Chapter 3

Dynamics of a System of Particles

$$\vec{F} = M\vec{a}$$

$$\vec{F} = \frac{d}{dt}\left(M\vec{v}\right)$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

The force on the j^th particle is:

$$\vec{f}_j = \frac{d\vec{p}_j}{dt}$$

Dynamics of a System of Particles

The force on the jth particle can be split into:

$$\vec{f}_j = \vec{f}_{j\,\mathrm{int}} + \vec{f}_{j\,ext}$$

Now, the equation of motion becomes:

$$\vec{f}_{j\,\mathrm{int}} + \vec{f}_{j\,ext} = \frac{d\vec{p}_j}{dt}$$

Now, if we focus on the entire system:

$$\vec{f}_{1^{\text{int}}} + \vec{f}_{1^{ext}} = \frac{d\vec{p}_1}{dt}$$

....

$$\vec{f}_{j^{\text{int}}} + \vec{f}_{j^{ext}} = \frac{d\vec{p}_j}{dt}$$

....

$$\vec{f}_{N^{\text{int}}} + \vec{f}_{N^{ext}} = \frac{d\vec{p}_N}{dt}$$

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Now, by adding all the above equations:

$$\sum \vec{f}_{j^{\text{int}}} + \sum \vec{f}_{j^{ext}} = \sum \frac{d\vec{p}_j}{dt}$$

The summation extend over all particles,

$$j = 1, \ldots\ldots, N.$$

Dynamics of a System of Particles

Now, the second term :

$$\sum \vec{f}_{j^{ext}}$$

is the sum of all external forces acting on all the particles.

Therefore, we have:

$$\sum \vec{f}_{j^{ext}} = \vec{F}_{ext}$$

Dynamics of a System of Particles

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The first term :

$$\sum \vec{f}_{j^{\text{int}}} = 0$$

Therefore, we have:

$$\vec{F}_{ext} = \sum \frac{d\vec{p}_j}{dt}$$

Finally, we have

$$\vec{F}_{ext} = \sum \frac{d\vec{p}_j}{dt}$$

$$= \left(\frac{d}{dt}\right)\sum \vec{p}_j$$

$$= \frac{d\vec{P}}{dt}$$

Dynamics of a System of Particles

Hence, the total external force applied to a system equals the rate of change of the systems momentum.

Center of Mass

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$\vec{F} = M\ddot{\vec{R}}$$

Center of Mass

$$M\ddot{\vec{R}} = \frac{d\vec{P}}{dt} = \sum m_j \ddot{\vec{r}}_j$$

$$M\ddot{\vec{R}} = \sum m_j \ddot{\vec{r}}_j$$

$$\ddot{\vec{R}} = \frac{1}{M} \sum m_j \ddot{\vec{r}}_j$$

Center of Mass

$$\vec{R} = \frac{1}{M} \sum m_j r_j$$

➢***R*** is a vector from the origin to the point called
the Center of Mass.

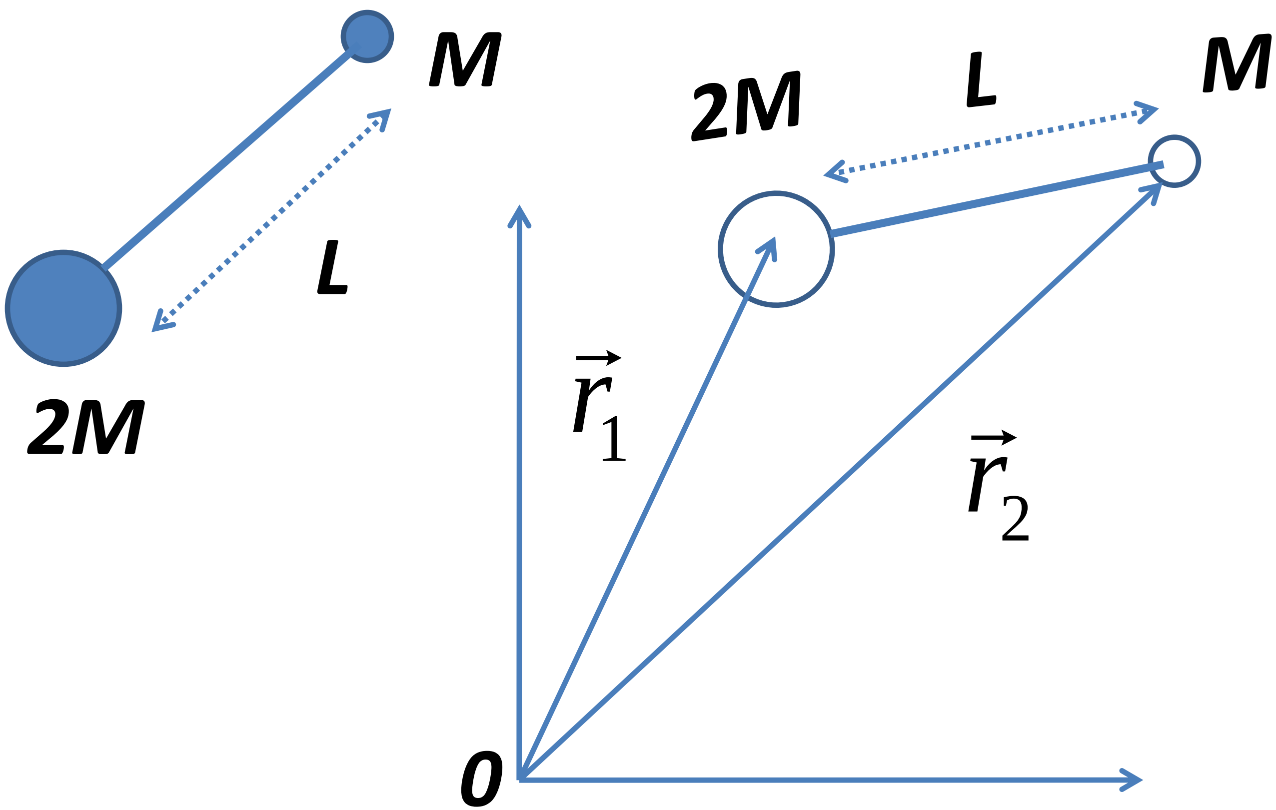
➢The system behaves as if all the mass is concentrated at the Center of Mass and all the external forces act at that point.

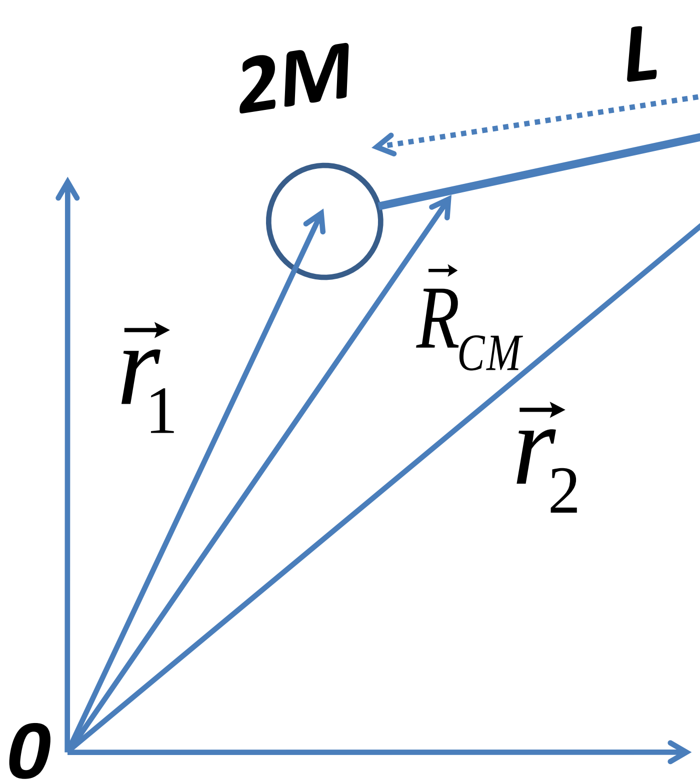
Center of Mass

- The relation $\vec{F} = M\ddot{\vec{R}}$ describes only the translation of the body i.e. the motion of its center of mass.

- It does not describe the body's orientation in the space.

