MATHEMATICS-I
(MATH F111)

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Section 11.4

Graphing in Polar Coordinates

Symmetry Tests

- **Symmetry about x-axis:** If (r,θ) lies on the graph, then $(r,-\theta)$ or $(-r,\pi-\theta)$ also lies on the graph.

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- **Symmetry about origin:** If (r,θ) lies on the graph, then $(-r,\theta)$ or $(r,\pi+\theta)$ also lies on the graph.
- **Symmetry about line $y=x$:** If (r,θ) lies on the graph, then $\left(r,\frac{\pi}{2}-\theta\right)$ or $\left(-r,-\frac{\pi}{2}-\theta\right)$ also lies on the graph.

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- The parametric equations of $r = f(\theta)$ are $x = r\cos\theta,\ y = r\sin\theta$.
- The slope of the curve $r = f(\theta)$ at any point (r,θ) is given by

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \left.\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

provided $\frac{dx}{d\theta} \neq 0$ at any point (r,θ).

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Tracing the Curve in Polar Coordinate

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- $r = 0$ gives $\cos\theta = -1 \implies \theta = \pi$. Thus $\theta = \pi$ is a tangent to the curve at pole.

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- $\frac{dr}{d\theta} > 0 \Rightarrow \sin\theta < 0$, thus no value of θ in between 0 and π.

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- $\frac{dr}{d\theta} < 0 \Rightarrow \sin\theta > 0 \Rightarrow 0 < \theta < \pi$, thus r decreases in the interval $[0, \pi]$.

  Clearly $\max r = 2$ at $\theta = 0$ and $\min r = 0$ at $\theta = \pi$.

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At $\theta = 0$, $r = 2$ and $\tan\theta_1 = \infty \implies \theta_1 = \frac{\pi}{2}$.

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At $\theta = 0$, $r = 2$ and $\tan\theta_1 = \infty \implies \theta_1 = \frac{\pi}{2}$.

At $\theta = \frac{\pi}{2}$, $r = 1$ and $\tan\theta_1 = 1 \implies \theta_1 = \frac{\pi}{4}$.

Step 5. Make a table θ vs r for different values of θ.

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θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π
r	2	$1+\frac{1}{\sqrt{2}}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$1-\frac{1}{\sqrt{2}}$	0

Step 6. Plot the curve while considering the above steps.

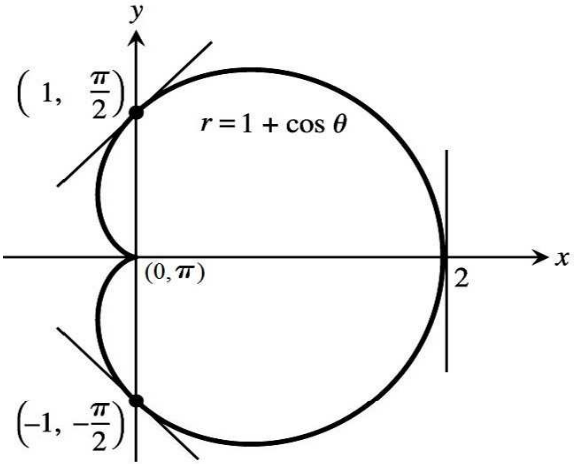

Figure: $r = 1 + \cos\theta$: The cardioid

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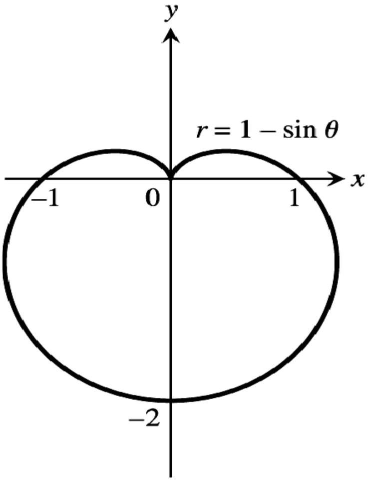

Figure: $r = 1 - \sin\theta$: The cardioid

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Remark

Note that it is enough to consider either $r = \sqrt{\sin 2\theta}$ or $r = -\sqrt{\sin 2\theta}$. Symmetry will give the graph corresponding to other.

