BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: TEMPORAL LOGICS

Linear Temporal Logic: Semantics of Formulas

Formulas and Interpretation

- The meaning of a formula in Linear Temporal Logic is obtained by
  - evaluating the formula under a given model $M = (S, \rightarrow, L)$
  - in a specific path $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$
  - where $s_i \in S$ for each $i >= 1$

Formulas and Interpretation

- We will start with the known connectives (from propositional logic):
  - Let $M = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in M.
  - Then define the *satisfaction relation* $|=$ as follows:
    - $\pi \models \text{TRUE}$
    - $\pi \not\models \text{FALSE}$ (or $\pi \not\models \perp$)
    - $\pi \models p$ iff $p \in L(s_1)$
    - $\pi \models \neg\phi$ iff $\pi \not\models \phi$
    - $\pi \models \phi_1 \wedge \phi_2$ iff $\pi \models \phi_1$ and $\pi \models \phi_2$
    - $\pi \models \phi_1 \vee \phi_2$ iff $\pi \models \phi_1$ or $\pi \models \phi_2$
    - $\pi \models \phi_1 \dashrightarrow \phi_2$ iff $\pi \models \phi_2$ whenever $\pi \models \phi_1$

BITS Pilani
Pilani Campus

innovate achieve lead

CS/IS F214
Logic in Computer Science

MODULE: **TEMPORAL LOGICS**

Linear Temporal Logic: Semantics

- Unary Operators and Their Semantics

Formulas and Interpretation

- Now let us consider some ***temporal*** connectives (and formulas) :
  - **X** ϕ /* read ne**X**t ϕ */
  - **G** ϕ /* read **G**lobal ϕ or **H**enceforth ϕ */
  - **F** ϕ /* read **F**uture ϕ or **E**ventually ϕ */

- Semantics of these temporal operators:
  - Let $M = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in M.
  - Then define the *satisfaction relation* $|=$ as follows:
    - $\pi \mid= X\ \phi$ **iff** $\pi^2 \mid= \phi$
    - $\pi \mid= G\ \phi$ **iff for all** $i>=1$ $\pi^i \mid= \phi$
    - $\pi \mid= F\ \phi$ **iff for some** $i>=1$ $\pi^i \mid= \phi$

innovate achieve lead

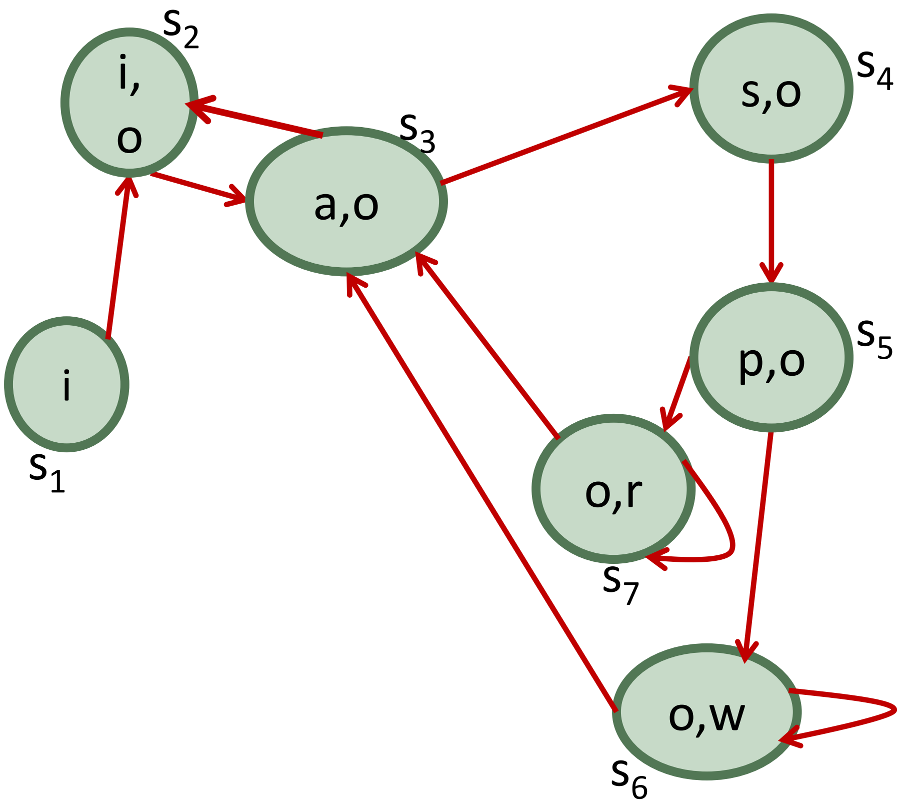
BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: **TEMPORAL LOGICS**

Expressing Formulas in LTL: Examples

Examples



Let π_w be

$s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4 \rightarrow s_5 \rightarrow s_6 \rightarrow s_3$, … i.e.

where $\mathbf{s_3} \rightarrow \mathbf{s_4} \rightarrow \mathbf{s_5} \rightarrow \mathbf{s_6} \rightarrow$ is repeated *ad infinitum*.

