Mechanics, Waves & Oscillations
PHY F111

Instructor

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Books:

1. An Introduction to Mechanics by Kleppner/Kolenkow

2. Problems in General Physics by I.E Irodov

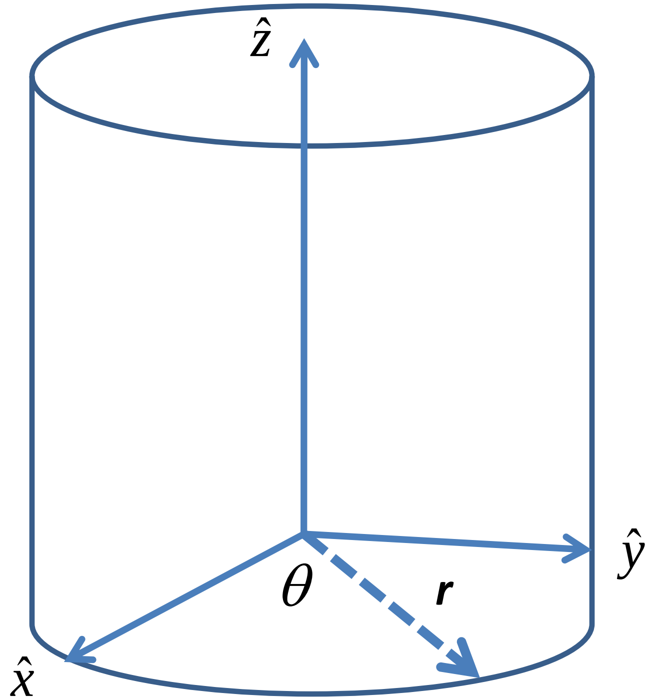
3. Vibrations & Waves by A.P French

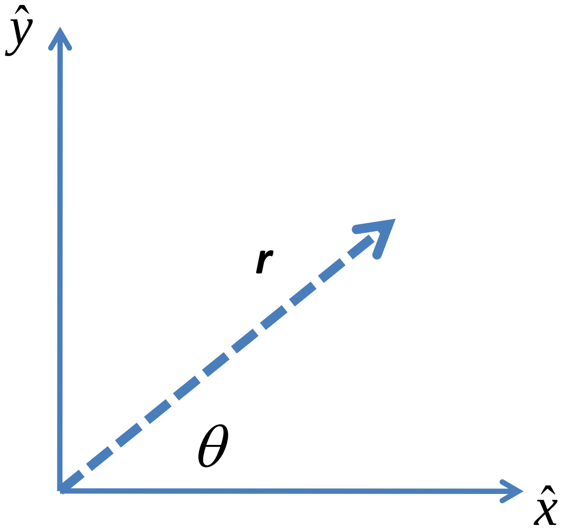
A Few Mathematical Preliminaries

Coordinate Systems:

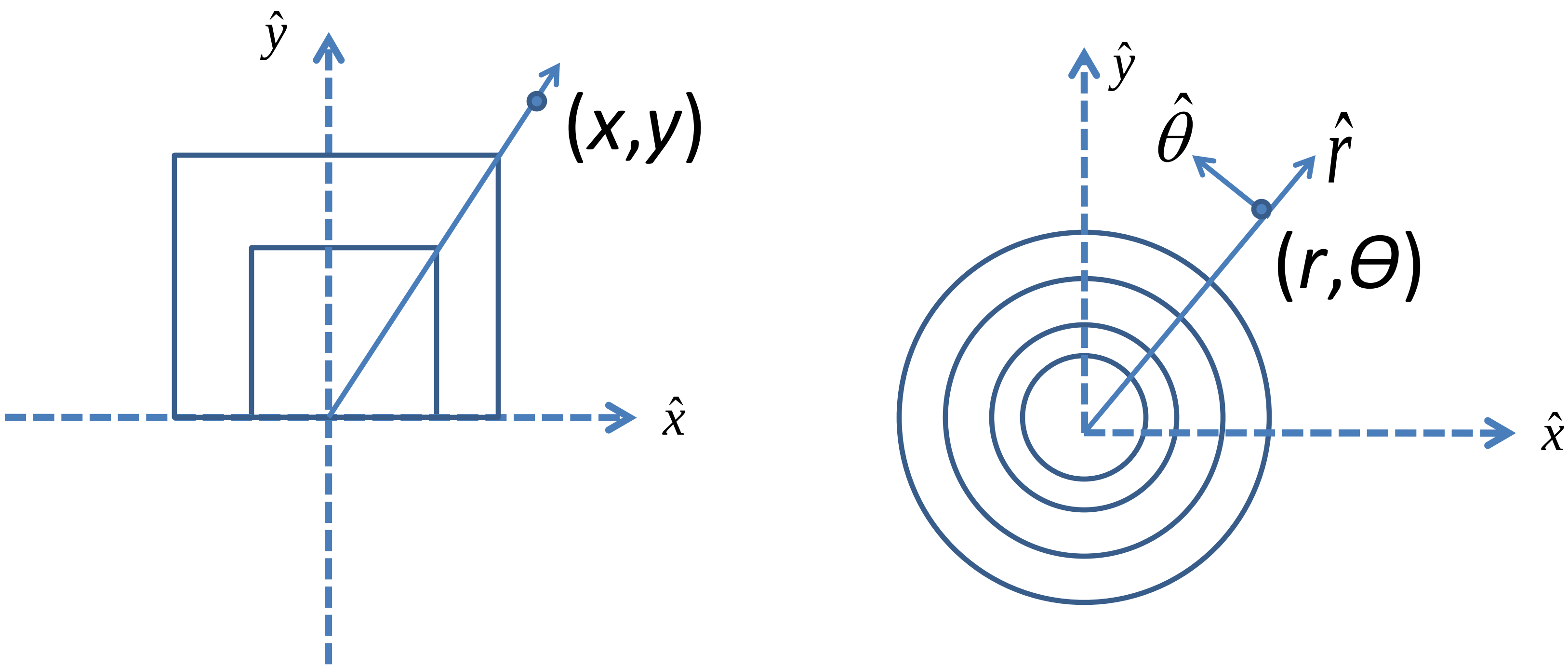
1. **Cartesian System**
2. **Spherical Polar Coordinates**
3. **Cylindrical Coordinate Systems**

