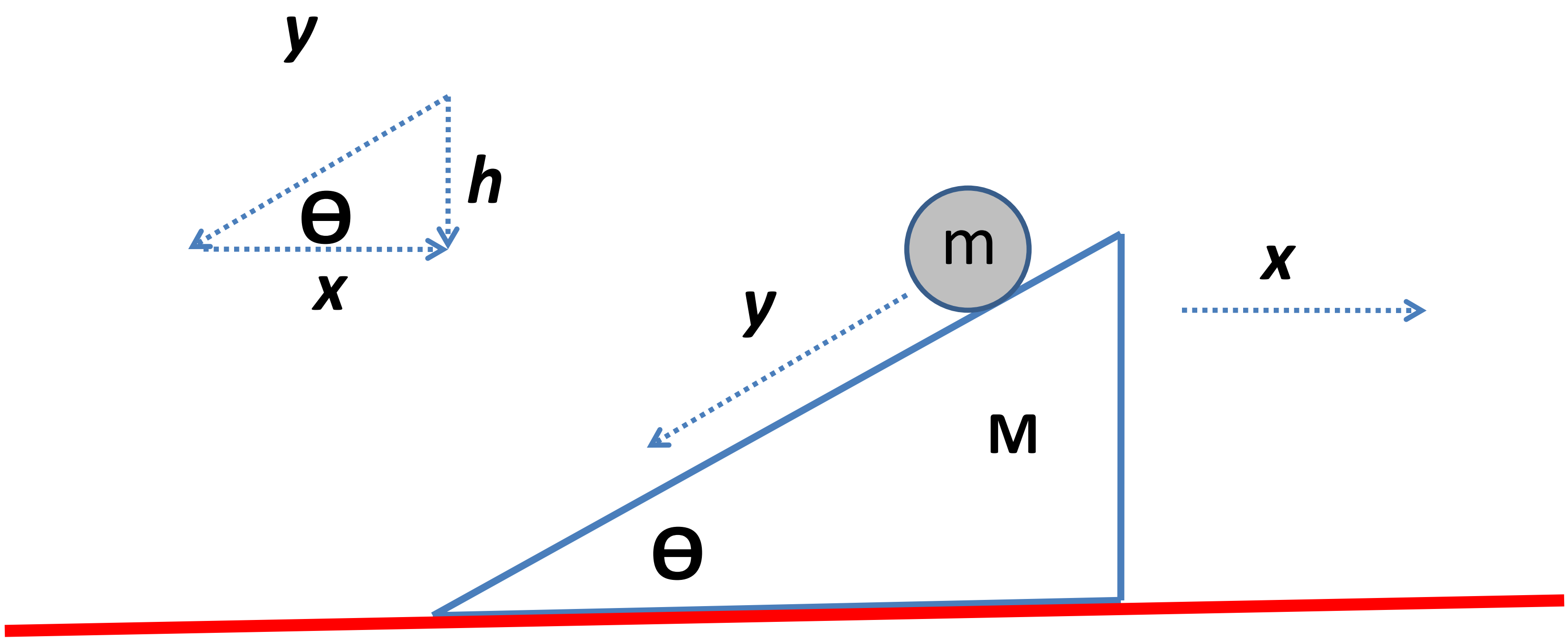
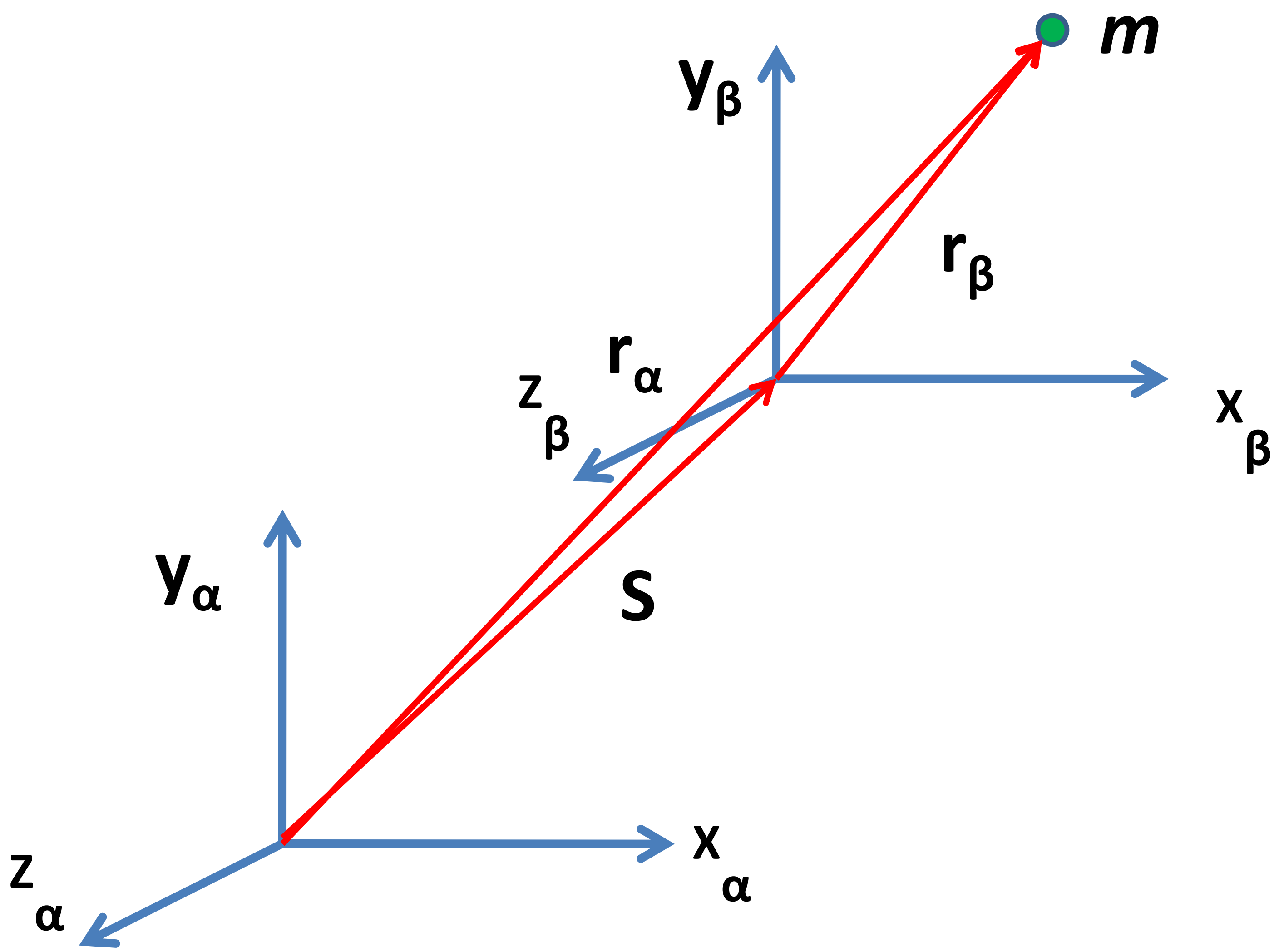
Problem: A spherical shell rolls down without slipping along the inclined plane of the wedge which is kept on smooth horizontal plane. Calculate the expression of the acceleration of the wedge.



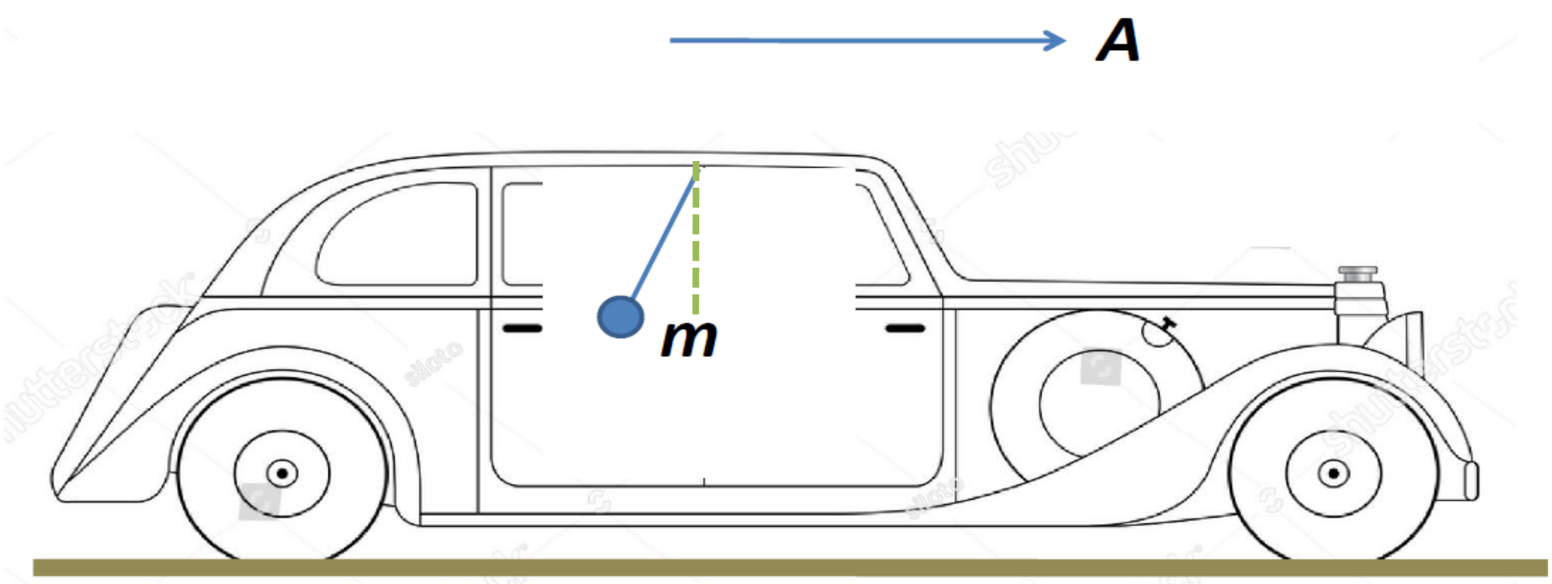
Chapter 8:
NONINERTIAL SYSTEM AND FICTITIOUS FORCES

The Galilean Transformation

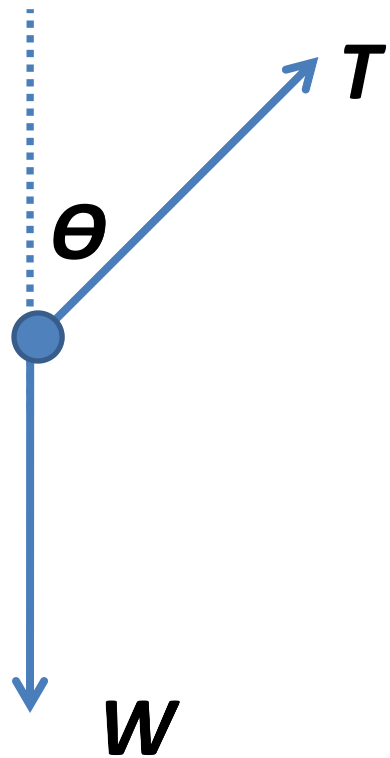


Uniformly Accelerating Systems

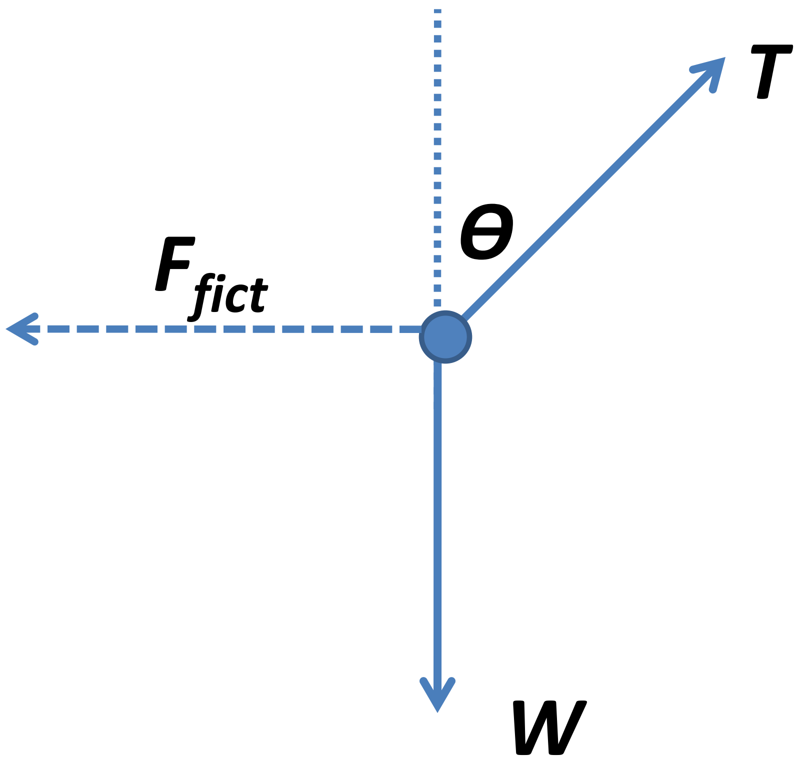
Problem: A small weight of mass m hangs from a string in a car which accelerates at rate A. What is the static angle of the string from the vertical and what is its tension



Inertial System

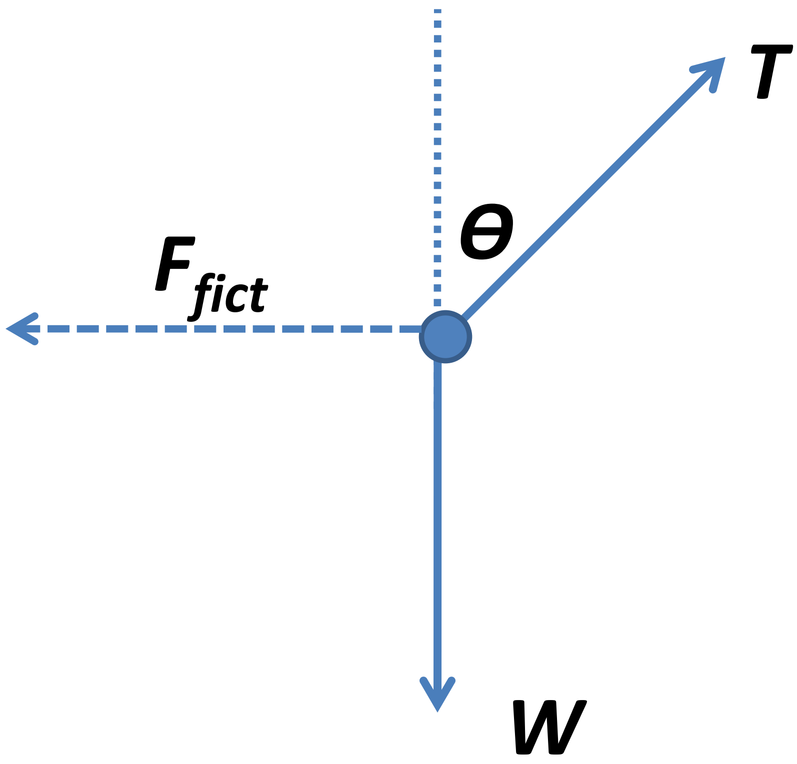


System Accelerating with the Car



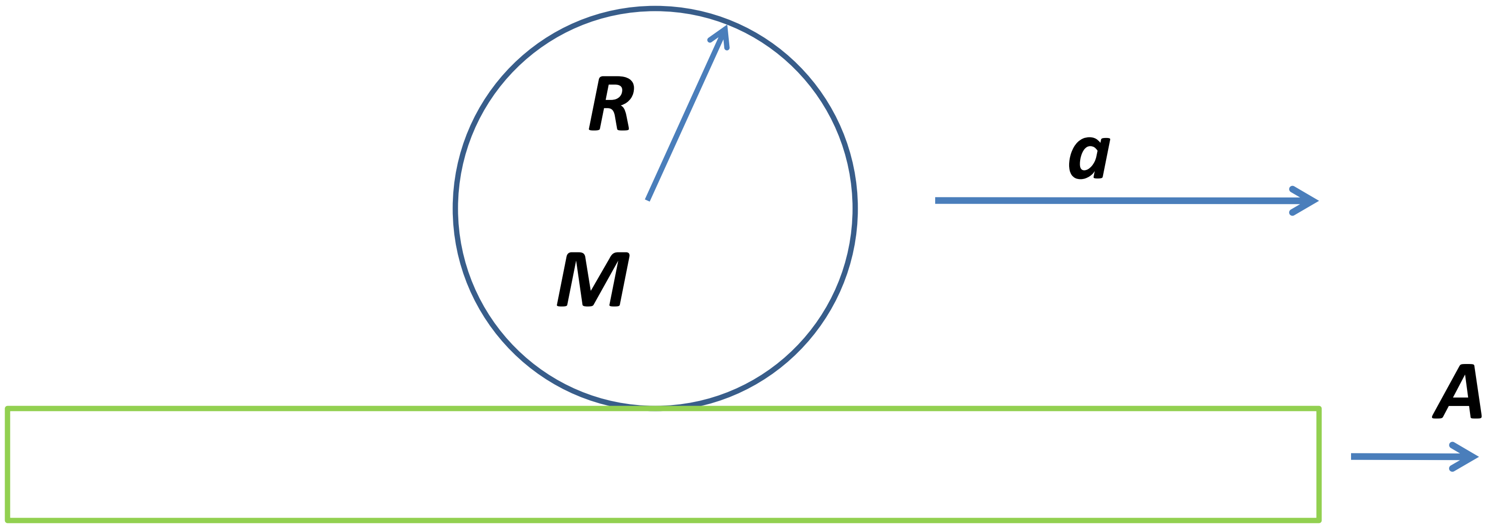
$A=0$

System Accelerating with the Car



$A=0$

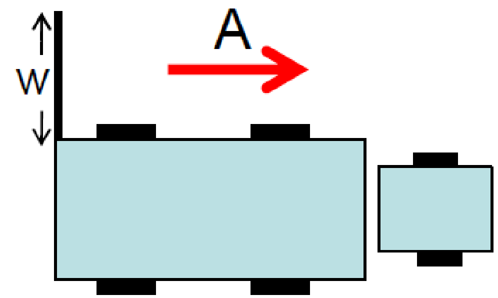
Problem: Cylinder on an Accelerating Plank



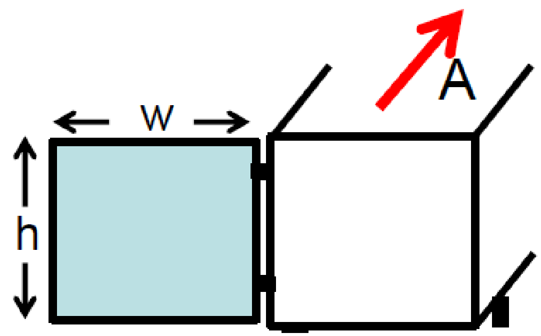
Prob. 8.2

A truck at rest has its back door fully open.

The truck accelerates forward at constant rate A, and the door begins to swing shut. The door is uniform and solid, has total mass M, height h, and width w.



Top View

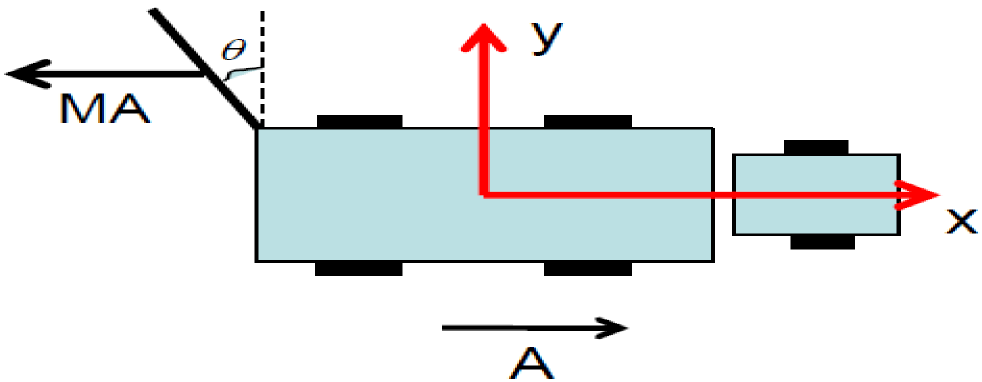


Rear View

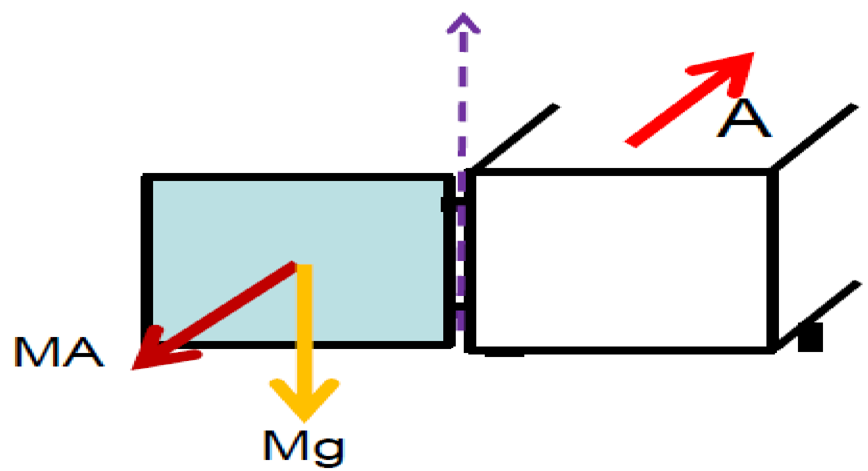
a. Find the instantaneous ang. velocity of the door about its hinges, when it has swung through 90 degrees.

b. Find the horizontal force on the door when it has swung through 90 deg.

