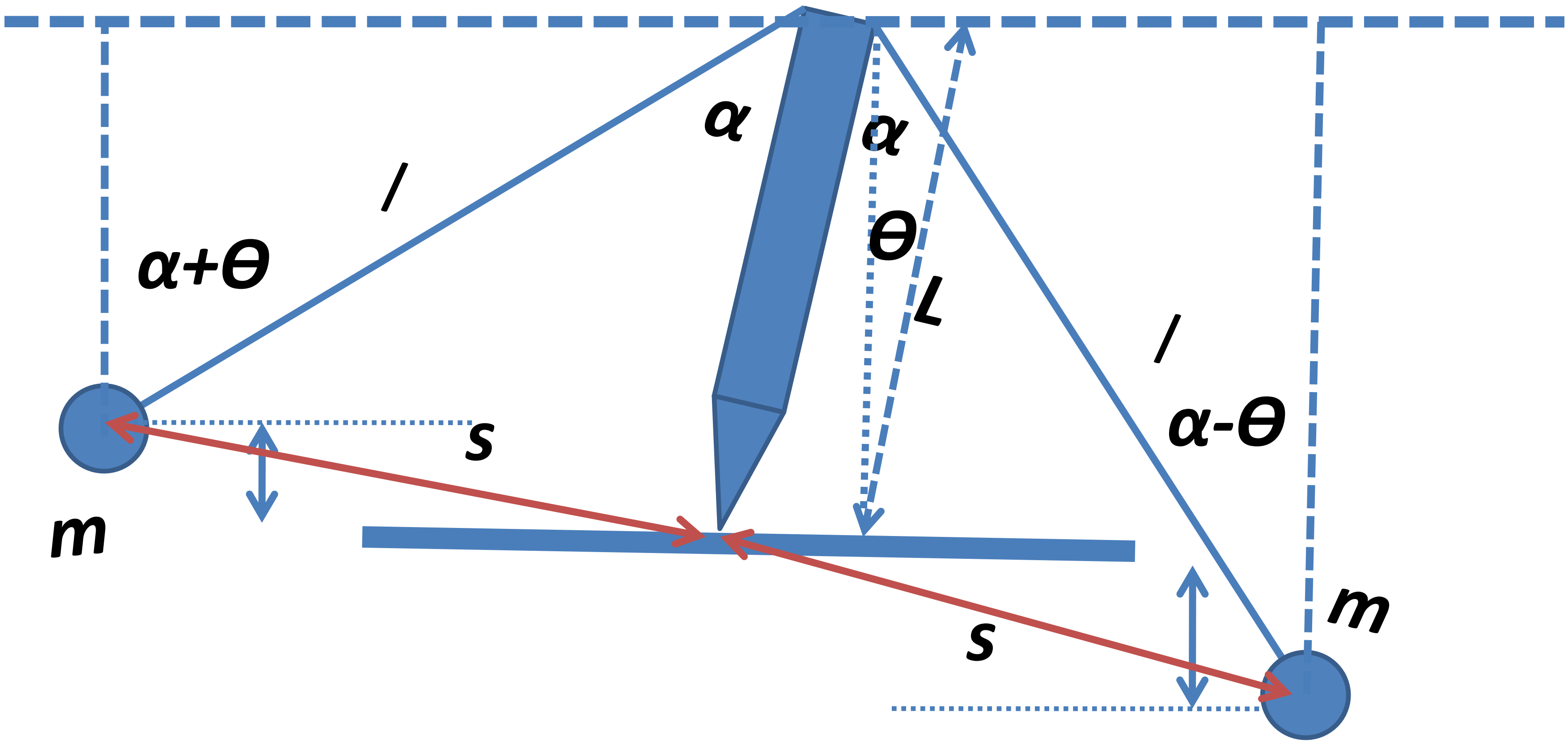
Pendulum



Problem: Energy and Stability –Teeter Toy



Problem: Energy and Stability –Teeter Toy



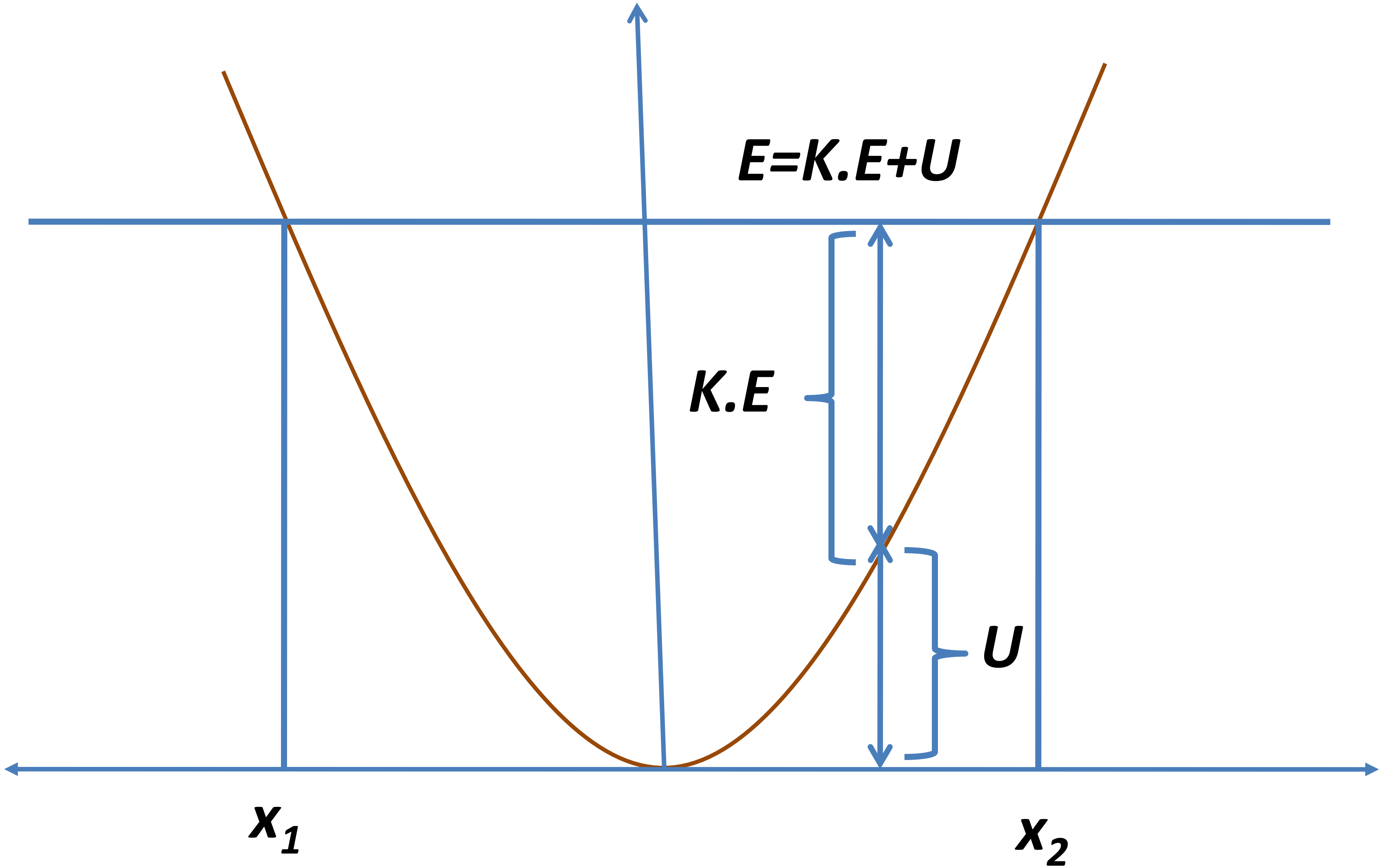
Problem: In a certain two-dimensional field of force the potential energy of a particle has the form $\boldsymbol{U=\alpha x^2+\beta y^2}$, where α and β are positive constants whose magnitudes are different. Find out:

