DISCRETE STRUCTURE FOR COMP. SCI. (CS F222)

Practice Problems

(Induction, Recursion and Relations)

Induction

1. Prove, by Mathematical Induction, that n(n + 1)(n + 2)(n + 3) is divisible by 24, for all natural numbers n.
2. Prove using mathematical induction that for all n >= 1,
   a. 1 + 4 + 7 + … + (3n-2) = n(3n-1) / 2
3. Prove, by Mathematical Induction, that

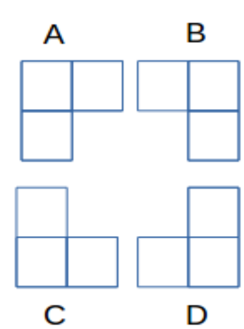
$$(n+1)^2 + (n+2)^2 + (n+3)^2 + \ldots + (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$$

   is true for all natural numbers n.

4. Prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all n >= 0.
5. Consider the sequence of real numbers defined by the relations
   $x_1 = 1$ and $x_{n+1} = \sqrt{(1 + 2x_n)}$ for n ≥ 1.
   Use the Principle of Mathematical Induction to show that $x_n < 4$ for all n ≥ 1.
6. $f_1f_2 + f_2f_3 + f_3f_4 + \ldots + f_{2n-1}f_{2n} = f_{2n}^2$ for all n >= 1.
7. Show that $n! > 3^n$ for n ≥ 7.
8. Prove the binomial theorem using induction. This states that for all n >= 1,

$$(x + y)^n = \sum_{r=0}^{n} ({}^nC_r)\, x^{n-r} y^r$$

9. Prove that for any positive integer n, a 2^nx2^n checkerboard C with any one square removed can be tiled using triominoes. (figure shows the four different orientations of triominoes)



10. Prove that every amount of postage that is at least 12 cents can be made from 4-cent and 5-cent stamps.

11. In the parlour game Nim, there are two players and two piles of matches. At each turn, a player removes some (non-zero) number of matches from one of the piles. The player who removes the last match wins. Prove that if the two piles contain the same number of matches at the start of the game, then the second player can always win.

12. Prove that every positive integer n, n≥2, can be expressed as the product of one or more prime numbers.

13. Suppose that $p_0 = 1$ and $p_x = \alpha p_{x+1}$ for all x = 1, 2, ... Prove by mathematical induction that $p_n = 1/\alpha^n$ for n = 0, 1, 2,. ...

14. Prove the correctness of insertion sort (given below) using mathematical induction.

```
// Sort an arr[] of size n
insertionSort(arr, n)
Loop from i = 1 to n-1.
……a) Pick element arr[i] and insert it into sorted sequence arr[0…i-1]
```

15. Consider the famous Fibonacci sequence $\{x_n\}$, defined by the relations $x_1 = 1$, $x_2 = 1$, and $x_n = x_{n-1} + x_{n-2}$ for n ≥ 3. Use an extended Principle of Mathematical Induction in order to show that for n ≥ 1,

$$x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Recursion

1. A computer system considers a string of decimal digits a valid code-word if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let a_n be the number of valid n-digit code-words. Find a recurrence relation for a_n.

2. Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0's.

3. Solve the following recurrence relations
   (i) $a_n = 5a_{n-1} - 4a_{n-2}$, $a_0 = 1,\ a_1 = 0$
   (ii) $a_n = 3na_{n-1}$, $a_0 = 2$
   (iii) $a_n = 2a_{n-1} + 1$

4. Find a recurrence relation with initial condition(s) satisfied by the following sequences. Assume a_0 is the first term of the sequence.
   (i) $a_n = 2^n$.
   (ii) $a_n = 3n - 1$.

5. Consider a savings plan in which $10 is deposited per month, and a 6% / year interest rate given with payments made every month. If P_n represents the amount in the account after n months, find a recurrence relation for P_n.

6. Give the recursive definition with initial condition for the function $f(n) = 5n + 2$, $n = 1,2,3, ...$

7. Consider the following recursive function f(x; y) s.t. $f(x; 0) = 0$; for all x, and $f(x; y) = f(x; y - 1) + x$, where x; y are non-negative integers. What does f(x; y) calculate?

