BITS Pilani
Pilani Campus

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CS/IS F214
Logic in Computer Science

MODULE: **PROPOSITIONAL LOGIC**

Natural Deduction: Derived Rules, Proofs - Examples

Modus Tollens

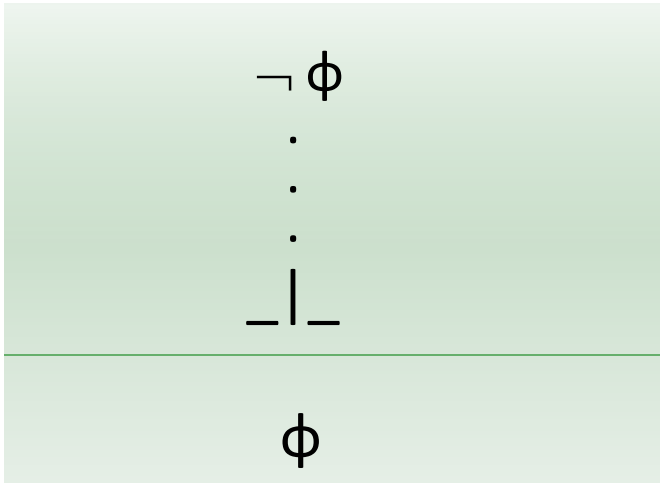
$$\frac{\phi \dashrightarrow \psi \qquad \neg \psi}{\neg \phi}$$

MT (modus tollens)

Questions: What is the relation between this and *modus ponens*?

How do you derive this rule (from the basic rules)?

Proof by Contradiction



PbC

- One can infer anything from a contradiction.
  - But in this case the contradiction resulted from an assumption i.e. ¬ ϕ
  - Therefore it is meaningful to infer ϕ that the assumption led to the contradiction
- In fact one must infer ϕ to eliminate the assumption ¬ ϕ.
  - Why?

Exercise: Derive PbC from the ¬ i rule.

Exercises

- Prove the following:
  1. $\neg p \vee q$ |-- p-->q
  2. q --> (p --> r), $\neg r$, q |-- $\neg p$
  3. p-->q |-- $\neg p \vee q$
  4. Use 3. to prove LEM

[These examples were worked out in class – on the board]

Take-home Exercises

Prove the following sequents

1. $\neg(p \wedge q) \quad |\!-\!- \quad \neg p \vee \neg q$
2. $q \dashrightarrow (p \dashrightarrow r),\ \neg r,\ q \quad |\!-\!- \quad \neg p$
3. $p \dashrightarrow q,\ r \dashrightarrow s \quad |\!-\!- \quad p \vee r \dashrightarrow q \vee s$
4. $(p \dashrightarrow r) \wedge (q \dashrightarrow r) \quad |\!-\!- \quad p \wedge q \dashrightarrow r$