Center of Mass





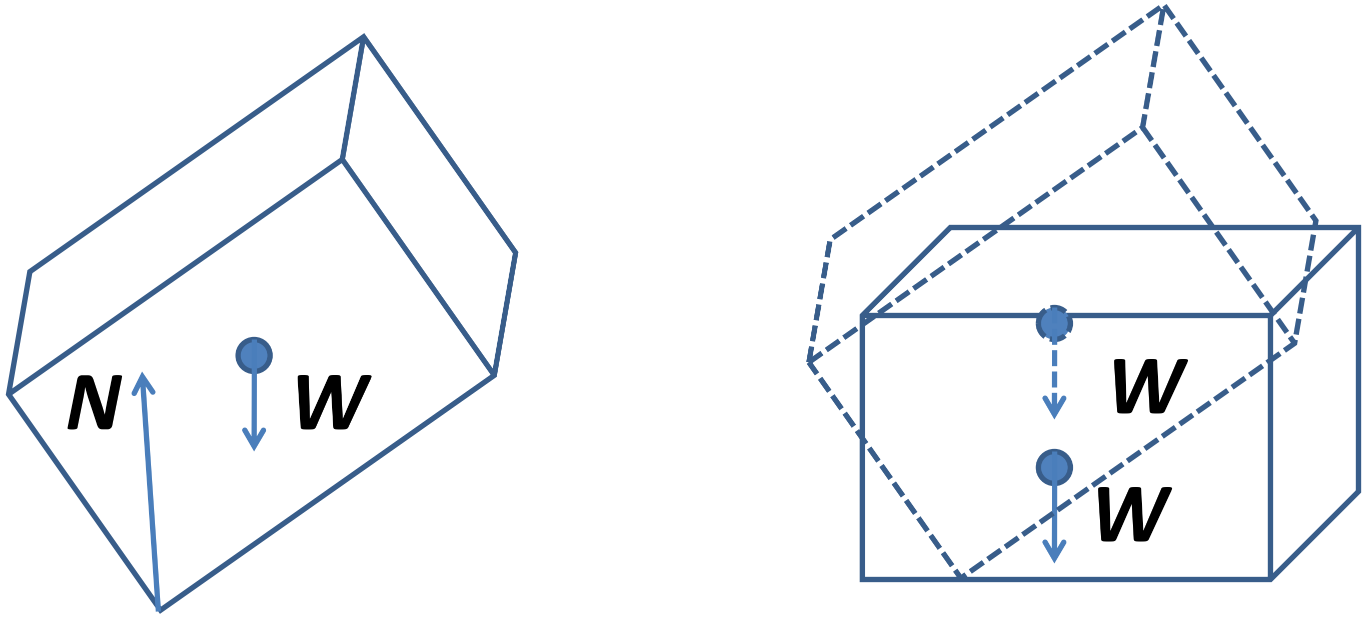
$$\vec{R}_{CM} = \frac{2M\vec{r}_1 + M\vec{r}_2}{3M}$$

$$3M\ddot{\vec{R}}_{CM} = 2M\ddot{\vec{r}}_1 + M\ddot{\vec{r}}_2 = 3M\vec{g}$$

$$\ddot{\vec{R}}_{CM} = \vec{g}$$

- **The Center of Mass will follow a parabolic trajectory of a single mass in a uniform gravitational field.**

Center of Mass Motion



- If the box is released from the rest, then its center falls straight down.

Conservation of Momentum

$$\vec{F} = \frac{d\vec{P}}{dt}$$

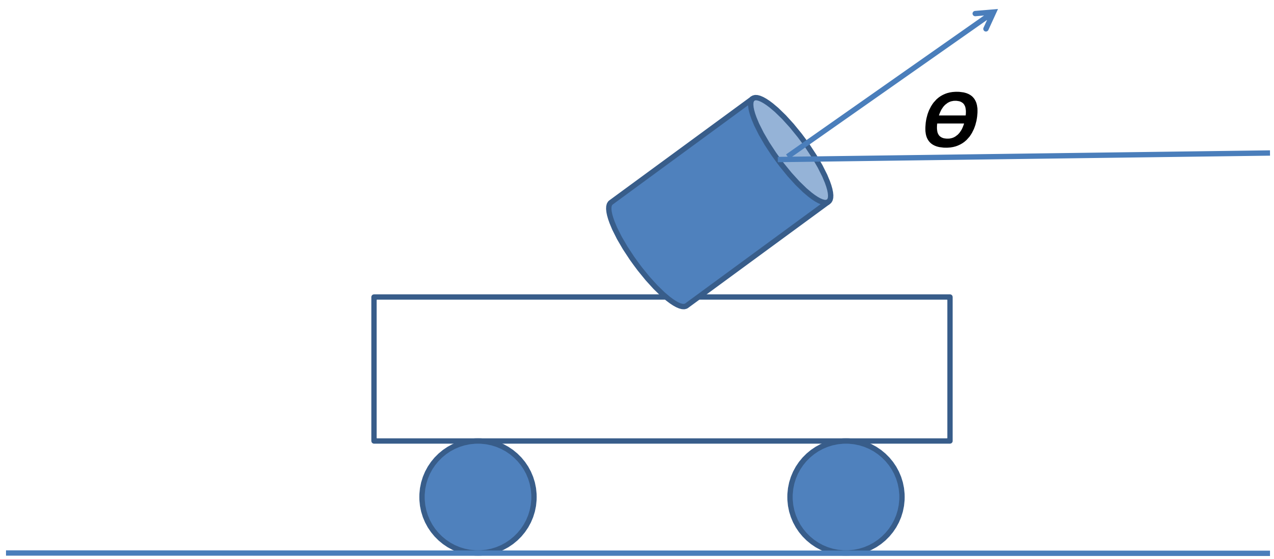
If $\boldsymbol{F=0}$

$$\frac{d\vec{P}}{dt} = 0$$

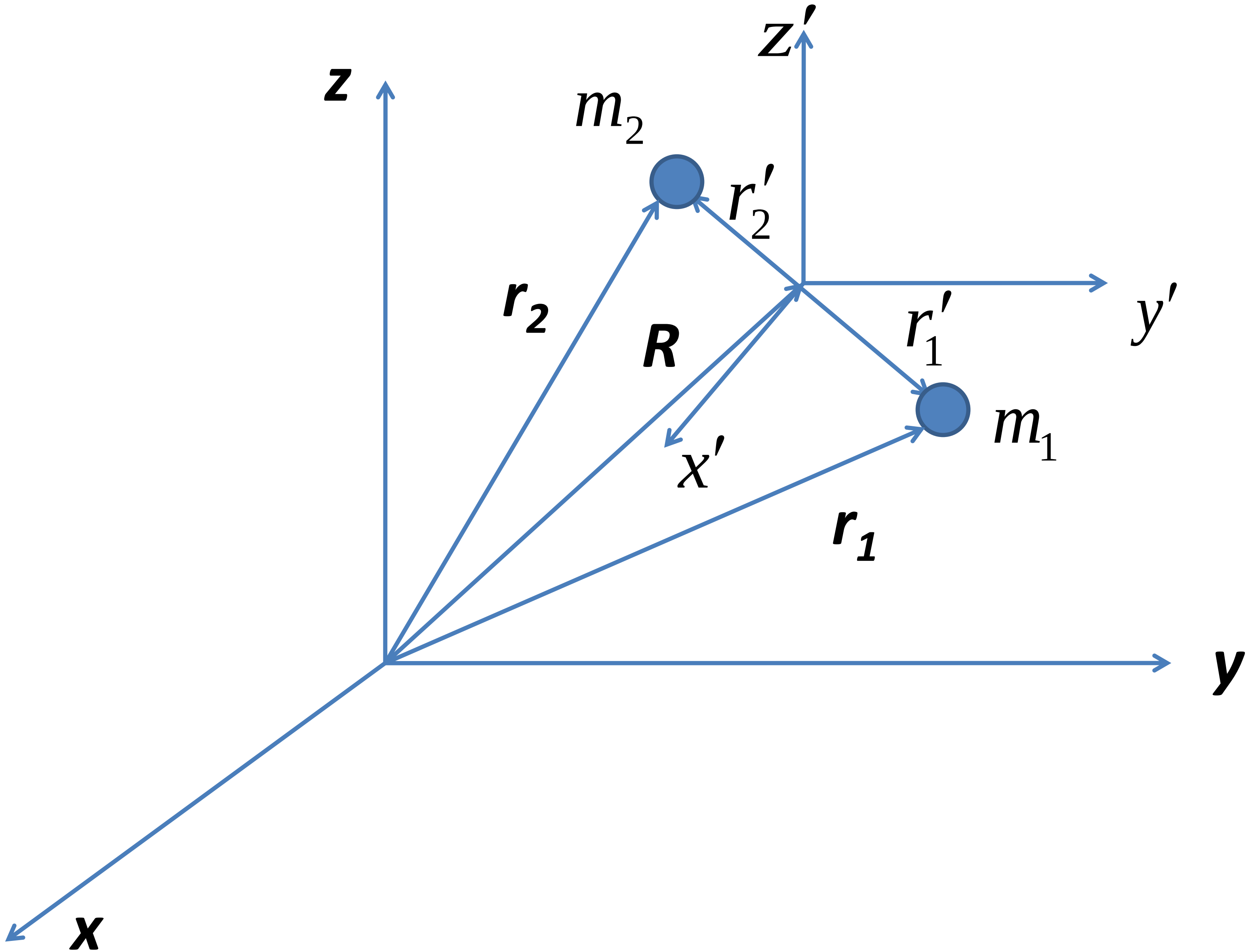
This is the law of conservation of momentum