8. Let T(n) be the number of comparisons required to find the minimum and maximum integers from a list of n positive integers. Which of the following value of T(n) is correct?

   (A) $T(n) = T(n - 1) + 1$
   (B) $T(n) = T(n - 1) + 2$
   (C) $T(n) = T(n/2) + 1$
   (D) $T(n) = T(n/2) + 2$

9. Write a code snippet to find gcd of 2 numbers a and b using recursion (a>b).

10. Tower of Hanoi is a mathematical puzzle where we have three rods and n disks. The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

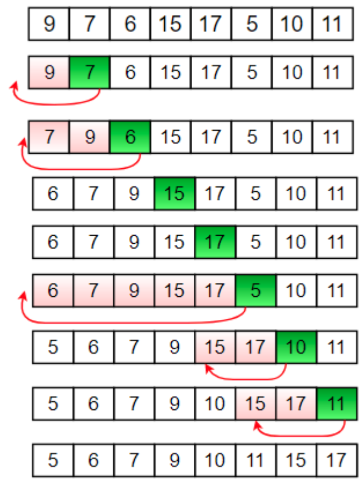
    a. Only one disk can be moved at a time.
    b. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
    c. No disk may be placed on top of a smaller disk.

    Solve the problem using recursion.

11. Given a "2 x n" board and tiles of size "2 x 1", count the number of ways to tile the given board using the 2 x 1 tiles. A tile can either be placed horizontally i.e., as a 1 x 2 tile or vertically i.e., as 2 x 1 tile.

12. Write a recursive code snippet to implement the following sorting algorithm

```
// Sort an arr[] of size n
insertionSort(arr, n)
Loop from i = 1 to n-1.
……a) Pick element arr[i] and insert it into sorted sequence arr[0…i-1]
```



The above sorting algorithm is known as insertion sort.

13. What does fun() do in general i.e. which mathematical operation does it implement (+, -, *, / , x^y, &, |, ~, etc.)?

```c
int fun(int a, int b) {

    if (b == 0) {
        return 0;
    }
    if (b % 2 == 0) {
        return fun(a+a, b/2);
    }

    return fun(a+a, b/2) + a;
}
```

14. Predict the output of fun(99). What does the following fun() do in general?

```c
int fun(int n)
{
    if (n > 100) {
        return n - 10;
    }
    return fun(fun(n+11));
}
```

Hint: This function is known as McCarthy 91 function.

15. What does the following recursive algorithm do?

```
void fun2(int arr[], int start_index, int end_index)
{
  if(start_index >= end_index)
     return;
  int min_index;
  int temp;

  /* Assume that minIndex() returns index of minimum value in
    array arr[start_index...end_index] */
  min_index = minIndex(arr, start_index, end_index);

  temp = arr[start_index];
  arr[start_index] = arr[min_index];
  arr[min_index] = temp;

  fun2(arr, start_index + 1, end_index);
}
```

16. Find the number of ways that a given integer, X can be expressed as the sum of the Nth powers of unique, natural numbers. For example, if X=13 and N=2 and, we have to find all combinations of unique squares adding up to 13. The only solution is 2^2 + 3^2.

17. In k-partition problem, we need to partition an array of positive integers into k disjoint subsets that all have equal sum and they completely covers the set.

For example, consider below set
S = { 7, 3, 5, 12, 2, 1, 5, 3, 8, 4, 6, 4 }

1. S can be partitioned into 2 partitions each having sum 30.

S_1 = { 5, 3, 8, 4, 6, 4 }
S_2 = { 7, 3, 5, 12, 2, 1 }

2. S can be partitioned into 3 partitions each having sum 20.

S_1 = { 2, 1, 3, 4, 6, 4 }
S_2 = { 7, 5, 8 }
S_3 = { 3, 5, 12 }

3. S can be partitioned into 4 partitions each having sum 15.

S_1 = { 1, 4, 6, 4 }
S_2 = { 2, 5, 8 }
S_3 = { 12, 3 }
S_4 = { 7, 3, 5 }

4. S can be partitioned into 5 partitions each having sum 12.

S_1 = { 2, 6, 4 }
S_2 = { 8, 4 }
S_3 = { 3, 1, 5, 3 }
S_4 = { 12 }
S_5 = { 7, 5 }

18. Given an integer, write a recursive function that returns true if the given number is palindrome, else false. For example, 12321 is palindrome, but 1451 is not palindrome.