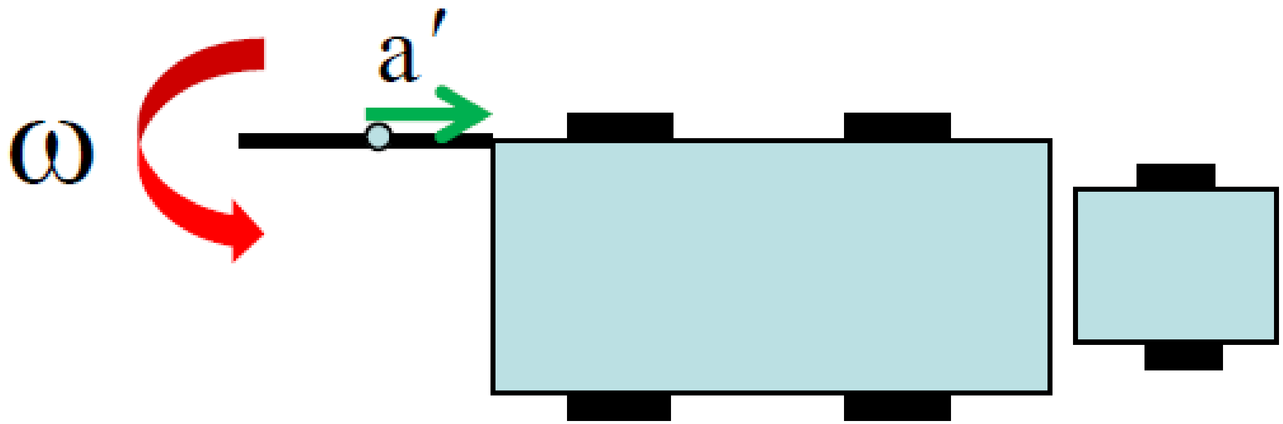
a. Find the instantaneous ang. velocity of the door about its hinges, when it has swung through 90 degrees.



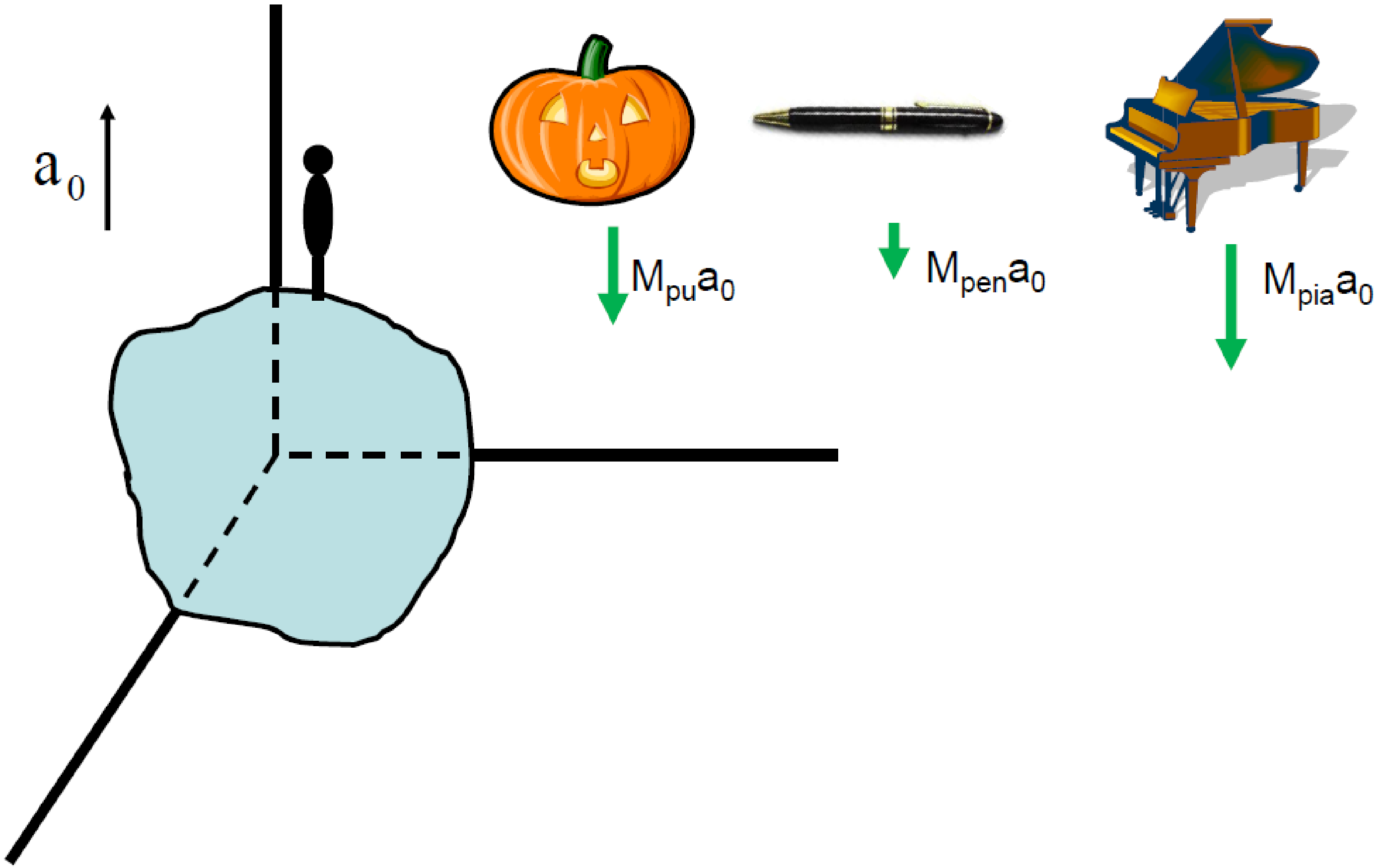
Truck's Frame



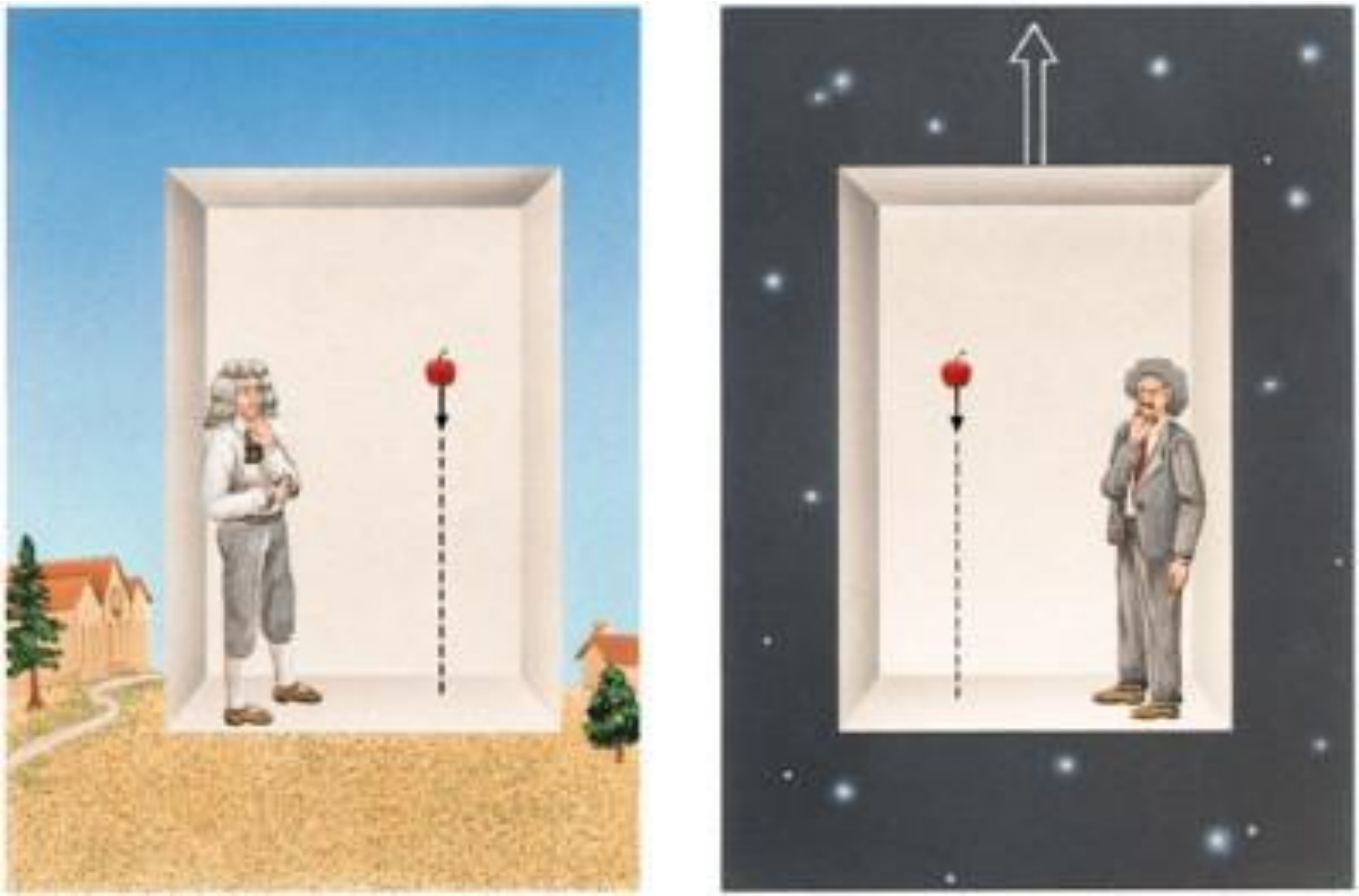
b. Find the horizontal force on the door when it has swung through 90 deg.



Fictitious forces (inertial forces) on objects in an accelerated frame are proportional to their masses!

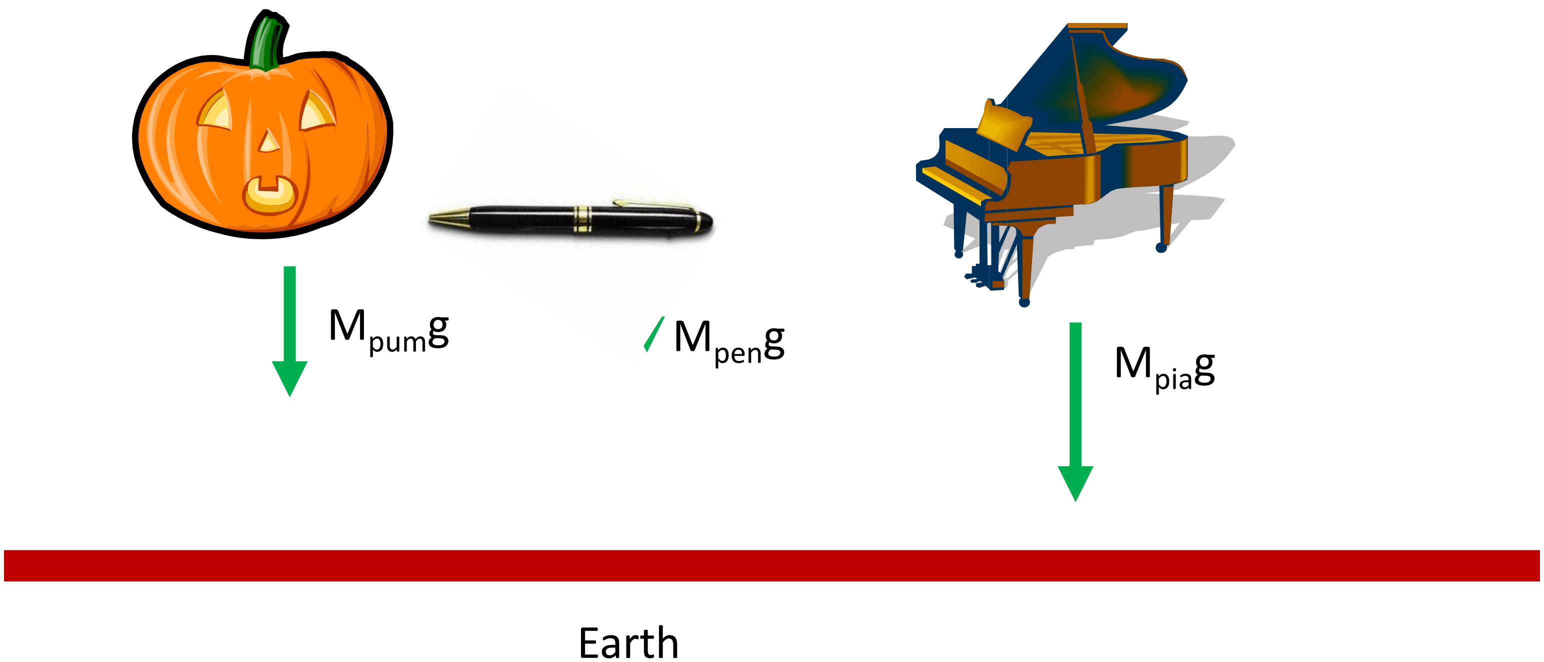


The Principle of Equivalence



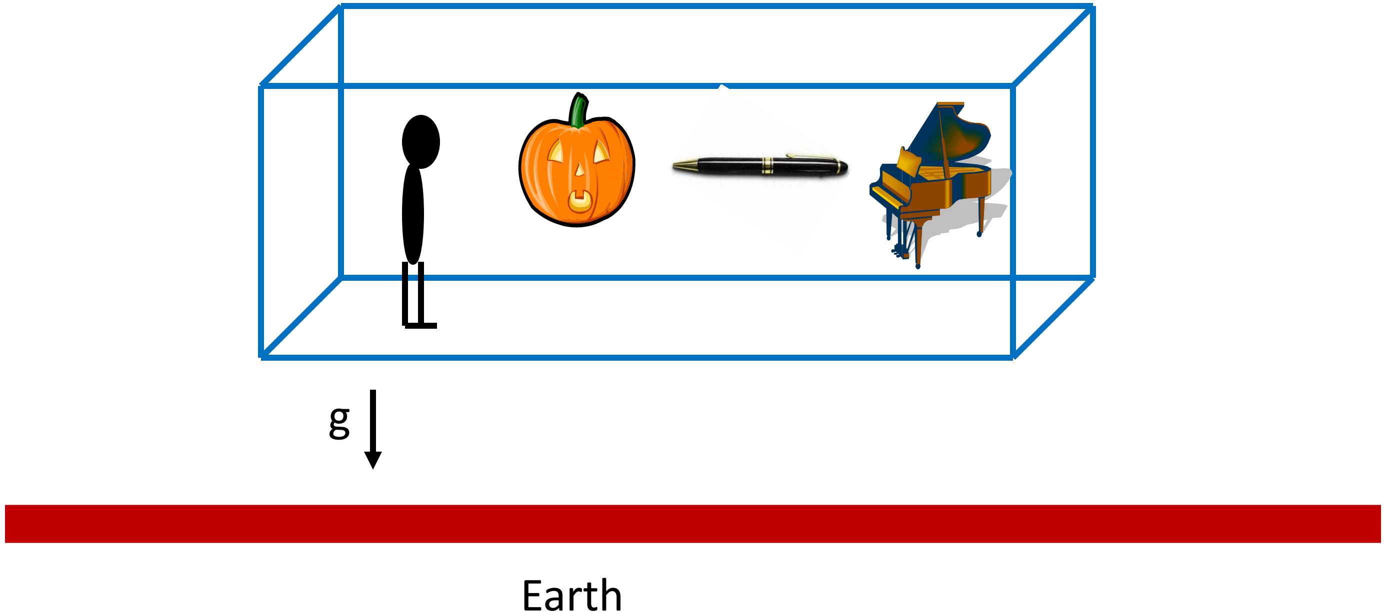
The Principle of Equivalence

Accelerating Frames are Similar to Uniform Gravity!



The two forces, inertial and uniform gravity are totally indistinguishable

One can make uniform gravity disappear by accelerating his frame in an appropriate manner

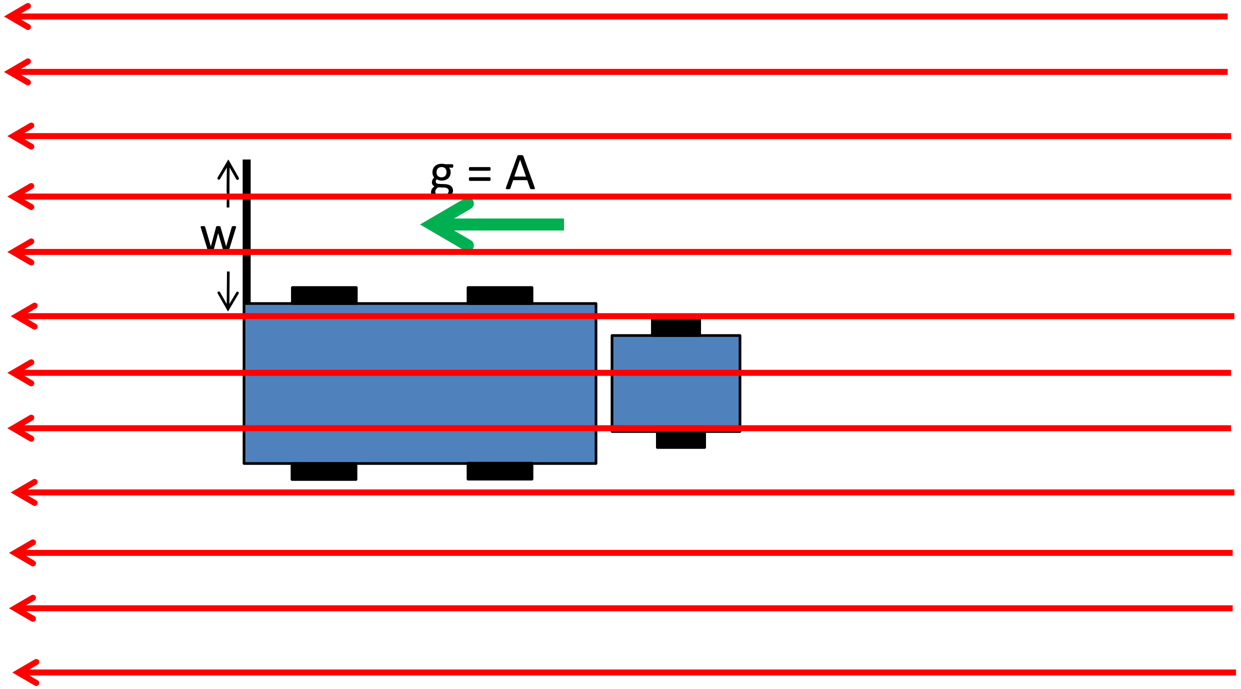


Prob. 8.2 Revisited

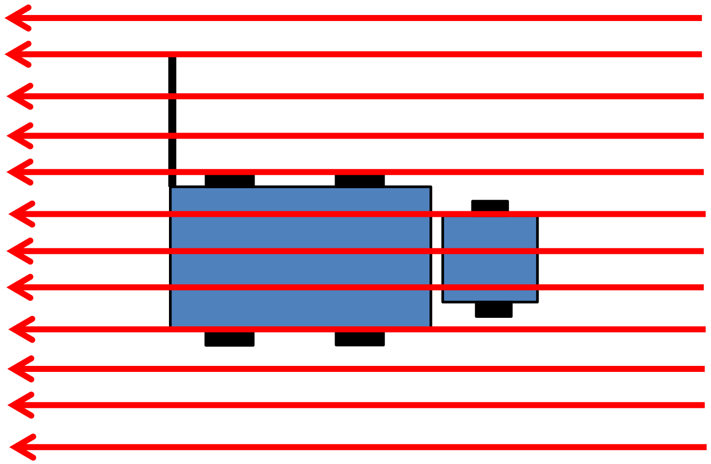
Alternative Approach (Energy Conservation)

We have seen that the fictitious force developed in an accelerating frame, is indistinguishable from uniform gravity.

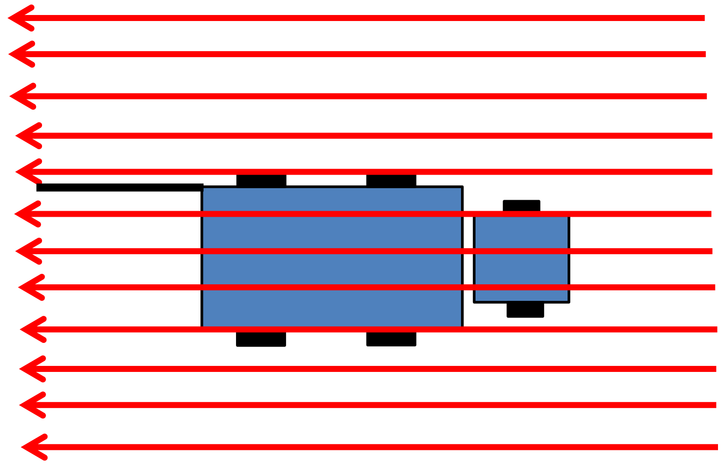
Thus, in the truck's frame, there will be a uniform horizontal gravity, directed opposite to the acceleration of the truck. The g value of this gravitational field is A, the acceleration of the truck.



