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BITS Pilani
Pilani Campus

MATH F111 (Mathematics-I)

Lecture 11 (Chapter-11.7)
Conics in Polar Coordinate

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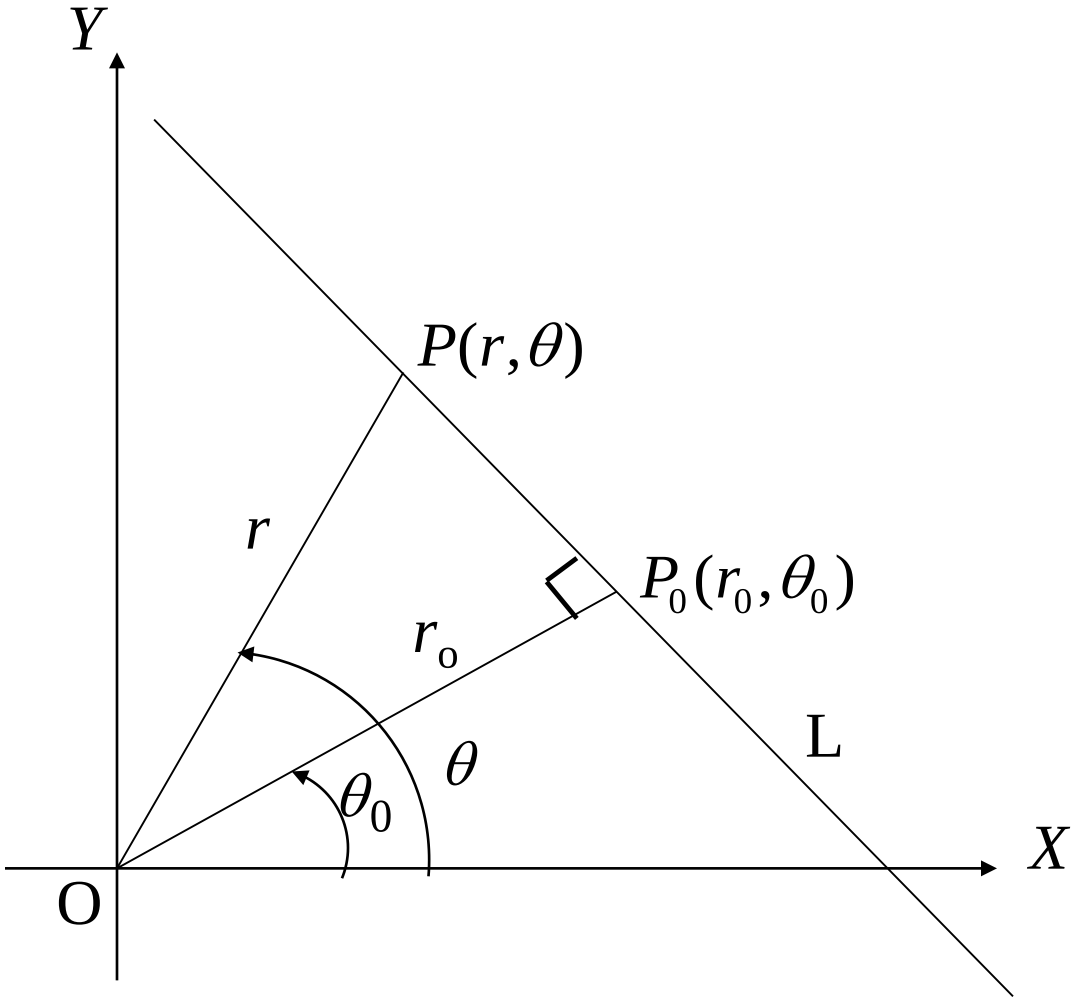
Polar Equation of Straight Line



Polar Equation of Straight Line

If the point $P_0(r_0, \theta_0)$ is the foot of perpendicular from the origin on the line L, then an equation for L is:

$$r\cos(\theta - \theta_0) = r_0,$$

where (r, θ) is any point on the line L.

Ex. 11.7/Q.47 Sketch the line and find the Cartesian Equation for :