Cylindrical Coordinate Systems

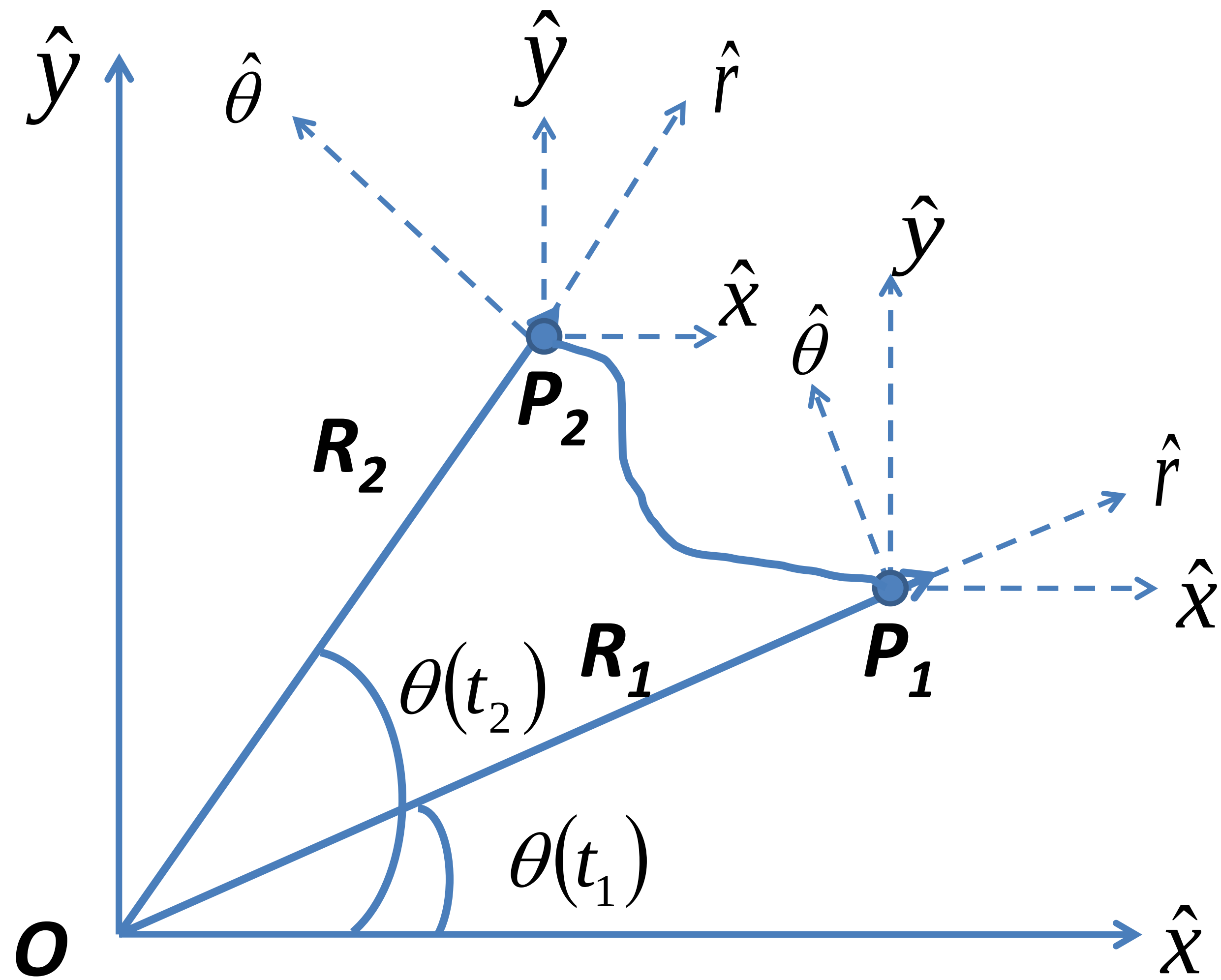




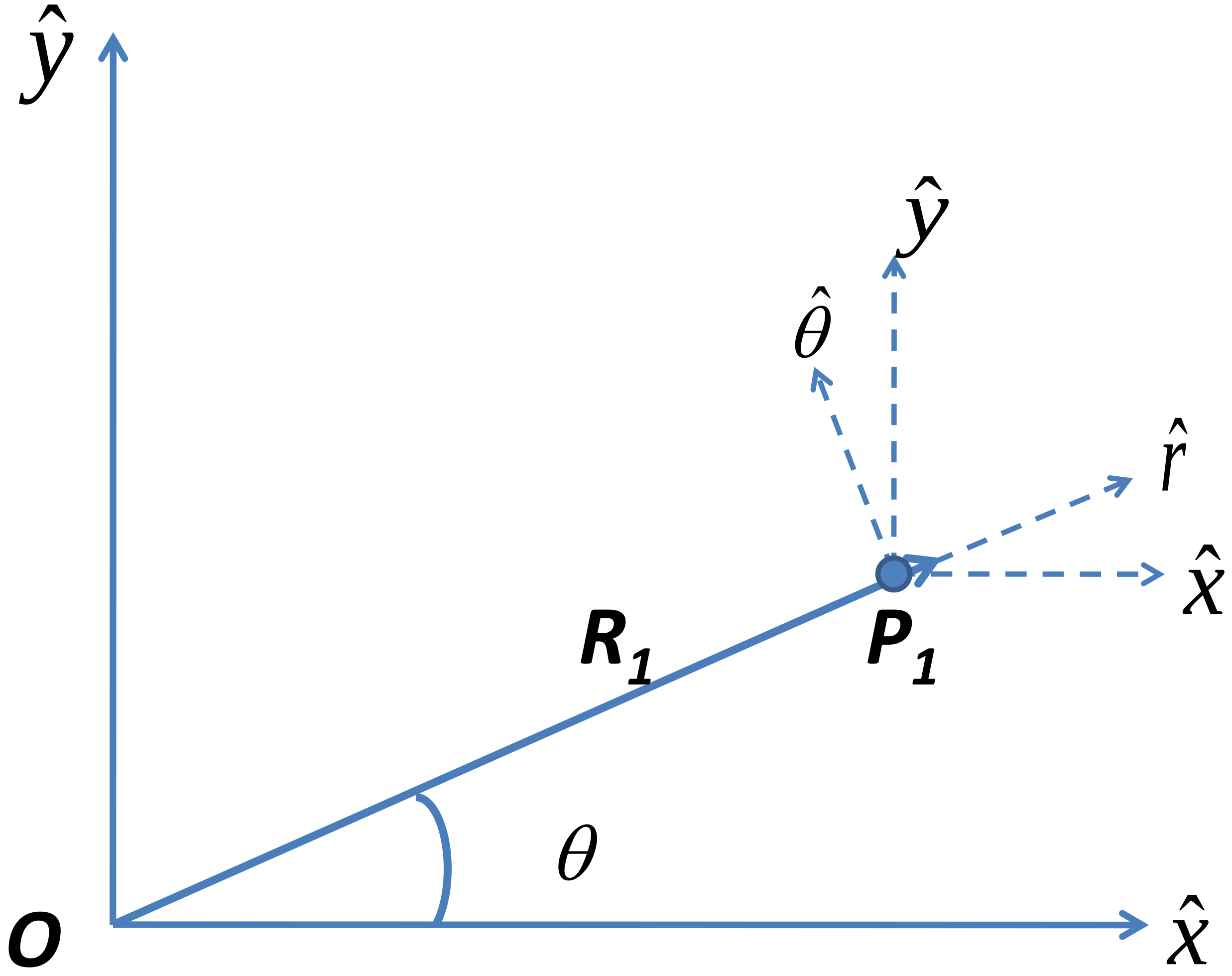
Cartesian Coordinate
vs.
Plane Polar



Motion in Plane Polar Coordinates



Motion in Plane Polar Coordinates



Velocity in Plane Polar Coordinates

