MATH F113 (Probability and Statistics)

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In Lecture 16

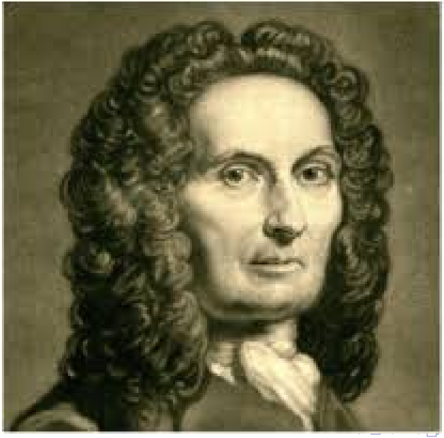
Exponential Distribution
Chi Squared Distribution
Recap of the Distribution

Normal Distribution

(Gaussian Distribution)

Distribution that underlies many of the statistical methods used in data analysis

The normal distribution was first introduced by Abraham de Moivre in an article in 1733



It was reprinted in the second edition of his The Doctrine of Chances, 1738 in the context of approximating (limiting) certain binomial distributions for large n or infinite

His result was extended by Laplace and Gauss with problem of astronomy (describe the behavior of errors in astronomical measurements) in his book (1812), and is now called De Moivre-Laplace theorem

Normal Distribution

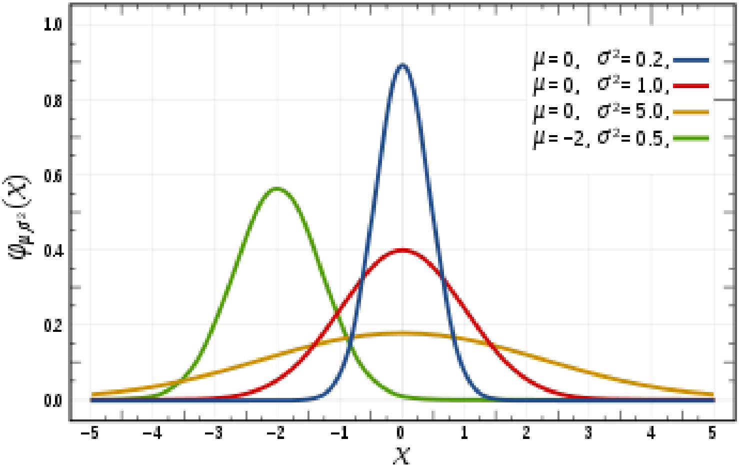
A random variable X with density

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

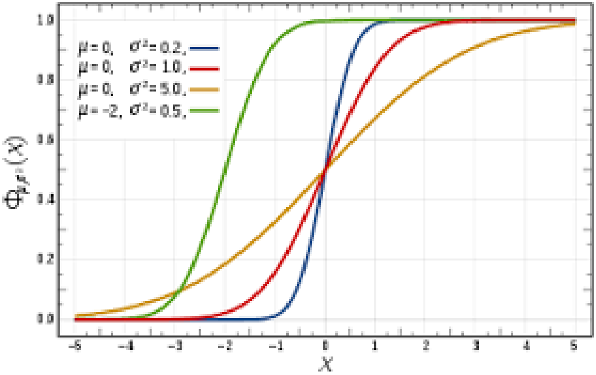
$$x, \mu \in (-\infty, \infty), \sigma > 0$$

is said to have normal distribution with parameter μ and σ

Probability Density Function



Cumulative Distribution Function



Let X be normally distributed with parameters μ and σ, then the moment generating function for X is given by

$$m_x(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

$$E[X] = \frac{dm_x(t)}{dt}|_{t=0} = \mu$$

$$E\left[X^2\right] = \frac{d^2 m_x(t)}{dt^2}|_{t=0} = \sigma^2 + \mu^2$$

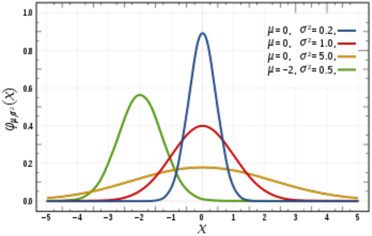
$$Var[X] = \sigma^2$$

Mean and Standard deviation for Normal distribution

Theorem

Let X be a normal random variable with parameters μ and σ. Then μ is the mean of X and σ is its standard deviation.

Normal Distribution



- Symmetric at mean
- Bell shaped curve centered at its mean.
- The points of inflection occur at $\mu \pm \sigma$

Standard Normal Distribution

Standard Normal Distribution

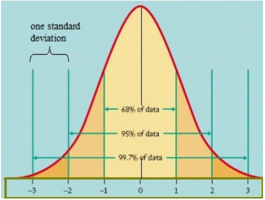
Standard Normal Distribution

- The standard normal distribution is the normal distribution with a mean of zero and a variance of one.

Standard Normal Distribution

- Carl Friedrich Gauss became associated with this set of distributions when he analyzed astronomical data using them and defined the equation of its probability density function.

Standard Normal Distribution



- It is often called the bell curve because the graph of its probability density resembles a bell.

Standard Normal Distribution

Let X be normal with mean μ and standard deviation σ. The variable

$$Z = \frac{X - \mu}{\sigma}$$

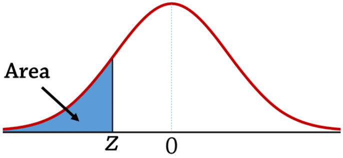
is standard normal. Z has mean 0 and standard deviation 1.

The probability density function is given by

$$\phi(z) = f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad z \in (-\infty, \infty)$$

The corresponding distribution function is given by

$$\begin{aligned}\Phi(z) = F(z) = P(Z \leq z) &= \int_{-\infty}^{z} f(z)dz \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz\end{aligned}$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.0
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.00
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.00
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.00
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.00
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.00
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.00
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.00
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.00
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.00
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.00

Important Results

(1) $\Phi(-z) = 1 - \Phi(z)$

Proof:

$$\Phi(-z) = P\left(Z \leq -z\right) = P\left(Z \geq z\right)$$

$$1 - P\left(Z \leq z\right) = 1 - \Phi(z)$$

(2)

$$P(a \leq X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

Other Notations

Given a particular probability r, we shall need to find the point with r of the area to its right. This point is denoted by z_r. Thus, notationally, z_r denotes that point associated with a standard normal random variable such that

$$P\left[Z \geq z_r\right] = r$$

Exercise 39/sec 4.4

Using table, find the values of

- $P[Z \leq 1.57]$
- $P[Z < 1.57]$
- $P[Z = 1.57]$
- $P[Z > 1.57]$

Normal Distribution (Cont...)

- $P[-1.25 \leq Z \leq 1.75]$
- $z_{.10}$
- $z_{.90}$
- **The point z such that** $P[-z \leq Z \leq z] = 0.95$
- **The point z such that** $P[-z \leq Z \leq z] = 0.90$