19. What does the following recursive algorithm do?

```c
/* Assume that n is greater than or equal to 0 */
void fun2(int n)
{
  if(n == 0)
    return;

  fun2(n/2);
  printf("%d", n%2);
}
```

20. Given a string S, find count of all contiguous substrings starting and ending with same character.

```
Input  : S = "abcab"
Output : 7
There are 15 substrings of "abcab"
a, ab, abc, abca, abcab, b, bc, bca
bcab, c, ca, cab, a, ab, b
Out of the above substrings, there
are 7 substrings : a, abca, b, bcab,
c, a and b.

Input  : S = "aba"
Output : 4
The substrings are a, b, a and aba
```

21. What does the following recursive algorithm do? Also give the output.

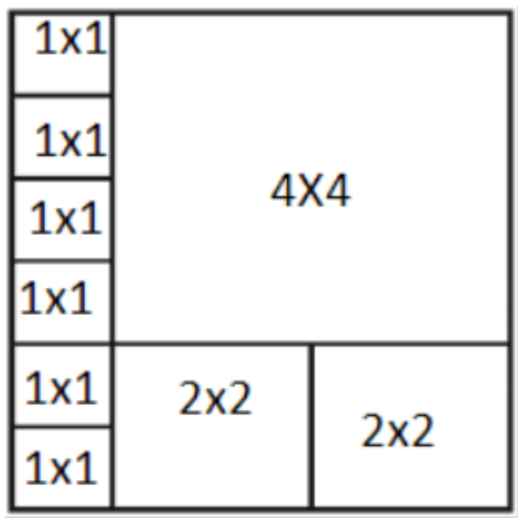
```c
int fun(int i)
{
  if ( i%2 ) return (i++);
  else return fun(fun( i - 1 ));
}

int main()
{
  printf(" %d ", fun(200));
  getchar();
  return 0;
}
```

22. Given an area of **N X M**. You have infinite number of tiles of size 2^i X 2^i, where i = 0, 1, 2… so on. The task is to find minimum number of tiles required to fill the given area with tiles.

```
Input : N = 10, M = 5.
Output : 14

Input : N = 5, M = 6.
Output : 9
Area of 5 X 6 can be covered with minimum 9 tiles.
6 tiles of 1 X 1, 2 tiles of 2 X 2, 1 tile of 4 X 4.
```



Relations

1. Which of the following relations are reflexive, symmetric and antisymmetric?

R1 = {(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)}

R2 = {(1,1),(1,2),(2,1)}

R3 = {(1,1),(1,2), (1,4),(2,1),(2,2),(3,3),(4,1),(4,4)}

R4 = {(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)}

R5 = {(1,1),(1,2), (1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)}

R1 = {(3,4)}

2. Let A and B be the set of all students and the set of all courses at a school, respectively. Suppose that R1 consists of all ordered pairs (a, b), where a is a student who has taken course b, and R2 consists of all ordered pairs (a, b), where a is a student who requires course b to graduate. What are the relations
    a. R1 ∪ R2
    b. R1 ∩ R2
    c. R1 ⊕ R2
    d. R1 − R2
    e. R2 − R1?

3. Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ∘ R.

4. Let R be the relation on the set of people consisting of pairs (a, b), where a is a parent of b. Let S be the relation on the set of people consisting of pairs (a, b), where a and b are siblings (brothers or sisters). What are S∘R and R∘S?

5. Suppose that the relation R on a set is represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

   Is R reflexive, symmetric, and/or antisymmetric?

6. Suppose that the relations R1 and R2 on a set A are represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

   What are the matrices representing R1 ∪ R2 and R1 ∩ R2?