Conservation of Momentum

Gun Recoil



Center of Mass Coordinates



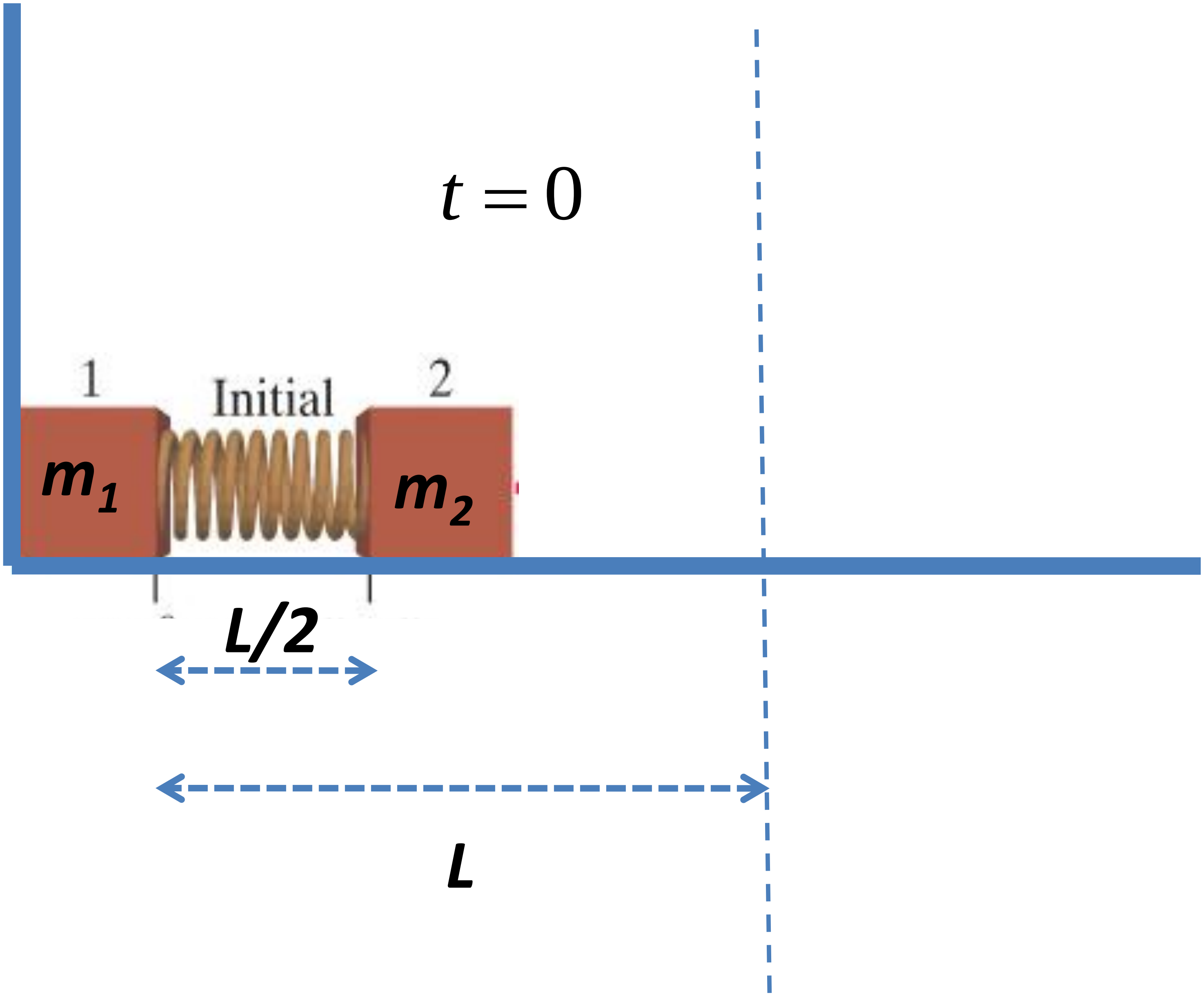
Center of Mass Coordinates

The center of mass coordinates for the two particles are:

$$\vec{r}_1{}' = \vec{r}_1 - \vec{R}$$

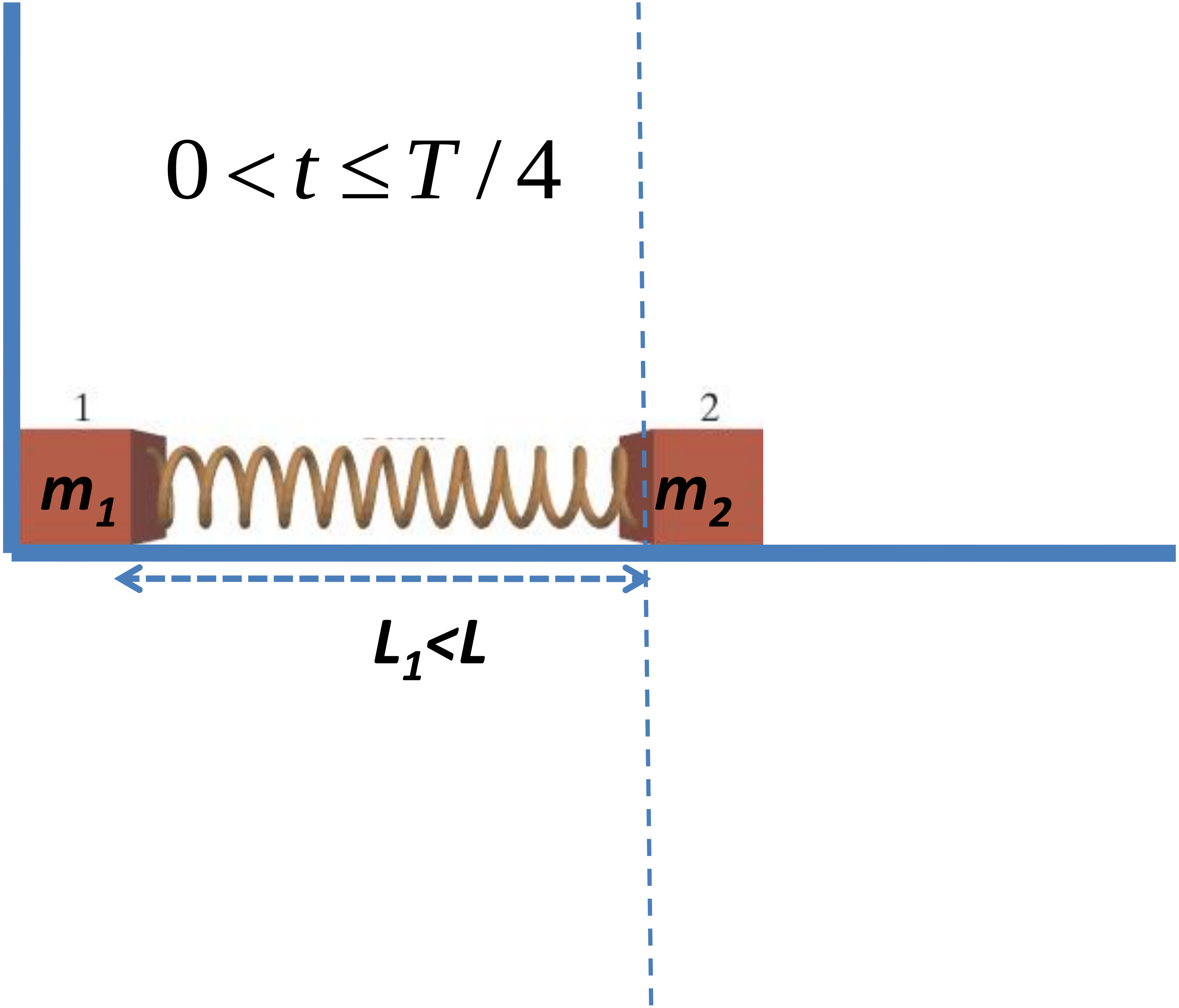
$$\vec{r}_2{}' = \vec{r}_2 - \vec{R}$$

Problem: 3.7 A system is composed of two blocks of mass m_1 and m_2 connected by a massless spring with spring constant *k*. The blocks slide on a friction--less plane. The unstretched length of the spring is *L*. Initially m_2 is held so that the spring is compressed to *L/2* and m_1 is forced against a stop as shown. m_2 is released at t=0. Find the motion of the center of mass of the system as a function of time.

