Mechanics Oscillations and Waves

(MEOW!)

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The Course

- Mechanics: Rishikesh Vaidya ($\sim$ 21 lectures)
  Text: Introduction to Mechanics by Kleppner and Kolenkow
- Oscillations and Waves:
  Prof. Tapomoy Guha Sarkar ($\sim$ 20 lectures)
  Text: Vibrations and Waves by A.P.French

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Two simple questions

(1). What is the physical content of Newton's laws?

What do they buy us? Are there any assumptions? What are their limitations?

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(2). I push this duster and it displaces in the direction of the force. Simple and intuitive. Why is then a circular motion so defiant? Why the earth doesn't collapse on to the Sun?

Simple answer: Force is proportional to acceleration and not displacement. But then why are the displacement and acceleration aligned in the rectilinear motion and orthogonal in circular motion? The simplicity of mathematical answers is always deceptive.

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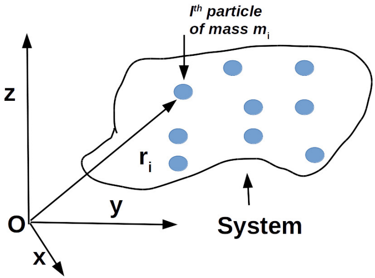
The Problem of Mechanics

Given a set of bodies m_i ($i = 1, 2, 3, ...N$), at different locations ($\vec{r}_i(t = 0)$), interacting with each other as well as the environment, predict their future course of evolution ($\vec{r}_i(t)$).



Newton's laws provide an answer to this question when the bodies in the system of interest are of macroscopic size that we see around.

What good is first law?

Two not so illuminating perspectives.

First law is at best a tautology

Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it doesn't.
– Sir Arthur Eddington (British Astrophysicist)

A special case of second law?

In the absence of any force 2^{nd} law says: $\vec{F} = 0 = \frac{d\vec{p}}{dt}$ implying $\vec{p}$ = constant.
Let us accept this tautology and move on until we are compelled to revisit the first law.

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The Second Law

Law of Causality

The rate of change of momentum of a body is proportional to the (net) force acting on it and takes place in the direction of straight line along which the force acts.

$$\vec{F} = \frac{d\vec{p}}{dt}(= m\vec{a} \text{ when m} = \text{constant}).$$

Net force is obtained from law of parallelogram of vector addition.

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Several questions

- Why is the rate of change of momentum (or velocity when m is constant) an absolute measure of change?
- How do we quantify motion before we quantify change in motion?
- What is the measure of no-change? How do we measure change?
- What is the role of mass here?
- What are the assumptions about the nature and role of space and time?
- What is an ideal measurement?

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Before we go full throttle!

It is a commonplace experience for passengers standing in a moving bus to suddenly find themselves in the lap of a sitting passenger, as the bus navigates a turn.

We have all experienced this change (the fall!) with no causative agent lurking in the sight.

Before we quantify signatures of change, we must know the signatures of no-change. Revisit the first law.

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2^{nd} law stands on the crutches of 1^{st} law!

- The connection between cause and effect quantified in 2^{nd} law applies only to a specific class of reference frames called inertial frames.
- First law quantifies the state of no change and its signature — a body free from the influence of causative agents (forces) maintains its state of rest or that of a uniform rectilinear motion.
- A valid or inertial reference frame is that in which this tell tale of signature of freedom from forces, that is a state of rest or of uniform rectilinear motion is observed.

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A million dollar question

Is there an attribute by virtue of which any body resists crazy spontaneous changes without being compelled by external forces?

Inertia is that attribute which mandates application of force for effecting change.

A million dollar question

This inertia is nothing but the quantity of matter and is the only player possible in the equation between cause (force) and effect (acceleration). When written in terms of rate of change of momentum, inertia goes in the definition of momentum, that is quantity of motion ($\vec{p}$) is equal to the product of quantity of inertia (m) and velocity ($\vec{v}$).

A million dollar question

Nothing is free. Even to exert inertia it costs you mass! In fact mass is inertia, or is it?

Three simple yet profound lessons

1. state of rest = state of uniform motion
2. Force as a causative agent for absolute change
3. Inertia is the attribute by which any body resists spontaneous change (in absence of force)

The first law is thus, in part a definition of valid (inertial) reference frames and in part an assertion that they exist.

