Ch. 8: Non Inertial Systems and Fictitious Forces

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There is no way to distinguish locally between a uniform gravitational acceleration $\vec{g}$ and an acceleration of the coordinate system $\vec{A} = -\vec{g}$.
Locally here means a sufficiently small region like that of an elevator) where you can assume $\vec{g}$ to be practically constant.

For instance if your elevator is in a state of free fall under earth's gravity and you drop an apple, then the apple will float in front of you($m_i = m_g$)

$$\vec{F}' = \underbrace{-m_g\vec{g}}_{real\ \ force} + \underbrace{m_i\vec{g}}_{Fictitious\ \ force} = 0$$

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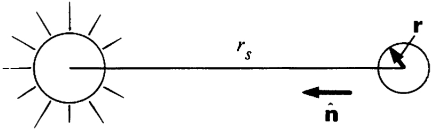
That means inside of a sufficiently giant elevator since $\vec{g}$ would vary from point to point, I can see departures from this equivalence as cancellation won't be exact.

Earth is in a state of free-fall towards Sun.



$$\vec{G}_0 = G\,M_s \frac{\hat{n}}{r_s^2}$$

If $\vec{G}(\vec{r})$ is the gravitational field of Sun at some location $\vec{r}$ on earth. Then,

$$\vec{F} = m\,\vec{G}(\vec{r})$$

The apparent force to an earthbound observer is

$$\vec{F}' = F - m\,\vec{A} = m\,[\vec{G}(\vec{r}) - \vec{G}_0] = m\,\vec{G}'(\vec{r})$$

Earth as a giant elevator

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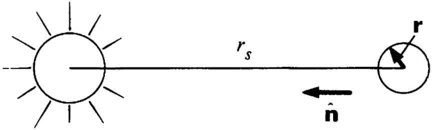

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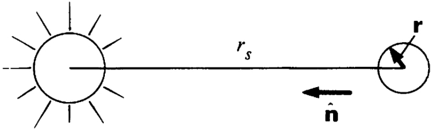

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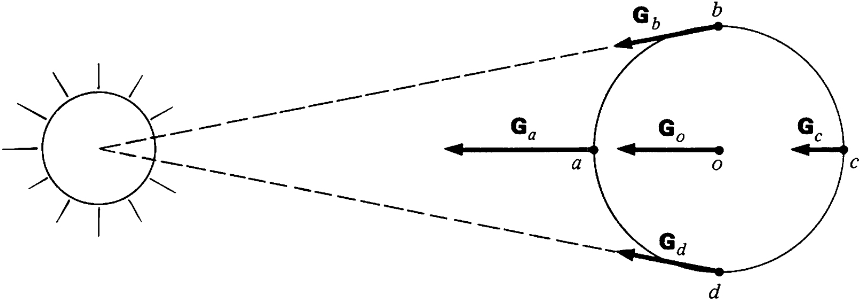
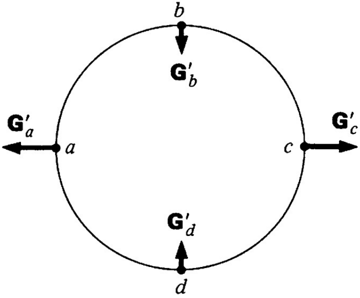
Sun's gravitational field $\vec{G}(\vec{r})$ at different points on Earth



Apparent field $\vec{G}'(\vec{r})$ at different points on earth's surface



$$G_a = \frac{G\,M_s}{(r_s - R_e)^2}$$

Calculation of G'_a and G'_c:

$$\begin{aligned} G'_a &= G_a - G_0 \\ &= \frac{G\,M_s}{(r_s - R_e)^2} - \frac{G\,M_s}{r_s^2} \\ &= \frac{G\,M_s}{r_s^2}\left[\frac{1}{[1-(R_e/r_s)^2]} - 1\right] \end{aligned}$$

Apparent field $\vec{G}'(\vec{r})$ at different points on earth's surface

Since $\frac{R_e}{r_s} = \frac{6.4\times 10^3 \ km}{1.5\times 10^8 \ km} = 4.3 \times 10^{-3} << 1$, we have

$$\begin{aligned} G'_a &= G_0 \left[\left(1 - \frac{R_e}{r_s} \right)^{-2} - 1 \right] \\ &= G_0 \left[1 + 2\frac{R_e}{r_s} + \ldots - 1 \right] \\ &= 2\, G_0 \frac{R_e}{r_s} \end{aligned}$$

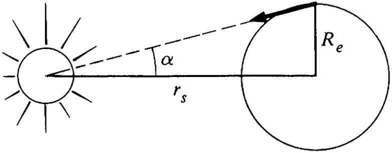
where we have neglected terms of order $(R_e/r_s)^2$ and higher.

