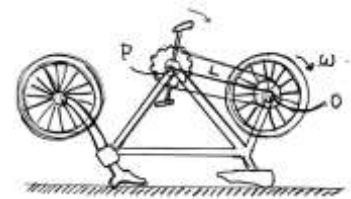


***Instruction(s): All questions are compulsory. Each question carries 3 marks. Write only the answer of the question in the appropriate box provided. There is no negative marking.***

1. A small disk slides down in a frictionless groove along a slope which makes an angle  $45^\circ$  with horizontal. The slope can rotate about z-axis which is passing through the lower end of the slop. What should be the minimum rotational speed ( $\omega$ ) (in inertial reference frame) of the slope such that the disk is about to move up along the slop when it is at a height  $h$  from the horizontal?
2. A bead slides on a long frictionless horizontal bar with constant speed  $v_0$  relative to the bar. The bar rotates about the z-axis with a constant angular speed  $\omega_0$ . At time  $t=0$  the position of the bead on the bar is,  $r = r_0$ . What is the **total acceleration** of the bead (inertial reference frame) at any instant of time  $t$  in polar coordinate system?
3. A rocket of initial mass  $M_0$  (including fuel mass) is at rest on earth at  $t = 0$ . The rocket begins to move upwards by ejecting mass backwards at the rate  $R$  with a speed  $u$  with respect to the rocket. What is the rocket velocity after time  $t$ ?
4. For a particle moving in the x-y plane has the following trajectory  $x(t) = v_0 t$  and  $y(t) = y_0$  where  $y_0$  is constant. What are the radial and angular components of velocity in terms of given parameters  $v_0, y_0$  and  $t$ ?
5. Potential energy of a 1 kg particle free to move along x-axis is given by,  $U(x) = (0.25x^4 - 0.5x^2)$  Joule. If the total mechanical energy of the particle is 2 Joule, what is the maximum speed of the particle in m/sec?
6. Sand runs from a hopper at constant rate ' $k$ ' kg/sec onto a horizontal conveyor belt driven at constant speed  $v$  by a motor. How much power is required to drive the belt?
7. Mass  $m$  is tied to one end of a rope whose other end is wound over a flywheel that is free to spin about its axis. The rope is almost in-extensible and of negligible mass. The flywheel has mass  $M$ , radius  $R$  and moment of inertia ( $I = Mk^2$ ). What is the speed  $v$  of mass  $m$  as it descends through height  $h$  in terms of  $M, g, h, R, k$ .
8. Consider a bicycle put upside down and resting on its handle bar and seat. Rotating the paddle sets the back wheel into rotation. If the rear wheel has mass  $M$ , radius  $R$ , and is spun with angular speed  $\omega$ , about its axle what is the angular momentum of the wheel about an axis passing through the point P which is the axle of the paddle. The point P is at a distance  $L$  from axel of the wheel. Take, M.I. of wheel as  $MR^2$ .
9. A guitar of mass  $M$  is hung from a pivot that is distance  $L$  from its center of mass and performs small amplitude oscillations of time period  $T$ . What is the moment of inertia of guitar about the center of mass in terms of  $M, g, L, T$ ?