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It takes two to tango

In the absence of more than one body there is no such thing as body of interest. A lonesome isolated body is hardly interesting for it does nothing for eternity (never say never though – modern understanding of particle physics revises the notion of free isolated particle which can spontaneously decay).

- Consider a body of interest that suffers a change when acted upon by a force that originates in an external agency.
- However, this division of Universe in to "body of interest" and the rest of the Universe where forces "reside" is artificial and arbitrary.
- According to the third law, the causal connection is bi-directional. If body 1 is acted upon by a body 2 with a force $\vec{F}_{12}$, then the body 2 also suffers an equal and opposite force $\vec{F}_{21} = -\vec{F}_{12}$ due to body 1.

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It takes two to tango!

Third law is a profound prescription of going from the effect on 'the body of interest' to mutual interaction of many bodies.

So inertial mass is a source of massive inertia – no tautology right?

Well, a little detail here. There are two kinds of masses – literally, figuratively, physically as well as politically. There are masses that resist change (inertial) and there are masses that (easily) gravitate! Talking of physics,

- **Inertial mass m_i:** Measure of how much a body hates to accelerate
- **Gravitational massm_g:** Measure of how much a body can gravitate
- *A priori* there is no reason why $m_i = m_g$ just as electric charge q is not a measure of inertia.
- Equality $m_i = m_g$ follows from a profound law – Einstein's principle of equivalence. More in chap. 8.

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How do we verify Newton's second law?

- Measure LHS and RHS of the law separately and see if LHS = RHS
- Equipped with inertial frames, you can measure $\vec{a}$.
- How do you measure m? Balance ? Thats m_g. Take your balance to outer space and a mouse will balance an elephant. Doesn't help.
- $m = \frac{F}{a}$? That's circular argument
- We can compare two inertial masses if we apply same force on them and then compare their accelerations. How do you ensure F is same?

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3^{rd} law to the rescue of 2^{nd} law

If we isolate two bodies then the only force they feel is the force of mutual interaction. Following the 3^{rd} law:

$$\begin{aligned} \vec{F}_{12} + \vec{F}_{21} &= 0 \\ \frac{d}{d}(\vec{P}_1 + \vec{P}_2) &= 0 \end{aligned} \tag{1}$$

Total momentum of a closed system is conserved.

3^{rd} law to the rescue of 2^{nd} law

If we examine the collision of two masses m_1 and m_2 comprising a closed system then from the conservation of momentum:

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= m_1 v_1' + m_2 v_2' \\ \frac{m_1}{m_2} &= \frac{v_2' - v_2}{v_1 - v_1'} \end{aligned} \tag{2}$$

Since you can measure the RHS, it fixes the ratio of masses. Taking one of them to be unit test mass, you can now determine any mass. Now the second law defines the force for you.

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$\vec{F} = m\vec{a}$ vs. $\vec{F} = \frac{d\vec{p}}{dt}$

- $\vec{F} = \frac{d\vec{p}}{dt}$ is a more general and powerful formulation.
- It applies to mass varying systems including the relativistic increase in mass with velocity
- In so formulating his second law he defined a very important notion of momentum and paved way for powerful conservation laws. Conservation laws in turn have deep connections with the fundamental properties of space and time, namely space is homogeneous and isotropic and time is homogeneous.

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So.....

Three laws of Newton together form a logically complete and consistent system of laws to answer the fundamental problem of mechanics. Have they stood the test of time?

For a theory to become a solid science logical consistency is not enough. Experimental vindication is the ultimate arbiter.

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Questionable assumptions

- Space and time are independent of each other and absolute
  Truth: according to special Relativity space and time are inter-mixed and relative
- Space and time are non-participatory in dynamics and the geometry of space is flat(euclidean)
  Truth: according to general relativity geometry of space-time is determined by the matter and energy-density. What we called gravitation is nothing but the geometry of space-time.

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Questionable assumptions

- Space and time are continuous
  According to one of the modern theories for gravity space may be granular. But this is only one among few alternate quantum theory for gravity and we do not know if this is really true.
- Instantaneous action at a distance
  Truth: According to special relativity forces are communicated with a finite speed that is equal to the speed of light c which is a universal constant

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  Truth: According to quantum mechanics ideal non-destructive measurements are inherently not possible. However, for the physics of the macroscopic everyday objects, the errors incurred are negligible.
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An advertisement for theoretical physics

Are there other formulations of mechanics?

