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CS/IS F214
Logic in Computer Science

MODULE: INTRODUCTION

A Brief – and Selective - History of Logic – Part II: Intuitionism

Intuitionism

2G Scam – Proof 1

Assume that *the Govt. acted honestly*:

- Then they should have *wanted to increase revenue for Govt*.
- Auctioning spectrum would have brought more revenue than awarding it on **First-Come-First-Serve** basis.
- Spectrum was awarded on a **FCFS** basis.
    - This is a contradiction.
- Therefore we infer *that the Govt. was dishonest*

2G Scam – Proof 2

- Company X hired lobbyist Y.
- Lobbyist Y paid Rs *rrrrrrrr* to Minister Z.
- Company X received preferential allocation of spectrum.

- Therefore we infer that *the Govt. was dishonest*.

Proof Techniques: *Proof by Contradiction*

- ***Proof by Contradiction* [PbC]** :
  - Assume that the *negation of a statement X* is true.

  …
  - Derive a contradiction
    - Conclude that X is true

- e.g. (see previous slide)
  - Consider the first "proof" of 2G scam

Brouwer's Intutionism

- Intuitionism is a school of thought in logic and mathematics:
  - founded by Brouwer
  - in the early 20th century as a response to Hilbert's program
- Intuitionists rejected ***Proof by Contradiction [PbC]***
- Why did intuitionists reject **PbC**?
  - Consider the two "proofs" of 2G scam in the next two slides.
  - Intuition (no pun intended):
    - Negation of a negation does not prove existence!

Brouwer's Intutionism – Another Illustration

- Consider the claim:
  - There exist irrational numbers a and b such that a^b is rational.
- Consider a proof of this claim:
  - Let $c = \sqrt{2}$
  - Consider c^c – it is either rational or irrational.
  - If it is: then let c=a and c=b and we are done.
  - If not: let $a=c^c$ and b=c then a^b is rational.
- What is wrong with this proof?
  - How do we avoid such proofs?
  - Should we avoid such proofs?

Brouwer's Intutionism

- The proof in the last side is non-constructive:
  - i.e. it proves that *there exist irrational a and b such that a^b is rational but it does not produce (i.e. construct) such a and b.*
- Brouwer and intuitionists denounced such "***non-constructive" proofs*** and argued that such proofs should be disallowed:
  - They identified the "**Law of Excluded Middle (LEM)"** as the root cause of such proofs:
    - **LEM:** *for any proposition A : A OR (NOT A) is true*
- Exercise:
  - Identify where LEM is used in the previous proof?

Brouwer's Intutionism

- The philosophical underpinning of Brouwer's argument is this:
  - while A OR (NOT A) is always true it may not always be proven or provable for a specific A
  - and therefore it may not be used in a proof (by itself) unless one has proven A or one has proven NOT A.

- What is common in the critique of LEM and that of PbC by intuitionists?