SET-X

Birla Institute of Technology and Science, Pilani

Mid Semester Examination, II Semester 2017-2018

MATH F112 (Mathematics II)

PART-A (Closed Book)

Max. Marks: 30 **Max. Time: 30 Min.**

Name:	**ID:**

NOTE 1. All the questions are multiple choice questions. Write the most appropriate answer in the box provided below and nowhere else. Each question carries **3 marks**. **One mark will be deducted for each wrong answer.**

2. **Overwriting/cutting is not allowed and considered as "question not attempted"**.

3. Do the rough work only on the back pages of answer sheet of Part-B, nowhere else and cross the rough work.

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	**A**	**C**	**B**	**C**	**B**	**D**	**C**	**D**	**D**	**B**

Q.1 Let D be an upper triangular square matrix of order n with all non-zero entries on the main diagonal, and b is any $n \times 1$ matrix. Then which of the following statements is true about the existence of solution of $Dx = b$.

[A] Solution exists and it is unique
[B] The given system is inconsistent
[C] The given system is consistent and it has infinite number of solutions
[D] None of these.

Q.2 Let V be the real vector space of all 2×2 matrices. Let W be the subspace of matrices of the form $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$. Then dimension of W is equal to

[A] 1 **[B]** 2 **[C]** 3 **[D]** 4

Q.3 Let $S = \{(3,3,1),(1,1,0),(0,0,1)\}$ be a subset of $\mathbb{R}^3$. Then which of the following statements is true

[A] S is linearly independent
[B] S is linearly dependent
[C] S is linearly independent but S does not span $\mathbb{R}^3$
[D] None of these

Q.4 Let $\mathbb{P}_4$ be the vector space of all polynomials with real coefficients of degree at most 4, and $W = \{p(x) \in \mathbb{P}_4 : p(1) = p(-1) = 0\}$ is a subspace of $\mathbb{P}_4$.Then dimension of W is equal to

[A] 1 **[B]** 2 **[C]** 3 **[D]** 4

Q.5 The rank of the matrix $M = \begin{bmatrix} 1 & 4 & -1 & -5 \\ 2 & 8 & 5 & 4 \\ -1 & -4 & 2 & 7 \\ 6 & 24 & -1 & -20 \end{bmatrix}$ is equal to

[A] 1 **[B]** 2 **[C]** 3 **[D]** 4

Q.6 Consider $L: \mathbb{P}_2 \to \mathbb{P}_4$ given by $L(p(x)) = x^2 p(x)$, then

[A] $ker\,(L)$ = {all constant polynomials} **[B]** $ker\,(L)$ = {all polynomials of degree ≤ 4}

[C] $ker\,(L)$ = {all quadratic polynomials} **[D]** $ker\,(L)$ = {zero polynomial}

Q.7 Let A be the square matrix such that $A^2 = A$. Then the greatest eigenvalue of A is

[A] -1 **[B]** 0 **[C]** 1 **[D]** 2

Q.8 Let $L: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator that performs a counterclockwise rotation through an angle of $\pi/6$. Then the matrix for L with respect to the standard basis for $\mathbb{R}^2$ is

[A] $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **[B]** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **[C]** $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **[D]** $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Q.9 Let $m > n$, then select true statement

[A] There is an isomorphism from $\mathbb{R}^n$ to $\mathbb{R}^m$.

[B] There is an onto linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

[C] There is no one-to-one linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

[D] There is no onto linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

Q.10 Let X be a fixed $n \times n$ matrix, and consider $L: \mathbb{M}_{nn} \to \mathbb{M}_{nn}$ given by $L(Y) = XY - YX$, then

[A] L is an isomorphism **[B]** L is one-to-one but not onto

[C] L is onto but not one-to-one **[D]** L is neither one-to-one nor onto

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Ans.	**C**	**D**	**C**	**A**	**A**	**D**	**C**	**D**	**D**	**B**

Q.1 The rank of the matrix $M = \begin{bmatrix} 1 & 4 & -1 & -5 \\ 2 & 8 & 5 & 4 \\ -1 & -4 & 2 & 7 \\ 6 & 24 & -1 & -20 \end{bmatrix}$ is equal to

[A] 4 **[B]** 3 **[C]** 2 **[D]** 1

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[A] $ker\ (L)$ = {zero polynomial} **[B]** $ker\ (L)$ = {all polynomials of degree ≤ 4}

[C] $ker\ (L)$ = {all quadratic polynomials} **[D]** $ker\ (L)$ = {all constant polynomials}

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[A] 4 **[B]** 3 **[C]** 2 **[D]** 1