Relations & Functions

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Topics

- Relations
- Representation of Relations
- n-ary Relations
- Equivalence relations
- Partially ordered set
- Totally ordered set
- Hasse diagrams
- Well ordered set
- Functions

Introduction

- Functions are important in many areas of Mathematics
- Elementary algebra starts to differ from arithmetic, when the concepts of a function is developed
- Calculus is study of functions, and of certain ways of associating new functions with a given one
- We start by talking about binary relations (generalization of functions)

Relations

- A (binary) relation ρ from a set S to a set T is a rule that stipulates, given any element s of S and any element t of T, whether s bears a certain relationship to t (written $s \rho t$) or not (written $s \neg\rho t$)
- A relation is the subset of the cartesian product of S & T.
- S= set of living males

  T= set of living females

  ρ= is the son of

  if s denotes a certain male and t denotes a certain woman, we write $s \rho t$ if s is son of t, and $s \neg\rho t$, otherwise

Relations

- Examples of Binary Relations:

1. Set of students and set of courses
2. Set of all cities of India and set of all states of India
3. $A=\{0,1,2\}$ & $B=\{a,b\}$

Functions as Relations:

f from A to B is a relation which assigns exactly one element of B to each element of A.

Relations are generalizations of Functions

Relations

- A relation on a set A is a relation from A to A.
- A={1,2,3,4}. R={(a,b)|a divides b}

Problem:

$R1 = \{(a, b) \mid a \leq b\}$,

$R2 = \{(a, b) \mid a > b\}$,

$R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,

$R4 = \{(a, b) \mid a = b\}$,

$R5 = \{(a, b) \mid a = b + 1\}$,

$R6 = \{(a, b) \mid a + b \leq 3\}$.

Which of these relations contain each of the pairs (1, 1), (1, 2), (2, 1), (1,−1), and (2, 2)?

Relations

- How many relations are there on a set A with n elements?

Properties of Relations

- Reflexive
- Symmetric
- Transitive

Properties of Relations

- Reflexive

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Consider the following relations on $\{1, 2, 3, 4\}$:

$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,

$R2 = \{(1, 1), (1, 2), (2, 1)\}$,

$R3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$R4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$R5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,

$R6 = \{(3, 4)\}$.

Which of these relations are reflexive?

Properties of Relations

- Reflexive

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

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Properties of Relations

- Reflexive

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Is the Divides relation on a set of positive integers Reflexive?

All integers?

Properties of Relations

- Symmetric

A relation R on a set A is called *symmetric* if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$. A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called *antisymmetric*.

Properties of Relations

- Transitive

A relation R on a set A is called *transitive* if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Anti-symmetric Property

A relation R on a set A such that for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ is called anti-symmetric.

Consider the following relations on {1, 2, 3, 4}:

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},

R2 = {(1, 1), (1, 2), (2, 1)},

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},

R6 = {(3, 4)}.

Which of these relations are antisymmetric?

Anti-symmetric Property

A relation R on a set A such that for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ is called anti-symmetric.

$R1 = \{(a, b) \mid a \leq b\}$,

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$R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,

$R4 = \{(a, b) \mid a = b\}$,

$R5 = \{(a, b) \mid a = b + 1\}$,

$R6 = \{(a, b) \mid a + b \leq 3\}$.

Which of these relations are anitsymmetric?

Combining Relations

- Union, Intersection, and difference of relations