MATH F113
(Probability and Statistics)

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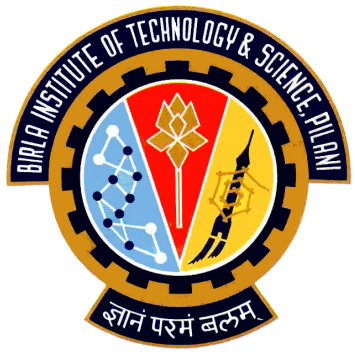
Associate Professor



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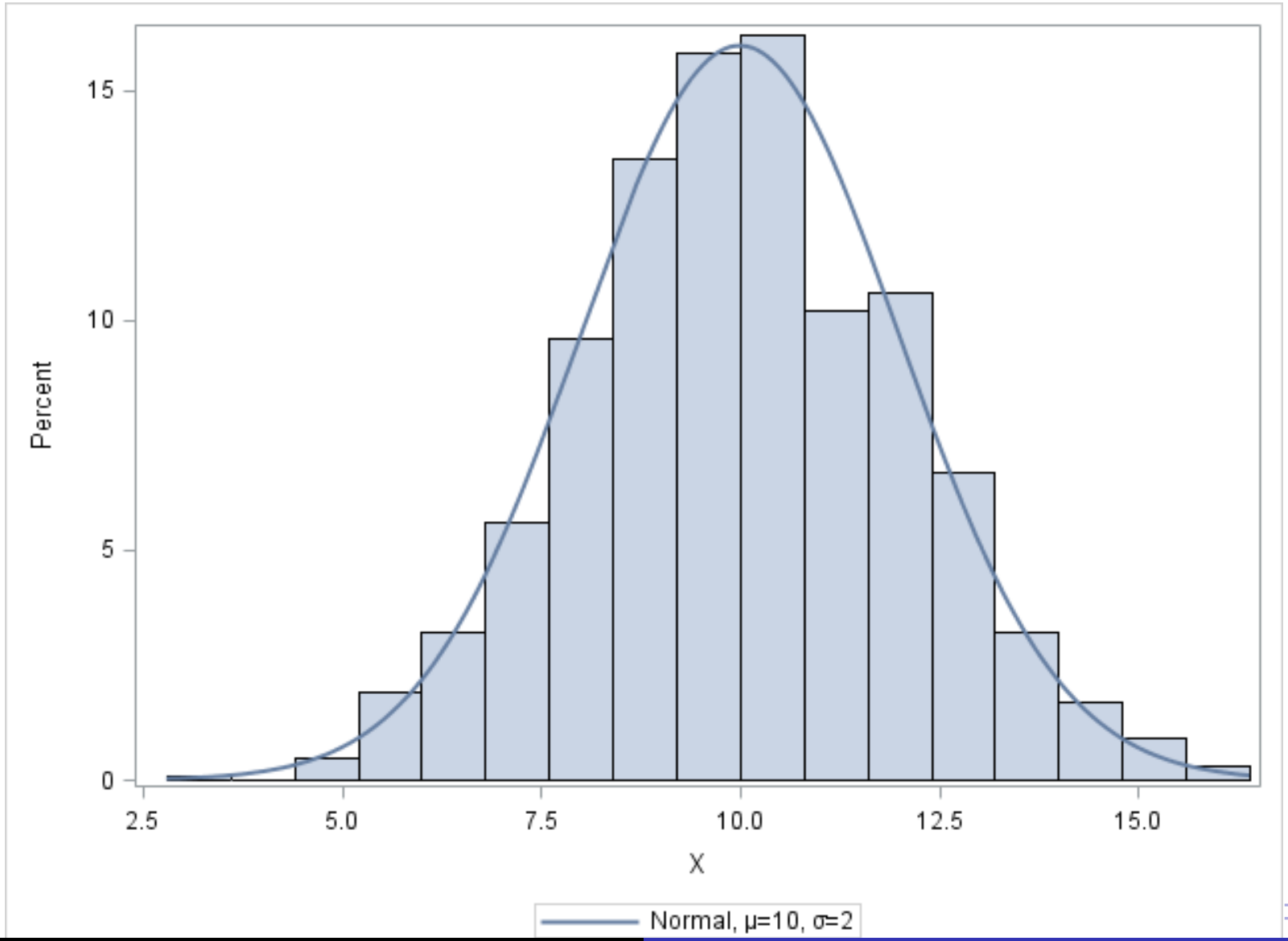
In Lecture 19