Uniform horizontal gravitational field in the truck's frame, directed left.



Position 1



Position 2

In going from position 1 to position 2, the CM of the door falls in this uniform gravitational field through a distance $\frac{w}{2}$

Conserving energy,

$$\frac{1}{2} I\omega^2 = MA\frac{w}{2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{MAw}{I}}$$

What about non-uniform gravity?

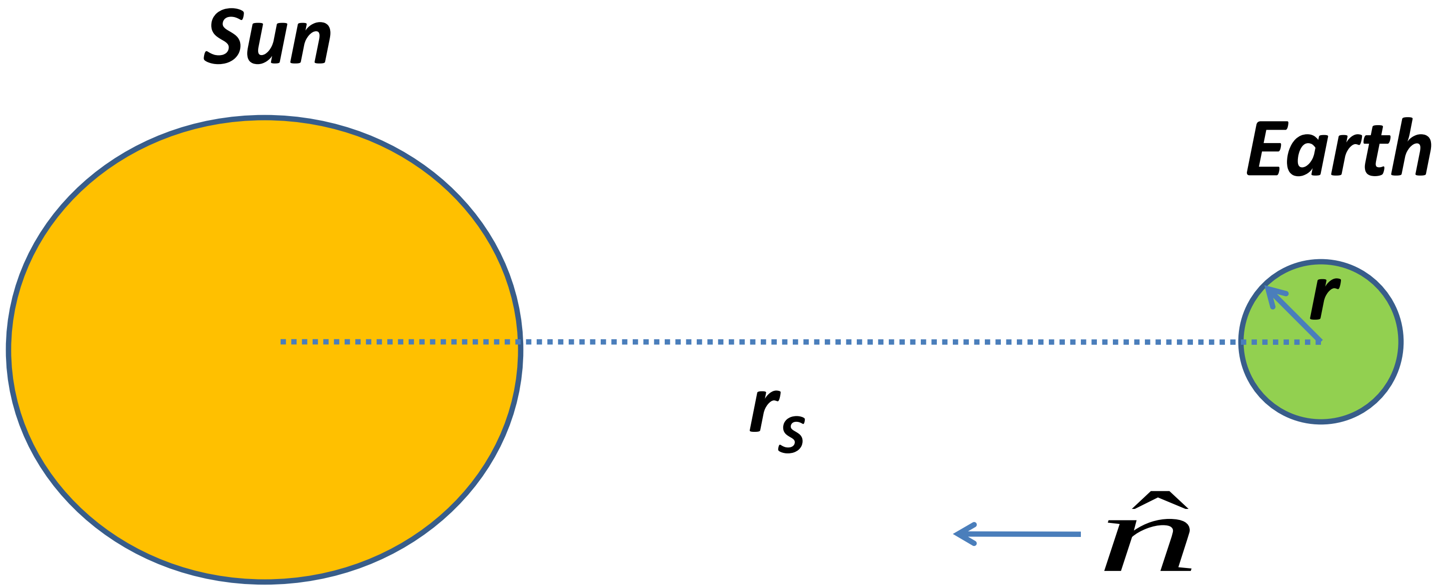
Can it be also be mimicked by an accelerating frame?

Can it be made to vanish?

Ans : The major part of it can be made to vanish. However, a residual part will still remain, and this residual gravity is the well known Tidal Force