7. Find the matrix representing the relation S ∘R, where the matrices representing R and S are:

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

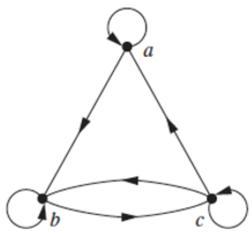
8. Find the matrix representing the relation R^2, where the matrix representing R is:

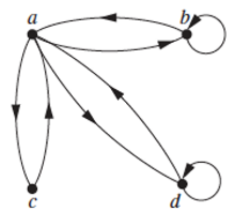
$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

9. Let R be the relation on N given by xRy iff x divides y. Determine which of the following properties applies to each relation.

   (i) Reflexive (ii) Irreflexive (iii) Symmetric (iv) Antisymmetric (v) Asymmetric (vi) Transitive

10. Determine whether the relations for the directed graphs shown in the following figures are reflexive, symmetric, antisymmetric, and/or transitive.





MCQs

11. The number of relations on an "n" element set that are symmetric is:

(A) 2^{n^2} (B) $2^{n(n-1)}$ (C) $2^{\frac{n(n+1)}{2}}$ (D) $3^{\frac{n(n-1)}{2}}$

12. Let R be the relation on R given by xRy if and only if x < y + 1.

(A) Reflexive, but not symmetric and not transitive.
(B) Reflexive, symmetric and not transitive.
(C) Not Reflexive, not symmetric and not transitive.
(D) Reflexive, but not symmetric and transitive.

13. Which of the relations on the given sets are antisymmetric?
(S1) A = {1, 2, 3, 4, 5}, R = {(1, 3), (1, 1), (2, 4), (3, 2), (5, 4), (4, 2)}
(S2) set of real numbers xRy iff $x^2 = y^2$.
(A) Only S1 (B) Only S2 (C) Both S1 and S2 (D) Nither S1 nor S2

14. Which relation R is not transitive?

(A) {(1, 1), (2, 2)} (B) {(1, 2), (2,3), (1, 3)}

(C) {(1, 2), (2, 1), (1, 1), (2,2)} (D) {(1, 2), (3, 3)}

15. Which of the following is equivalence relation?
(A) <= on Z.
(B) R = {(1, 2), (2, 3), (3, 1)} on the set {1, 2, 3}.
(C) | on Z, i.e. divide on Z
(D) R = {1, 2, 3} × {1, 2, 3} on the set {1, 2, 3}.

16. Which matrix represents an equivalence relation?

(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

17. Let A = {2, 4, 5, 10}. Which relation R is an equivalence relation?

(A) R = {(a,b) | a mod 2 = b mod 2} (B) R = {(a,b) | a mod 2 ≠ b mod 2}

(C) R = {(a,b) | a mod b = 0} (D) R = {(a,b) | a mod b = 2}

18. Consider the equivalence relation R
R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} on {1, 2, 3, 4}. The equivalence class for [1] is

(A) {1} (B) {1, 2} (C) {1,2,3} (D) {1,2,4}

19. If R is the equivalence relation defined on the set B={1,2,3,4} by
R = {(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4)} then the number of equivalence classes is:
(A) 1 (B) 2 (C) 3 (D) 4

20. Circle all which are equivalence relation.

(A) f(A, b) if a and b speak a common language.

(B) f(x, y) if x and y are bit strings of length 3 or more that agree at 3 or more bits

(C) f(f, g) if f(x) – g(x) = C for some constant C and for every x, where f and g are functions that map integers to integers

(D) f(a, b) if a and b earn the same final letter grade, where a and b are students

21. Let A = {2,3,57,8}. Which relation is an equivalence relation?
(A) R = {(a,b) | a < 2b} (B) R = {(a,b) | a mod 3 = b mod 2}
(C) R = {(a,b) | b mod a = 0} (D) R = {(a,b) | a + b is even}

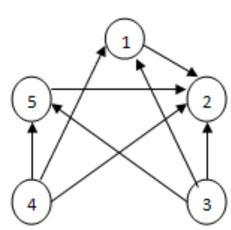
22. How many different equivalence relations are there on the set A = {a, b, c}