$$\vec{v} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

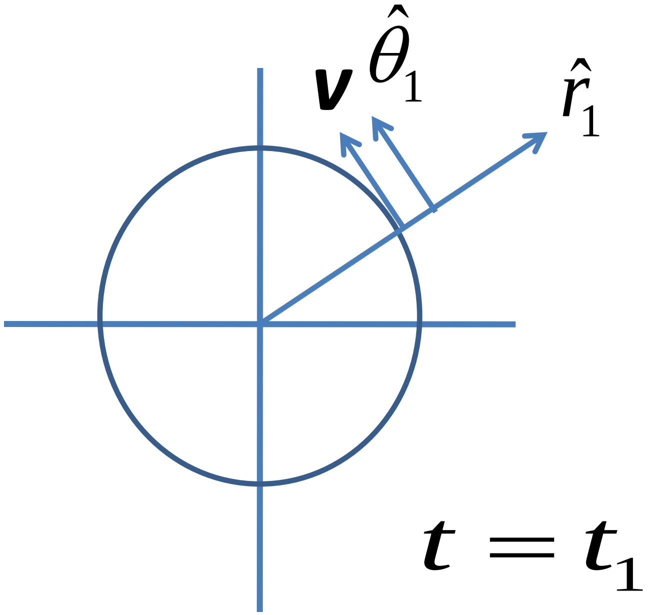
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Important Results:

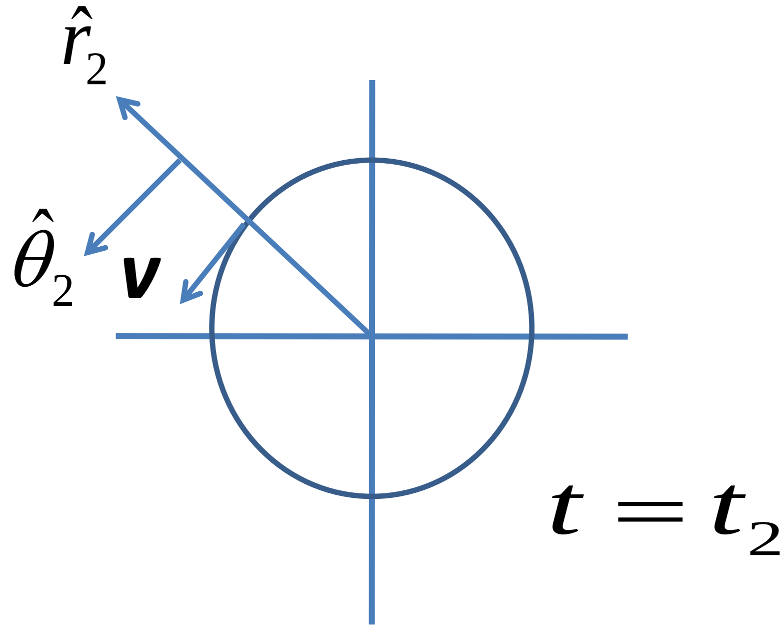
$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