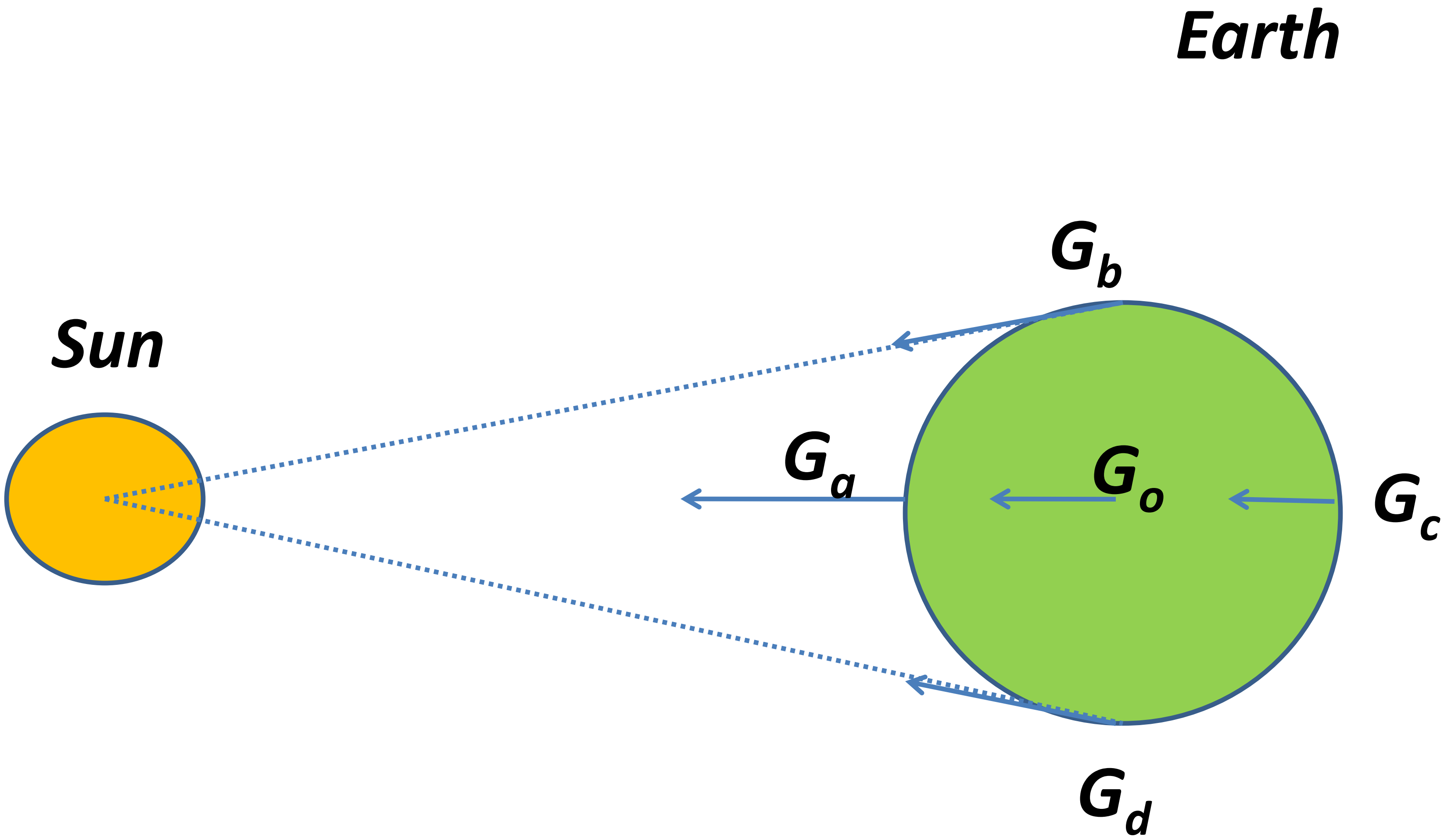
Tidal Force

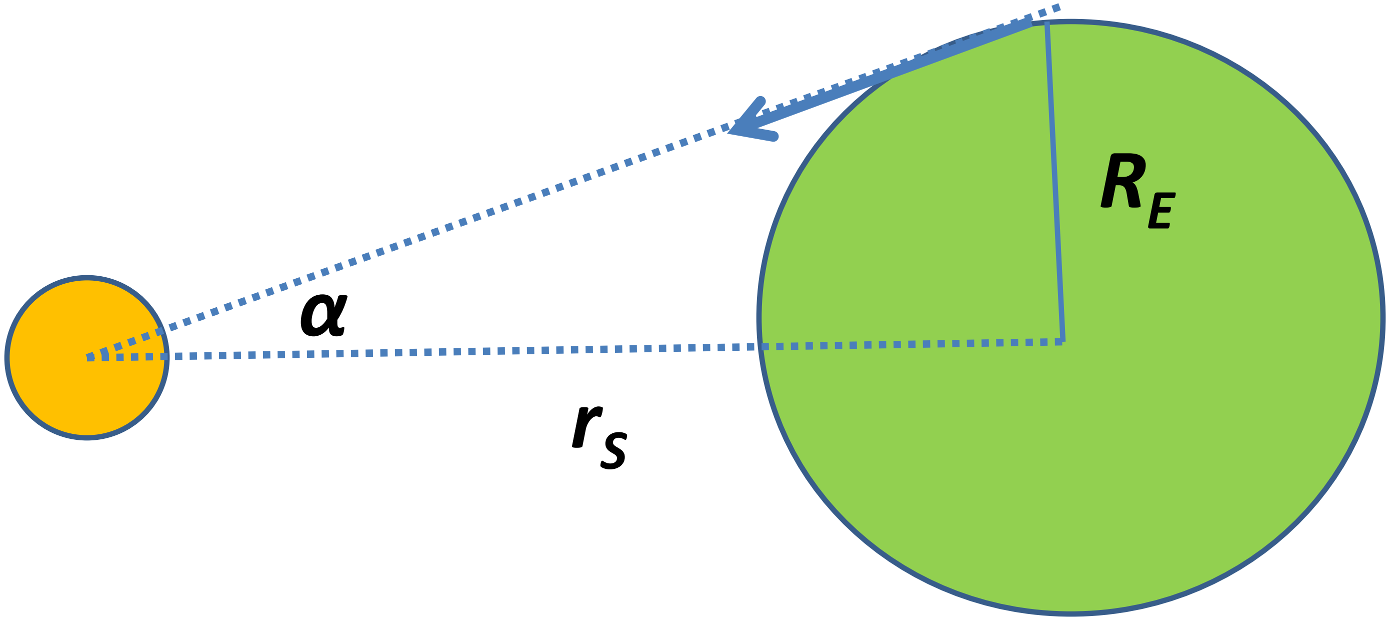
Earth –Sun System

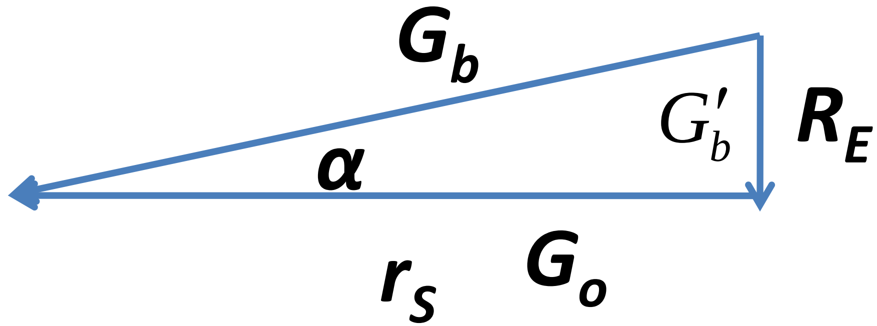


$$G_0 = GM_s \frac{\hat{n}}{r_s^2}$$

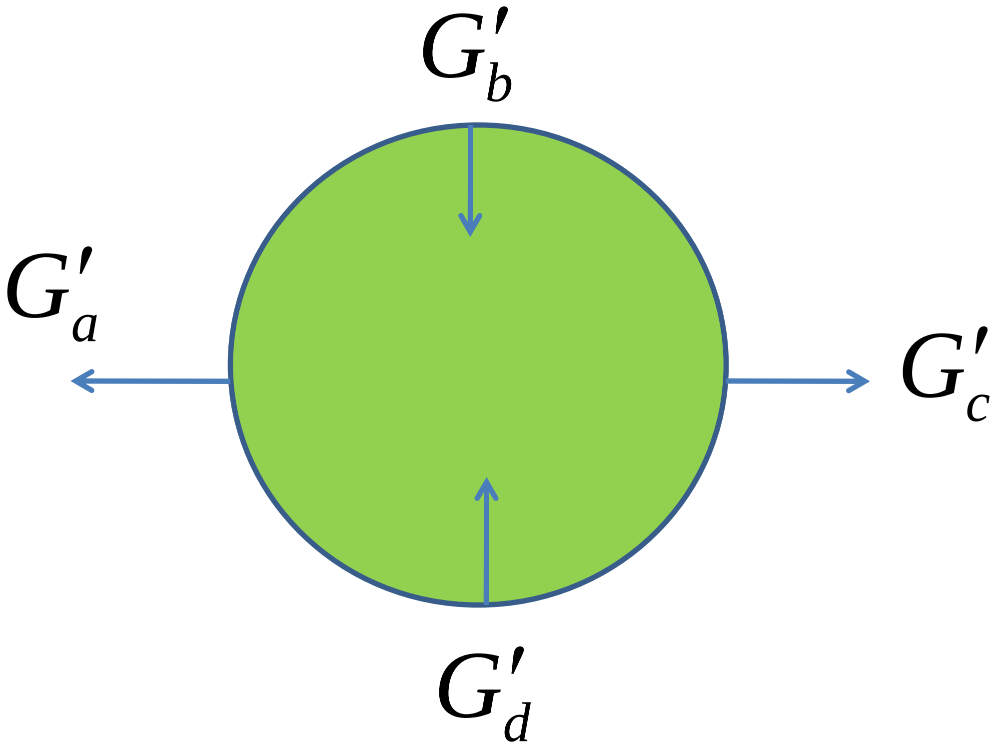
Earth –Sun System



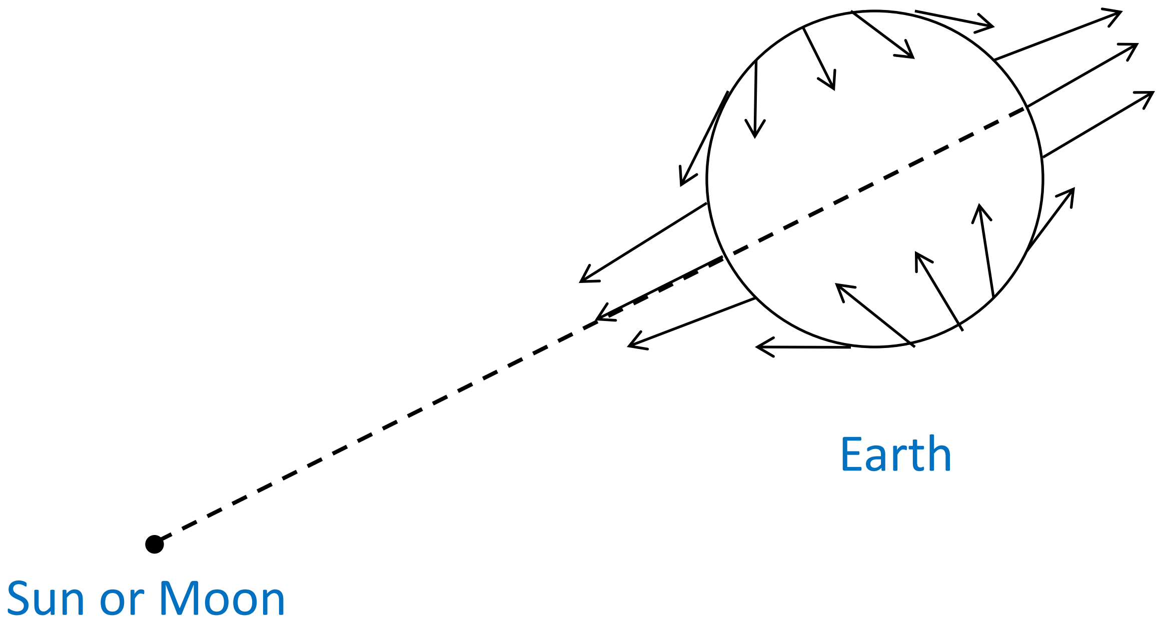

α
R_E
r_S

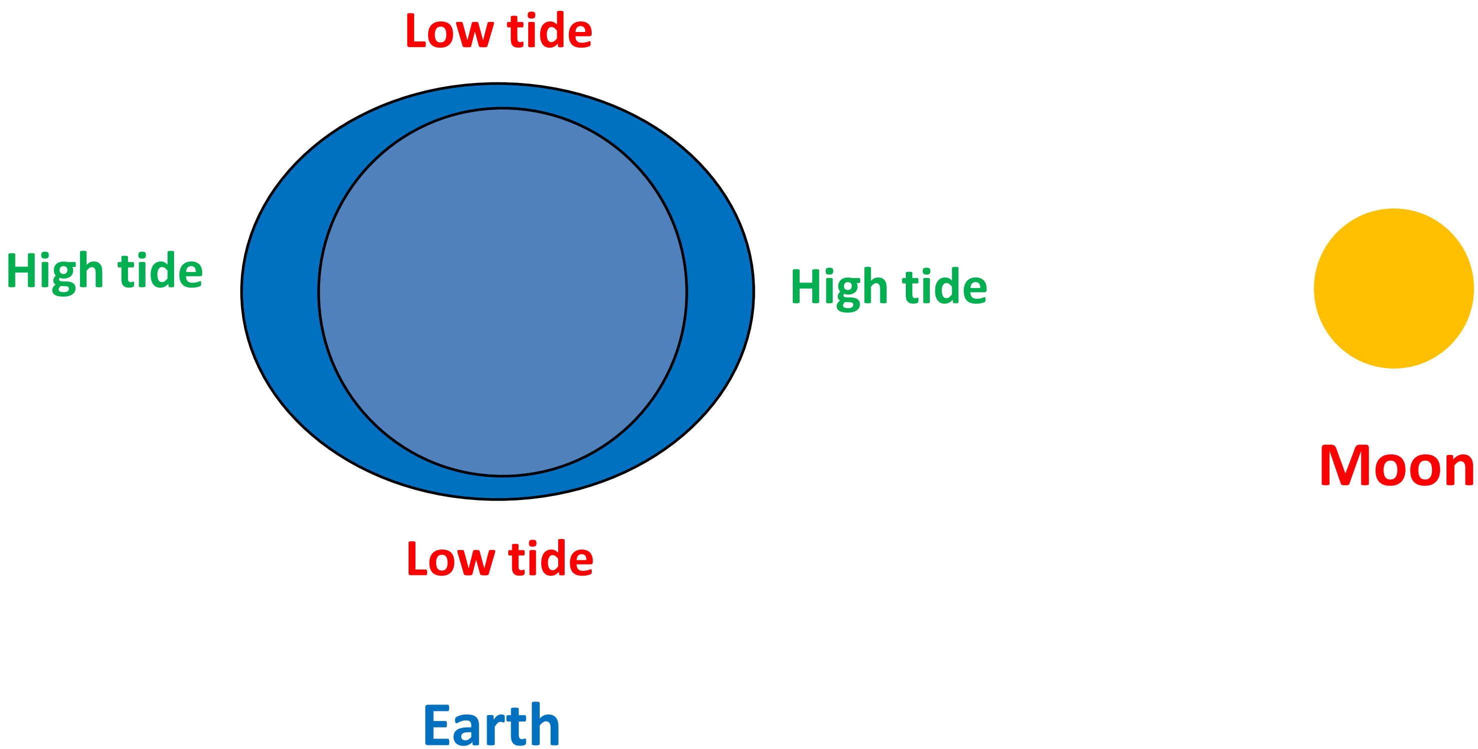

G_b
G_b'
R_E
α
r_S
G_o

The Apparent Field

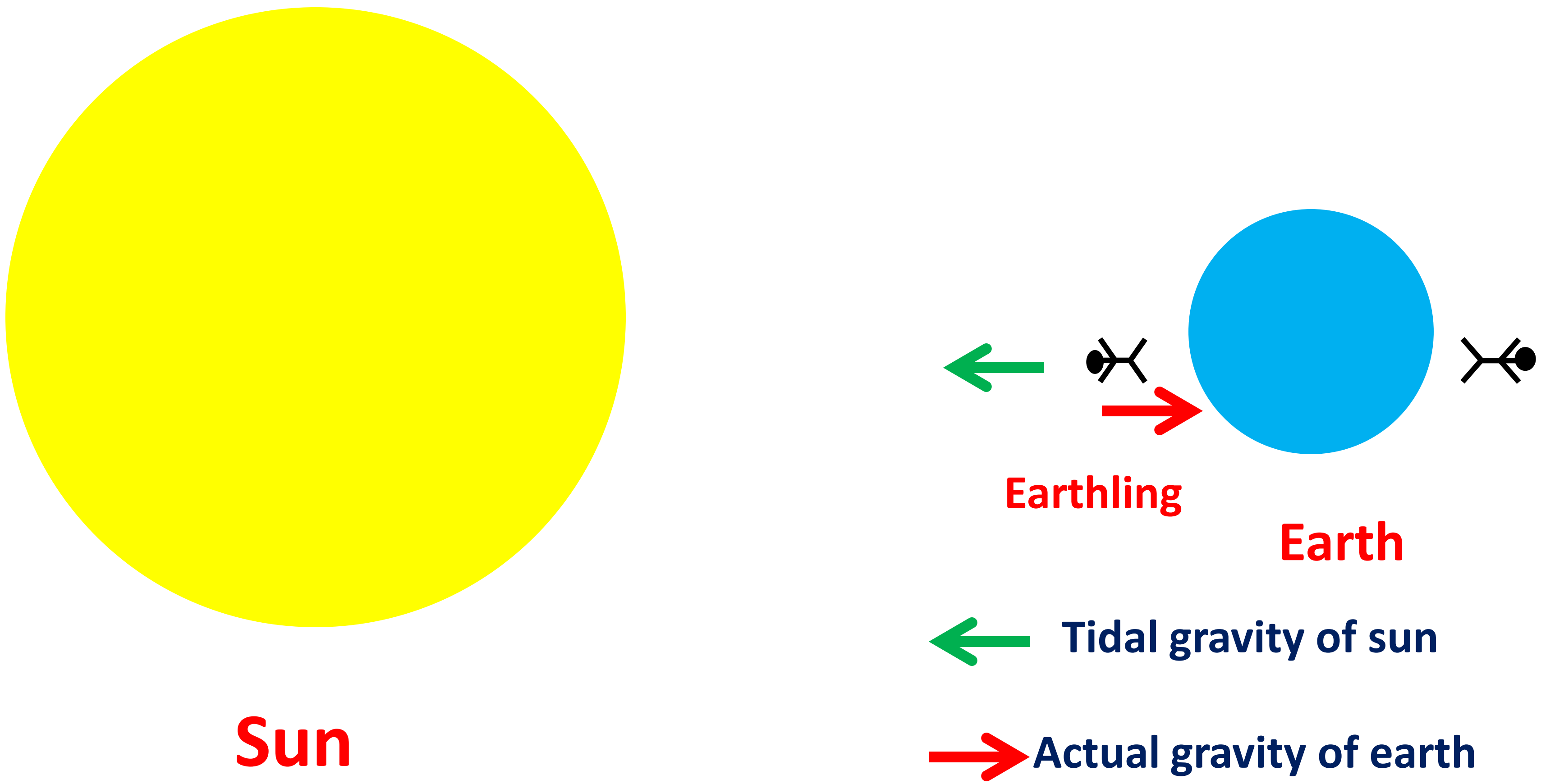


Tidal Force Distribution on the Surface of Earth

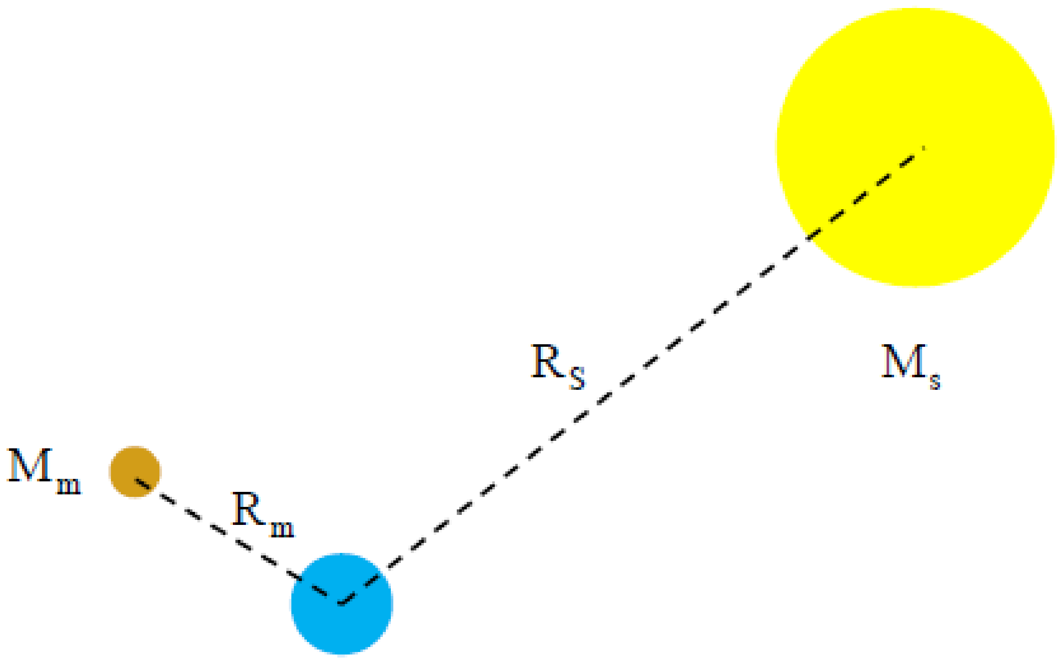



Low tide
High tide
High tide
Moon
Low tide
Earth

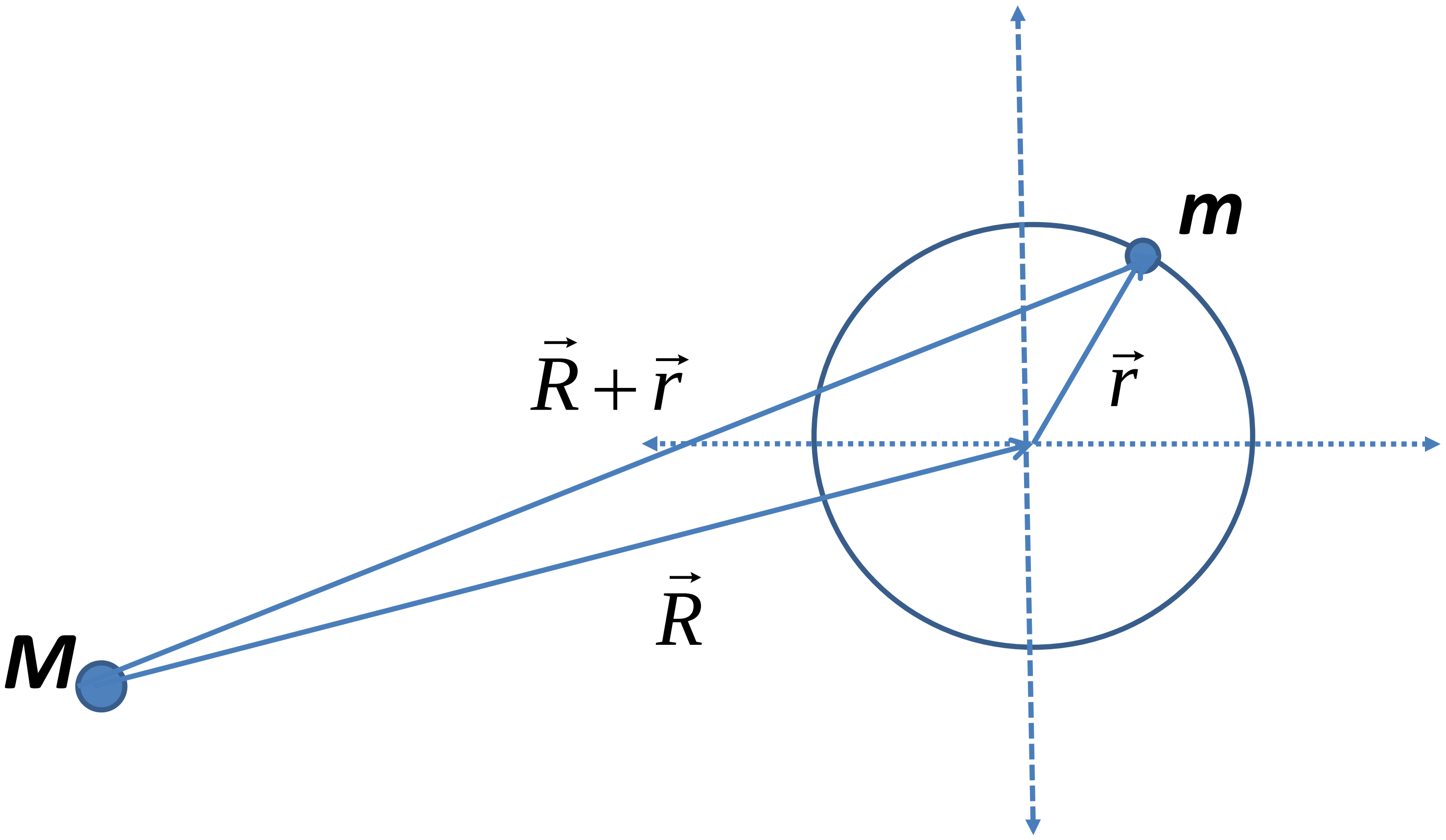
Q. How massive does the sun have to be for it to snatch away people from earth?



Tidal Force of Sun vs Moon

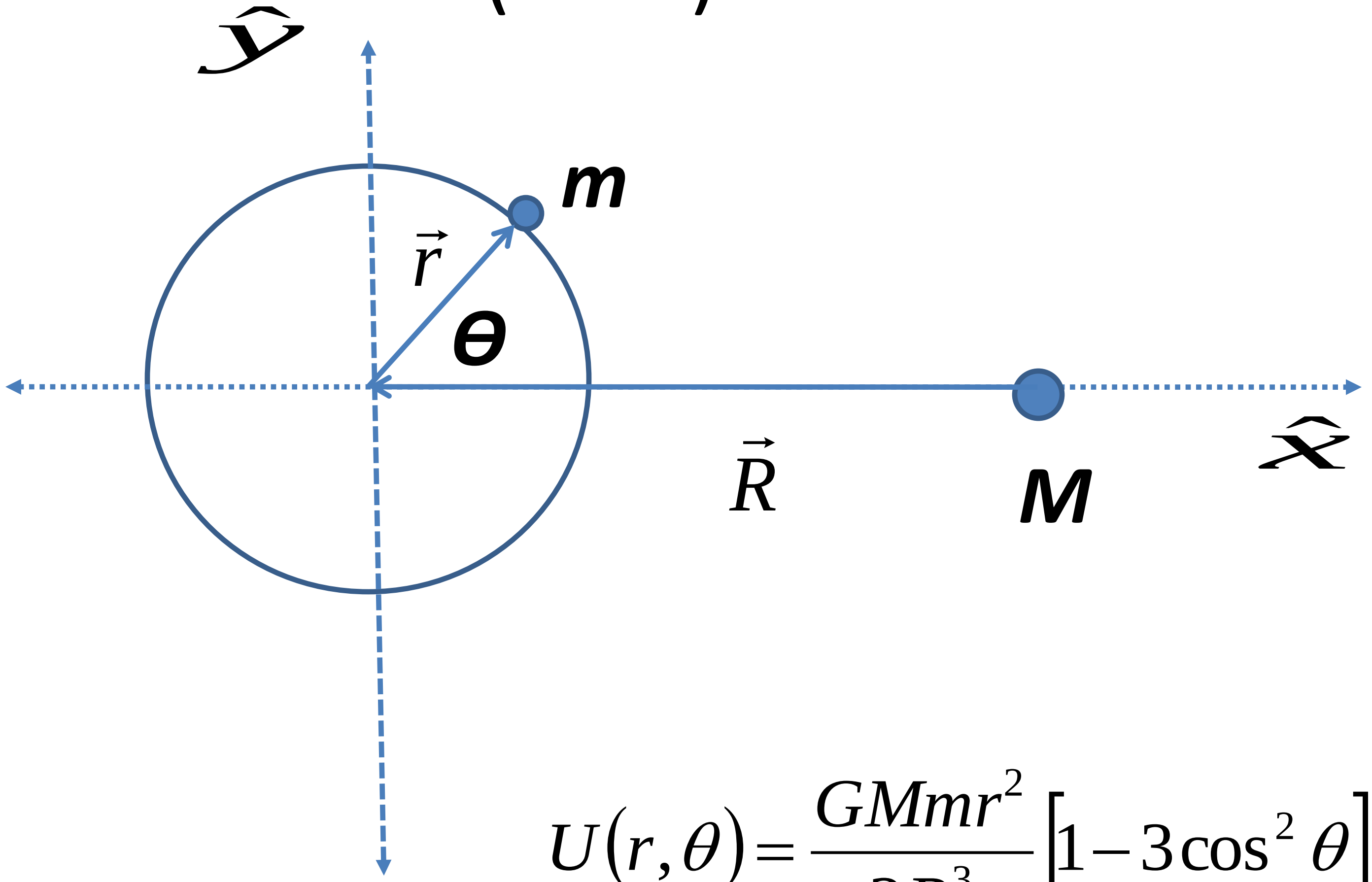


General Expression of Tidal Force (R>>r)



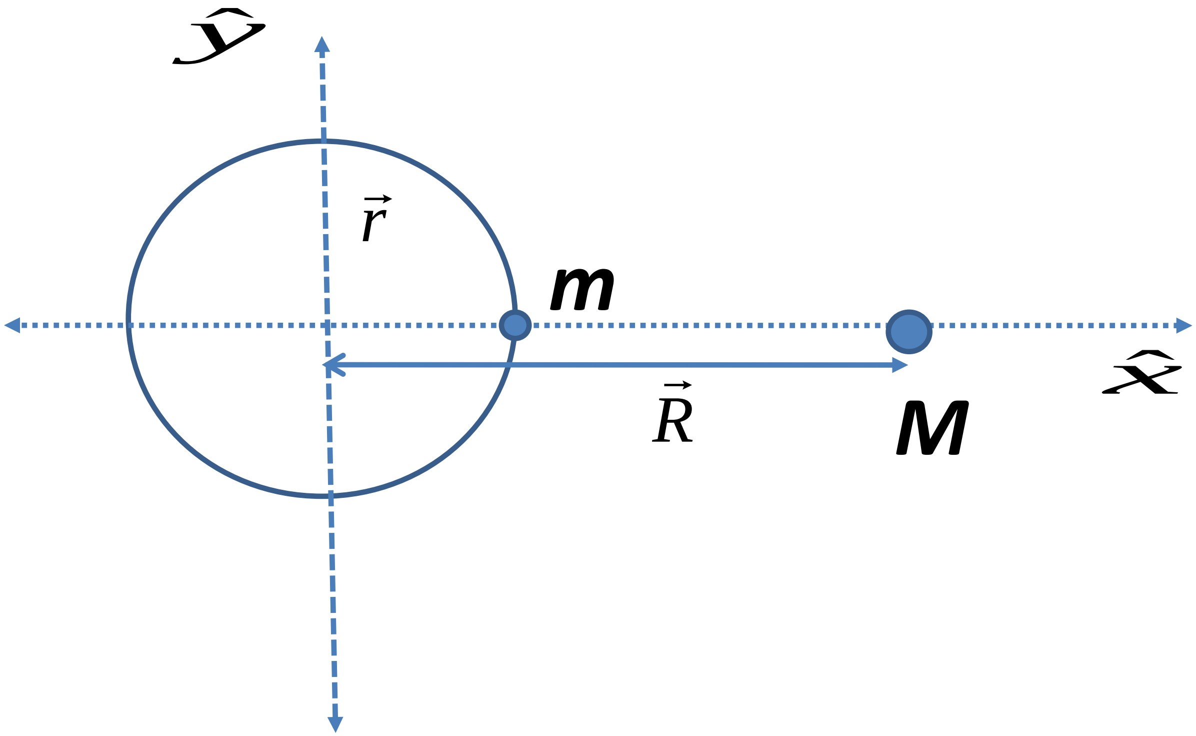
$$\vec{F}_{Tidal} \approx \frac{GMm}{R^3}\left[3\hat{R}\left(\hat{R}\bullet\vec{r}\right)-\vec{r}\right]$$

Expression for the Potential for the Tidal Force (R>>r)

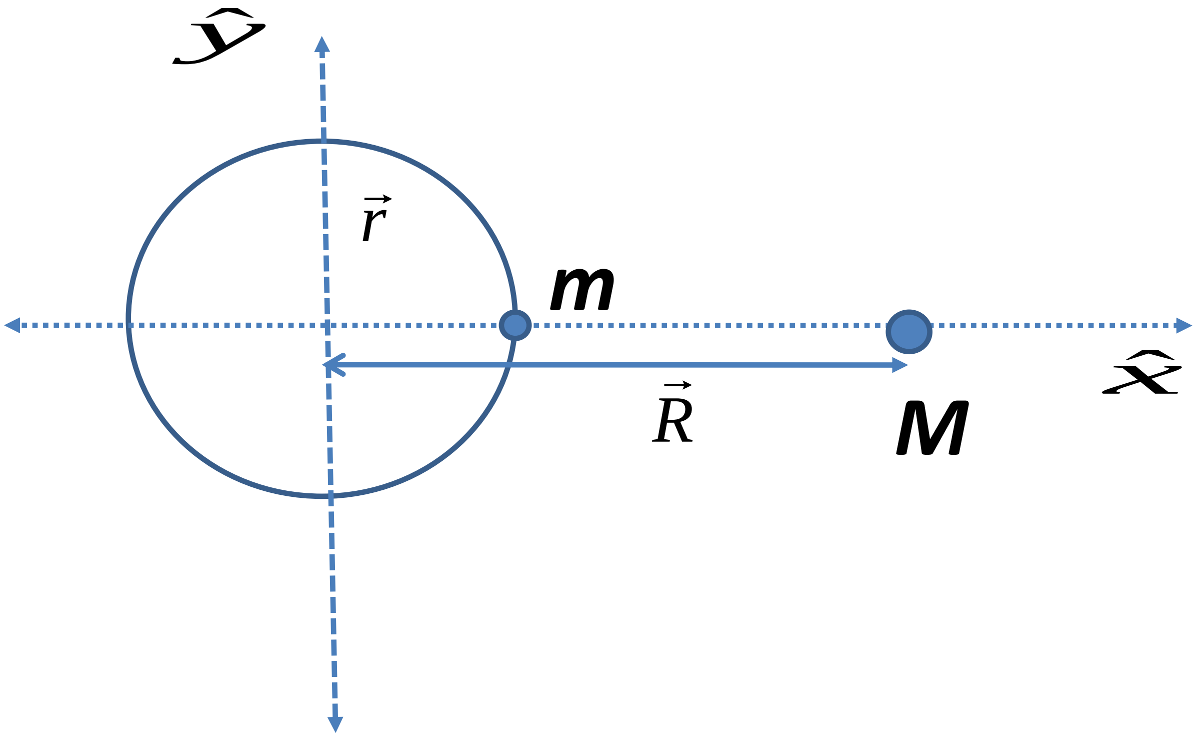


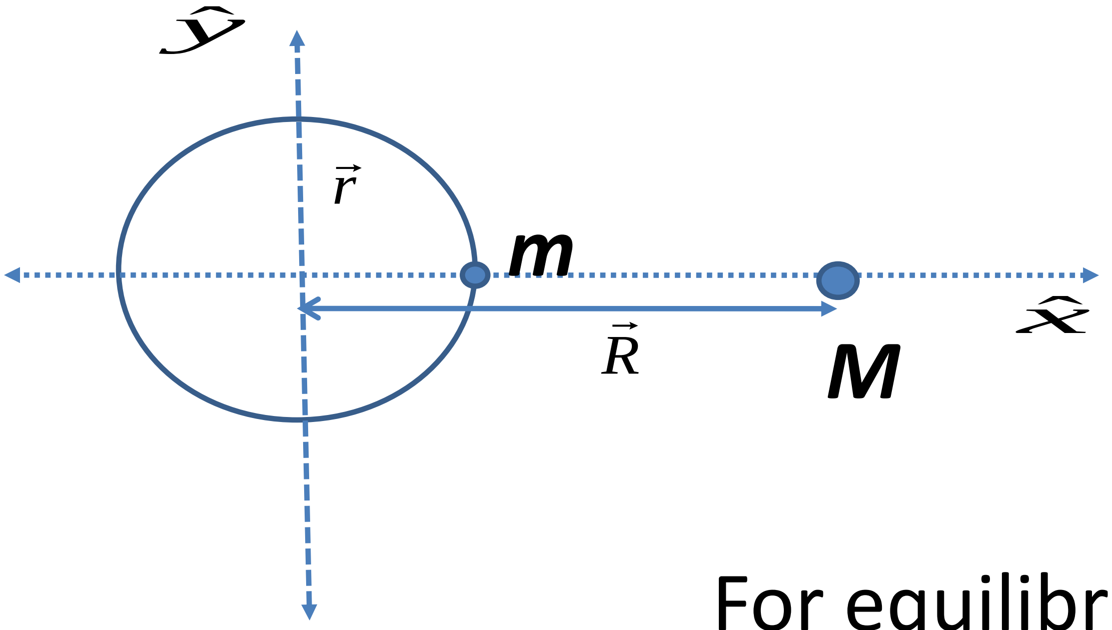
$$U(r,\theta)=\frac{GMmr^2}{2R^3}\left[1-3\cos^2\theta\right]$$

Problem: A bead of mass m is constrained to move on a frictionless hoop of radius r that is Located a distance R from an object of mass M. Assume that R>>r, and assume that M is much Larger than the mass of the hoop, which is much larger than m.



Problem: If the hoop freely falls towards M and the bead starts at a point close to the right most point, What is the frequency of small oscillations? (Assume that you grab M and move it to the right to keep it a Distance R from it.)