$$r\cos(\theta + \pi/3) = 2$$

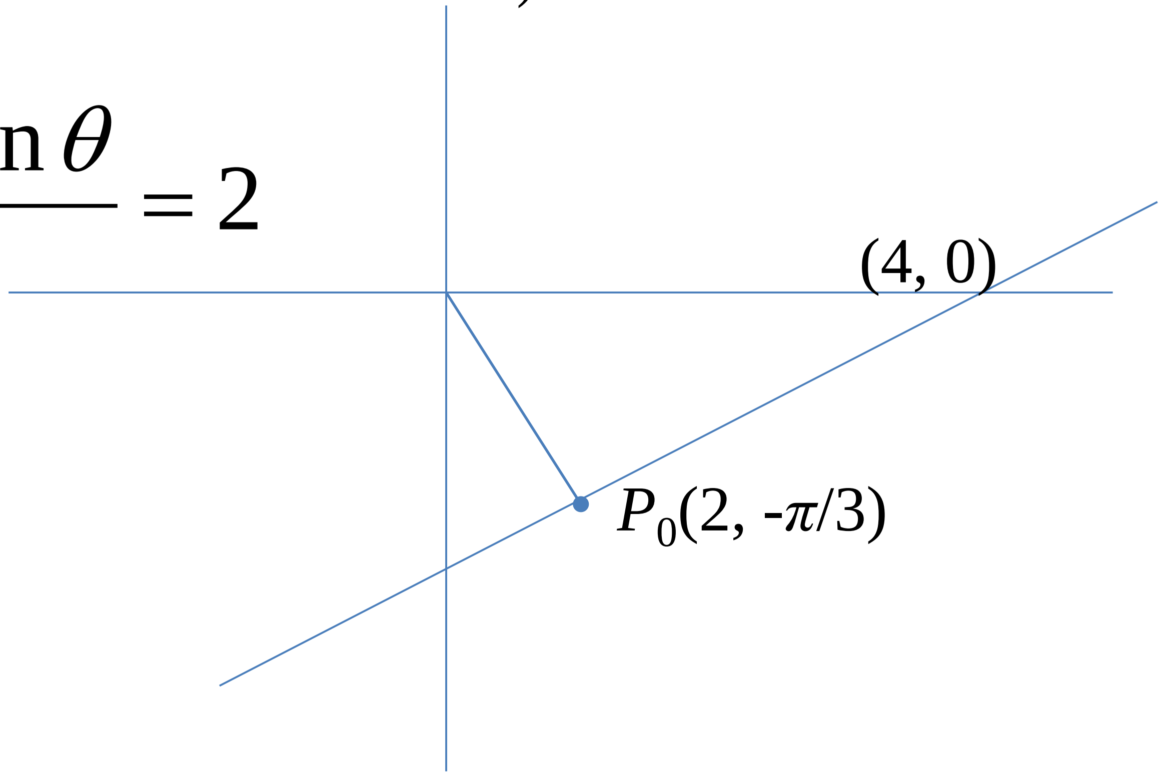
Polar Equation of Straight Line

$$\text{Ans}: r\cos(\theta+\pi/3)=2$$

$$\Rightarrow r\left(\cos\theta\cos\pi/3-\sin\theta\sin\pi/3\right)=2$$

$$\Rightarrow \frac{r\cos\theta-\sqrt{3}r\sin\theta}{2}=2$$

$$\Rightarrow x-\sqrt{3}y=4$$



Polar Equation of Straight Line

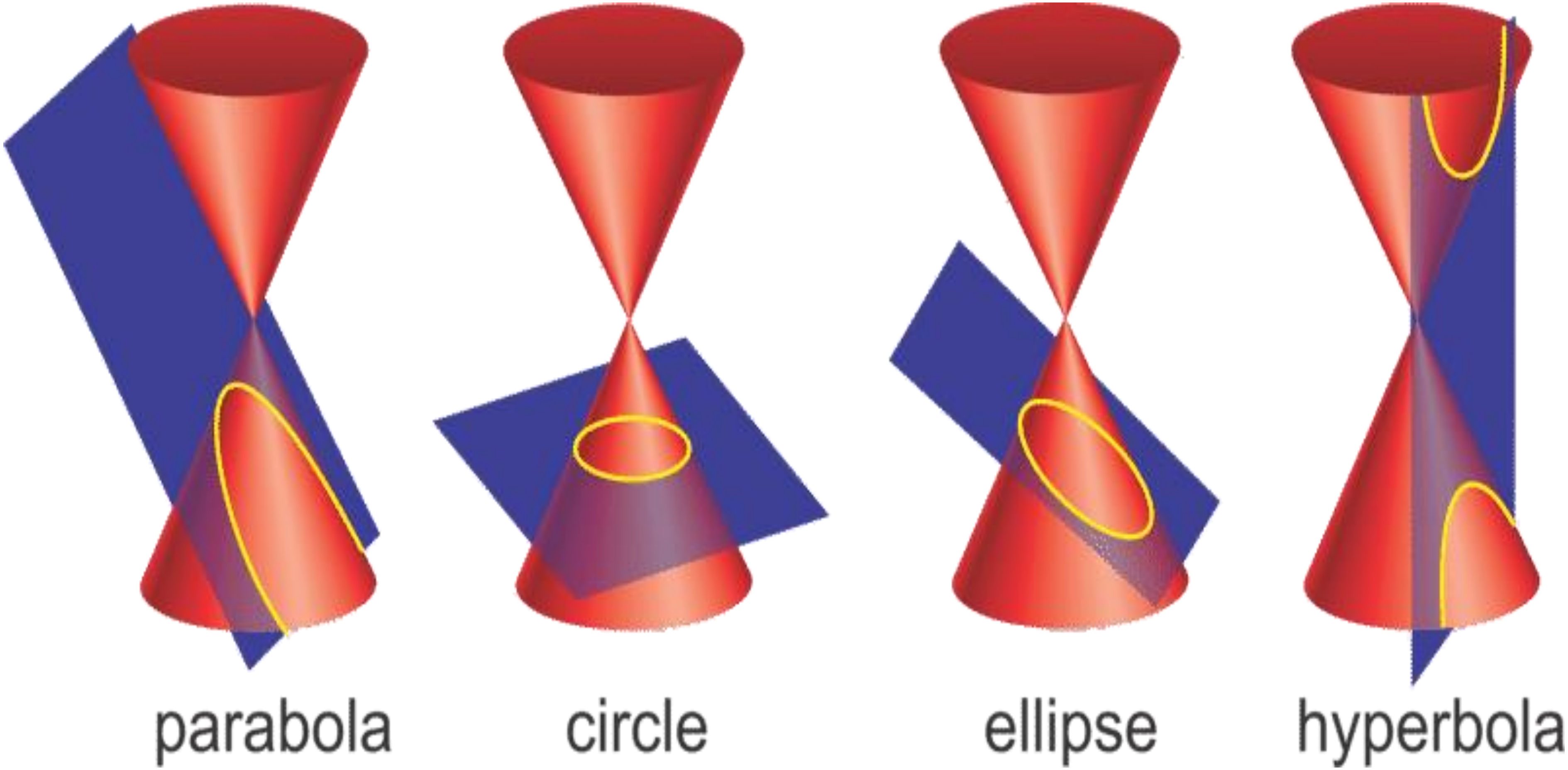
Ex.11.7/Q.50. Find the polar equation for

$$\sqrt{3}\,x - y = 1$$

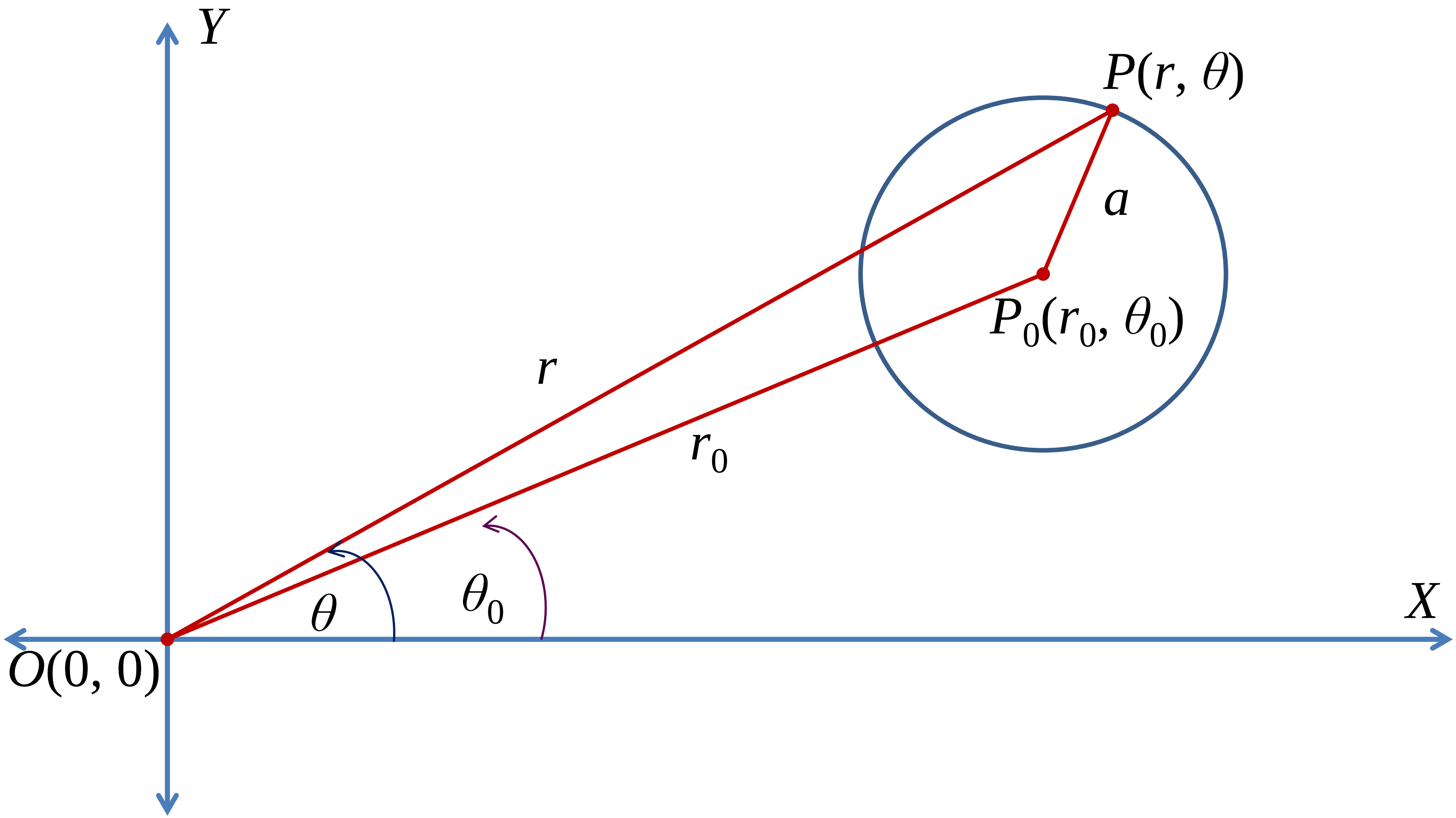
in the form $r\cos(\theta - \theta_0) = r_0$.

