

CS/IS F214 Logic in Computer Science

MODULE: PREDICATE LOGIC

Predicate Logic

Proofs and Proof Rules: Introduction

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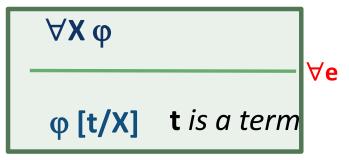
Universally Quantified Variables and Inference

- Given a statement of the form:
 - ∀X (bird(X) --> fly(X))
 - one can infer the following:
 - bird(fancy_piggy) --> fly(fancy_piggy)
 - bird(seagull(jonathon)) --> fly(seagull(jonathon))
- This amounts to an <u>inference rule</u>:
 - If a <u>statement with a universally quantified variable</u>, say
 X, is true
 - then <u>any instance of that statement</u> i.e. the <u>statement with any term substituted for X</u> – should be true.



Inference Rule for Universally Quantified Formulas

- The inference rule can be stated formally as follows:
 - Proof Rule (<u>Elimination of universal quantifier</u>):

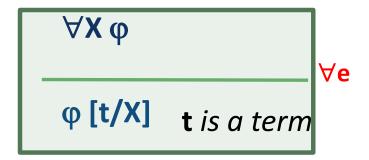


- where $\varphi[t/X]$ denotes
 - the <u>formula</u> ϕ in which each <u>occurrence of variable **X**</u> has been replaced with term **t**.



Inference Rule for Universally Quantified Formulas

Proof Rule (<u>Elimination of universal quantifier</u>):



- φ[t/X] is usually read as:
 - φ with **t** (*substituted*) for **X**
- Note that φ[t/X] is not a formula (in our language):
 - it denotes the substitution operation that results in a formula



Existentially Quantified Variables and Inference

- Given a statement of the form:
 - mammal(platypus) ∧ lays_eggs(platypus)
 - one can infer the following:
 - ∃X (mammal(X) ∧ lays_eggs(X))
- Given a statement of the form:
 - ugly(duckling) \(\triangle \text{becomes(swan(duckling))} \)
 - one can infer the following:
 - ∃X (ugly(X) ∧ becomes(swan(X)))



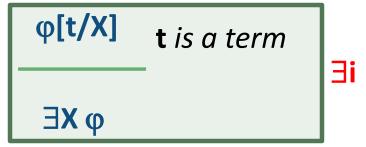
Existentially Quantified Variables and Inference

- From: mammal(platypus) ∧ lays_eggs(platypus)
 - infer : $\exists X \text{ (mammal(X)} \land lays_eggs(X))$
- From: ugly(duckling) \(\triangle \text{becomes(swan(duckling))} \)
 - infer : ∃X (ugly(X) ∧ becomes(swan(X)))
- This kind of reasoning amounts to an <u>inference rule</u>:
 - If a statement is true for a concrete value
 - then the <u>same statement existentially quantified by a</u> <u>variable</u>, say X, <u>is true</u>
 - where X replaced by the concrete instance yields the original statement.



Inferring Existentially Quantified Formulas

- The inference rule can be stated formally as follows:
 - Proof Rule (Introduction of existential quantifier):



- where denotes $\phi[t/X]$ the formula ϕ in which each occurrence of variable X has been replaced with term t.
- Note that the *substitution* operation referred here is the same one in the proof rule $\forall e$
 - but this proof rule uses it in reverse