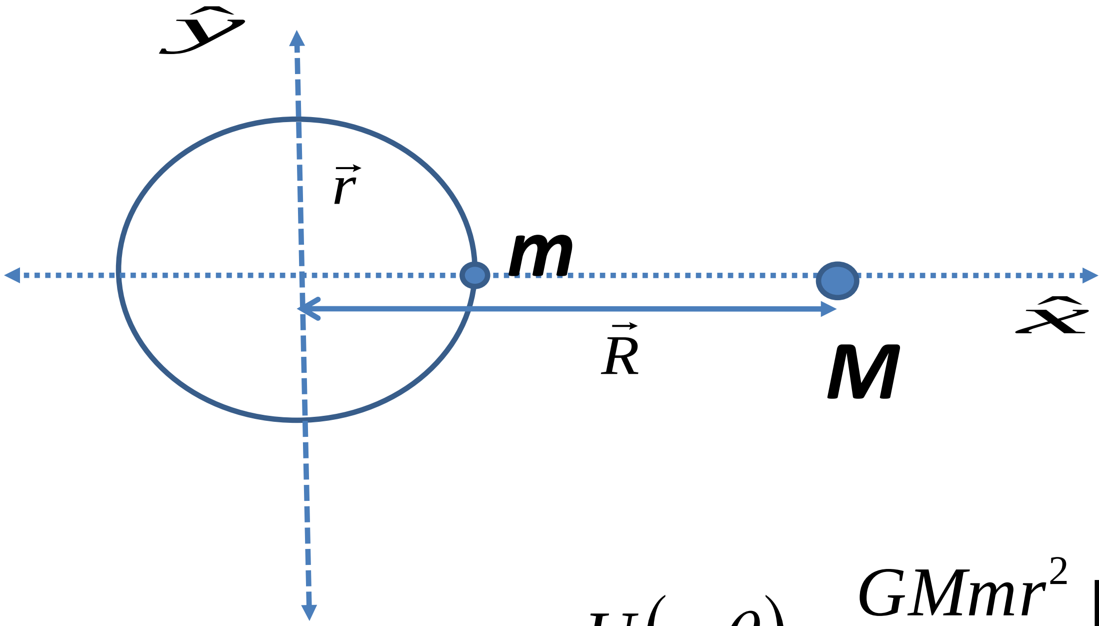


For equilibrium

$$U(r,\theta) = \frac{GMmr^2}{2R^3}\left[1 - 3\cos^2\theta\right]$$

$$\frac{dU}{d\theta} = \frac{GMmr^2}{2R^3}(3\sin 2\theta) = 0$$

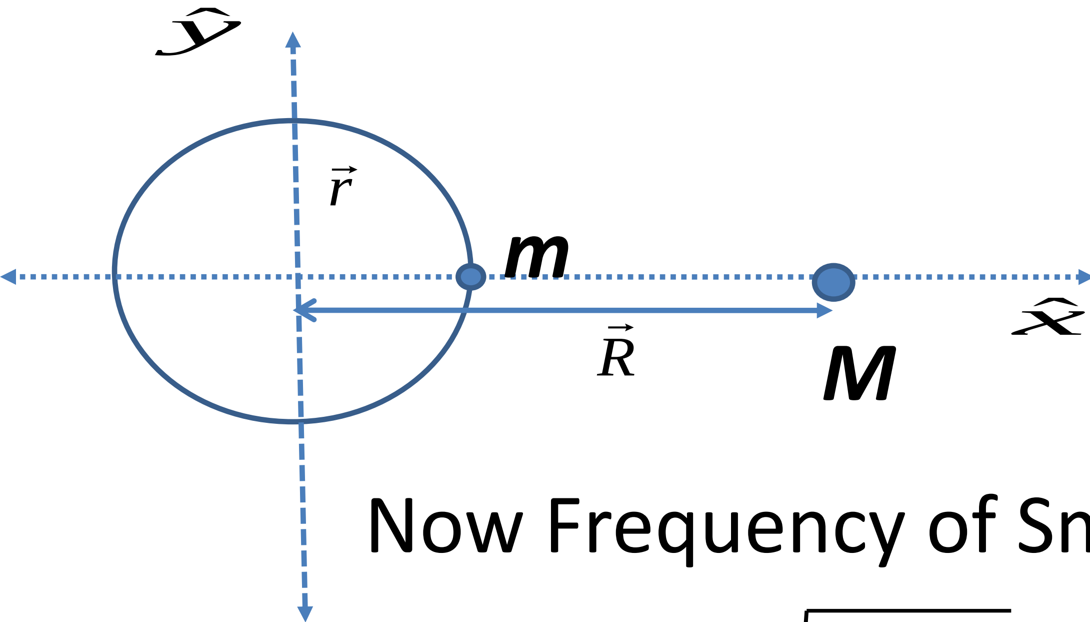
$$\Rightarrow \theta = 0$$



$$U(r,\theta) = \frac{GMmr^2}{2R^3}\left[1 - 3\cos^2\theta\right]$$

$$\frac{d^2U}{d\theta^2} = \frac{GMmr^2}{R^3}(3\cos 2\theta) > 0$$

It is a Stable Equilibrium at ϴ=0

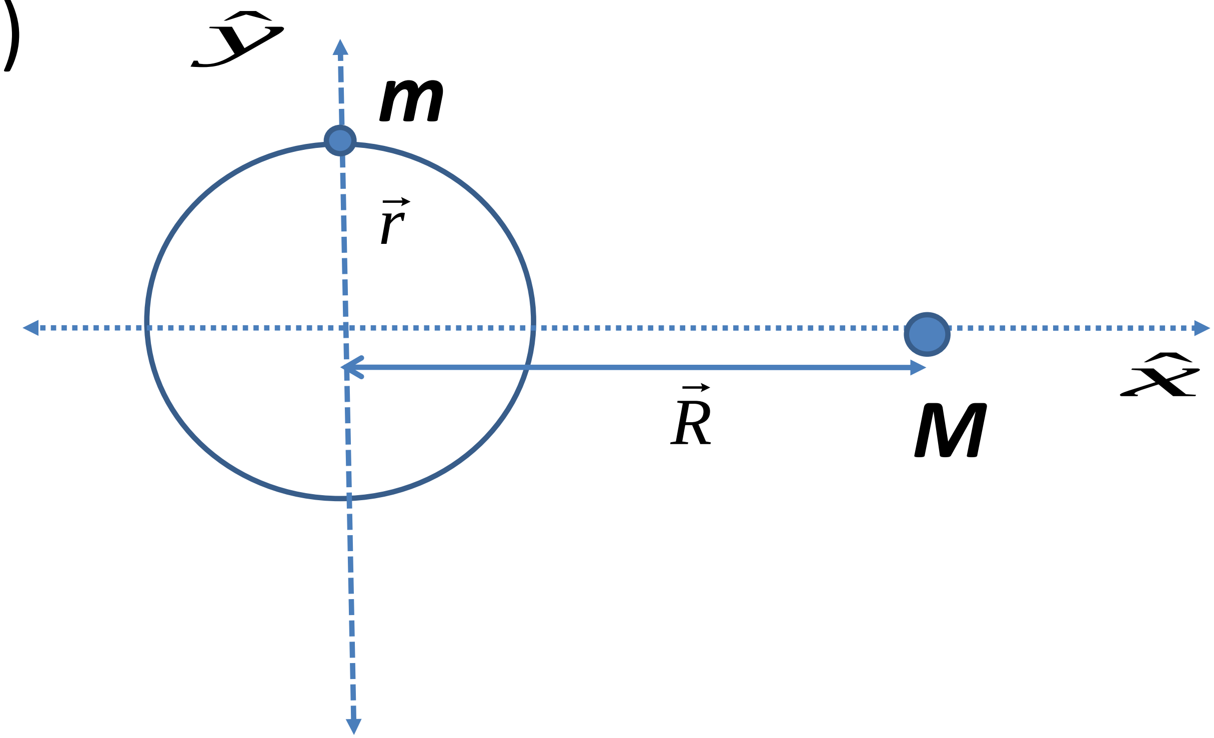


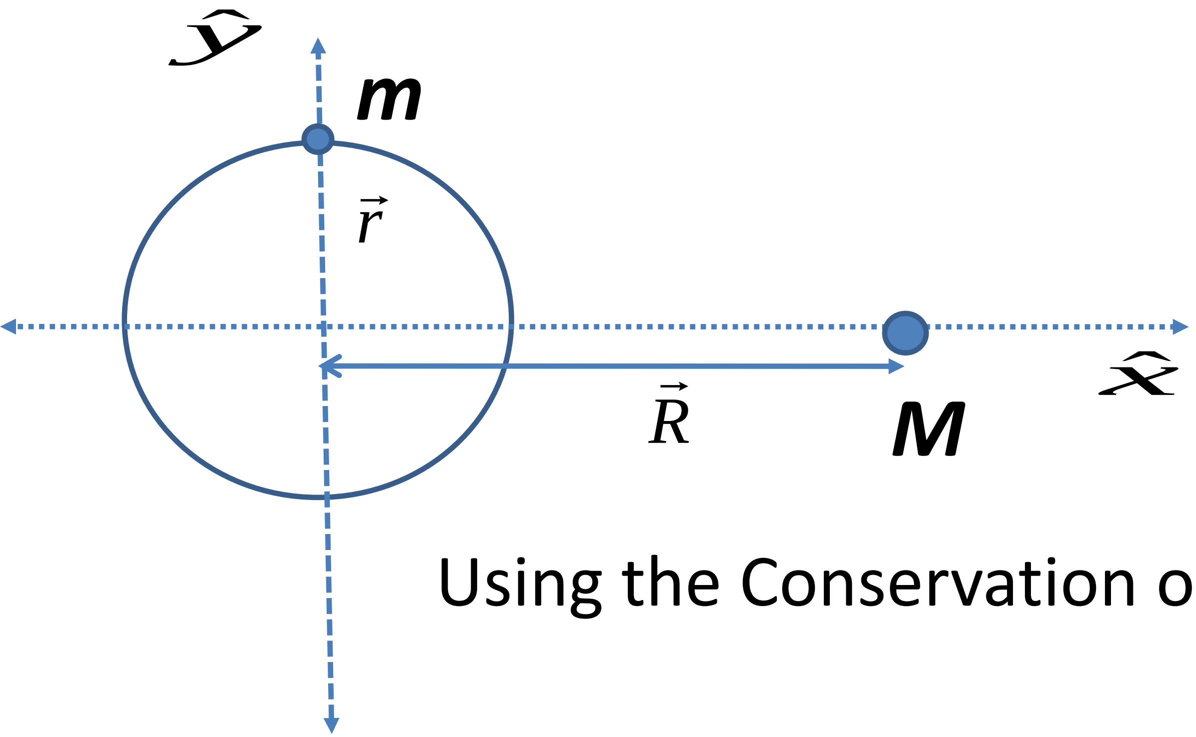
Now Frequency of Small Oscillations will be

$$\omega = \sqrt{\frac{\frac{d^2U}{d\theta^2}}{I}} = \sqrt{\frac{3GM}{R^3}}$$

at Θ=0 . Here, ***I=mr²***

Problem: Now, the bead starts at a point just slightly to the right of the top point, what is its Speed with respect to the hoop when it gets to the Right most point of the hoop.?(Assume that you grab M and move it to the right to keep it a Distance R from it.)





Using the Conservation of Mechanical Energy:

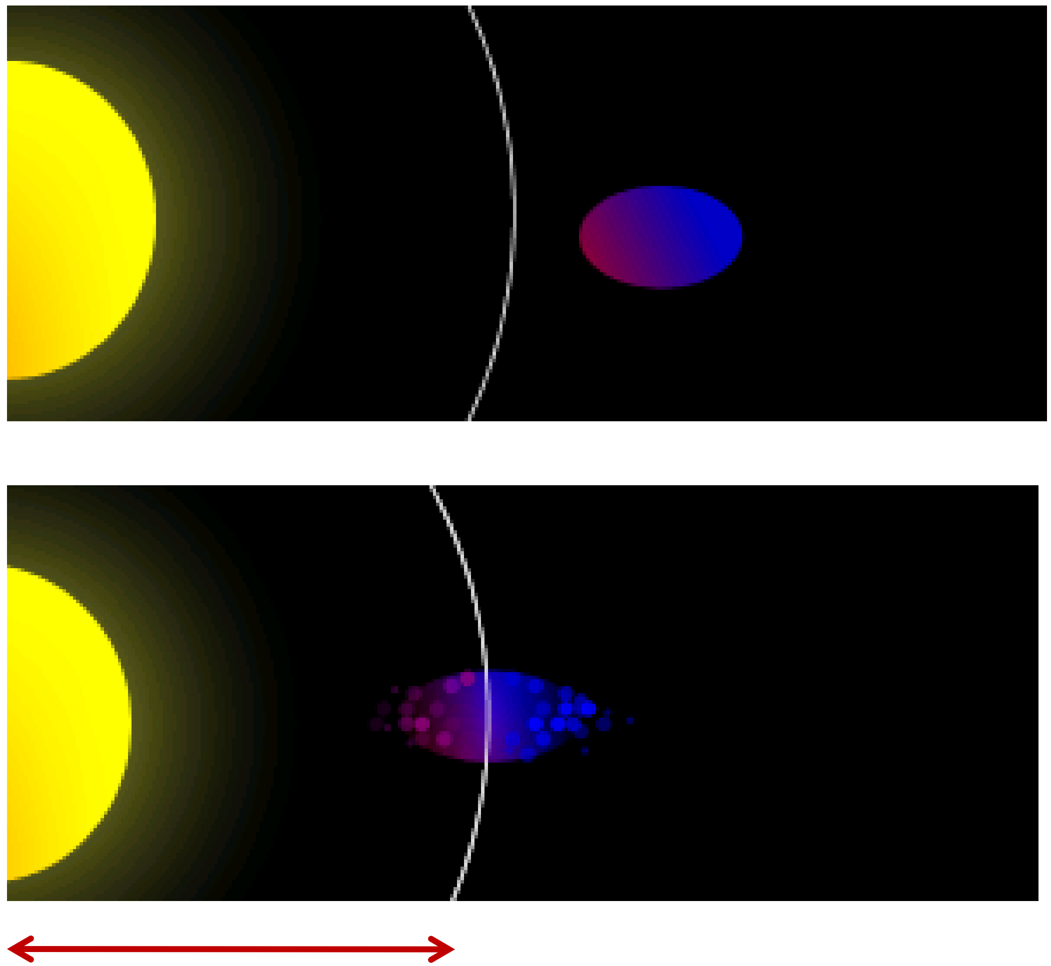
$$\frac{GMmr^2}{2R^3}\left(1-3\cos 90^0\right)=\frac{GMmr^2}{2R^3}\left(1-3\cos 0^0\right)+\frac{1}{2}mv_o^2$$

$$v_0=\sqrt{\frac{3GMr^2}{R^3}}$$

Problem: Roche Limit: A small spherical rock covered with sand falls in radially toward a planet. Let the planet have radius R and density ρ_P , and let the rock have density ρ_r . It turns out that when the rock gets close enough to the planet, the tidal force ripping the sand off the rock will be larger than the gravitational force attracting the sand of the rock. The cutoff distance is called the Roche Limit. Show that it is given by

$$d = R\left(\frac{2\rho_P}{\rho_r}\right)^{\frac{1}{3}}$$

Roche Limit

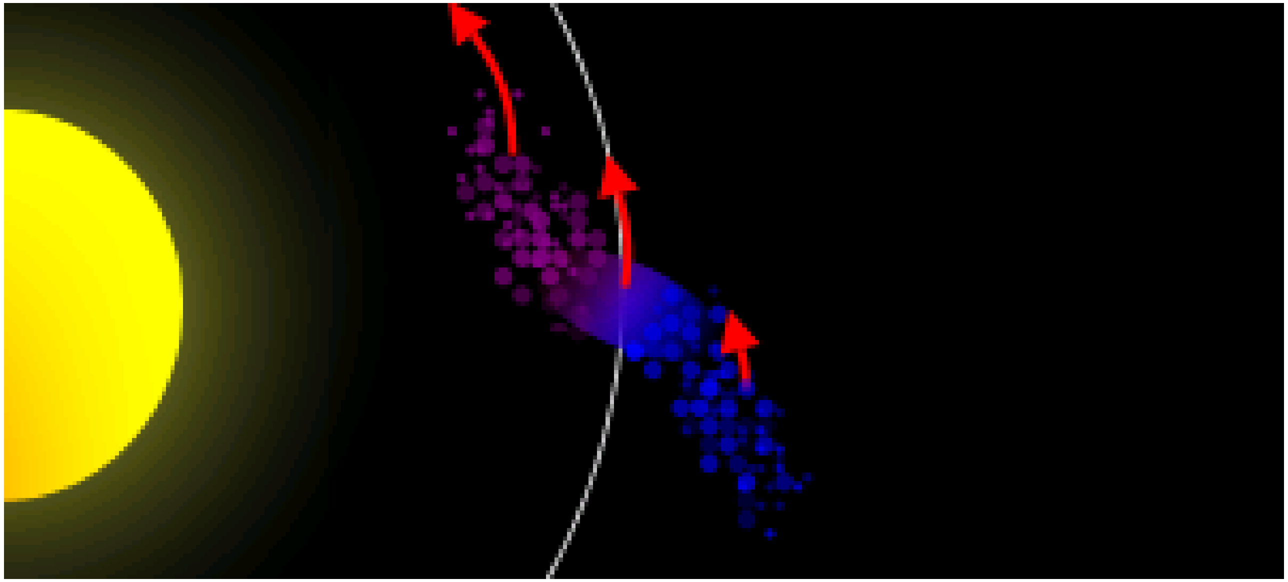


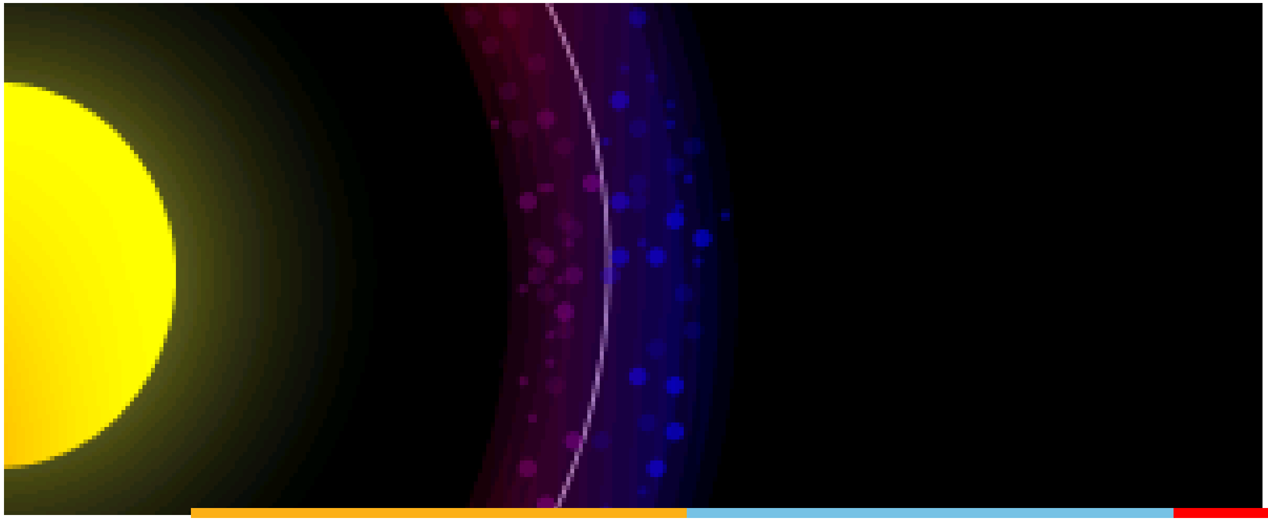
Roche Limit

If the tidal gravity of the primary over a secondary becomes stronger than the self gravity of the secondary, the secondary is torn apart.