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- $\frac{dr}{d\theta} < 0$ when $\cos 2\theta$ and $\sin 2\theta$ are of opposite sign, which is not possible for any value of θ.

Clearly max value of r is 1 at $\theta = \pi/4$ and min value of r is 0 at $\theta = 0$.

Step 4.

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$= \frac{\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}\sin\theta + \sqrt{\sin 2\theta}\cos\theta}{\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}\cos\theta - \sqrt{\sin 2\theta}\sin\theta}$$

$$= \frac{\cos 2\theta\sin\theta + \sin 2\theta\cos\theta}{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}$$

$$= \frac{\sin 3\theta}{\cos 3\theta} = \tan 3\theta$$

Thus $\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \tan(3\pi/4)$. Thus slope of the tangent at $\theta = \pi/4$ is $3\pi/4$.

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Step 5. Table θ vs r: Not required.

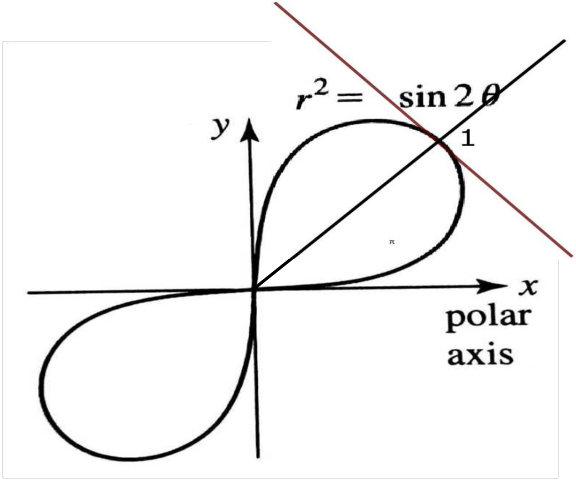

Figure: Lemniscate: Two leaved rose

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Hence, it is enough to consider the region $0 \leqslant \theta \leqslant \frac{\pi}{4}$.

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  Clearly Max $r = 1$ at $\theta = 0$ and min $r = 0$ at $\theta = \frac{\pi}{4}$.

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Step 5. Table

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$
r	1	0.86	0.5	0

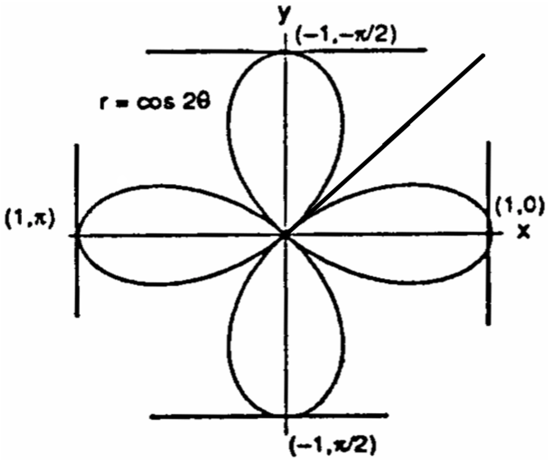

Figure: $r = \cos 2\theta$: Four leaved rose

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Step 2. $r = 0$ gives $\sin\theta = -1/2 \implies \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$. Thus, $\theta = \frac{7\pi}{6}$ and $\theta = \frac{11\pi}{6}$ are tangents to the curve at pole.

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  Clearly Max $r = 3$ at $\theta = \frac{\pi}{2}$ and min $r = -1$ at $\theta = \frac{3\pi}{2}$.

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$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = 0, \quad \left.\frac{dy}{dx}\right|_{\theta=\pi} = -1/2, \quad \left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = 0.$$

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At $\theta = \frac{\pi}{2}$, $r = 3$ and $\tan\theta_1 = 0 \implies \theta_1 = 0$.

Step 4.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = 0, \quad \left.\frac{dy}{dx}\right|_{\theta=\pi} = -1/2, \quad \left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = 0.$$

At $\theta = \frac{\pi}{2}$, $r = 3$ and $\tan\theta_1 = 0 \implies \theta_1 = 0$.

At $\theta = \pi$, $r = 1$ and

$\tan\theta_1 = -1/2 \implies \theta_1 = -26.57$.