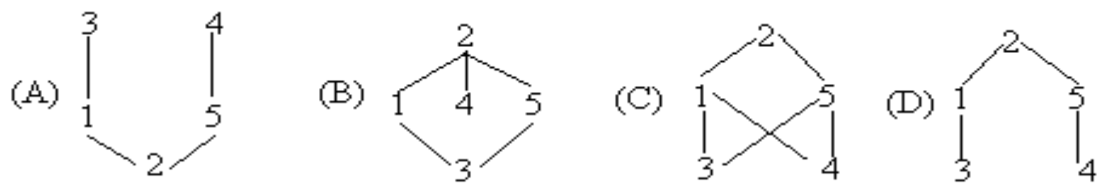
(A) 3 (B) 4 (C) 5 (D) 6

23. Which matrix represents a partial order relation?

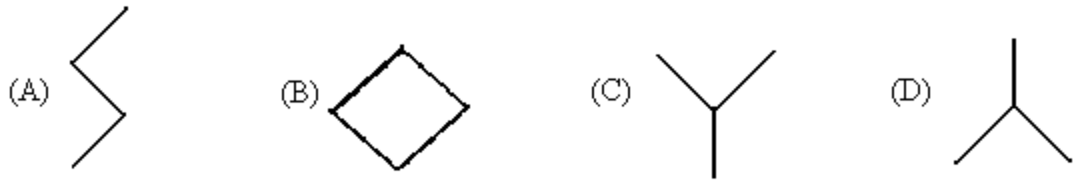
(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

24. Which one is the Hasse diagram for the following digraph?





25. A = {4,8,12,24} and R = {(a,b) | a divides b} ⊆ A x A. The Hasse diagram is



26. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be partially ordered by the division relation (that is, for a, b ∈ A, we say that a R b if a is a divisor of b). How many maximal elements are there for this partial order relation?

    (A) 5 (B) 2 (C) 3 (D) 4

27. Let A be a set. Consider the partial order ⊆ on P(A). Let C and D be subsets of A. Consider the following statement

    S1: The least upper bound of {C,D} is CUD

    S2: The greatest lower bound of {C,D} is C ∩ D.

    Which of the following statement is correct?

    (A) Only Si is true (B) Only S2 is true

    (C) Both S1 & S2 are true (D) Neither S1 nor S2 are true

28. Let A = {1, 2, 3, 4,5}. Which of the following is a partial order relation on A?

    (A) R = {(a, b) | b mod a = 3} (B) R = {(a, b) | a mod b = 0}

    (C) R = {(a, b) | a + b is even} (D) R = {(a, b) | a mod 3 = b}

29. Let be a relation R is defined as all even number are less than all odd numbers and the usual ordering is applied between the evens and the odds. Is R a total ordering relations. Also, give the order of the elements.

30. For which sets A of P(A) with set inclusion (⊆) a total ordering?

    (i) ∅ (ii) {a} (iii) {a, b} (iv) {a, b, c}

    (A) i & ii (B) ii and iii (C) iii and iv (D) i, ii, iii, iv

31. Let (S, ≤) be a partial order with two minimal elements a and b, and a maximum element c. Let P : S →{True, False} be a predicate defined on S. Suppose that P(a) = True, P(b) = False and P (x) ⇒ P(y) for all x, y ∈S satisfying x ≤ y, where ⇒ stands for logical implication. Which of the following statements

CANNOT be true?

(A) $P(x)$ = True for all $x \in S$ such that $x \neq b$

(B) $P(x)$ = False for all $x \in S$ such that $x \neq a$ and $x \neq c$

(C) $P(x)$ = False for all $x \in S$ such that $b \leq x$ and $x \neq c$

(D) $P(x)$ = False for all $x \in S$ such that $a \leq x$ and $b \leq x$

32. Let R be a binary relation on the set of all strings of 0's and 1's such that R={(a,b)| a and b are strings that have same number of 0s}. Which of the following statement is correct?
    - A) R is reflexive but not symmetric
    - B) R symmetric but not anti-symmetric
    - C) R is anti-symmetric but not transitive
    - D) None of the above

33. Let A be the set of all strings of 0′s, 1′s, and 2′s of length 4. Define a relation R on the set A as follows: $\forall s,t \in A, sRt$ iff the sum of the characters in s equals the sum of the characters in t. For example, the string "0121" is related to "2200". Which of the following statement is correct?
    - A) R is reflexive but not symmetric and transitive
    - B) R is reflexive and symmetric but not transitive
    - C) R is reflexive, symmetric and transitive
    - D) None of the above