(a) whether the field is central;

(b) what is the shape of the equipotential surfaces and also of the surfaces for which the magnitude of the vector of force ***F=const.***

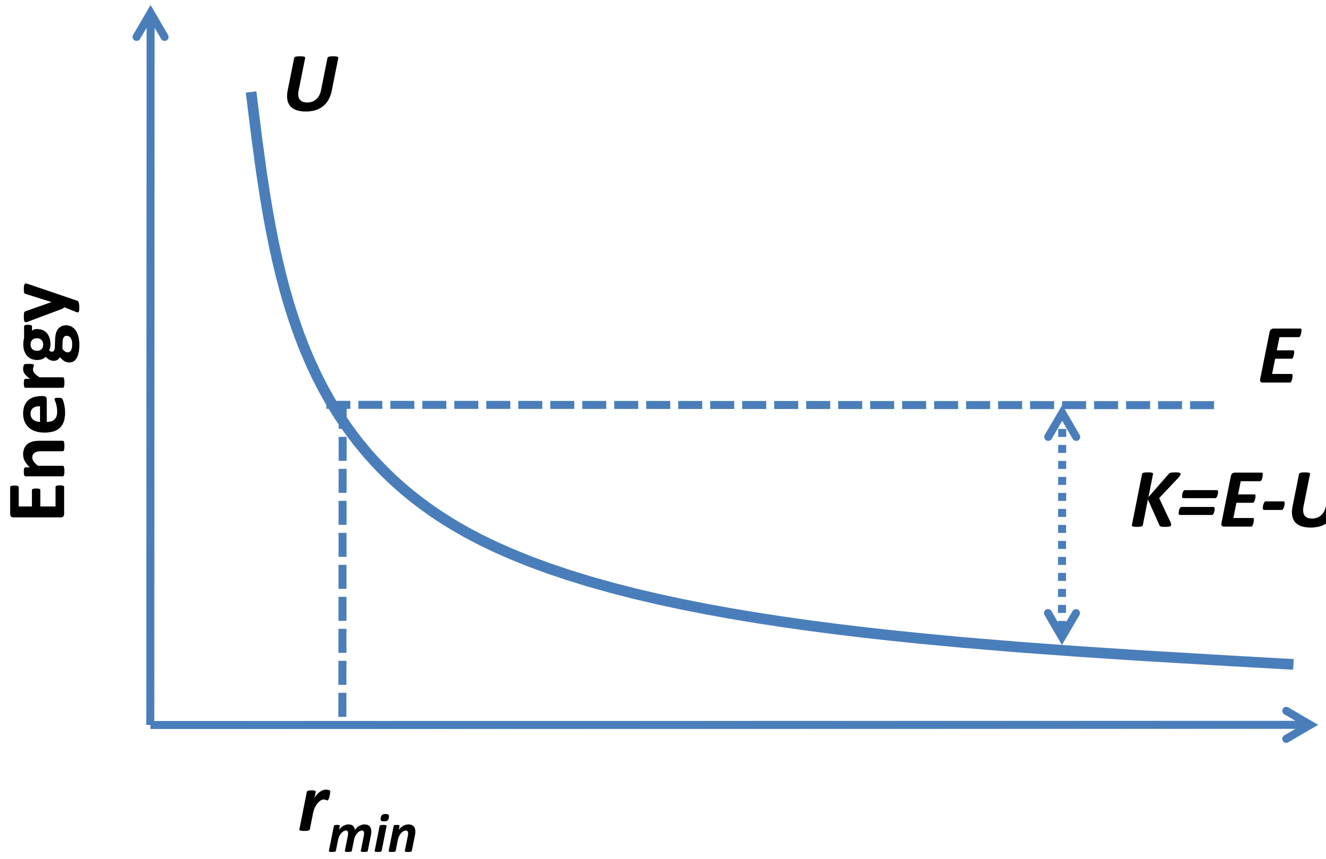
Problem: There are two stationary fields of force ***F=ayi*** and ***F=axi+byj*** where i and j are the unit vectors of the x and y axes, and a and b are constants. Find out whether these forces have a potential.