Let π_{rr} be

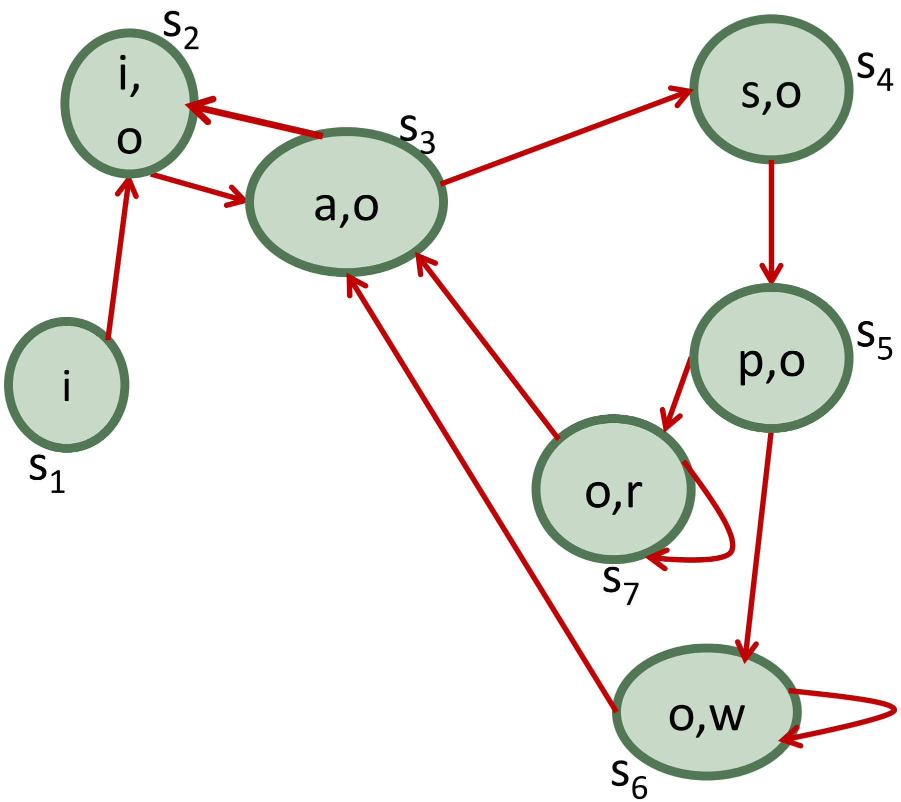
$s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4 \rightarrow s_5 \rightarrow s_7 \rightarrow s_7 \rightarrow s_3$, … i.e.

where $\mathbf{s_3} \rightarrow \mathbf{s_4} \rightarrow \mathbf{s_5} \rightarrow \mathbf{s_7} \rightarrow \mathbf{s_7} \rightarrow$ is repeated *ad infinitum*.

Encode the following in LTL:

- state s_6 will be reached eventually in π_w
- state s_3 will be reached eventually in π_{rr} irrespective of where you start after s_1

Examples



Encode the following and argue whether they are true or false:

a) a *read* or *write* will eventually happen (in π_w)

b) the system will eventually be *idle* and *rotating* (in π^2_w)

c) a *write* will eventually happen (in π^2_w)

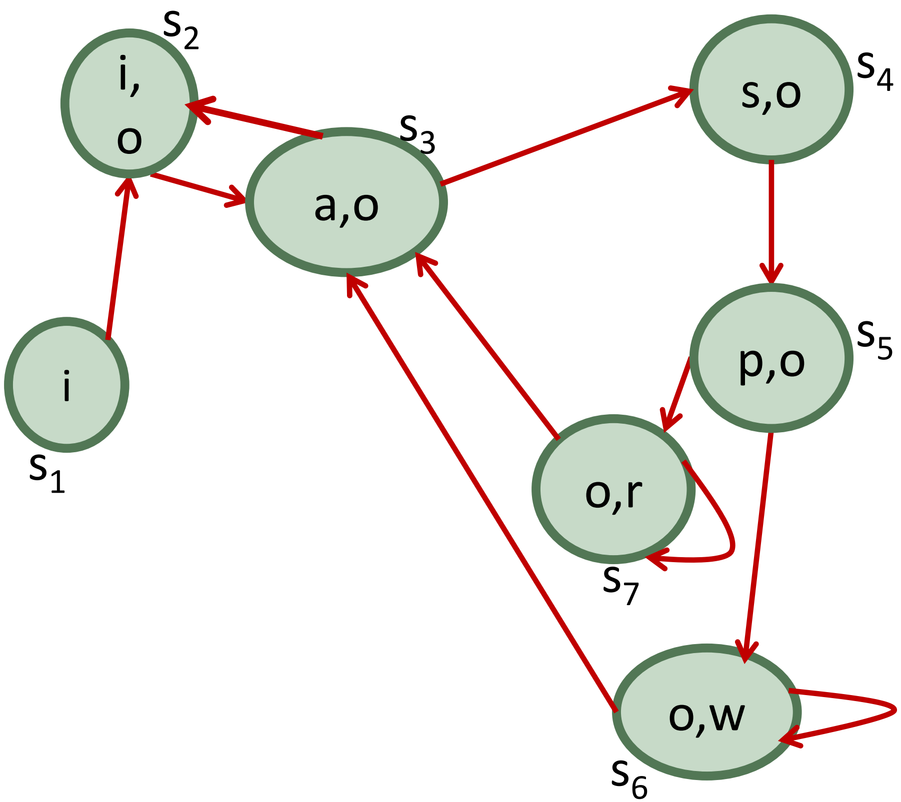
d) the disk will always be *rotating* (in π_{rr})

e) the disk will always be *rotating* (in π^2_{rr})

f) there will be a <u>sequence of two *reads*</u> eventually (in π^2_{rr})

g) there will be a <u>sequence of four *reads*</u> eventually (in π^2_{rr})

Examples



Encode the following and argue whether they are true or false:

a) if the head is seeking in the next state
then eventually a read will happen (in π^2_{rr})

b) if the head is seeking in the next state
then eventually a read will happen (in π^3_{w})

c) if the head is seeking in the next state
an operation will be pending in the state after that (in π^2_{w})