Circular Motion in Polar Coordinates

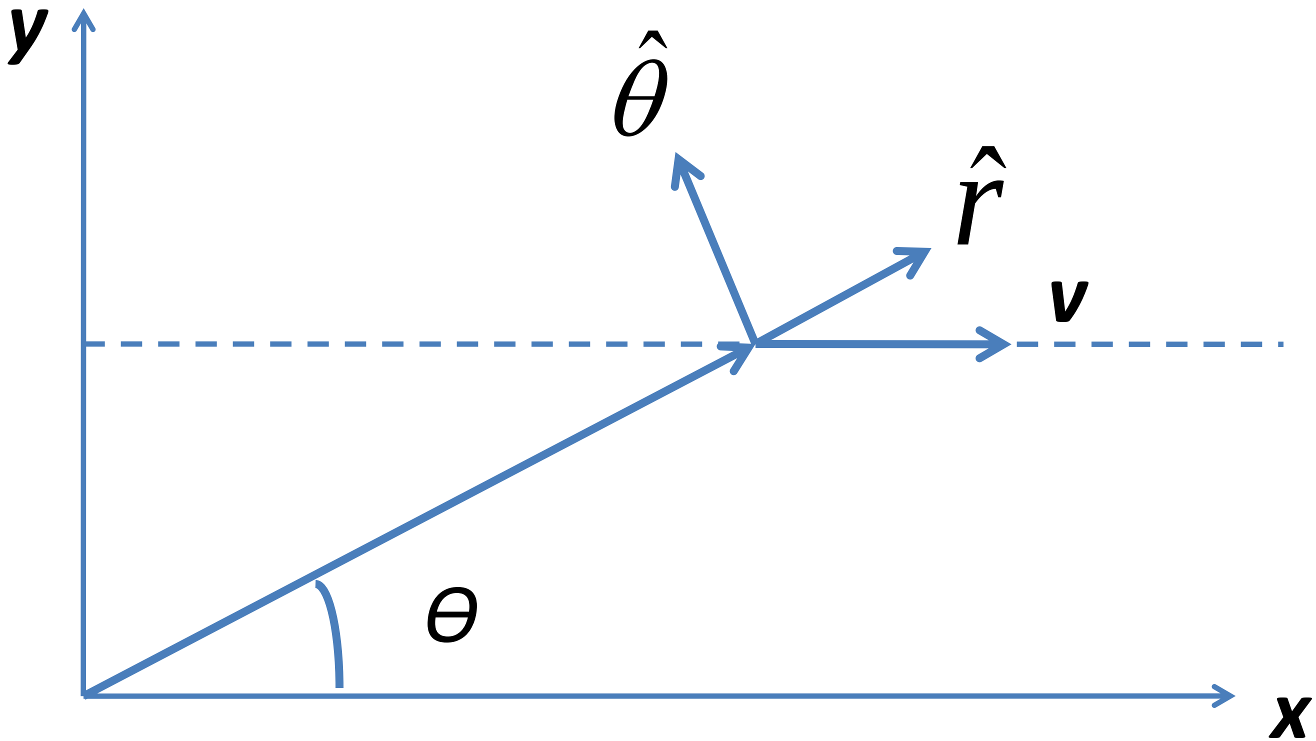


(a)