$$\text{Ans}: r\cos(\theta + \pi/6) = 1/2.$$

Conic Section



Polar Equation of Circle

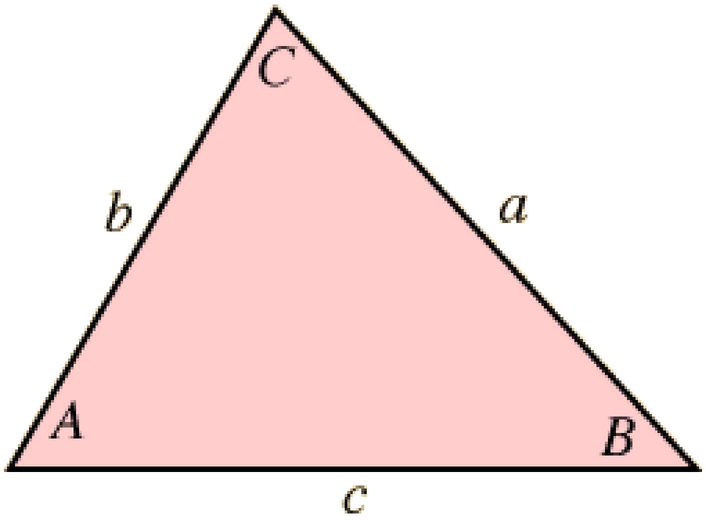


Polar Equation of Circle

The polar equation of a circle of radius a and centered at (r_0, θ_0) is (using cosines law on ΔOPP_0):

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$r^2 + r_0^2 - 2rr_0\cos(\theta - \theta_0) = a^2$$



Polar Equation of Circle (Special Cases)

Case I : **If the circle passes through the origin**, then $r_0 = a$ and the equation simplifies to:

$$r = 2a\cos(\theta - \theta_0)$$

Case II : **Equation of a circle centered at $(a,0)$ and radius a**. If the center lies on positive x-axis then the equation becomes:

$$r = 2a\cos\theta$$

Polar Equation of Circle (Special Cases)

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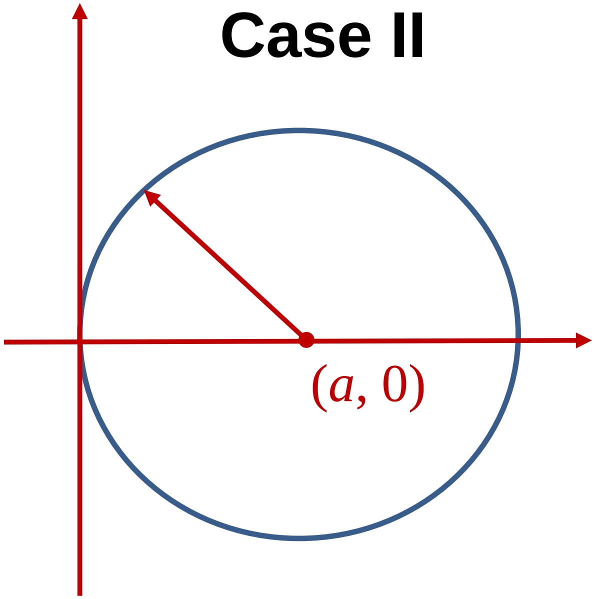
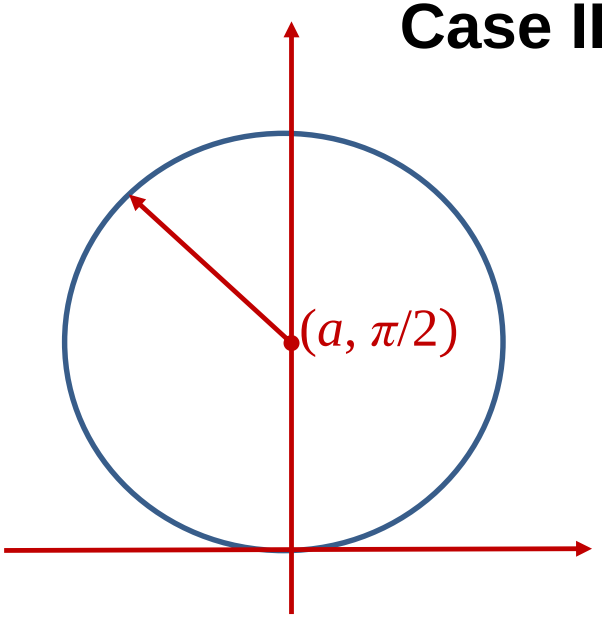
Case II



Case III



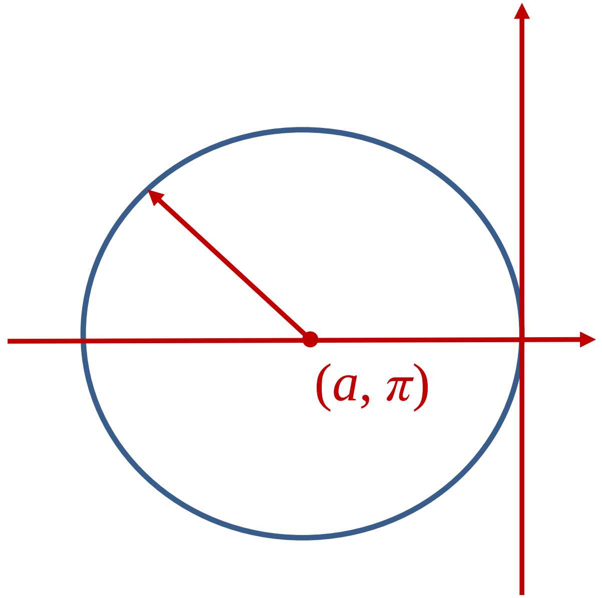
Case III: **Equation of a circle centered at $(a, \pi/2)$ and radius a**. If the center lies on positive y-axis then the equation becomes:

$$r = 2a \sin\theta$$

Polar Equation of Circle (Special Cases)