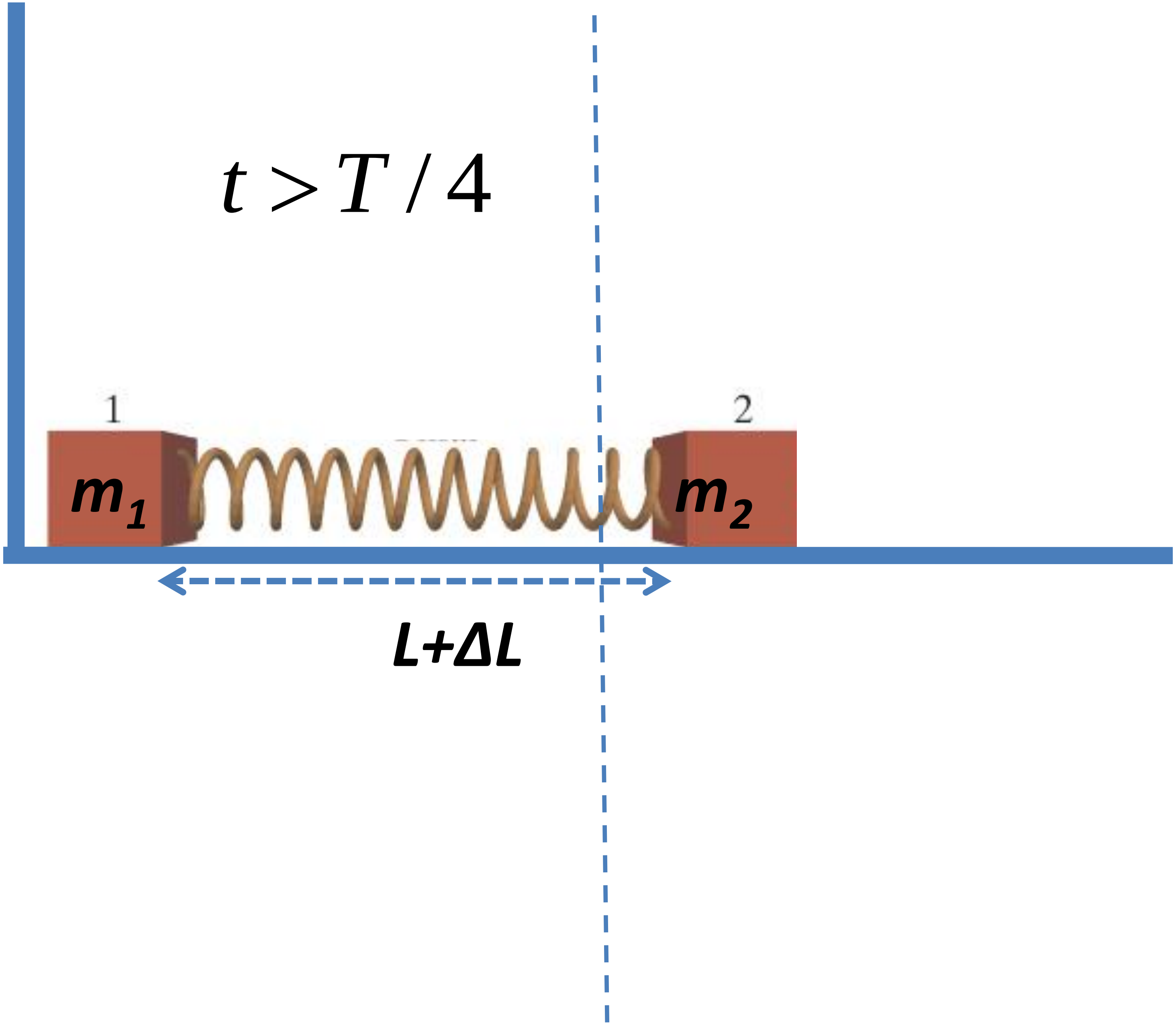


$t = 0$
1
Initial
2
m_1
m_2
$L/2$
L

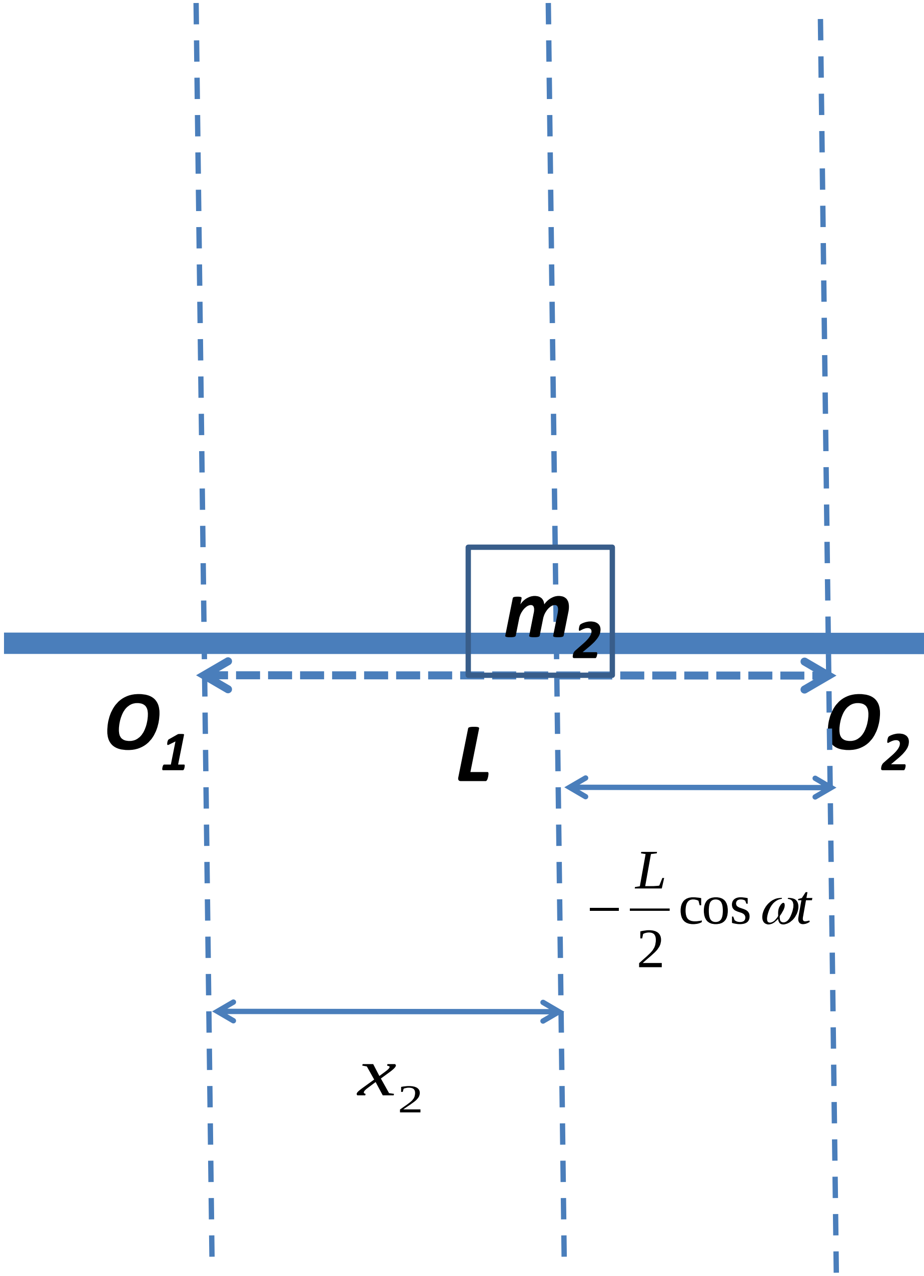
$$0 < t \leq T/4$$



$$t > T/4$$



$$0 < t \leq T/4$$



$$x_2' = -\frac{L}{2}\cos\omega t$$

$$x_2 = L - \frac{L}{2}\cos\omega t$$

Case 1: $0 < t \leq T/4$

Position of m_2 measured from the wall:

$$x_2 = \frac{L}{2}\left(2 - \cos\omega t\right)$$

Position of the CM measured from the wall:

$$X(t) = \frac{m_2 x_2}{m_1 + m_2}$$

$$= \frac{m_2 L\left(2 - \cos\omega t\right)}{2\left(m_1 + m_2\right)}$$

Case 1: $0 < t \leq T/4$

Velocity of CM:

$$V_{CM}(t) = \frac{m_2 L\omega \sin \omega t}{2(m_1 + m_2)}$$

Case 2: $t > T / 4$

The CM will move with constant velocity:

$$v_2(t = T / 4) = \dot{x}_2' = \frac{L\omega}{2} \sin \omega t$$

$$= \frac{L\omega}{2}$$

The velocity of the CM :