Energy Diagrams

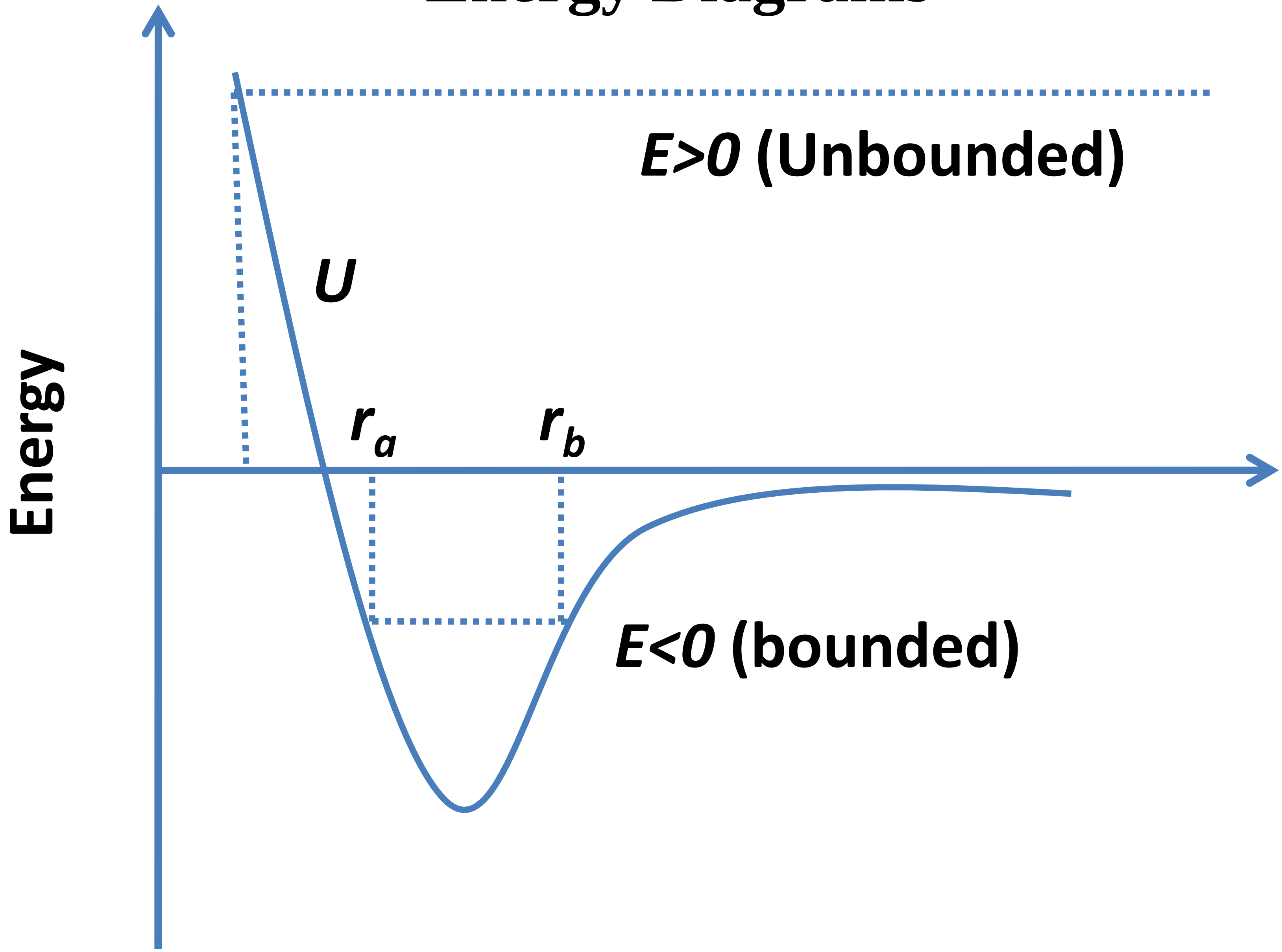


Energy Diagram for a Harmonic Oscillator

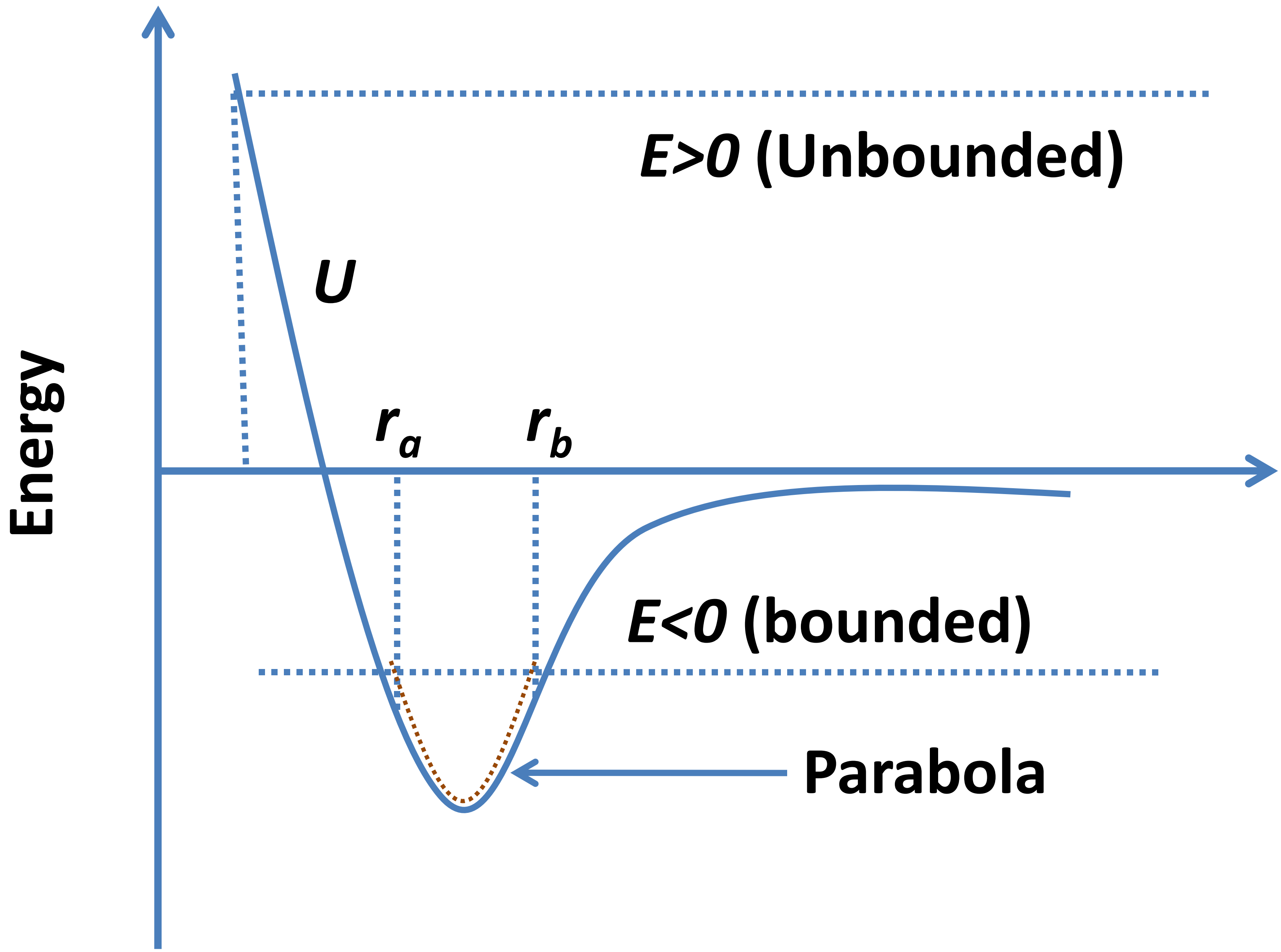
Energy Diagrams



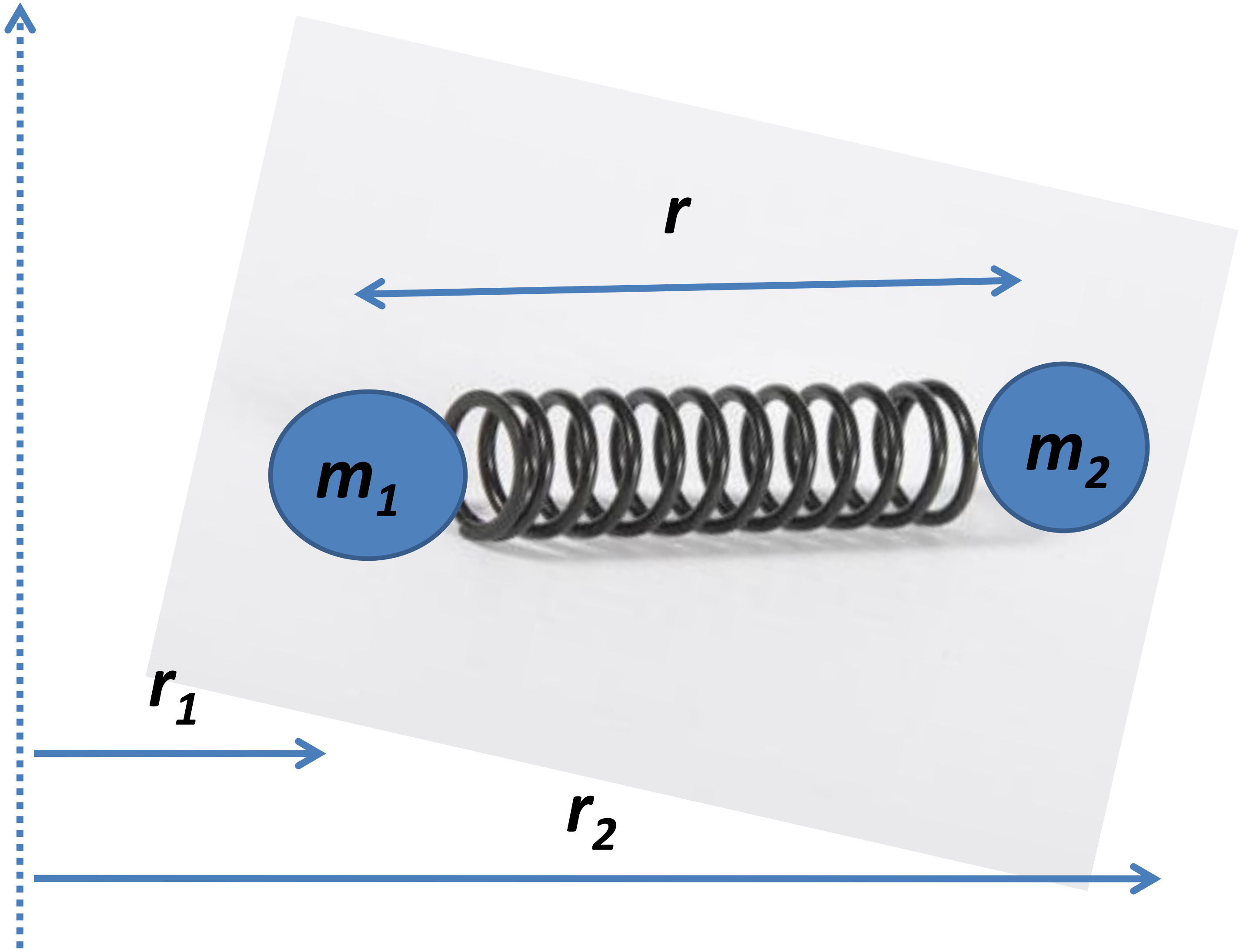
Energy Diagrams



Small Oscillations in a Bound System



Small Oscillations in a Bound System



Problem 4.15 A particle of mass m moves in one Dimension along the positive x axis. It is acted on by a constant force directed toward the origin with Magnitude ***B***, and an inverse square law repulsive force with magnitude A/x^2.