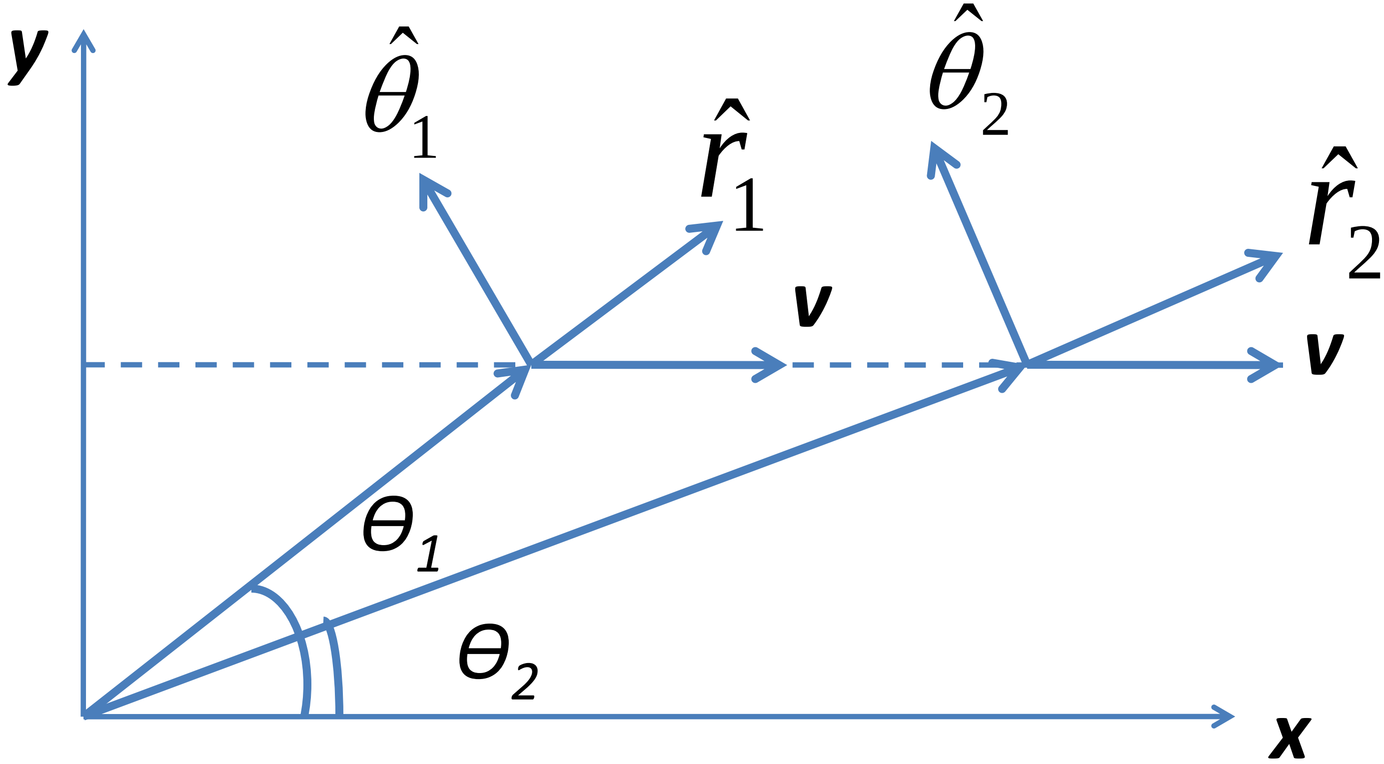


(b)

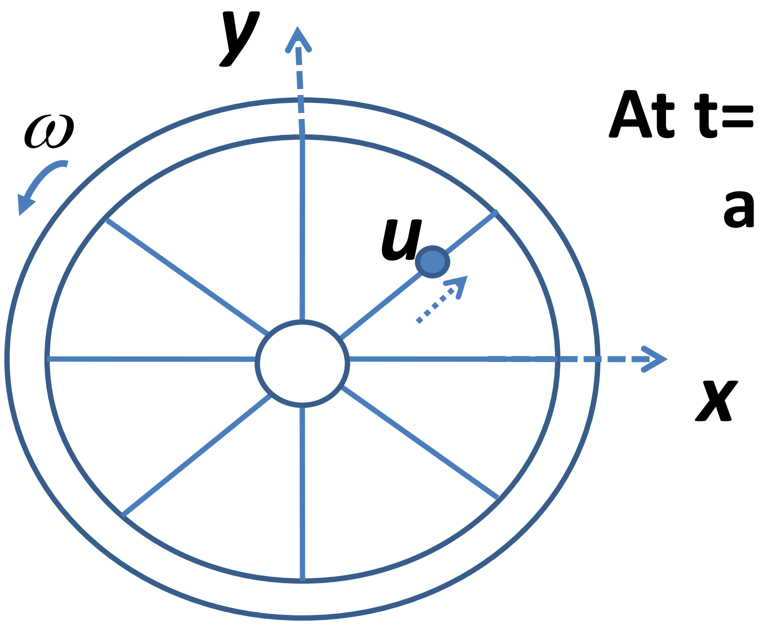
Straight line
Motion in Polar Coordinates



Straight line
Motion in Polar Coordinates



Velocity of a Bead on a Spoke



At t=0, the spoke is along the x-axis and the bead is at the origin.

Find the velocity of the bead at time t :

(a)In Polar Coordinates

(a)In Cartesian Coordinates

Acceleration in Polar Coordinates

$$\vec{a} = \frac{d}{dt}\vec{v}$$

$$= \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right)$$

$$= \ddot{r}\hat{r} + \dot{r}\frac{d}{dt}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d}{dt}\hat{\theta}$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

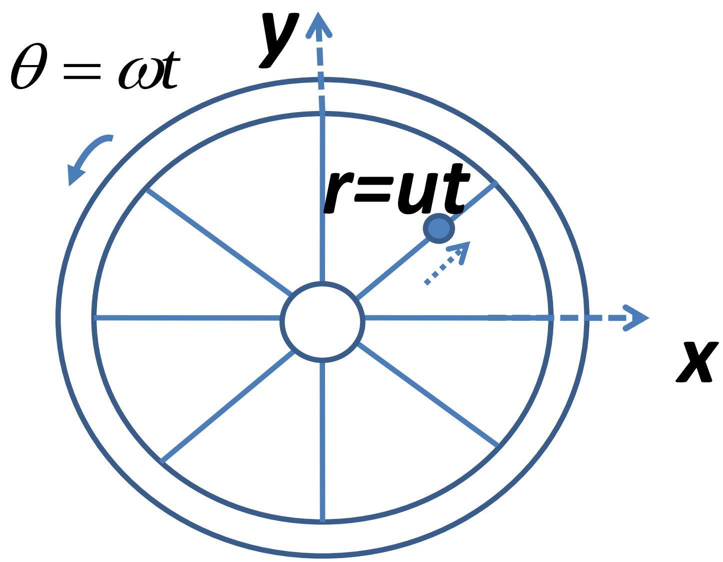
Linear
Acceleration
In the Radial Direction

Centripetal
Acceleration

Linear
Acceleration
the Tangential Direction

Coriolis
Acceleration

Acceleration of a Bead on a Spoke



At t=0, the spoke is along the x-axis and the bead is at the origin.

Newton's Laws of Motion

Newton's First Law:

- It is always possible to find a coordinate system with respected to which Isolated bodies move uniformly.
- Newton's fist law of motion is an assertion that inertial systems exist.

