MATHEMATICS-II (MATH F112)

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Section 5.5

Isomorphism

Isomorphism

A LT $L : V \to W$ that is both one-to-one and onto is called as isomorphism from V to W.

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Similarly, $L(cp) = cL(p)$ for all real c and $p \in P_n$. Hence, L is a linear operator.

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Q:. Check if the linear operator $L:\mathbb{R}^3 \to \mathbb{R}^3$ given by $L(e_1) = e_1 + e_2, L(e_2) = e_2 + e_3, L(e_3) = e_1 + e_2 + e_3$ is an isomorphism.

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Sol. For $u = [x, y, z] \in \mathbb{R}^3$, $L(u) = L([x, y, z]) =$ $L([xe_1 + ye_2 + ze_3]) = xL(e_1) + yL(e_2) + zL(e_3) =$ $[x + z, x + y + z, y + z]$.

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Now $L([x, y, z]) = [0, 0, 0] \implies [x + z, x + y + z, y + z] = [0, 0, 0] \implies x = y = z = 0$. Since $\mathbf{ker}(L) = \{0_{\mathbb{R}^3}\}$, L is one-to-one.L is also onto onto since $\mathbf{dim}(\text{range}(L)) = 3 - 0 = 3 = \mathbf{dim}(\mathbb{R}^3)$, from Dimension Theorem. Hence, L is an isomorphism.

Example 3

Q:. Check if the LT $L : P_4 \to \mathbb{R}^3$ given by $L(p) = [p(-1), p(0), p(1)]$ is an isomorphism.

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Sol. Basis for $\ker(L)$ is $\{x(x^2-1), x^2(x^2-1)\}$.Hence, $\dim(\ker(L)) = 2$.

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Sol. Basis for $\ker(L)$ is $\{x(x^2-1), x^2(x^2-1)\}$.Hence, $\dim(\ker(L)) = 2$.

Hence, L is not one-to-one, i.e., it is not an isomorphism.

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Q:. Show that the linear operator $L : P_2 \longrightarrow P_2$ given by $L(a+bt+ct^2) = (b+c)+(a+c)t+(a+b)t^2$ is an isomorphism.

Q:. Show that the linear operator $L : M_{mn} \longrightarrow M_{nm}$ given by $L(A) = A^T$ is an isomorphism.

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Such a function M, denoted by L^{-1}, is called an inverse of L.

Theorem: A LT $L : V \to W$ is an isomorphism if and only if L is an invertible LT.

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Sol. It can be easily verified that L is both one-to-one and onto. Hence, invertible.

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$$\implies x = a, y = \frac{b+c-2a}{2}, z = \frac{c-b}{2}.$$

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Hence, $L^{-1}(a+bt+ct^2) = [a, \frac{b+c-2a}{2}, \frac{c-b}{2}]$.

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On solving, we get
$a_1 = \frac{1}{2}(b+c-a), b_1 = \frac{1}{2}(a+c-b), c_1 = \frac{1}{2}(a+b-c).$

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Hence,
$L^{-1}(a+bt+ct^2) = \frac{1}{2}(b+c-a) + \frac{1}{2}(a+c-b)t + \frac{1}{2}(a+b-c)t^2.$

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Sol. $\ker(L) = \{p \in P_2 | L(p) = 0_{\mathbb{R}^2}\} \implies$

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$p = (x-1)(ax-a) \implies p = a(x^2 - 2x + 1)$.

Now, $p(1) = 0 \implies p = (x-1)(ax+b)$. Also, $p'(1) = 0 \implies a = -b$. Hence, $p = (x-1)(ax-a) \implies p = a(x^2-2x+1)$. Thus $\ker(L) = \{a(x^2-2x+1) | a \in \mathbb{R}\}$.

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Is L invertible. If yes, find L^{-1}.

Sol. $L^{-1}([x, y, z]) = [y - z, y - x, x - y + z].$

Isomorphism

Let V and W be vector spaces. Then V is isomorphic to W, denoted by $V \cong W$, if and only if there exists an isomorphism $L : V \to W$.

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