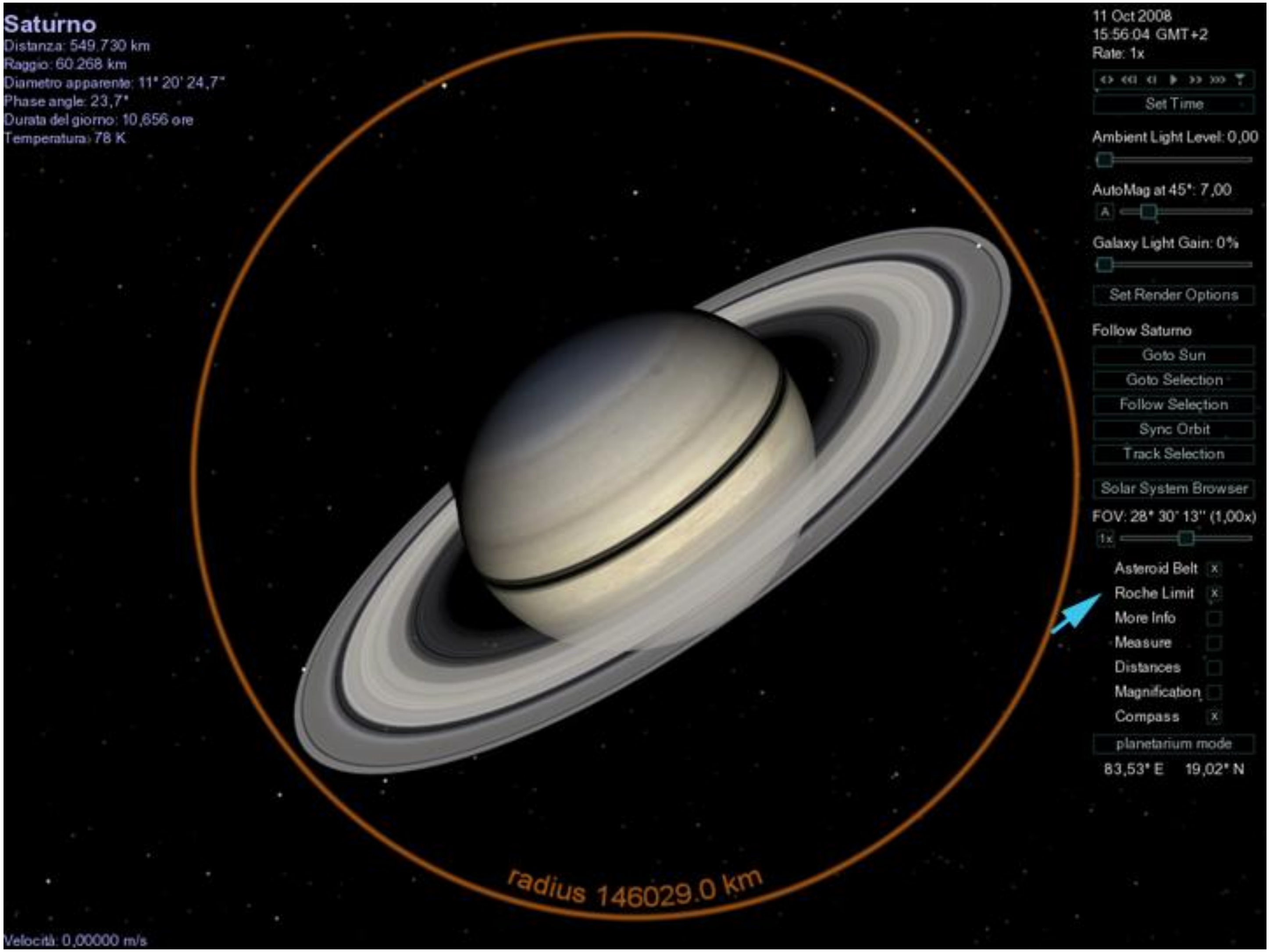
Inside the Roche Limit, no object can be held together by its gravitational attraction alone

Formation of Rings of Planets





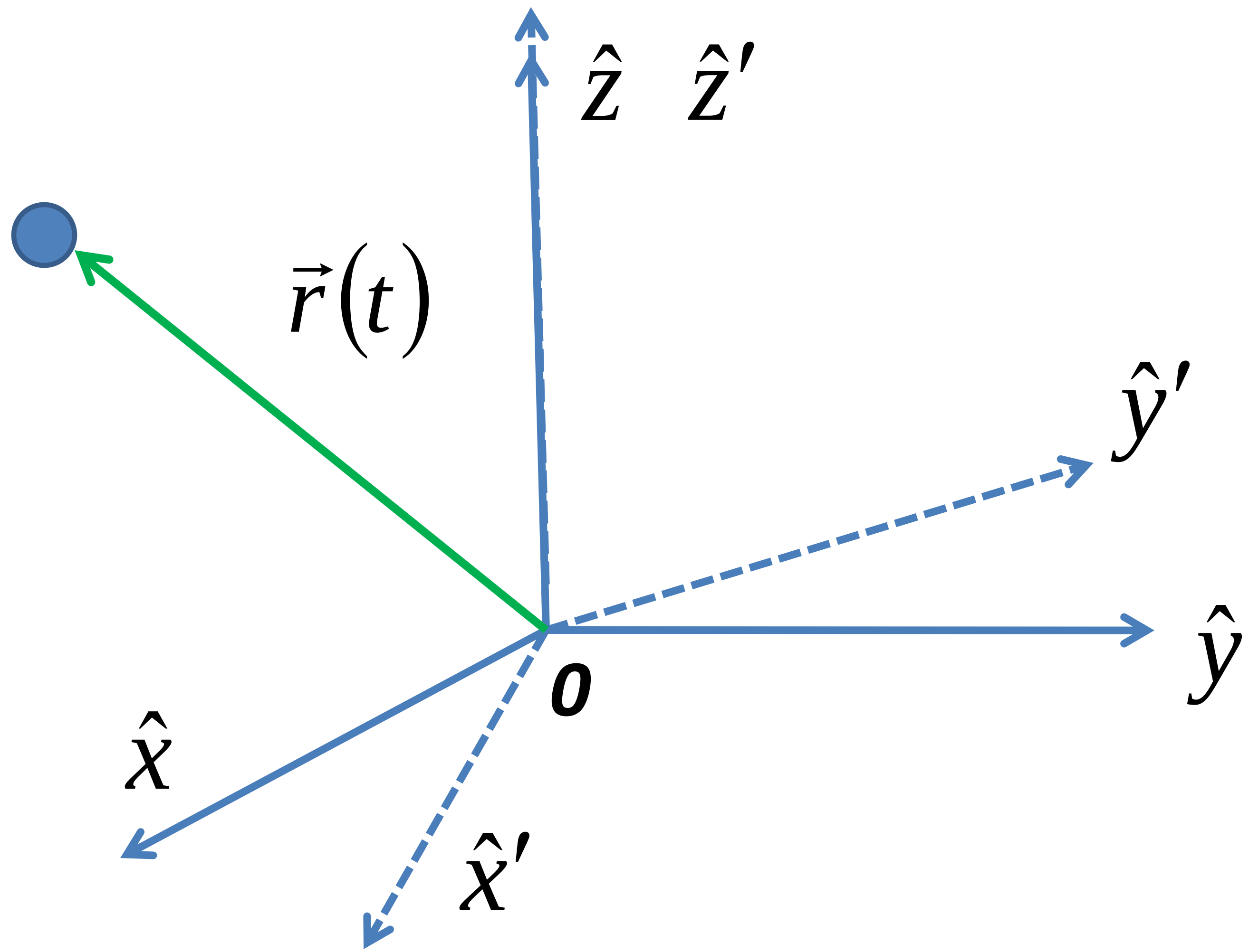
Roche Limit for Saturn



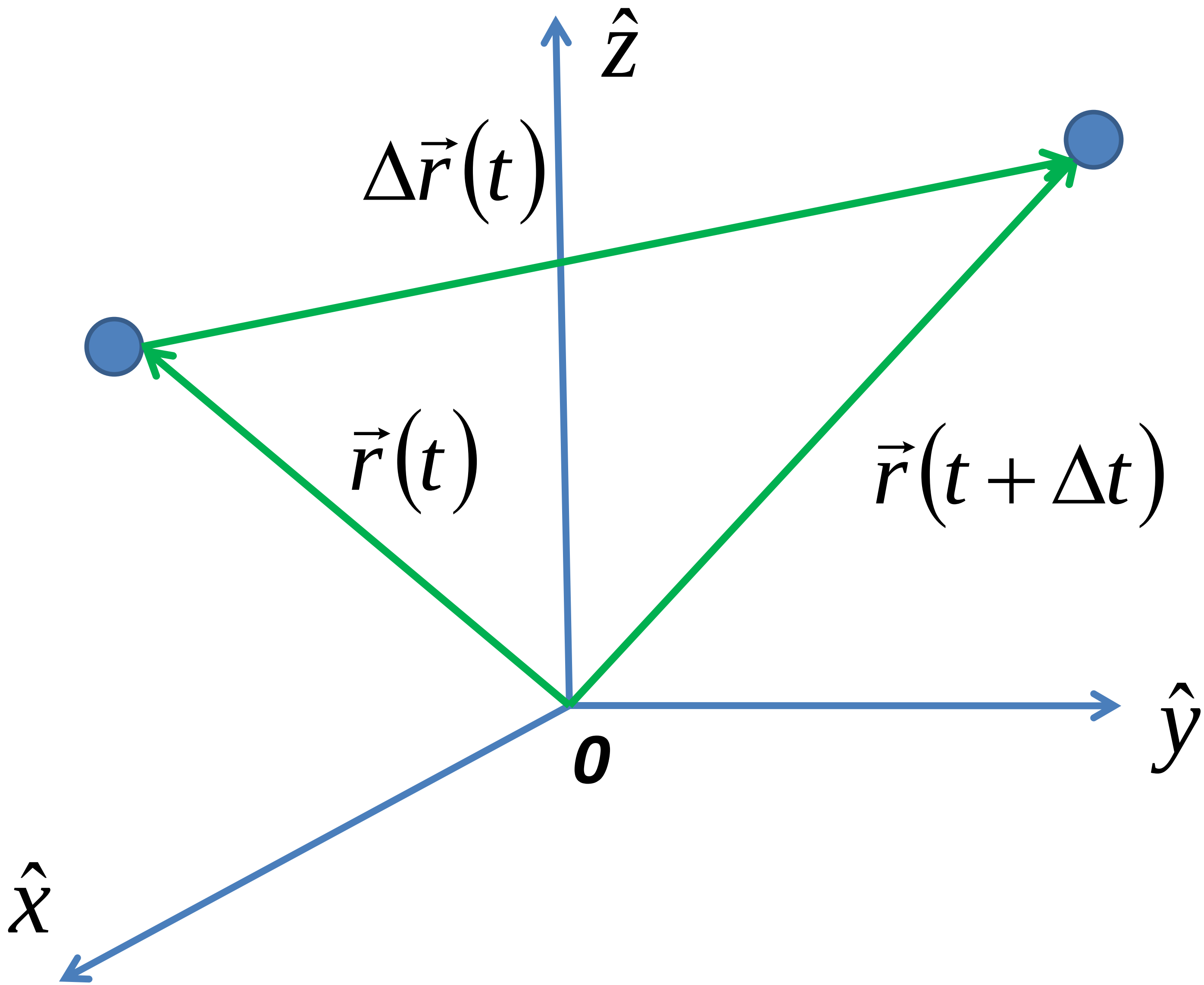
Problem: Roche Limit: If the object orbits the planet in such a way that the same side always faces the planet, then the Roche Limit is given by:

$$d = R\left(\frac{3\rho_P}{\rho_r}\right)^{\frac{1}{3}}$$

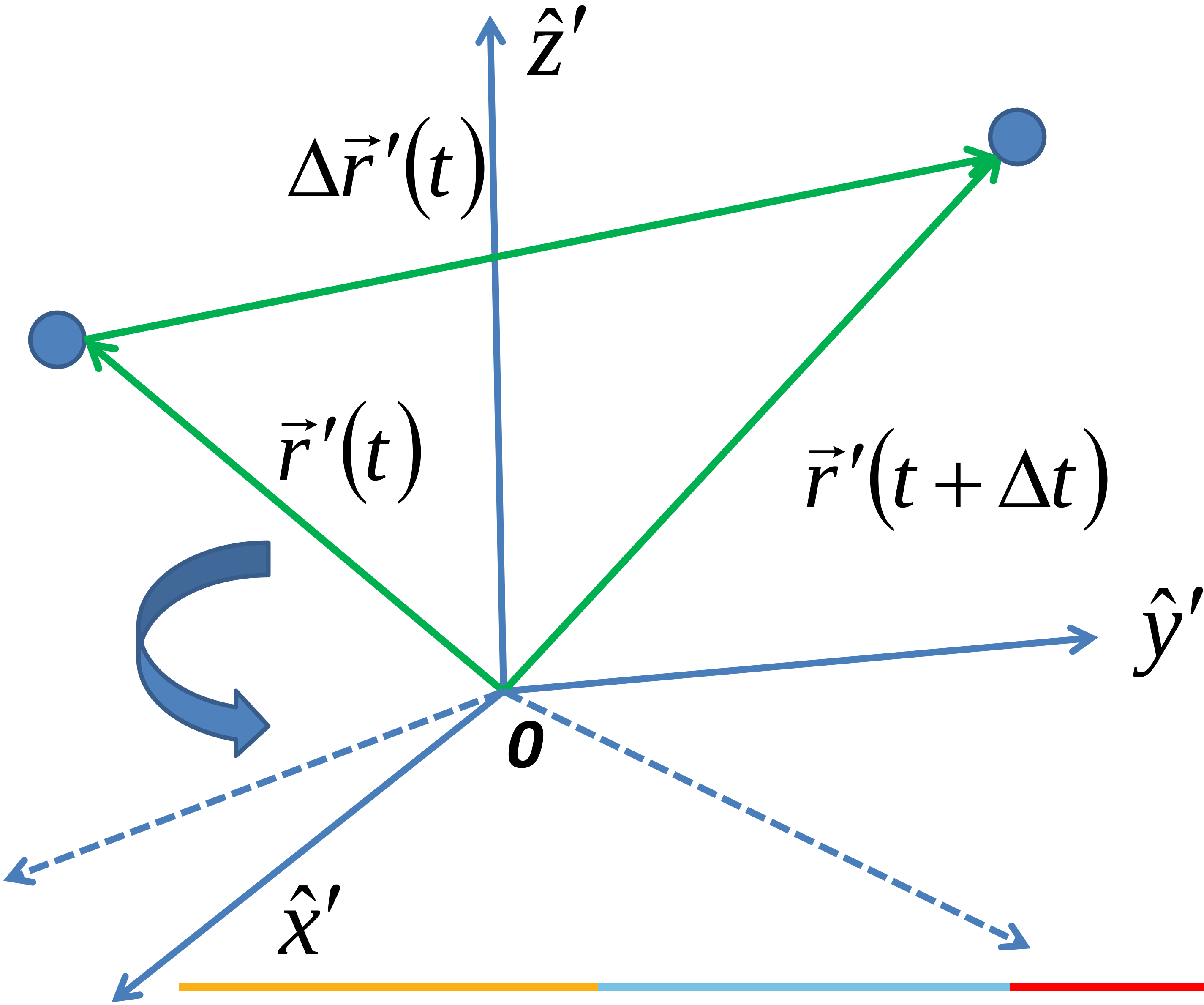
Physics in a Rotating Frame: Time Derivatives & Rotating Coordinates



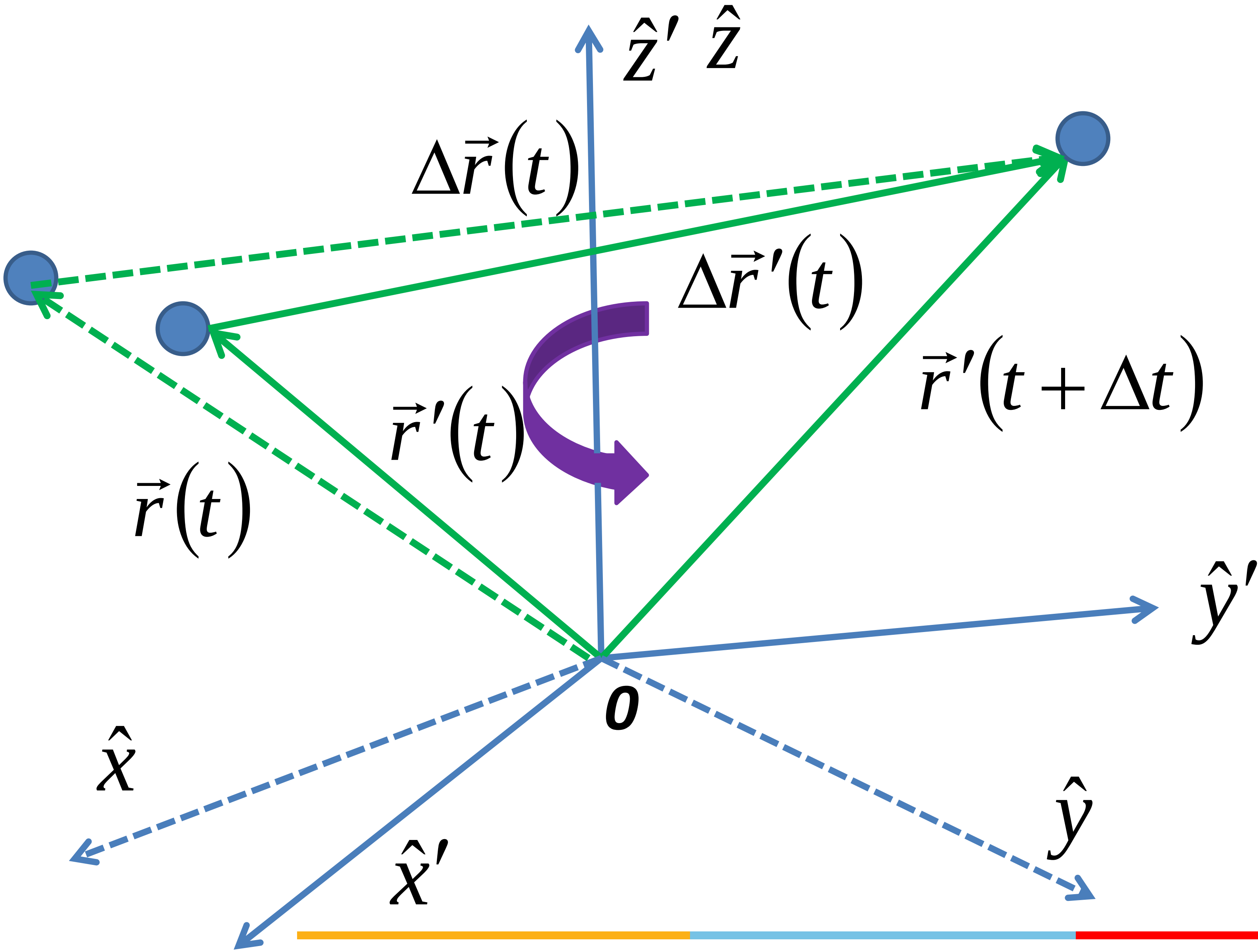
Fixed Frame

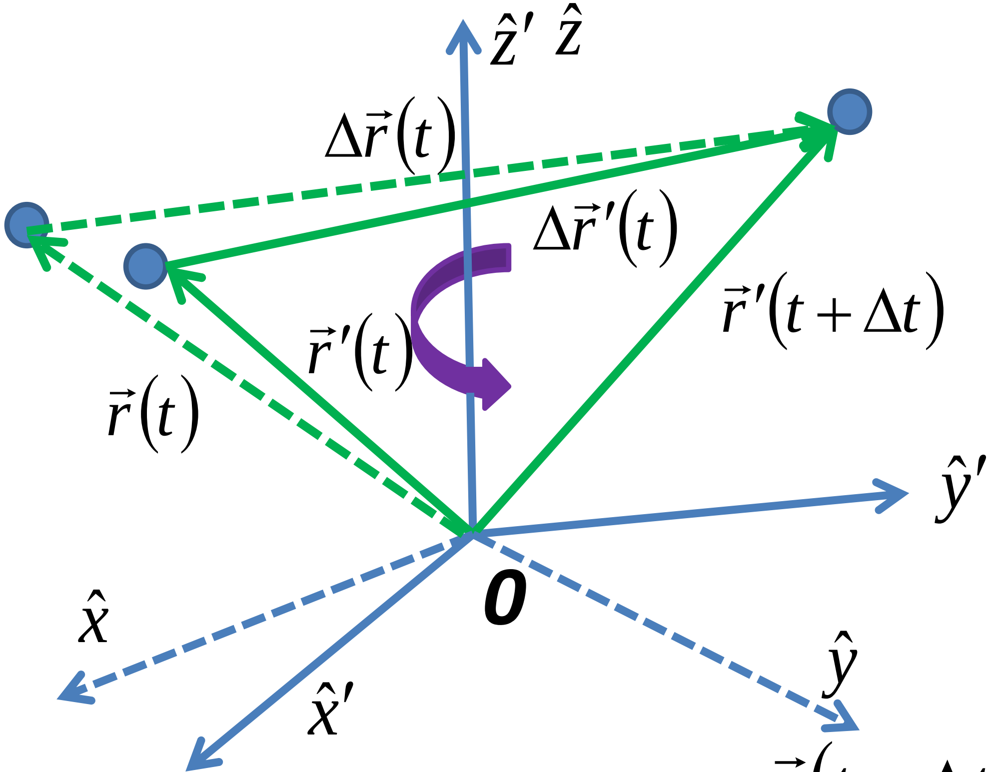


Rotating Frame



Fixed Frame vs Rotating Frame





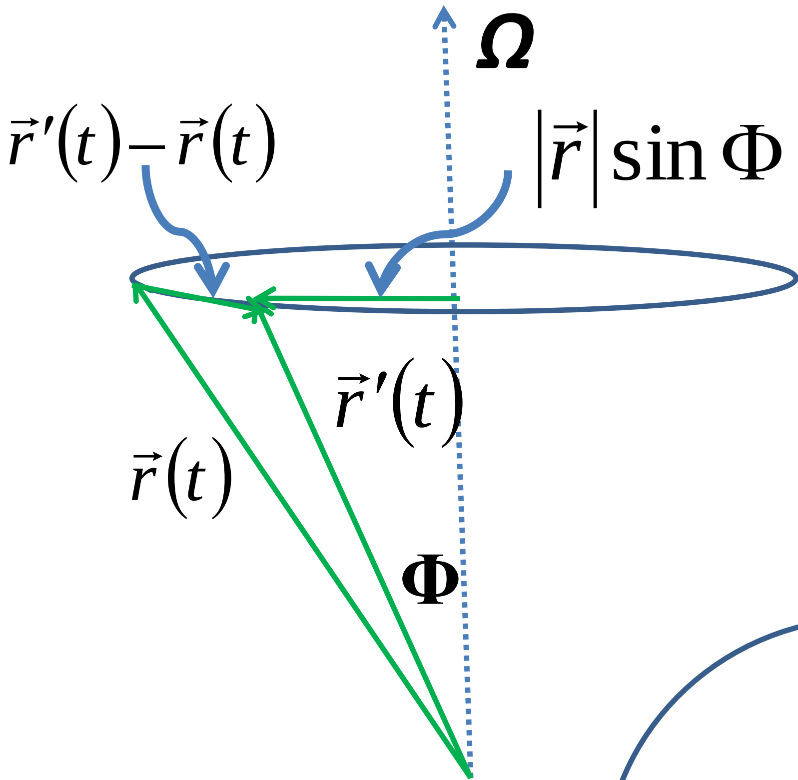
$$\vec{r}(t+\Delta t)=\vec{r}'(t)+\Delta\vec{r}'$$