Case IV: **Equation of a circle centered at (a, π) and radius a**. If the center lies on negative x-axis then the equation becomes:

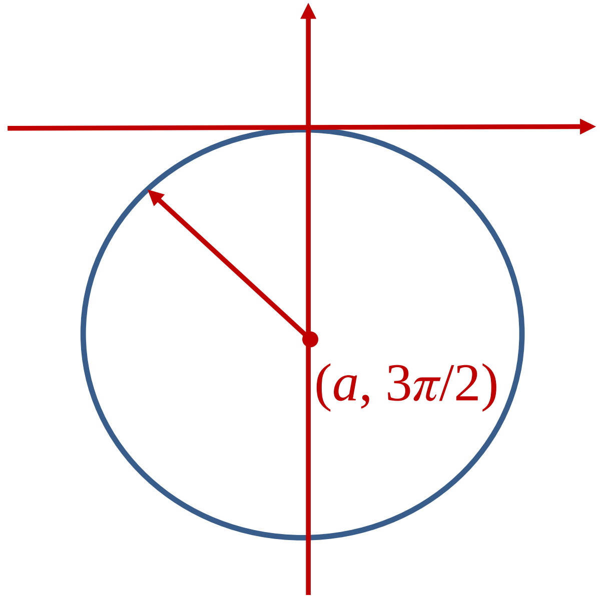
$$r = -2a\cos\theta$$



Polar Equation of Circle (Special Cases)

Case V: **Equation of a circle centered at (a, $3\pi/2$) and radius a.** If the center lies on negative y-axis then the equation becomes:

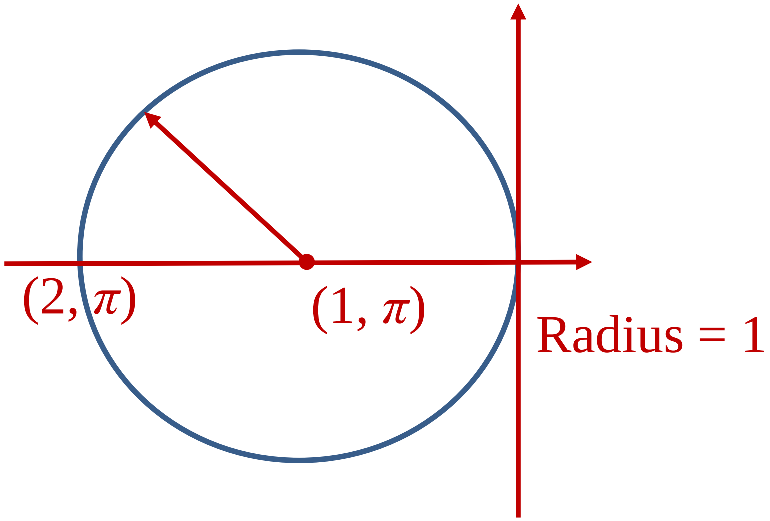
$$r = -2a\sin\theta$$



Polar Equation of Circle

Ex.11.7/Q.55 Sketch the circle $r = -2\cos\theta$. Find polar coordinate of the center and identify the radius.

Sol. Compare with $r = -2a\cos\theta$, we get radius $a = 1$. Therefore the polar coordinate of the center is $(1, \pi)$



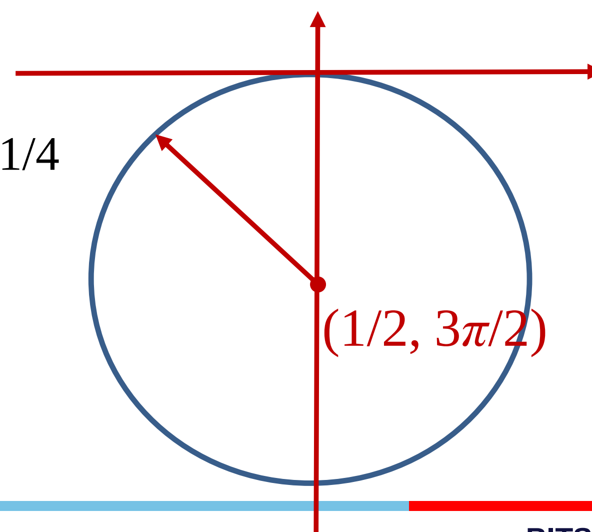
Polar Equation of Circle

Ex.11.7/Q.60 Find polar equation for the circle $x^2 + y^2 + y = 0$. Sketch the circle and label it with both its Cartesian and polar equations.

Sol. Compare with $(x - x_0)^2 + (y - y_0)^2 = a^2$. The center is $(0, -1/2)$ and radius is $a = 1/2$. Therefore the polar coordinate of center is $(1/2,\ 3\pi/2)$ and polar equation is $r = -\sin\theta$.

$x^2 + (y + ½)^2 = 1/4$

$r = -\sin\theta$



Eccentricity "*e*"