Step 4.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = 0, \quad \left.\frac{dy}{dx}\right|_{\theta=\pi} = -1/2, \quad \left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = 0.$$

At $\theta = \frac{\pi}{2}$, $r = 3$ and $\tan\theta_1 = 0 \implies \theta_1 = 0$.

At $\theta = \pi$, $r = 1$ and

$\tan\theta_1 = -1/2 \implies \theta_1 = -26.57$.

At $\theta = \frac{3\pi}{2}$, $r = -1$ and $\tan\theta_1 = 0 \implies \theta_1 = 0$.

Step 4.

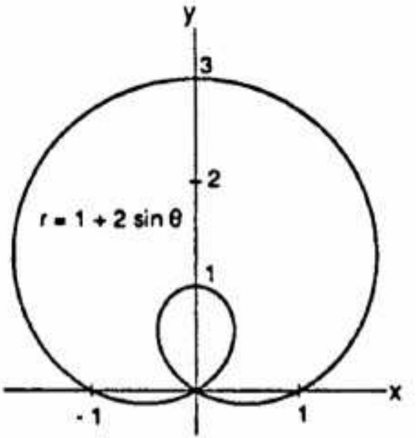
$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}=0,\quad \left.\frac{dy}{dx}\right|_{\theta=\pi}=-1/2,\quad \left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}}=0.$$

At $\theta=\frac{\pi}{2}$, $r=3$ and $\tan\theta_1=0 \implies \theta_1=0$.

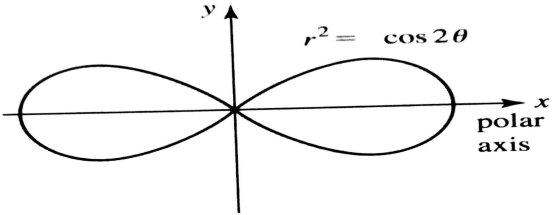
At $\theta=\pi$, $r=1$ and
$\tan\theta_1=-1/2 \implies \theta_1=-26.57$.

At $\theta=\frac{3\pi}{2}$, $r=-1$ and $\tan\theta_1=0 \implies \theta_1=0$.

Step 5. Table


y
3
2
$r = 1 + 2\sin\theta$
1
x
-1
1

Trace the curve $r^2 = \cos 2\theta$



Q:. Trace the curve $r = 1 - \cos\theta$.

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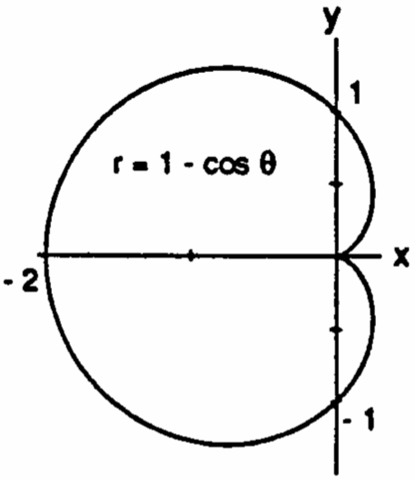

Figure: $r = 1 - \cos\theta$: The cardioid

Sol. We can write $r = 1 - \cos\theta = 1 + \cos(\theta - \pi)$.

Sol. We can write $r = 1 - \cos\theta = 1 + \cos(\theta - \pi)$.
Thus the curve $r = 1 - \cos\theta$ is obtained from $r = 1 + \cos\theta$ by replacing θ by $\theta + \pi$. Therefore, to obtain the curve of $r = 1 - \cos\theta$, we just need to rotate the curve of $r = 1 + \cos\theta$ by an angle π.

Faster Graphing

Steps for Faster Graphing

The polar equation can quickly be captured by plotting $r = f(\theta)$ in cartesian θr-plane.

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Step 1: Compute $\theta - r$ table for θ in the interval $[0, 2\pi]$.

- Consider the curve $r = \cos 2\theta$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	1	0	-1	0	1	0	-1	0	1

Step 2: Graph $r = f(\theta)$ in the cartesian θr-plane (that is, x-axis for θ and y-axis for r) in the interval $[0, 2\pi]$.