$$V_{CM} = \frac{m_2 v_2}{m_1 + m_2} = \frac{m_2 L\omega}{2(m_1 + m_2)}$$

Case 2: $t > T/4$

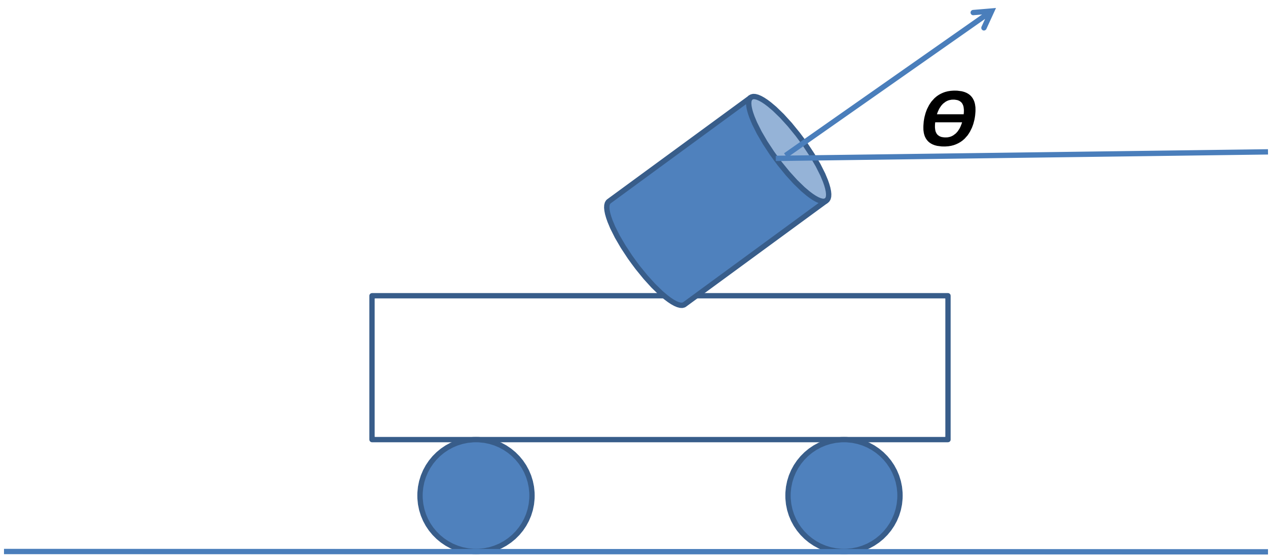
The Position of CM:

$$X_{CM}(t) = X_{CM}(T/4) + V_{CM}(t - T/4)$$

$$= \frac{m_2 L}{m_1 + m_2} + \frac{m_2 L \omega}{2(m_1 + m_2)}(t - T/4)$$

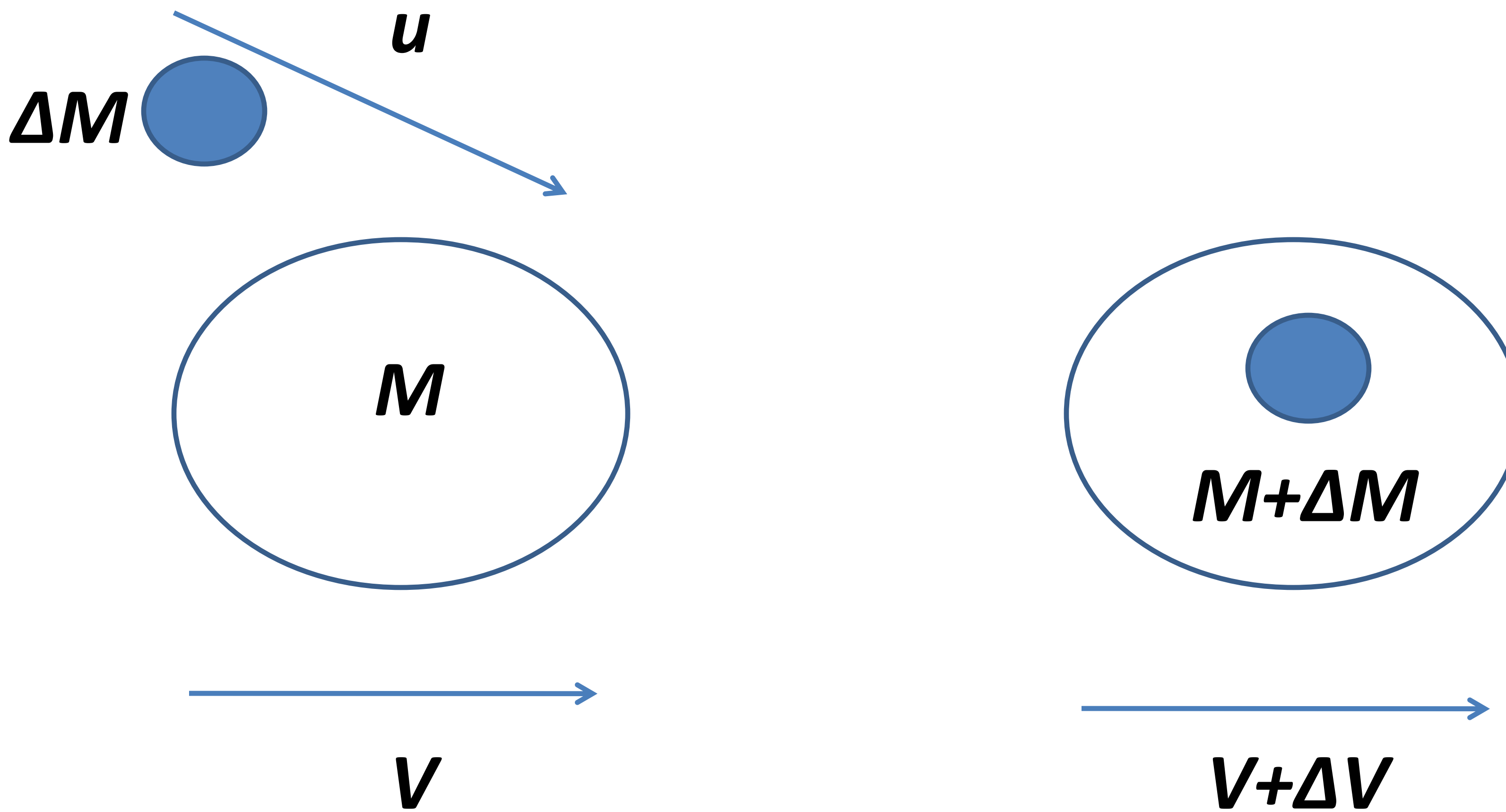
$$= \frac{m_2 L}{2(m_1 + m_2)}\left(2 + \omega t - \frac{\pi}{2}\right)$$