Yes, there are more powerful and elegant formulations of mechanics due to Lagrange and Hamilton. Newton's formulation is based on the primacy of vectorial force. Lagrange and Hamilton's formulation are based on the primacy of energy which is a scalar.

An advertisement for theoretical physics

Are there other formulations of mechanics?

Yes, there are more powerful and elegant formulations of mechanics due to Lagrange and Hamilton. Newton's formulation is based on the primacy of vectorial force. Lagrange and Hamilton's formulation are based on the primacy of energy which is a scalar.

Question 2

Why does the duster move in the direction of force but motion of earth is orthogonal to the direction of force?

Some remarks:
(a) Acceleration ($\vec{a}$) is sometimes in the same direction as velocity ($\vec{dr}/dt$) but can sometimes be orthogonal to it.
(b) Very specific initial conditions are implicit in this question. The duster is at rest whereas earth is already orbiting the Sun. If Earth and Sun stood still in isolation, they would unabashedly crash into each other along the line joining their center of mass and not revolve around common center of mass.
(c) Since $\vec{a} = \vec{dv}/dt$, we must investigate the meaning of the derivative of a vector and what are the different ways in which a vector can change. How different can a derivative of a vector be from the derivative of a scalar?

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Upshot of the blackboard work is

A vector can change in two independent ways

1. **Pure scaling:** change purely in magnitude without changing direction
2. **Pure rotation:** change purely in direction without changing magnitude

This sounds interesting. Does there exist a coordinate system to which such a resolution is native? If yes, how different would things look in such a system? Would it simplify or complicate things?

It is all about choosing horses for courses.

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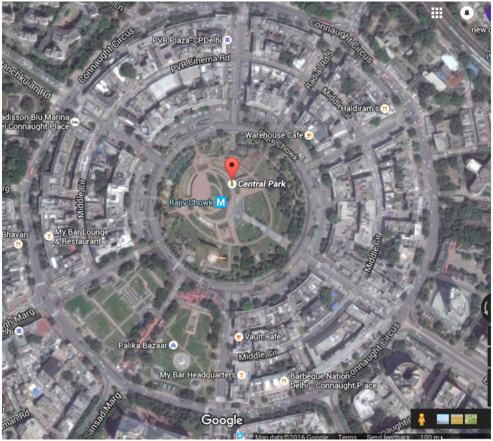
Arial shot of Downtown Chicago

Rectangular geometry of the roads lends itself very naturally to cartesian coordinate system.

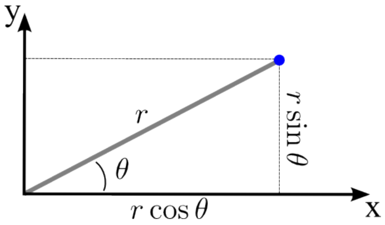
Arial shot of Cannought Place New Delhi



Circular/radial geometry of the roads lends itself naturally to the plane polar coordinate system.

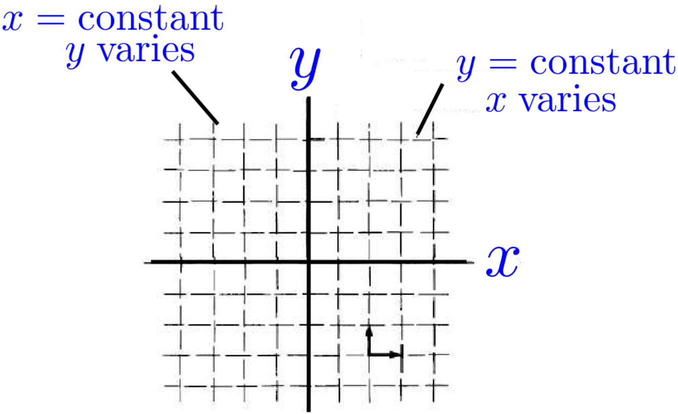
Cartesian vs. Polar coordinates

You need two numbers to specify a location in 2-D plane.
Cartesian: (x, y)
Polar: (r, θ)
r is distance from the origin and theta is the angle from the reference direction (x-axis).