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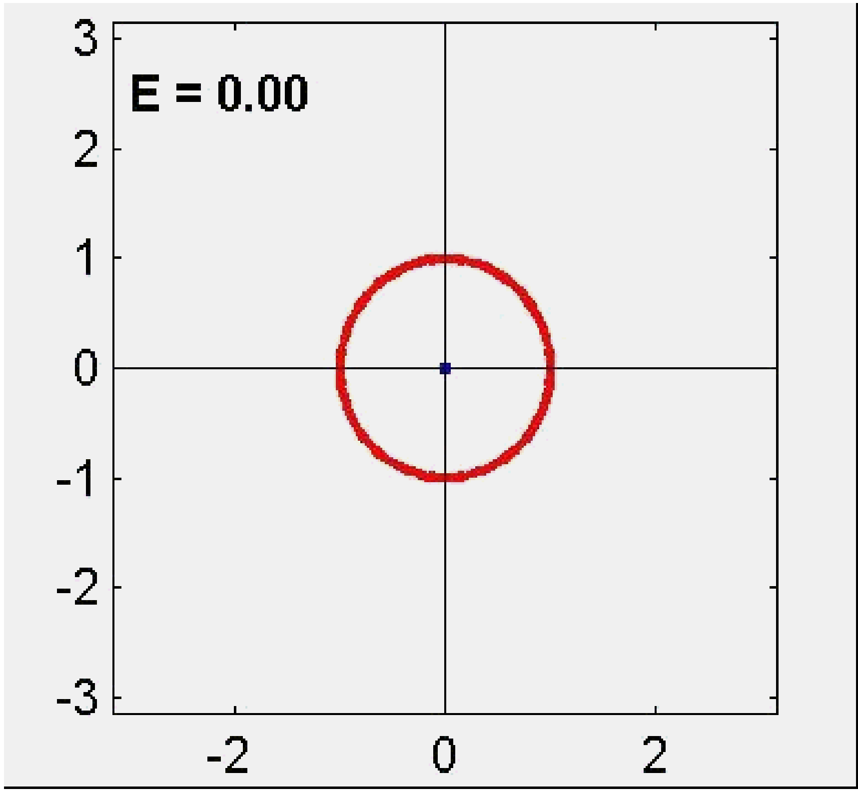
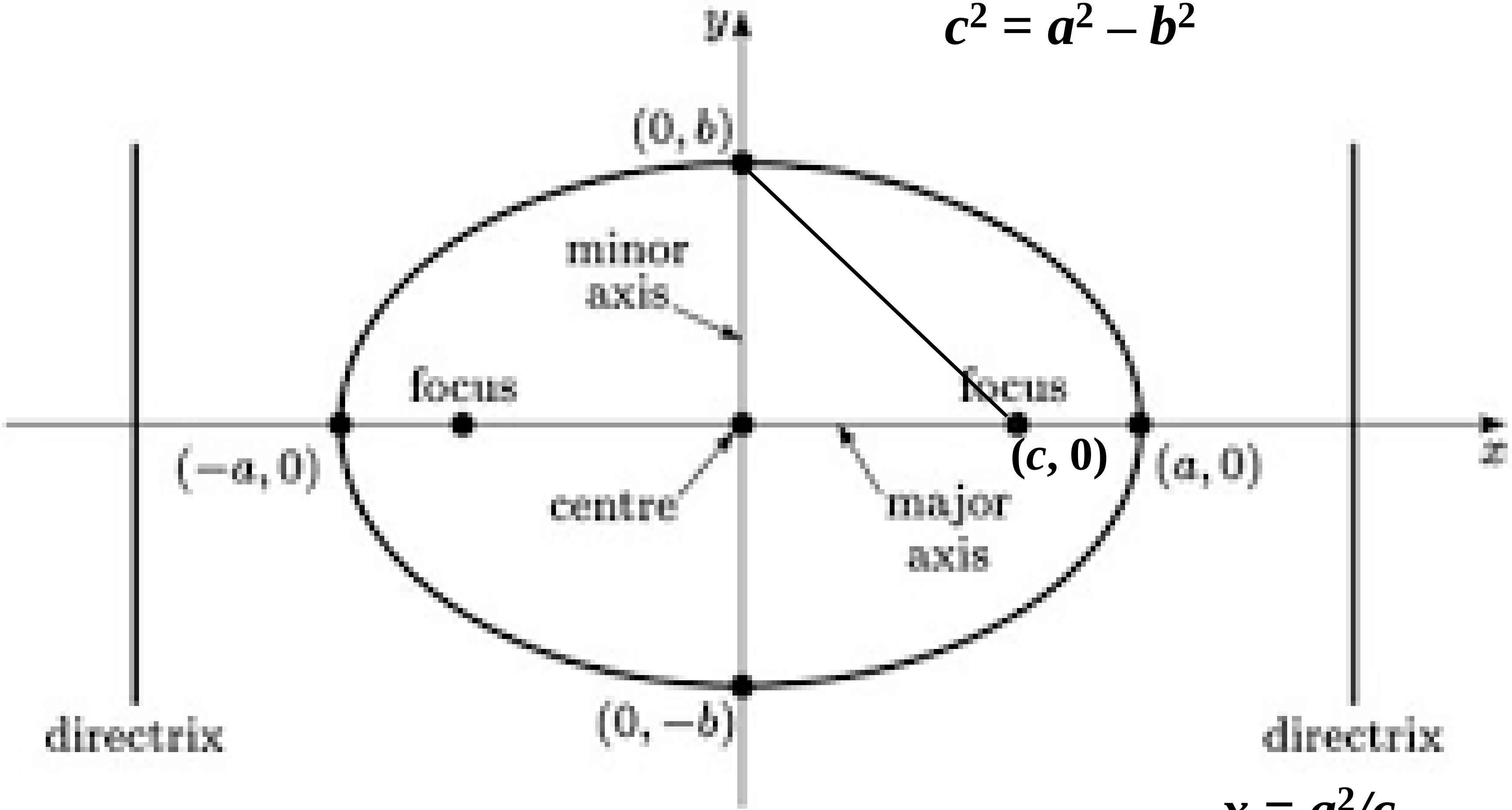
A measure of the "roundness" of a conic



A circle has an **eccentricity of zero**, so the eccentricity shows you how "un-circular" the curve is. Bigger eccentricities are less curved.

Eccentricity "*e*"

$c^2 = a^2 - b^2$



$x = a^2/c$

Eccentricity "*e*"

The ***eccentricity*** of ellipse $x^2/a + y^2/b = 1$

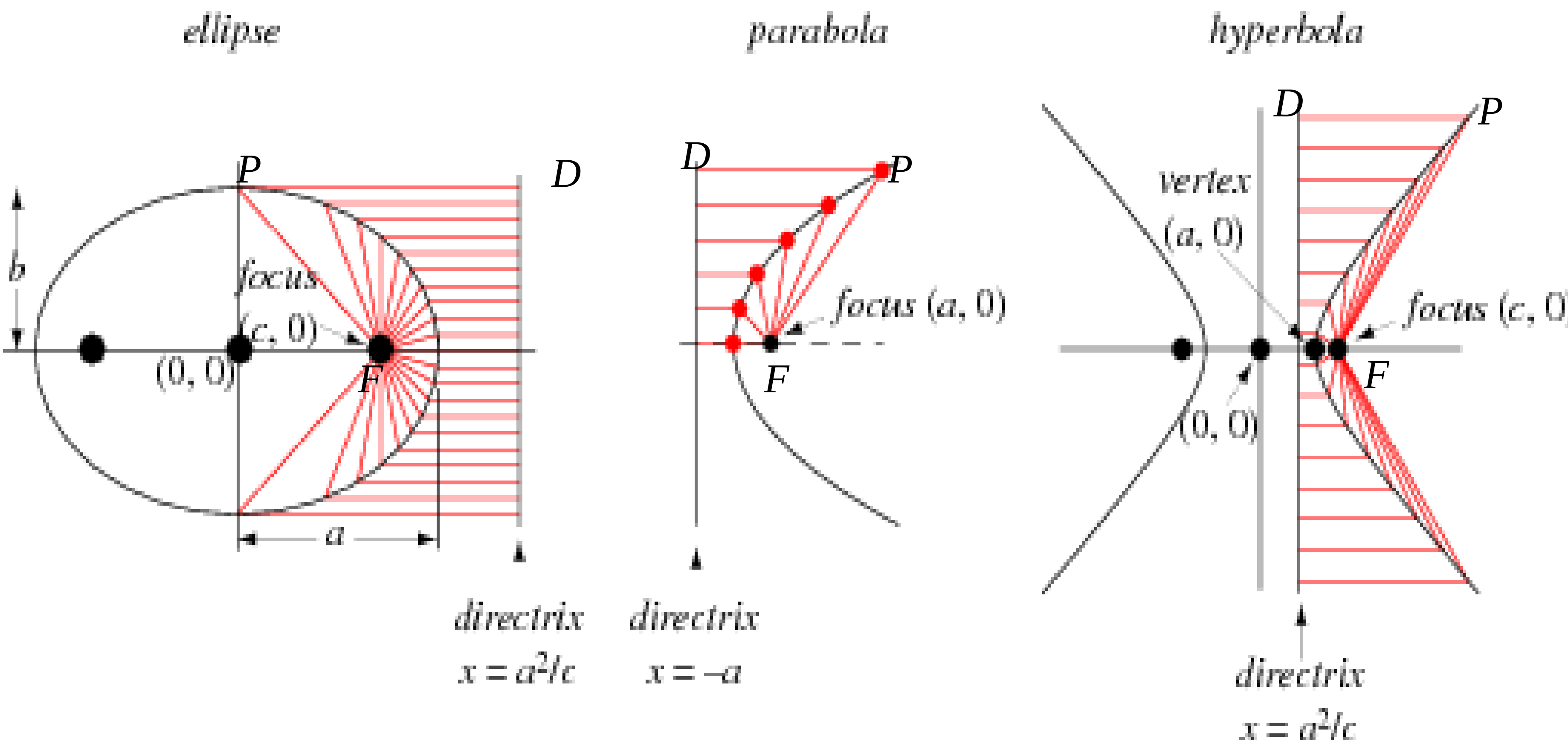
$$\text{is } e = c/a = \sqrt{(a^2 - b^2)}/a$$