Conservation of Momentum
Gun Recoil



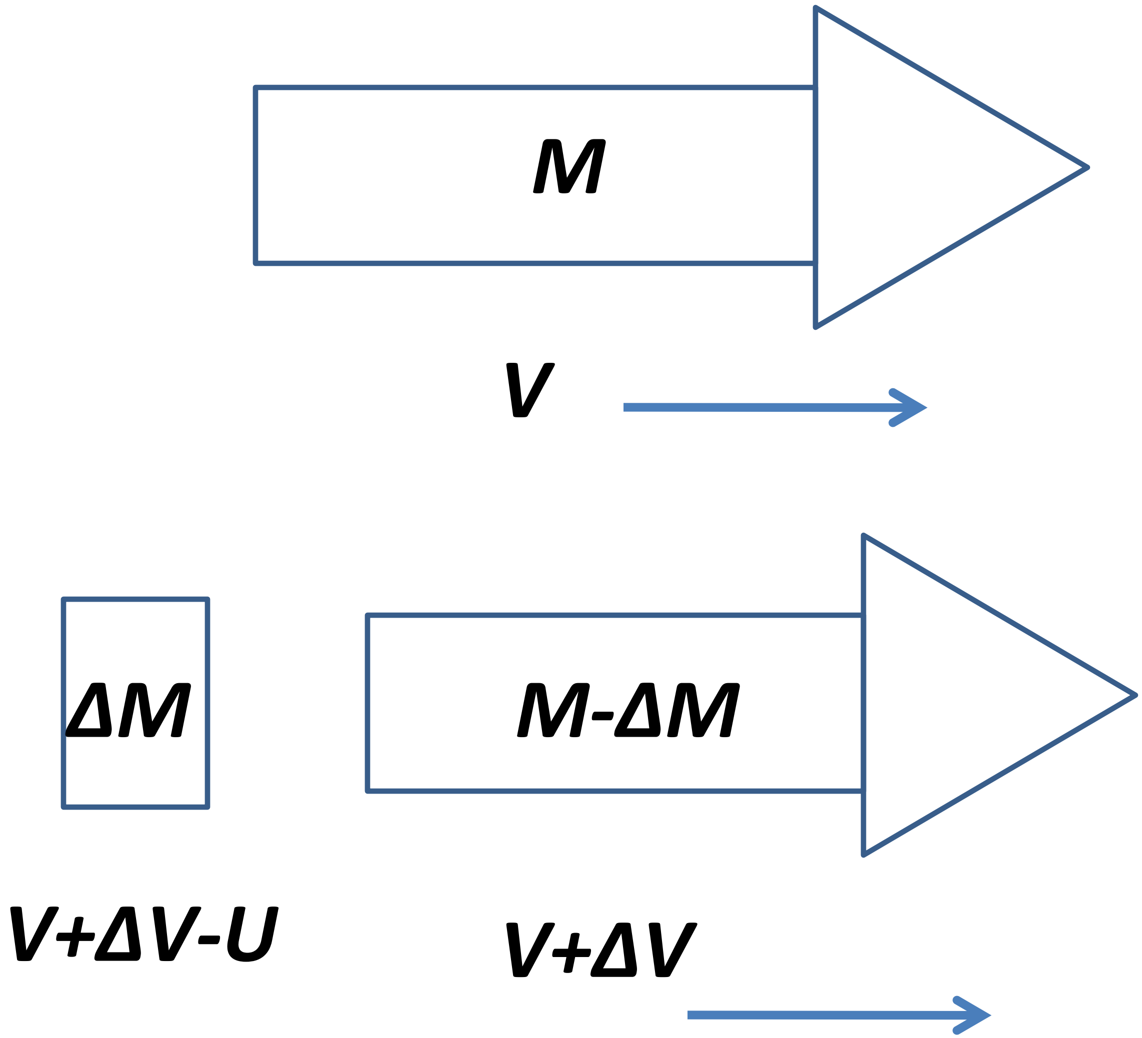
Find the angle to the horizontal at which the shell emerges from the gun.

Problem of Variable Mass System:

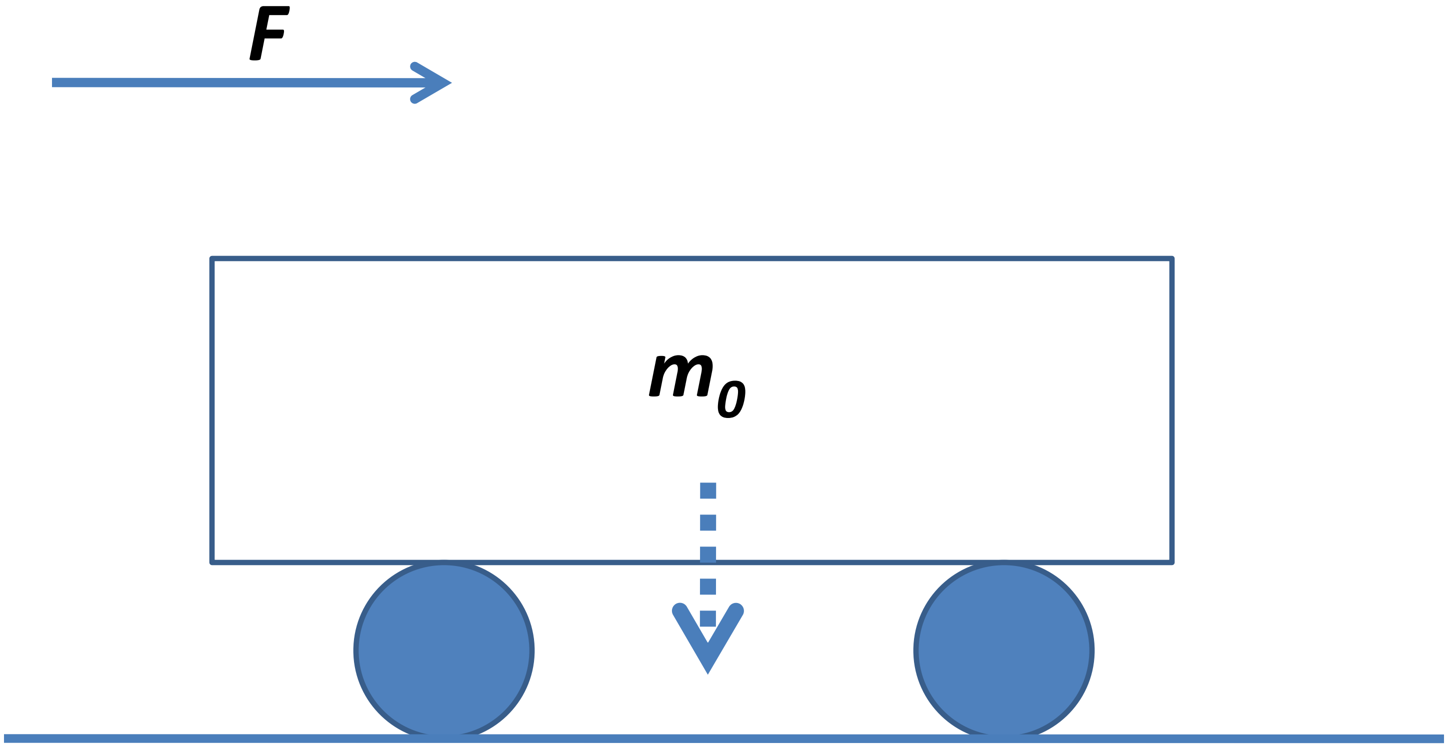


Problem 3.20 A rocket ascends from rest in a uniform gravitational field by ejecting exhaust with constant speed u. Assume that the rate at which Mass is expelled is given by $dm/dt=\Upsilon m$, where m is the instantaneous mass of the rocket and Υ is a constant, and that the rocket is retarded by air resistance with a force mbv, where b is a constant. Find the velocity of the rocket as a function of time.

Rocket Motion:

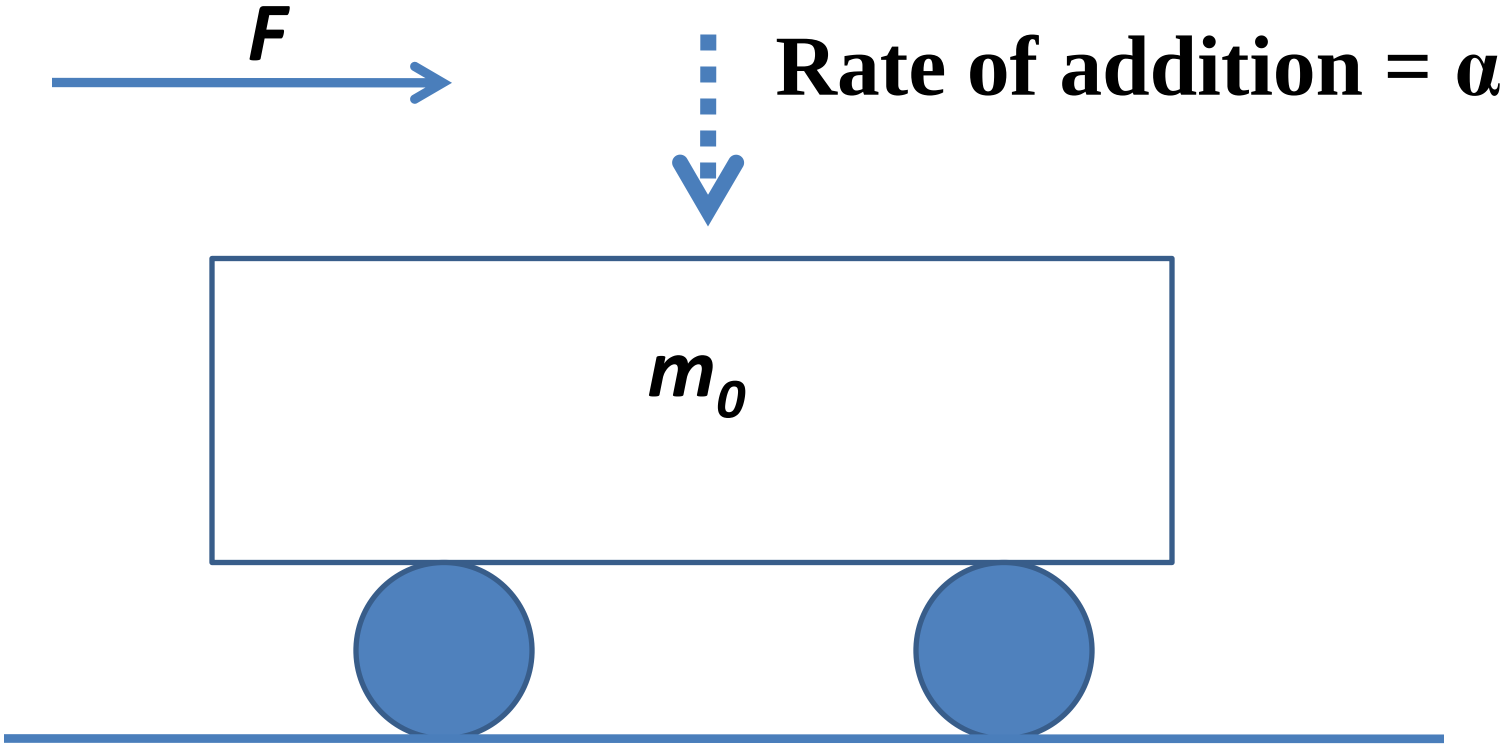


Problem:



Rate of leakage = α

Problem:



Problem 3.19 A raindrop of initial mass M_0 starts Falling from the rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity:

$$\frac{dM}{dt} = kMV$$

where *k* is a constant.