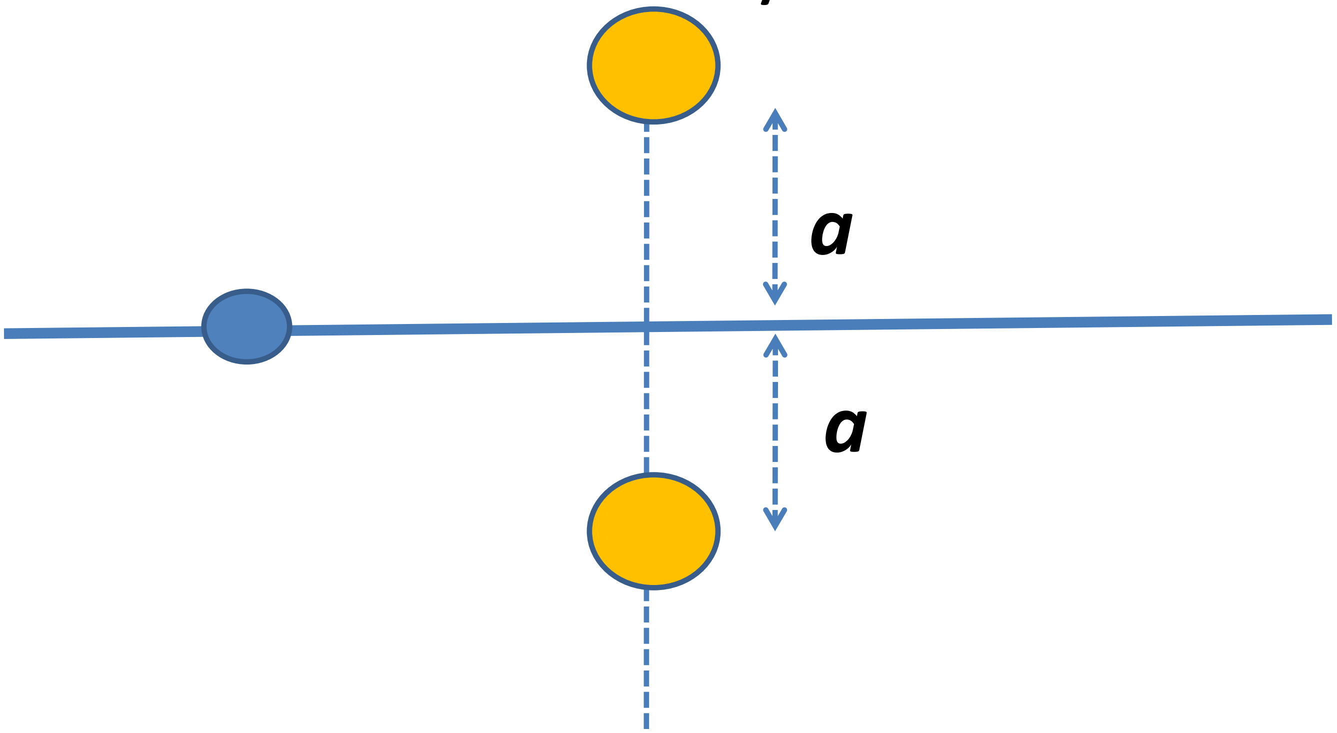
(a) Find the potential energy function *U(x)*.

(b) Sketch the energy diagram for the system when the maximum kinetic energy is $\frac{1}{2}MV_0^2$

(c) Find the equilibrium position x_0.

(d) What is the frequency of small oscillations about x_0.

Problem 4.14 A bead of mass m slides without friction on a smooth rod along the x-axis. Two spheres Each of mass Mare located at x=0, $y = \pm a$, and attract the bead Gravitationally.



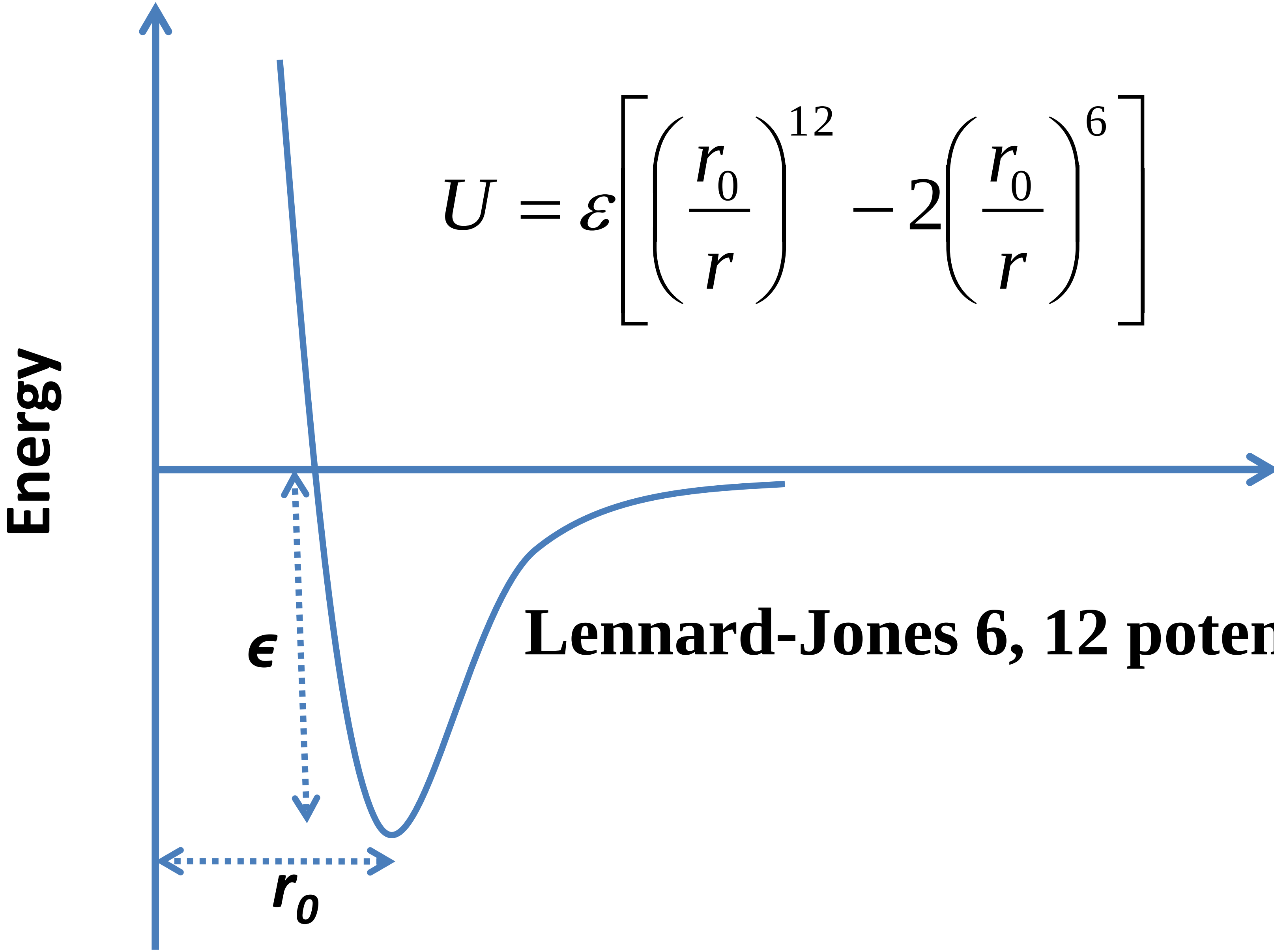
Find the frequency of small oscillations about x=0.