Newton's Laws of Motion

Newton's Second Law:

- The Total Force *F* on a body of mass *m* is :
- $$\vec{F} = \sum_i \vec{F}_i$$
- where $\vec{F}_i$ is the ith applied force.

Newton’s Laws of Motion

Newton’s Second Law:

- **If ‘a’ is the net acceleration, and a_i the acceleration due to F_i alone, then we have :**

$$\vec{F} = \sum_i \vec{F}_i = \sum_i m\vec{a}_i = m\sum_i \vec{a}_i = m\vec{a}$$

$$\vec{F} = m\vec{a}$$

Newton's Laws of Motion

Newton's Third Law:

- **The fact that force is necessarily the result of an interaction between two systems is made explicitly by Newton's third law.**

- **The Third law states that forces always appear in pairs;**

- **If body 'b' exerts force F_a on body 'a', then there must be a force F_b acting on body 'b', due to body 'a', such that**

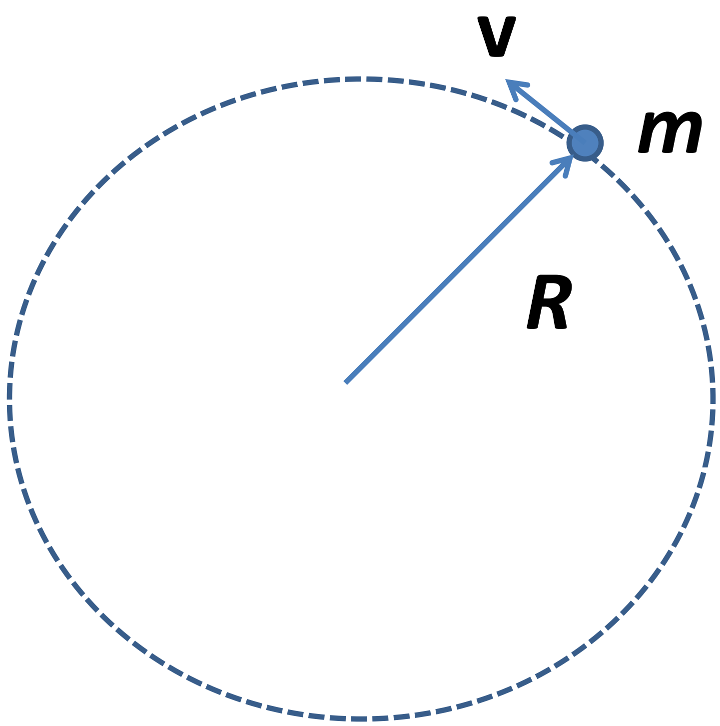
$$\vec{F}_b = -\vec{F}_a$$

Applying Newton’s Law:

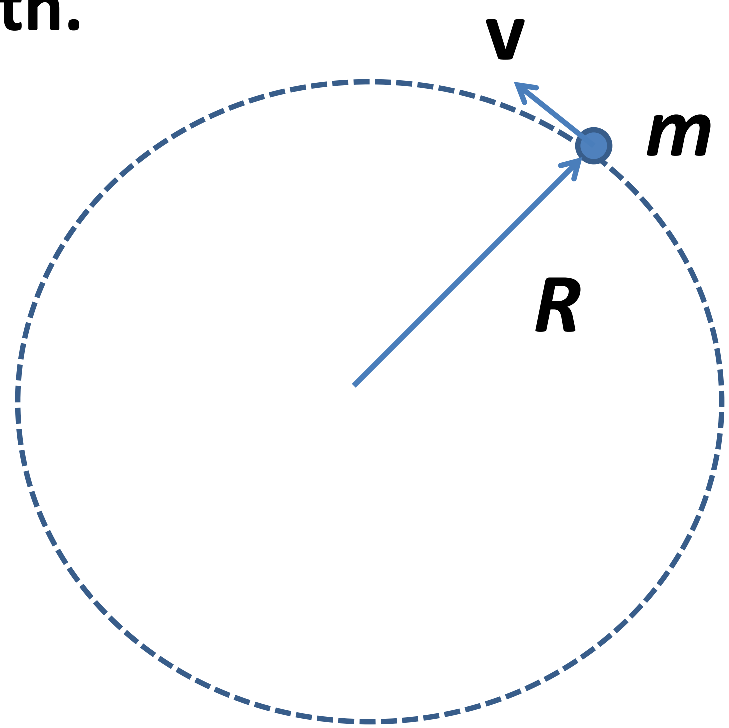
- **Mentally divide the system into smaller systems, each of which can be treated as a point mass.**

- **Draw a force diagram for each mass.**

Problem: Mass m whirls with constant speed v at the end of a string of length R. Find the force on m in the absence of gravity or friction

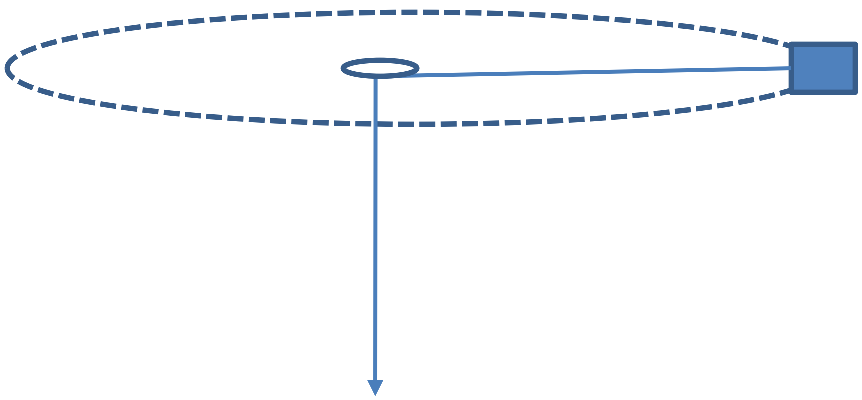


Problem: Mass m whirls with constant speed v at the end of a string of length R. Find the force on m if the motion is in a vertical plane in the gravitational field of the earth.

