

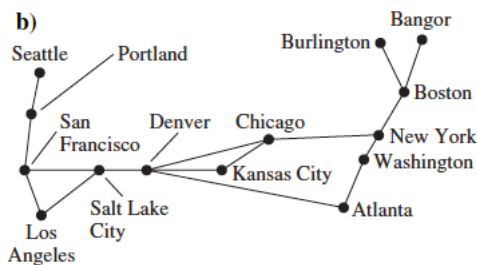
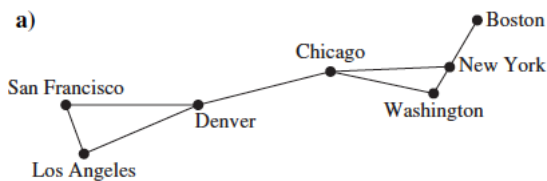
DISCRETE STRUCTURE FOR COMP. SCI. (CS F222)

Practice Problems

Modelling using Graphs

(problems are taken from reference books)

1. Draw the acquaintanceship graph that represents that Tom and Patricia, Tom and Hope, Tom and Sandy, Tom and Amy, Tom and Marika, Jeff and Patricia, Jeff and Mary, Patricia and Hope, Amy and Hope, and Amy and Marika know each other, but none of the other pairs of people listed know each other. It is also known that for how long the pairs know each other.
2. Suppose that there are five young women and six young men on an island. Each woman is willing to marry some of the men on the island and each man is willing to marry any woman who is willing to marry him. Suppose that Anna is willing to marry Jason, Larry, and Matt; Barbara is willing to marry Kevin and Larry; Carol is willing to marry Jason, Nick, and Oscar; Diane is willing to marry Jason, Larry, Nick, and Oscar; and Elizabeth is willing to marry Jason and Matt.
 - a) Model the possible marriages on the island using graphs
 - b) Find a matching of the young women and the young men on the island such that each young woman is matched with a young man whom she is willing to marry.
 - c) Is the matching you found in part (b) a complete matching? Is it a maximum matching?
3. A communications link in a network should be provided with a backup link if its failure makes it impossible for some message to be sent. For each of the communications networks shown here in (a) and (b), determine those links that should be backed up.



4. Use a graph model and a path in your graph, to solve the jealous husbands problem. Two married couples, each a husband and a wife, want to cross a river. They can only use a boat that can carry one or two people from one shore to the other shore. Each husband is extremely jealous and is not willing to leave his wife with the other husband, either in the boat or on shore. How can these four people reach the opposite shore?
5. Gray Codes The position of a rotating pointer can be represented in digital form. One way to do this is to split the circle into 2^n arcs of equal length and to assign a bit string of length n to each arc. Two ways to do this using bit strings of length three are shown in Figure 1 below. The digital representation of the position of the pointer can be determined using a set of n contacts. Each contact is used to read one bit in the digital representation of the position. This is illustrated in Figure 2 for the two assignments from Figure 1. When the pointer is near the boundary of two arcs, a mistake may be made in reading its position. This may result in a

major error in the bit string read. For instance, in the coding scheme in Figure 1(a), if a small error is made in determining the position of the pointer, the bit string 100 is read instead of 011. All three bits are incorrect! To minimize the effect of an error in determining the position of the pointer, the assignment of the bit strings to the 2^n arcs should be made so that only one bit is different in the bit strings represented by adjacent arcs. This is exactly the situation in the coding scheme in Figure 1(b). An error in determining the position of the pointer gives the bit string 010 instead of 011. Only one bit is wrong. A Gray code is a labeling of the arcs of the circle such that adjacent arcs are labeled with bit strings that differ in exactly one bit. The assignment in Figure 1(b) is a Gray code. We can find a Gray code by listing all bit strings of length n in such a way that each string differs in exactly one position from the preceding bit string, and the last string differs from the first in exactly one position. Model the Gray codes using Hamiltonian paths/circuits.

Figure 1

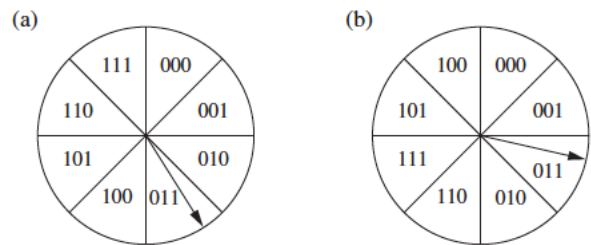
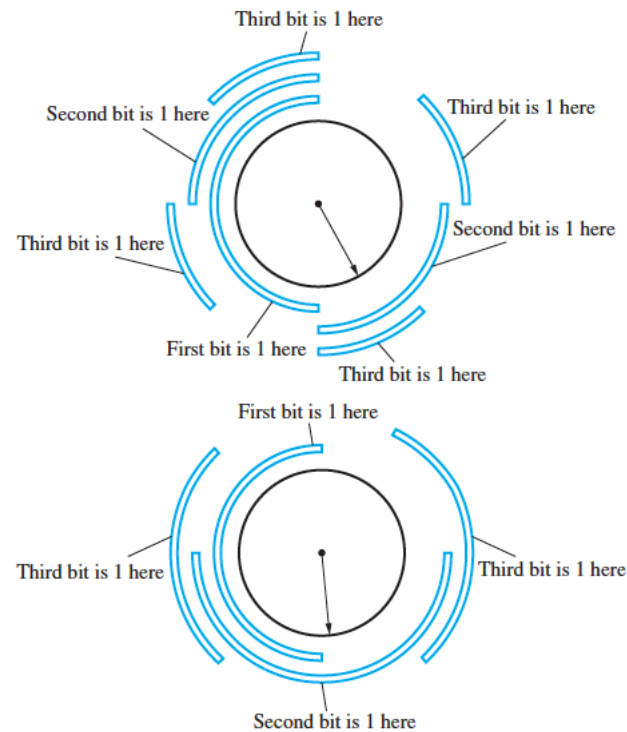


Figure 2



- Because traffic is growing heavy in the central part of a city, traffic engineers are planning to change all the streets, which are currently two-way, into one-way streets. Explain how to model this problem.