Problem: 4.13 A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones 6, 12 potential

$$U = \varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$$

(a) Show that the radius at the potential minimum is r_0, and that the depth of the potential well is ϵ.
(b) Find the frequency of small oscillations about equilibrium for two identical atoms of mass m bound to each other by Lennard-Jones interaction potential.

$$U = \varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$$



Lennard-Jones 6, 12 potential

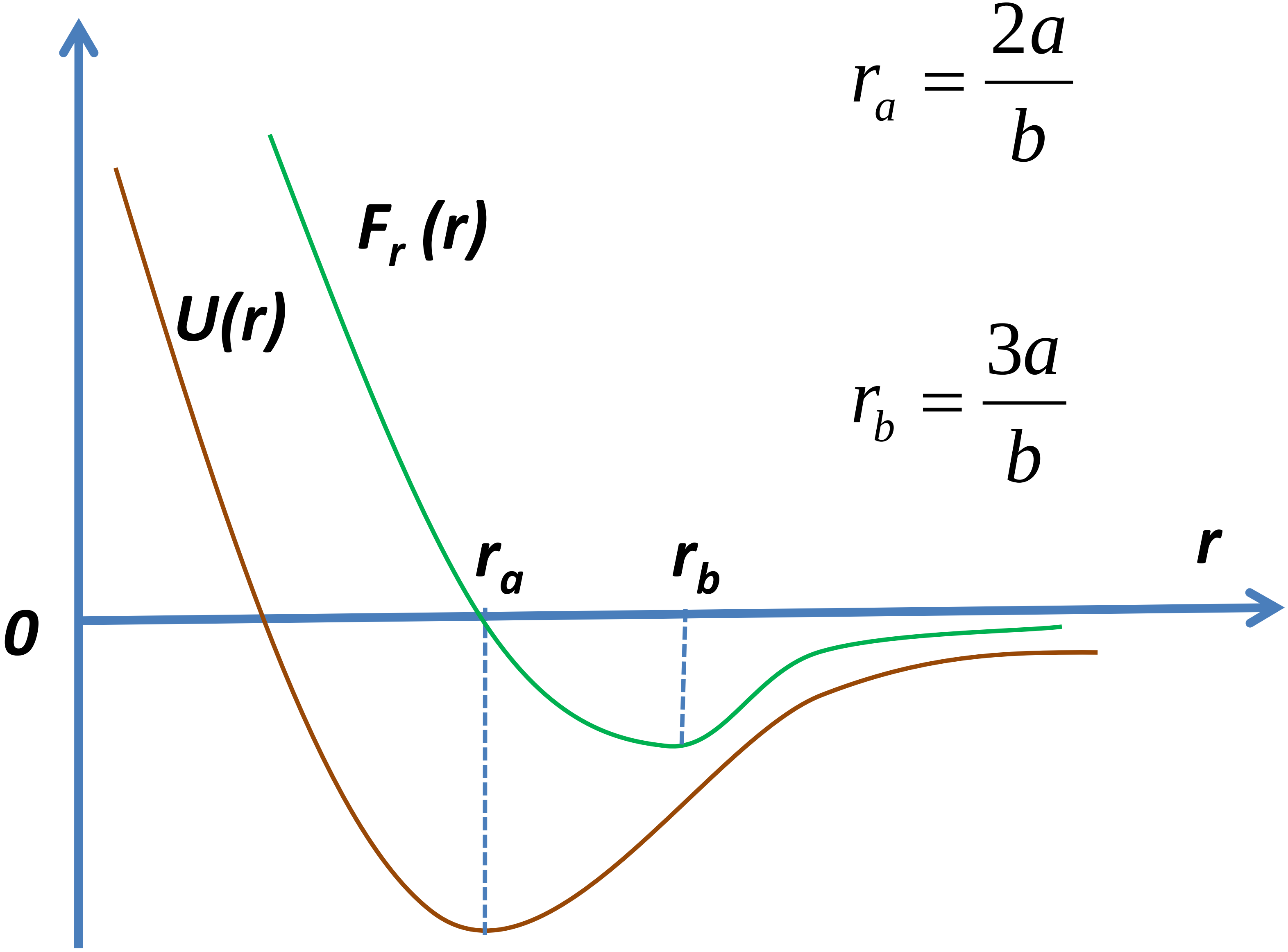
Problem: The potential energy of a particle in a certain field has the form

$$U = \frac{a}{r^2} - \frac{b}{r}$$

where a and b are positive constants, r is the distance from the origin of the field. Find:

(a) the value of r_0 corresponding to the equilibrium position of the particle; examine whether this position is steady.

(b) The maximum magnitude of the attractive force.