34. The number of relation on a three element set that are both symmetric and antisymmetric is:
    - A) 3^2
    - B) 7
    - C) 2^6
    - D) 2^3

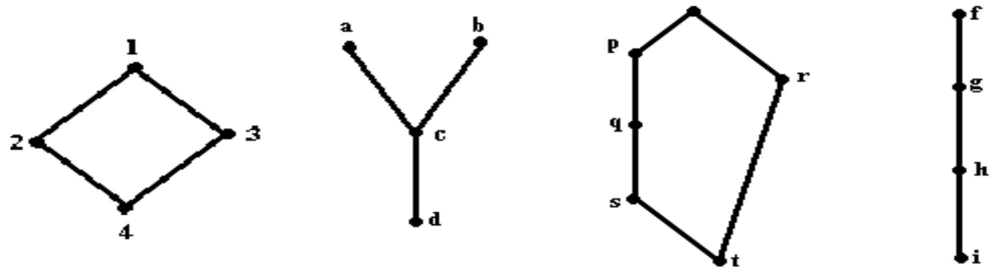
35. The number of relation on a three element set that are both symmetric and antisymmetric is:
    - A) 3^
    - B) 2^6
    - C) 0^
    - D) 5^

36. Which of the following statement is true?
    - A) the transitive closure of a symmetric relation is symmetric
    - B) the symmetric closure of a transitive relation is transitive
    - C) the reflexive closure of a transitive relation is transitive
    - D) the transitive closure of an antisymmetric relation is antisymmetric

37. Let r(R), s(R) and t(R) be the reflexive, symmetric and transitive closures of a relation R respectively. Which of the following statement is NOT correct?
    - A) r(s(R)) = s(r(R))
    - B) s(t(R)) = t(s(R))

C) $r(t(R)) = t(r(R))$
D) $t(s(r(R))) = r(t(s(R)))$

38. Which of the following Hasse diagrams represent lattices?



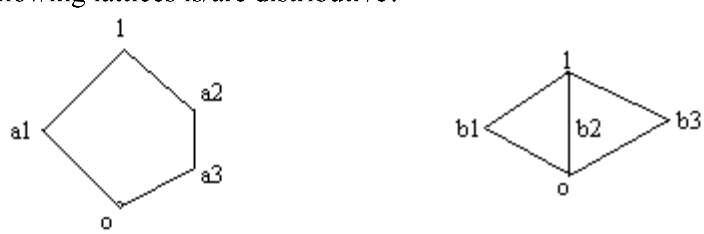
(I) **(II)** **(III)** **(IV)**

(A) I and III only
(B) I, III and IV only
(C) II and IV only
(D) I, II, III and IV only.

39. Let A be any set such that A = {1, 2, 3, 4, 5, 6} and R be any relation defined as $(a, b) \in R$ if a divides b then

(A) R forms lattice over A
(B) R doesn't form lattice over A but R forms poset
(C) Neither R form lattice over A nor it forms poset over A
(D) None of the above

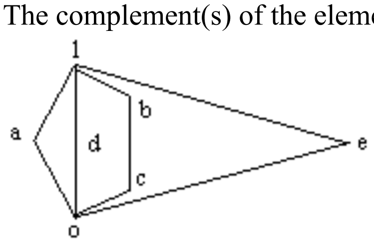
40. Which of the following lattices is/are distributive?



I **II**

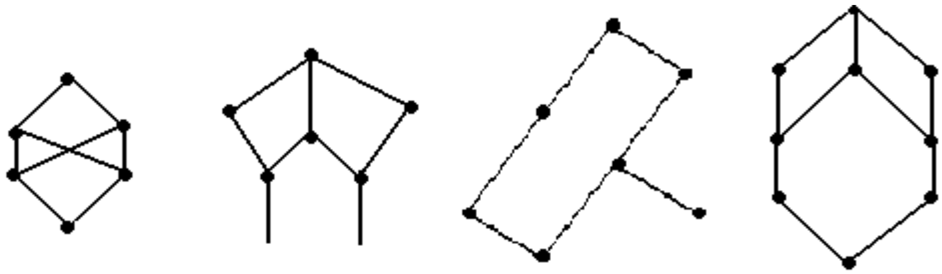
(A) I (B) II (C) both (D) none

41. The complement(s) of the element 'a' in the lattice shown in fig.



(A) e only (B) b, c and e only (C) b, c, d and e only (D) none

42. Which of the following Hasse diagram represent lattice?



(A) (B) (C) (D)