Apparent field $\vec{G}'(\vec{r})$ at different points on earth's surface

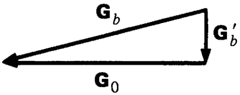
Similarly for point c, the distance from the Sun changes to $r_s + R_e$. Hence,

$$G'_c = -2\, G_0 \frac{R_e}{r_s}$$

Calculation of G'_b and G'_d:



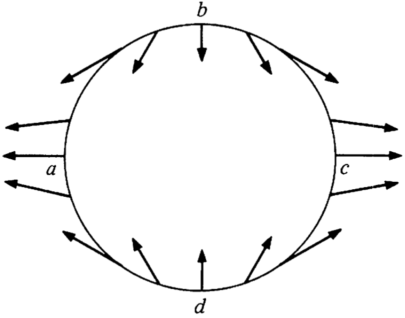
Points b and d are approximately same distance from the Sun as the center of the earth. However, $\alpha \approx R_e/r_s = 4.3 \times 10^{-5} << 1$.



$$
\begin{aligned}
G'_b &\approx G_0\,\alpha \\
&\approx G_0 \frac{R_e}{r_s}
\end{aligned}
$$

By Symmetry G'_d is equal but opposite to G'_b.

Apparent field $\vec{G}'(\vec{r})$ at different points on earth's surface

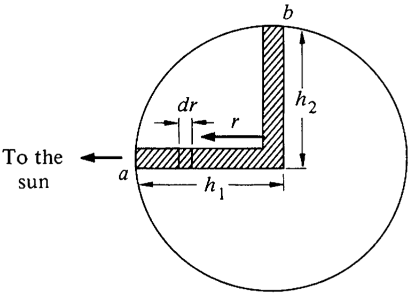


Tidal Forces

Forces at *a* and *c* tend to lift the ocean

Forces at *b* and *d* tend to depress them.

Can you see why there are two tides in a day?



Two orthogonal wells

Pressure due to short column of water of height dr is $\rho g(r)dr$ where $g(r)$ is the effective gravitational field at r. For equilibrium:

$$\int_0^{h_1} \rho\, g_1(r)dr = \int_0^{h_2} \rho\, g_2(r)dr$$

The idea is to calculate $\Delta h = h_1 - h_2$, the height of tide due to Sun.

Effective field toward the center of earth in column 1:

$$g_1(r) = \underbrace{g(r)}_{Earth's\ gravitational\ field} - \underbrace{G_1'(r)}_{Apparent\ field\ of\ Sun}$$

Borrowing from G_a' (substitute r for R_e):

$$\begin{aligned} G_1'(r) &= \frac{2\,G\,M_s\,r}{r_s^3} \\ &= 2\,C\,r \end{aligned}$$

where $C = G\,M_s/r_s^3$.

Thus,

$$\begin{aligned} g_1(r) &= g(r) - 2\,C\,r \\ g_2(r) &= g(r) + G_2'(r) \\ &= g(r) + C\,r \end{aligned}$$

Condition for equilibrium is:

$$\begin{aligned} \int_0^{h_1} [g(r) - 2\,c\,r]dr &= \int_0^{h_2} [g(r) + C\,r]dr \\ \int_0^{h_1} g(r)dr - \int_0^{h_2} g(r)dr &= \int_0^{h_1} 2\,C\,rdr + \int_0^{h_2} C\,r\,dr \\ \int_{h_2}^{h_1} g(r)dr &= \int_0^{h_1} 2\,C\,rdr + \int_0^{h_2} C\,r\,dr \end{aligned}$$

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Equilibrium Height of a Tide: Newton's Model

Since $h_1 \approx h_2 \approx R_e$, $g(r) \approx g(R_e) = g$ above equation reduces to

$$g\Delta h_s = \frac{3}{2} C R_e^2$$

$$\Delta h_s = \frac{3}{2}\frac{M_s}{M_e}\left(\frac{R_e}{r_s}\right)^3 R_e \qquad \left[g = \frac{GM_s}{R_e^2} \quad C = \frac{GM_s}{r_s^3}\right]$$

Using the data

$$M_s = 1.99 \times 10^{33}\ \mathrm{g} \qquad r_s = 1.49 \times 10^{13}\ \mathrm{cm}$$

$$M_e = 5.98 \times 10^{27}\ \mathrm{g} \qquad R_e = 6.37 \times 10^{8}\ \mathrm{cm},$$

we obtain

$$\Delta h_s = 24.0 cm$$

Equilibrium Height of a Tide: Newton's Model

An identical calculation for moon yields:

$$\Delta h_m = \frac{3}{2}\frac{M_m}{M_e}\left(\frac{R_e}{r_m}\right)^3 R_e$$
$$= 53.5\,cm$$

Since $\Delta h \to 1/r^2$, the distance factor more than kills whatever advantage Sun has due to its mass. Strongest tides (spring tides) occur when moon and Sun act along the same line. Weak (neap tides) occur midway between, at the quarters of the moon.

$$\frac{\Delta h_{spring}}{\Delta h_{neap}} = \frac{\Delta h_m + \Delta h_s}{\Delta h_m - \Delta h_s} \approx 3$$

Equilibrium Height of a Tide: Newton's Model