10. What is the time period (**in hour**) of the plane of oscillation of a Foucault Pendulum on the Northern Hemisphere at a latitude of  $30^\circ$ ?
11. A circular platform rotates with a constant angular velocity  $\omega_0$  counter-clockwise. A man of mass  $m$  runs away from the origin of the platform at time  $t=0$  towards its circumference on a straight line which is painted on the platform without slipping at a constant speed  $v_0$ . What are expressions of the fictitious forces on the man at time  $t = t_F$ ?
12. A platform is executing simple harmonic motion in a vertical direction with an amplitude of 5 cm and a frequency of  $10/\pi$  vibrations per second. A block is placed on the platform at the lowest point of its path. At what distance, measured from the lowest point, will the block leave the platform?  **$g=10\text{m/s}^2$** .
13. A damped harmonic oscillator is governed by the equation,  $\ddot{x} + 0.1\dot{x} + 100x = 0$ . Find out the number of cycles completed by the oscillator when the amplitude decays to  $1/e$  of its initial value.
14. A critically damped system is set into motion from its equilibrium position with a velocity 10 cm/s. It takes 0.1 sec to reach 0.5 cm from its equilibrium position. Find out  $\gamma$ .
15. An object of mass 2 kg hangs from a spring with spring constant 800 N/m. The top end of the spring is oscillated up and down in SHM with an amplitude of 2 mm. The angular frequency of the free undamped oscillations is 20 rad/s. The  $Q$  of the system is 20. Assume,  $g = 10 \text{ m/s}^2$ . Calculate the amplitude of forced oscillations  $B$  (in cm) at resonance.
16. A weakly damped harmonic oscillator is driven by a force  $F = F_0 \cos \omega t$  whose amplitude  $F_0$  is kept constant but its angular frequency is varied. It is observed that the amplitude of the steady-state oscillations is 0.1 mm at very low values of  $\omega$  and attains a maximum value of 10 cm when  $\omega = 100 \text{ rad/s}$ . Find the  $Q$  of the system.
17. An object of mass 0.1kg is hung from a spring with a spring constant 100 N/m. A resistive force  $-bv$  acts on the mass, where  $b = 1 \text{ Ns/m}$ . The object is subjected to a harmonic driving force  $F = F_0 \cos \omega t$ , where  $F_0 = 2 \text{ N}$  and  $\omega = 50 \text{ rad/s}$ . In the steady-state, what is the value of  $\tan \delta$  (where  $\delta$  is the phase relative to that of the applied force)?
18. Two identical pendulums are connected by a light coupling spring. Each pendulum has a length of 0.4m, and they are at a place where  $g = 9.8\text{m/s}^2$ . With the coupling spring connected, one pendulum is clamped and the period of the other is found to be 1.25 sec exactly. With neither pendulum clamped, what are the time periods of two normal modes (in seconds)?
19. Two masses 0.02 kg and 0.05 kg are connected by a massless spring of spring constant 12 N/m. Find the frequency ( $\nu$ ) of oscillation (in Hz) if the masses are free to move along the line joining their centers.
20. A string of length  $L$  and negligible mass is attached to two fixed supports at its ends. The tension in the string is  $T$ . A particle of mass  $m$  is attached at a distance of  $L/4$  from one end of the string. Find the period of small transverse oscillations of mass  $m$ .

**Instruction(s): All questions are compulsory. Each question carries 3 marks. Write only the answer of the question in the appropriate box provided. There is no negative marking.**

1. An object of mass 0.1kg is hung from a spring with a spring constant 100 N/m. A resistive force  $-bv$  acts on the mass, where  $b = 1$  Ns/m. The object is subjected to a harmonic driving force  $F = F_0 \cos \omega t$ , where  $F_0 = 2$  N and  $\omega = 50$  rad/s. In the steady-state, what is the value of  $\tan \delta$  (where  $\delta$  is the phase relative to that of the applied force)?
2. Two identical pendulums are connected by a light coupling spring. Each pendulum has a length of 0.4m, and they are at a place where  $g = 9.8 \text{ m/s}^2$ . With the coupling spring connected, one pendulum is clamped and the period of the other is found to be 1.25 sec exactly. With neither pendulum clamped, what are the time periods of two normal modes (in seconds)?
3. Two masses 0.02 kg and 0.05 kg are connected by a massless spring of spring constant 12 N/m. Find the frequency ( $\nu$ ) of oscillation (in Hz) if the masses are free to move along the line joining their centers.
4. A string of length  $L$  and negligible mass is attached to two fixed supports at its ends. The tension in the string is  $T$ . A particle of mass  $m$  is attached at a distance of  $L/4$  from one end of the string. Find the period of small transverse oscillations of mass  $m$ .
5. A small disk slides down in a frictionless groove along a slope which makes an angle  $45^\circ$  with horizontal. The slope can rotate about  $z$ -axis which is passing through the lower end of the slop. What should be the minimum rotational speed ( $\omega$ ) (in inertial reference frame) of the slope such that the disk is about to move up along the slop when it is at a height  $h$  from the horizontal?
6. A bead slides on a long frictionless horizontal bar with constant speed  $v_0$  relative to the bar. The bar rotates about the  $z$ -axis with a constant angular speed  $\omega_0$ . At time  $t=0$  the position of the bead on the bar is,  $r = r_0$ . What is the **total acceleration** of the bead (inertial reference frame) at any instant of time  $t$  in polar coordinate system?
7. A rocket of initial mass  $M$  (including fuel mass) is at rest on earth at  $t = 0$ . The rocket begins to move upwards by ejecting mass backwards at the rate  $R$  with a speed  $u$  with respect to the rocket. What is the rocket velocity after time  $t$ ?
8. For a particle moving in the  $x$ - $y$  plane has the following trajectory  $x(t) = v_0 t$  and  $y(t) = y_0$  where  $y_0$  is constant. What are the radial and angular components of velocity in terms of given parameters  $v_0, y_0$  and  $t$ ?
9. A damped harmonic oscillator is governed by the equation,  $\ddot{x} + 0.1\dot{x} + 100x = 0$ . Find out the number of cycles completed by the oscillator when the amplitude decays to  $1/e$  of its initial value.
10. A critically damped system is set into motion from its equilibrium position with a velocity 10 cm/s. It takes 0.1 sec to reach 0.5 cm from its equilibrium position. Find out  $\gamma$ .