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43. A relation R is defined on ordered pairs of integers as follows (x, y) R(u, v) if x < u and y> v. Then R is

    Equivalence relation, Total Order relation, Partial Order relation?

44. Consider the set S = {a, b, c, d}. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$, on

    S : $\pi_1 = \overline{\{abcd\}}$, $\pi_2\{\overline{ab}, \overline{cd}\}$, $\pi_3 = \overline{\{abc}, \overline{d}\}$, $\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$

    Let $\prec$ be the partial order on the set of partitions S' = ($\pi_1, \pi_2, \pi_3, \pi_4$) defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for (S', $\prec$) is

45. Draw the hasse diagram of relation R = {(1,1), (2,2), (3,3), (4,4), (5,5), (3,1), (1,4), (5,1), (3,4), (5,4)}

46. In a partially ordered set, a chain is a totally ordered subset. For example, in the set 1, 2, 3, 4, 5, 6, the divisibility relation is a partial order and 1, 2, 4 and 1, 3, 6 are chains. What is the longest chain on the set {1, 2 . . . n} using the divisibility relation?

47. What is the longest chain on the power set of a set A with |A| = n with the ⊆ relation?

48. Given Poset ({3,5,9,15,24,45},/).
    a) Find the maximal elements.
    b) Find the minimal elements.
    c) Is there a greatest element?
    d) Is there a least element?
    e) Find all upper bounds of {3,5}.
    f) Find the least upper bound of {3,5}, if it exists.
    g) Find all lower bounds of {15,45}.
    h) Find the greatest lower bound of {15,45}, if it exists.

49. Consider R with usual order ≤ :

    a). Find lub{x ∈ R : x < 73} b). Find lub{x ∈ R : x2 < 73} c). Is [R; ≤] a lattice?

50. For the elements x, y, z in a poset, show that if lub[x,y] = a and lub[a,z] = b, then lub[x,y,z] = b.

51. List all the binary relations on the set {0, 1}.

52. A company makes four kinds of products. Each product has a size code, a weight code, and a shape code. The following table shows these codes:

	Size Code	Weight Code	Shape Code
#1	42	27	42
#2	27	38	13
#3	13	12	27
#4	42	38	38

Find which of the three codes is a primary key. If none of the three codes is a primary key, explain why.

53. If X =(Fran Williams, 617885197, MTH 202, 248B West), find the projections P1,3(X) and P1,2,4(X).

54. R and S are relations on {a, b, c, d}, where R = {(a, b), (a, d), (b, c), (c, c), (d, a)} and S = {(a, c), (b, d), (d, a)}.
Find the following combination of relations.
i) R^2, ii) $R^{3,}$. iii) S^2. iv) S^3. v) R : S. vi) S : R.

55. Calculate R^{-1}, where R is the relation on {1, 2, 3, 4} such that a R b means $|a - b| <= 1$.
56. Determine the number of bitwise operations required to compute the transitive closure of a relation R defined on a set having n element, using:
    a. Boolean powers of $M_{R.}$
    b. Warshall's algorithm

Programming Problems

1. Write an algorithm, which would take a relation as an input and determine if it is reflexive, symmetric, antisymmetric, and transitive. Also, give some test cases that you would use to check your algorithm.

2. Write an algorithm for finding the transitive closure of a relation using Warshall's algorithm.

3. Write an algorithm which will output the composition of two given relations, R & S.

4. Write an algorithm, which would take a relation as an input and will output its reflexive, symmetric, and transitive closures. Also, give some test cases that you would use to check your algorithm.

5. Write an algorithm which will compute the transitive closure of a relation R, using the Boolean powers of M_{R}.