The ***eccentricity*** of hyperbola $x^2/a - y^2/b = 1$

$$\text{is } e = c/a = \sqrt{(a^2 + b^2)}/a$$

The ***eccentricity*** of parabola $e = 1$

Eccentricity "*e*"



PF /*PD* = ***e***

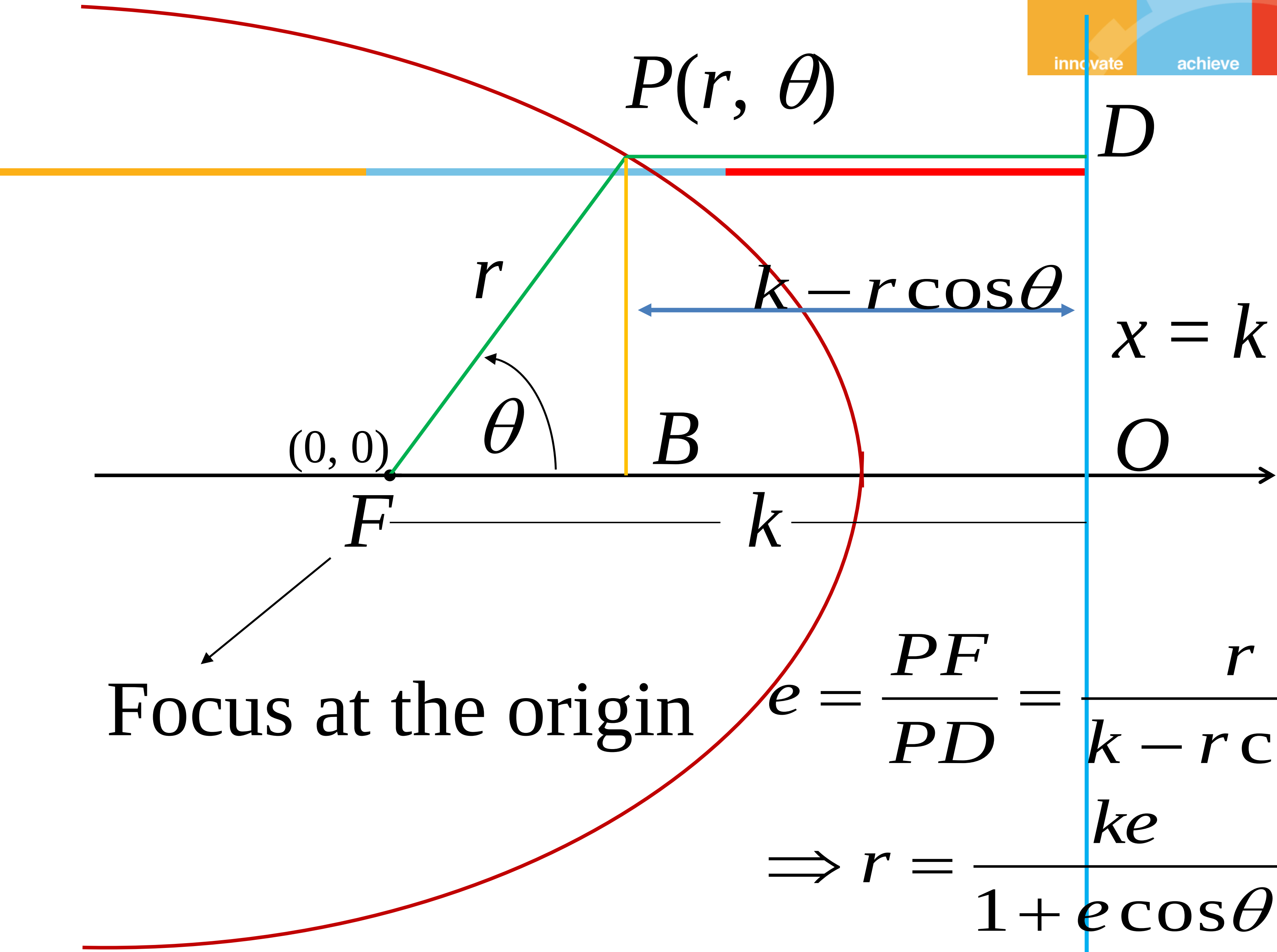
PF /*PD* = **1** = ***e***

PF /*PD* = ***e***

Polar Equation of a Conic

Case I: Let a focus of the conic section be at the origin and the corresponding directrix be $x = k$ ($k > 0$) (vertical, to the right of the origin).