**11.** An object of mass 2 kg hangs from a spring with spring constant 800 N/m. The top end of the spring is oscillated up and down in SHM with an amplitude of 2 mm. The angular frequency of the free undamped oscillations is 20 rad/s. The  $Q$  of the system is 20. Assume  $g = 10 \text{ m/s}^2$ . Calculate the amplitude of forced oscillations  $B$  (in cm) at resonance.

**12.** A weakly damped harmonic oscillator is driven by a force  $F = F_0 \cos \omega t$  whose amplitude  $F_0$  is kept constant but its angular frequency is varied. It is observed that the amplitude of the steady-state oscillations is 0.1 mm at very low values of  $\omega$  and attains a maximum value of 10 cm when  $\omega = 100 \text{ rad/s}$ . Find the  $Q$  of the system.

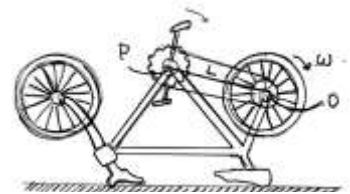
**13.** What is the time period (**in hour**) of the plane of oscillation of a Foucault Pendulum on the Northern Hemisphere at a latitude of  $30^\circ$ ?

**14.** Potential energy of a 1 kg particle free to move along x-axis is given by,  $U(x) = (0.25x^4 - 0.5x^2) \text{ Joule}$ . If the total mechanical energy of the particle is 2 Joule, what is the maximum speed of the particle in m/sec?

**15.** Sand runs from a hopper at constant rate ' $k$ ' kg/sec onto a horizontal conveyor belt driven at constant speed  $v$  by a motor. How much power is required to drive the belt?

**16.** Mass  $m$  is tied to one end of a rope whose other end is wound over a flywheel that is free to spin about its axis. The rope is almost in-extensible and of negligible mass. The flywheel has mass  $M$ , radius  $R$  and moment of inertia ( $I = Mk^2$ ). What is the speed  $v$  of mass  $m$  as it descends through height  $h$  in terms of  $M$ ,  $g$ ,  $h$ ,  $R$ ,  $k$ .

**17.** Consider a bicycle put upside down and resting on its handle bar and seat. Rotating the paddle sets the back wheel into rotation. If the rear wheel has mass  $M$ , radius  $R$ , and is spun with angular speed  $\omega$ , about its axle what is the angular momentum of the wheel about an axis passing through the point  $P$  which is the axle of the paddle. The point  $P$  is at a distance  $L$  from axle of the wheel. Take, M.I. of wheel as  $MR^2$ .



**18.** A guitar of mass  $M$  is hung from a pivot that is distance  $L$  from its center of mass and performs small amplitude oscillations of time period  $T$ . What is the moment of inertia of guitar about the center of mass in terms of  $M$ ,  $g$ ,  $L$ ,  $T$ ?

