Q.3

3) $L: R^4 \longrightarrow R^3$ defined by

$$L([x, y, z, t]) = [x-y+z+t,\ x+2z-t,\ x+y+3z-3t]$$

usual basis of $R^4 = \{[1,0,0,0], [0,1,0,0], [0,0,1,0], [0,0,0,1]\}$

Then,

$L([1,0,0,0]) = [1,1,1]$, $L([0,1,0,0]) = [-1,0,1]$

$L([0,0,1,0]) = [1,2,3]$ $L([0,0,0,1]) = [1,-1,-3]$

By simplified span method

let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -3 \end{bmatrix}$

$RREF(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ — (2m)

Basis of Range(L) is $B = \{[1,0,-1], [0,1,2]\}$ — (2M)

$\dim(\text{Range}(L)) = 2$ — (1m)

$\text{Range}(L) = \{[a, b, 2b-a];\ a, b \in R\}$ — (2m)

Second method By independence test method

$A = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

$RREF(A) = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Basis of Range$(L) = \{[1,1,1], [-1,0,1]\}$

Dim (Range $(L)) = 2$

Range $(L) = \{[a-b, a, a+b] ; a, b \in R\}$

b) $\ker(L) = \{ v \in R^4 \mid L(v) = 0 \}$

$L(v) = 0$ where $v = [x, y, z, t]$

i.e $L[x, y, z, t] = [x - y + z + t,\ x + 2z - t,\ x + y + 3z - 3t] = [0, 0, 0]$

$$m = [L \mid 0] = \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 2 & -1 & 0 \\ 1 & 1 & 3 & -3 & 0 \end{array}\right]$$

$$RREF(m) = \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ —— 2M

Hence

$x - y + z + t = 0$

$y + z - 2t = 0$

Basis of $\ker(L) = \{[2, 1, -1, 0], [1, 2, 0, 1]\}$ —— 2M

$\dim(\ker L) = 2$ —— 1M

$\ker(L) = \{[k - 2h,\ 2k - h,\ h,\ k] ; h, k \in R\}$ —— 2M

3 b)

$$T(f) = \begin{bmatrix} f(0) \\ |f(-2)| + |f(2)| \end{bmatrix}$$

let $f, g \in V$ s.t

$f(x) = 1$; $g(x) = x$ — (4m)

Then we have $(f+g)(x) = x+1$

$$T(f+g) = \begin{bmatrix} 1+0 \\ |-1| + |3| \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$ — (2m)

$$T(f) + T(g) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$ — (2m)

$T(f+g) \neq T(f) + T(g)$ — (1m)

T is not linear transformation — (2m)