$$\vec{r}(t+\Delta t)=\vec{r}(t)+\Delta\vec{r}$$

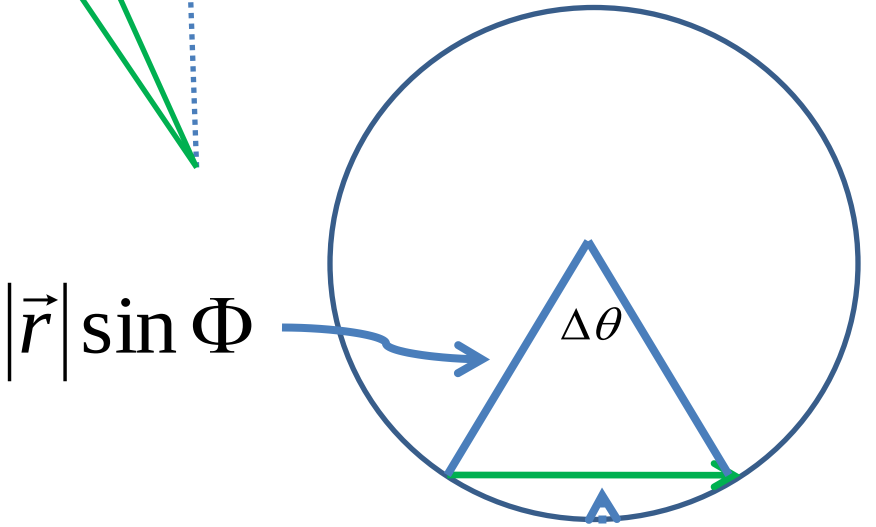
$$\vec{r}(t+\Delta t)=\vec{r}(t)+\Delta\vec{r}=\vec{r}'(t)+\Delta\vec{r}'$$

$$\vec{r}'(t)-\vec{r}(t)=\Delta\vec{r}-\Delta\vec{r}'$$

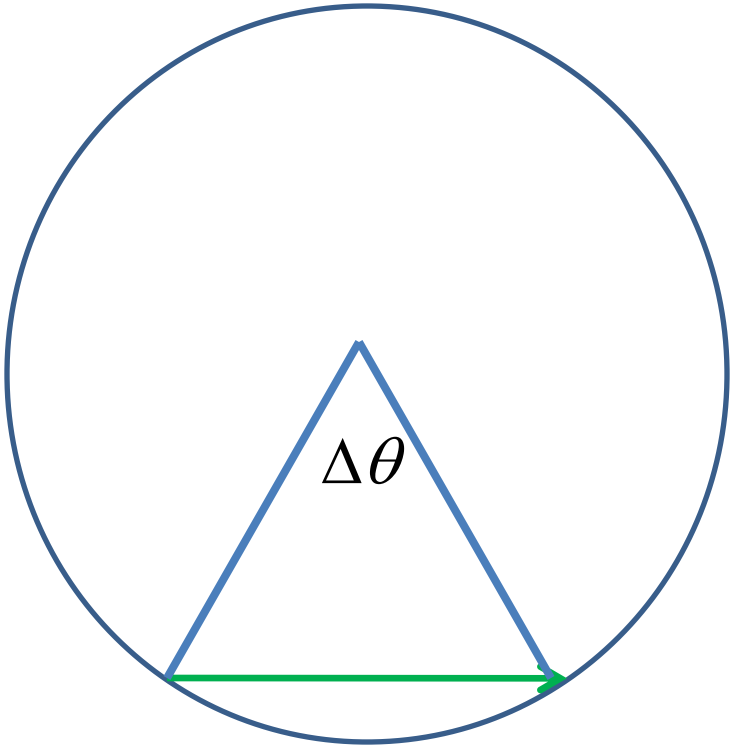
$$\Delta\vec{r}=\Delta\vec{r}'+\vec{r}'(t)-\vec{r}(t)$$



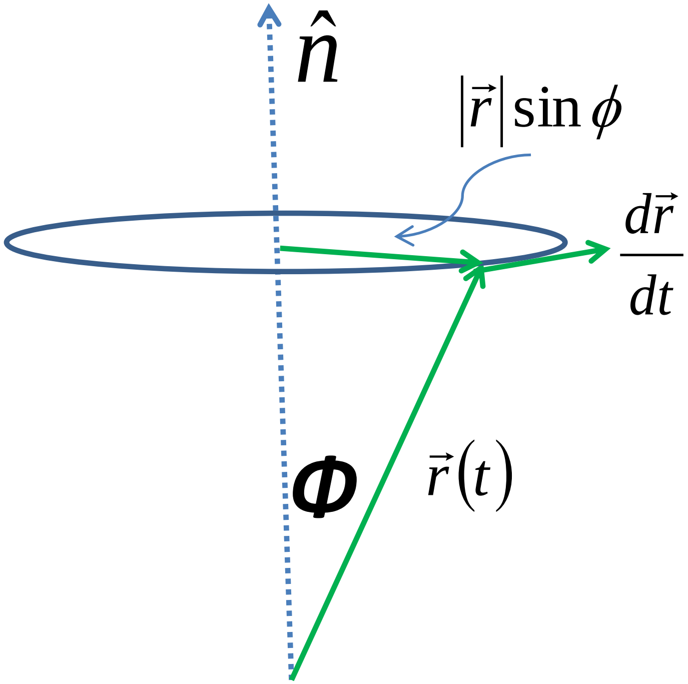
$$\Delta\vec{r} = \Delta\vec{r}\,' + \vec{r}\,'(t) - \vec{r}(t)$$



$$\left|\vec{r}\,'(t)-\vec{r}(t)\right| \approx r\sin\Phi\,\Delta\theta$$



$$Lt_{\Delta t\to 0}\left|\frac{\vec{r}'(t)-\vec{r}(t)}{\Delta t}\right| = \frac{r\sin\phi\Delta\theta}{\Delta t} = r\sin\Phi\,\Omega$$



$$Lt_{\Delta t\to 0}\left|\frac{\Delta r}{\Delta t}\right| = \frac{r\sin\phi\Delta\theta}{\Delta t} = r\sin\phi\Omega$$

$$\frac{d\vec{r}}{dt} = \Omega\times\vec{r}$$

$$\Delta\vec{r} = \Delta\vec{r}\,' + \vec{r}\,'(t) - \vec{r}(t)$$

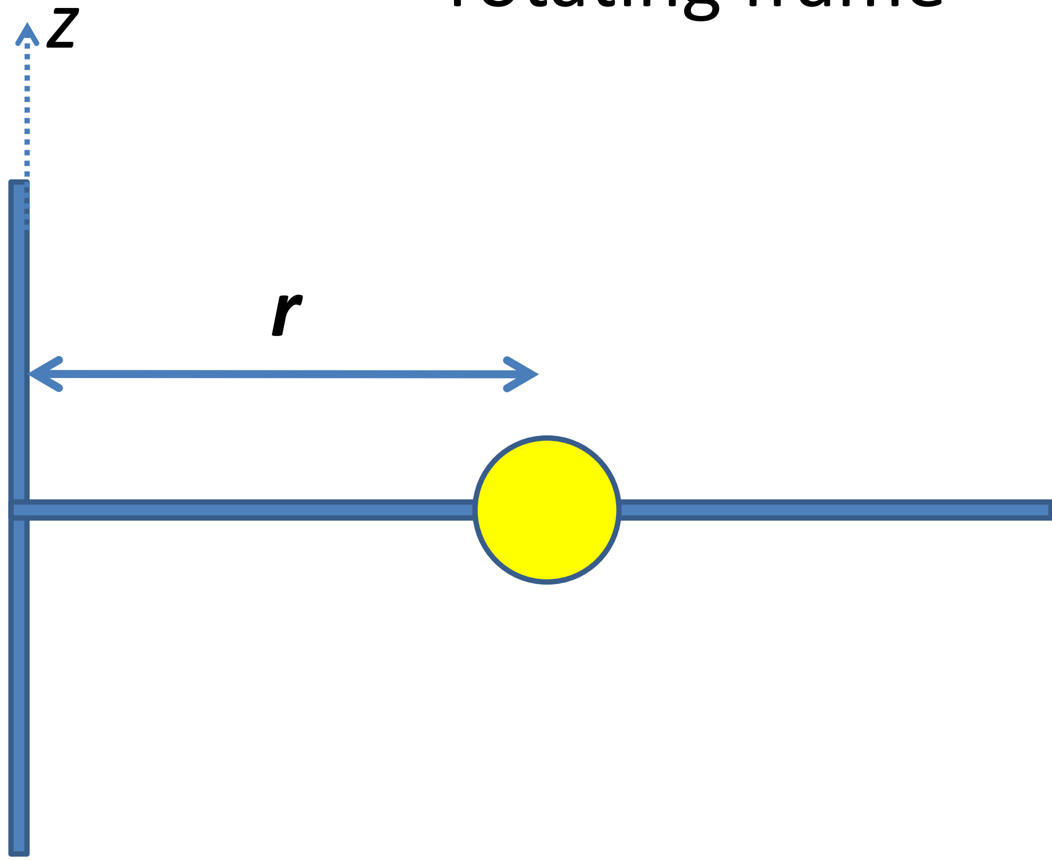
$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \Omega \times \vec{r}$$

$$\vec{v}_{Inertial} = \vec{v}_{Rot.} + \Omega \times \vec{r}$$

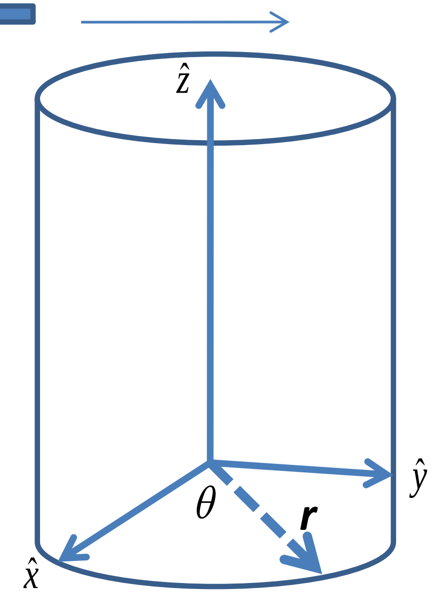
$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \left(\Omega \times \vec{r}\right)$$

$$\left(\frac{d}{dt}\right)_{Inertial} = \left(\frac{d}{dt}\right)_{Rotating} + \left(\Omega \times\right)$$

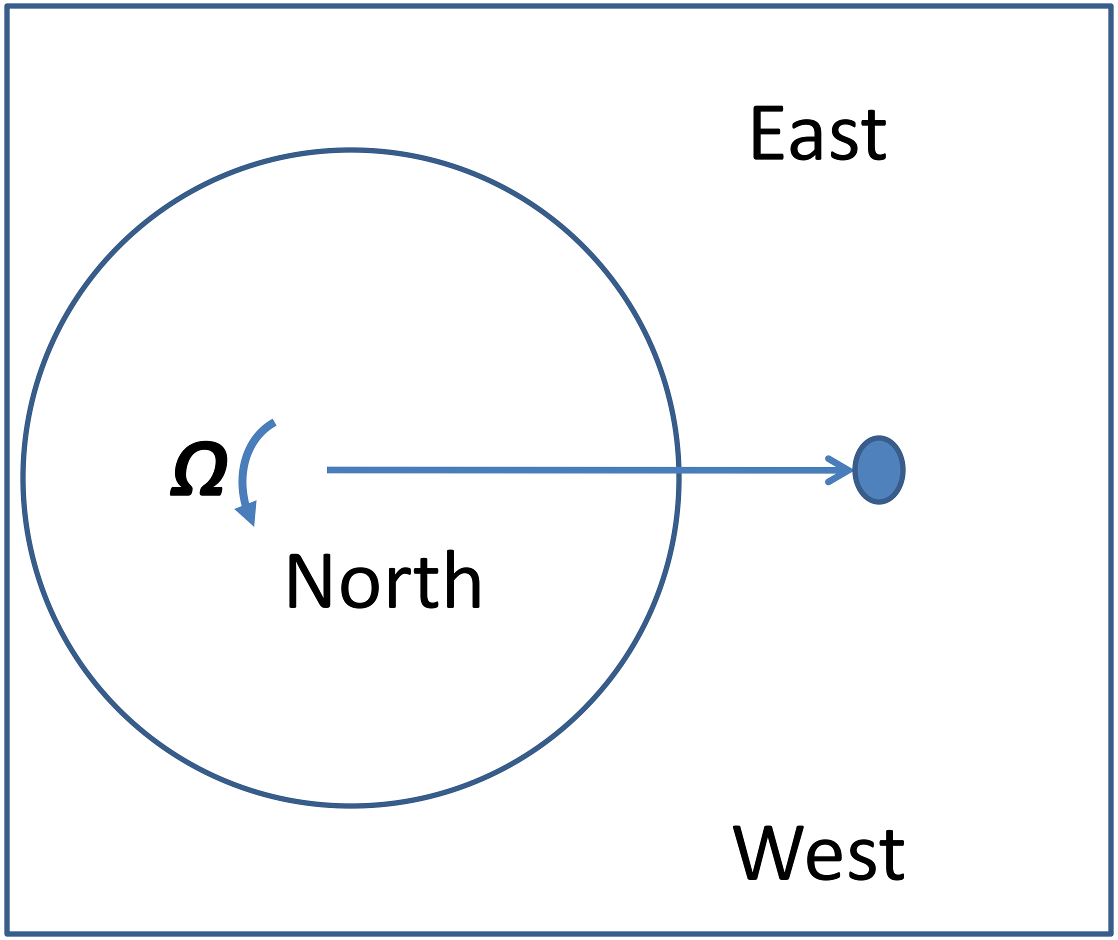
Problem: Solve the bead-rod problem in the rotating frame

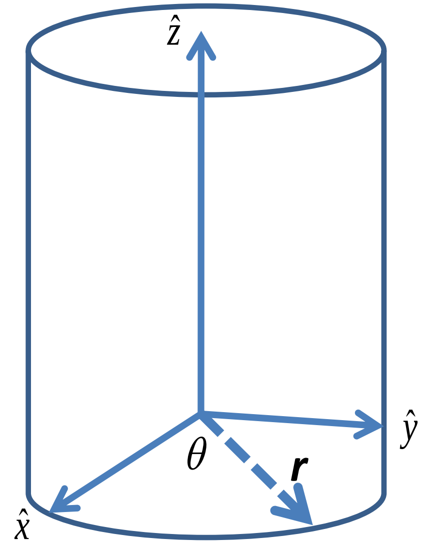


Cylindrical Coordinate Systems



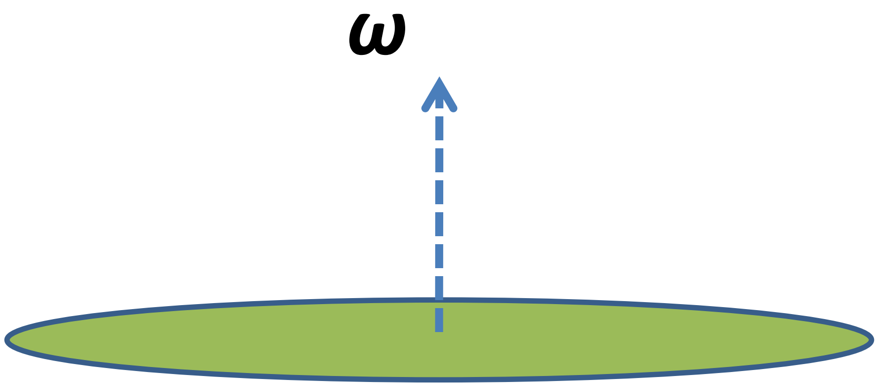
Problem: Deflection of a Falling Mass Dropped from a Tower of height h at Equator.



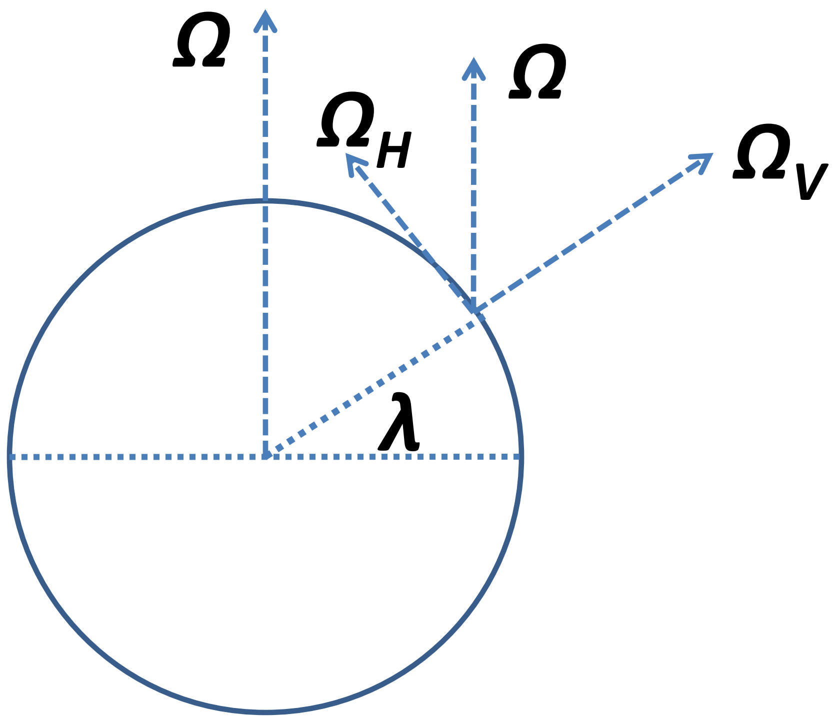


Cylindrical Coordinate Systems

Problem: Man Standing on a Rotating Disc



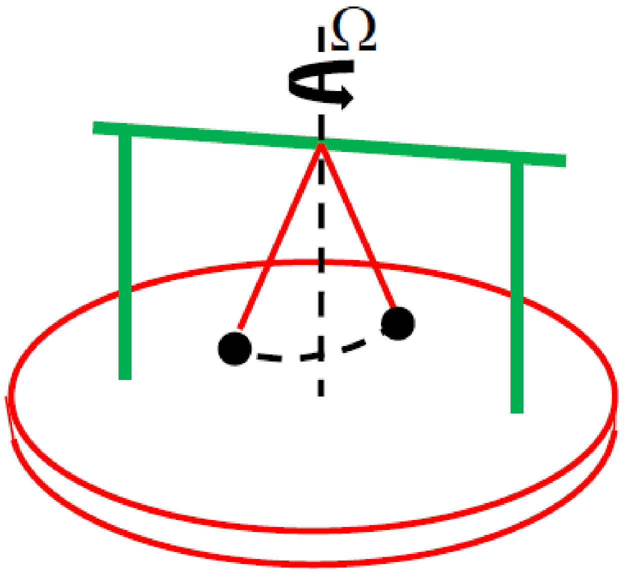
Problem: Motion on the Rotating Earth



Problem 8.10 : The acceleration due to gravity measured in an earthbound coordinate system is denoted by $\boldsymbol{g}$. However, because of the earth's rotation, $\boldsymbol{g}$ differs from the true acceleration due to the acceleration due to gravity, $\boldsymbol{g_0}$. Assuming that the earth is perfectly round, with radius $\boldsymbol{R_e}$ and angular velocity Ω_e , find $\boldsymbol{g}$ as a function of latitude λ.

Problem 8.7 : Find the difference in the apparent acceleration of gravity at the equator and the poles, assuming that the earth is spherical.