Show that the speed of the drop eventually becomes Effectively constant, and give the expression for the terminal speed.

Impulse and a Restatement of the Momentum Relation

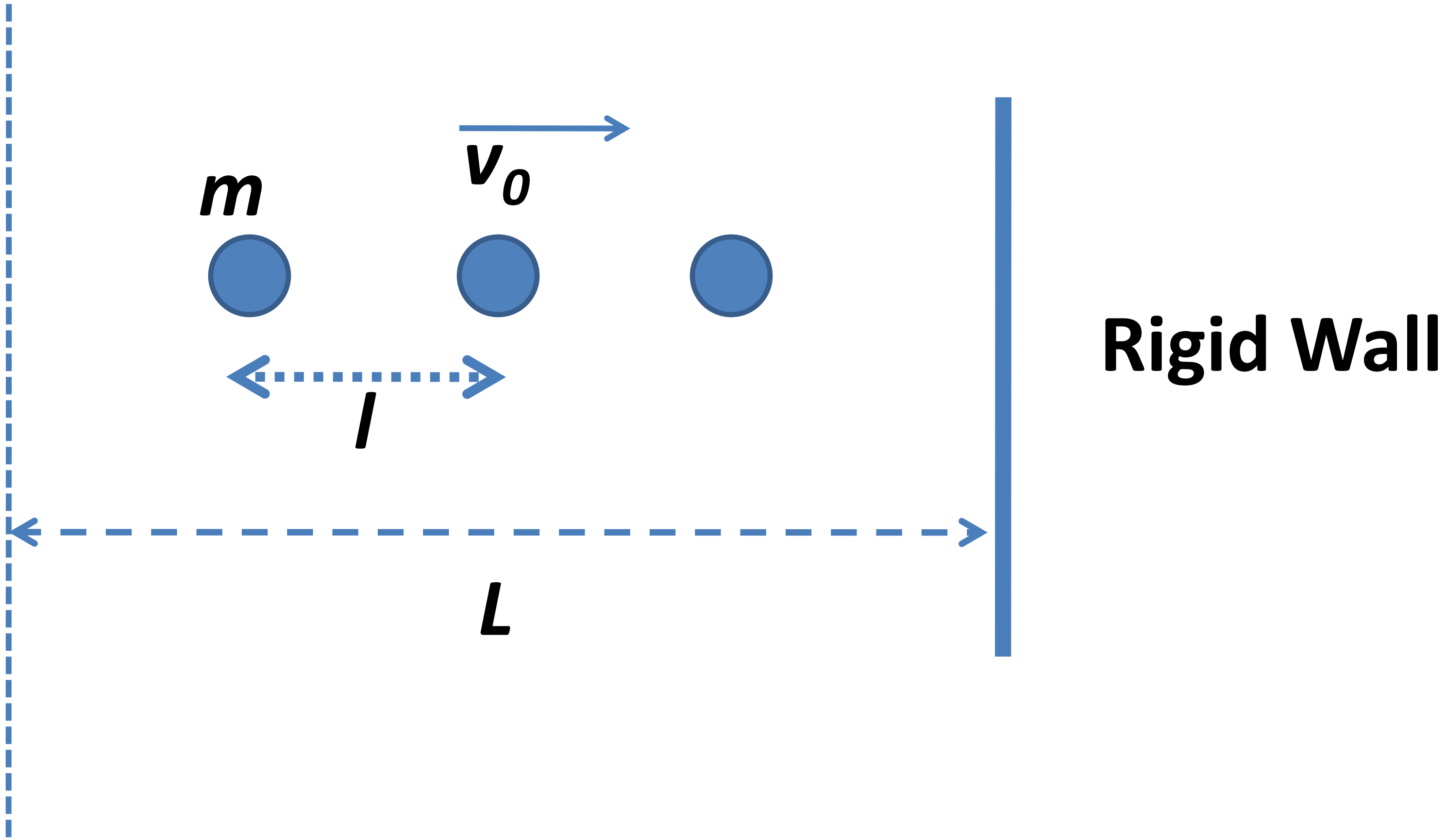
The relation between force and momentum is:

$$\vec{F} = \frac{d\vec{P}}{dt}$$

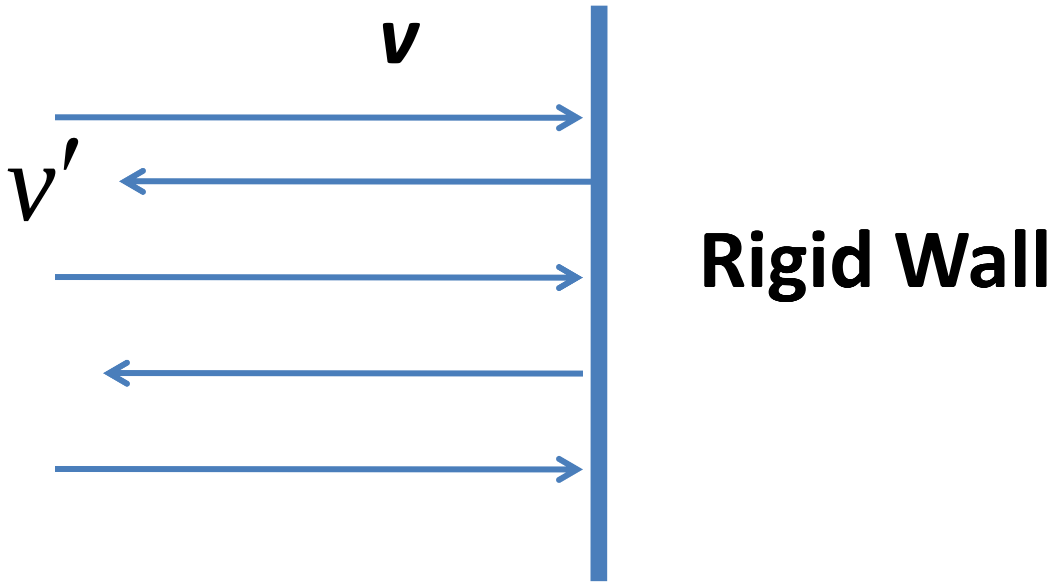
The Integral form of the force-momentum relationship is:

$$\int_0^t \vec{F}dt = \vec{P}(t) - \vec{P}(0)$$

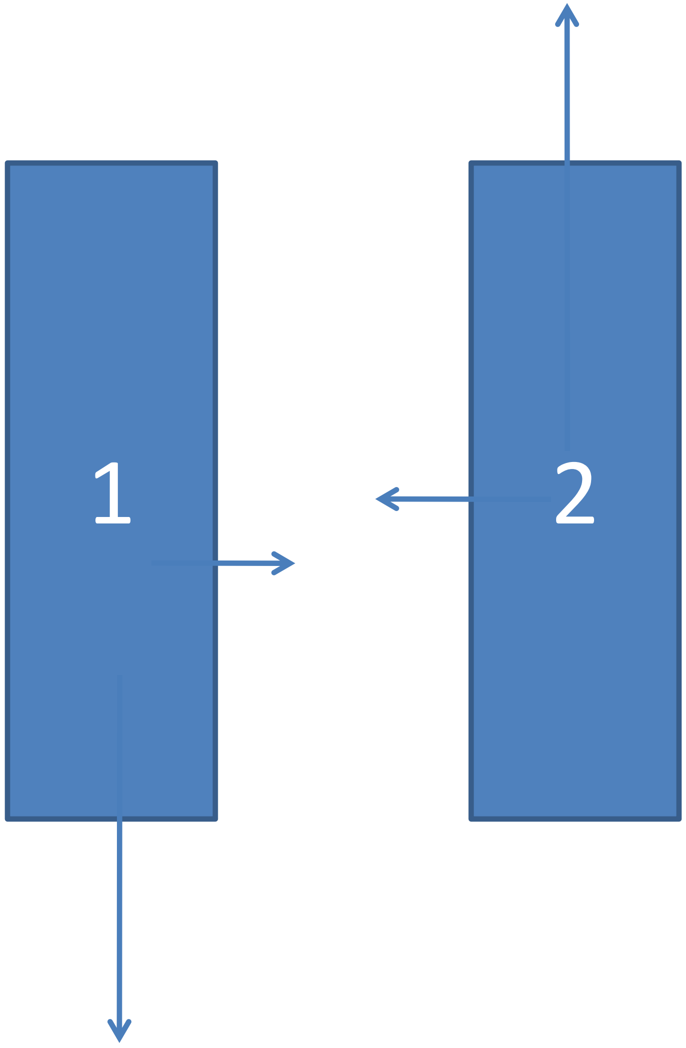
Momentum Transport:



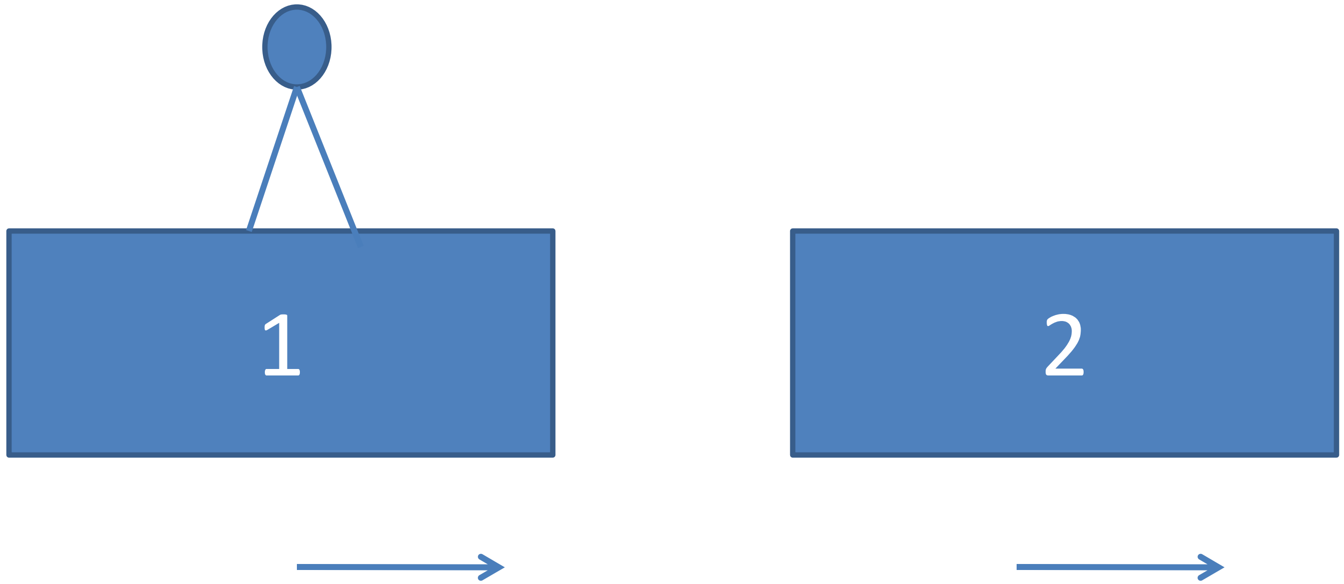
Problem:



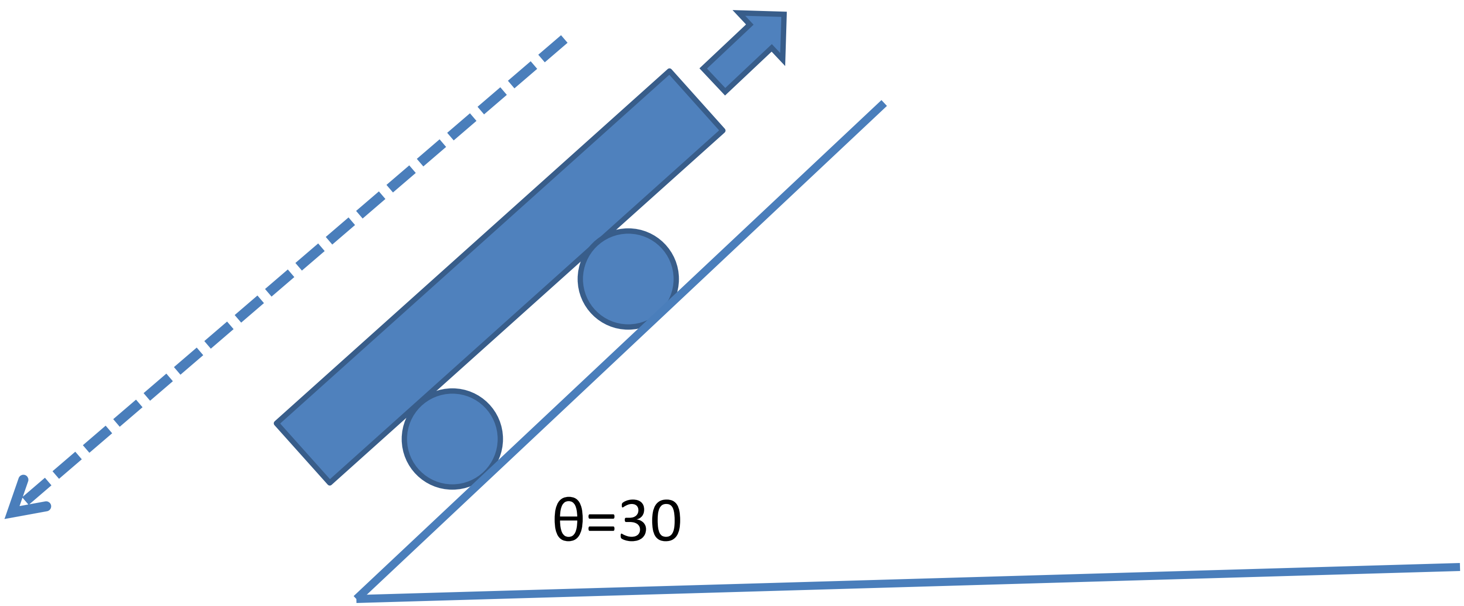
Problem: Two identical wagons 1 and 2 with one man in each move without friction due to inertia along the parallel rails toward each other. When the wagons get opposite each other, the men exchange their places by jumping in the direction perpendicular to the motion direction. As a consequence, wagon 1 stops and wagon 2 keeps moving in the same direction, with its velocity becoming equal to *V*. Find the initial velocities of the wagons if mass of each wagon is M and the mass of each man is m.


1
2

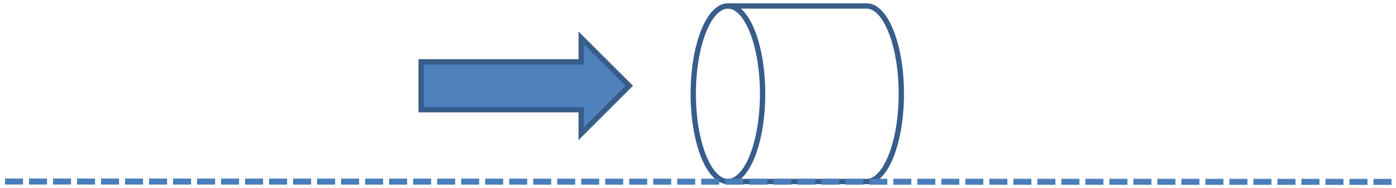
Problem: Two identical wagons move one after other due to inertia (without friction) with same velocity V_0. A man of mass m rides the rear wagon. At a certain moment the man jumps into the front wagon with a velocity u relative to his wagon. If the mass of each wagon is M, find the velocities with which the wagons will move after that.


1
2

Problem: A sand-filled wagon slides down a smooth inclined plane, starting from rest at time t=0. The inclination of the plane to the horizontal is $\theta=30^0$. The initial mass of the wagon and its contents is M_0=5000 Kg. The sand is ejected backwards from the wagon at a speed equaling the wagon's speed, the rate of ejection being μ=50 kg/s. Find the speed of the wagon at t=4 s.



Problem: An empty drum of mass M is initially at rest. From time t=0 onwards, a jet of water issues from a pipe at a rate μ with velocity u, strikes the drum and is retained by it. If the drum moves in the direction Of the jet with velocity V at a subsequent instant, (a) Calculate the acceleration and the loss of energy up to this instant.



Problem: An ramp with a belt of rope along two pulleys as show is driven by a motor moves with a steady speed of 1.0 m/s. If persons of average mass 70 kg takes the ramp every 5 secs on average. Calculate the average force provided by the motor