The Grid

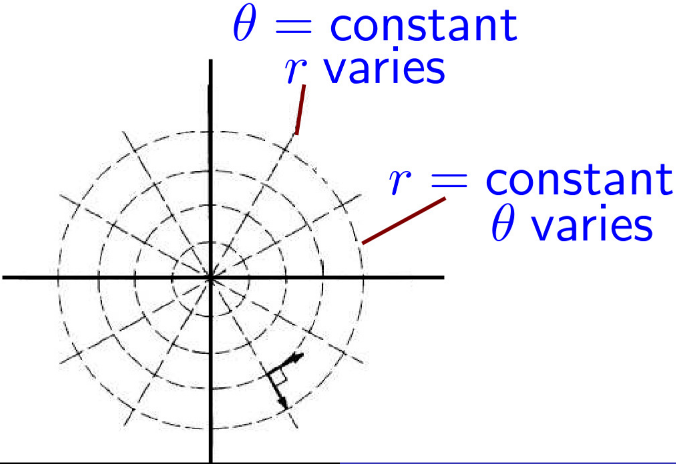
The cartesian grid



Cartesian coordinates

The Grid

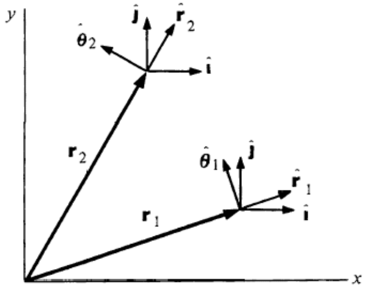
The plane-polar grid



So what is the difference?

Cartesian coordinates: Unit vectors are constant vectors (constant in magnitude as well as direction).

Polar coordinates Unit vectors are constant in magnitude (after all unit magnitude) but point in different directions at every location.



Block on String (no gravity)

Example 2.5 Find the force on the whirling mass m in the absence of gravity.

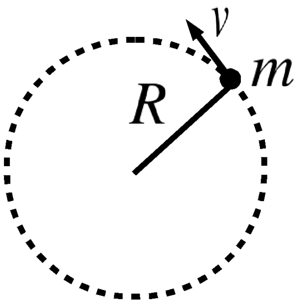


Figure: Mass m whirls at constant speed v at the end of string of length R

Block on String (now with gravity!)

Example 2.6 Find the force on the whirling mass *m* in the vertical plane in gravity.

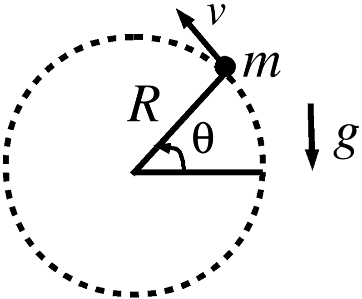


Figure: Now the forces are: Weight (Mg) downwards and Tension (T) radially inwards.

Bead on the frictionless rod

Prob. 2.33 A particle of mass m is free to slide on a thin rod. The rod rotates in a plane about one end with a constant velocity ω. Show that the motion is given by $r = Ae^{-\alpha t} + Be^{+\alpha t}$, where α is a constant which you must find and A and B are arbitrary constant. Neglect gravity.

Show that for a particular choice of initial conditions (that is $r(t=0)$ and $v(t=0)$, it is possible to obtain a solution such that r decreases continually in time, but for any other choice r will eventually increase

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Figure for problem 2.33

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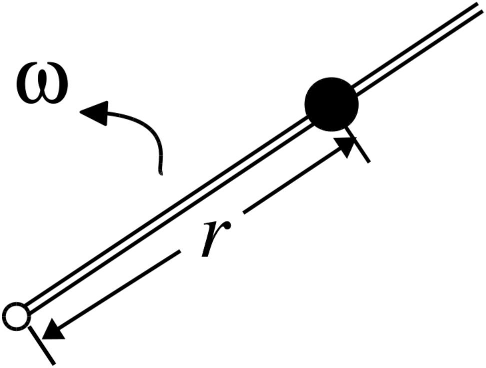

Lessons you learn from 2.33

- Formulate problems in polar coordinates
- A generic solution will have two abitrary constants and they are fixed by specific initial conditions.
- How to reset a mercury thermometer or drain clean a standard garden hose.
- How to clean egg from your face, or, how do you explain $\ddot{r} \neq 0$ despite $F_r = 0$.

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Pilani Campus

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Pull radially and yet accelerate tangentially!

Prob. 2.34 A mass *m* whirls around on a string which passes through a ring as shown. Neglect gravity. Initially the mass is at a distance r_0 from the center and is revolving at an angular speed of ω_0. The string is pulled with a constant velocity V starting at $t = 0$ so that the radial distance to the mass decreases. Draw a force diagram and obtain a differential equation for ω. Find (a) $\omega(t)$ (b) The force needed to pull the string.