Normal Probability Rule
Chebyshev's Inequality

Normal Approximation to the Binomial Distribution: Let X be binomial with parameters n and p. For large n, X is approximately normal with mean np and variance $np(1-p)$;

$$X \sim N\left(np, \sqrt{np(1-p)}\right)$$

Normal Approximation (Cont...)



Normal Approximation (Cont...)

(a) $p \leq 0.5$ **and** $np > 5$ **i.e** $n \geq 10$

(b) $p > 0.5$ **and** $n(1-p) > 5$ $n \geq 10$

Correction for continuity

(1)

$$P(a < X \leq b) = P(a - 0.5 < X \leq b + 0.5)$$

(2)

$$P(X \leq b) = P(-\infty < X \leq b)$$

$$P(-\infty < X \leq b + 0.5) = P(X \leq b + 0.5)$$

(3)

$$P(X \geq a) = P(a \leq X < \infty)$$

$$P(a - 0.5 \leq X < \infty) = P(X \geq a - 0.5)$$

(4)

$$P(X = a) = P(a - 0.5 < X < a + 0.5)$$

The number 0.5 is called the half unit correction for continuity.

Exercise 52/4.5/pp.148 Let X be binomial with $n = 20$ and $p = 0.3$. Use the normal approximation to each of the following. Compare the results with values obtained from Table I of App. A

(a) $\mathbf{P}[X \leq 3]$

(b) $\mathbf{P}[3 \leq X \leq 6]$

(c) $\mathbf{P}[X \geq 4]$

(d) $\mathbf{P}[X = 4]$

Exercise 53/4.6/pp.148 Although errors are likely when taking measurements from photographic images, these errors are often very small. For sharp images with negligible distortion, errors in measuring distances are often no larger then 0.0004 inch. Assume that the probability of a serious measurement error is 0.05. A series of 150 independent measurements are made

Normal Approximation (Cont...)

Let X denote the number of serious errors made

(a) In finding the probability of making at least one serious error, is the normal approximation appropriate? If so, approximate the probability using this method