Then $$r = \frac{ke}{1 + e\cos\theta}.$$

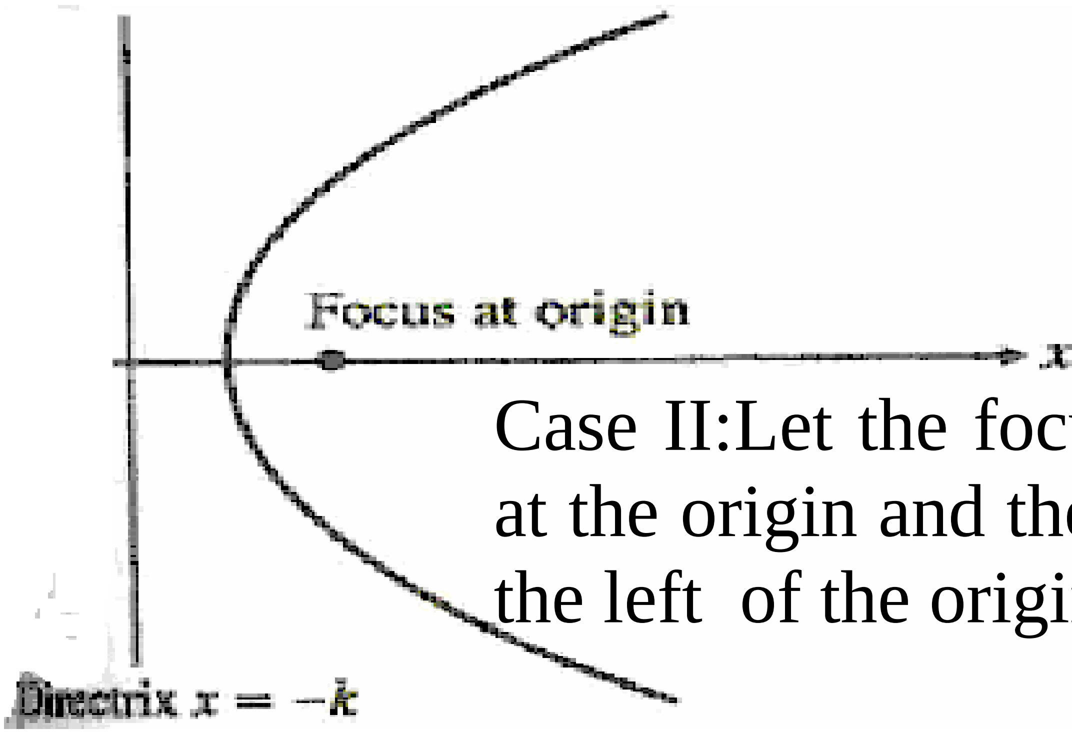


Focus at the origin

$$e = \frac{PF}{PD} = \frac{r}{k - r\cos\theta}$$

$$\Rightarrow r = \frac{ke}{1 + e\cos\theta}.$$

Polar Equation of a Conic



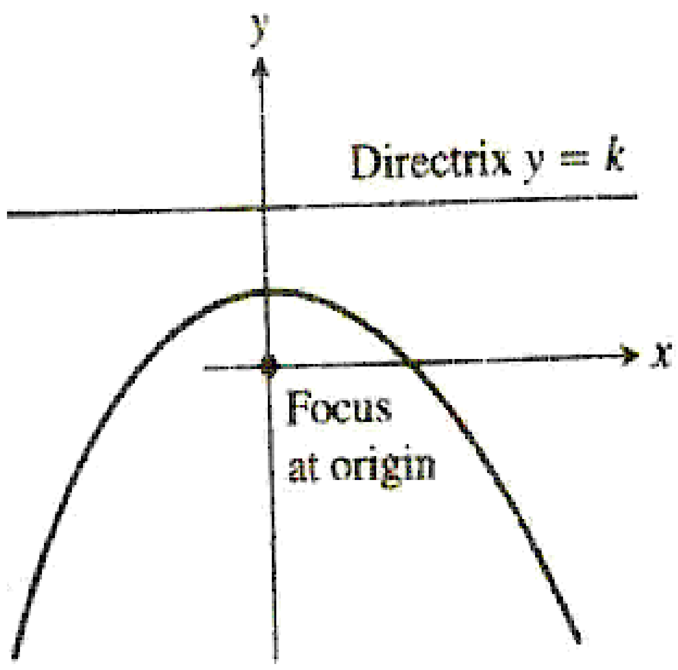
Case II: Let the focus of the conic section is at the origin and the directrix $x = -k$ is to the left of the origin ($k > 0$).

Then $r = \dfrac{ke}{1 - e\cos\theta}$.

Polar Equation of a Conic

Case III: Focus at the origin and directrix is $y = k$.

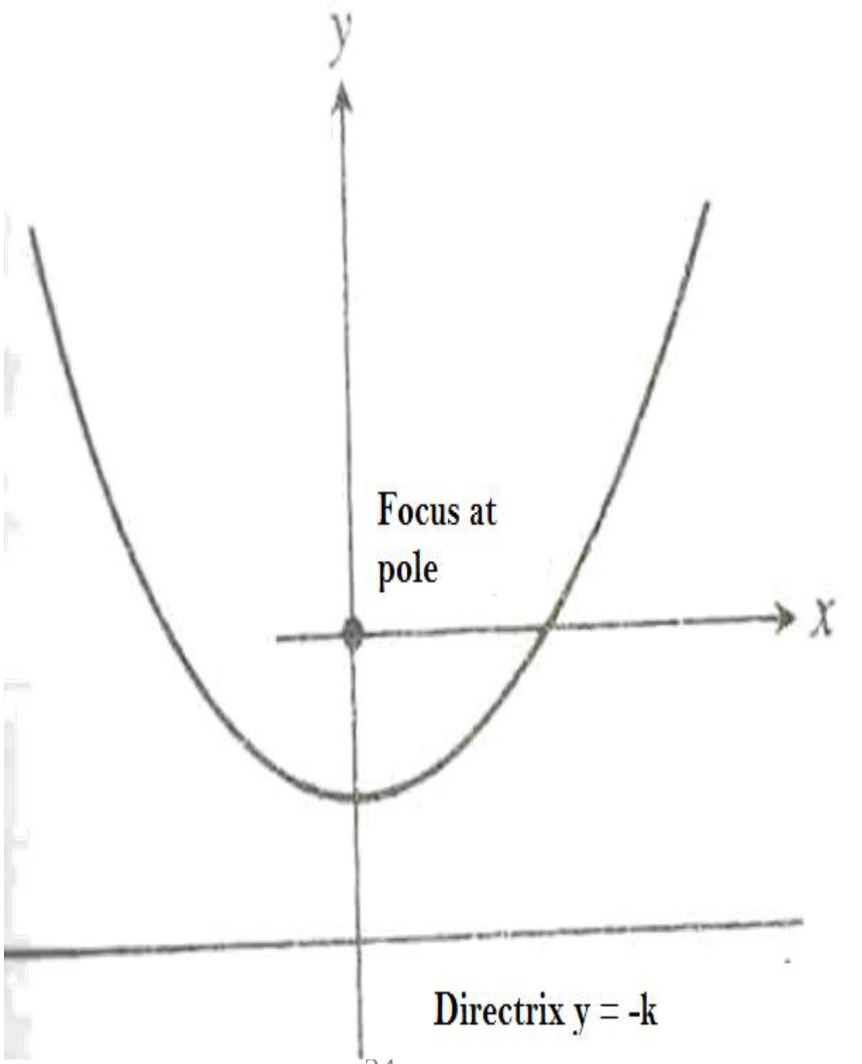
Then $$r = \frac{ke}{1 + e\sin\theta}.$$