$$r_a = \frac{2a}{b}$$

$$r_b = \frac{3a}{b}$$

Power

Power is the Rate of Doing Work

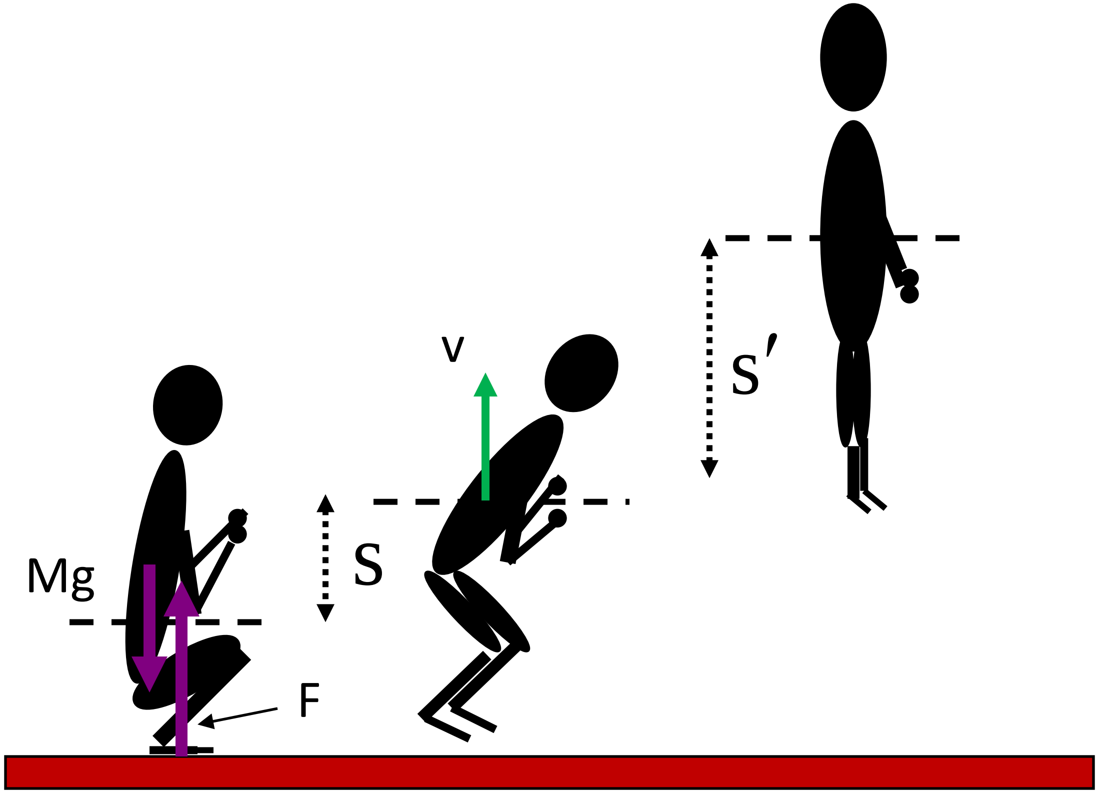
$$P = \frac{dW}{dt} = \vec{F} \bullet \frac{d\vec{r}}{dt}$$

The Unit of Power in the S.I system is the watt(W)

1 W= 1J/s

1 hp ≈ 746 W (S.I units)
1 hp ≈ 550 ft.lb/s (cgs units)

Problem: 4.18 A 160-lb man leaps into the air from a crouching position. His center of gravity rises 1.5 ft before he leaves the ground, and then rises 3.0 ft to the top of his leap. What power does he develop assuming that he pushes ground with constant force?

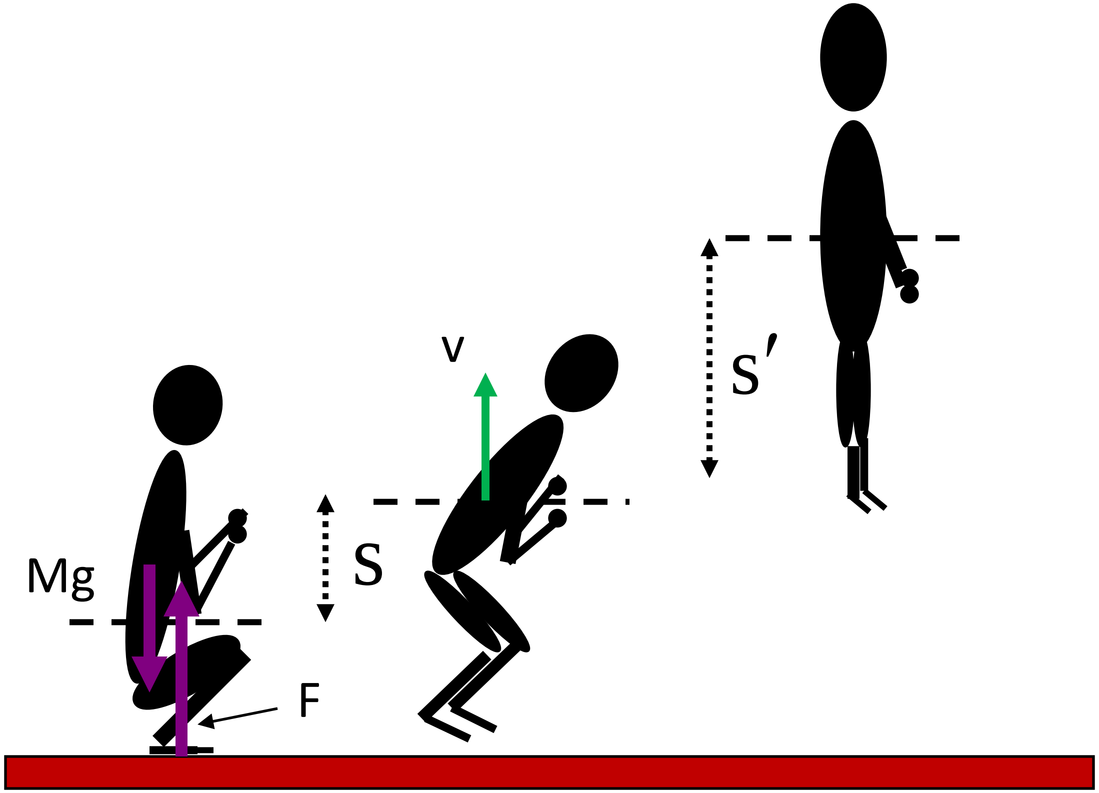


$S = 1.5$ ft

$S' = 3$ ft

(This figure is made by Prof. R.R Mishra)

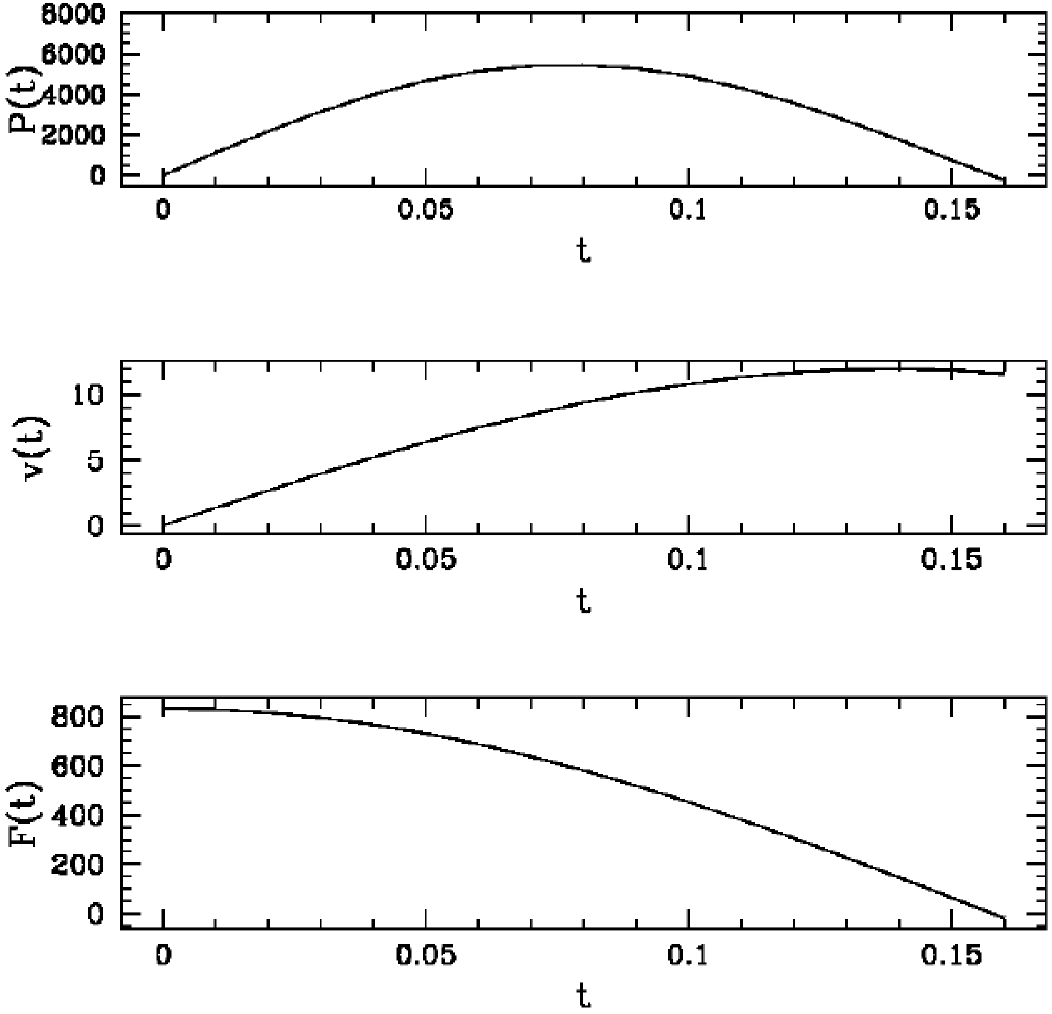
Problem: 4.19 The man in the preceding problem again leaps into the air, but this time the force he applies decreases from a maximum at the beginning of the leap to zero at the moment he leaves the ground. As a reasonable approximation, take the force to be $F=F_0\cos\omega t$, where F_0 is the peak force, and the contact with the ground ends when $\omega t=\pi/2$. Find the peak power the man develops during the jump.