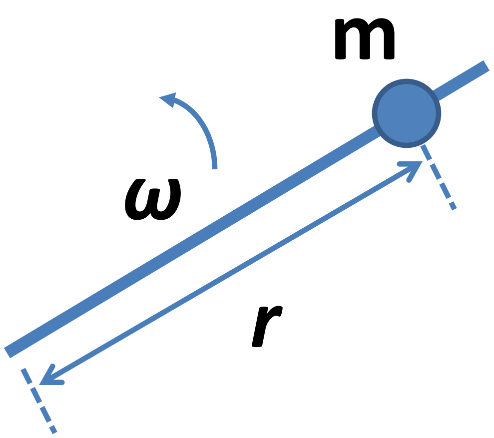


Problem 2.34: A mass m whirls around on a string which Passes through a ring as shown. Neglect Gravity. Initially the mass is distance r_0 from the center and is revolving at angular velocity ω_0. The string is pulled with constant velocity V starting at t=0 so that the radial distance to the mass decreases. Draw a force diagram and obtain a differential equation for ω.

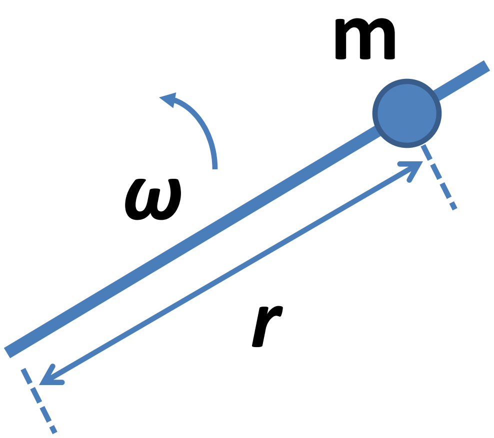
Find (a) ω(t) and (b) The force needed to pull the string.



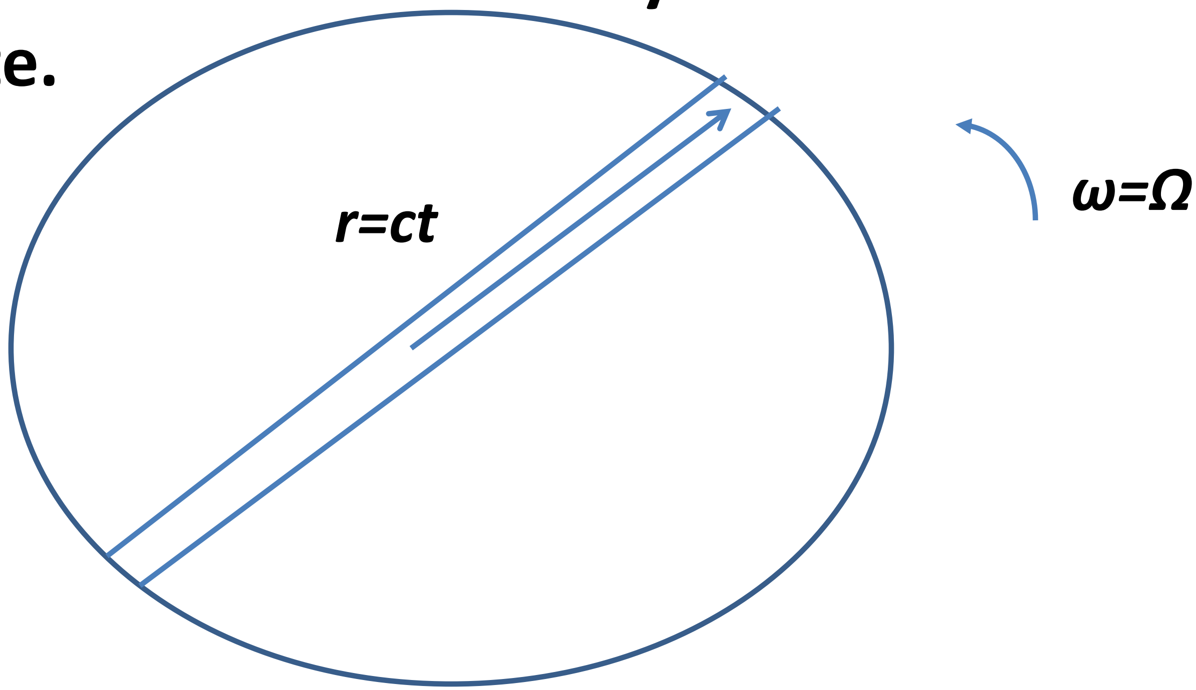
Problem 2.33: A particle of mass m is free to slide on a thin rod. The rod rotates in a plane about one end at constant angular velocity ω. Show that the motion is given by $r = Ae^{-\Upsilon t} + Be^{\Upsilon t}$, where ϒ is a constant which you must find and A and B are arbitrary constants. Neglect Gravity.



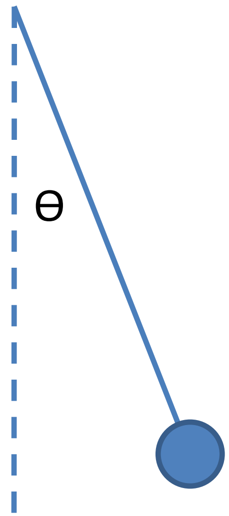
Problem 2.33:Show that for a particular choice of initial Conditions [that is, r(t=0) and v(t=0)], it is possible to obtain a solution such that *r* decreases continually in time, but that for any other choice *r* will eventually increase.