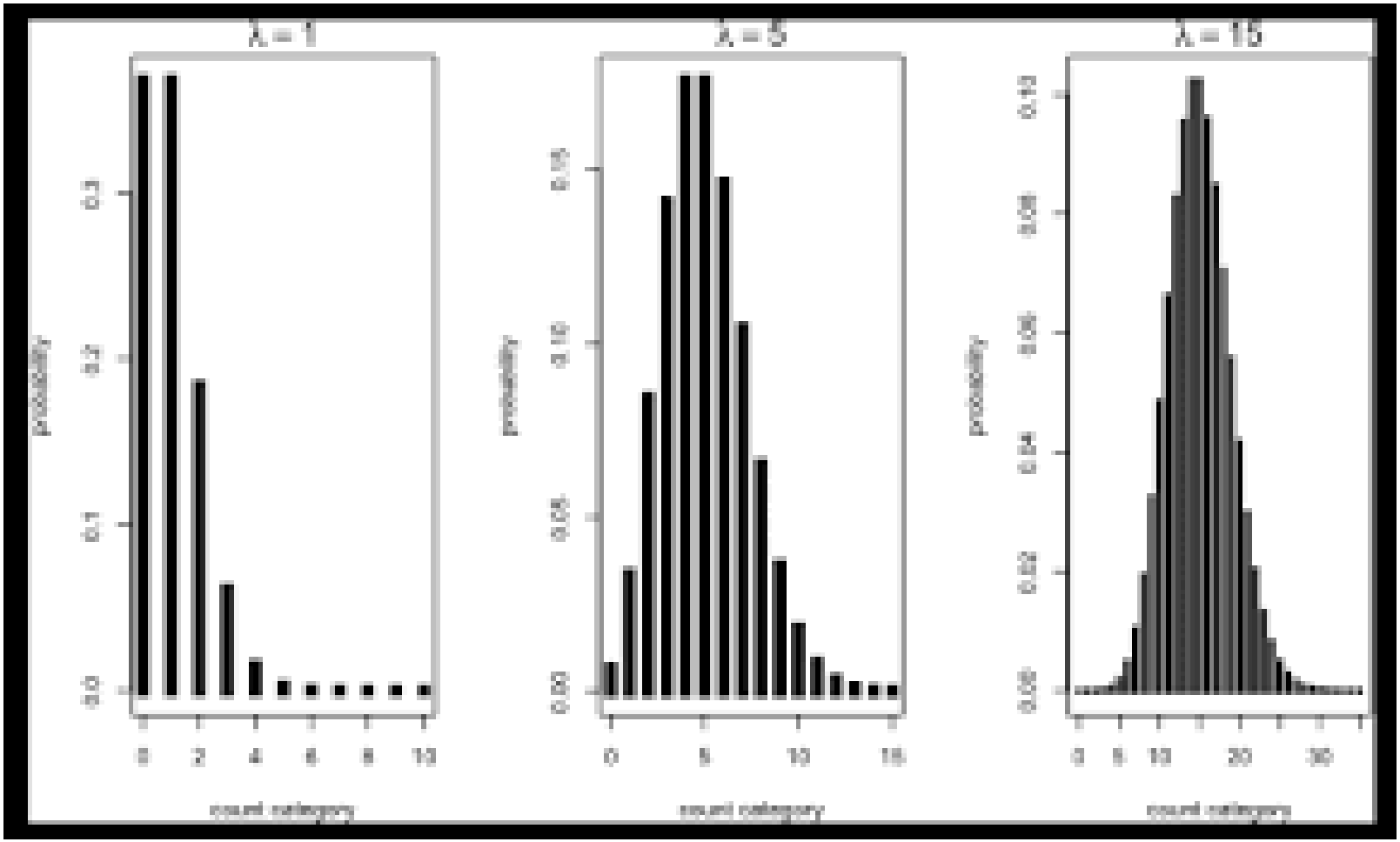
(b) Approximate the probability that at most three serious errors will be made

Normal Approximation to the Poisson Distribution Let X be Poisson with parameter λs

$$f(x) = \frac{e^{-\lambda s}(\lambda s)^x}{x!} \qquad x = 0, 1, 2, 3...$$

Then for large value of λs X is approximately normal with mean λs and variance λs

Normal Approximation (Cont...)



Correction for continuity

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Exercise 56/4.6/pp.149 Let X be a Poisson random variable with parameter $\lambda s = 15$. Find $P(X \leq 12)$ from table II of App. A. Approximate this probability using normal curve. Be sure to employ the half-unit correction.

Exercise 57/4.6/pp.149 The average number of its either arriving at or departing from O'Hare Airport is one every 40 seconds.

(a) What is the approximate probability that at least 75 such flights will occur during a randomly select hour?

(b) What is the probability that fewer than 100 flights will take place in an hour?

Acknowledgment: I am thankful to Mr. Amit Kumar, Research Scholar, Department of Mathematics for his sincere help in developing these lecture slides.

For any typographical errors, queries and feedbacks, please feel free to write at chandrashekhar@pilani.bits-pilani.ac.in

Probability and Statistics (Cont...)