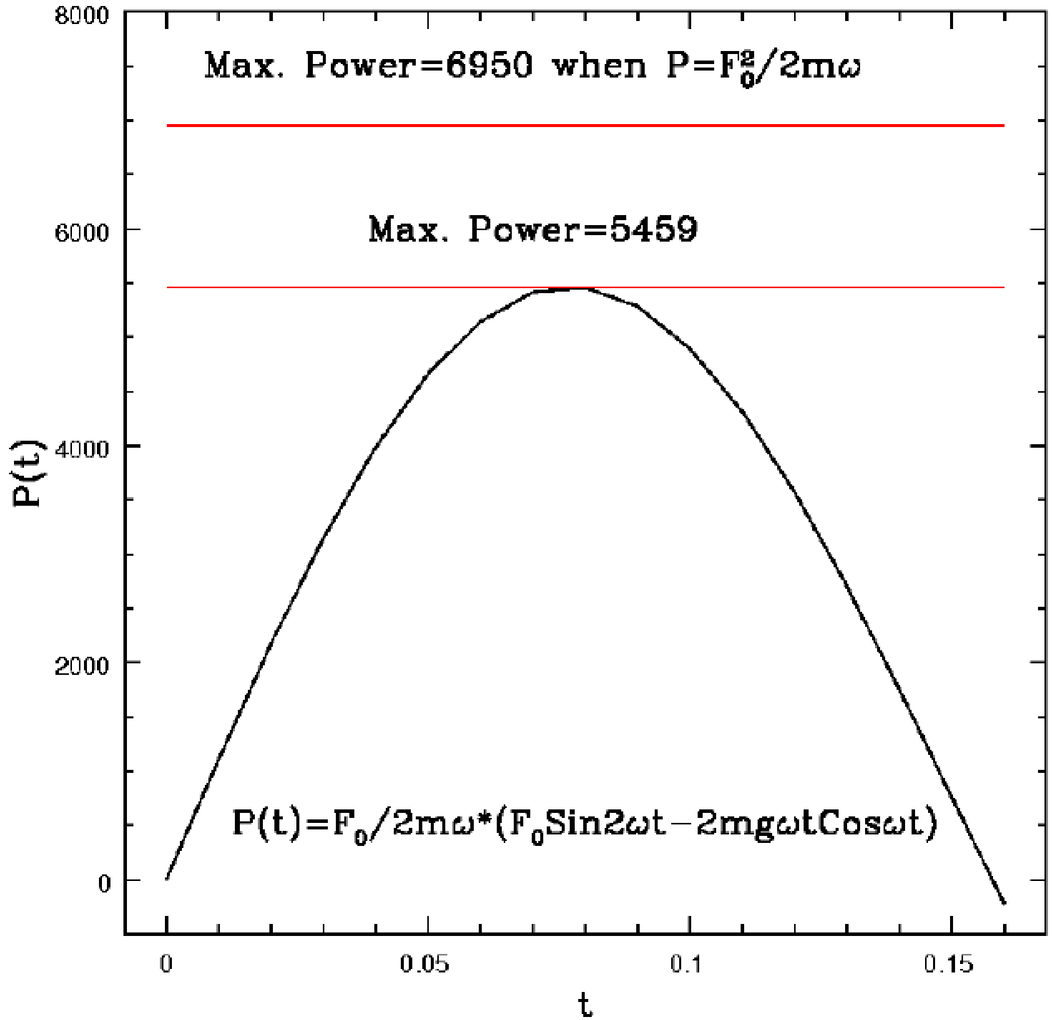
$S = 1.5$ ft

$S' = 3$ ft

(This figure is made by Prof. R.R Mishra)

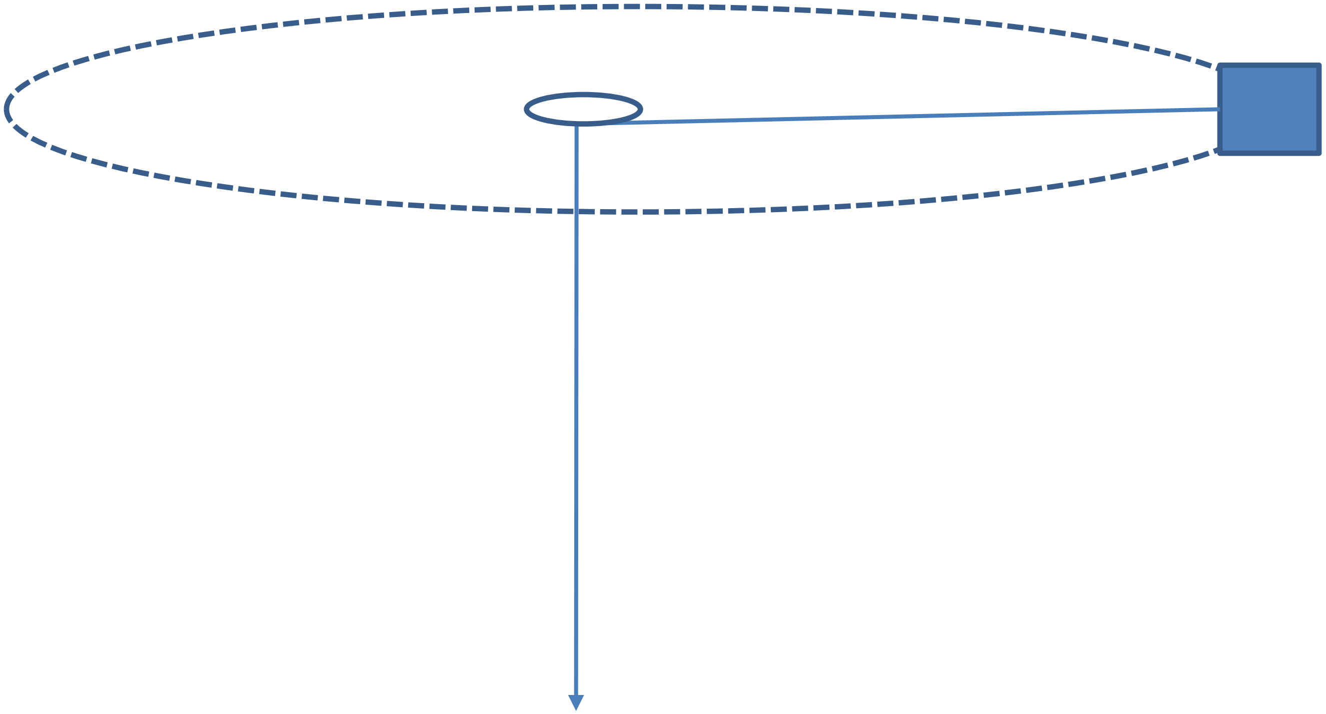


(This graph is plotted by Prof. Rishikesh Vaidya)



(This graph is plotted by Prof. Rishikesh Vaidya)

Problem: 4.5 Mass m whirls on a frictionless table, held to circular motion by a string which passes through a hole in the table. The string is slowly pulled through the hole so that the radius of the circle changes from L_1 to L_2. Show that the work done in pulling the string equals the increase in the kinetic energy of the mass.

