Chapter 3: K&K

Lecture: 7, 20, 24;   Tut: 9, 14, 18; Suggested: 5,10,11,13,15,17,19

3.9. *Variable mass problem.*

Let at any instant of time, the mass of the freight car + sand is M.

The constant rate of loosing sand is $b = \dfrac{dm}{dt} = const.$

The velocity of the car is V and the velocity of the sand is u. Here $V = u$

$$\therefore F = External\ force = M\frac{dV}{dt} - V_{rel}\frac{dm}{dt} = M\frac{dV}{dt}$$

$$Here, M = M(t) = M_{car} + (m - bt) = (M_{cat} + m) - bt$$

$$\therefore (M_{car} + m - bt)\frac{dV}{dt} = F$$

$$\Rightarrow \int_0^{V(T)} dV = \int_0^T \frac{F}{(M_{car} + m - bt)}dt = -\frac{F}{b}\ln(M_{car} + m - bt)\Big|_0^T = \frac{F}{b}\ln\left(\frac{M_{car} + m}{M_{car}}\right)$$

Here, T is the time when, $m = bT$. So, $V_f = \dfrac{F}{b}\ln\left(\dfrac{M_{car} + m}{M_{car}}\right)$

3.14. *N – Men problem*

(a) *Since there is no external force, therefore, $P_i = P_f$ and $P_i = 0$.*

Let the final velocity of the flat car is V (w.r.t. ground) when all men jump off together and u is the velocity of each man w.r.t. flat car.

$$So,\ P_f = MV + Nm(V - u) = P_i = 0$$

$$\therefore V = \frac{Nmu}{M + Nm} ----(1)$$

(b) *Let V_i is the velocity of the flat car when ith man jump of the car. Just before that, when the $i-1$th man jumps off the car, the velocity of the flat car is V_{i-1}.*

$$So, \left[M + m\{N - (i-1)\}\right]V_{i-1} = \left[M + m(N - i)\right]V_i + m(V_i - u)$$

$$\Rightarrow V_i = V_{i-1} + \frac{mu}{M + (N - i + 1)m}$$

$$\Rightarrow \Delta V_i = V_i - V_{i-1} = \frac{mu}{M + (N - i + 1)m}$$

$$\therefore V_f = \sum_{i=1}^{N}\Delta V = \frac{mu}{M + Nm} + \frac{mu}{M + (N-1)m} + \frac{mu}{M + (N-2)m} + ... + \frac{mu}{M + m} > N.\frac{mu}{M + Nm}$$

$$\therefore V_f\Big|_{2nd\ case} > V_f\Big|_{1st\ case}$$

3.10.

$$F = M\frac{dV}{dt} - V_{rel}\frac{dm}{dt} = M\frac{dV}{dt} - V_{rel}\frac{dM}{dt}; V_{rel} = u - V = -V$$

$$\therefore F = M\frac{dV}{dt} + V\frac{dM}{dt} = \frac{d\left(MV\right)}{dt} \Rightarrow Fdt = d\left(MV\right) = F\int_0^T dt = \int_0^{(M+m)V} d\left(MV\right)$$

$$\Rightarrow V = \frac{Ft}{M+m} = \frac{Fm}{b\left(M+m\right)}; T = \frac{m}{b}$$

3.18. *Let* $M\left(t\right)$ *and* $V\left(t\right)$ *are the mass and velocity of the water drop at any instant 't'.*
At time instant $t+\Delta t$ *the same are* $M+\Delta M$ *and* $V+\Delta V$.
So, change in momentum: $\Delta P = \left(M+\Delta M\right)\left(V+\Delta V\right) - MV \approx M\Delta V + V\Delta M$

$$\therefore \frac{dP}{dt} = M\frac{dV}{dt} + V\frac{dM}{dt} = M\frac{dV}{dt} + kMV^2$$

$$\therefore Mg = M\frac{dV}{dt} + kMV^2 \Rightarrow \frac{dV}{dt} = g - kV^2 \Rightarrow \int_0^V \frac{dV}{g - kV^2} = \int_0^t dt$$

$$\therefore t = \frac{1}{k}\int_0^V \frac{dV}{\left(\frac{g}{k}\right) - V^2} = \frac{1}{k}\int_0^V \frac{dV}{a^2 - V^2} = \frac{1}{2ak}\ln\left[\left|\frac{a+V}{a-V}\right|\right]_0^V$$

$$\Rightarrow \ln\left[\frac{a+V}{a-V}\right] = 2akt \Rightarrow \frac{a+V}{a-V} = e^{2akt} \Rightarrow V = a\frac{e^{2akt}+1}{e^{2akt}-1} \Rightarrow V_T = a.\lim_{t\to\infty}\frac{e^{2akt}+1}{e^{2akt}-1} = a = \sqrt{\frac{g}{k}}$$

$$\therefore \boxed{V_T = \sqrt{\frac{g}{k}}}$$