BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: **TEMPORAL LOGICS**

Linear Temporal Logic –Temporal Operators: Adequate Sets

Formulas and Interpretation

- Semantics of binary temporal operators:
  - Let $M = (S, \to, L)$ be a model and $\pi = s_1 \to s_2 \to$ be a path in M.
  - Then define the *satisfaction relation* $|=$ as follows:
    - $\pi \models \phi \mathbf{U} \psi$ iff there is some i such that $\pi^i \models \psi$ and for all j = 1,…, i-1, $\pi^j \models \phi$
    - $\pi \models \phi \mathbf{W} \psi$ iff there is some i such that $\pi^i \models \psi$ and for all j = 1,…, i-1, $\pi^j \models \phi$; or all k>=1 $\pi^k \models \phi$
    - $\pi \models \phi \mathbf{R} \psi$ iff for some i>=1 $\pi^i \models \phi$ and for all j = 1,…, i, $\pi^j \models \psi$; or all k>=1 $\pi^k \models \psi$
- Q:
  - Is there a relation between **U** and **R**?
  - Do we need all three (of **U**, **W**, and **R**)?
  - Do we need all three unary operators?

Adequate / Complete Set of Operators

- **Q:** *Do we need all three binary operators*?
  - ***Weak-until*** can expressed using ***until*** :

    $$\phi\ \mathbf{W}\ \psi \equiv (\phi\ \mathbf{U}\ \psi) \vee (\mathbf{G}\ \phi)$$

    and vice-versa

    $$\phi\ \mathbf{U}\ \psi \equiv (\phi\ \mathbf{W}\ \psi) \wedge (\mathbf{F}\ \psi)$$

  - ***Release*** is the dual of ***until*** **:**

    $$\phi\ \mathbf{U}\ \psi \equiv \neg(\neg\phi\ \mathbf{R}\ \neg\psi)$$

Adequate / Complete Set of Operators

- **Q:** *Do we need all three unary operators*?
  - G and F are duals (of each other):
    - $G\,\phi \equiv \neg(F\,\neg\phi)$
  - Can X be expressed using the other operators?
    - No. How do you argue (or prove) this?

- X is a dual of itself:
    - $\neg(X\,\phi) \equiv (X\,\neg\phi)$

Adequate / Complete Set of Operators

- **Q:** *Do we need any unary operators*?
  - X cannot be expressed in terms of other operators.
  - What about F ?
    - **F $\phi \equiv$ True U ϕ**
  - Use this to derive G in terms of the binary operators!

Adequate / Complete Set of Operators

- *Thus each of the following sets would be adequate:*
  - *{ X, U }*
  - *{ X, R }*
  - *{ X, W }*

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Linear Temporal Logic –Temporal Operators: Properties

Distributive Properties

- $F(\phi \vee \psi) \equiv (F\phi) \vee (F\psi)$
  - Prove this.
    - $\equiv$ denotes semantic equivalence

- $F(\phi \wedge \psi) \not\equiv (F\phi) \wedge (F\psi)$
  - Prove this:
    - $F(\phi \wedge \psi) \dashrightarrow (F\phi) \wedge (F\psi)$ is always TRUE

  but

    - $(F\phi) \wedge (F\psi) \dashrightarrow F(\phi \wedge \psi)$ is not always TRUE
      - Provide a counter-example.

Distributive Properties

- Since
  - $G\,\phi \equiv \neg(F\,\neg\phi)$
- we can derive this:
  - $G\,(\phi \wedge \psi) \equiv \neg\,F(\neg(\phi \wedge \psi))$

$$\equiv \neg\,F(\neg\phi \;\vee\; \neg\psi))$$

$$\equiv \neg\,((F\;\;\neg\phi) \vee (F\,\neg\psi))$$

$$\equiv \neg\,(F\;\;\neg\phi) \wedge \neg(F\,\neg\psi)$$

$$\equiv\;(G\,\phi)\;\;\wedge\;\;(G\,\psi)$$