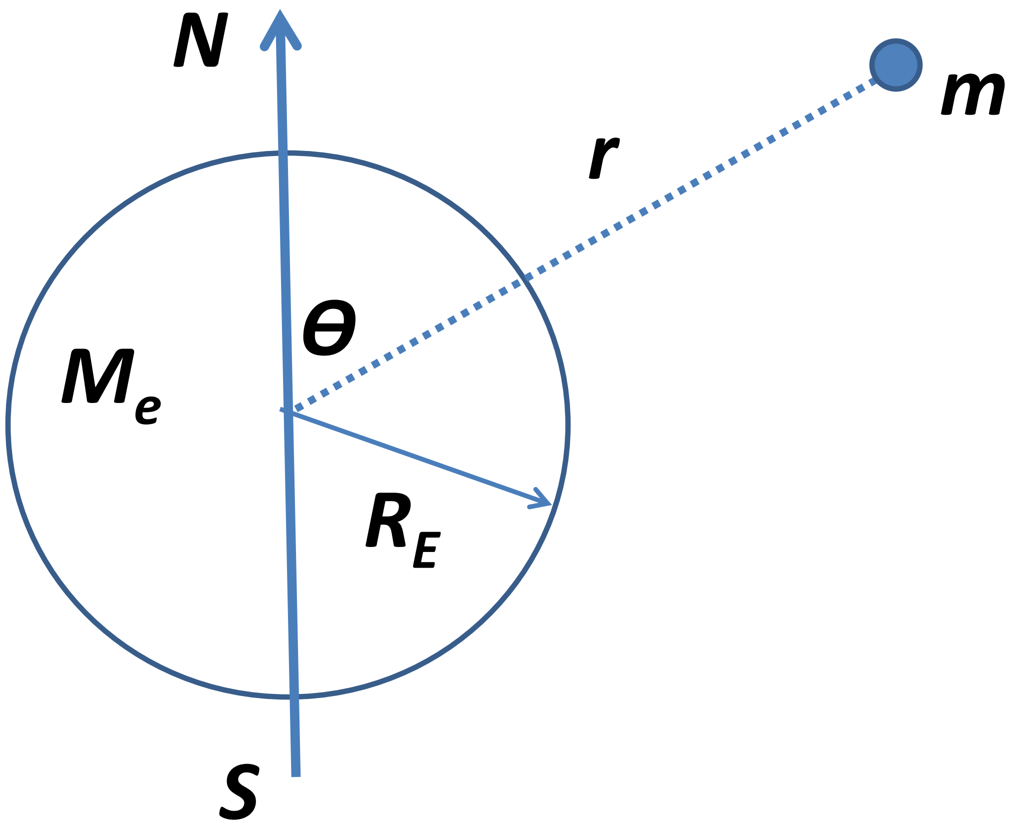


Non-Conservative Forces

Problem: When the flattening of the earth at the poles is taken into account, it is found that the gravitational potential energy of a mass *m* a distance *r* from the center of the earth is approximately

$$U = -\frac{GM_e m}{r}\left[1 - 5.4\times10^{-4}\left(\frac{R_e}{r}\right)^2\left(3\cos^2\theta - 1\right)\right]$$

where θ is measured from the pole. Show that there is a small tangential gravitational force on *m* except above the poles and equator. Find the ratio of this force to $GM_e m/r^2$ for $\Theta = 45^0$ and $r = R_e$.

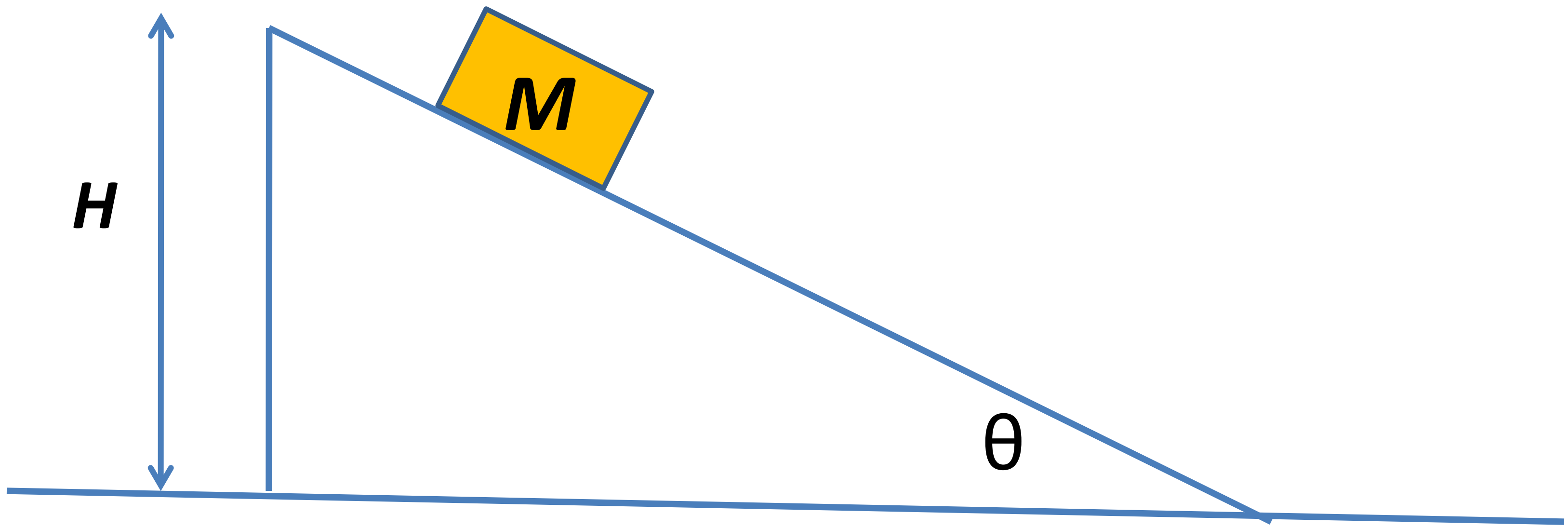


$$U = -\frac{GM_e m}{r}\left[1 - 5.4\times10^{-4}\left(\frac{R_e}{r}\right)^2\left(3\cos^2\theta - 1\right)\right]$$

Non-conservative Forces

The Work-Energy Theorem

Non-conservative Forces



Block Sliding Down an Incline Plane with Friction

Problem: A body of mass m is slowly hauled up the hill by a force *F* which at each point is directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is *h*, the length of its base L, and the coefficient of friction is *k*.