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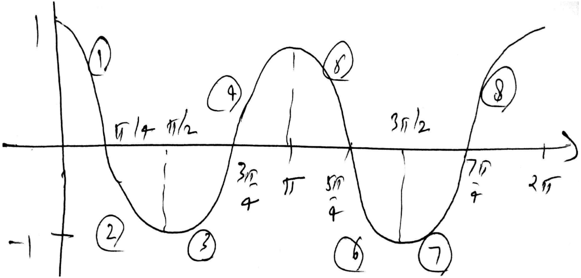

Remark

The value of θ where the curve crosses (or touches) θ-axis is a tangent to the curve at pole.

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The value of θ where the curve crosses (or touches) θ-axis is a tangent to the curve at pole. For example, $\theta = \frac{\pi}{4}$ is a tangent to the curve $r = \cos 2\theta$.

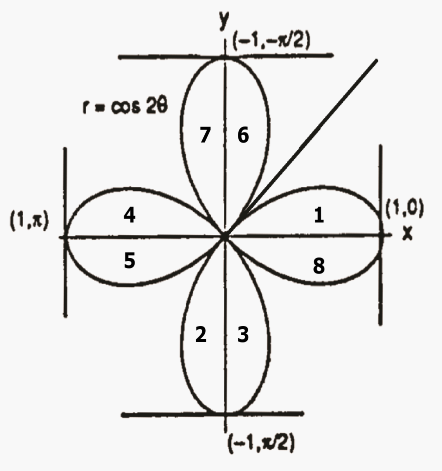

Figure: $r = \cos 2\theta$: Four leaved rose

Q:. Trace the curve $r = \sin 3\theta$ using faster graphing.

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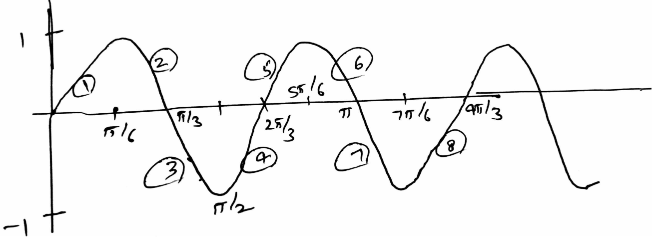
θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$4\pi/3$
r	0	1	0	-1	0	1	0	-1	0



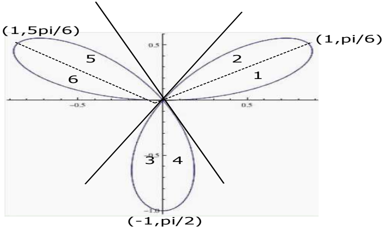


Figure: $r = \sin 3\theta$

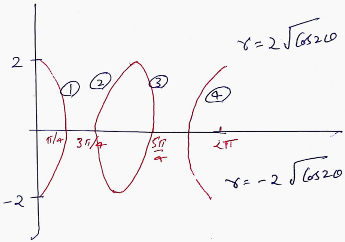
$r^2 = f(\theta)$

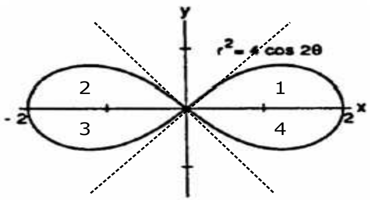
For the equation of the form $r^2 = f(\theta)$ first trace $r^2 = f(\theta)$ in the cartesian θr^2-plane (that is, x-axis for θ and y-axis for r^2) in the interval $[0, 2\pi]$ and then graph $r = f(\theta)$ in the cartesian θr-plane (that is, x-axis for θ and y-axis for r) in the interval $[0, 2\pi]$.

Q:. Trace the curve $r^2 = 4\cos 2\theta$ using faster graphing.

Q:. Trace the curve $r^2 = 4\cos 2\theta$ using faster graphing.

θ	r^2	r
0	4	± 2
$\pi/4$	0	0
$\pi/2$	$-$	$-$
$3\pi/4$	0	0
π	4	± 2
2π	4	± 2




y
$r^2 = 4\cos 2\theta$
2
1
- 2
x
2
3
4

Remarks

- The only drawback of faster graphing is that we can not find the slopes at particular points. But the slope is not important in the case when we evaluate the areas or lengths of curves.

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- The only drawback of faster graphing is that we can not find the slopes at particular points. But the slope is not important in the case when we evaluate the areas or lengths of curves.
- More examples over Fasting Graphing will be covered in Next Section.