**19.** A circular platform rotates with a constant angular velocity  $\omega_0$  counter-clockwise. A man of mass  $m$  runs away from the origin of the platform at time  $t=0$  towards its circumference on a straight line which is painted on the platform without slipping at a constant speed  $v_0$ . What are expressions of the fictitious forces on the man at time  $t = t_f$ ?

**20.** A platform is executing simple harmonic motion in a vertical direction with an amplitude of 5 cm and a frequency of  $10/\pi$  vibrations per second. A block is placed on the platform at the lowest point of its path. At what distance, measured from the lowest point, will the block leave the platform?  $g=10 \text{ m/s}^2$ .

**Name:** \_\_\_\_\_

**ID:** \_\_\_\_\_

**Tut Section Number:** \_\_\_\_\_

**Marks obtained**

Recheck (if any)

**Answer Sheet**

1	$\omega = \sqrt{g/h}$	11	$m\omega_0^2 v_0 t_0$ & $2m\omega_0 v_0$
2	$\vec{a} = -r\omega_0^2 \hat{r} + 2v_0\omega_0 \hat{\theta}$	12	7.5 cm
3	$u \ln \left( \frac{M_0}{M_0 - Rt} \right) - gt$	13	31.81 ≈ 32
4	$v_r = \frac{v_0^2 t}{\sqrt{v_0^2 t^2 + y_0^2}}; v_\theta = \frac{-v_0 y_0}{\sqrt{v_0^2 t^2 + y_0^2}}$	14	$20 \ln 2 = 13.862$
5	$v_{max} = \frac{3}{\sqrt{2}} m/sec$	15	$B = 4cm$
6	$Power = kv^2$	16	$Q = 1000$
7	$v = \sqrt{2mgh / \left( m + \frac{Mk^2}{R^2} \right)} = \sqrt{2gh / \left( 1 + \frac{Mk^2}{mR^2} \right)}$	17	$\tan \delta = -1/3$
8	$mR^2 \omega$	18	1.27 sec; 1.23 sec
9	$M \left[ \frac{gLT^2}{4\pi^2} - L^2 \right] = \frac{ML}{4\pi^2} [gT^2 - 4\pi^2 L]$	19	4.612 Hz
10	48 hours	20	$2\pi\sqrt{3mL/16T}$

**Name:** \_\_\_\_\_

**ID:** \_\_\_\_\_

**Tut Section Number:** \_\_\_\_\_

**Marks obtained**

Recheck (if any)

**Answer Sheet**

1	$\tan \delta = -1/3$	11	$B = 4cm$
2	$1.27 sec; 1.23 sec$	12	$Q = 1000$
3	$4.612 Hz$	13	$48 hours$
4	$2\pi\sqrt{3mL/16T}$	14	$v_{max} = \frac{3}{\sqrt{2}} m/sec$
5	$\omega = \sqrt{g/h}$	15	$Power = kv^2$
6	$\vec{a} = -r\omega_0^2\hat{r} + 2v_0\omega_0\hat{\theta}$	16	$v = \sqrt{2mgh / \left(m + \frac{Mk^2}{R^2}\right)} = \sqrt{2gh / \left(1 + \frac{Mk^2}{mR^2}\right)}$
7	$u \ln\left(\frac{M_0}{M_0 - Rt}\right) - gt$	17	$mR^2\omega$
8	$v_r = \frac{v_0^2 t}{\sqrt{v_0^2 t^2 + y_0^2}}; v_\theta = \frac{-v_0 y_0}{\sqrt{v_0^2 t^2 + y_0^2}}$	18	$M \left[ \frac{gLT^2}{4\pi^2} - L^2 \right] = \frac{ML}{4\pi^2} [gT^2 - 4\pi^2 L]$
9	$31.81 \approx 32$	19	$m\omega_0^2 v_0 t_0 \text{ \& } 2m\omega_0 v_0$
10	$20 \ln 2 = 13.862$	20	$7.5 cm$