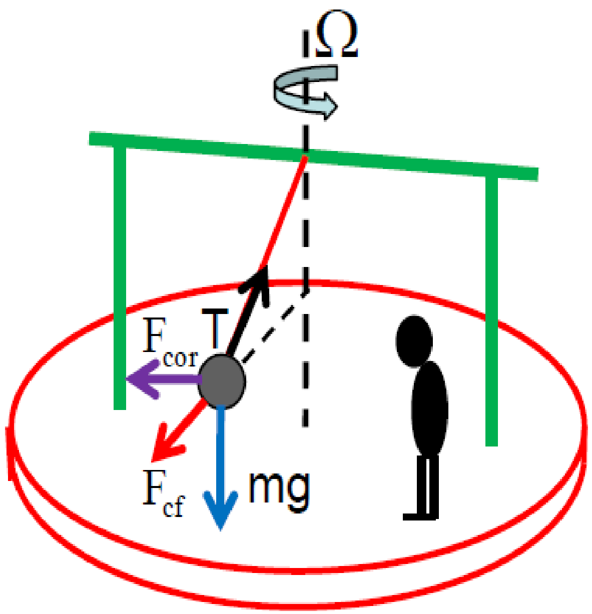
Prob. 8.12 : A pendulum is fixed on a revolving platform as shown. It can swing only in a plane perpendicular to the horizontal axle. Find the frequency of the pendulum



M : Mass of pendulum

L : Length of massless rod

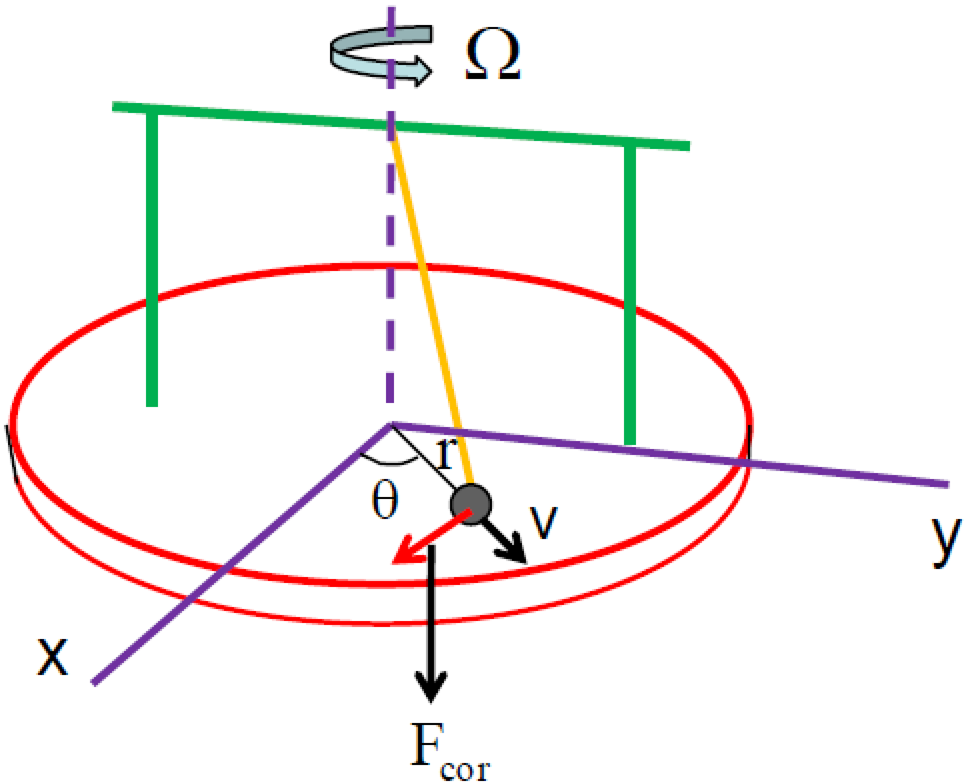
Ω : Const. Ang. Vel. of platform



In the frame of the rotating platform all the forces acting on the pendulum are shown.

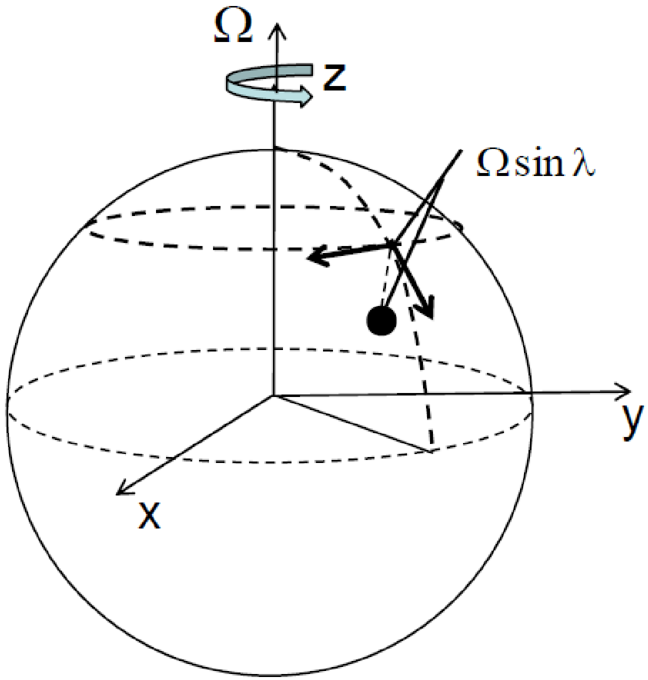
Foucault Pendulum

Foucault pendulum on a rotating platform

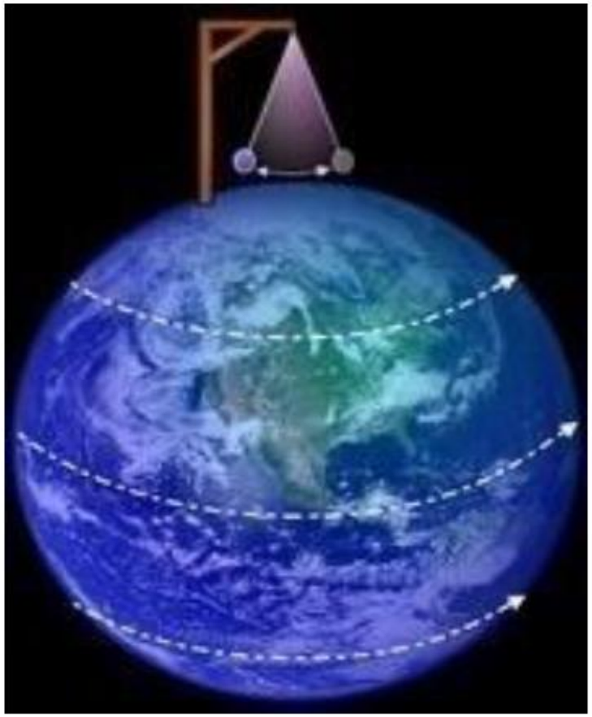


Foucault Pendulum on Rotating Earth

On the surface of the earth, at latitude λ, the rate of rotation is : $\Omega \sin \lambda$, where Ω is earth's angular velocity. The rotation is clockwise.



At the north pole, the plane of the pendulum, rotates once a day.



Prob. 8.8 : Derive the familiar expression for velocity in plane polar coordinates :

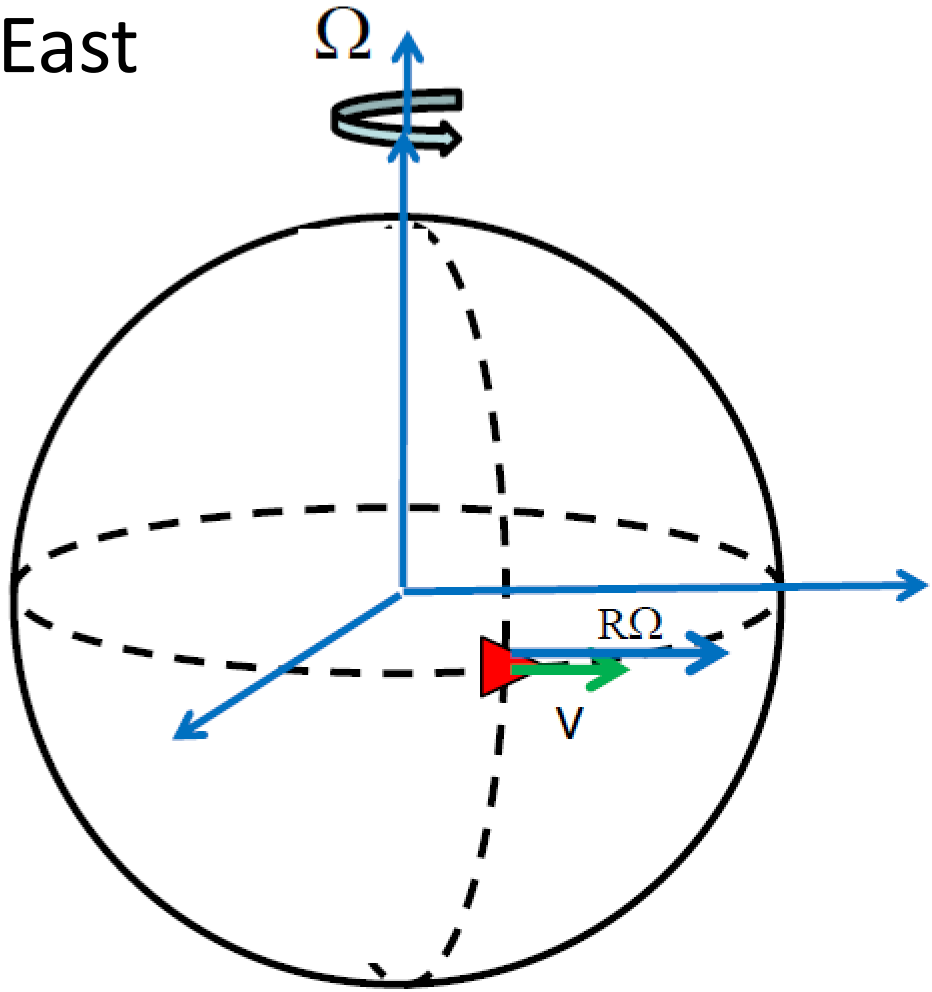
$$\vec{v} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$$

by examining the motion in a coordinate system in which the instantaneous velocity is in the radial direction.

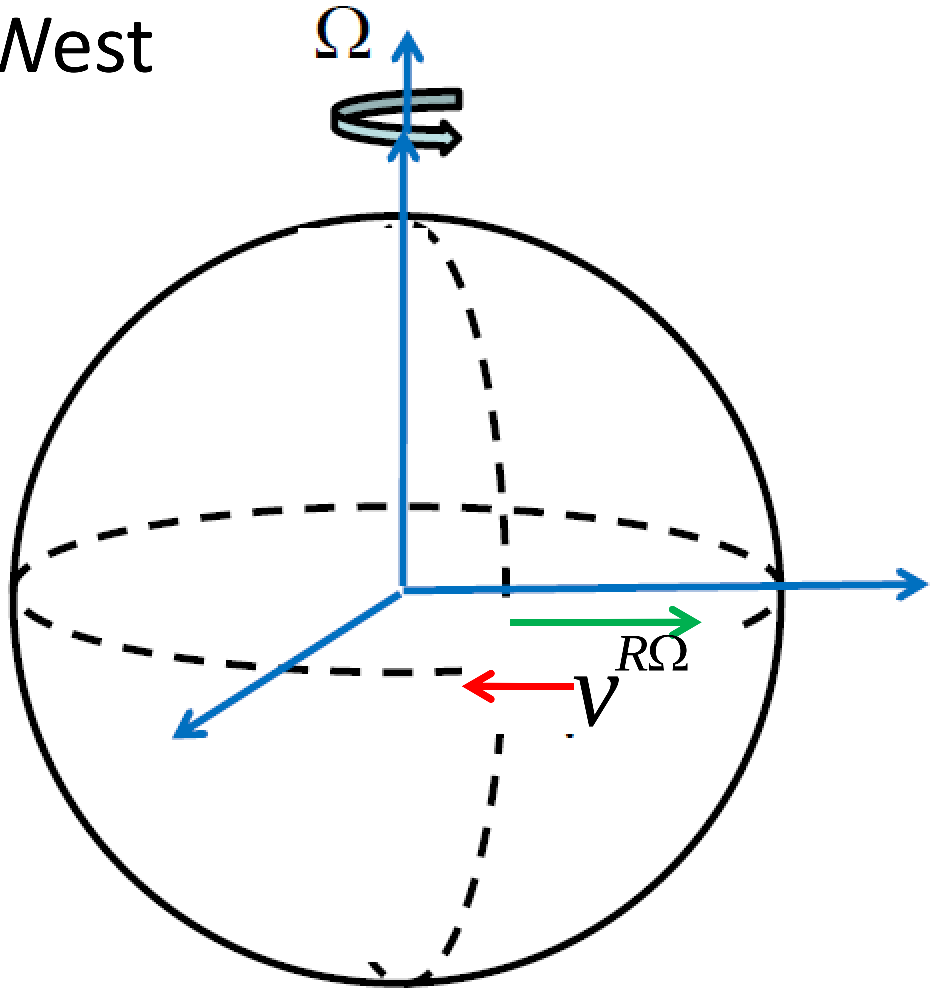
Problem: 8.12 A high speed hydrofoil races across the ocean at the equator at a speed of 200 mi/h. Let the acceleration of gravity for an observer at rest on the earth be g. Find the fractional change in gravity , $\Delta g/g$, measured by a passenger on the hydrofoil when the hydrofoil heads in the following directions:

a) East
b) West
c) South
d) North

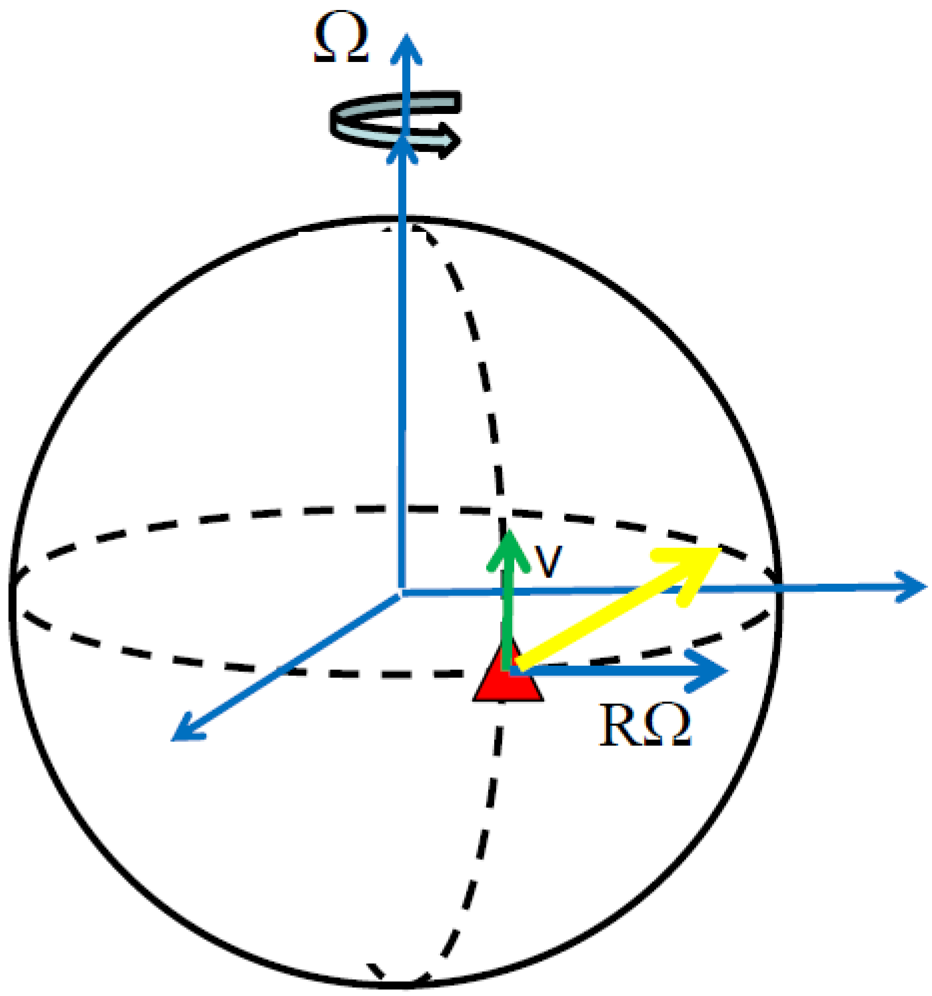
a) Due East



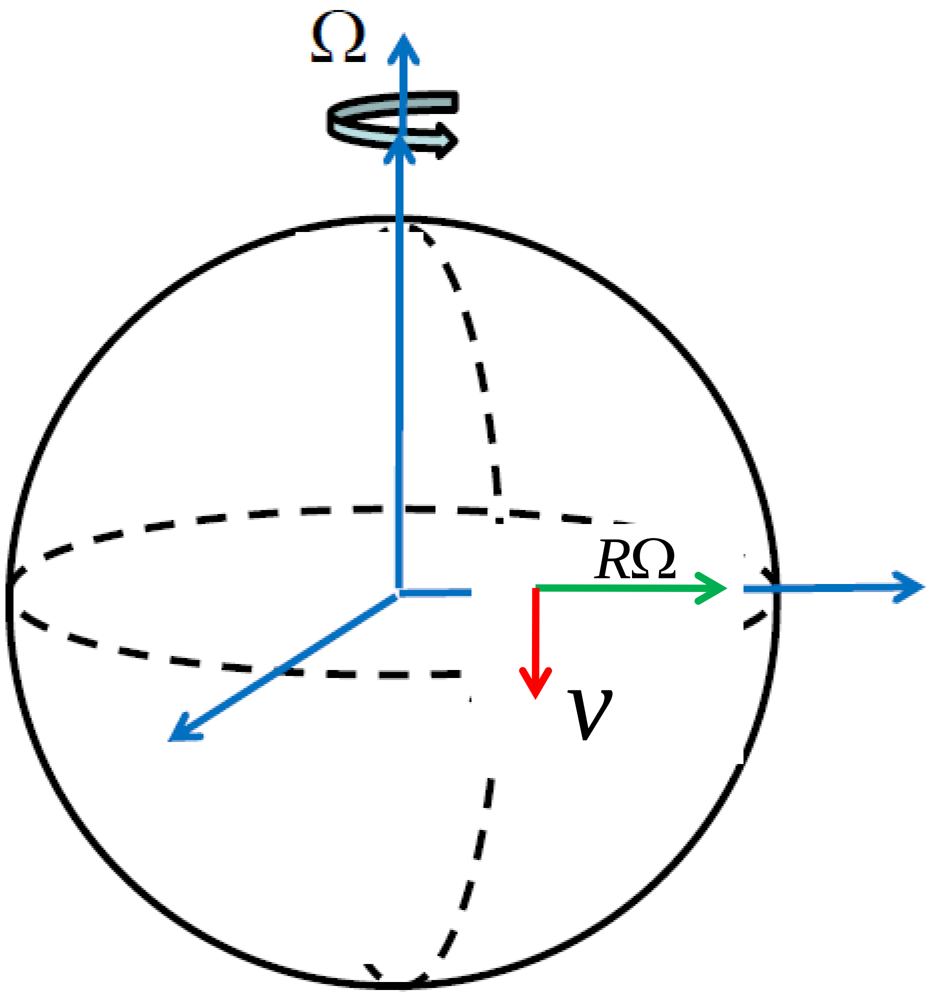
b) Due West



c) Due North



c) Due South

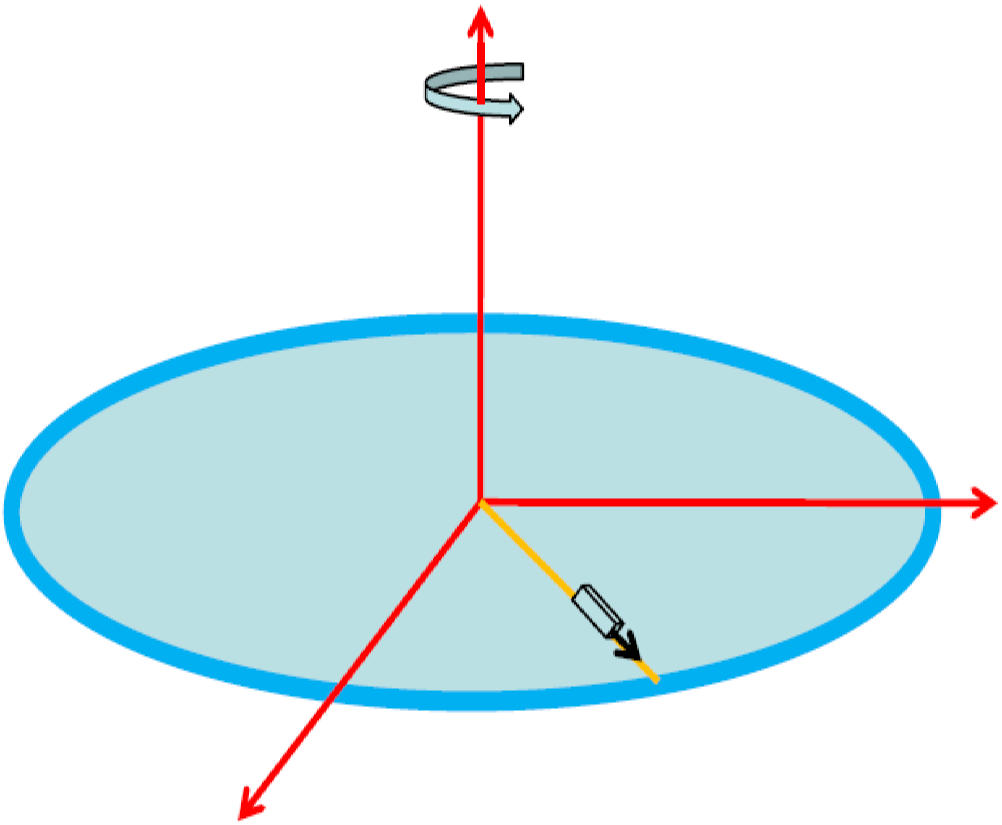


Problem 2.9 (Modified): A car is driven on a large revolving platform which rotates with constant angular speed ω. At $t=0$,a driver leaves the origin and follows a line painted radially outward on the platform with constant speed v_0. The total weight of the car is W, and the coefficient of friction between the car and stage is μ.

(a) Find the acceleration of the car in the rotating Frame.

(b) Find the time at which the car just starts to slide.

(c) Find the direction of the friction force with respect to the instantaneous position vector ***r*** just before the car starts to slide.



Direction of the friction force with respect to the instantaneous position vector ***r*** just before the car starts to slide.