Problem: A particle sliding along a radial groove in a rotating turntable has polar coordinates at time t given by $r=ct$, $\theta=\Omega t$, where c and Ω are positive constants. Find the velocity and acceleration vectors of the particle at time t and find the speed of the particle at time t. Deduce that, for $t>0$, the angle between the velocity and acceleration vectors is always acute.



Problem: The mass of a certain pendulum moves on a vertical circle of radius b and, when the string makes an angle θ with the downward vertical, the circumferential velocity v of the bob is given by $v^2=2gb\cos\theta$, where g is a positive constant. Find the acceleration of the mass when the string makes an angle θ with the downward vertical.



An aircraft flies on a horizontal trajectory such that its polar coordinates (r, θ) at time t are given as:

$$r = \frac{\alpha t}{\beta^2}(2\beta - t) \quad \text{and} \quad \theta = \frac{t}{\beta} \quad \text{for} \quad (0 \le t \le 2\beta),$$

where α and β are positive constants.

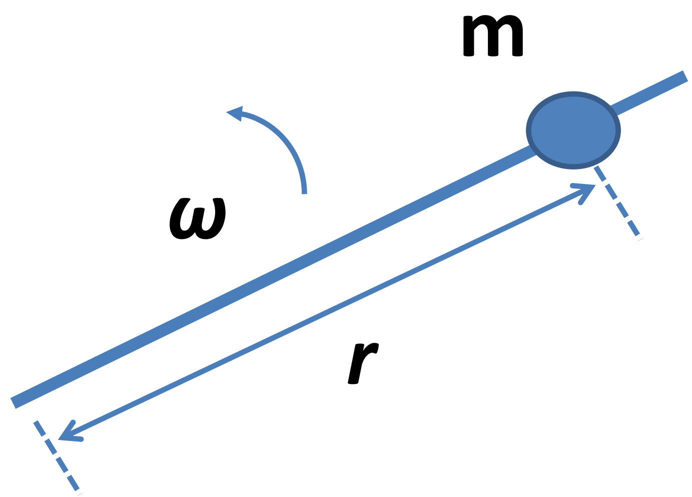
(a) Write the expression for the velocity in polar coordinates for the aircraft at time t.

(b) Calculate the expression for the minimum speed achieved by the aircraft.

(c) Calculate the expression for the acceleration when the speed of the aircraft is minimum.

Problem: Consider a particle that feels an angular force only, of the form $F_\theta = m\dot{r}\dot{\theta}$. Show that $\dot{r} = \sqrt{A \ln r + B}$, where A and B are constants of Integration, determined by the initial conditions.

Problem: Consider a particle that feels an angular force only, of the form $F_\theta = 3m\dot{r}\dot{\theta}$. Show that $\dot{r} = \pm\sqrt{Ar^4 + B}$, where A and B are constants of Integration, determined by the initial conditions.

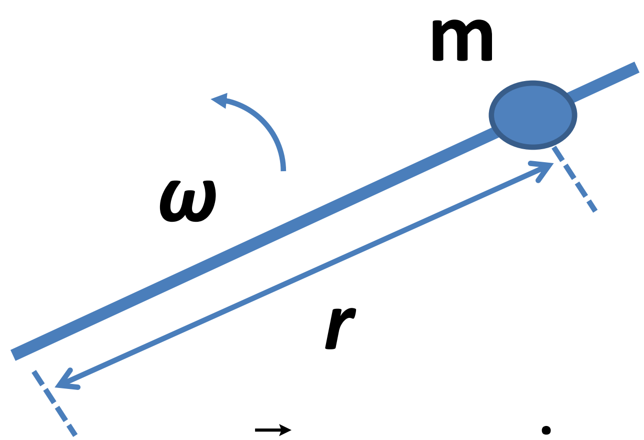


$$\vec{F}_\theta = 2m\dot{r}\dot{\theta}$$

$$\dot{\theta} = \omega$$

Angular velocity is constant in this case

$$\dot{r}(t) = -\omega A\exp(-\omega t) + \omega B\exp(\omega t)$$

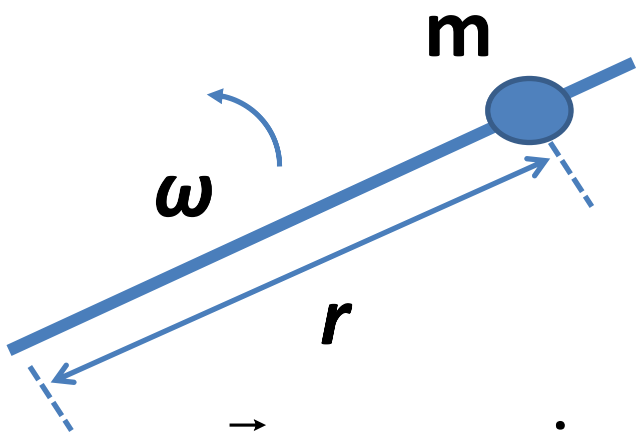


$$\vec{F}_\theta = m\dot{r}\dot{\theta}$$

$$\dot{\theta} = \frac{D}{r}$$

D is a constant in this case

$$\dot{r} = \sqrt{A\ln r + B}$$



$$\vec{F}_{\theta} = 3m\dot{r}\dot{\theta}$$

$$\dot{\theta} = Dr$$

D is a constant in this case

$$\dot{r} = \pm\sqrt{Ar^{4} + B}$$