Polar Equation of a Conic

Case IV: Focus at the origin and directrix is
$y = -k.$

Then $r = \dfrac{ke}{1 - e\sin\theta}.$



Polar Equation of a Conic

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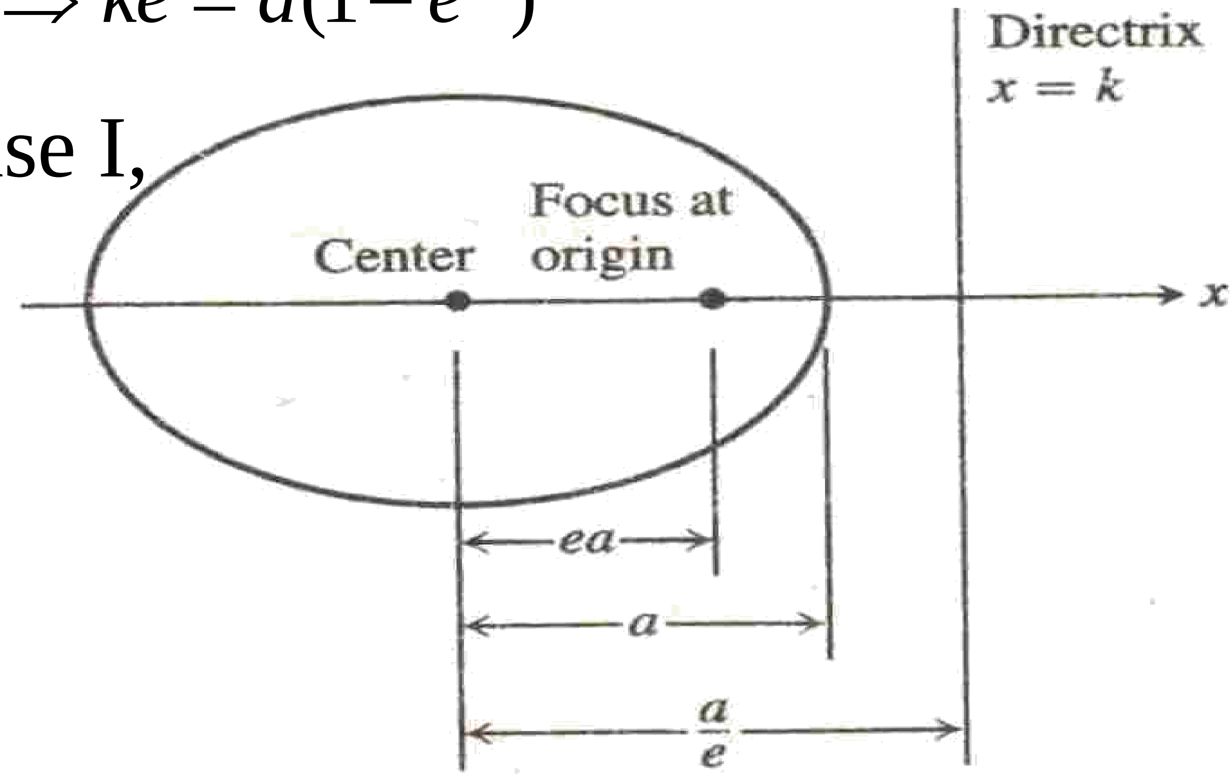
NOTE: For an ellipse with semimajor axis a and eccentricity e (with focus at the origin), we have: $k = \frac{a}{e} - ea \quad \Rightarrow ke = a(1-e^2)$

Hence from Case I,

$$r = \frac{a(1-e^2)}{1+e\cos\theta}.$$



Polar Equation of a Conic

Ex.11.7/Q.31 If $e = 5$, and $y = -6$, then find equation of the conic section (Assume one focus at the origin).

Solu.

$$r = \frac{ke}{1 - e\sin\theta} = \frac{30}{1 - 5\sin\theta}$$

Nature of conic?

Polar Equation of a Conic

Ex.11.7/Q.35 If $e = 1/5$, and $y = -10$, then find equation of the conic section (Assume one focus at the origin).

Solu.

$$r = \frac{ke}{1 - e\sin\theta} = \frac{10}{5 - \sin\theta}$$

Nature of conic?

We can find a $\quad a = \frac{ke}{1-e^2} = \frac{2}{1 - 1/25} = \frac{25}{12}$

Now center is $(ae, \pi/2) = (5/12, \pi/2)$

Polar Equation of a Conic

Ex.11.7/Q.41 Sketch $r = 400/(16 + 8\sin\theta)$. Include the directrix that corresponds to the focus at the origin. Label the vertices with appropriate polar coordinates. Label the center in the case of ellipse.

Solu. $r = \dfrac{25}{1+(1/2)\sin\theta}$

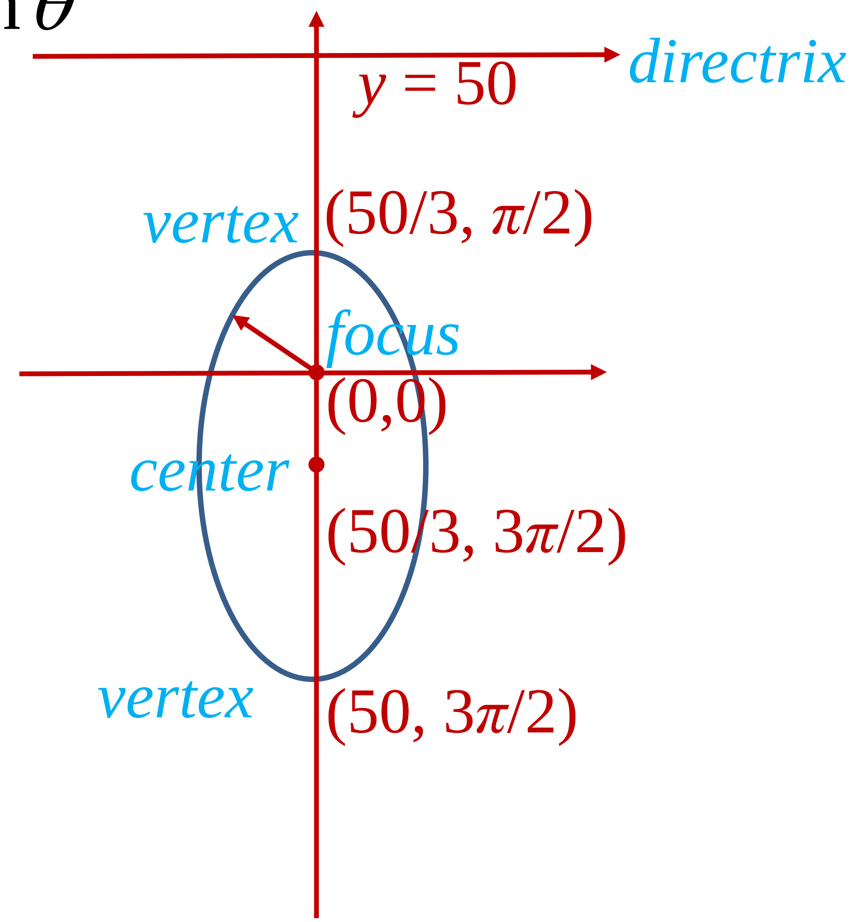
So $e = 1/2$ and $ke = 25 \Rightarrow k = 50$ and so $y = 50$ is the directrix. Since $e < 1$ so the curve is an ellipse.

$$a = \frac{ke}{1-e^2} = \frac{25}{1-1/4} = \frac{100}{3}$$

Now center is $(ae, 3\pi/2) = (50/3, 3\pi/2)$

Polar Equation of a Conic

$$r = \frac{25}{1 + (1/2)\sin\theta}$$



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